Gyökvonásos példák jönnek.

848.

$$\sqrt{\frac{x^2-16}{|x|-1}} \geq 0.$$

$$\frac{x^2-16}{|x|-1} \geq 0 \text{ lesz igaz.}$$

I.

$x^2 - 16 \geq 0$

$|x| - 1 > 0.$

$x^2 \geq 16 \rightarrow \underline{x \geq 4}$ vagy $\underline{x \leq -4}$

$|x| > 0 + 1$

$|x| > 1$

$\underline{x > 1}$ $\quad$ $\underline{x < -1}$ $\Rightarrow x \in [4, \infty) \cup (-\infty, -4]$

II.

$x^2 - 16 < 0 \quad \Rightarrow -4 < x < 4$

$|x| - 1 < 0 \quad -1 < x < 1$

$x \in (-1, 1)$

Megoldás: $x \in (-\infty, -4] \cup (-1, 1) \cup [4, \infty).$

$$\sqrt{\frac{x^2-7x+6}{-x^2+11x-24}} \geq 0 \rightarrow \sqrt{\frac{(x-1)(x-6)}{-(x-3)(x-8)}} \geq 0$$

II. $(x-1)(x-6) \geq 0$ $\quad$ $(x-1)(x-6) < 0$

$-(x-3)(x-8) \geq 0$ $\quad$ $-(x-3)(x-8) < 0$

$x \leq 1$ vagy $x \geq 6.$ $\quad$ $1 < x < 6$

$x < 3$ vagy $x > 8$

és $3 < x < 8$

$\boxed{8 > x \geq 6}$ $\quad$ $\boxed{3 > x > 1}$

$x \in [1, 3) \cup [6, 8)$

882./A.

$$(2-\sqrt{3})\cdot\sqrt{2+\sqrt{3}} + (2+\sqrt{3})\cdot\sqrt{2-\sqrt{3}} =$$

$$\sqrt{(2-\sqrt{3})^2\cdot(2+\sqrt{3})} + \sqrt{(2+\sqrt{3})^2\cdot(2-\sqrt{3})} =$$

$$= \sqrt{(4-3)(2-\sqrt{3})} + \sqrt{(2+\sqrt{3})(4-3)} = \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} =$$

= ésennyi.

882./B.

$$\left(\frac{\sqrt{p+q}-\sqrt{p-q}}{\sqrt{p+q}+\sqrt{p-q}} - \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}}\right) : \frac{4}{\underbrace{q\sqrt{p^2-q^2}}_{A}} =$$

$$= \frac{(\sqrt{p+q}-\sqrt{p-q})^2 - (\sqrt{p+q}+\sqrt{p-q})^2}{(\sqrt{p+q}+\sqrt{p-q})(\sqrt{p+q}-\sqrt{p-q})} : \frac{4}{q\sqrt{p^2-q^2}} =$$

$$= \frac{\left[(\cancel{\sqrt{p+q}}-\sqrt{p-q}) - (\cancel{\sqrt{p+q}}+\sqrt{p-q})\right]\left[(\sqrt{p+q}-\cancel{\sqrt{p-q}}) + (\sqrt{p+q}+\cancel{\sqrt{p-q}})\right]}{(\sqrt{p+q})^2-(\sqrt{p-q})^2} : A =$$

$$= \frac{-2\sqrt{p-q}\cdot 2\sqrt{p+q}}{\cancel{p}+q-\cancel{p}+q} : \frac{4}{q\sqrt{p^2-q^2}} = \frac{-\cancel{4}\sqrt{p^2-q^2}}{2\cancel{q}}\cdot\frac{\cancel{q}\sqrt{p^2-q^2}}{\cancel{4}} =$$

$$\cancel{= \frac{2\sqrt{p^2-q^2}}{2}} \; \cancel{= \sqrt{p^2-q^2}}$$

$$= \frac{-(p^2-q^2)}{2}$$

882./g.

