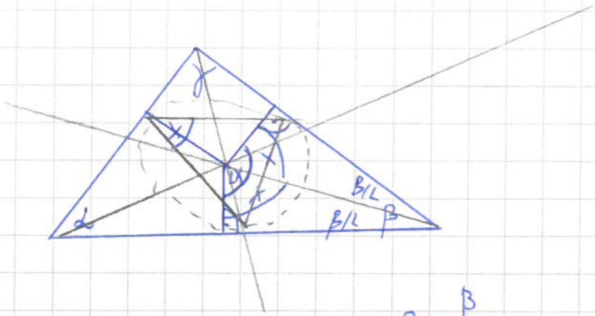
Geometriai bizonyításokkal megyünk tovább.

909.



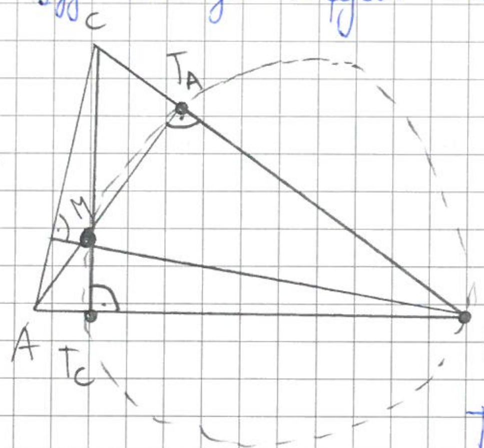
$$x = 90° - \frac{\beta}{2}$$

$$z = 90° - \frac{\alpha}{2}$$

$$y = 90° - \frac{\gamma}{2}$$

Így lehetséges kifejezni őket.

979.



T_A, M, T_C és B egy körön vannak.

Thálesz-tétel miatt.

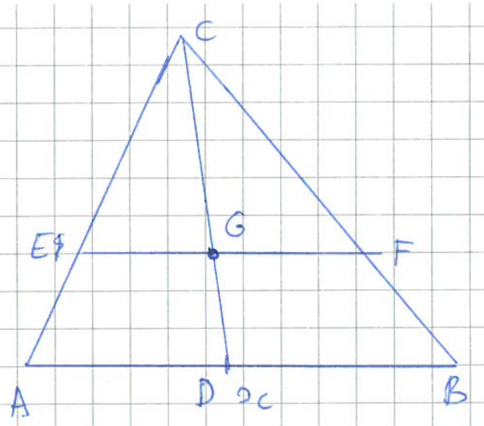
MT_CT_AB □ húrnégyszög-e?

Húrnégyszög, mert a szemközti szögeinek összege 180°.

T_C-nél ∟ van, T_A-nál is ∟ van } Σ 180° ✓

Ezért M∡ és B∡ is 180°.

1085.



$EF \parallel AB$

$\Downarrow$

$EG = GF$. Ezt kell belátni.

$AD = \cancel{AB}\ DB$, mert a súlyvonal miatt D felezőpont.

$\Downarrow$

EF∥AB miatt
- CEG△ hasonló CAD △-hez.
- CGF△ hasonló CDB △-höz.

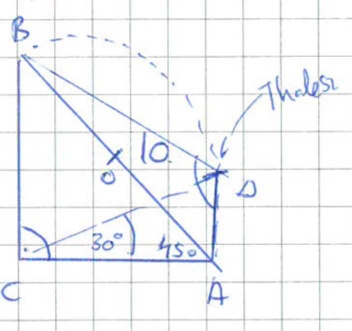
$AD = DB$

$\Downarrow$

$EG = GF$.

Ennyi volt a bizonyítás.

2954. TRIGONOMETRIA



Thalesz tétel miatt b.

Koszinusztétel: CD-hez:

$CD^2 = AD^2 + AC^2 - 2\,AD\,AC\cos 105°$

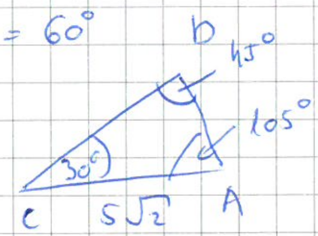
$CD \approx 9,66$

ACD △ oldalait keressük.

