Kovariancia, korreláció

PÉLDAMEGOLDÁS

(66)

B = bevétel, $E(B) = 120$, $D(B) = 30$.

K = kiadás, $E(K) = 80$, $D(K) = 20$.

P = profit $\quad P = B - K$.

a) B és K függetlenek.

$$E(P) = E(B-K) = E(B) - E(K) = 40.$$

$$\mathrm{Var}(P) \underset{\text{függetlenség.}}{=} \mathrm{Var}(\underset{\text{2-esen emelődik ki.}}{B-K}) = \mathrm{Var}(B) + \mathrm{Var}(K) = 30^2 + 20^2 = 1300.$$

$$D(P) = \sqrt{\mathrm{Var}(P)} = \sqrt{1300} = \ldots$$

b) Legyen $\mathrm{corr}(B,K) = 0{,}8$.

$E(P) = 40$ továbbra is.

$$\mathrm{Var}(P) = \mathrm{Var}(B-K) = \mathrm{Var}(B) + \mathrm{Var}(K) - 2\underbrace{\mathrm{Cov}(B,K)}_{\mathrm{Corr}(B,K)\cdot D(B)\cdot D(K)} = 30^2 + 20^2 - 2\cdot 0{,}8\cdot 30\cdot 20 = \underline{\underline{340}}$$

$D(P) = \sqrt{340}$. (Kevesebb, mert hamadára csökken ez.)

(67) lehet hf. is.

(65.) Z_1 és Z_2 két független kockadobás.

$$X = Z_1 + Z_2, \quad Y = Z_1 \cdot Z_2.$$

$$\mathrm{Cov}(X,Y) = ?$$

$$\mathrm{Cov}(X,Y) = \mathrm{Cov}(Z_1+Z_2,\ Z_1\cdot Z_2) = \mathrm{Cov}(X,Y) = E\big((Z_1+Z_2)Z_1Z_2\big) -$$

$$- E(Z_1+Z_2)E(Z_1Z_2) =$$

$$= E(Z_1^2Z_2) + E(Z_1Z_2^2) - \big(E(Z_1)+E(Z_2)\big)\, E(Z_1)E(Z_2)$$

$$= \cancel{E(Z_1^2)E(Z_2)} + \cancel{E(Z_1)E(Z_2^2)} - \cancel{E^2(Z_1)E(Z_2)} - \cancel{E^2(Z_2)E(Z_1)}$$

$$= \underbrace{E(Z_2)}_{\frac{7}{2}} Var(Z_1) + \underbrace{E(Z_1)}_{\frac{7}{2}} Var(Z_2) = \dots$$

Véletlen vektorváltozók.

2 lap 32-es magyar kártyából.

$X = (X_1, X_2)$ X_1: ászok száma;

X_2: piros lapok száma.

$X_1 \in \{0,1,2\}$

$X_2 \in \{0,1,2\}$

$\Downarrow$

$X \in \{0,1,2\}^2$.

Táblázat:

$X_2 \backslash X_1$	0	1	2	
0	$\frac{210}{496}$	$\frac{63}{496}$	$\frac{3}{496}$	
1	$\frac{147}{496}$	$\frac{42}{496}$	$\frac{3}{496}$	
2	$\frac{21}{496}$	$\frac{7}{496}$	0	
	$\frac{378}{496}$	$\frac{[illegible]}{496}$	$\frac{6}{496}$	1

X_1 marginális eloszlás $\Rightarrow$

$$P(X_1=2, X_2=0) = \frac{\binom{3}{2}\cdot 1}{\binom{32}{2}} = \frac{3}{496}$$

$$P(X_1=0, X_2=0) = \frac{\binom{21}{2}}{\binom{32}{2}} = \frac{105}{248} = \frac{210}{496}.$$

63

$$P(X_1=1, X_2=0) = \frac{\binom{3}{1}\binom{21}{1}}{\binom{32}{2}} = \frac{63}{496}$$

$$P(X_1 = 0, X_2 = 1) = \frac{\binom{7}{1}\binom{21}{1}}{\binom{32}{2}} = \underline{\underline{\frac{147}{496}}}$$

$$P(X_1 = 1, X_2 = 1) = \frac{\cancel{1}\,\binom{21}{1} + \binom{3}{1}\binom{7}{1}}{\binom{32}{2}} = \underline{\underline{\frac{42}{496}}}$$

$$P(X_1 = 2, X_2 = 1) = \frac{\binom{3}{1}}{\binom{32}{2}} = \underline{\underline{\frac{3}{496}}}$$

$$P(X_1 = 0, X_2 = 2) = \frac{\binom{7}{2}}{\binom{32}{2}} = \underline{\underline{\frac{21}{496}}}$$

$$P(X_1 = 1, X_2 = 2) = \frac{1 \cdot \binom{7}{1}}{\binom{32}{2}} = \underline{\underline{\frac{7}{496}}}$$

$$P(X_1 = 2, X_2 = 2) = 0$$

Most rendben vagyunk.