$$\left(\frac{\sqrt{a^3}-\sqrt{b^3}}{\sqrt{a}-\sqrt{b}}+\sqrt{ab}\right):\left(\frac{a-b}{\sqrt{a}-\sqrt{b}}\right)^2=\frac{\sqrt{a^3}-\sqrt{b^3}+\sqrt{ab}\cdot(\sqrt{a}-\sqrt{b})}{\cancel{\sqrt{a}-\sqrt{b}}}\cdot\left(\frac{\cancel{\sqrt{a}-\sqrt{b}}}{(a-b)^2}\right)^3=$$

$$=\frac{(a\sqrt{a}-b\sqrt{b}+a\sqrt{b}-b\sqrt{a})(\sqrt{a}-\sqrt{b})}{(a-b)^2}=$$

$$=\frac{(a(\sqrt{a}+\sqrt{b})-b(\sqrt{a}+\sqrt{b}))\cdot(\sqrt{a}-\sqrt{b})}{(a-b)^2}=$$

$$=\frac{(\sqrt{a}+\sqrt{b})(\cancel{a-b})\cdot(\sqrt{a}-\sqrt{b})}{(a-b)^{\cancel{2}}}=\frac{a-b}{a-b}=\underline{\underline{1}}$$

898.

a) $\sqrt[k]{a^{k+1}}=a^{\frac{k+1}{k}}=a\cdot a^{\frac{1}{k}}=\underline{a\sqrt[k]{a}}$

b) $\sqrt[n]{a^{n+2}\cdot b^{n+1}}=\sqrt[n]{a^{n+2}}\cdot\sqrt[n]{b^{n+1}}=\sqrt[n]{a^n\cdot a^2}\cdot\sqrt[n]{b^n\cdot b}=$

$=\underline{\underline{ab\sqrt[n]{a^2b}}}$

907. $\sqrt[4]{\frac{a^6b^5}{c^9}}:\sqrt[4]{\frac{a^2b}{c^5}}=\sqrt[4]{\frac{a^{\cancel{6}\,4}b^{\cancel{5}\,4}\cancel{c^5}}{c^{\cancel{9}\,4}\cancel{a^2}\cancel{b}}}=\sqrt[4]{\frac{a^4b^4}{c^4}}=\underline{\underline{\frac{ab}{c}}}$

909. $\left(\sqrt[3]{p^2}-\sqrt[3]{pq}+\sqrt[3]{q^2}\right)\cdot\left(\sqrt[3]{p}+\sqrt[3]{q}\right)=$

$\sqrt[3]{p^3}-\cancel{\sqrt[3]{p^2q}}+\cancel{\sqrt[3]{pq^2}}+\cancel{\sqrt[3]{p^2q}}-\cancel{\sqrt[3]{pq^2}}+\sqrt[3]{q^3}=$

$=\sqrt[3]{p^3}+\sqrt[3]{q^3}=\underline{p+q}$

Ennyi ıtze.

913

$$\sqrt[3]{\frac{x}{y}\sqrt{\frac{y}{x}\sqrt[4]{\frac{x^3}{y^2}}}} = \left(\frac{x}{y}\cdot\left(\frac{y}{x}\cdot\left(\frac{x^3}{y^2}\right)^{1/4}\right)^{1/2}\right)^{1/3} =$$

$$= \left(\frac{x}{y}\right)^{1/3}\cdot\left(\frac{y}{x}\right)^{1/6}\cdot\left(\frac{x^3}{y^2}\right)^{1/24} = x^{1/3-1/6+3/24}\cdot y^{-1/3+1/6-2/24} =$$

$$= x^{\frac{8-4+3}{24}}\cdot y^{\frac{-8+4-2}{24}} = x^{\frac{7}{24}}\cdot y^{-\frac{6}{24}} = x^{\frac{7}{24}}\cdot y^{-\frac{1}{4}} = \sqrt[24]{x^7}\cdot\frac{1}{\sqrt[4]{y}}$$

