— o —

$$z^{(3)} + (1+\alpha+\alpha m)\, z^{(2)} + (\beta+\alpha m)\, z^{(1)} + \alpha\beta(1+m)\, z + \alpha\beta(1+m)\, d = 0.$$

Áll.: $-d$ megoldás.

Biz.:

$$d := \begin{cases} \dfrac{m_0 - m_1}{1+m_1} & x < -1 \\ 0 & -1 \le x \le 1 \\ \dfrac{m_1 - m_0}{1+m_1} & x > 1 \end{cases}$$

$z(t) = -d.$ $\quad z^{(1)} = z^{(2)} = z^{(3)} = 0.$

a) $x < 1$: $\qquad d = \dfrac{m_0 - m_1}{1+m_1}, \quad m = m_1.$

Kell: $\alpha\beta(1+m_1)(-d) + \alpha\beta(1+m_1)\, d = 0$ ✓.

b) $x > 1$: $\qquad d = \dfrac{m_1 - m_0}{1+m_1}, \quad m = m_1.$

Kell: $\alpha\beta(1+m_1)(-d) + \alpha\beta(1+m_1)\, d = 0$ ✓.

c) $-1 \le x \le 1$: $\qquad d = 0, \quad m = m_0$

$0 = 0$ triv. ✓ $\quad \square$

$$z^{(3)} + (1+\alpha+\alpha m)\, z^{(2)} + (\beta+\alpha m)\, z^{(1)} + \alpha\beta(1+m)\, z + \alpha\beta(1+m)\, d = 0$$

$\alpha, \beta, m, d \in \mathbb{R}$.

Karakterisztikus egyenlet:

$$\lambda^3 + (1+\alpha+\alpha m)\lambda^2 + (\beta+\alpha m)\lambda + \alpha\beta(1+m) = 0$$

(1.) $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$, $\lambda_1 \neq \lambda_2 \neq \lambda_3$.

$$z(t) = k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t} + k_3 e^{\lambda_3 t} - d$$

$$y(t) = -\frac{1}{\beta} z^{(1)}(t)$$

$$z^{(1)}(t) = \lambda_1 k_1 e^{\lambda_1 t} + \lambda_2 k_2 e^{\lambda_2 t} + \lambda_3 k_3 e^{\lambda_3 t}$$

$$y(t) = -\frac{1}{\beta}\lambda_1 k_1 e^{\lambda_1 t} - \frac{1}{\beta}\lambda_2 k_2 e^{\lambda_2 t} - \frac{1}{\beta}\lambda_3 k_3 e^{\lambda_3 t}.$$

$$y^{(1)}(t) = -\frac{1}{\beta}\lambda_1^2 k_1 e^{\lambda_1 t} - \frac{1}{\beta}\lambda_2^2 k_2 e^{\lambda_2 t} - \frac{1}{\beta}\lambda_3^2 k_3 e^{\lambda_3 t}$$

és $y^{(1)} = x - y + z \iff x = y^{(1)} + y - z.$

$$x(t) = -\frac{1}{\beta}\lambda_1^2 k_1 e^{\lambda_1 t} - \frac{1}{\beta}\lambda_2^2 k_2 e^{\lambda_2 t} - \frac{1}{\beta}\lambda_3^2 k_3 e^{\lambda_3 t} - \frac{1}{\beta}\lambda_1 k_1 e^{\lambda_1 t} - \frac{1}{\beta}\lambda_2 k_2 e^{\lambda_2 t} - \frac{1}{\beta}\lambda_3 k_3 e^{\lambda_3 t} -$$
$$- k_1 e^{\lambda_1 t} - k_2 e^{\lambda_2 t} - k_3 e^{\lambda_3 t} + d$$

$$x(t) = -\frac{1}{\beta} k_1 (\lambda_1^2 + \lambda_1 + \beta) e^{\lambda_1 t} - \frac{1}{\beta} k_2 (\lambda_2^2 + \lambda_2 + \beta) e^{\lambda_2 t} - \frac{1}{\beta} k_3 (\lambda_3^2 + \lambda_3 + \beta) e^{\lambda_3 t} + d$$

