



Experiment No. 2.

Active Bandpass Filters; Frequency Shift Keying Communications

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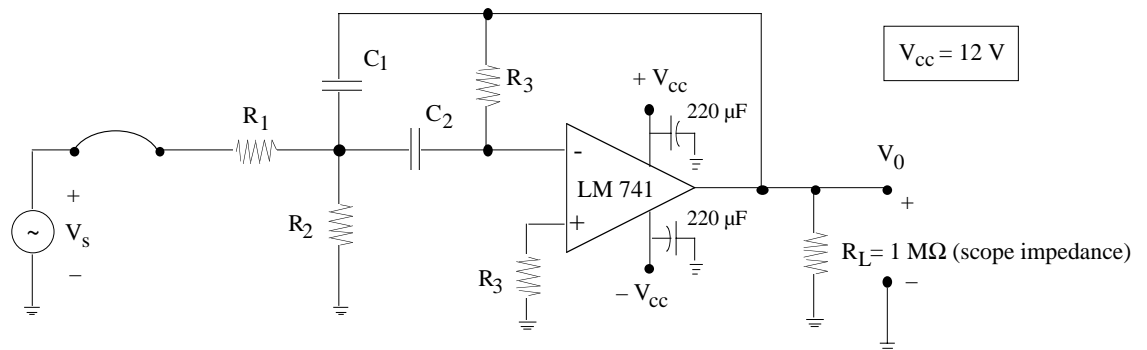
Purpose

To measure the response of a bandpass filter for different Q, in time and frequency domain. Also, to use the bandpass filter as a simple FSK receiver in a digital system.

- ☐ Read this experiment and answer the pre-lab questions before you come to the lab.

1.0 Active Bandpass Filters:

An active multiple-loop bandpass filter is shown below:



The filter response is given by:

$$\frac{V_0}{V_s} = H(s) = \frac{-\left(\frac{1}{R_1 C_2}\right)s}{s^2 + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3}s + \frac{1}{R_3 C_1 C_2} \left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \quad s = j\omega$$

with

$$|K| = \frac{R_3}{R_1} \frac{C_1}{C_1 + C_2} \quad Q = \frac{\sqrt{\frac{R_3}{R_2} \left(1 + \frac{R_2}{R_1}\right)}}{\sqrt{\frac{C_1}{C_2}} + \sqrt{\frac{C_2}{C_1}}}$$

$$\omega_0 = \sqrt{\frac{1}{R_3 C_1 C_2} \left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \quad \Delta\omega = \frac{1}{R_3} \left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$



Notice the negative sign in the numerator. This is an *inverting* active filter.

Where

$$\begin{aligned} |K| &= \text{Gain at resonance } (\omega_0) \\ \omega_0 &= \text{Resonant frequency} \\ \Delta\omega &= \text{3-dB Bandwidth} \\ Q &= \text{Quality factor} \approx \frac{\omega_0}{\Delta\omega} \text{ (for small } \Delta\omega) \end{aligned}$$

In order to simplify the design, choose $C_1 = C_2 = C$ and:

$$\begin{aligned} R_1 &= \frac{Q}{|K| \omega_0 C}, & R_3 &= \frac{2Q}{\omega_0 C} \\ R_2 &= \frac{Q}{(2Q^2 - |K|) \omega_0 C} & \text{with } |K| < 2Q^2 \end{aligned}$$

which results in:

$$|K| = \frac{1}{2} \frac{R_3}{R_1}, \quad \omega_0 = \frac{1}{C} \left(\sqrt{\frac{1}{R_3} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)} \right), \quad \Delta\omega = \frac{2}{R_3 C}$$

and

$$Q = \frac{\sqrt{\frac{R_3}{R_2} \left(1 + \frac{R_2}{R_1} \right)}}{2}$$

This filter topology is good for low ($Q \sim 1$) to medium Q ($Q < 50$). For high Q , this topology gives a very large spread of resistance values and the value of ω_0 and Q become very sensitive to component values. Notice also that the filter peak gain, $|K|$, is always smaller than $2Q^2$. This means that low Q filters using this topology will have a gain around -3 dB to +3 dB.

For $C_1 = C_2 = C$ and relatively large Q and K ($R_2 \ll R_1$ and $R_3 \gg R_1$), we have:

$$|K| = \frac{1}{2} \frac{R_3}{R_1} \quad Q \approx \frac{1}{2} \sqrt{\frac{R_3}{R_2}}$$

$$\omega_0 \approx \frac{1}{C \sqrt{R_2 R_3}} \quad \Delta\omega = \frac{2}{R_3 C}$$

**Low-Q Design:**

1. Assemble the circuit on the breadboard using $C_1 = C_2 = C = 2.2 \text{ nF}$ and:

$$R_1 = 9.1 \text{ K}\Omega$$

$$R_2 = 9.1 \text{ K}\Omega$$

$$R_3 = 20 \text{ K}\Omega$$

This should result in $f_0 \sim 7.6 \text{ KHz}$, $Q \sim 1$ and $|K| \sim 1$.


