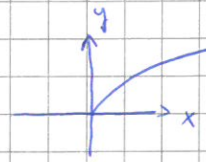
Matematikai alapismeretek — 4. konzultáció — 2017. 11. 10.

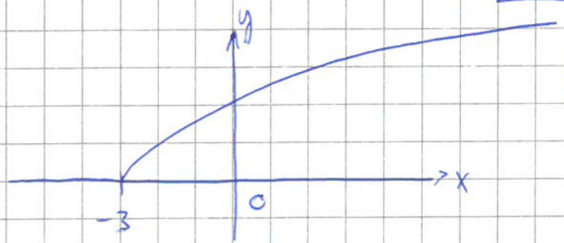
Gyökös függvények

$\sqrt{x}$



714. $a(x) = 2\sqrt{x+3}$

É.T.: $x+3 \geq 0 \to \boxed{x \geq -3}$

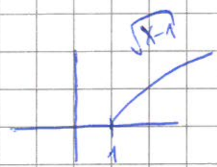


É.K.: $a(x) \in [0, \infty)$.

Tengelymetszet: x-tengelyen (zérushely) $\left.\begin{array}{l} x = -3 \\ y = 0 \end{array}\right\}$

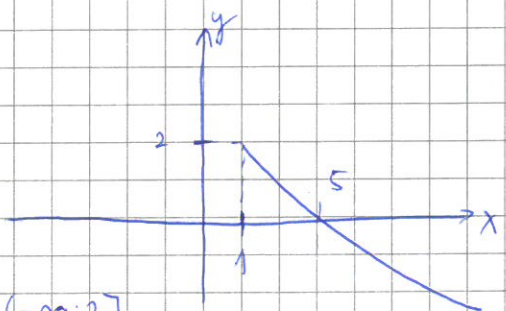
y-tengely: $\left.\begin{array}{l} 2\sqrt{3} = y \\ 0 = x \end{array}\right\}$

$b(x) = -0,5 \cdot \sqrt{x-1} + \cancel{8}\, 2$



① É.T.: $x \geq 1$.

② Grafikon



kb. ez.

É.K.: $b(x) \in (-\infty; 2]$

Tengelymetszetek: x: $(5, 0)$

y: nincsen. (É.T. miatt).

1531. a) $\sqrt{3x-5} \cdot \sqrt{4x-3} = 3x-1$

É.T.: $3x-5 > 0$, $4x-3 > 0$

$\boxed{x > \frac{5}{3}}$!! Ez.

$\boxed{x > \frac{3}{4}}$

$(3x-5)(4x-3) = (3x-1)^2$

$3 \cdot 4x^2 - 9x - 20x + 15 = 9x^2 - 6x + 1$

és $3x-1 \geq 0$! $x \geq \frac{1}{3}$

$3x^2 - 23x + 14 = 0$

$\boxed{x_1 = 7}$ $\quad x_2 = \frac{2}{3}$ $\quad x_2$ nem megoldás!

$x = 7$ a helyes megoldás.

c) $\frac{x+3}{\sqrt{x-1}} = \sqrt{2x+6}$ $\qquad x - 1 > 0$

$\boxed{x > 1}$!!!

$2x + 6 \geq 0$

$\frac{x^2+6x+9}{x-1} = 2x+6$ $\qquad x + 3 \geq 0$

$\boxed{x \geq -3}$

$x^2 + 6x + 9 = \cancel{29}\ 2x^2 + 6x - 2x - 6$

$x^2 - 2x - 15 = 0$ $\qquad x_1 = -3$ NEM MEGOLDÁS!

