Agilent Technologies

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Experiment No. 1.
Filters

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Filters, or the selective amplification or attenuation of frequencies, are the most common element in analog electronic circuits. Filters are not only used in audio systems for tone control (EECS 210). Actually, they are used in <u>all</u> systems. Every time you "tune" in a radio/TV station, you are using at least 3 filters (and in high quality systems, 5 filters). The word "tune" means that you "pass" certain frequencies (the radio/TV channel you want) and "attenuate" or "reject" other frequencies. It is very common to have a total rejection of 80 dB for the next FM or TV station which is only 200 KHz (FM) and 5 MHz (TV) away. We will talk about AM and FM modulation and radio receivers later in the lab manual.

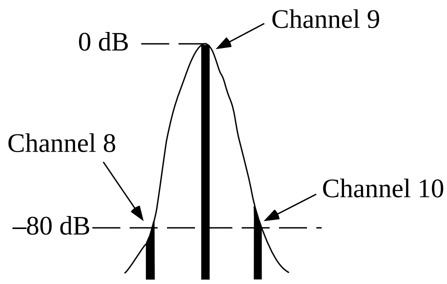


Figure 1: Combined effect of several filters in a TV receiver.

As mentioned in class, filters can be low-pass, high-pass, band-pass or band-reject (Fig. 2a). The tone control filter in Exp. #6 of EECS 210 were LP/Agilent filters depending on the boost/cut setting. Also, midband audio filters (or equalizers) are bandpass/band-reject filters depending on the boost/cut setting (Fig. 2b). Most filters used for radios/TV receivers are of the bandpass type; selecting one channel and rejecting others.

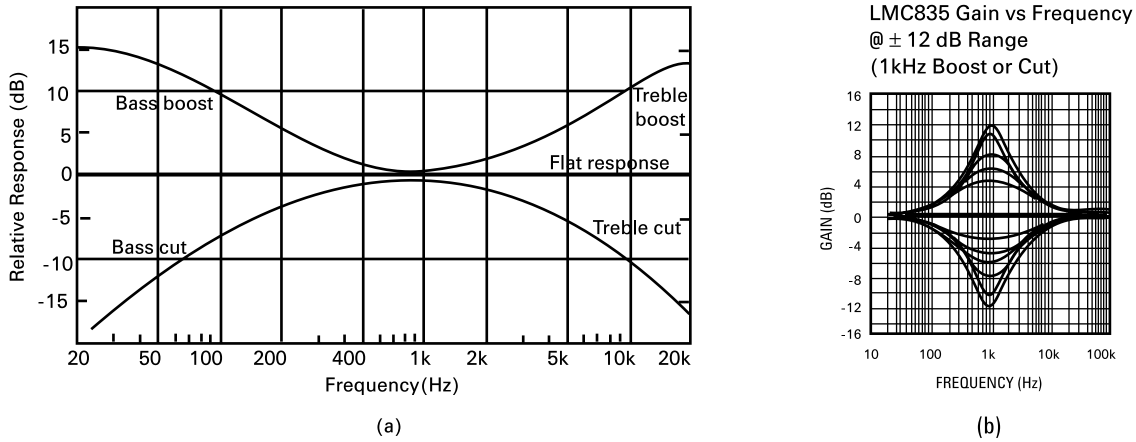


Figure 2: (a) Action of typical bass and treble controls. From *The Science of Sound, Second Edition*, Rossing, p. 444. (b) A midrange filter response. From *Application Specific Analog Products Databook, 1995 Edition*, p. 1-229.

Analog filter theory is quite complicated but very elegant. Actually, there are entire books and courses of this subject alone! Filters can be designed to give a maximally-flat response (Butterworth), a small ripple in the passband but sharper attenuation curve (Chebysher) or a very sharp attenuation curve with a small ripple in the passband and stop-band (elliptic).

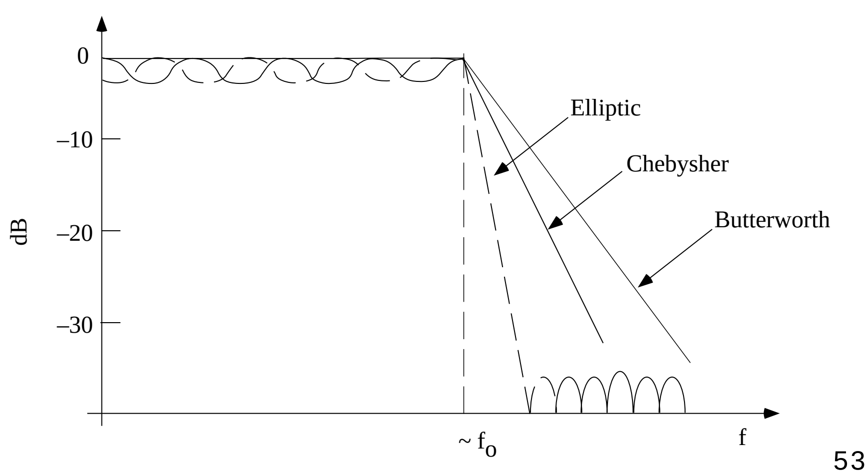


Figure 3: Low-Pass Filter Prototypes of Butterworth, Chebysher and Elliptic.

The filter response can be designed to have a specific bandwidth (from ~100% to 0.1%) and a certain attenuation curve (or roll-off). The roll-off is determined by the number of poles and zeros in filter function. In a Butterworth (maximally flat response) design, every pole results in (1/ω) roll-off which is equivalent to -20 dB/decade.

One-pole	$H(s) \propto \frac{1}{\omega}$	$\omega >> \omega_O$	–20 dB/dec or –6 dB/Oct
Two-pole	$H(s) \propto \left(\frac{1}{\omega}\right)^2$	$\omega >> \omega_O$	–40 dB/dec or –12 dB/Oct
N-pole	$H(s) \propto \left(\frac{1}{\omega}\right)^N$	$\omega >> \omega_O$	–20xN dB/dec or –6xN dB/Oct

In radio receivers (FM, TV, wireless telephones, …), it is very typical to result in an effective 10-pole (or higher) bandpass filter having a narrow bandwidth and an elliptic roll-off curve of 100^+ dB/Octave.

Tradeoffs: Filter design is always full of tradeoffs, as are most engineering problems. Typically, the steeper the cutoff slope, the greater the chance for ringing and overshoot. As we'll see in Experiment 1, we can adjust the "Q" of the filter to get faster risetime, but a higher Q also means more ringing and more variability with small changes in component values."

