Experiment No. 6 Pre-Lab

Frequency Modulation (FM), Generation and Detection

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1.0 Frequency Modulation:

Frequency modulation (FM) is the standard technique for high-fidelity communications as is evident in the received signals of the FM band (88-108 MHz) vs. the AM band (450-1650 KHz). The main reason for the improved fidelity is that FM detectors, when properly designed, are not sensitive to random amplitude variations which are the dominant part of electrical noise (heard as static on the AM radio). Frequency modulation is not only used in commercial radio broadcasts, but also in police and hospital communications, emergency channels, TV sound, wireless (cellular) telephone systems, and radio amateur bands above 30 MHz.

The basic idea of an FM signal vs. an AM signal is shown in Fig. 1. In an FM signal, the frequency of the signal is changed by the modulation (baseband) signal while its amplitude remains the same. In an AM signal, we now know that it is the amplitude (or the envelope) of the signal that is changed by the modulation signal. The FM signal can be summarized as follows:

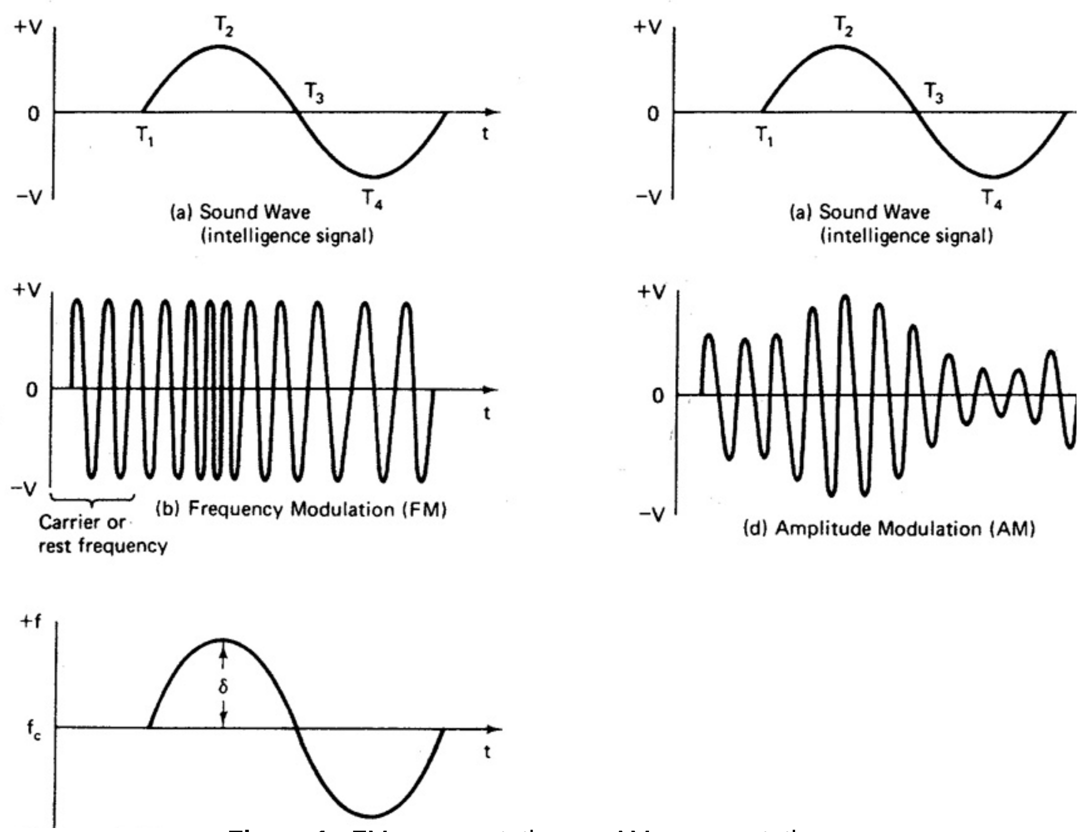


Figure 1: FM representation vs. AM representation.