$x_2 = 5$

$\boxed{x = 5}$

1533. $\sqrt{x-11} + \sqrt{15-x} = 2$ $\qquad \left.\begin{matrix} x \geq 11 \\ x \leq 15 \end{matrix}\right\} \to \boxed{15 \geq x \geq 11}$

$\sqrt{x-11} = 2 - \sqrt{15-x}$

$x - 11 = 4 - 4\sqrt{15-x} + 15 - x$

$4\sqrt{15-x} = 30 - 2x$

$2\sqrt{15-x} = 15 - x$

$4(15-x) = 225 - 30x + x^2$

$60 - 4x = 225 - 30x + x^2$

$x^2 - 26x + 165 = 0$ $\qquad \left.\begin{matrix} x_1 = 15 \\ x_2 = 11 \end{matrix}\right\}$ ✓

d) $\sqrt{x-\frac{7}{3}}+\sqrt{\frac{3}{4}-x}=3$ $\qquad x \geq \frac{7}{3}$ $\qquad \frac{3}{4}-x \geq 0$

$\frac{3}{4} \geq x$

$\sqrt{x-\frac{7}{3}}=3-\sqrt{\frac{3}{4}-x} \quad /\,{}^2$

$\Uparrow$

NINCS MEGOLDÁS.

1534. $\quad \sqrt{5x+4}-\sqrt{4x-3}=\sqrt{3x+1}$ $\qquad \left.\begin{array}{l} x \geq -\frac{4}{5} \\ \boxed{x \geq \frac{3}{4}} \\ x \geq -\frac{1}{3} \end{array}\right\}$

$\cancel{5}x+4-\cancel{2}\sqrt{20x^2-15x+16x-12}+\cancel{4}x-\cancel{3}=\sqrt{3x+1}$

$6x=2\sqrt{20x^2+x-12}$

$9x^2=20x^2+x-12$

$11x^2+x-12=0 \qquad x_1=1 \qquad \underline{\underline{x_2=-\frac{12}{11}}}$

1539.

a) $4-x=\sqrt{16-x\sqrt{16+x^2}}$ $\qquad x \leq 4$

$\cancel{16}-8x+x^2=\cancel{16}-x\sqrt{16+x^2}$ $\qquad \boxed{x=0}$ ✓ megoldás!

$8-x=\sqrt{16+x^2}$

Ell: behelyettesítni!

$64-16x+\cancel{x^2}=16+\cancel{x^2}$

$16x-48=0$

$x-3=0 \qquad \boxed{x=3}$ ✓

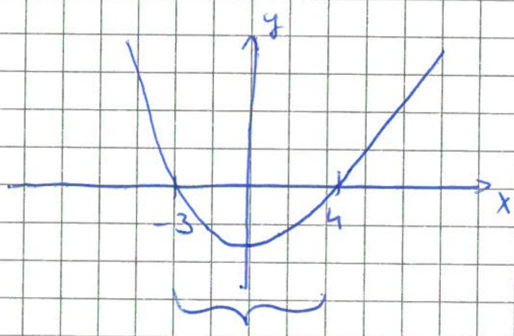
1557. $\quad \sqrt{5x+16}>x+2$ $\qquad x \geq -\frac{16}{5}$