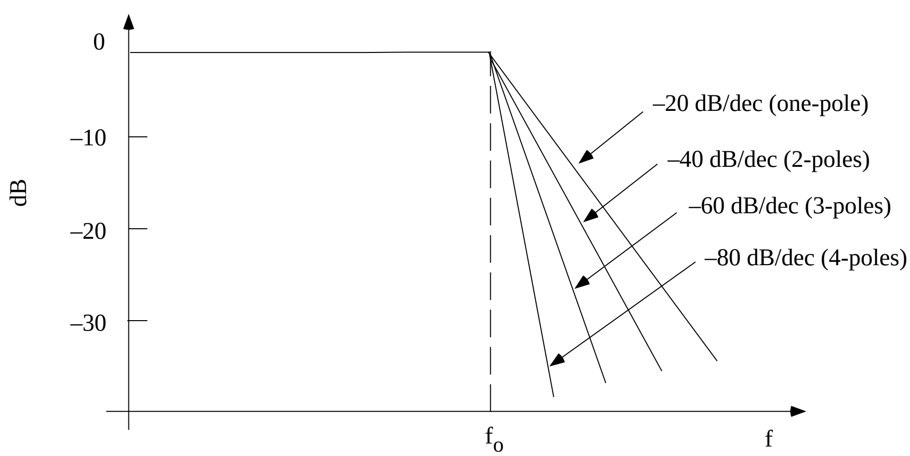
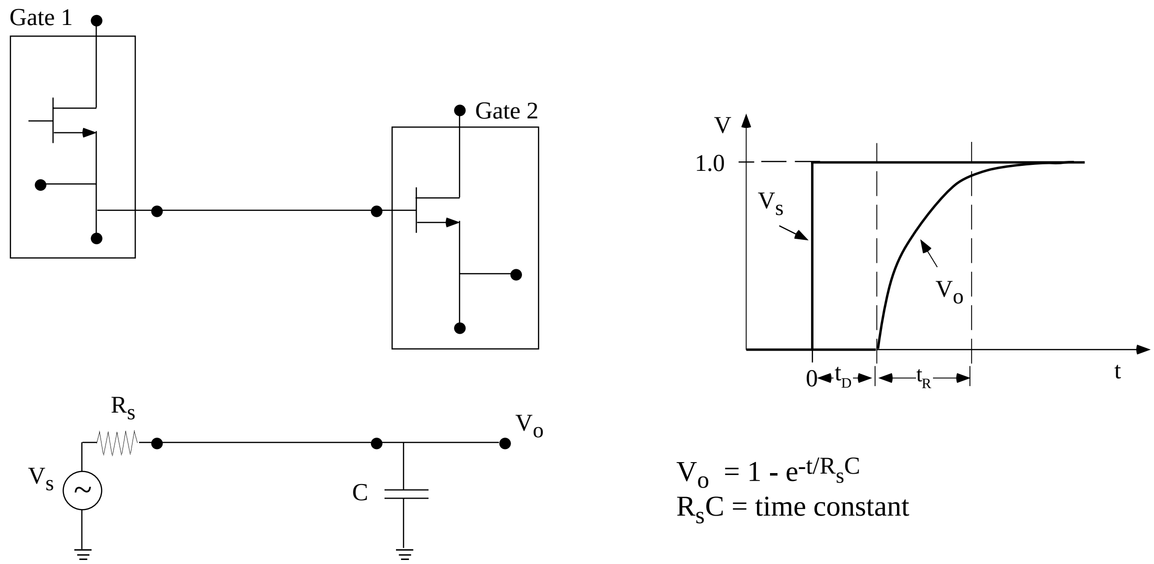


Figure 4: Filter response vs. number of poles.

First and Second-Order Circuits in Digital Systems:

The main interest in digital circuits is in the time domain response of pulses. As mentioned in class, the input of a digital circuit is modeled by a parallel RC circuit, and the output of a digital circuit is modeled by a series resistor. The digital connection between gates 1 and 2 is therefore:



$V_o = 1 - e^{-t/R_sC}$
R_sC = time constant

Figure 5: Risetime in digital circuits.

Two important things happen here: One is that there is a time delay (called propagation delay, t_D) between the two gates given by the speed of the pulse and the distance between the gates. The speed of the pulse is given approximately by $c/\sqrt{\varepsilon_r}$, where c is the speed of light ($3x10^8$ m/s) and ε_r is the relative dielectric constant of the material (EECS 230). In fiberglass digital boards with ε_r=4, the speed of the pulse is around c/2, which is $1.5x10^8$ m/s. Inside of a silicon chip (ε_r=11.9), the speed of the pulse is around $0.8x10^7$ m/s. This delay needs to be considered when designing very high speed circuits on large digital boards (clock frequency > 200 MHz), but is generally not the limiting factor in 100's MHz clocks.

Another more limiting factor is the risetime of the pulse due to the input capacitance of the digital gate (t_R). For a first-order circuit (Fig. 5), the risetime is given by $1/R_SC$ and is a simple exponential. We see that the gate capacitance of the input transistor (or gate) and the series resistance of the output transistor (or gate) are the most important factor in limiting the risetime! One way to solve this is to build transistors with very small gates (submicron dimensions) and large ($^W/_L$) ratios for low series resistances. The smaller dimensions result in smaller area taken by the transistor which reduces the risetime of the input pulse (for the same resistance), and therefore increases the highest operating speed. In the early 1980's, a 2 µm silicon process was the standard dimension in silicon technology. In 1997, the 0.35 µm silicon process is the norm and some companies have even a 0.16 µm silicon process resulting in GHz speed systems.

Sometimes in high-speed digital boards, the input pulse shows some ringing (Fig. 6). This is quite bad since it delays the settling time of the pulse and therefore slows down the digital circuit. This is the response of a second-order circuit and is due to a small inductance which is in series with the input gate capacitance. The inductance can be due to very thin lines on the digital board, or to not-so-good ground connections between the digital chips and the board. It is very hard to measure the value of this inductance but it can be accurately estimated from the oscillation frequency. Also, the Q of the circuit (and therefore the resistance) can be estimated from the level of the first peak. One should always try to maintain a Q of less than 1.5 to keep the overshoot below 25% of the pulse level (which is not that easy in very high speed circuits (f > 300 MHz)).

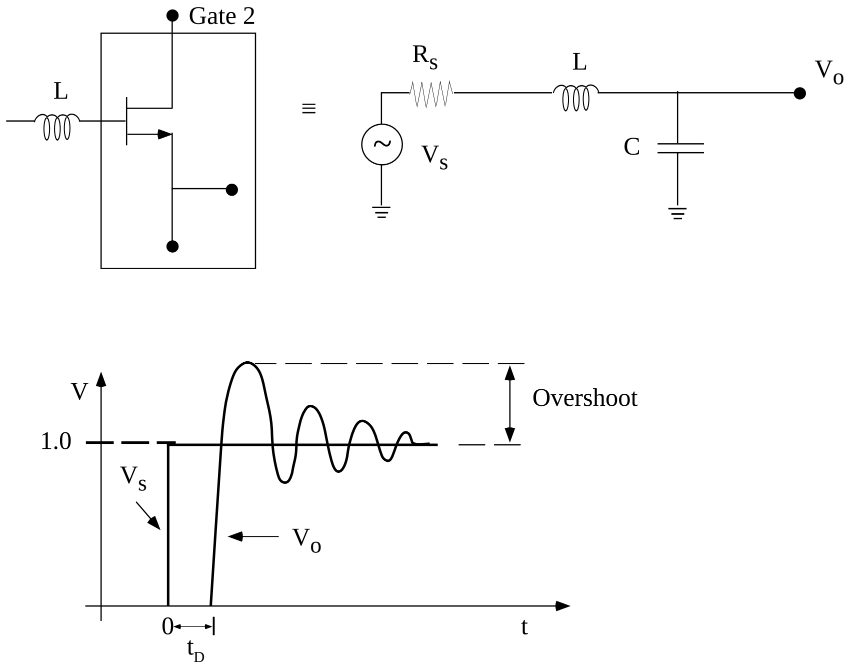


Figure 6: Ringing in digital circuits.