Logaritmus:

$$\cancel{4^{\log_{16}9}} = \cancel{4^{\frac{\log_4 16}{\log_4 9}}} = \cancel{4^{\log_4 16}\cdot 4^{\log_4 9}} =$$

947/A.

$$100^{\lg 2+\lg 3} = 100^{\lg 6} = 10^{2\cdot\lg 6} = 10^{\lg 36} = \underline{\underline{36}}$$

b) $$4^{2+\log_{16}9} + 36^{1-\log_6 2} + 10^{-\lg 2} =$$

$$= 4^2\cdot 4^{\log_{4^2}9} + 6^{2(1-\log_6 2)} + 2^{-1} =$$

$$= \underbrace{16\cdot\underbrace{4^{\frac{\log_4 9}{\log_4 16}}}_{\frac{9}{16}}}_{\underline{\underline{9}}}$$

$$4^{\frac{\log_4 9}{\log_4 16}} = \left(4^{\log_4 9}\right)^{\frac{1}{\log_4 9}} = 9^{\frac{1}{\log_4 9}} = 9^{-\log_4 9} =$$

$$= \left(9^{\log_4 9}\right)^{-1} = \underline{\underline{3}}$$

Kissé itt eltévedtünk.

957

$$X = \frac{2a^3}{3b^2} \quad / \lg$$

$$\lg X = \lg\left(\frac{2a^3}{3b^2}\right) = \lg(2a^3) - \lg(3b^2) =$$

$$= \lg(2) + \lg(a^3) - \lg(3) - \lg(b^2) =$$

$$= \lg(2) \cdot 3\lg(a) - \lg(3) - 2\lg(b)$$

$$\lg \frac{p^2\sqrt[3]{q}}{q^3(p+q)} = \lg(p^2\sqrt[3]{q}) - \lg(q^3(p+q)) =$$

$$= \lg(p^2) + \lg(\sqrt[3]{q}) - \lg(q^3) - \lg(p+q) =$$

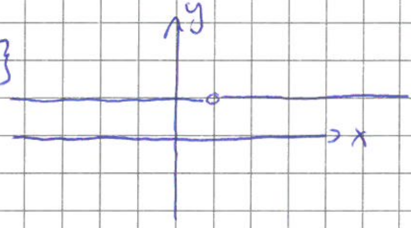
$$= 2\lg(p) + \frac{1}{3}\lg(q) - 3\lg(q) - \lg(p+q).$$

$\leftarrow y = ax^0$

Nullad és elsőfokú függvények

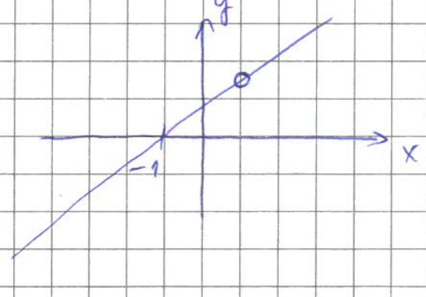
$\boxed{625.}$

a) $a(x) = \frac{x-1}{x-1} \Rightarrow D_f = \mathbb{R}\setminus\{1\}$



$b(x) = \frac{x^2-1}{x-1} \rightarrow D_f = \mathbb{R}\setminus\{1\}$

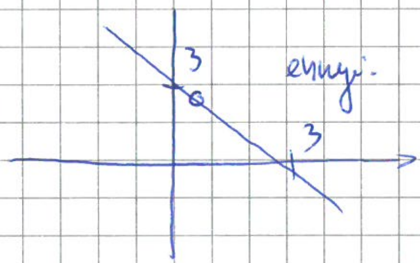
$$\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = \underline{\underline{x+1}}$$



d) $d(x) = \dfrac{-2x^2+7x-3}{2x-1} = \dfrac{-2(x-3)(x-0{,}5)}{2x-1} = \dfrac{-(x-3)\cancel{(2x-1)}}{\cancel{2x-1}} =$

