# Minta ZH

## 5. feladat

$$B = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 2 \end{pmatrix}$$

Legyen $\underline{x} = (x_1 \quad x_2 \quad x_3)$ és $\underline{y}^T = (y_1 \quad y_2 \quad y_3)$

Tudjuk, hogy $x_i, y_i > 0 \quad \forall i \in \{1,2,3\}$.

(1) alsóérték = 2, felsőérték = 3 $\Rightarrow$ nincs nyeregpont $\Rightarrow$ $2 \leq v \leq 3$.

(2) nincs dominált sor, nincs dominált oszlop $\Rightarrow$ valódi 3x3-as m. játék.

(3)

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} v \\ v \\ v \end{pmatrix}$$

$$\left.\begin{array}{l} y_1 + y_2 + 3y_3 = v \quad a) \\ y_1 + 3y_2 + 2y_3 = v \quad b) \\ 3y_1 + 2y_2 + 2y_3 = v \quad c) \end{array}\right\} \Rightarrow \left.\begin{array}{l} a) - b): \ -2y_2 + y_3 = 0 \\ b) - c): \ -2y_1 + y_2 = 0 \\ y_1 + y_2 + y_3 = \cancel{0}\ 1 \end{array}\right\}$$

$+ \quad y_1 + y_2 + y_3 = 1.$

$$\left(\begin{array}{ccc|c} 0 & -2 & 1 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right).$$

$$D = \begin{vmatrix} 0 & -2 & 1 \\ -2 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -2 \\ -2 & 1 \end{vmatrix} =$$

$$= (-1) - (2) + (-4) = \underline{\underline{-7}}$$

$$y_1 = \begin{vmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} \Big/ D = \frac{\begin{vmatrix} -2 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix}}{-7} = \underline{\underline{\frac{1}{7}}}$$

$$y_2 = \frac{\begin{vmatrix} 0 & 0 & 1 \\ -2 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \frac{\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ -2 & 0 \end{vmatrix}}{-7} = \underline{\underline{\frac{2}{7}}}$$

$$y_3 = \begin{vmatrix} 0 & -2 & 0 \\ -2 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} / D = \frac{\begin{vmatrix} -2 & 0 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 0 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -2 \\ -2 & 1 \end{vmatrix}}{-7} = \underline{\underline{\frac{4}{7}}}$$

$$y^T = (1/7 \quad 2/7 \quad 4/7). \qquad \left( \sum_{i=1}^{3} y_i = 1 \ \checkmark \right)$$

<u>A játék értéke:</u> $v = y_1 + y_2 + 3y_3 = \frac{1}{7} + \frac{2}{7} + \frac{12}{7} = \underline{\underline{\frac{15}{7}}}$

$\left( \text{Ell:} \quad \frac{15}{7} \approx 2{,}14 \quad \checkmark \right)$

$$(x_1\, x_2\, x_3) \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 2 \\ 3 & 2 & 2 \end{pmatrix} = \begin{pmatrix} v \\ v \\ v \end{pmatrix} \Rightarrow \left.\begin{aligned} x_1 + x_2 + 3x_3 &= v \\ x_1 + 3x_2 + 2x_3 &= v \\ 3x_1 + 2x_2 + 2x_3 &= v \\ + \quad x_1 + x_2 + x_3 &= 1 \end{aligned}\right\} \begin{aligned} &a) \\ &b) \\ &c) \end{aligned}$$

$a) - b): \quad -2x_2 + x_3 = 0$

$b) - c): \quad -2x_1 + x_2 \cancel{+ x_3} = 0 \quad \Rightarrow \left(\begin{array}{ccc|c} 0 & -2 & 1 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right)$

$x_1 + x_2 + x_3 = 1$

$$D = \begin{vmatrix} 0 & -2 & 1 \\ -2 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} -2 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -2 \\ -2 & 1 \end{vmatrix} =$$

$$= (-1) - (2) + (-4) = \underline{\underline{-7}}$$

$$x_1 = \frac{\begin{vmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \frac{\begin{vmatrix} -2 & 1 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix}}{-7} = \underline{\underline{\frac{1}{7}}}$$

$$x_2 = \frac{\begin{vmatrix} 0 & 0 & 1 \\ -2 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \frac{\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ -2 & 0 \end{vmatrix}}{-7} = \underline{\underline{\frac{2}{7}}}$$