Low-Pass Filters

A general low-pass filter has a transfer function given by

$$H(s) = \frac{K\omega_o^2}{s^2 + (\omega_o / Q)s + \omega_0^2} \qquad s = j\omega \tag{1}$$

with a low-frequency ($\omega \to 0$) gain of K, a "resonant" frequency of ω_o, and a quality factor Q. The frequency response is shown in Figure 1. For Q > 0.5, ω_o is called the "resonant" frequency, while for Q < 0.5, ω_o is called the "mean" frequency, i.e., it is the mean frequency between two poles ω_1 and ω_2 ($\omega_o = \sqrt{\omega_1\omega_2}$, see Fig. 1). At $\omega = \omega_o, |H(\omega_o)| = KQ$ and $\angle H(\omega_o) = -90^\circ$. The roll-off for large ω is proportional to $1/\omega^2$ which is -40 dB/decade. It is easy to determine K, ω_o, and Q from the frequency domain measurements as shown in Figure 1.

Determining K, ω_O, and Q from Time-Domain Measurements:

If a step function, Au(t), is impressed on the low-pass filter, the output voltage (see Fig. 2) is (Q > 0.5):

$$V_o(t) = AK\left[1+\frac{1}{\sqrt{1-\frac{1}{4Q^2}}}e^{\frac{-\omega_o t}{2Q}}\sin\left(\sqrt{1-\frac{1}{4Q^2}}\,\omega_o t-\tan^{-1}\left(\sqrt{4Q^2-1}\right)\right)\right]u(t) \qquad (2)$$

Determining K: The value of K can be measured from the final value of the response and knowledge of A.

$$K = \frac{V_o(t\to\infty)}{A} = \frac{V_f}{A}$$

Determining ω_O: The oscillation frequency of the decaying sinusoid is

$$\omega_R = 2\pi f_R = 2\pi/t_R = \omega_o\sqrt{1-\frac{1}{4Q^2}}$$

For $Q>3, \omega_R \cong \omega_o$ with less than 1.5% error.

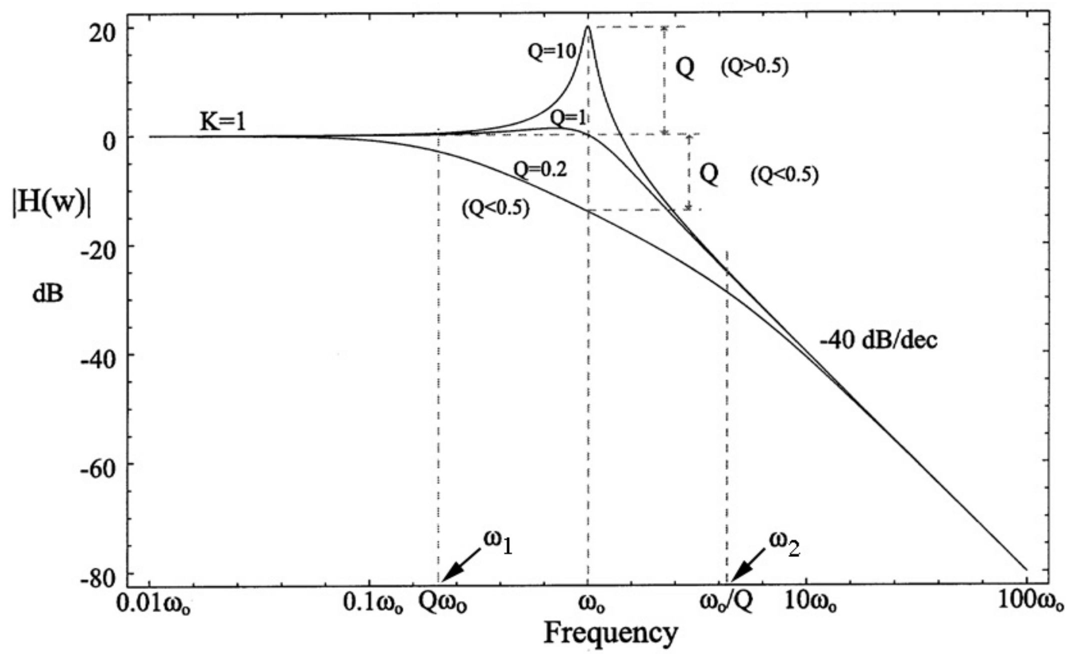


Figure 1a. Low-pass frequency response, magnitude vs. frequency. This is called a Bode-Plot.