$AB = AC\sqrt{2}$ Pitagorasz:

$AC = \frac{AB}{\sqrt{2}} = \frac{AB\sqrt{2}}{2} = 5\sqrt{2}$

AOD△ egyenlő oldalú, mert AD-hez tartozó középponti szög: $AOD\sphericalangle = 2ACD\sphericalangle = 60°$



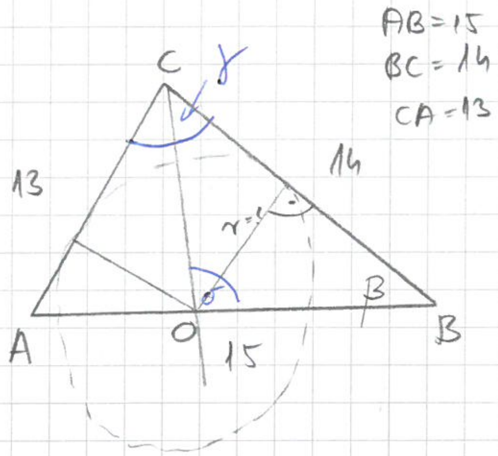
Szinusztétel ACD-re:

$\frac{AC}{\sin 45°} = \frac{DA}{\sin 30°}$

$AD = \sin 30° \cdot \frac{AC}{\sin 45°} = 5$

3002.

13, 14, 15



$AC^2 = AB^2 + BC^2 - 2AB\,BC\cos\beta$

$$\cos\beta = \frac{AC^2 - AB^2 - BC^2}{-2AB\cdot BC} = \frac{13^2 - 14^2 - 15^2}{-2\cdot 15\cdot 14} = \underline{\underline{\frac{3}{5}}}$$

$\beta \approx \underline{\underline{53{,}13^\circ}}$

$$\frac{AB}{\sin\gamma} = \frac{AC}{\sin\beta}$$

$$\sin\gamma = \frac{\sin\beta}{AC}\cdot AB = \underline{\underline{0{,}923}}$$

$\gamma \cong 67{,}37^\circ$

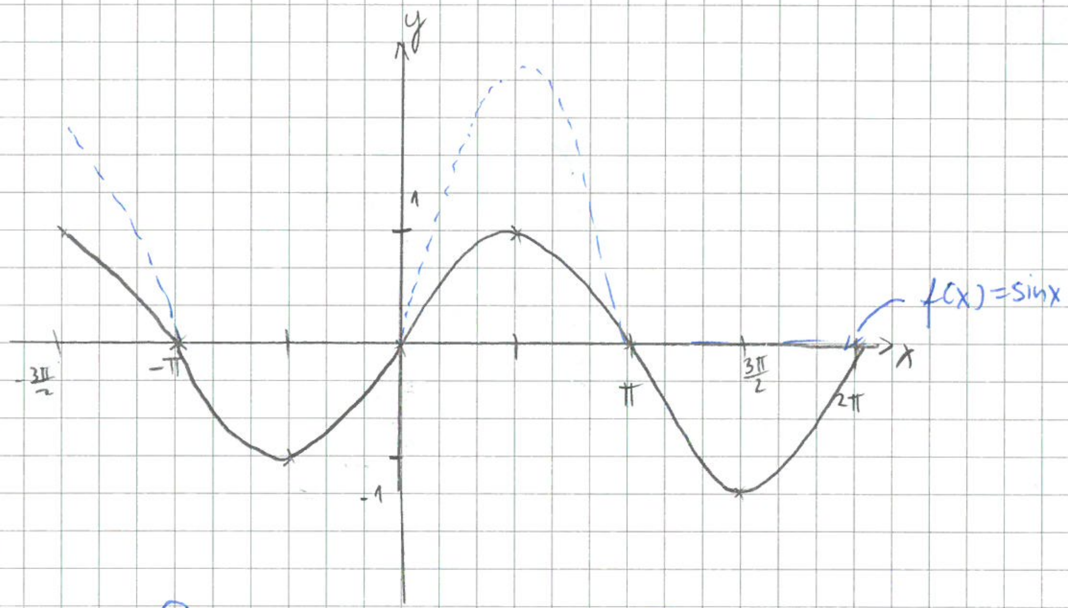
$$\frac{OB}{\sin\frac{\gamma}{2}} = \frac{CB}{\sin\sigma} \qquad \sigma = 180^\circ - \beta - \frac{\gamma}{2} = \underline{\underline{93{,}185^\circ}}$$

$$OB = \frac{14\cdot\sin\frac{67{,}37^\circ}{2}}{\sin\sigma} \cong \underline{\underline{7{,}78}}$$

$\cancel{\frac{r}{\sin\beta} =}$ $\quad \sin\beta = \frac{r}{OB} \Rightarrow r = OB\sin\beta = \underline{\underline{6{,}22}}$

2753.

a) $f(x) = |\sin x| + \sin x$ ábrázolja!