1. The amplitude of the modulation signal determines the amount of the frequency change from the center frequency.

2. The frequency of the modulation signal determines the rate of the frequency change from the center frequency.

3. The amplitude of the FM signal is constant at all times and is independent of the modulation signal.

Mathematically, an FM signal is written as:

$$V = A\sin\left(\omega_c t + m_f \sin\omega_m t\right)$$

where A = the amplitude of the signal

ω_c = center frequency (frequency for no modulation signal)

ω_m = modulation frequency

and m_f = FM modulation index = δ/f_m

where δ = maximum frequency shift caused by the modulation signal

f_m = frequency of the modulation signal.

The spectrum of an FM signal is quite complicated and is dependent on m_f. Actually, it follows a Bessel Function (you will study this in a senior-level math course) and is given by:

FM Spectrum =	$J_0\left(m_f\right)\cos\left(\omega_c t\right)$	Center Frequency $\left(\omega_c\right)$
	$-J_1\left(m_f\right)\left[\cos\left(\omega_c-\omega_m\right)t-\cos\left(\omega_c+\omega_m\right)t\right]$	Components at $\left(\omega_c+/-\omega_m\right)$
	$+J_2\left(m_f\right)\left[\cos\left(\omega_c-2\omega_m\right)t+\cos\left(\omega_c-2\omega_m\right)t\right]$	Components at $\left(\omega_c+/-2\omega_m\right)$
	$-J_3\left(m_f\right)\left[\cos\left(\omega\omega_c-3\omega_m\right)t-\cos\left(\omega_c+3\omega_m\right)t\right]$	Components at $\left(\omega_c+/-3\omega_m\right)$
	+	

where $J_0\left(m_f\right), J_1\left(m_f\right)$ etc. are in volts, and are the levels of the frequency components of the FM signal for A = 1 V.

The spectrum is dependent on m_f, the modulation index, and Table 1 gives the values of the Bessel-functions J_0, J_1, J_2, etc.... for m_f = 0 to 15. Figure 2 gives some spectrum representation for various modulation indices. Notice from Table 1 that for m_f = 2.4, there is no power in the center frequency component (J_0(2.4) =0). This also occurs at m_f = 5.5, 8.6, ... This does not mean that there is no power transmitted in the signal. All that it means is that for m = 2.4, 5.5, ..., there is no power at the center frequency and all of the power is in the sidebands.

The bandwidth of an FM signal depends on the modulation index (m_f), and is approximated by the well-known Carson's Rule:

$$BW = 2\left(\delta + f_{m(max)}\right)$$

where $f_{m(max)}$ is the maximum frequency of the modulating signal. The factor (2) in the equation is to account for both the upper and lower sidebands (left and right of the carrier). This equation gives the bandwidth which contains 98% of the signal power.

x	n OR ORDER																
	(CARRIER)	Sidebands															
(m_f)	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—	—	—
9.0	−0.09	0.24	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03	0.01	—	—	—
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	—	—
12.0	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	−0.01	0.21	0.04	−0.19	−0.12	0.13	0.21	0.03	−0.17	−0.22	−0.09	0.10	0.24	0.28	0.25	0.18	0.12

Source: E. Cambi, *Bessel Functions*, Dover Publications, Inc., New York, 1948. Courtesy of the publisher.

Table 1: FM spectrum levels for m_f = 0 to 15 (not in dB). A negative sign means a phase of 180˚ with respect to other components.