$5x+16>x^2+4x+4$

$x^2-x-12<0$

$x_1=-3$

$\underline{\underline{x_2=4}}$

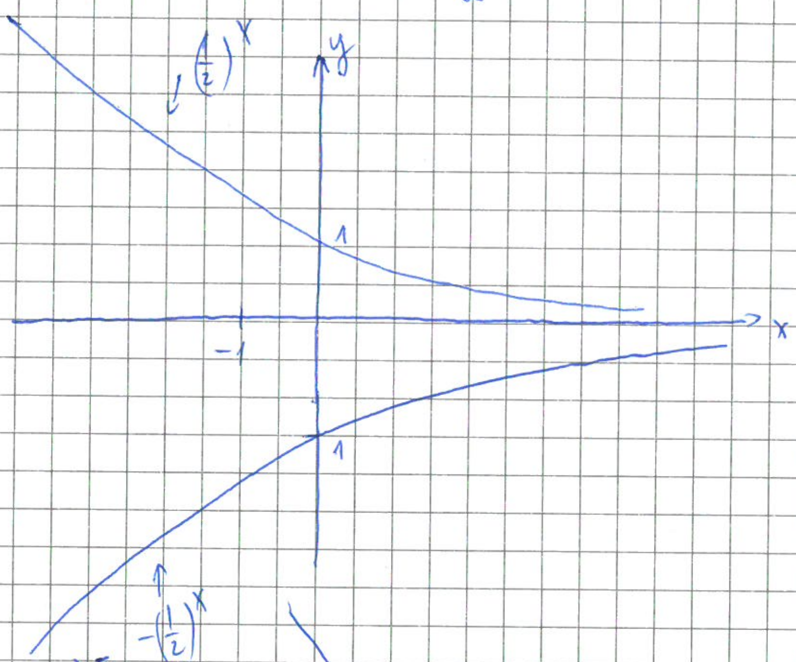


$x \in (-3, 4)$

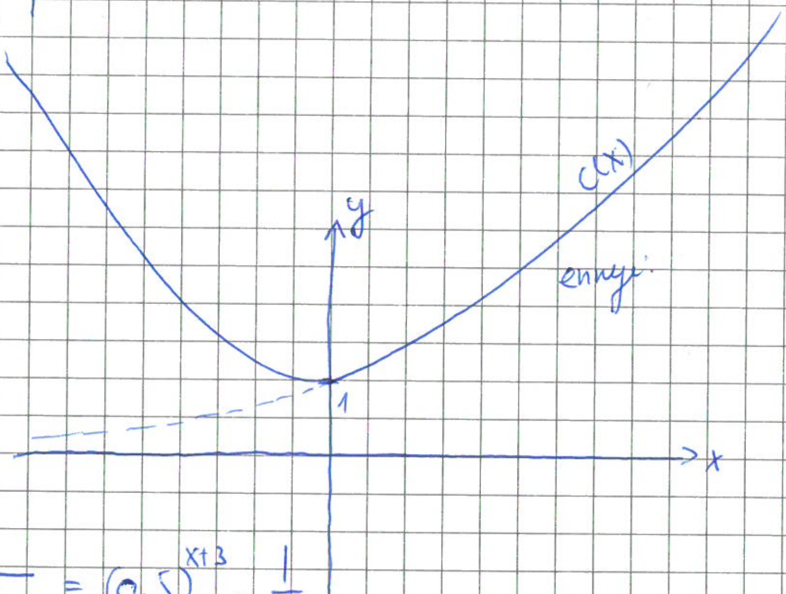
itt teljesül a feltétel.

(726.) Exponenciális fgvny.

$$a(x) = 2^{1-x} - \frac{3}{2^x} = 2\cdot 2^{-x} - 3\cdot 2^{-x} = -2^{-x} = -\frac{1}{2^x} = -\left(\frac{1}{2}\right)^x$$



$c(x) = 2^{|x|}$



(1610.) $$\frac{8}{\sqrt{32^{3x+5}}} = (0{,}5)^{x+3}\cdot\frac{1}{64}$$

$$\frac{2^3}{\left(\left(2^5\right)^{3x+5}\right)^{\frac{1}{4}}} = \frac{1}{2^{x+3+6}} \Rightarrow 2^{3-\frac{5(3x+5)}{4}} = 2^{-x-9}$$

$\Downarrow$

$$\Rightarrow \quad 3 - \frac{5(3x+5)}{4} = -x-9$$

$$12 - 15x - 25 = -4x - 36$$

$$11x - 23 = 0$$

$$\underline{\underline{x = \frac{23}{11}}} \quad \text{ennyi.}$$

1614.

