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$(x-u)^2+(y-v)^2=r^2$

$f(x)$	$f'(x)$
$c \in \mathbb{R}$	0
x	1
x^n	$n\,x^{n-1}$
e^x	e^x
$a^x,\ a>0$	$a^x \cdot \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\operatorname{arcctg} x$	$-\frac{1}{1+x^2}$

$f(x)$	$\int f(x)dx$
1	$x+c$
x^n	$\frac{x^{n+1}}{n+1}+c$
e^x	e^x+c
a^x	$\frac{a^x}{\ln a}+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
$\sin x$	$-\cos x+c$
$\cos x$	$\sin x+c$
$\frac{1}{\cos^2 x}$	$\tan x+c$
$\frac{1}{\sin^2 x}$	$-\cot x+c$
$\frac{1}{1+x^2}$	$\arctan x+c$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x+c$

$\int f'g\,dx = fg - \int fg'\,dx.$

$(fg)' = f'g+fg',\ \left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$

$\int \frac{f'}{f}dx = \ln\lvert f\rvert + c$

$f'(g) = f'(g)\,g'.$

$\int f'\cdot f^{\alpha}dx = \frac{f^{\alpha+1}}{\alpha+1}+c$

$\int f(ax+b)\,dx = \frac{F(ax+b)}{a}+c.$

T.P.: $P_n(x)=\sum_{i=0}^{n}\frac{f^{(i)}(a)}{i!}(x-a)^i$, T.S: $\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$.

$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n,\ \forall \lvert x\rvert<1,\quad e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!},\ \forall x\in\mathbb{R},\quad \sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!},\quad \cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!},\quad (1+x)^{\alpha}=\sum_{n=0}^{\infty}\binom{\alpha}{n}x^n,\ \forall x<1.$

$a^2-b^2=(a-b)(a+b)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^4-b^4=(a-b)(a^3+a^2b+ab^2+b^3)$

$a^5-b^5=(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4).$

$\cos^2\varphi+\sin^2\varphi=1$

$2\cos\varphi\sin\varphi=\sin 2\varphi.$

S: $\forall\varepsilon>0,\ \exists r(\varepsilon),\ n>r(\varepsilon),\ \lvert a_n-A\rvert<\varepsilon.$

KONV $\Rightarrow$ KORL., KONV $\not\Leftarrow$ KON.

$(\exists$ F.K $\wedge$ H.N.$)\vee(\exists$ A.K. $\wedge$ H.CS.$)\Rightarrow$ KONV.

$\lim_{n\to\infty}q^n=\begin{cases}+\infty, & q>1\\ 1, & q=1\\ 0, & -1<q<1\\ \nexists, & q\le -1\end{cases}$

$\lim_{n\to\infty}\sqrt[n]{n}=1$

$\forall a>0,\ \lim_{n\to\infty}\sqrt[n]{a}=1.$

R.S: $(a_k)_{k=1}^{\infty}$ T.P. B-W: $\forall$ k.s $\exists$ k.r.s.

Cauchy: $\forall\varepsilon>0,\ \exists N\in\mathbb{N}.$ $n,m>N:\ \lvert a_n-a_m\rvert<\varepsilon.$

$\Updownarrow$

KONV.

S: $\sum_{n=0}^{\infty}a_n.\quad S=\sum_{n=0}^{\infty}a_n.\quad S_n=\sum_{k=0}^{n}a_k.$

$\sum_{n=0}^{\infty}q^n$ konv $\Leftrightarrow \lvert q\rvert<1,\ =\frac{1}{1-q}.$

$\sum a_n$ konv $\Rightarrow a_n\to 0,\ n\to\infty.$

I.K.: f folyt., $\searrow$, $\oplus$. $\sum_{n=k}^{\infty}a_n\sim\int_k^{\infty}f(x)dx.$

$\sum_{n=1}^{\infty}\frac{1}{n^p}$ konv $\Leftrightarrow p>1.$

$\sum a_n,\ \sum b_n,\ 0\le a_n\le b_n.$ MA: $\sum b_n$ KON. $\Rightarrow \sum a_n$ KON.

MI: $\sum a_n$ DIV. $\Rightarrow \sum b_n$ DIV.

$\lim_{n\to\infty}\frac{a_n}{b_n}=L>0\Rightarrow \sum a_n\sim\sum b_n.$

GY: $\exists\lim_{n\to\infty}\sqrt[n]{\lvert a_n\rvert}.$ $\lim_{n\to\infty}\sqrt[n]{\lvert a_n\rvert}<1\to\sum a_n$ AK.

$\lim_{n\to\infty}\sqrt[n]{\lvert a_n\rvert}>1\to\sum a_n$ D.

