Név: Unger Tamás István
Neptun: FTD1YJ
Web: http://maxwell.sze.hu/~ungert

Kalkulus I., negyedik házi feladat

Függvények határozatlan integráljának meghatároz(gat)ása

1.) $$\int \left(x^2-3x+2\right)\ dx = \int x^2\ dx - 3\int x\ dx + 2\int\ dx = \frac{x^3}{3}-\frac{3x^2}{2}+2x+\mathrm{C}.$$

2.) $$\int \left(\sqrt{y}-\frac{1}{\sqrt[3]{y}}+\frac{1}{y}\right)\ dy = \int y^{\frac{1}{2}}\ dy - \int y^{-\frac{1}{3}}\ dy + \int y^{-1}\ dy =$$
$$= \frac{y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{y^{\frac{2}{3}}}{\frac{2}{3}} + \ln|y| + \mathrm{C} = \frac{2}{3}\sqrt{y^3}-\frac{3}{2}\sqrt[3]{y^2}+\ln|y|+\mathrm{C}.$$

3.) $$\int \frac{1-z+z^5}{z^2}\ dz = \int \frac{1}{z^2}\ dz - \int \frac{1}{z}\ dz + \int z^3\ dz = \int z^{-2}\ dz - \int z^{-1}\ dz +$$
$$+\int z^3\ dz = \frac{z^{-1}}{-1} - \ln|z| + \frac{z^4}{4} + \mathrm{C} = -\frac{1}{z} - \ln|z| + \frac{1}{4}z + \mathrm{C}.$$

4.) $$\int (2u+3)^3\ du = \int \left(4u^4+12u+9\right)(2u+3)\ du =$$
$$= \int \left(8u^3+12u^2+24u^2+36u+18u+27\right)\ du = \int \left(8u^3+36u^2+54u+27\right)\ du =$$
$$= \frac{8u^4}{4}+\frac{36u^3}{3}+\frac{54u^2}{2}+27u+\mathrm{C} = u\left(2u^3+12u^2+27u+27\right)+\mathrm{C}.$$

4.)* $$\int (2u+3)^3\ du = \frac{(2u+3)^4}{4\,(2u+3)'}+\mathrm{C} = \frac{(2u+3)^4}{4\cdot 2}+\mathrm{C} = \frac{(2u+3)^4}{8}+\mathrm{C}.$$

5.) $$\int \sqrt[3]{3-v}\ dv = \int (3-v)^{\frac{1}{3}}\ dv = \frac{(3-v)^{\frac{4}{3}}}{-\frac{4}{3}}+\mathrm{C} = -\frac{3}{4}\sqrt[3]{(3-v)^4}+\mathrm{C}.$$

6.) $$\int \frac{1}{t+2}\ dt = \frac{\ln|t+2|}{1}+\mathrm{C} = \ln|t+2|+\mathrm{C}.$$

7.) $$\int \frac{1}{2s-1}\ ds = \frac{\ln|2s-1|}{2}+\mathrm{C} = \frac{1}{2}\ln|2s-1|+C.$$

8.) $\int \frac{1-p}{2-p}\ dp = \frac{2-p-1}{2-p}\ dp = \int 1 - \frac{1}{2-p}\ dp = \int 1\ dp - \int \frac{1}{2-p}\ dp =$

$= p - \frac{\ln|2-p|}{-1} + \mathrm{C} = p + \ln|2-p| + \mathrm{C}.$

9.) $\int \frac{1}{1+4y^2}\ dy = \int \frac{1}{1+(2y)^2}\ dy = \frac{\arctan(2y)}{2} + \mathrm{C} = \frac{1}{2}\arctan(2y) + \mathrm{C}.$

10.) $\int \frac{1}{4+u^2}\ du = \frac{1}{4}\int \frac{1}{1+\frac{u^2}{4}}\ du = \frac{1}{4}\int \frac{1}{1+\left(\frac{u}{2}\right)^2}\ du = \frac{\frac{1}{4}\arctan\left(\frac{1}{2}u\right)}{\frac{1}{2}} + \mathrm{C}$

