Játékelmélet,

vizsgadolgozat

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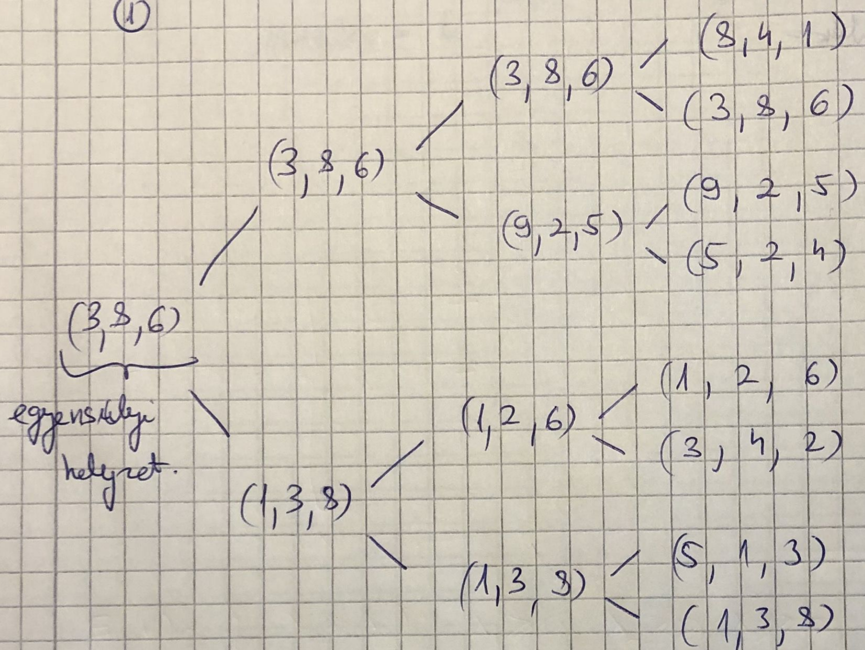
FTD1YJ

①




Válasz: (3, 8, 6).

②  $A = \begin{pmatrix} 2 & 8 \\ 6 & 3 \end{pmatrix}$  alsóé = 3, felsőé = 6 => nincs nyeregpont.

$a = 2$

$b = 8$

$c = 6$

$d = 3$

=> $a < b$, $a < c$, $d < c$, $d < b$ eset.

$$x = \frac{d-c}{a-b-c+d} = \frac{3-6}{2-8-6+3} = \frac{-3}{-9} = \frac{1}{3}$$

$$y = \frac{d-b}{a-b-c+d} = \frac{3-8}{2-8-6+3} = \frac{-5}{-9} = \frac{5}{9}$$

$\underline{x} = (1/3 \quad 2/3)$ és $y^T = (5/9 \quad 4/9)$.

③

$$B = \begin{pmatrix} 3 & 5 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 9 \end{pmatrix} \qquad \underline{x} = ?, \quad y^T = ?, \quad v = ?$$

1) alsóérték = 2
felsőérték = 4 $\Rightarrow$ nincs nyeregpont, $2 \leq v \leq 4$.

2) Nincs dominált sor, nincs dominált oszlop $\Rightarrow$
valódi 3x3-as mátrixjáték.

3) Tfh $y^T = (y_1 \ y_2 \ y_3)$, $y_1, y_2, y_3 > 0$, $\sum_{i=1}^{3} y_i = 1$.

$$\begin{pmatrix} 3 & 5 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 9 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} v \\ v \\ v \end{pmatrix}$$

I) $3y_1 + 5y_2 + 2y_3 = v$
II) $4y_1 - 3y_2 + 3y_3 = v$ $\Rightarrow$
III) $y_1 + 3y_2 + 9y_3 = v$

I) − II): $-y_1 + 8y_2 - y_3 = 0$
II) − III): $3y_1 - 6y_2 - 6y_3 = 0$
$y_1 + y_2 + y_3 = 1.$