$D_d = \mathbb{R} \setminus \{0{,}5\}$

$= \dfrac{-(x-3)}{(3-x)}$



f) $$f(x) = \frac{2x-3}{(x-2)^2-(2-x)^2} = \frac{\cancel{2x-3}}{\cancel{(x-2)(2-x)+}}$$

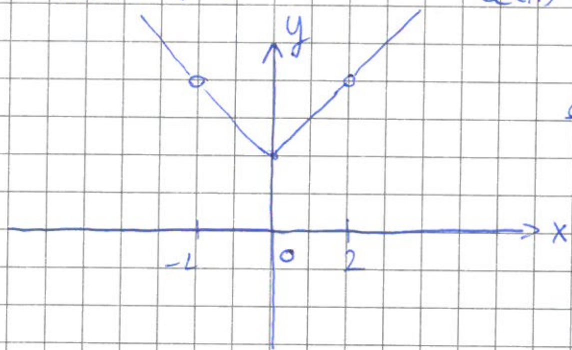
$$= \frac{2x-3}{\underbrace{(x-2+2-x)}_{\emptyset}(x-2-2+x)}$$

=> innentől kezdve kár vele foglalkozni, értelmetlen.

649.

a) $$a(x) = \frac{x^2-4}{|x|-2} = \frac{(x-2)(x+2)}{|x|-2}$$

$x \geq 0$ | $x < 0$
$x \neq 2$ | $x \neq -2$
$a(x) = x+2$ | $a(x) = -x+2$



ennyi.

b) $$b(x) = \frac{x^2+2|x|+1}{|x|+1}, \quad x \in \mathbb{R}$$

1) $x \geq 0$

$$b(x) = \frac{x^2+2x+1}{x+1} = \frac{(x+1)^2}{x+1} = \underline{\underline{x+1}}$$

2) $x < 0$

$$b(x) = \frac{x^2-2x+1}{-x+1} = \frac{(x-1)^2}{-\cancel{(x-1)}} = \underline{\underline{-x+1}}$$

innentől ugyanaz. Ábrázolható.

Más megold: $b(x) = \frac{(|x|+1)^{\cancel{2}}}{\cancel{|x|+1}} = \underline{\underline{|x|+1}}$ Ez is ok.

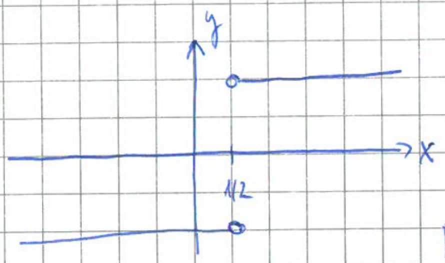
↑ $|x|$ eltolva.

c) $c(x) = \frac{\sqrt{4x^2-4x+1}}{2x-1}$, $x \in [-3,3]$.

$c(x) = \frac{\sqrt{(2x-1)^2}}{2x-1} = \frac{|2x-1|}{2x-1}$

↗ 1 ha $x > \frac{1}{2}$

↘ -1 ha $x < \frac{1}{2}$



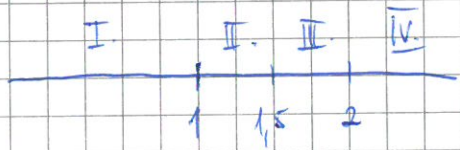
ennyi.

Hasonló, mint sgn(x).

650/6.

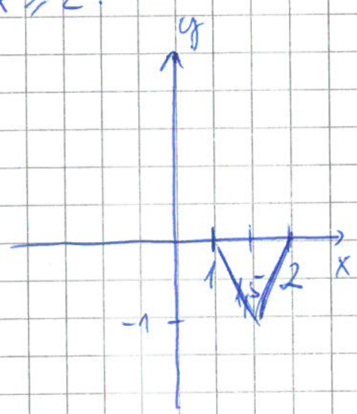
$x = \frac{3}{2} = 1{,}5$ $x=1$ $x=2$

b) $b(x) = |2x-3| - |x-1| - |x-2|$



1) $x < 1$ 2) $1 \le x < 1{,}5$ 3) $1{,}5 \le x < 2$

4) $x \geq 2$.