$= \frac{1}{2}\arctan\left(\frac{u}{2}\right) + \mathrm{C}.$

11.) $\int \frac{1}{z^2+2z+2}\ dz = \frac{1}{(z+1)^2+1}\ dz = \arctan(z+1) + \mathrm{C}.$

12.) $\int \frac{x-y}{y+1}\ dx = \int x \cdot \frac{1}{y+1}\ dx - \int \frac{y}{y+1}\ dx = \frac{1}{y+1}\int x\ dx - \frac{y}{y+1}\int\ dx =$

$= \frac{1}{y+1}\frac{x^2}{2} - \frac{xy}{y+1} + \mathrm{C}.$

13.) $\int \frac{x-y}{y+1}\ dy = \int x \cdot \frac{1}{y+1}\ dy - \int \frac{y}{y+1}\ dy = x\int \frac{1}{y+1}\ dy - \int \frac{y+1-1}{y+1}\ dy =$

$x\ln|y+1| - \int 1 - \frac{1}{y+1}\ dy = x\ln|y+1| - y + \ln|y+1| + \mathrm{C}.$

14.) $\int 2x\sin x^2\ dx = \int \sin u\ du = -\cos u + \mathrm{C} = -\cos x^2 + \mathrm{C}.$

$u = x^2$

$\frac{du}{dx} = 2x \to du = 2xdx$

15.) $\int x\sin x^2\ dx = \frac{1}{2}\int \sin u\ du = -\frac{1}{2}\cos u + \mathrm{C} = -\frac{1}{2}\cos x^2 + \mathrm{C}.$

$u = x^2$

$\frac{du}{dx} = 2x \to du = 2xdx$

16.) $\int \cos^3 y \sin y\ dy = -\int u^3\ du = -\frac{u^4}{4} + \mathrm{C} = -\frac{1}{4}\cos^2 y + \mathrm{C}.$

$u = \cos y$

$\frac{du}{dy} = -\sin y \to du = -dy\sin y$

17.) $\displaystyle\int ze^{-z^2}\ dz = -\frac{1}{2}\int e^p\ dp = -\frac{1}{2}e^p + \mathrm{C} = -\frac{1}{2}e^{-z^2} + \mathrm{C}.$

$p = -z^2$

$\displaystyle\frac{dp}{dz} = -2z \to dp = -2zdz$

18.) $\displaystyle\int t\sqrt{t^2-2}\ dt = \frac{1}{2}\int \sqrt{x}\ dx = \frac{1}{2}\int x^{\frac{1}{2}}\ dx = \frac{1}{2}\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \mathrm{C} = \frac{1}{3}\sqrt{(t^2+2)^3} + \mathrm{C}.$

$x = t^2 - 2$

$\displaystyle\frac{dx}{dt} = 2t \to dx = 2tdt$

19.) $\displaystyle\int \frac{\sin\omega}{\cos^2\omega}\ d\omega = -\int \frac{1}{\xi^2}\ d\xi = -\int \xi^{-2}\ d\xi = -\frac{\xi^{-1}}{-1} + \mathrm{C} = \frac{1}{\cos\omega} + \mathrm{C}.$

$\xi = \cos\omega$

$\displaystyle\frac{d\xi}{d\omega} = -\sin\omega \to d\xi = -\sin\omega d\omega$

20.) $\displaystyle\int \frac{1+z}{z^2+2z+1}\ dz = \frac{1}{2}\int \frac{1}{x}\ dx = \frac{1}{2}\ln|x| + \mathrm{C} = \frac{1}{2}\ln\left|(z+1)^2\right| + \mathrm{C} = \ln|z+1| + \mathrm{C}.$

$x = z^2 + 2z + 1$

$\displaystyle\frac{dx}{dx} = 2z + 2 \to dx = (2z+2)\,dz = 2\,(z+1)\,dz$

21.) $\displaystyle\int xe^{-x}\ dx = -xe^{-x} + \int 1\cdot e^{x}\ dx = -xe^{-x} - e^{-x} + \mathrm{C}.$