$x(0)$, $y(0)$ és $z(0)$ adott.

$$z(0) = z(0)$$
$$z^{(1)}(0) = -\beta y(0)$$
$$z^{(2)}(0) = -\beta \dot{y}(0) = -\beta\left(x(0) - y(0) + z(0)\right)$$

– o –

$$\left.\begin{aligned} z(0) &= k_1 + k_2 + k_3 - d \\ z^{(1)}(0) &= \lambda_1 k_1 + \lambda_2 k_2 + \lambda_3 k_3 \\ z^{(2)}(0) &= \lambda_1^2 k_1 + \lambda_2^2 k_2 + \lambda_3^2 k_3 \end{aligned}\right\} \Rightarrow \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{pmatrix}^{-1} \begin{pmatrix} z(0) + d \\ -\beta y(0) \\ -\beta(x(0) - y(0) + z(0)) \end{pmatrix}$$

② $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$, $\overbrace{\lambda_1 = \lambda_2}^{\lambda_{12}} \neq \lambda_3$.

$$z(t) = k_1 e^{\lambda_{12} t} + k_2 t e^{\lambda_{12} t} + k_3 e^{\lambda_3 t} - d$$

$$y(t) = -\frac{1}{\beta} z^{(1)}(t)$$

$$z^{(1)}(t) = \lambda_{12} k_1 e^{\lambda_{12} t} + \lambda_{12} k_2 t e^{\lambda_{12} t} + k_2 e^{\lambda_{12} t} + \lambda_3 k_3 e^{\lambda_3 t}$$

$$y(t) = -\frac{1}{\beta} \lambda_{12} k_1 e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_{12} k_2 t e^{\lambda_{12} t} - \frac{1}{\beta} k_2 e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_3 k_3 e^{\lambda_3 t}$$

$$y^{(1)}(t) = -\frac{1}{\beta} \lambda_{12}^2 k_1 e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_{12}^2 k_2 t e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_{12} k_2 e^{\lambda_{12} t} - \frac{1}{\beta} k_2 \lambda_{12} e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_3^2 k_3 e^{\lambda_3 t}$$

$$y^{(1)}(t) = -\frac{1}{\beta} \lambda_{12}^2 k_1 e^{\lambda_{12} t} - \frac{2}{\beta} \lambda_{12} k_2 e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_{12}^2 k_2 t e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_3^2 k_3 e^{\lambda_3 t}.$$

$$y^{(1)} = x - y + z \iff x = y^{(1)} + y - z$$

$$x(t) = -\frac{1}{\beta} \lambda_{12}^2 k_1 e^{\lambda_{12} t} - \frac{2}{\beta} \lambda_{12} k_2 e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_{12}^2 k_2 t e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_3^2 k_3 e^{\lambda_3 t} -$$
$$- \frac{1}{\beta} \lambda_{12} k_1 e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_{12} k_2 t e^{\lambda_{12} t} - \frac{1}{\beta} k_2 e^{\lambda_{12} t} - \frac{1}{\beta} \lambda_3 k_3 e^{\lambda_3 t} -$$
$$- k_1 e^{\lambda_{12} t} - k_2 t e^{\lambda_{12} t} - k_3 e^{\lambda_3 t} + d.$$

$$x(t) = -\frac{1}{\beta} \left(\lambda_{12}^2 k_1 + 2\lambda_{12} k_2 + \lambda_{12}^2 k_2 t + \lambda_{12} k_1 + \lambda_{12} k_2 t + k_2 + \beta k_1 + \beta k_2 + \beta k_3 \right) e^{\lambda_{12} t}$$
$$- \frac{1}{\beta} k_3 \left(\lambda_3^2 + \lambda_3 + \beta \right) e^{\lambda_3 t} + d$$

$x(0)$, $y(0)$ és $y(0)$ adott.