1) $b(x) = -\cancel{2x}+3 \cancel{+x}-1 \cancel{+x}-2 = \underline{\underline{0}}$

2) $b(x) = -2x\cancel{+3} \cancel{-x}\cancel{+1} \cancel{+x}\cancel{-2} = \underline{\underline{-2x+2}}$

3) $b(x) = 2x\cancel{-3} \cancel{-x}\cancel{+1} \cancel{+x}\cancel{-2} = \underline{\underline{2x-4}}$

4) $b(x) = \cancel{2x}\cancel{-3} \cancel{-x}\cancel{+1} \cancel{-x} \cancel{+2} = \underline{\underline{0}}$

Egyenletek.

1007. $x \neq \frac{2}{3}$

$$\frac{12x-9}{3x-2} + \frac{6x-5}{2-3x} = 2x-2 \quad / \cdot (3x-2)$$

$$12x - 9 - 6x + 5 = (2x-2)(3x-2)$$

$$6x - 4 = 6x^2 - 4x - 6x + 4$$

$$6x - 4 = 6x^2 - 10x + 4$$

$$6x^2 - 16x + 8 = 0.$$

$$3x^2 - 8x + 4 = 0.$$

$$x_{1,2} = \frac{8 \pm \sqrt{64 - 4 \cdot 3 \cdot 4}}{6} \nearrow \quad x_1 = 2 \quad \cancel{x_2 = \frac{2}{3}}$$

Csak 1 megoldás van. $x = 2.$

k.) $$\frac{x+2}{x+6} : \left(\frac{1}{2} + \frac{1}{x}\right) = \frac{2x}{3}$$ $\boxed{\begin{matrix} x \neq -6 \\ x \neq 0 \end{matrix}}$

$x \neq -2$

$$\frac{x+2}{x+6} : \left(\frac{x+2}{2x}\right) = \frac{2x}{3}$$

$$\frac{\cancel{x+2}}{x+6} \cdot \frac{2x}{\cancel{x+2}} = \frac{2x}{3}$$

$$\frac{2x}{x+6} = \frac{2x}{3} \quad / : 2x$$

$$\frac{1}{x+6} = \frac{1}{3}$$

$$x+6 = 3 \quad \boxed{x = -3}$$

1008/f.

$$(x-1)^3+2=x^3+3x-2$$

$$\cancel{x^3}-3x^2+\cancel{3x}-\cancel{1}+\cancel{2}=\cancel{x^3}\cancel{+3x}\cancel{-2}$$

$$-3x^2+3=0 \quad /:-3$$

$$x^2-1=0$$

$$x^2=1 \quad \nearrow \underline{\underline{x_1=1}} \quad \searrow \underline{\underline{x_2=-1}}$$

659.

$$f(x)=x^2+6x+c$$

a) $x\ 2$-vel

b) $x\,(-3)$-al

c) $f(x)\pm c$, $c>0$

d) $f^2(x)$

e) $\frac{1}{f(x)}$

f) $|f(x)|$

a) A szélsőértékhely nem változik, csak a szélsőérték értéke.

Ezeken el lehet gondolkodni.

672.