b) $$\left(\frac{2}{3}\right)^{x} \cdot \left(\frac{4}{9}\right)^{x+1} \cdot \left(\frac{81}{16}\right)^{x-1} = \left(\frac{3}{2}\right)^{\sqrt{6-x}}$$

$$\left(\frac{2}{3}\right)^{x} \cdot \left(\frac{2}{3}\right)^{2x+2} \cdot \left(\frac{2}{3}\right)^{-4x+4} = \left(\frac{2}{3}\right)^{-\sqrt{6-x}}$$

$$\left(\frac{2}{3}\right)^{-x+6} = \left(\frac{2}{3}\right)^{-\sqrt{6-x}} \qquad \text{É.T.:} \ \boxed{x \le 6}$$

$$-x + 6 = -\sqrt{6-x}$$

$$x^2 - 12x + 36 = 6 - x$$

$$x^2 - 11x + 30 = 0$$

$$\boxed{\begin{aligned} x_1 &= \underline{\underline{6}} \\ x_2 &= \underline{\underline{5}} \end{aligned}}$$

Mindkettő megoldás OK.

1618.

a) $$1024^{x} + 2^{5x+1} = 8$$

$$2^{10x} + 2^{5x+1} = 8$$

$$\left(2^{5x}\right)^2 + 2 \cdot 2^{5x} - 8 = 0$$

$$2^{5x} = t \qquad t^2 + 2t - 8 = 0$$

$t_1 = 2$ $\Rightarrow$ $5x = 1 \Rightarrow \underline{\underline{x = \frac{1}{5}}}$

$t_2 = -4 \Rightarrow$ nincs megoldás.

b) $\sqrt[3]{10^{2x+3}} - 1 = 9 \cdot 10^{\frac{x}{3}}$

$10^{\frac{2x+3}{3}} - 1 = 9 \cdot 10^{\frac{x}{3}}$

$10^{2\frac{x}{3}+1} - 1 = 9 \cdot 10^{\frac{x}{3}}$

$10 \cdot \left(10^{\frac{x}{3}}\right)^2 - 1 = 9 \cdot 10^{\frac{x}{3}}$ $\qquad t = 10^{\frac{x}{3}}$

$10t^2 - 1 = 9t$

$\vdots$

c) $6 \cdot 3^x - 13 \cdot 3^{\frac{x}{2}} \cdot 2^{\frac{x}{2}} + 6 \cdot 2^x = 0 \quad / : \left(3^{\frac{x}{2}} \cdot 2^{\frac{x}{2}}\right)$

$6 \cdot \frac{3^{\frac{x}{2}}}{2^{\frac{x}{2}}} - 13 + 6 \cdot \frac{2^{\frac{x}{2}}}{3^{\frac{x}{2}}} = 0$

$\left(\frac{2}{3}\right)^{\frac{x}{2}} = t$

$\frac{6}{t} - 13 + 6t = \emptyset.$

$t_1 = \frac{3}{2} \implies \underline{\underline{x_1 = 2}}$

$t_2 = \frac{2}{3} \qquad \underline{\underline{x_2 = -2}}$

kikötés nem volt.

1620

$$\left.\begin{aligned} 8\cdot 6^{x+2} + 2\cdot 3^{y-1} &= 10 \\ 2\cdot 6^{x+3} - 3^{y+2} &= -15 \end{aligned}\right\}$$

$6^{x+2} = a$
$3^{y-1} = b$ legyen.

$$\left.\begin{aligned} 8a + 2b &= 10 \\ 12a - 27b &= -15 \end{aligned}\right\}$$

G–J elimináció:

$$\left(\begin{array}{cc|c} 8 & 2 & 10 \\ 12 & -27 & -15 \end{array}\right) \sim \left(\begin{array}{cc|c} 4 & 1 & 5 \\ 12 & -27 & -15 \end{array}\right) \sim \left(\begin{array}{cc|c} 4 & 1 & 5 \\ 0 & -30 & -30 \end{array}\right)$$

$x_2 = 1$

$4x_1 + 1 = 5$

$x_2 = 4$ $\qquad x_1 = 1$

$$\left.\begin{aligned} a &= 1 \\ b &= 1 \end{aligned}\right\} \begin{aligned} &\to x+2=0 \quad \boxed{x=-2} \\ &\to y-1=0 \quad \boxed{y=1} \end{aligned}$$