$$x_3 = \frac{\begin{vmatrix} 0 & -2 & 0 \\ -2 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \frac{\begin{vmatrix} -2 & 0 \\ 1 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 0 \\ -2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -2 \\ -2 & 1 \end{vmatrix}}{-7} = \underline{\underline{\frac{4}{7}}}$$

$$\underline{x} = (1/7 \quad 2/7 \quad 4/7). \ (= y^T.)$$

## 4. feladat

$$A = \begin{pmatrix} 6 & 3 & 2 \\ -2 & 1 & 6 \end{pmatrix}$$

$$\left.\begin{array}{l} \text{alsóérték} = 2 \\ \text{felsőérték} = 3 \end{array}\right\} \Rightarrow \text{nincs nyeregpont.}$$

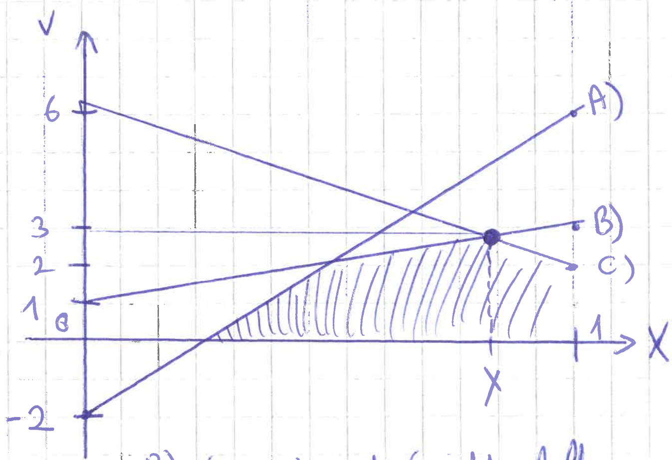
Se domináns sor, se domináns oszlop.

$\underline{x} = (x \quad 1-x) \qquad \underline{y}^T = (y_1 \quad y_2 \quad y_3)\ , \quad \sum_{i=1}^{3} y_i = 1$

$$(x \quad 1-x)\begin{pmatrix} 6 & 3 & 2 \\ -2 & 1 & 6 \end{pmatrix} \geq v$$

$$\left.\begin{array}{l} 6x - 2 + 2x \geq v \\ 3x + 1 - x \geq v \\ 2x + 6 - 6x \geq v \end{array}\right\} \Rightarrow \begin{array}{l} A)\ 8x - 2 \geq v \\ B)\ 2x + 1 \geq v \\ C)\ -4x + 6 \geq v. \end{array}$$

Ábrázol!




B) és C) metszéspontja kell.

$$2x + 1 = -4x + 6 \Leftrightarrow 6x = 5 \Leftrightarrow \underline{\underline{x = \frac{5}{6}}}$$

ezért $\underline{\underline{\underline{x} = (5/6 \quad 1/6)}}$

$\cancel{v = 8x - 2 = \frac{40}{6} - \frac{12}{6} = \frac{28}{6}}$

$$v = 2x + 1 = \frac{10}{6} + 1 = \underline{\underline{\frac{8}{3}}}$$

Kérdés még $\underline{y}^T$.

$$\begin{pmatrix} 6 & 3 & 2 \\ -2 & 1 & 6 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 8/3 \\ 8/3 \\ 8/3 \end{pmatrix} \qquad + : y_1 + y_2 + y_3 = 1.$$

megoldható.

$$\left(\begin{array}{ccc|c} 6 & 3 & 2 & 8/3 \\ -2 & 1 & 6 & 8/3 \\ 1 & 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -4 & -10/3 \\ 0 & 3 & 8 & 14/3 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -4 & -10/3 \\ 0 & 0 & 4 & 4/3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 3 & 4 & 10/3 \\ 0 & 0 & 1 & 1/3 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2/3 \\ 0 & 3 & 0 & 6/3 \\ 0 & 0 & 1 & 1/3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2/3 \\ 0 & 0 & 1 & 1/3 \end{array}\right) \Rightarrow \begin{array}{l} y_1 = 0 \\ y_2 = 2/3 \\ y_3 = 1/3 \end{array}$$

$$\boxed{y^T = \begin{pmatrix} 0 & 2/3 & 1/3 \end{pmatrix}.}$$