$\lim_{n\to\infty}\sqrt[n]{\lvert a_n\rvert}=1\Rightarrow$ T.V.

H: $\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert\ \exists.$

$\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert<1\to\sum a_n$ AK.

$\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert>1\to\sum a_n$ DIV.

$\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert=1\to$ T.V.

ALI: $a_n>0,\ \sum(-1)^n a_n.$ $\left.\begin{array}{l}a_n\to 0\\ a_n\searrow\end{array}\right\}\sum(-1)^n a_n$ FELT.K.

T.: $d(\underline{x},\underline{y})\ge 0,\ d(\underline{x},\underline{y})=0\Leftrightarrow\underline{x}=\underline{y}.$

$d(\underline{x},\underline{y})=d(\underline{y},\underline{x}),\ d(\underline{x},\underline{y})\le d(\underline{x},\underline{z})+d(\underline{z},\underline{y}).$

szokt.: $d(\underline{x},\underline{y})=\sqrt{(x_1-y_1)^2+\dots+(x_n-y_n)^2}$

csob.: $d(\underline{x},\underline{y})=\max\{\lvert x_1-y_1\rvert,\lvert x_2-y_2\rvert,\dots,\lvert x_n-y_n\rvert\}$

mank.: $d(\underline{x},\underline{y})=\sum_{i=1}^{n}\lvert x_i-y_i\rvert.$

B.P.: $\exists\delta>0$, hg $U(a,\delta)\subset H$, lakópont.

$\lim_{\underline{x}\to\underline{a}}f(\underline{x})=A:(\forall\varepsilon>0)(\exists\delta>0)(\forall x\in D)(0\le d(\underline{x},\underline{a})\le\delta)$

$\Downarrow$

$\lvert f(\underline{x})-A\rvert<\varepsilon.$

$f'_x(a,b)=\lim_{x\to a}\frac{f(x,b)-f(a,b)}{x-a},\quad f'_y(x,y)=\lim_{y\to b}\frac{f(a,y)-f(a,b)}{y-b}$

Érintősík: $z(x,y)=f(a,b)+f'_x(a,b)\cdot(x-a)+f'_y(a,b)(y-b).$

T.D.: (a,b) b.p. $\exists q_1,q_2\in\mathbb{R},\ \exists\omega_1,\omega_2: D\to\mathbb{R}$,

$\exists\wedge$ folyt. $\lim_{(x,y)\to(a,b)}\omega_1(x,y)=\dots=0.$ és

$f(x,y)-f(a,b)=q_1(x-a)+\dots+\omega_1(x,y)(x-a).$

$\forall(x,y)\in D.$

Láncszabály: $\frac{d}{dt}F(t)\Big|_{t=a}=\sum_{i=1}^{k}\frac{\partial}{\partial x_i}g(f_1(a),f_2(a),\dots,f_k(a))\cdot\frac{d}{dt}f_i(a)$

Szélsőérték: $(\forall\delta>0)(\forall x\in D)(d(\underline{x},\underline{a})<\delta\to f(\underline{x})\le f(\underline{a})).$

$\nabla f(a,b)=(f'_x,f'_y).\ (=0)$

$D=\begin{vmatrix}f''_{xx} & f''_{xy}\\ f''_{yx} & f''_{yy}\end{vmatrix}.$

$D>0$: van

$f''_{xx}>0\Rightarrow$ sl. h. min.

$f''_{xx}<0\Rightarrow$ sl. h. max.

$F(x,y,\lambda)=f(x,y)-\lambda g(x,y)$

$x=r\cdot\cos\varphi$

$y=r\cdot\sin\varphi$

$r=\sqrt{x^2+y^2},\quad \varphi=\arctan\frac{y}{x}.$

IFT: $(a,b)\in D$, F.D.

$f(a,b)=0,\ f'_y(a,b)\ne 0.$

$(\exists\delta>0)(\exists\varphi:(a-\delta,a+\delta)\ \varphi(a)=b.$

$\varphi'(a)=-\frac{f'_x(a,b)}{f'_y(a,b)}.\quad f(x,\varphi(x))=0.$

$[\underline{a},\underline{b}]=\{\underline{x}\in\mathbb{R}^n,\ \underline{a}\le\underline{x}\le\underline{b}\}.$

$\underline{S}(f,P)=\sum m_j\mu(I_j)\quad\overline{S}(f,P)=\sum M_j\mu(I_j).$

$\mu(I_j)=\prod_{i=1}^{n}(b_i-a_i)$

$\underline{\int_I}f(\underline{x})d\underline{x}=\inf\{\overline{S}(f,P)\}$ bezd.

$\int_D f\,d\underline{x}=\int_{\alpha}^{\beta}S\,dx.$