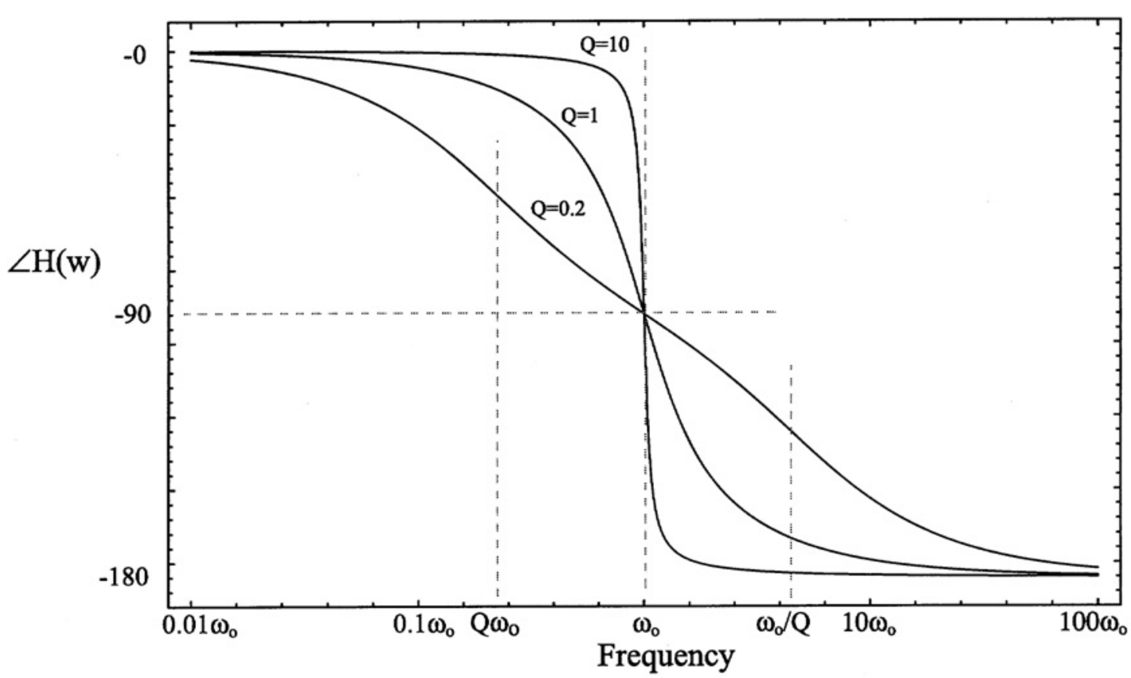


Figure 1b. Low-pass frequency response, phase vs. frequency.

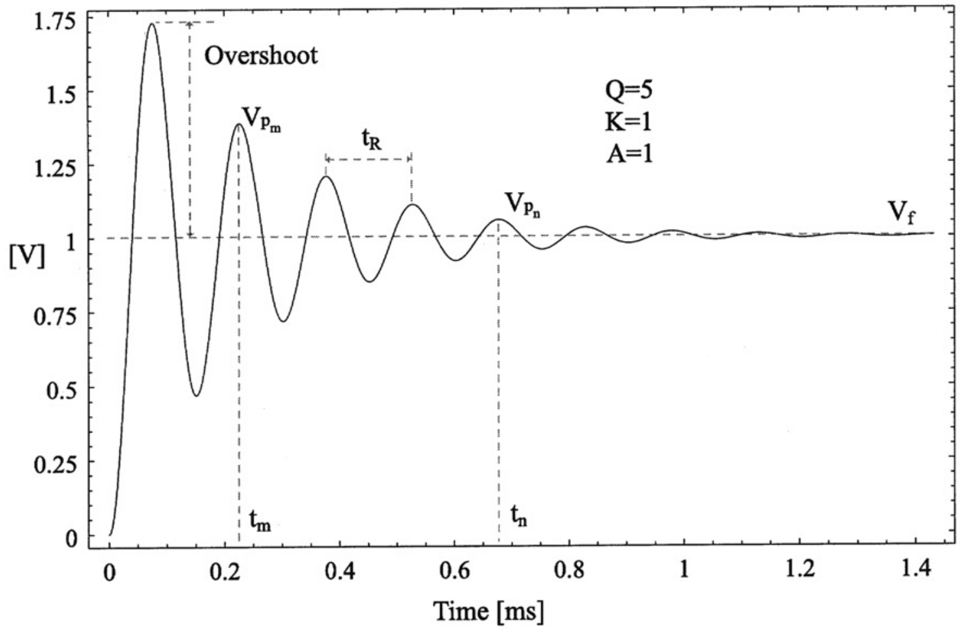


Figure 2. Time domain step response of a low-pass filter with K = 1, Q = 5 and A = 1.

Determining Q:

Method 1: The Overshoot Approach:

The first, and highest peak occurs at t = t_1 when $dV_o/dt = 0$. To find the peak overshoot value, first find t_1 and replace it in equation (2) above. When all is done, we find:

$$V_p = K\left(1+e^{\frac{-\pi}{\sqrt{4Q^2-1}}}\right)u(t)$$

The overshoot value is defined as $V_p - V_o(t \rightarrow \infty)$ and is:

$$Overshoot = V_p - V_f = AKe^{\frac{-\pi}{\sqrt{4Q^{2-1}}}}$$
$$= e^{\frac{-\pi}{\sqrt{4Q^2-1}}} \qquad for\ K=1,\ A=1.$$

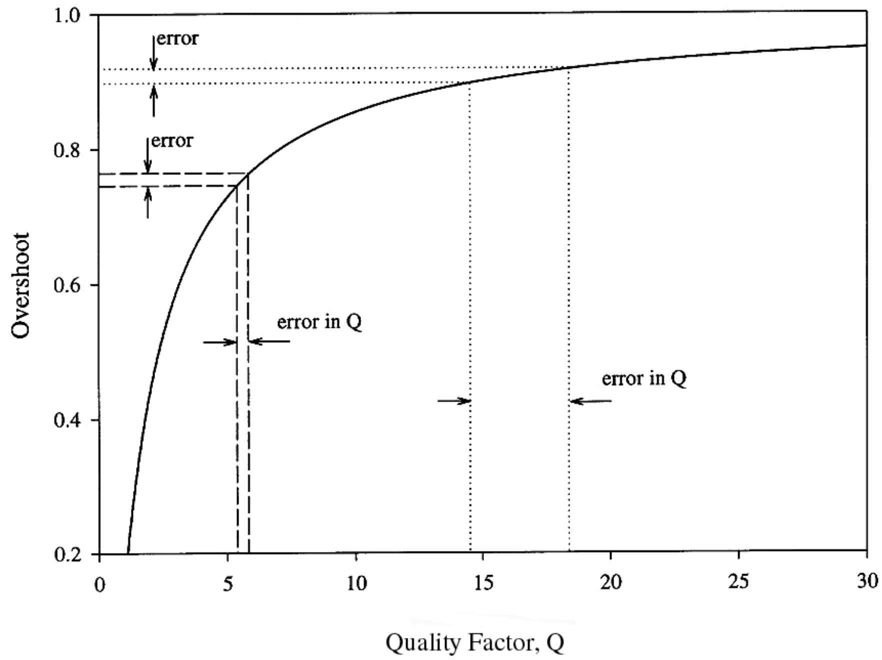


Figure 3. Overshoot vs. Q. Notice the error in Q for a small measurement error when Q > 8.

Figure 3 shows the value of the overshoot vs. Q for K = 1, A = 1. Notice that for Q > 5, any measurement error in the value of the overshoot voltage results in a large error in the determination of Q. For this reason, this method is accurate for 0.5 < Q < 5 and is risky for Q > 5.

There are also other reasons why this method should be used with caution in high Q systems. The op-amp could be *slew-rate* limited (the output voltage will not change fast enough to arrive to the first peak). This is especially true in high frequency filters (f_o > 500 kHz) where the op-amp bandwidth might limit the total voltage swing. Also, for small-signal op-amps, it could be in a current-limiting mode, and therefore will not be able to drive the necessary currents in the capacitors in the circuit. Since $i = C\, dV_o(t)/dt$, this will limit the slope of the voltage and result in reduced first peak. Again, this may occur for high frequency filters (dt is very small). Therefore, use this method with caution for high-*Q* filters and at high frequencies!

