gyakorlati jellegű accccs lesz.

(1) $(a+b)(b+c)(c+a) = \underbrace{ab+bc}+ca$ igazoljuk.

$\underbrace{(a+b)(a+c)(b+c)}$ $\quad b(a+c)+ca$

$\qquad\qquad\qquad\qquad\qquad\qquad\parallel$

$a+bc\ \cancel{\cdot}\ (b+c)$ $\quad b(a+c)+ac$

$(a+bc)(b+c) = \cancel{ab}\quad ab+ac+bbc+bcc = ab+ac+\underbrace{bc+bc}_{bc}$ $\square$

(2) Igazoljuk: $\cancel{(s+r)}(p+q)\cancel{(q+r)(q+s)} \cancel{= ps+qr}$

$\cancel{(q+r)(q+s)}\ \cancel{lz}$

$(p+q)(p+r)(q+s)(r+s) = ps+qr$

$(p+qr)(s+qr)$

$qr+ps.$ $\square$

(3) $(a+b)(a'+c)(b+c) = \underbrace{(b+a)(b+c)}(a'+c)$

$ac+a'b+bc$ $\qquad (b+ac)(a'+c)$

$(a+b)(a'+c)$ $\qquad\qquad \parallel$

$ac+a'b$ $\qquad a'b+bc+\underbrace{aa'}_{0}c+acc =$

$= \boxed{a'b+bc+ac}$

$\parallel$

$ac+a'b+bc$ $\qquad b(a'+c)+ac$

$(a+b)(a'+c) =$

$= \underbrace{aa'}_{0}+ac+a'b+bc = \boxed{a'b+bc+ac}$

$ac+a'b = ??$

Igazoljuk:

$$\left.\begin{aligned} 0 \cdot x &= 0 \\ 0 + x &= x \end{aligned}\right\}$$

$1 \cdot x = x \Rightarrow (x + x')\, x = x$ ✓

$1 + x = 1$

$\Downarrow$

$x + x' + x = x + x' = 1$ ✓

$x + x' = 1$

$x x' = 0$

$0 + x = x$

$x x' + x = x + x x' = x$ ✓

$x x' x = x x x' = x x' = 0.$ ✓ $\square$

– o –

Trükk: $ac + a'b + bc\underbrace{(a + a')}_{\text{eggyel szorzunk.}} =$

$$= \underbrace{ac + acb}_{} + \underbrace{a'b + a'bc}_{} \quad \text{elnyelődés}$$

$= ac + a'b.$ $\square$

(4.) $0 \leq x \leq 1$ Igazoljuk.

(1) $0 \leq x \Leftrightarrow 0 + x = x \Leftrightarrow 0 \cdot x = 0.$ ✓

(2) $x \leq 1 \Leftrightarrow x + 1 = x \Leftrightarrow x \cdot 1 = x$ ✓ $\square$

(5.) Ha egy Boole-algebra nem 1-elemű, akkor nincs olyan x elem, amelyre $x' = x$.

Tfh Boole-algebra nem 1 elemű és $(\forall x)(x' = x)$.

$x + x' = x + x = x = 1.$ $\quad x' + x' = x' + x' = x' = 1$

②

x és y, $x \neq y$.

$x(x+y) = x$

Tfh x, y, $x \neq y$ és $x' = x$.

$$x + x' = (x + xy) + x' = x + x' + xy = x + x + xy = x + xy = x = 1,$$

$$x x' = x(x+y)x' = x x' (x+y) = x x (x+y) = x(x+y) = x = 0 \text{ ↯}$$

⊕ a részbenrendezés $\underbrace{0 \leq x \leq 1}$

$0 = 1$ esetén 1-elemű a Boole-algebra.

⑥ <u>Igazoljuk:</u> $ax = bx$ és $ax' = bx'$ $\Rightarrow$ $a = b$.

$$\left.\begin{aligned} ax &= bx \\ ax' &= bx' \end{aligned}\right\}$$

$$a = a(x + x') = ax + ax' = bx + bx' = b(x + x') = b. ✓$$

$ax = bx$

⑦ SZORGALMI: $a + x = b + x$ és $a + x' = b + x'$ $\Rightarrow$ $a = b$.

otthon megondolható.

⑧ $ax = ay$ és $a + x = a + y$ $\Rightarrow$ $x = y$.

$$x = x(a + a') = ax + a'x + \underbrace{a'a}_{0} = ax + a'(a + x) = ay + a'(a + y) =$$

$$= ay + a'a + a'y = ay + a'y = y(a + a') = y. \square$$

⑨ $\boxed{a + x = 1 \quad \text{és} \quad ax = 0 \quad \Rightarrow \quad x = a'.}$

$\underset{a+a'}{\|}$ $\qquad \underset{aa'}{\|}$

~~$a x$ =~~ $\cancel{x = a + a' + x = a + x + a' = 1 + a' = 1}$

$$\cancel{x} = \underbrace{0 + x = ax + x = x(a+1)}_{} =$$

hogy ancz

(10) $(x')' = x$ volt ugye.

$x' + (x')' = 1$ az utolsó axióma: $x' + \underline{\underline{x}} = 1$

$x' \cdot (x')' = 0$ $x' \underline{\underline{x}} = 0$

(11) $\boxed{x' = y' \Rightarrow x = y.}$

$x' + (x')' = y' + (y')' = 1 \Rightarrow x + x' = y + y' = 1$

$x'(x')' = y' \cdot (y')' = 0$ $xx' = yy' = 0$

~~$x' = x(x+x') = xx + xx' = x + xx' = x + xy' = x(1+y) =$~~

$x' = y' \Rightarrow (x')' = (y')'$

$\Downarrow$

$x = y.$ □

(12) $0' = 1$ miért?

$0 + 1 = 1$ ✓

$0 \cdot 1 = 0$ ✓ } ezt kell ellenőrizni $\Rightarrow 1 = 0'$.

ezek igazak, leveszteik

(13) $0 = 1'$ (az előző konjugálásával)

(14) $x + y = x + x'y$

$x + y = x + y(x + x') = x + x'y + xy = x + xy + x'y = x + x'y.$ □

De-Morgan belátása:

① $(x+y)' = x'y'$

$$x+y+x'y' = 1$$

$$\Downarrow$$

$$x+y+(x+y)' =$$

$$\|$$

$$x+x'y+x'y' =$$

$$x+x'\underbrace{(y+y')}_{1} = x+x' = 1.$$

$$(x+y)\,x'y' = 0$$

$$(x+x'y)\,x'y' =$$

$$= \underbrace{xx'}_{0}y' + x'x'yy' =$$

$$= \cancel{y'} + \cancel{x} = 0.$$

$\square$

$(xy)' = x'+y'$

~~$xy+(x'+y')=1$~~ ~~$(xy)(x'+y')=0$~~

$$(x'+y')' = (x')'\cdot(y')' = xy \quad /'$$

$$(xy)' = x'+y' \quad \square$$

Igaz még: $xy = x(x'+y) = \underbrace{xx'}_{0} + xy = xy.$ $\square$