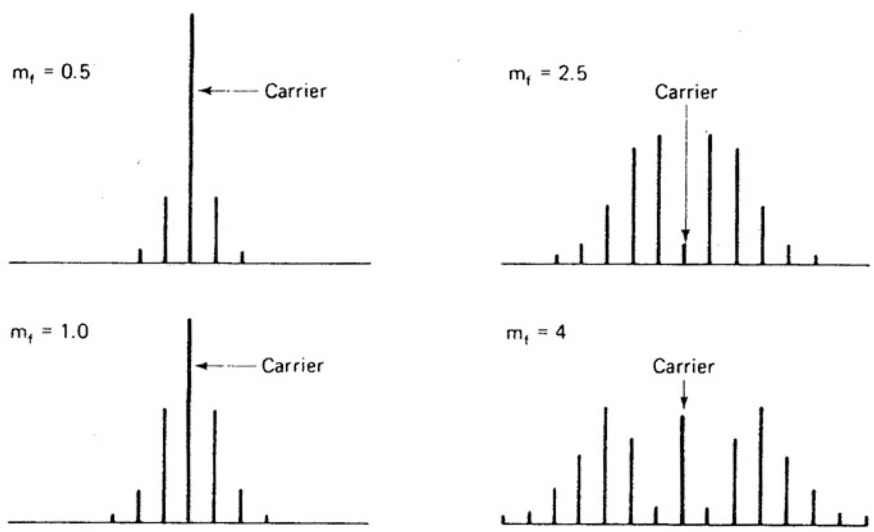


Figure 2: Frequency spectrum of FM signals with different m_f and same modulating frequency.

The best way to understand FM signals is to consider a real life example. Let us take an Ann Arbor station, 107.1 FM, broadcasting at 107.1 MHz with a power of 50 KW. The bandwidth of the modulation signal is from 30 Hz to 15 KHz which is excellent for high-fidelity broadcast. The maximum deviation set by the FCC, (δ), is 75 KHz. The range of the modulation index is therefore:

$m_{f(min)} = \delta/f_{m(max)}$ = 75 KHz/15 KHz= 5 (for f_m= 15 KHz)

to

$m_{f(max)} = \delta/f_{m(min)}$ = 75 KHz/30 Hz = 2500! (for f_m= 30 Hz)

Notice that the modulation index changes a lot with the modulation frequency (from 2,500 to 5). For the 15 KHz signal, m_f = 5 and from Table 1, we see that the frequency components are up high up to J_6 and drop quickly afterwards. This means that the bandwidth of the signal is 6x15 KHz = 90 KHz on each side of the center frequency (a total bandwidth of 180 KHz). We can also use Carson's rule and, BW = 2 (δ+ $f_{m(max)}$) = 180 KHz or 90 KHz on each side of the center frequency.

For the 30 Hz signal and m_f = 2,500, a huge number of sidebands exist but remember that there are now spaced at only 30 Hz apart. The total bandwidth of the signal is BW = 2 (δ+ $f_{m(min)}$) = 150 KHz or 75 KHz on each side of the center frequency. This means that the bandwidth of the FM signal changes from +/- 75 KHz to +/- 90 KHz from the carrier depending on the modulation frequency.