Method 2: The Decaying Peaks Approach:

A much better way to determine Q in high-Q filters is from the decay values of the sinusoidal waveform peaks. First, measure the values of several peaks and their corresponding times (V_{p1}, t_1; V_{p2}, t_2; ...). see Fig. 2. The peak values occur when the sinusoid in equation (2) is equal to 1, so it can be removed from the analysis. The Q can then be calculated from the peak values and times of two peaks (*m, n*) at times (t_m, t_n), where $0 < t_m < t_n$ (one of the peaks could very well be the first peak).

At t_m,
$$Overshoot(m) = V_{p_m} - V_f = AK\frac{e^{\frac{-\omega_o t_m}{2Q}}}{\sqrt{1-\frac{1}{4Q^2}}}$$

At tn,
$$Overshoot(n) = V_{p_n} - V_f = AK\frac{e^{\frac{-\omega_o t_n}{2Q}}}{\sqrt{1-\frac{1}{4Q^2}}}$$

Dividing;
$$\frac{V_{p_n} - V_f}{V_{p_m} - V_f} = \frac{e^{\frac{-\omega_o t_n}{2Q}}}{e^{\frac{-\omega_o t_m}{2Q}}}$$

and
$$Q = \frac{\frac{1}{2}\omega_o \Delta t}{\ell n\left(\frac{V_{p_m} - V_f}{V_{p_n} - V_f}\right)} \qquad \Delta t = t_n - t_m$$

The two peaks (m, n) should be chosen carefully. One one side, they should be spaced far apart to minimize errors, but one the other side, the peaks must be well defined above the final value. For example, in Fig. 3, the peaks after t = 0.9 ms should not be used since they are very small and not well defined. Therefore, this method is not very accurate for small Q filters (Q < 2) where all the peaks (after the first one) are poorly defined.

Revisiting ω_o for 0.5 < Q < 2:

In this case, the natural resonant frequency (ω_R) is different from the resonant frequency (ω_o). From equation (2), we have $\omega_R = \omega_o\sqrt{1-\frac{1}{4Q^2}}$. The best way to solve it is to first determine Q from the overshoot method with $\omega_R \cong \omega_o$, and then use Q to determine ω_o from the measurement of ω_R.

What if Q < 0.5?:

For Q < 0.5, the frequency response is similar to a single pole system with $\omega_1 \cong \omega_o Q$, (Fig. 1). The risetime of the step response can be used to determine ω_1 accurately (problem 3 in pre-lab). The final value of the output voltage is still AK and this can be used to determine K. We cannot really determine Q and ω_2 from the time domain measurements *alone*, but this is not important. The system performance is limited by ω_1, and we know how to find its value!

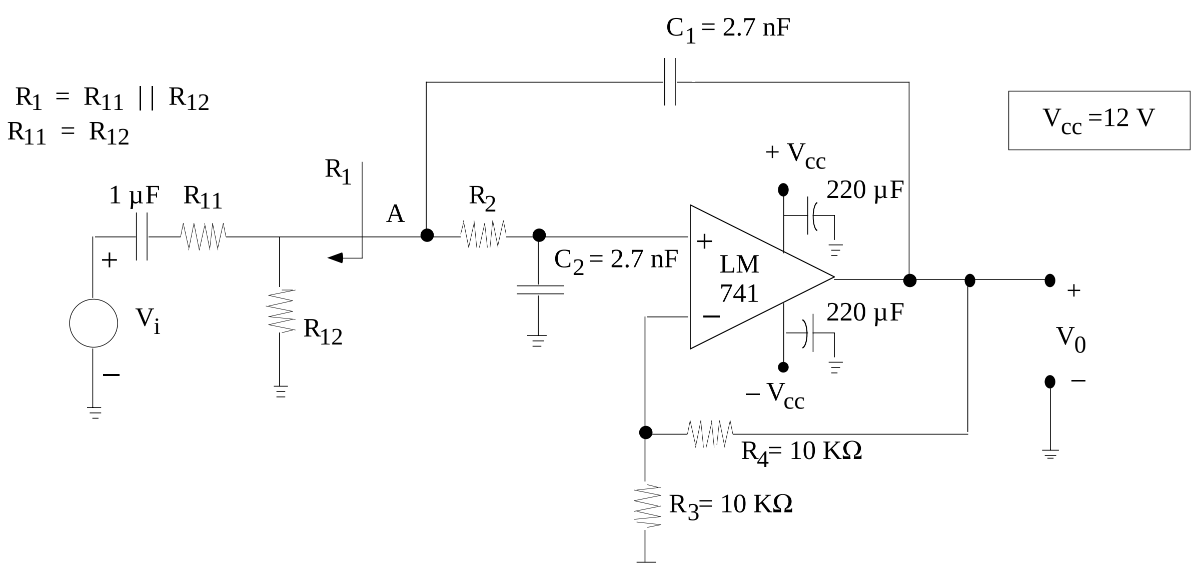
Experiment No. 1.
Low-Pass Filters; Step Response vs. Q

Goal: To determine the frequency response of an active second order low-pass filter circuit for underdamped (Q > 0.5) and overdamped (Q < 0.5) cases, and to see the effect of Q in time domain using the step-response.

Equipment:
- Agilent E3631A Triple output DC power supply
- Agilent 33120A Function Generator
- Agilent 34401A Multimeter
- Agilent 54645A Oscilloscope

1.0 Low-Pass Filter Implementation:

A Sallen-Key non-inverting low-pass filter is shown below:



The transfer function is given by:

$$\frac{V_o}{V_i} = \frac{\dfrac{1}{R_1 R_2 C_1 C_2}\left(1+\dfrac{R_4}{R_3}\right)\left(\dfrac{R_{12}}{R_{12}+R_{11}}\right)}{s^2 + \left(\dfrac{1}{R_2 C_1} + \dfrac{1}{R_1 C_1} - \dfrac{R_4}{R_2 R_3 C_2}\right)s + \dfrac{1}{R_1 R_2 C_1 C_2}} \qquad (s = j\omega)$$

where $\left(\dfrac{R_{12}}{R_{12}+R_{11}}\right)$ is the input voltage divider, and $\left(1+\dfrac{R_4}{R_3}\right)$ is the non-inverting op-amp low-frequency gain when C1 and C_2 are open-circuited. The 1 µF capacitor at the input is a DC-block capacitor and will act as a short circuit for f > 100 Hz (EECS 210).