$$z^{(2)}(t) = \lambda_{12}^2 k_1 e^{\lambda_{12} t} + \lambda_{12}^2 k_2 t e^{\lambda_{12} t} + \lambda_{12} k_2 e^{\lambda_{12} t} + \lambda_{12} k_2 e^{\lambda_{12} t} + \lambda_3^2 k_3 e^{\lambda_3 t}$$

$$z(0) = z(0)$$
$$z^{(1)}(0) = -\beta y(0)$$
$$z^{(2)}(0) = -\beta \dot{y}(0) = -\beta (x(0) - y(0) + z(0))$$

— o —

$$\left.\begin{aligned} z(0) &= k_1 + k_3 - d \\ z^{(1)}(0) &= \lambda_{12} k_1 + k_2 + \lambda_3 k_3 \\ z^{(2)}(0) &= \lambda_{12}^2 k_1 + 2\lambda_{12} k_2 + \lambda_3^2 k_3 \end{aligned}\right\} \Rightarrow \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ \lambda_{12} & 1 & \lambda_3 \\ \lambda_{12}^2 & 2\lambda_{12} & \lambda_3^2 \end{pmatrix}^{-1} \begin{pmatrix} z(0) + d \\ -\beta y(0) \\ -\beta(x(0) - y(0) + z(0)) \end{pmatrix}.$$

③ $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$.

$$z(t) = k_1 e^{\lambda t} + k_2 t e^{\lambda t} + k_3 t^2 e^{\lambda t} - d.$$

$$y(t) = -\frac{1}{\beta} z^{(1)}(t)$$

$$z^{(1)}(t) = k_1 \lambda e^{\lambda t} + k_2 \lambda t e^{\lambda t} + k_2 e^{\lambda t} + k_3 \lambda t^2 e^{\lambda t} + k_3 2t e^{\lambda t}$$

$$z^{(2)}(t) = k_1 \lambda^2 e^{\lambda t} + k_2 \lambda^2 t e^{\lambda t} + k_2 \lambda e^{\lambda t} + k_2 \lambda e^{\lambda t} + k_3 \lambda^2 t^2 e^{\lambda t} +$$
$$+ k_3 \lambda 2t e^{\lambda t} + k_3 \lambda 2t e^{\lambda t} + 2k_3 e^{\lambda t}.$$

$$y(t) = -\frac{1}{\beta}\left(k_1\lambda + k_2 \lambda t + k_2 + k_3 \lambda t^2 + k_3 2t\right) e^{\lambda t}.$$

$$y^{(1)}(t) = -\frac{1}{\beta}\left(k_2 \lambda + 2k_3 \lambda t + 2k_3\right) e^{\lambda t} - \frac{1}{\beta}\left(k_1 \lambda^2 + k_2 \lambda^2 t + k_2 \lambda + k_3 \lambda^2 t^2 + k_3 \lambda 2t\right) e^{\lambda t}$$

$$y^{(1)} = x - y + z \iff x = y^{(1)} + y - z.$$

$$x(t) = -\frac{1}{\beta}\left(k_2 \lambda + 2k_3 \lambda t + 2k_3 + k_1 \lambda^2 + k_2 \lambda^2 t + k_2 \lambda + k_3 \lambda^2 t^2 + k_3 \lambda 2t\right) e^{\lambda t} -$$
$$- \frac{1}{\beta}\left(k_1 \lambda + k_2 \lambda t + k_2 + k_3 \lambda t^2 + k_3 2t\right) e^{\lambda t} - \left(k_1 + k_2 t + k_3 t^2\right) e^{\lambda t} + d.$$

$x(0)$, $y(0)$ és $z(0)$ adott.

$$z(0) = z(0)$$
$$z^{(1)}(0) = -\beta y(0)$$
$$z^{(2)}(0) = -\beta \dot{y}(0) = -\beta\left(x(0) - y(0) + z(0)\right)$$