$|\sin x| = \left\{ \text{felírható 2 szakaszra} \right\}$

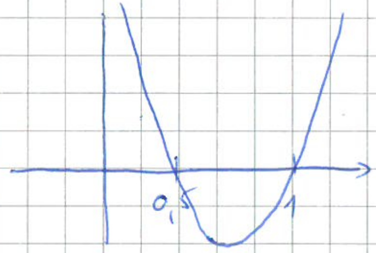
$$f(x)=\begin{cases} 2\sin x, & \text{ha } \sin x \geq 0 \\ 0, & \text{ha } \sin x < 0 \end{cases}$$ egyszerű.

Nem túl szép az ábra, de érthető...

2898. $\sin^2 x - \frac{3}{2}\sin x + \frac{1}{2} \geq 0$ $\quad t = \sin x$

$t^2 - \frac{3}{2}t + \frac{1}{2} \geq 0$

$(t-0{,}5)(t-1) \geq 0$



$t \leq \frac{1}{2}$ vagy $t \geq 1$

II. $\sin x \leq \frac{1}{2}$ $\qquad$ I. $\sin x \geq 1$

$\sin 30° = \frac{1}{2}$

$x \in \left[\frac{5\pi}{6} + 2k\pi ; \frac{13\pi}{6} + 2k\pi\right]$ $\qquad$ $x = \frac{\pi}{2} + 2k\pi$

$k \in \mathbb{Z}$.

2904.

$\sin^2 x + \cos^2 x = 1.$

a) $8\cos^4 x \geq 2\sin^2 x + 1$

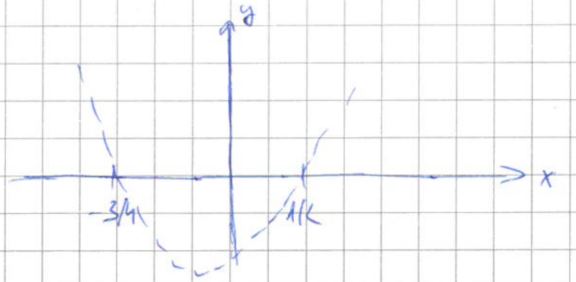
$8\cos^4 x \geq 2(1-\cos^2 x)+1$

$8\cos^4 x + 2\cos^2 x - 3 \geq 0.$ $\quad t = \cos^2 x$

$8t^2 + 2t - 3 \geq 0$

$t_1 = \frac{1}{2}$ $\quad \cancel{t_2 = -\frac{3}{4}}$

I. $\cos^2 x \geq \frac{1}{2}$ $\quad$ II. $\cos^2 x = -\frac{3}{4}$ nincs megoldás $\mathbb{R}$ felett.



$\cos x \geq \frac{\sqrt{2}}{2}$ vagy $\cos x \leq -\frac{\sqrt{2}}{2}$

$x \geq \frac{\pi}{4}$

$-\frac{\pi}{4} + 2k\pi \leq x \leq \frac{\pi}{4} + 2k\pi$

3394.

a) $\left.\begin{aligned} \sin^2 x - \cos y &= 1 \\ \cos^2 x + \cos y &= 1 \end{aligned}\right\} +$

$\sin^2 x + \cos^2 x = 2$

Ellentmondó.

b) $\left.\begin{aligned} \sin^2 x + \cos^2 y &= \frac{3}{2} \\ \cos^2 x - \sin^2 y &= \frac{1}{2} \end{aligned}\right\} +$ $\quad \underbrace{\sin^2 x + \cos^2 x}_{1} + \cos^2 y - \sin^2 y = 2$

$\cos^2 y - \underbrace{\sin^2 y}_{1-\cos^2 y} = 1$

$2\cos^2 y = 2 \quad \cos^2 y = 1$

$\cos y = 1$ $\qquad\qquad$ $\cos y = -1$

$y = 0 + 2k\pi$ $\qquad\qquad$ $y = \pi + 2k\pi$

$y = k\pi \quad , \quad k \in \mathbb{Z}$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm \frac{\sqrt{2}}{2}$

$x_1 = \frac{\pi}{4} + 2k\pi$ $\qquad$ $x_3 = \frac{5\pi}{4} + 2k\pi$

$x_2 = \frac{3\pi}{4} + 2k\pi$ $\qquad$ $x_4 = \frac{7\pi}{4} + 2k\pi$

$x = \frac{\pi}{4} + k\frac{\pi}{2}$ Trigonometrikus kör alapján.