For $C_1 = C_2 = C$, we have:

$$K = \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_{12}}{R_{11} + R_{12}}\right) \quad \textit{low frequency filter gain}$$

$$\omega_0 = \frac{1}{C\sqrt{R_1 R_2}} \qquad \textit{cut-off frequency (also called corner frequency)}$$

$$R_2 = \frac{1}{2Q\omega_0 C}\left(1 + \sqrt{1 + 4Q^2(K' - 2)}\right) \qquad K' = \left[1 + \frac{R_4}{R_3}\right] \qquad K' \geq 2$$

$$R_1 = \frac{1}{\omega_0^2 C^2 R_2}$$

and $Q = \dfrac{1}{\sqrt{\dfrac{R_2}{R_1}} + \sqrt{\dfrac{R_1}{R_2}} - \dfrac{R_4}{R_3}\sqrt{\dfrac{R_1}{R_2}}} = \dfrac{1}{\sqrt{\dfrac{R_2}{R_1}} + \sqrt{\dfrac{R_1}{R_2}}\left(1 - \dfrac{R_4}{R_3}\right)}$

For the case of $R_4 = R_3$ and $R_{11} = R_{12}$, we have $K' = 2$ and:

$$K = 1, \quad \omega_0 = \frac{1}{C\sqrt{R_1 R_2}} \quad \textit{and} \quad Q = \sqrt{\frac{R_1}{R_2}}$$

In reality 0.95 < K < 1.5 due to resistance values inaccuracies (±5%).

Low-Q Active Filter:

1. Assemble the circuit as shown above with:

   $R_{11} = R_{12} = 2.4$ KΩ and $R_2 = 33$ KΩ

   $C_1 = C_2 = C = 2.7$ nF

   This is the low-Q case with Q ~ 0.19, f_o ~ 9.3 KHz. (Again, f_o can change by ±10% due to capacitor values inaccuracies of ±10%).

   As indicated by Problem 5 in the Pre-Lab, the transfer function has effectively 2-poles, one at $f_1 \simeq Q f_o$, and one at $f_2 \simeq f_o/Q$.

   ❑ Draw the filter circuit in your notebook.

2. ❑ Measure the op-amp DC voltages, V(-), V(+) and V_o and make sure that they are all in the mV levels. Write them in your notebook.

Frequency Response:

3. Connect V_o to scope channel 1 and V_i to channel 2.

   a. ❑ Set the function generator to give a sinusoidal voltage of Vppk = 1 V and measure the frequency response (V_o/V_i) from 50 Hz to 200 KHz (amplitude only). For the low-Q case, you should do it at 100, 200, 500, 1000 Hz, etc. Above 20 KHz, the output voltage will be very low and you should increase the input voltage to 10 V ppk in order to measure accurately V_o.

   ❑ b. Measure the phase delay between the input and output waveform in the region around 8-10 KHz and determine exactly the frequency where it is -90˚. This is ω_o ($= 2\pi f_o$)!

   ❑ c. Measure V_o/V_i at this frequency ($|H(\omega_o)| = K Q \simeq Q$ for $K \simeq 1$). This is your Q!

   (Remember, that under the Measure [Time] menu, you will find a softkey at the bottom of the screen which measures the phase delay between Channel 1 and Channel 2.)

Step Response:

4. ❑ Set the function generator to give a square wave voltage of f = 500 Hz and Vppk = 1 V. Measure the risetime of the output waveform (10% - 90% of peak value). Draw the waveform on your lab notebook.

   (Remember that under the Measure [Time] menu, you can find a Risetime softkey at the bottom of the screen.)

5. ❑ The risetime is dominated by the first pole at $f_1 = 1/2\pi C_2 R_2 \sim 1.8 KHz$. Therefore C_1 has virtually no effect on this pole. Remove C_1 from the circuit and measure the risetime again. What do you notice?

6. ❑ Now, put C_1 back into the circuit. Change the input square-wave frequency to 4 KHz and measure $V_o$ppk. Plot V_i and V_o and label the ppk voltages. Why does V_o look like a triangular wave?

High-Q Active Filter:

1. Change the values of R_{11} and R_{12}, and R_2 to be:

   $R_{11} = R_{12} = 120\ K\Omega$ and $R_2 = 1\ K\Omega$

   Keep $C_1 = C_2 = C = 2.7$ nF.

   This is the high Q case with $Q_{(ideal)} = \sqrt{R_1/R_2} = 7.7$. However, since C_1 is not identical to C_2 and also there are some parasitic capacitances in the circuit, you will find a Q between 3 and 12. The resonant frequency is:

   $f_o = \frac{1}{2\pi C\sqrt{R_1 R_2}} \sim 7.6\ KHz$ but could change ($\pm$10%) due to capacitor values inaccuracies.

   The low frequency gain, K, is still equal to 1 (0.95 < K < 1.05 due to resistance inaccuracies).

   ❑ Draw the circuit in your lab notebook.

Frequency Response:

2. a. ❑ Set the function generator to a sinewave with V_{ippk} = 1 V. Measure the frequency response from 50 Hz to 200 KHz.

   Be careful, the frequency response changes very quickly around f_o! First determine f_o $\left(\max V_o/V_i\right)$ by a quick frequency scan (using the knob), and then measure the frequency response taking several more points around f_o. (For example; the -3 dB, -6 dB, -10 dB points of H(ω) max).

   b. ❑ Measure the phase of V_o/V_i around f_o and determine the frequency where the phase delay is -90˚. This frequency will be very close to where |H (ω)| is maximum and is the exact ω_o.

   c. ❑ Measure V_o/V_i at this frequency. ($|H(\omega_o)| = K\,Q \simeq Q$ for $K \simeq 1$). This is your Q!

3. Set the function generator for a square wave of frequency f_o (whatever you have measured, ~ 7.6 KHz) and Vppk = 1 V.

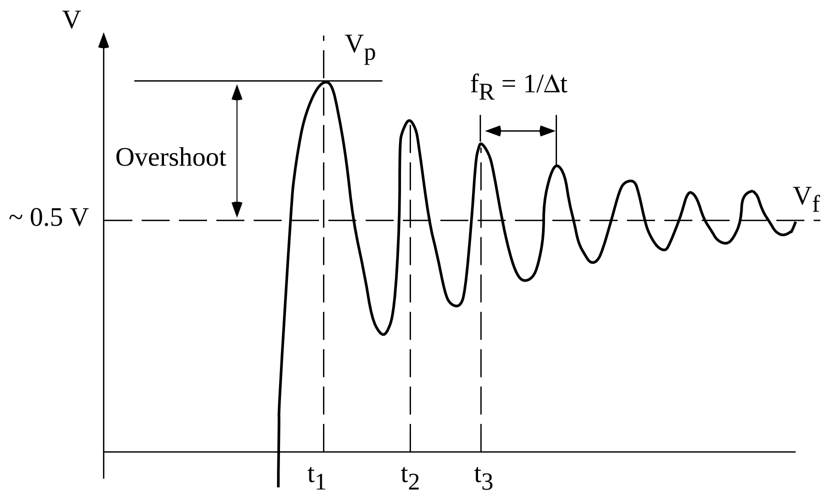
   a. ❑ Measure the output voltage. Plot V_i and V_o and label the ppk voltages. What is the shape of the output voltage waveform?

   b. ❑ Measure V_i and V_o in frequency domain in dB (f_o, $3f_o$, $5f_o$ and $7f_o$). Be careful, choose at least a 100 KHz frequency span so as not to get aliasing on the FFT.