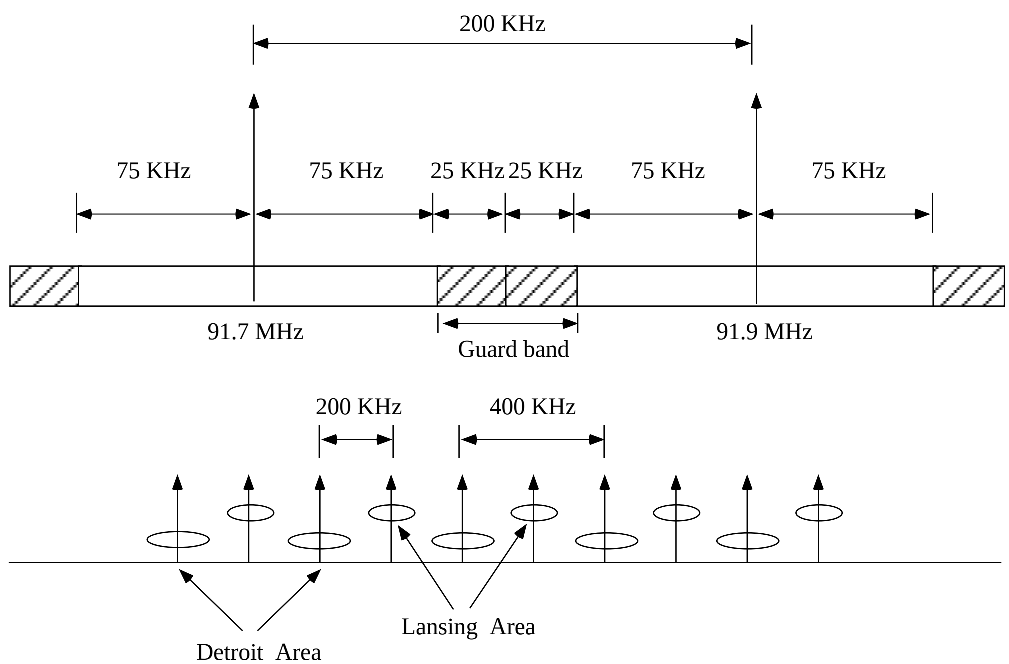


Figure 3: Commercial FM station assignment.

Commercial FM stations are therefore spaced 200 KHz apart to avoid interference for all modulating frequencies. In order to even isolate the stations further, FCC only assigns alternate stations for a certain area. For example, in the Detroit/Ann Arbor area, the stations are 107.1, 107.5 (and 93.1, 93.5, 93.9, ...) spaced 400 KHz apart. In adjoining areas, such as Toledo to the south (or Lansing to the north, but very far from Toledo), the stations are also centered at 400 KHz, but they are 107.3, 107.7, etc... (and 93.3, 93.7, 94.1 etc...). This allows inexpensive radios with bad-to-acceptable selectivity to receive FM stations without interference from adjoining stations (since they are 400 KHz away and not only 200 KHz away). The 200 KHz-away stations are very far and therefore their signals would appear as noise in the receiver. However, as mentioned before, FM receivers have excellent noise rejection and therefore are not affected by the far-away stations.

2.0 FM Generation:

FM generation is quite easy. All that is needed is a linear voltage-to-frequency converter. In EECS 311 and EECS 413, you will study in detail amplifiers, feedback and oscillators. If a voltage-controlled capacitor is placed in the feedback loop of an oscillator and the modulation signal is applied across this capacitor, then the oscillator will change in frequency and the resulting output signal is an FM signal.

In this experiment, we will use a famous chip, the LM 566C to generate the FM signal. The LM 566 is a linear voltage-to-frequency converter which can generate an FM signal up to 1

MHz and for a +/- 10% deviation from the center frequency, it has an FM distortion of less than 0.2%. The center frequency (f_o) is set by a resistor (R_o) and a capacitor (C_o), where:

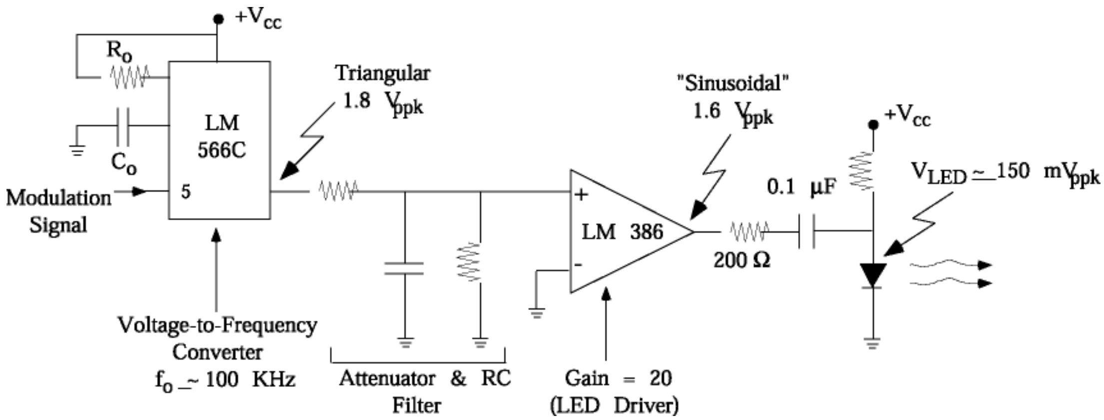


Figure 4: FM generation using the LM 566C. The LM 386 drives the LED.