– o –

$$\left.\begin{aligned} z(0) &= k_1 - d \\ z^{(1)}(0) &= \lambda k_1 + k_2 \\ z^{(2)}(0) &= \lambda^2 k_1 + 2\lambda k_2 + 2k_3 \end{aligned}\right\} \Rightarrow \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \lambda & 1 & 0 \\ \lambda^2 & 2\lambda & 2 \end{pmatrix}^{-1} \begin{pmatrix} z(0) + d \\ -\beta y(0) \\ -\beta(x(0) - y(0) + z(0)) \end{pmatrix}$$

(4.) $\lambda_1 = \gamma + j\omega, \quad \lambda_2 = \gamma - j\omega, \quad \lambda_3 \in \mathbb{R}.$

$$z(t) = k_1 e^{\gamma t}\cos\omega t + k_2 e^{\gamma t}\sin\omega t + k_3 e^{\lambda_3 t}$$

$$z^{(1)}(t) = k_1\gamma e^{\gamma t}\cos\omega t - k_1\omega e^{\gamma t}\sin\omega t + k_2\gamma e^{\gamma t}\sin\omega t + k_2\omega e^{\gamma t}\cos\omega t + k_3\lambda_3 e^{\lambda_3 t}.$$

$$z^{(2)}(t) = k_1\gamma^2 e^{\gamma t}\cos\omega t - k_1\gamma\omega e^{\gamma t}\sin\omega t - k_1\omega\gamma e^{\gamma t}\sin\omega t - k_1\omega^2 e^{\gamma t}\cos\omega t + k_2\gamma^2 e^{\gamma t}\sin\omega t +$$
$$+ k_2\gamma\omega e^{\gamma t}\cos\omega t + k_2\gamma\omega e^{\gamma t}\cos\omega t - k_2\omega^2 e^{\gamma t}\sin\omega t + k_3\lambda_3^2 e^{\lambda_3 t}.$$

$$y(t) = -\frac{1}{\beta} z^{(1)}(t).$$

$$y(t) = -\frac{1}{\beta} e^{\gamma t}(k_1\gamma + k_2\omega)\cos\omega t - \frac{1}{\beta} e^{\gamma t}(-k_1\omega + k_2\gamma)\sin\omega t - \frac{1}{\beta} k_3\lambda_3 e^{\lambda_3 t}.$$

$$y^{(1)}(t) = -\frac{1}{\beta}(k_1\gamma + k_2\omega)(\gamma e^{\gamma t}\cos\omega t - e^{\gamma t}\omega\sin\omega t) - \frac{1}{\beta}(-k_1\omega + k_2\gamma)(\gamma e^{\gamma t}\sin\omega t + e^{\gamma t}\omega\cos\omega t)$$
$$- \frac{1}{\beta} k_3\lambda_3^2 e^{\lambda_3 t}.$$

$$y^{(1)} = x - y + z \iff x = y^{(1)} + y - z.$$

$x(t) = \ldots$ számolható $y^{(1)}$, y és z értékéből a fentiek szerint.

(Amúgy hosszadalmas.)

$x(0)$, $y(0)$ és $z(0)$ adott.

$$z(0) = z(0)$$
$$z^{(1)}(0) = -\beta y(0)$$
$$z^{(2)}(0) = -\beta \dot{y}(0) = -\beta(x(0) - y(0) + z(0)).$$

—o—

$$\left.\begin{aligned} z(0) &= k_1 + k_3 \\ z^{(1)}(0) &= k_1\gamma + k_2\omega + k_3\lambda_3 \\ z^{(2)}(0) &= k_1\gamma^2 - k_1\omega^2 + k_2\gamma\omega + k_2\gamma\omega \\ &= k_1(\gamma^2 - \omega^2) + 2\gamma\omega k_2 + k_3\lambda_3^2. \end{aligned}\right\} \Rightarrow \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ \gamma & \omega & \lambda_3 \\ \gamma^2-\omega^2 & 2\gamma\omega & \lambda_3^2 \end{pmatrix}^{-1} \begin{pmatrix} z(0) + d \\ -\beta y(0) \\ -\beta(x_0 - y_0 + z_0) \end{pmatrix}$$