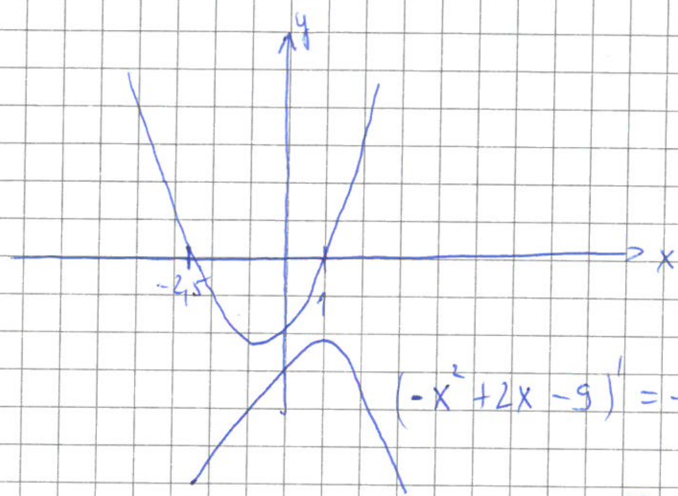
$$\left.\begin{aligned} f(x) &= 2x^2+3x-5 \\ g(x) &= -x^2+2x-9 \end{aligned}\right\}$$

$f(x)\le g(x)$ a megoldandó.

$$2x^2+3x-5\le -x^2+2x-9$$

$$\underbrace{3x^2+x+4}_{\text{valós számokon nincs megoldása}}\le 0$$

Ábrával:



$$(-x^2+2x-9)' = -2x+2$$

$$-2x+2=0$$

$$-2x=-2$$

$$\underline{\underline{x=1}}$$

itt lesz szélsőérték.

6 85.

a) $a(x) = x^2 - 2|x| - 8$

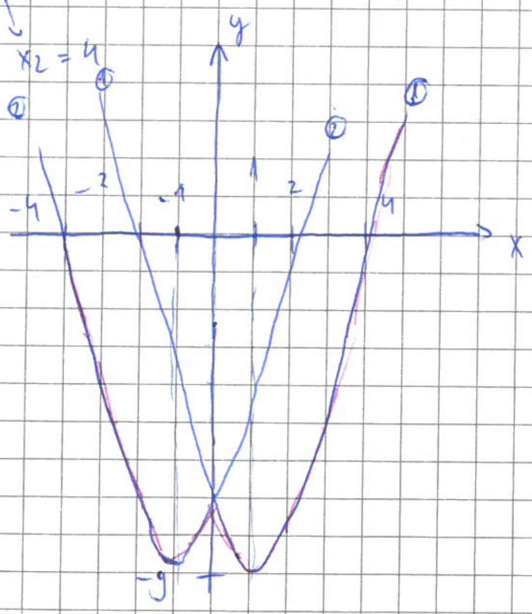
$x \geq 0$: x^2-2x-8

$x_1 = -2$ $\quad x_2 = 4$

$x < 0$: x^2+2x-8

$x_1 = 2$

$x_2 = -4$



c) $c(x) = \left|x^2 - 2|x| - 8\right|$ ezt alapján megoldható

x-re tükrözéssel, ahol f(x) negatív.

1245/b.

$(x^2-2x-3)(x^2-2x+1)=45$

$(t-3)(t+1)=45$ $\boxed{t=x^2-2x}$

$t^2-2t-3=45$

$t^2-2t-48=0$

$t_1=\underline{\underline{8}}$ $t_2=\underline{\underline{-6}}$

$\downarrow$ $\downarrow$

$8=x^2-2x$ $x^2-2x+6=\emptyset$

$x^2-2x-8=0$ $x_3=$ $x_4=$

nincs valós.

$x_1=4$ $x_2=-2$

1277.

$ax^2+bx+c=0$ $(a\neq 0)$

u és v a gyökök.

$$\left.\begin{aligned} u+v&=-\frac{b}{a}\\ u\cdot v&=\frac{c}{a}\end{aligned}\right\}\text{ Ezt tudjuk.}$$

a) $(u-v)^2=?$

$u=-\frac{b}{a}-v$

$\left(-\frac{b}{a}-v\right)\cdot v=\frac{c}{a}$

$-v^2+\frac{b}{a}v-\frac{c}{a}=0$

Nem vezet eredményre.

<u>De kifejezhető.</u>