$$E(\mathbb{X}) = (E(X_1), E(X_2)) = \underline{\underline{\left(\frac{124}{496}, \frac{248}{496}\right)}}$$

$$\mathrm{Cov}(\mathbb{X}) = \begin{bmatrix} \mathrm{Var}\, X_1 & \mathrm{Cov}(X_1, X_2) \\ \mathrm{Cov}(X_1, X_2) & \mathrm{Var}\, X_2 \end{bmatrix}$$

$$\mathrm{Cov}(X_1, X_2) = E(X_1 X_2) - E(X_1)E(X_2)$$

$$E(X_1 \cdot X_2) = 1 \cdot \frac{42}{496} + 2 \cdot \left(\frac{7}{496} + \frac{3}{496}\right) - 62/496 = \underline{\underline{\frac{1}{8}}}$$

$$\mathrm{Cov}(X_1, X_2) = \frac{1}{8} - \frac{\cancel{124}}{\cancel{496}}\,\frac{1}{2} = \cancel{-\frac{3}{554}}\ \underline{\underline{0}}.$$
$$\frac{1}{4}$$

$\Rightarrow$ X_1 és X_2 korrelálatlan, de nem független.

$$P(X_1 = 2, X_2 = 2) = 0 \neq P(X_1 = 2) \cdot P(X_2 = 2).$$

71.

a) $f(x,y) = x+y\,, \quad (x,y) \in [0,1] \times [0,1]$

Marginális sűrűség függvények: $\mathbb{X} = (X,Y)$

$$f_{\mathbb{X}}(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^1 x+y\,dy =$$

$$= [xy]_0^1 + \left[\frac{y^2}{2}\right]_0^1 = \underline{\underline{x+\frac{1}{2}}}\,, \quad x \in [0,1].$$

Igaz hogy $\int_0^1 \left(x+\frac{1}{2}\right)dx = 1.$

$$f_y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{-\infty}^{\infty} (x+y)\,dx = \left[\frac{x^2}{2}+xy\right]_0^1 =$$

$$= \underline{\underline{y+\frac{1}{2}}}.$$

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_0^1 x\left(x+\frac{1}{2}\right)dx = \int_0^1 \left(x^2+\frac{x}{2}\right)dx =$$

$$= \frac{1}{3}+\frac{1}{4} = \underline{\underline{\frac{7}{12}}}$$

$$E(Y) = \underline{\underline{\frac{7}{12}}} \qquad E(\mathbb{X}) = \left(\frac{7}{12}, \frac{7}{12}\right) \text{ lesz.}$$

$$\mathrm{Cov}(\mathbb{X}) = \begin{pmatrix} \mathrm{Var}(X) & \mathrm{Cov}(X,Y) \\ \mathrm{Cov}(X,Y) & \mathrm{Var}(X) \end{pmatrix} =$$

$$= ? \qquad \mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y) =$$

$$E(XY) = ?$$

$$E(XY) = \int\int_{-\infty}^{\infty} xy\, f(x,y)\, dx\, dy =$$

$$= \int_0^1 \int_0^1 xy\,(x+y)\, dx\, dy = \int_0^1 \int_0^1 (x^2 y + x y^2)\, dx\, dy =$$

$$= \int_0^1 \left[\frac{x^3 y}{3} + \frac{x^2 y^2}{2} \right]_0^1 dy = \int_0^1 \left(\frac{y}{3} + \frac{y^2}{2} \right) dy =$$

$$= \int_0^1 \frac{y}{3}\, dy + \int_0^1 \frac{y^2}{2}\, dy = \left[\frac{y^2}{6} \right]_0^1 + \left[\frac{y^3}{6} \right]_0^1 = \frac{1}{6} + \frac{1}{6} = \underline{\underline{\frac{2}{6}}}$$

$$\mathrm{Cov}(X,Y) = \frac{1}{3} - \left(\frac{7}{12} \right)^2 = \frac{1}{3} - \frac{49}{144} = \underline{\underline{-\frac{1}{144}}}$$