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FTD1YJ

20. feladat

$$A = \begin{pmatrix} 6 & 0 & 6 \\ 8 & -2 & 0 \\ 4 & 6 & 5 \end{pmatrix}, \quad \underline{x} = ?, \quad \overline{y} = ?, \quad v = ?.$$

1) alsóérték = 4, felsőérték = 6 $\Rightarrow$ nincs nyeregpont, $4 \le v \le 6$.

2) Nincs dominált sor, se dominált oszlop.

3) TFH $y = (y_1\ y_2\ y_3)^T$, $y_1, y_2, y_3 > 0$, $\sum_{i=1}^{3} y_i = 1$.

$$\begin{pmatrix} 6 & 0 & 6 \\ 8 & -2 & 0 \\ 4 & 6 & 5 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} v \\ v \\ v \end{pmatrix}$$

$$\left.\begin{aligned} 6y_1 + 6y_3 &= v \\ 8y_1 - 2y_2 &= v \\ 4y_1 + 6y_2 + 5y_3 &= v \\ y_1 + y_2 + y_3 &= 1 \end{aligned}\right\}$$

$$\left(\begin{array}{ccc|c} 6 & 0 & 6 & v \\ 8 & -2 & 0 & v \\ 4 & 6 & 5 & v \\ 1 & 1 & 1 & 1 \end{array}\right) \begin{matrix} \text{I)} \\ \text{II)} \\ \text{III)} \end{matrix}$$

$$\left.\begin{aligned} \text{I)} - \text{II)}: \quad -2y_1 + 2y_2 + 6y_3 &= 0 \\ \text{II)} - \text{III)}: \quad 4y_1 - 8y_2 - 5y_3 &= 0 \\ y_1 + y_2 + y_3 &= 1. \end{aligned}\right\}$$

$$\left(\begin{array}{ccc|c} -2 & 2 & 6 & 0 \\ 4 & -8 & -5 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right).$$

$$D = \begin{vmatrix} -2 & 2 & 6 \\ 4 & -8 & -5 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ -8 & -5 \end{vmatrix} - \begin{vmatrix} -2 & 6 \\ 4 & -5 \end{vmatrix} + \begin{vmatrix} -2 & 2 \\ 4 & -8 \end{vmatrix} =$$

$$= (-10 + 48) - (10 - 24) + (16 - 8) =$$

$$= 38 + 14 + 8 = \underline{\underline{60}}$$

$$y_1 = \frac{\begin{vmatrix} 0 & 2 & 6 \\ 0 & -8 & -5 \\ 1 & 1 & 1 \end{vmatrix}}{60} = \frac{\begin{vmatrix} 2 & 6 \\ -8 & -5 \end{vmatrix}}{60} = \frac{-10 + 48}{60} = \underline{\underline{\frac{38}{60}}}$$

$$y_2 = \frac{\begin{vmatrix} -2 & 0 & 6 \\ 4 & 0 & -5 \\ 1 & 1 & 1 \end{vmatrix}}{60} = \frac{-\begin{vmatrix} -2 & 6 \\ 4 & -5 \end{vmatrix}}{60} = \frac{-(10-24)}{60} = \frac{14}{60}$$

$$y_3 = \frac{\begin{vmatrix} -2 & 2 & 0 \\ 4 & -8 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{60} = \frac{\begin{vmatrix} -2 & 2 \\ 4 & -8 \end{vmatrix}}{60} = \frac{16-8}{60} = \frac{8}{60}.$$

Ell.: $y_1 + y_2 + y_3 = \frac{38+14+8}{60} = 1.$ ✓

ekvt $\underline{y}^T = (38/60 \quad 14/60 \quad 8/60)$.

A játék értéke 1) alapján: $6y_1 + 6y_3 = \frac{6 \cdot 38 + 6 \cdot 8}{60} = 4{,}6$

Ell: $4 \le V = 4{,}6 \le 6$. ✓

Tfh $\underline{x} = (x_1 \; x_2 \; x_3)$, $x_1, x_2, x_3 > 0$, $\sum_{i=1}^{3} x_i = 1$.

$$(x_1 \; x_2 \; x_3)\begin{pmatrix} 6 & 0 & 6 \\ 8 & -2 & 0 \\ 4 & 6 & 5 \end{pmatrix} = \begin{pmatrix} V \\ V \\ V \end{pmatrix} \Rightarrow \begin{array}{l} 6x_1 + 8x_2 + 4x_3 = V \quad \text{I)} \\ -2x_2 + 6x_3 = V \quad \text{II)} \\ 6x_1 + 5x_3 = V \quad \text{III)} \\ x_1 + x_2 + x_3 = 1 \end{array}$$

I − II): $6x_1 + 10x_2 - 2x_3 = 0$

II) − III): $-6x_1 - 2x_2 + x_3 = 0$

$x_1 + x_2 + x_3 = 1$

$$\Rightarrow \left(\begin{array}{ccc|c} 6 & 10 & -2 & 0 \\ -6 & -2 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right)$$

$$D = \begin{vmatrix} 6 & 10 & -2 \\ -6 & -2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 10 & -2 \\ -2 & 1 \end{vmatrix} - \begin{vmatrix} 6 & -2 \\ -6 & 1 \end{vmatrix} + \begin{vmatrix} 6 & 10 \\ -6 & -2 \end{vmatrix} =$$

$= (10 - 4) - (6 - 12) + (-12 + 60) = 6 + 6 + 48 = 60.$

$$x_1 = \frac{\begin{vmatrix} 0 & 10 & -2 \\ 0 & -2 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{60} = \frac{\begin{vmatrix} 10 & -2 \\ -2 & 1 \end{vmatrix}}{60} = \frac{10-4}{60} = \frac{6}{60}$$

$$x_2 = \frac{\begin{vmatrix} 6 & 0 & -2 \\ -6 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{60} = \frac{-\begin{vmatrix} 6 & -2 \\ -6 & 1 \end{vmatrix}}{60} = \frac{-(6-12)}{60} = \frac{6}{60}.$$

$$x_3 = \frac{\begin{vmatrix} 6 & 10 & 0 \\ -6 & -2 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{60} = \frac{\begin{vmatrix} 6 & 10 \\ -6 & -2 \end{vmatrix}}{60} = \frac{-12+60}{60} = \frac{48}{60}$$

Ell: $x_1 + x_2 + x_3 = \frac{6+6+48}{60} = \frac{60}{60} = 1.$ ✓

$\underline{X} = (6/60 \quad 6/60 \quad 48/60)$.