$f = x \quad g' = e^{-x}$

$\displaystyle f' = 1 \quad g = \int e^{-x}\ dx = -e^{-x} + \mathrm{C}$

22.) $\displaystyle\int \sqrt{s}\ln s\ ds = \ln s\cdot\frac{2}{3}\sqrt{s^3} - \int \frac{1}{s}\cdot\frac{2}{3}s^{\frac{3}{2}}\ ds = \frac{2\ln s\sqrt{s^3}}{3} - \frac{2}{3}\int s^{\frac{1}{2}}\ ds =$

$f = \ln s \quad g' = s^{\frac{1}{2}}$

$\displaystyle f' = \frac{1}{s} \quad g = \int s^{\frac{1}{2}}\ ds = \frac{s^{\frac{3}{2}}}{\frac{3}{2}} + \mathrm{C}$

$\displaystyle = \frac{2\ln s\sqrt{s^3}}{3} - \frac{2}{3}\frac{s^{\frac{3}{2}}}{\frac{3}{2}} + \mathrm{C} = \frac{2\ln s\sqrt{s^3}}{3} - \frac{4}{9}\sqrt{s^3} + \mathrm{C}.$

23.) $\displaystyle\int \ln\omega\ d\omega = \int 1\cdot\ln\omega\ d\omega = \ln\omega\cdot\omega - \int \frac{1}{\omega}\omega\ d\omega = \omega\ln\omega - \omega + \mathrm{C}.$

$f = \ln\omega \quad g' = 1$

$\displaystyle f' = \frac{1}{\omega} \quad g = \int\ d\omega = \omega + \mathrm{C}$

24.) $\int e^x \cos x \; dx = e^x \sin x - \int e^x \sin x \; dx = e^x \sin x - \left(-e^x \cos x + \int e^x \cos x \; dx\right).$

$f = e^x \quad g' = \cos x \qquad f^* = e^x \quad (g^*)' = \sin x$

$f' = e^x \quad g = \int \cos x \; dx = \sin x + \mathrm{C} \qquad (f^*)' = e^x \quad g^* = \int \sin x \; dx = -\cos x + \mathrm{C}$

Azt kaptuk, hogy $\int e^x \cos x \; dx = e^x (\sin x + \cos x) - \int e^x \cos x \; dx.$

$\rightarrow 2 \int e^x \cos x \; dx = e^x (\sin x + \cos x) \rightarrow \int e^x \cos x \; dx = \frac{e^x (\sin x + \cos x)}{2} + \mathrm{C}.$

25.) $\int \frac{v+1}{v^3+v} \; dv = \int \frac{v+1}{v(v^2+1)} \; dv.$

$$\frac{v}{v(v^2+1)} = \frac{\mathrm{A}}{v} + \frac{\mathrm{B}v + \mathrm{C}}{v^2+1} = \frac{\mathrm{A}v^2 + \mathrm{A} + \mathrm{B}v^2 + \mathrm{C}v}{v(v^2+1)} = \frac{(\mathrm{A}+\mathrm{B})v^2 + \mathrm{C}v + \mathrm{A}}{v(v^2+1)}.$$

$\mathrm{A} + \mathrm{B} = 0, \rightarrow \mathrm{B} = -1$

$\mathrm{C} = 1$

$\mathrm{A} = 1$

$\int \frac{v+1}{v^3+v} \; dv = \int \frac{1}{v} \; dv + \int \frac{1-v}{v^2+1} \; dv = \ln|v| + \int \frac{1-v}{v^2+1} \; dv.$

$\int \frac{1-v}{v^2+1} \; dv = -\frac{1}{2} \int \frac{-2+2v}{v^2+1} \; dv = \int \frac{1}{v^2+1} \; dv - \frac{1}{2} \int \frac{2v}{v^2+1} \; dv =$

$f = v^2 + 1, \; f' = 2v$

$= \arctan(v) - \frac{1}{2} \ln\left|v^2+1\right| + \mathrm{C}.$

Rakjuk össze: $\int \frac{v+1}{v^3+v} \; dv = \ln|v| + \arctan(v) - \frac{1}{2} \ln\left|v^2+1\right| + \mathrm{C}.$