1622

a) $$\left.\begin{aligned} \sqrt{2^x + 3^y - 6} &= 5 \\ 9^y - 4^x &= 23(2^x + 3^y) \end{aligned}\right\}$$

$2^x = a$
$3^y = b$

$\sqrt{a+b-6} = 5 \to a+b-6 = 25$

$a+b = 31$

$a^2 - b^2 = 23(a+b)$ $\qquad \boxed{b=4}$

$(a-b)\cancel{(a+b)} = 23\cancel{(a+b)}$ $\qquad a+b \neq 0!$

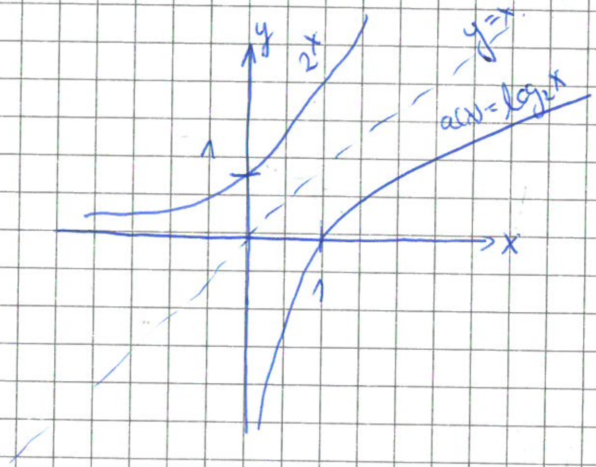
$a - b = 23$

↗ pont fordítva $\qquad 2a = 54 \quad \boxed{a=27}$

Logaritmus

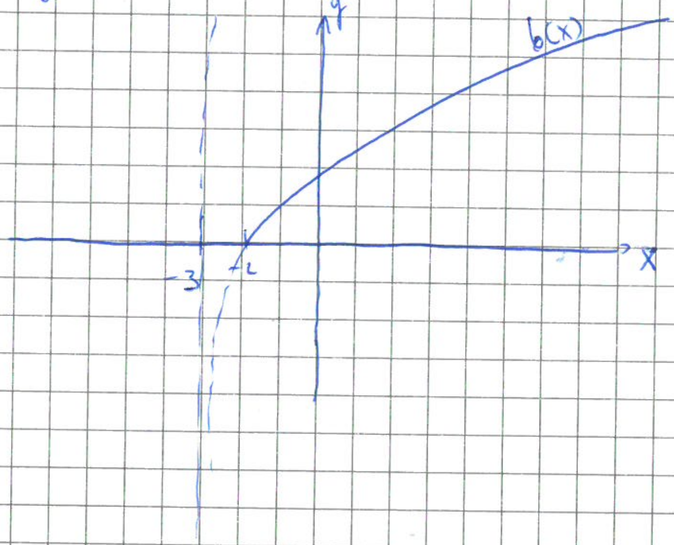
731.