Így
$$\left(\begin{array}{ccc|c} -1 & 8 & -1 & 0 \\ 3 & -6 & -6 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right).$$

$$D = \begin{vmatrix} -1 & 8 & -1 \\ 3 & -6 & -6 \\ 1 & 1 & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} 8 & -1 \\ -6 & -6 \end{vmatrix} - \begin{vmatrix} -1 & -1 \\ 3 & -6 \end{vmatrix} + \begin{vmatrix} -1 & 8 \\ 3 & -6 \end{vmatrix} =$$

$$= (-54) - (9) + (-18) = \underline{\underline{-81}}$$

Herlitz

$$y_1 = \frac{\begin{vmatrix} 0 & 8 & -1 \\ 0 & -6 & -6 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \frac{\begin{vmatrix} 8 & -1 \\ -6 & -6 \end{vmatrix}}{-81} = \frac{-54}{-81} = \underline{\underline{\frac{2}{3}}}$$

$$y_2 = \frac{\begin{vmatrix} -1 & 0 & -1 \\ 3 & 0 & -6 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \frac{-\begin{vmatrix} -1 & -1 \\ 3 & -6 \end{vmatrix}}{-81} = \frac{-9}{-81} = \underline{\underline{\frac{1}{9}}}$$

$$y_3 = \frac{\begin{vmatrix} -1 & 8 & 0 \\ 3 & -6 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \frac{\begin{vmatrix} -1 & 8 \\ 3 & -6 \end{vmatrix}}{-81} = \frac{-18}{-81} = \underline{\underline{\frac{2}{9}}}$$

Ezért $y^T = (2/3 \quad 1/9 \quad 2/9)$ és

I)-ből: $V = 3 \cdot \frac{2}{3} + 5 \cdot \frac{1}{9} + 2 \cdot \frac{2}{9} = \underline{\underline{3}}$

Ell: $2 \leq V = 3 \leq 4$ OK.

Megnézzük a pluszpontot:

Tfh $\underline{x} = (x_1 \quad x_2 \quad x_3)$, $x_1, x_2, x_3 > 0$, $\sum_{i=1}^{3} x_i = 1$.

$$(x_1 \quad x_2 \quad x_3) \begin{pmatrix} 3 & 5 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 9 \end{pmatrix} = (V \quad V \quad V)$$

I) $3x_1 + 4x_2 + x_3 = V$ $\quad$ I)-II): $-2x_1 + 7x_2 - 2x_3 = 0$

II) $5x_1 - 3x_2 + 3x_3 = V$ $\Rightarrow$ II)-III): $3x_1 - 6x_2 - 6x_3 = 0$

III) $2x_1 + 3x_2 + 9x_3 = V$ $\quad$ $x_1 + x_2 + x_3 = 1$.

Így $\left(\begin{array}{ccc|c} -2 & 7 & -2 & 0 \\ 3 & -6 & -6 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right)$ Megoldjuk

$$D = \begin{vmatrix} -2 & 7 & -2 \\ 3 & -6 & -6 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 7 & -2 \\ -6 & -6 \end{vmatrix} - \begin{vmatrix} -2 & -2 \\ 3 & -6 \end{vmatrix} + \begin{vmatrix} -2 & 7 \\ 3 & -6 \end{vmatrix} = (-54) - (18) + (-9) =$$

$$= \underline{\underline{-81}} \quad \text{(stimmel)}$$

$$x_1 = \frac{\begin{vmatrix} 0 & 7 & -2 \\ 0 & -6 & -6 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \frac{\begin{vmatrix} 7 & -2 \\ -6 & -6 \end{vmatrix}}{-81} = \frac{-54}{-81} = \underline{\underline{\frac{2}{3}}}$$

$$x_2 = \frac{\begin{vmatrix} -2 & 0 & -2 \\ 3 & 0 & -6 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \frac{-\begin{vmatrix} -2 & -2 \\ 3 & -6 \end{vmatrix}}{-81} = \frac{-18}{-81} = \underline{\underline{\frac{2}{9}}}$$

$$x_3 = \frac{\begin{vmatrix} -2 & 7 & 0 \\ 3 & -6 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \frac{\begin{vmatrix} -2 & 7 \\ 3 & -6 \end{vmatrix}}{-81} = \frac{-9}{-81} = \underline{\underline{\frac{1}{9}}}$$

ezért $\underline{x} = (2/3 \quad 2/9 \quad 1/9)$. $\square$

4.

$$A = \begin{pmatrix} 6 & 1 \\ 4 & 3 \end{pmatrix}, \quad A' = \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}$$

<u>bimátrixjáték.</u>

Leggyen $\underline{x} = (x \;\; 1-x)$ és $y^T = (y \;\; 1-y)$.