$$f_o = \frac{2.4}{R_o C_o}\left(1 - \frac{V_5}{V^+}\right) \qquad 0.75V^+ < V_5 < V^+$$

$$2\,K\Omega < R_o < 20\,K\Omega$$

Notice that V_5 must be between $0.75V^+$ and V^+ for proper operation of the LM 566, and 2 KΩ < R_o < 20 KΩ.

The LM 566 delivers either an FM square wave or an FM triangular wave. However, this is not important, since the square/triangular wave can be converted into a "good" sinewave with the use of a simple RC filter. The input impedance for the modulation signal is 1 MΩ and the output impedance of the square/triangular wave is 50 Ω.

In both cases above, if the modulating signal is a digital waveform, then the resulting FM signal is a frequency-shift-keying signal (FSK) which you have seen in experiment #2.

3.0 FM Detection (FM Discriminators):

FM detection is the secret for the success of FM broadcasts. Basically, one can design an FM detector, called a frequency discriminator, which is immune to any amplitude variations (noise) in the signal. Therefore, it can be used with weak signals and can reject strong interfering stations. There are several FM discriminator designs and we will cover some of them below.

1. Slope Detector:

   Any filter (low-pass, high-pass, bandpass) can be turned into an FM detector. The FM signal is chosen to be in the cutoff region of the filter (Fig. 6) and therefore any frequency deviation is translated into an amplitude variation. The filter is then followed by an AM detector (Experiment #4) and the modulation signal is recovered.