Step Response:

4. ❑ Set the function generator to a 200 Hz square wave with Vppk = 1 V. Sketch the input and output voltage in your lab notebook for one period and label the output voltage when the ripple settles down, (V_f).

a. ❑ For the positive portion of the waveform, measure the exact time of the first peak (t_1) and its value (V_p). (You can do this using the Measure [Cursors] menu.) The overshoot value is (V_p - V_f).



b. ❑ From the waveform determine the ripple frequency, f_R. (Again, you can do this quickly under the Measure [Cursors] menu).

c. ❑ If you have a Q lower than 4-5, measure the values and times of the second, third and fourth peaks. You will need this to calculate Q in the post-lab report. If you have a high-Q (Q > 4-5), then measure the third, fifth and seventh peaks (value & time). This will result in a more accurate determination of Q.

Experiment No. 1.
Low-Pass Filters; Step Response vs. Q

Pre-Lab Assignment

1. Using the Golden Rules (Ideal OP-Amp):

   (Ignore the 1 µF DC-block capacitor. Assume it is a short-circuit at all frequencies.)

   a. In the low-pass filter, why is $R_1 = R_{11} \| R_{12}$? (Disconnect the circuit to the right of the arrow and determine the Thevenin's equivalent of the source circuit.)

   b. Draw the filter circuit at $\omega << \omega_o$ (caps. are open-circuit) and determine V_o/V_i.

   c. Draw the filter circuit at $\omega >> \omega_o$ and determine V_o/V_i.

   d. What is the input impedance of the circuit (seen by V_i) for $\omega << \omega_o$?

2. a. Check that the resistor/capacitor values given in the experiment result in the quoted f_o, Q, K for the low-Q and high-Q cases.

   b. In the low-pass filter, the circuit will <u>never</u> work if R_{12} is not present (and V_s is a pure <u>ac</u> source). Explain why (think of the non-ideal op-amp properties).

3. The frequency response of a first order filter is given by:

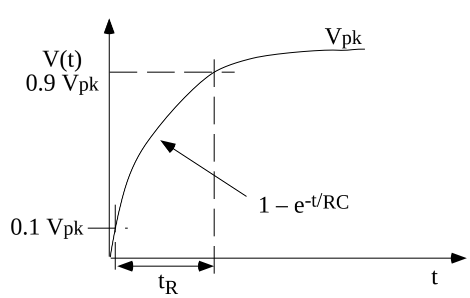
$$\frac{V_o}{V_i} = \frac{1}{\left(1+\frac{s}{\omega_1}\right)} \quad \text{where } \omega_1 = 1/RC \quad \text{and} \quad f_1 = \frac{1}{2\pi RC}$$

   The risetime of a function (t_R) is defined as the time from 0.1 Vpk to 0.9 Vpk.

   The time response to a step function of amplitude 1 is given by:

$$V_o(t) = 1 - e^{-t/RC} = 1 - e^{-t/\tau}$$

   where $\tau = RC \equiv$ time constant.



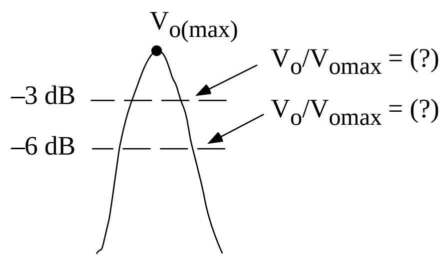
a. Determine t_R as a function of τ (the time constant).

b. Express t_R as a function of f_1.

(This expression is very useful, since once you know the risetime in seconds, you can quickly determine the corner frequency in Hz (and vice versa).)

4. Calculate $V_o/V_{o(max)}$ (for a constant V_i) when the filter response (transfer function) drops by: 3 dB (really –3 dB), 6 dB, 10 dB, 15 dB, 20 dB, 30 dB, 40 dB.

Graphically, this means



5. A low-pass filter transfer function is given by:

$$\frac{V_o}{V_s}(s) = H(s) = \frac{K\,\omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2} \qquad s = j\omega \qquad f_o = 10\,KHz$$

and $$|H(\omega)| = \frac{K\,\omega_o^2}{\sqrt{\left[\omega_0^2 - \omega^2\right]^2 + \left[(\omega o/Q)\,\omega\right]^2}}$$ and $K \equiv$ low frequency gain

a. Derive $|H(\omega)|$.

b. For K = 1 and using MATLAB, plot on the same graph (dB, log f) the filter response for Q = 0.2, Q = 1 and Q = 10. The horizontal scale should be from 0.01 f_o (100 Hz) to 100 f_o (1 MHz). The vertical scale should be from +20 dB to -60 dB. Reminder: dB = 20 log $|H(\omega)|$.

c. Derive the equation of the phase of $H(\omega)$ and plot on the same graph the phase for Q = 0.2, Q = 1 and Q = 10 over the above mentioned frequency range. What is the phase of $H(\omega)$ as $\omega \to 0$, $\omega \to \infty$ and $\omega = \omega_o$.

d. Prove that at ω_o, $|H(\omega)| = KQ$ and the phase of $H(\omega_o)$ is -90˚.

e. For the special case of Q << 1 (Q = 0.2), the transfer function can be very well approximated by two poles at $\omega_1 = Q\omega_o$ and $\omega_2 = \omega_o/Q$. For Q << 1, ω_1 dominates the response. For the case of Q = 0.2, plot

$$H_1(s) = \frac{1}{\left(1 + \frac{s}{\omega_o Q}\right)\left(1 + \frac{s}{\omega_o/Q}\right)}$$

and compare it with the function of part (b) with Q = 0.2 (on the same graph).

f. Prove mathematically that, for Q << 1, $H_1(s)$ is nearly equal to H(s). You can do this by expanding the denominator of $H_1(s)$, determining which factor you must ignore so that $H_1(s)$ = H(s), and proving that for Q << 1, this factor is insignificnat.

Experiment No. 1.
Low-Pass Filters; Step Response vs. Q

Lab-Report Assignment

1. a. Draw the filter circuit and neatly summarize all your measured data for the low-Q and high-Q cases (R values, V^+, V^-, V_o (DC), f_o, Q, f_1, t_R, overshoot, t_1, etc. ..., everything except the frequency response data).

   b. Using Matlab, plot the measured transfer function for the low-Q, high-Q filters on the same Bode-plot (f: 50 Hz–200 KHz, dB: +10 or +20 dB–depending on your measured high-Q- to -60 dB minimum). Label clearly the low frequency gain (K), the '"resonant" frequency (f_o) and the Q of the filter for the low-Q and high-Q cases.