$\boxed{2824}$

$tg^2 x - 5 = \frac{1}{\cos x}$ megoldás:

$\left(\frac{\sin x}{\cos x}\right)^2 - 5 = \frac{1}{\cos x}$

$\frac{1 - \cos^2 x}{\cos^2 x} - 5 = \frac{1}{\cos x}$ $\qquad$ $t = \cos x$

$\frac{1 - t^2}{t^2} - 5 = \frac{1}{t} \quad / \cdot t^2$

$1 - t^2 - 5t^2 = t$

$-6t^2 - t + 1 = 0$

$t_1 = \frac{1}{3}$ $\qquad$ $t_2 = -\frac{1}{2}$

$\cos x = \frac{1}{3}$ $\qquad\qquad$ $\cos x = -\frac{1}{2}$

$x_1 = \pm \arccos\left(\frac{1}{3}\right) + 2k\pi$ $\qquad$ $x_2 = \frac{2\pi}{3} + 2k\pi$

$x_3 = \frac{4\pi}{3} + 2k\pi$

Haladunk tovább.

808

a) $a(x) = x^2 - 4x + 3$

$a'(x) = 2x - 4 \to$ $2x - 4 = 0$

$2x = 4$ $\quad \underline{\underline{x = 2}}$

Ha $x > 2$ Ha $x = 0$

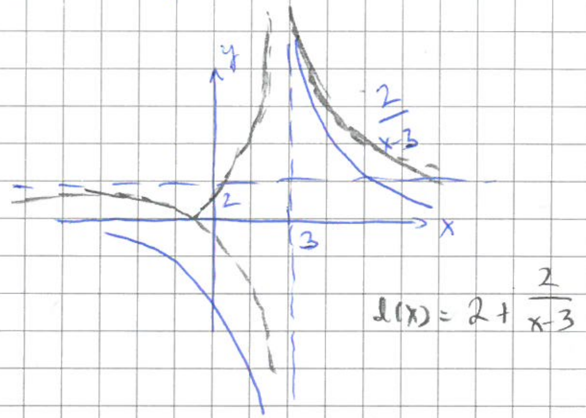
$a'(x=3) = 6 - 4 = \underline{\underline{2}}$ $\quad a'(x=0) = \underline{\underline{-4}}$

$x \in (-\infty; 2]$

b) $b(x) = |2x - 4|$ u.a. $x \in (-\infty, 2]$

d) $d(x) = \left|\frac{2x-4}{x-3}\right|$ $\quad x \neq 3, \quad D_d = \mathbb{R} \setminus \{3\}$

$d(x) = \left|2 + \frac{2}{x-3}\right|$



$x \in (-\infty, 2] \cup (3, \infty)$

PÁROS : $f(x) = f(-x)$

$a(x) = x + \sin x$

$a(-x) = -x + \sin(-x) = -\sin x - x = -(x + \sin x)$ Páratlan.

$a(x) = x + \sin x - 1$

$a(-x) = -x - \sin x - 1 = -(x + \sin x + 1)$ nincs paritás.

$g(x) = x \cos x$ $\quad g(-x) = -x \cdot \cos x \Rightarrow$ páratlan

$\boxed{-g(x)}$

$$j(x) = \frac{2^{x} + 2^{-x}}{2}$$

$$j(-x) = \frac{2^{-x} + 2^{x}}{2} = j(x) \quad \text{páros.}$$

789.

$$a(x) = x+2$$

$$y = x+2 \rightarrow x = y-2 \qquad \boxed{a^{-1}(x) = x-2}$$

$$d(x) = |x+2| \qquad \text{Ha } x \le 0$$

$$d(x) = -x-2$$

$$y = -x-2 \qquad -x = y+2$$

$$x = -y-2$$

$$d^{-1}(x) = \underline{\underline{-x-2}}$$

e) $e(x) = x^2 - 4x \quad , \quad x \geq 2$

$$y = x^2 - 4x \qquad y = x(x-4)$$

$$x^2 - 4x - y = \emptyset. \qquad x = \frac{4 \pm \sqrt{16+4y}}{2} \quad \text{Lehet egyszerűsíteni}$$

$$\cancel{x = 4 \pm 4\sqrt{}}$$

$$i(x) = \sqrt[3]{x-2}$$

$$y = \sqrt[3]{x-2}$$

$$y^3 = x-2 \qquad \underline{\underline{i^{-1}(x) = x^3+2}}$$

$$x = y^3+2$$

SOROZATOK.

