2018. 10. 27. 4. konzultáció Valszám

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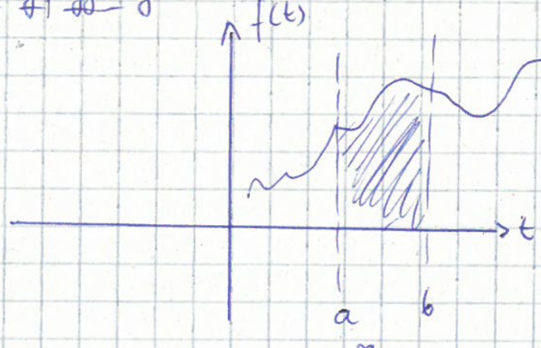
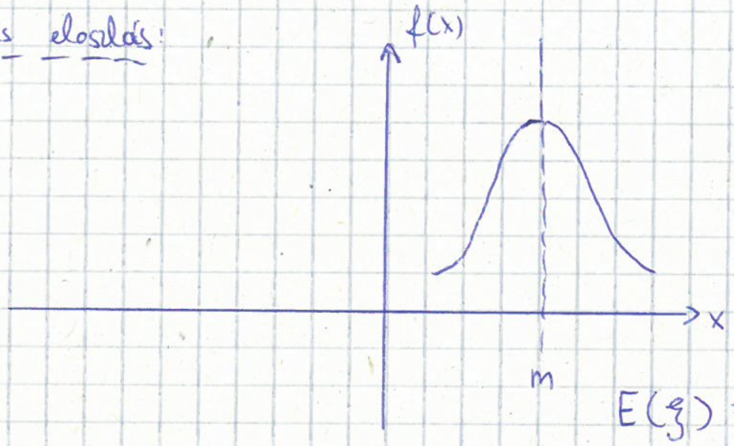
Sűrűség függvény:



$$E(\xi^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx.$$

Normális eloszlás:



$E(\xi) = m$ várható érték.

$D^2(\xi) = \sigma^2$ szórásnégyzet.

tipikus eloszlás, ugye.

standard - n. e.: $\left.\begin{matrix} m = 0 \\ \sigma^2 = 1. \end{matrix}\right\}$ lesz.

$\phi(-x) = 1 - \phi(x)$. világos.

pl. $\sigma = 10$ [ml]

$m = 1000$ [ml]

$\xi \sim N(1000, 10)$

$N(m, \sigma^2) = N(1000, 100)$.

$$P(980 \leq \xi \leq 1020) = P\left(\frac{980-1000}{10} \leq \frac{\xi - 1000}{10} \leq \frac{1020-1000}{10}\right)$$

1

$$Z := \frac{\xi - 1000}{10} \sim \mathcal{N}(0,1)$$

$$P(-2 \le Z \le 2) = \phi(2) - \phi(-2) = \phi(2) - (1 - \phi(2)) =$$

$$= 2\phi(2) - 1 = 2 \cdot 0{,}9772 - 1 = \underline{\underline{0{,}9544}}$$

$(\Omega, \mathcal{A}, P)$ $\quad A \in \mathcal{A}$ egy esemény.

<u>Binom:</u> $\xi$ $E(\xi) = np$ $\qquad D(\xi) = \sqrt{np(1-p)}$

$$\phi(b) - \phi(a) = P(a \le Z \le b) \qquad Z \sim \mathcal{N}(0,1).$$

$$P\left(a \le \frac{\xi - np}{\sqrt{np(1-p)}} \le b\right) \approx P(a \le Z \le b).$$

pl. $\quad \xi$

$\eta$ $\qquad \xi + \eta = 40\,000. \qquad \eta = 40\,000 - \xi$

$$P(-20 \le \xi - \eta \le 20) = P(-20 \le 2\xi - 40\,000 \le 20) =$$

$$= P(19\,990 \le \xi \le 20\,010)$$

$$\xi \sim \text{Binom}\left(40\,000, \tfrac{1}{2}\right)$$

pontosan: $\displaystyle\sum_{k=19990}^{20010} \binom{n}{k} p^k (1-p)^{n-k}$ így adható meg pontosan.

$$np = 40\,000 \cdot \frac{1}{2} = 20\,000 \qquad \sqrt{np(1-p)} = 100$$

$$P(19990 \le \xi \le 20010)$$

de-Moivre-Laplace-tétel, centrális határeloszlás tétele.

feladatok.