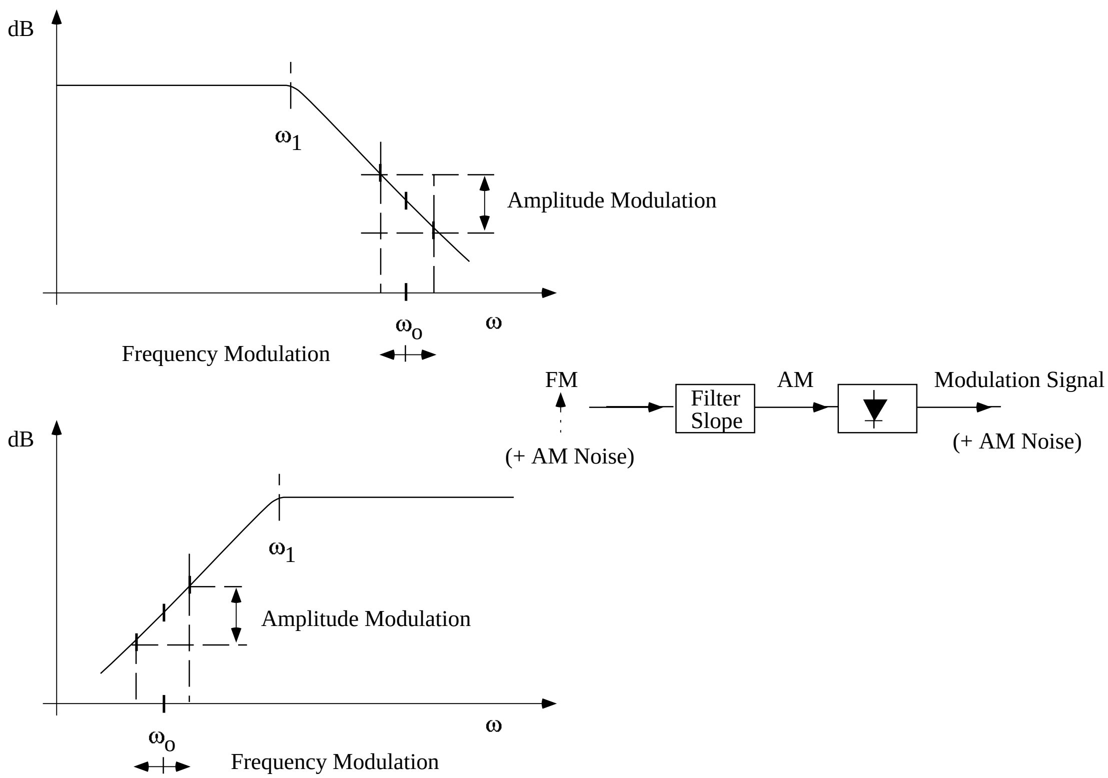


Figure 6: FM detection using a slope detector.

While this is the simplest technique available, it is non-linear since a first order low-pass filter response falls as 1/w and this results in second and third-order frequency components. For example, let us operate a low-pass filter (with a corner frequency ω_1) at ω_o and take a frequency deviation of $\Delta\omega$. The filter response is therefore:

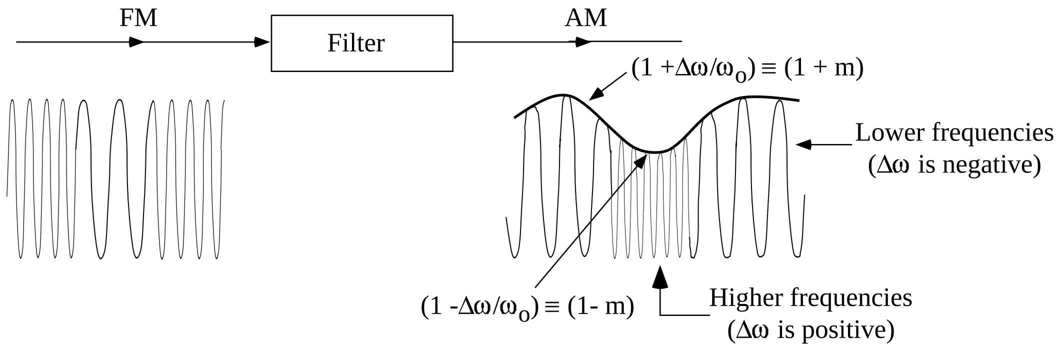
$$\frac{V_o}{V_i} = H(\omega) = \frac{1}{1 + j\omega/\omega_1} \approx \left| \frac{1}{j\omega/\omega_1} \right| = \frac{\omega_1}{\omega} \qquad \omega >> \omega_1$$

and for $\omega = \omega_o + \Delta\omega$,

$$\frac{V_o}{V_i} = |H(\omega)| = \frac{\omega_1}{\omega_o + \Delta\omega} \approx \frac{\omega_1}{\omega_o}\left(1 - \left(\frac{\Delta\omega}{\omega_o}\right) + \left(\frac{\Delta\omega}{\omega_o}\right)^2 - \left(\frac{\Delta\omega}{\omega_o}\right)^3 + \ldots\right)$$

(ω_1, ω_o are fixed).

The peak of the envelope occurs for a negative $\Delta\omega$ and is $V_i\left(\frac{\omega_1}{\omega_o}\right)\left(1 + \frac{|\Delta\omega|}{\omega_o}\right)$. The minimum of the envelope occurs for a positive $\Delta\omega$ and is $V_i\left(\frac{\omega_1}{\omega_o}\right)\left(1 - \frac{\Delta\omega}{\omega_o}\right)$. Therefore, the AM modulation factor, m, is $\Delta\omega/\omega_o$ for a first-order slope detector.



Notice that for a frequency deviation of +/- 10% of the center frequency ($\Delta\omega/\omega_o = 0.1$), the second order component is only 10x below the first order component resulting in a -20 dB distortion level. Therefore, this technique is valid only for a small frequency deviation from the center frequency ($\Delta\omega/\omega_o$ = 3%). Another pitfall of this technique is that it does not have any rejection of unwanted AM signals (noise).