$Q = 6 - 1 - 4 + 3 = \underline{\underline{4}}$, $q = 3 - 1 = \underline{\underline{2}}$,

$R = 1 - 3 - 3 + 2 = \underline{\underline{-3}}$, $r = 2 - 3 = \underline{\underline{-1}}$,

ezért $\frac{q}{Q} = \frac{2}{4} = \underline{\underline{\frac{1}{2}}}$ és $\frac{r}{R} = \frac{-1}{-3} = \underline{\underline{\frac{1}{3}}}$.

A következő esetek kellenek:

| | |
|---|---|
| ii) $Q > 0$ | vi) $R < 0$ |
| a) $x = 0$, $y \leq \frac{1}{2}$ | a') $x \geq \frac{1}{3}$ $y = 0$ |
| b) $x = 1$, $y \geq \frac{1}{2}$ | b') $x \leq \frac{1}{3}$ $y = 1$ |
| c) $0 < x < 1$, $y = \frac{1}{2}$ | c') $x = \frac{1}{3}$ $0 \leq y \leq 1$ |

c) és c') lehetséges egyszerre!

$$\left(0 < x < 1,\; y = \tfrac{1}{2}\right) \wedge \left(x = \tfrac{1}{3},\; 0 < y < 1\right)$$

itt $\underline{x} = (1/3 \;\; 2/3)$ és $y^T = (1/2 \;\; 1/2)$

Az első játékos kifizetése:

$$E_1(x,y) = E_1(1/3,\ 1/2) = XAY = \begin{pmatrix} 1/3 & 2/3 \end{pmatrix} \begin{pmatrix} 6 & 1 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} =$$

$$= \begin{pmatrix} 14/3 & 7/3 \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} = \frac{7}{2} = \underline{\underline{3{,}5}}$$

A második játékos kifizetése:

$$E_2(x,y) = E_2(1/3, 1/2) = XA'Y = \begin{pmatrix} 1/3 & 2/3 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} =$$

$$= \begin{pmatrix} 7/3 & 7/3 \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} = \frac{7}{3} \approx \underline{\underline{2{,}3\dot{3}}} \qquad \square$$

5.

$A = \{x, y, z, u\}$ 20 szavazó

$|A| = 4$

$x: \; 8 \cdot 4 + 7 \cdot 2 + 5 \cdot 2 = \underline{\underline{56}}$

$y: \; 8 \cdot 1 + 7 \cdot 4 + 5 \cdot 1 = \underline{\underline{41}}$

$z: \; 8 \cdot 3 + 7 \cdot 3 + 5 \cdot 4 = \underline{\underline{65}}$

$u: \; 8 \cdot 2 + 7 \cdot 1 + 5 \cdot 3 = \underline{\underline{38}}$

A sorrend: $\boxed{z \succ x \succ y \succ u}$

⑥

A játék háromszemélyes:

$$\gamma(|S|) = \begin{cases} 1/3 & \text{ha } |S| = 1, \\ 1/6 & \text{ha } |S| = 2, \\ 1/3 & \text{ha } |S| = 3. \end{cases}$$

$$\gamma(1) = \frac{0! \cdot 2!}{3!} = \frac{1}{3} \qquad \gamma(2) = \frac{1! \cdot 1!}{3!} = \frac{1}{6}$$

$$\gamma(3) = \frac{2! \cdot 0!}{3!} = \frac{1}{3}$$

Eddig OK.

A Shapley-érték komponensei:

$$\Phi_1(v) = \frac{1}{3} \cdot (2-0) + \frac{1}{6} \cdot (6-3) + \frac{1}{6} \cdot (8-0) + \frac{1}{3}(12-7) = \frac{25}{6}$$

$$\Phi_2(v) = \frac{1}{3}(3-0) + \frac{1}{6}(6-2) + \frac{1}{6}(7-0) + \frac{1}{3}(12-8) = \frac{25}{6}$$

$$\Phi_3(v) = \frac{1}{3}(0-0) + \frac{1}{6}(8-2) + \frac{1}{6}(7-3) + \frac{1}{3}(12-6) = \frac{11}{3} = \frac{22}{6}$$

A Shapley-érték: $\Phi(v) = (25/6 \quad 25/6 \quad 22/6)$ □