2. Low-Q Filter:

   a. From the bode-plot, determine the low frequency pole, f_1, (-3 dB pt). Knowing ω_o (or f_o) from the phase measurement and that $f_1 \simeq Qf_o$, determine Q.

   b. Using the risetime (t_R) from the time domain measurements, determine f_1 using the pre-lab problem #3. Determine Q (knowing ω_o). Does it agree with part (a) above?

   c. Knowing the low frequency pole, f_1, and for a square-wave of V_i = 1 V_{ppk} and f = 4 KHz, calculate V_{oppk} and compare with measurements of Section 6 of the low-Q design.

3. High-Q Filter:

   a. From the time domain measurements, determine f_o from the ripple frequency. Compare with frequency domain data.

   b. From the time domain measurements, determine Q from the overshoot value. Compare with frequency domain data.

   c. From the time domain measurements, determine Q from the ripple peaks. In one case, use the first two measured peaks (and times), and in another case, use the first and last measured peaks (and times). Compare both Q's with the frequency domain data.

4. a. For a square-wave of V_{pk} = 0.5V (V_{ppk} = 1 V) and f = f_o (~ 7.6 KHz–use your values), calculate the fundamental, third, fifth and seventh harmonic levels of the square-wave (in V and dB) before and after it passes by the high-Q low-pass filter. Compare with frequency-domain measurements for V_i and V_o. (Use Fourier-Series to calculate the frequency components of the input square-wave).

   b. For V_o, what is the value (in V and dB) of the third and fifth harmonic level compared to the fundamental? Is this a "clean" sinewave at f_o? Calculate the THD (total harmonic distortion) of the output signal (THD is defined in your EECS 210 lab manual).

Some Problems with the Active Sallen-Key Low-Pass Filter:

1. Oscillations:

The denominator of the transfer function $H(s) = V_o/V_i$ is:

$$s^2 + \left(\frac{1}{R_2C_1} + \frac{1}{R_1C_1} - \frac{R_4}{R_2R_3C_2}\right)s + \frac{1}{R_1R_2C_1C_2}$$

When $\left(\frac{R_4}{R_2R_3C_2}\right) > \left(\frac{1}{R_2C_1} + \frac{1}{R_1C_1}\right)$, the denominator has the form:

$$s^2 - a_1 s + a_o = 0$$

The solutions of this quadratic equation will always result in at least one positive pole, and therefore the system response will "blow" up. Since this is an active circuit, the filter will oscillate and generate a somewhat sinusoidal voltage limited by $\pm$Vcc. Therefore, always choose the components such that $\frac{R_4}{R_2R_3C_2} < \frac{1}{R_2C_1} + \frac{1}{R_1C_1}$ and you are guaranteed a good second-order low-pass filter. The above is especially true for high-Q filters where $R_1 >> R_2$. In this case, oscillations will not occur when:

$$\frac{R_4}{\cancel{R_2}R_3\cancel{C_2}} < \frac{1}{\cancel{R_2}\cancel{C_1}} + \cancel{\frac{1}{R_1C_1}}^{\;negligible} \qquad (C_1 = C_2 = C)$$

$$\Rightarrow \frac{R_4}{R_3} < 1.$$

Therefore, always maintain $R_4 < R_3$ for *stable* high-Q filters.

2. Uncertainty in Q for high-Q designs:

The Q of a Sallen-Key non-inverting low-pass filter (with $C_1 = C_2 = C$) is given by:

$$Q = \frac{1}{\sqrt{\frac{R_2}{R_1}} + \sqrt{\frac{R_1}{R_2}}\left(1 - \frac{R_4}{R_3}\right)}$$

which simplifiers to $Q = \sqrt{R_1/R_2}$ for $R_4 = R_3$.

However, in a high-Q design, $R_1 >> R_2$ and any slight non-equalities in R_4 and R_3 can change the value of Q by a large fraction. Let us consider the example where R_1 = 30 KΩ, R_2 = 1 KΩ, and:

$$Q = \frac{1}{0.183 + 5.48\left(1 - R_4 / R_3\right)}$$

which simplifies to Q = 5.5 for $R_4 = R_3$ exactly.

However, for $R_4 = 1.02\ R_3$, we have Q = 13.6, and for $R_4 = 0.98\ R_3$, we have Q = 3.4. Therefore, 3.4 < Q < 13.6, for a $\pm$2% change in the R_4/R_3 resistor ratio!

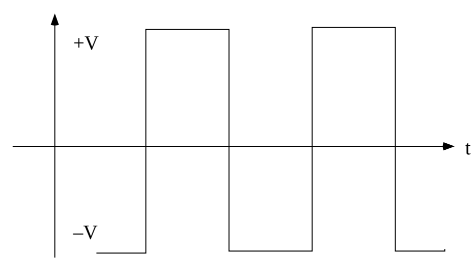
This holds true for $C_1 = C_2 = C$. However, $C_1 \neq C_2$ due to capacitor inaccuracies and parasitic capacitances in the circuit. Therefore, even if one maintains $R_4 = R_3$, Q can still vary between 3 and 14 due to capacitance variations alone. Therefore, the Sallen-Key filter is simply not a good design for high-Q filters!

In the above example, one could also ask what happens to Q when $R_4/R_3 = 1.05$ (with $C_1 = C_2 = C$). In this case, Q < 0 and the circuit will oscillate (see above). Therefore, it is always good to maintain $R_4 \leq R_3$ and use $\pm$ 1% resistors (instead of the standard $\pm$5% resistors) for R_4 and R_3 in Sallen-Key high-Q filters.

For the above reasons, a sensitivity analysis is always done in commercial engineering applications. The sensitivity analysis tells the engineer how much variation in output specifications to expect for a small change in component value. This is especially important in high-volume manufacturing, where a parts vendor could supply you with a whole run of components, all at the upper end of their tolerance limit."

Equations you may need for the lab report:

– The Fourier-Series of a square-wave signal is given by $V(t) = \sum_{n=1}^{\infty} \frac{4V}{n\pi} \sin\left(n\omega_o t\right)$ where V is the peak voltage (and <u>not</u> peak-to-peak).



– Total Harmonic Distortion, THD, is defined as:

$$\mathrm{THD}(\%) = \sqrt{\frac{\sum\left(\mathrm{P_{harmonics}}\right)}{\mathrm{P_{signal}}}}\ \mathrm{x}\ 100$$

where

$$\sum P_{harmonics} = \left(\frac{\mathrm{V_{2f_0}^2}}{\mathrm{R_L}}\right) + \left(\frac{\mathrm{V_{3f_0}^2}}{\mathrm{R_L}}\right) + \left(\frac{\mathrm{V_{4f_0}^2}}{\mathrm{R_L}}\right) + \left(\frac{\mathrm{V_{5f_0}^2}}{\mathrm{R_L}}\right) + \ldots$$

<u>V in rms!</u>

$$P_{signal} = \left(\frac{\mathrm{V_{f_0}^2}}{\mathrm{R_L}}\right)$$