Connect the power supply ($\pm 12 \text{ V}$) and make sure to use the $220 \text{ }\mu\text{F}$ noise canceling capacitors.

- ☐ Draw the circuit in your lab notebook.
2. Connect the Agilent 33120A waveform generator to the input and set it to $V_{ppk} = 1 \text{ V}$. Connect the output to the Agilent scope.

- ☐ Make a fast frequency sweep (with the knob) and determine f_0 and $|K|$.
3. ☐ Make an accurate frequency response measurement of this filter (you can do it using the V_{ppk} readout, or the FFT mode) from $\sim 0.1 f_0$ (500 Hz) to $\sim 10 f_0$ (100 KHz).

Be careful! With bandpass filters, you should not measure at 20 Hz, 50 Hz, 100 Hz, etc. as we did with standard amplifiers! The correct way to measure filters is to determine the frequency where the response falls by 3 dB, 6 dB, 10 dB, 15 dB, 20 dB, etc. ...

(You will find that the response is very board. However, this filter is still sharper than the typical midband audio tone controllers which have a Q of around 0.5.)

4. ☐ Calculate the 3-dB bandwidth and the Q of the filter, and show it to the T.A.
5. ☐ Measure the phase delay of V_O/V_S at $0.5 f_0$, f_0 , $2 f_0$ (you need to connect the output to Channel 1 and the input to Channel 2 and use the phase delay measurement under the Measure  menu).

High-Q Design:

1. Change the resistances to:

$$R_1 = 15 \text{ K}\Omega$$

$$R_2 = 240 \text{ }\Omega$$

$$R_3 = 300 \text{ K}\Omega$$

(keep $C_1 = C_2 = C = 2.2 \text{ nF}$).

This should result in $f_0 \sim 8.5 \text{ KHz}$, $Q \sim 18$, $|K| \sim 10$.

Set the Agilent 33120A to give a $V_{ppk} = 500 \text{ mV}$.

- ☐ Draw the circuit in your lab notebook.
2. ☐ Make a fast frequency sweep (with the knob) and determine f_0 and $|K|$.
(Be careful, the amplitude will change very quickly around 8-9 KHz and a high frequency resolution is needed). Determine f_0 to a 10 Hz resolution.
 3. ☐ Make an accurate frequency response measurement from $\sim 0.1 f_0$ ($\sim 1 \text{ KHz}$) to $\sim 10 f_0$ ($\sim 100 \text{ KHz}$). Again, determine the frequency where the response falls by 3 dB, 6 dB, 10 dB, 15 dB, 20 dB and after this at 5 dB steps (25, 30, 35, 40 dB, etc...until you reach $\sim 100 \text{ KHz}$).



(Make sure that you are not clipping the waveform on the display and that you are not measuring V_{ppk} using a "small" waveform on the scope display. For highest accuracy, always choose a vertical setting to "fill" your screen.)

You will find that the response is quite sharp and that the filter response varies quickly around f_0 .

4. ☐ Calculate the 3-dB bandwidth and the Q of the filter, and show it to the T.A.
5. ☐ Measure the phase delay of V_O/V_S at $0.8 f_0$, f_0 and $1.25 f_0$.

2.0 Time Domain Response of High-Q Bandpass Filters:

1. a. ☐ Set the Agilent 33120A to give a square-wave at f_0 (~8.5 KHz) and $V_{ppk} = 2$ V. Plot V_i and V_O and label the ppk values. Do you know why it is a sinusoid?
b. ☐ Measure V_i and V_O in dB in the frequency domain (f_0 , $3f_0$ and $5f_0$.)
2. ☐ Change the input to a square-wave frequency of 200 Hz and $V_{ppk} = 1$ V. Connect the sync. signal of the Agilent 33120A to the Ext. trigger of the scope, and set the trigger to External. Set the timebase to see at least one complete cycle of V_O . Plot V_i and V_O , and pay attention to the slope of output signal at the transition.
3. ☐ From the ripples, measure the oscillation frequency. Is it close to f_0 ?
4. ☐ From the ripples, take several (4 separated points) peak voltages (V_p) and time (t) measurements. This is needed to calculate the decay time or the Q of the filter.
5. ☐ Measure the final voltage of V_O (after the ripple dies down).