If a high pass filter is used, the demodulation process is much more linear since the high-pass function is:

$$\frac{V_o}{V_i} = H(\omega) = \frac{j\omega/\omega_2}{1 + j\omega/\omega_2} \approx \left| j\omega/\omega_2 \right| = \frac{\omega}{\omega_2} \qquad for\ \ \omega << \omega_2 (high-pass\ filter)$$

and for $\omega = \omega_o + \Delta\omega$,

$$\frac{V_o}{V_i} = \left|H(\omega)\right| = \frac{\omega_o + \Delta\omega}{\omega_2} = \frac{\omega_o}{\omega_2}\left(1 + \frac{\Delta\omega}{\omega_o}\right) \qquad (\omega_2, \omega_o \text{ are fixed})$$

which is a linear process. However, this technique still suffers from having no rejection to AM noise.

2. Matched Slope Detector:

   In this case, two filters are used in the FM discriminator and the center frequency is chosen at the intersection point of these filters. The filters are followed by two AM detectors and the outputs of the AM detectors are fed to a differential amplifier. This technique is more linear than the simple slope detector shown above and most important, is immune to AM noise.

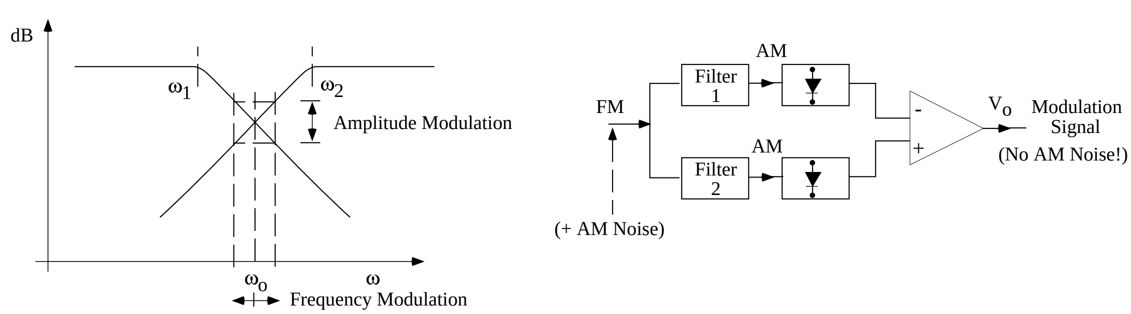


Figure 7: FM detection using a matched slope detector.

The frequency response of filters 1 and 2 are given above. For $\omega = \omega_O + \Delta\omega$,

$$\left|H_1(\omega)\right| = \frac{\omega_1}{\omega_o + \Delta\omega} \approx \frac{\omega_1}{\omega_o}\left(1 - \left(\frac{\Delta\omega}{\omega_o}\right) + \left(\frac{\Delta\omega}{\omega_o}\right)^2 - \left(\frac{\Delta\omega}{\omega_o}\right)^3 + \ldots\right)$$

$$\left|H_2(\omega)\right| = \frac{\omega_o + \Delta\omega}{\omega_2} = \frac{\omega_o}{\omega_2}\left(1 + \frac{\Delta\omega}{\omega_o}\right)$$

Choose $\omega_1/\omega_O = \omega_O/\omega_2$ or $\omega_2 = \sqrt{\omega_1 \omega_2}$ (i.e. operate at the intersection point of two filters), and pass the signals by a differential amplifier ($V_{02} - V_{01}$):

$$\frac{V_0}{V_i} = H_2(\omega) - H_1(\omega) = \frac{\omega_1}{\omega_o}\left\{2\frac{\Delta\omega}{\omega_o} - \left(\frac{\Delta\omega}{\omega_o}\right)^2 + \left(\frac{\Delta\omega}{\omega_o}\right)^3 + \ldots\right\} \quad (\omega_1, \omega_0 \text{ are fixed})$$

Note that the process has now second and third harmonic components, but since the signals of V_{02} and V_{01} are passed by a differential amplifier, any AM noise will be eliminated. Again, this technique is excellent for a small frequency deviation from the center frequency ($\Delta\omega/\omega_O$ < 5%).