887.

$a_8 = 8$ $\qquad$ $d = 3$

Hány tag van 500 és 700 között?

$$a_n = a_1 + (n-1)d$$

$$a_8 = a_1 + 7d \Rightarrow a_1 = a_8 - 7d = \underline{\underline{-13}}$$

$$a_n = 500$$

$$500 = -13 + (n-1)d$$

$$\frac{500+13}{3} = n-1 = 171$$

$$\underline{\underline{n = 172}}$$

$700 - 500 = 200$

198 a legközelebbi 3-mal osztható.

$198 = 3 \cdot 66$ $\quad$ 66 elem van közöttük.

890.

1 és 2 közé 10 szám kell

$1 \qquad\qquad 2$

$a_1 \qquad\qquad a_{12}$

a_{12} A kérdés! $\qquad a_{12} = a_1 + 11d$

$$d = \frac{2-1}{11} = \underline{\underline{\frac{1}{11}}}$$

897.

$a_1 = -210$

$a_n = 228$ $\qquad a_2 + a_3 + \dots + a_{n-1} = 45$

$n = ?$

$$S_n = n \cdot a_1 + d\left(\sum_{i=1}^{n-1} i\right) = n a_1 + d\,\frac{n(n-1)}{2} =$$

$$S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$ AZ ÖSSZEGKÉPLET.

$$S_n = 45 + (-210) + 228$$

$$S_n = 63 \qquad S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

$$a_n = a_1 + (n-1)d$$

$$63 = \frac{n}{2}\left(-420 + (n-1)d\right)$$

$$228 = -210 + (n-1)d$$

$$(n-1)d = 438$$

$$63 = \frac{n}{2}(\underbrace{-420 + 438}_{18})$$

$$\boxed{n = 7}$$

906.

$$a_1^2 + a_2^2 = 130$$

$$a_2^2 + a_3^2 = 202$$

$$a_1^2 + (a_1+d)^2 = 130$$

$$(a_1+d)^2 + (a_1+2d)^2 = 202$$

$$72 = (a_1+2d)^2 - a_1^2$$

$$72 = (\cancel{a_1} + 2d - \cancel{a_1}) \cdot (a_1 + 2d + a_1)$$

$$72 = 2d(a_1+d)$$

$$18 = d(a_1+d)$$

$$18 = a_1 d + d^2$$

$$\boxed{a_1 = \frac{18-d^2}{d}}$$

$$\left(\frac{18-d^2}{d}\right)^2 + \left(\underbrace{\frac{18-d^2+d^2}{d}}_{\left(\frac{18}{d}\right)^2}\right)^2 = 130$$

$$\left(\frac{18}{d} - d\right)^2$$

Negyed fokú lesz ez is.

953. Mértani

$$\sum_{i=1}^{3} a_i = 26$$

$$\sum_{i=5}^{7} a_i = 2106$$

$$a_n = a_1 \cdot q^{n-1}$$

$$\boxed{S_n = a_1 \frac{q^n - 1}{q - 1}}$$

$$S_{7_1} = 2 \cdot \frac{3^7 - 1}{3 - 1} = \cancel{728}\ 2186$$

$$S_{7_2} = \frac{26}{7} \cdot \frac{(-3)^7 - 1}{-3 - 1} =$$

$S_7 = ?$

$$\left.\begin{aligned} a_1 + a_1 q + a_1 q^2 &= 26 \\ a_1 q^4 + a_1 q^5 + a_1 q^6 &= 2106 \end{aligned}\right\}$$

$$\underline{\underline{a_1 = 2}}$$

$$\underline{\underline{a_2 = \frac{26}{7}}}$$

$$a_1 (1 + q + q^2) = 26$$

$$a_1 q^4 (1 + q + q^2) = \cancel{2106}\ 2106$$

$$q^4 = 81$$

$$\underline{\underline{q = 3}} \qquad \underline{\underline{q = -3}}$$