Lab Assignment #3

Sampling Theorem

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Objective

To study the sampling theorem using Agilent VEE.

Equipment

- Agilent VEE software

A. Theoretical Introduction

A typical time-dependent signal, for example AC voltage, is continuous with respect to magnitude and time. Such signals are called analog signals. Using a normal (analog) oscilloscope we get an analog representation of such a signal.

Today mainly digital equipment is used for electrical measurements. The original analog signal is converted to a digital signal. A digital signal is discrete with respect to the magnitude as well as to the time. Therefore, conversion of an analog to a digital signal means the value of the analog signal function F(t) is measured at discrete times $t_i = t_0 + i \cdot \Delta t$ during a time interval $t_m = t - t_0$, and the indicated values changes with steps ΔF. Mostly the measured value is represented by a binary number and the step ΔF depends on its number of positions, that means of the number of bits processed by the analog-to-digital converter (ADC).

The correct use of this measurement technology requires one to know how often the value of a signal must be measured during a given time interval t_m in order to reconstruct its shape correctly. The answer is given by the sampling theorem of time functions (Nyquist Criterion):

> If the given time function F(t) has a limited bandwidth B and the time interval of measurement t_m is infinite, a correct interpolation between the sampling points is possible when the function is sampled with a time step,
>
> $$\Delta t < \frac{1}{2B} ,$$
>
> i.e. with a sampling rate
>
> $$f_s = \frac{1}{\Delta t} > 2B .$$

Assuming a simple harmonic function this theorem says that it is necessary to sample the function more than two times per period. An important example is the digital processing of audio signals. Assuming a bandwidth of $B = 20\text{kHz}$ for such signals a sampling rate $f_s > 40\text{kHz}$ must be used (usually: $f_s = 44.1\text{kHz}$).

But in many cases a function is sampled only during a short time interval t_m and only a very simple interpolation algorithm is used. Therefore, in most cases the time step between two measured points must be shorter:

$$\Delta t < \frac{1}{(10...20) \cdot 2B} ,$$

i.e. an oversampling is necessary: The sampling rate must be 10 ... 20 times higher than the value given by the Nyquist Criterion (see Fig.1).

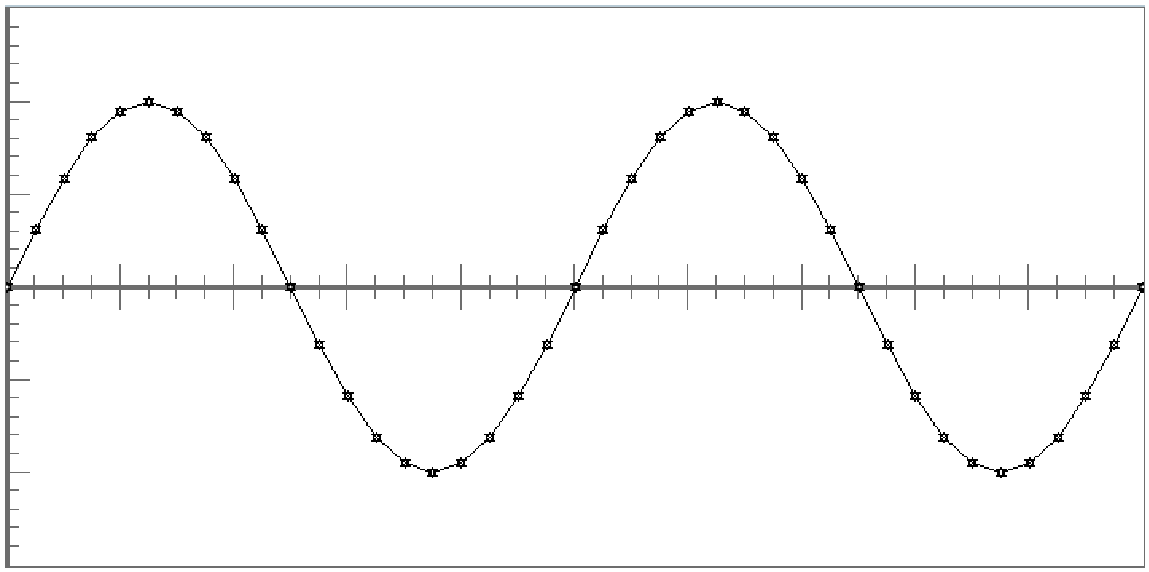

Fig.1: Sampled sine function (10 times oversampled and linear interpolated)

B. Simulation Experiments

The sampling theorem is a fundamental law for all digital equipment, and grave errors result from a disregard of this law. The following simulation programs illustrate different errors which can appear using this measurement method. To run this programs you need the run-time version of Agilent VEE (Version 5 or above).

1. Program *SAMPLING1.VXE*

A simple harmonic signal is sampled twice using different sampling rates ($f_{s2} = 5 \cdot f_{s1}$) and displayed using a linear interpolation.

Change the number N of sampling points and look to the shape of the both polylines!

The sampling rate f_{s1} disregards the sampling theorem, if N<7, and an aliasing error is generated. It is impossible to reconstruct shape and period of the original function. Also you can see that the higher the oversampling, the better the reconstruction.

2. Program *SAMPLING2.VXE*

This program compares two different displays of a sampled function: the sampled points (upper display) and a polyline using a linear interpolation between all points. In all cases the sampling theorem is fulfilled.

Change the signal frequency and compare the different displays!

At low frequencies the eyes interpolate correctly between the points in the upper display and we find no difference between the shapes. But, if the frequency will be higher and higher (especially for $f > 500\text{Hz}$), the interpolation made by the eyes will not be correct and a so-called perceptory aliasing appears: The eyes interpolate linearly between the nearest-neighbor points.

3. **Program *SAMPLING3.VXE***

This program demonstrates the results using different interpolation algorithms for the reconstruction of the shape of a function from a set of sampling points.
In the first case a unknown function is oversampled with a factor of two. All sampling points can be described correctly by using a linear interpolation, which results a triangular function shape, as well as by using a sine-wave or a square-wave interpolation. Knowing only the sampling points, it is impossible to decide which shape gives the best approximation to the original function. This decision is possible only by using a higher sampling rate.

Oversample using a factor of four and find out the correct interpolation!

For understanding this phenomenon remember that if the original function has a triangular or square-wave shape, it contains higher harmonics. If the function is two times oversampled, the sampling theorem is fulfilled only for the first harmonic f_1, but not for the second, because in both cases the frequency of the second harmonic is $f_2 = 3 \cdot f_1$. Using 4-fold oversampling the sampling theorem is fulfilled also for the second harmonic, and we can decide which interpolation is correct.