3. Foster-Sealy/Ratio Detector/Phase-Locked Loops:

   The Foster-Sealy and Ratio detectors will not be discussed here but suffice to say that they were the standard FM detectors till about 10 years ago. They operate on two tuned circuits at the center frequency and offer excellent linearity, both in amplitude and phase. Also, the ratio detector is immune to AM noise.

   As mentioned in experiment #2, a phase locked loop locks on a signal and tracks its frequency deviation. The tracking voltage is therefore a replica of the FM modulation signal. It is very easy to use and is integrated in a single IC. The LM 565 is a matching PLL for the LM 566 VCO (see attached data sheet). The use of PLLs is now very common in all FM detection systems.

4.0 FM Optical Link:

In experiment #6, you will build an FM optical link using the Agilent 3316 LED and a Radio-Shack phototransistor. The transmit portion of the link is composed of the LM 566C voltage-to-frequency converter with a triangular output waveform. The center frequency is set to 100 KHz. The LM 566 output is large (1.8 Vppk) and therefore it is attenuated by a factor of 20, and passed by a simple RC filter to result in a sinewave with a third harmonic component of -26 dB. Since the ac impedance of the LED is only 18 Ω, the LM 386 amplifier is used to isolate the RC filter from the LED and to provide the necessary current to drive the LED (see Fig. 4).

On the receive end, the phototransistor is configured to have a response time of around 12 µsec by choosing a 1 KΩ resistor at the emitter. The phototransistor corner frequency is around 20 KHz and therefore it acts as a low-pass filter and demodulates the FM signal at 100 KHz. The 100 KHz AM signal is amplified by the LM 386 with a gain of 50 and then passed by a simple RC high-pass filter to attenuate the 120 Hz pick up noise from the neon lighting system. After the high-pass filter, the signal is passed by an AM envelope detector. The output of the AM detector is amplified using the LM 380 audio amplifier and sent to the speakers.

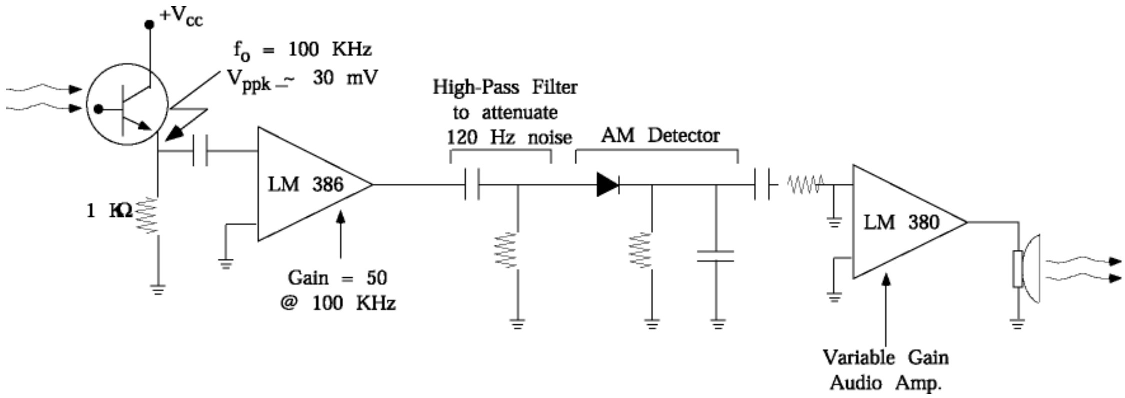


Figure 8: FM receiver using the phototransistor as a slope detector.

As is evident, this experiment does not use a PLL to detect the FM signal, neither does it use a matched slope detector to eliminate the AM noise. Therefore, the output has some noise due to the dark current of the transistor and the large bandwidth of the experiment. Still, it is a nice experiment and shows the operation of FM modulation and detection.