$a(x) = \log_2(x) \qquad D_f = (0, \infty)$

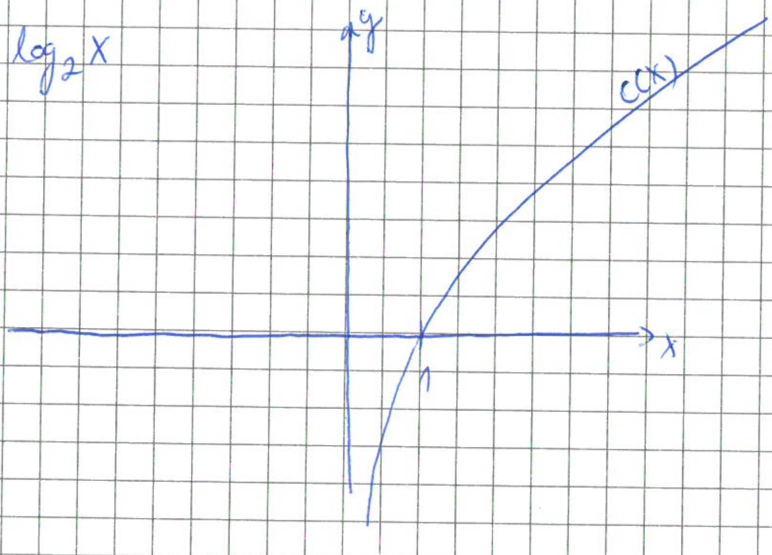


világos az összefüggés

$b(x) = \log_2(x+3)$



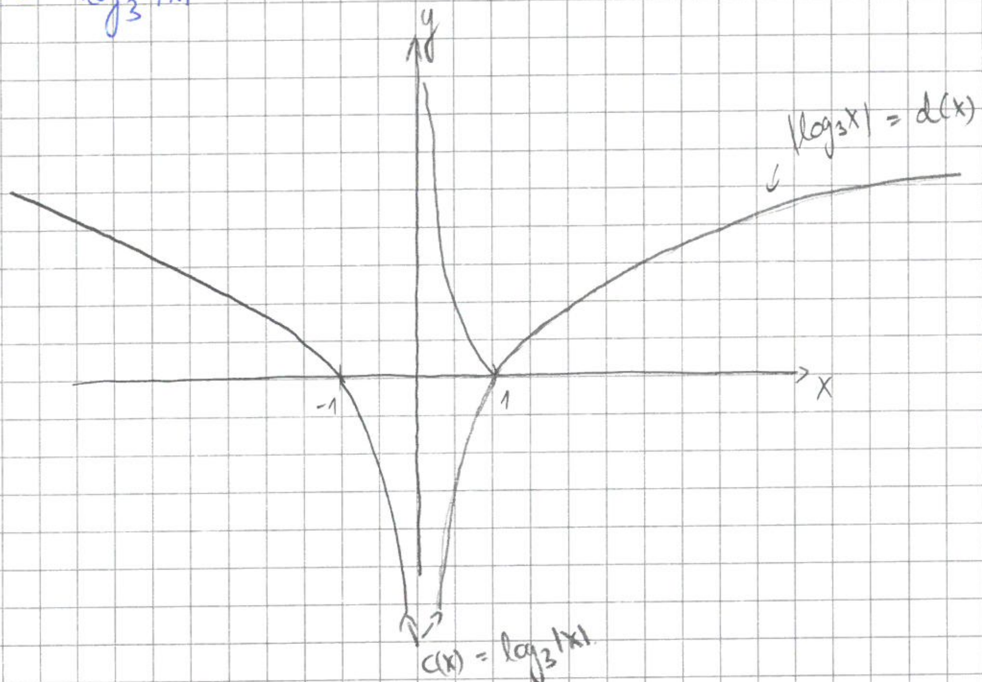
$c(x) = 2\log_2 x$



(734.)

$d(x) = |\log_3 x|$

$c(x) = \log_3 |x|$



(737.) É.T., É.K, tengelymetszetek

$$a(x) = \lg(x^2-5x+6) - \lg(x-2)$$

$x_1 = 2 \quad x_2 = 3$

É.T.: $x^2-5x+6>0$

$x-2>0$

$\boxed{x>2}$

~~x>2~~ ~~vagy~~

$$a(x) = \lg\left(\frac{x^2-5x+6}{x-2}\right) = \lg\left(\frac{(x-2)(x-3)}{x-2}\right) = x<2 \text{ v. } \boxed{x>3}$$

Ez jön!

$= \lg(x-3)$ Ennyi lesz a vége.

É.K.: $x \in \mathbb{R}$.

Tengelymetszetek: y-t nem metszi.

$\lg(x-3) = 0$

$\boxed{x=4}$

$b(x) = \log_2 x - 2\log_4 x + 6\log_8 x + \log_{1/2} x = ?$ É.T.: $x > 0$.