7.6/a.

$$\xi \sim \mathcal{N}(100, 15^2).$$

a) $P(90 \le \xi \le 120) = P\left(\frac{90-100}{15} \le \frac{\xi-100}{15} \le \frac{120-100}{15}\right) =$

$= P\left(-\frac{2}{3} \le \xi^* \le \frac{4}{3}\right) = P(-0{,}66 \le \xi^* \le 1{,}33) =$

$= \phi(1{,}33) - (1 - \phi(0{,}66)) =$

$= \phi(1{,}33) - 1 + \phi(0{,}66) = 0{,}9082 - 1 + 0{,}7486 =$

$= \underline{\underline{0{,}6568}}$

$\uparrow$

$0{,}9032 + (0{,}9192) \cdot \frac{0{,}033}{0{,}1}$

b) $P(\xi \ge 131) = 1 - P(\xi \le 131) =$

$= 1 - P\left(\frac{\xi-100}{15} \le \frac{131-100}{15}\right) =$

$= 1 - P\left(\frac{\xi-100}{15} \le \frac{31}{15}\right) = 1 - \phi\left(\frac{31}{15}\right) = 1 - 0{,}9808$

$\frac{31}{15} \approx 2{,}066$

$= \underline{\underline{0{,}0192}}$

c) $(100-x, 100+x)$ intervallum kell.

$0{,}95 = P(100 - x \le \xi \le 100 + x) =$

$= P\left(-\frac{x}{15} \le \boxed{\xi^*} \le \frac{x}{15}\right) = \phi\left(\frac{x}{15}\right) - \phi\left(-\frac{x}{15}\right) =$

$z$

$= \phi\left(\frac{x}{15}\right) - 1 + \phi\left(\frac{x}{15}\right) = 0{,}95$

$\phi\left(\frac{x}{15}\right) = \frac{1{,}95}{2} = \underline{\underline{0{,}975}}$

$2\phi\left(\frac{x}{15}\right) = 1{,}95$

$y = 1{,}96 = \frac{x}{15} \Rightarrow \boxed{x = 29{,}4}$

ezért

7.3 10 000 ügy fél

1% kárnsetvel.

$n = 10\,000$

$p = 0{,}01$

$q = 0{,}99 \quad (=1-p)$.

$\xi \sim \text{Binom}(10\,000,\ 0{,}01)$.

$E(\xi) = np = 100 \qquad D(\xi) = 3\sqrt{11} \approx 9{,}95$

$$P(85 \le \xi \le 115) = P\left(\frac{85-100}{3\sqrt{11}} \le Z \le \frac{115-100}{3\sqrt{11}}\right) =$$

$$= P\left(-\frac{5\sqrt{11}}{11} \le Z \le \frac{5\sqrt{11}}{11}\right) = \phi(1{,}507) - (1-\phi(1{,}507))$$

$-1{,}507 \qquad 1{,}507$

$$= 2\phi(1{,}507) - 1 = 0{,}869$$

$2 \cdot 0{,}9418$

$$0{,}1 = P(\xi \ge t) = 1 - P(\xi < t) \Leftrightarrow P(\xi \le t) = 0{,}9$$

$$P\left(\frac{\xi-100}{3\sqrt{11}} < \frac{t-100}{3\sqrt{11}}\right) = 0{,}9$$

$$P\left(\frac{\xi-100}{3\sqrt{11}} \le \frac{t-100}{3\sqrt{11}}\right) = 0{,}9$$

$$\phi\left(\frac{t-100}{3\sqrt{11}}\right) = 0{,}9$$

$$\frac{t-100}{3\sqrt{11}} = 1{,}28$$

$$t = 1{,}28 \cdot 3\sqrt{11} + 100$$

$$t = \underline{112{,}73}$$

7.12.

200 e – 400 e $\qquad \xi \sim$ egyenletes (200, 400)

$E(\xi) = 300 \qquad D(\xi) = \sqrt{Var} =$

$= \sqrt{\frac{(b-a)^2}{12}} = \underline{\underline{57,74}}$

$\{\xi_1, \xi_2, \ldots, \xi_{100}\}$ független

$S_{100} = \xi_1 + \ldots + \xi_{100}$

$E(S_{100}) = 100 \cdot 300 = \underline{30\,000}$

$D(S_{100}) = \sqrt{100} \cdot \frac{200}{\sqrt{12}} = \underline{\underline{577,\cancel{35}4}}$ (eFt)

$P(29\,000 \leq S_{100} \leq 31\,000) =$

$= P\left(\frac{29\,000 - 30\,000}{577,4} \leq Z \leq \frac{31\,000 - 30\,000}{577,4}\right) =$

$= P(-1,73 \leq Z \leq 1,73) \cong$

$\approx \phi(1,73) - \phi(-1,73) =$

$2 \cdot 0,9582 - 1 = \underline{\underline{0,9164}}$

$\cancel{P(30}$ $\quad P(30\,000 - x \leq \overset{S_{100}}{\cancel{\xi}} \leq 30\,000 + x) = 0,95$

$P\left(\frac{30\,000 - x - 30\,000}{577,4} \leq Z \leq \frac{x}{577,4}\right) \cong \phi\left(\frac{+x}{577,4}\right) - \phi\left(\frac{-x}{577,4}\right) =$

$= 2\phi\left(\frac{x}{577,4}\right) - 1 = 0,95 \iff \phi\left(\frac{x}{577,4}\right) = \frac{1,95}{2} = \underline{\underline{0,975}}$

$\boxed{x = 1131,704} \qquad \frac{x}{577,4} = 1,96$