$= \cancel{\log_2 x} - 2 \cdot \frac{1}{2} \cdot \log_2 x + 6 \cdot \frac{1}{3} \cdot \log_2 x - \cancel{\log_2 x} =$

$2\log_2 x - \log_2 x = \underline{\underline{\log_2 x}}$

É.K.: $x \in \mathbb{R}$.

Tengelymetszetek:

x: $(1, 0)$

y: nincs.

Egyenletek:

1636

a) $\log_x(x^3 + x^2 - 9x + 14) = 3$ $x \neq 1$, $x > 0$.

$\cancel{x^3} = \cancel{x^3} + x^2 - 9x + 14$

$x^2 - 9x + 14 = 0$ $x_1 = \underline{\underline{7}}$ ✓ $x_2 = \underline{\underline{2}}$ ✓

1642 $\log_{25}\left(\frac{1}{5}\log_3(2 - \log_{1/2} x)\right) = -\frac{1}{2}$

$25^{\frac{1}{2}} = \frac{1}{5}\log_3(2 - \log_{1/2} x)$

$\frac{1}{5} = \frac{1}{5}\log_3(2 - \log_{1/2} x)$

$1 = \log_3(2 - \log_{1/2} x)$

$3^1 = 2 - \log_{1/2} x$

$1 = -\log_{1/2} x$

$1 = \log_2 x$

$\boxed{x = 2}$

1658.

a) $\lg^2 x^2 + 3\lg x^2 = 40$

$t = \lg x^2$

$t^2 + 3t - 40 = 0$ $\quad t_1 = 5$ $\quad t_2 = -8$

$5 = \lg x^2$

$10^5 = x^2 \quad \to \quad x_{1,2} = \pm\sqrt{10^5}$

$\lg x^2 = -8$

$10^{-8} = x^2 \quad \Rightarrow \quad x_{3,4} = \pm\sqrt{10^{-8}}$

c) $9^{1/2 + \log_2 x} + 1 = 28 \cdot 3^{\log_2 x - 1}$

$3 \cdot 9^{\log_2 x} + 1 = 28 \dfrac{3^{\log_2 x}}{3}$

$3^{2\log_2 x}$

$t = 3^{\log_2 x}$

$3t^2 + 1 = 28 \dfrac{t}{3}$

$9t^2 - 28t + \cancel{1}\,3 = 0$

$t_1 = 3 \Rightarrow 3 = 3^{\log_2 x} \Rightarrow \boxed{x = 2}$

$t_2 = \frac{1}{9}$ $\quad \frac{1}{9} = 3^{\log_2 x}$

$-2 = \log_2 x \Rightarrow \boxed{x = \frac{1}{4}}$

1692.

$$\left.\begin{aligned} 3^x \cdot 2^y &= 576 \\ \log_{\sqrt{2}}(y-x) &= 4 \end{aligned}\right\}$$

K: $y > x$

$$3^x \cdot 2^y = 576$$

$$\underbrace{\sqrt{2}^{\,4}}_{4} = y - x$$

$$\boxed{y = 4 + x} = 6$$

$$3^x \cdot 2^{x+4} = 576$$

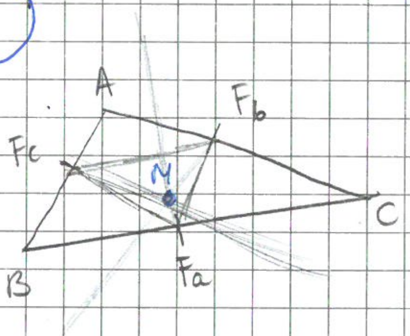
$$3^x \cdot 2^x \cdot 16 = 576$$

$$6^x = \frac{576}{16} = 36$$

$$6^x = 36$$

$$\boxed{x = 2}$$

GEOMETRIA



Biz.:

$MA = MB = MC$

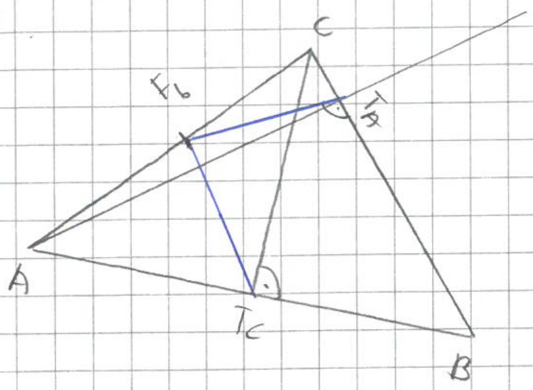
Ezt kell belátni.

$$\left.\begin{aligned} &F_bF_c \parallel BC \text{ és} \\ &F_aF_c \parallel AC \text{ és} \\ &F_aF_c \parallel AC. \end{aligned}\right\} \Rightarrow \begin{aligned} &m_a \perp BC \\ &m_b \perp AC \\ &m_c \perp AB \end{aligned}$$

Vagyis m_a, m_b és m_c az $ABC\triangle$ oldalfelező merőlegesei.

M pont a köréírt kör középpontja. $\Leftarrow$

531.



Biz: $F_b T_C = F T_A$!

AT_AC △ b-ű.

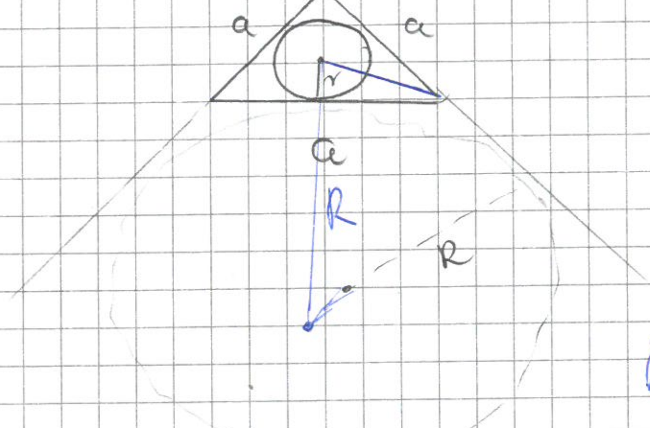
AT_CC △ b-ű.

Thálesz-tétel:

~~Ha c-beli~~ AC mindkét △ átfogója, melynek felezőpontja F_b.

F_b felé szerkesztett ○ (Thalesz-kör) tartalmazza a T_C és T_A pontokat => $F_bT_a = F_bT_c$.

573.



Biz.: $\boxed{R = 3r}$

körülírt

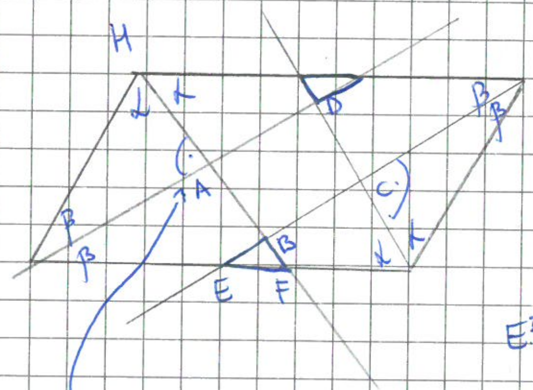
beírt

$$\left.\begin{aligned}(r+R)^2 &= \left(\frac{2}{3}h\right)^2 + x^2 \\ R^2 + \left(\frac{a}{2}\right)^2 &= x^2\end{aligned}\right\}$$

Van egy szebb út.

HF.

670.



Biz., hogy ABCD téglalap.

EFB △ ~ AGH △

⇓ ~~sz,~~ oldalak

BF ∥ AH

és

EB ∥ AB.

$2\alpha + 2\beta = 180°$

$\alpha + \beta = 90°$ ott derékszög van.

Ebből meg is van.