3.0 FSK Modulation and Digital Communication Systems:

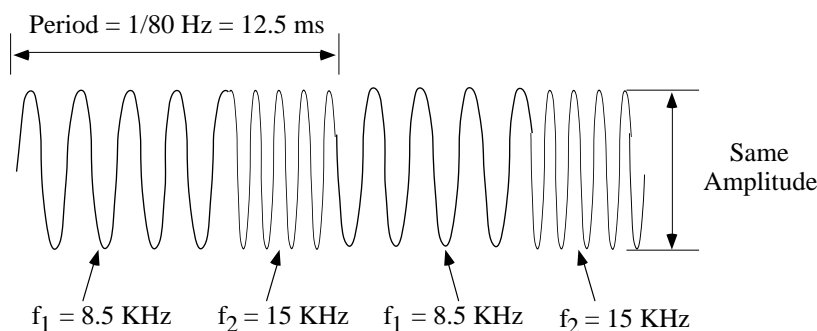
Set the Agilent 33120A to give an FSK signal with $V_{ppk} = 500$ mV.

$f_1 = f_0$ (~ 8.5 KHz) [First Frequency]

$f_2 = 1.7 f_0$ (~ 15 KHz) [Hop Frequency]

and FSK Rate = 80 Hz [FSK Rate]



The FSK signal should look like:









To Set an FSK Waveform:

1. First set f_1 normally for ~8.5 KHz (whatever your f_0 is) and $V_{ppk} = 500$ mV.











2. Press   to enter into FSK mode.

To Set the "Hop" Frequency:

3. a. Press   Recall Menu. You will see .
- b. Move down one level: Press . You will see .
- c. Enter 14 KHz (or whatever your frequency is) using the  command.

To Set the FSK Rate:

4. a. Press   Recall Menu. You will see .
- b. Move across one level: Press . You will see .
- c. Move down one level: Press . You will see .
- d. Enter 80 Hz using the  command.

Connect the SYNC output of the Agilent 33120A to the scope external trigger input. Set the scope trigger to "External". Now, look at the waveform with a long enough timebase to see the FSK signal. You should see clearly the FSK signal.

5. Connect the FSK signal to the input of the $Q = 18$, $K = 10$ filter.

The ~9 KHz signal should pass unattenuated and the ~15 KHz should be strongly attenuated (around -30 dB). Thus the 9 KHz is equivalent to a digital "1" and the 15 KHz is equivalent to a digital "0".

6. ☐ Draw the resulting waveform on your notebook and label the average peak voltage for "1" and the average peak voltage of "0".

Congratulations; you have built a simple but very useful digital FSK receiver. In this case, you are sending a stream of 101010... (square-wave) but it could be anything such as an ASCII code with 11000101...



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Pre-Lab Assignment

- Draw the bandpass filter circuit for $f \ll f_0$ and $f \gg f_0$, and calculate V_O/V_S at these frequencies.
 - Knowing that capacitors are "open" for $\omega \rightarrow 0$, and "short" for $\omega \rightarrow \infty$ calculate the input impedance of the active filter as seen by the source at $f \ll f_0$ and $f \gg f_0$.
 - Why is there a resistor R_3 connected to the (+) input of the op-amp (again, think non-ideal op-amp properties).
- Make sure that the values of R's and C's given in the experiment do result in the quoted f_0 , Q, K for the low-Q and high-Q cases.
- The response of a general bandpass filter with gain |K| at resonance is given by:

$$\frac{V_O}{V_S}(s) = H(s) = \frac{K(\omega_0/Q)s}{s^2 + (\omega_0/Q)s + \omega_o^2} \quad \begin{matrix} (s = j\omega) \\ f_o = 10 \text{ KHz} \end{matrix}$$

$$\text{and } |H(\omega)| = \frac{K \left(\frac{\omega_0}{Q} \right) (\omega)}{\sqrt{\left[\omega_0^2 - \omega^2 \right]^2 + \left[\left(\frac{\omega_o}{Q} \right) \omega \right]^2}} \quad (\text{Magnitude of } H(\omega))$$

- Derive $|H(\omega)|$
- For $K = 1$ and using MATLAB, plot on the same graph (dB, log f) the response of this filter for $Q = 0.4$, $Q = 2$ and $Q = 20$. The horizontal scale should be from $0.01 f_0$ to $100 f_0$. The vertical scale should be from 0 to -60 dB.
- From the plot, determine the 3-dB bandwidth ($\Delta\omega$) for each case, and determine Q from $Q = \omega_0/\Delta\omega$. What do you notice?
- Derive the equation of the phase of $H(\omega)$ and plot on the same graph the phase for $Q = 0.4$, $Q = 2$ and $Q = 20$ from $0.01 f_0$ to $100 f_0$. What is the phase of $H(\omega)$ as $\omega \rightarrow 0$, $\omega \rightarrow \infty$ and $\omega = \omega_0$?
- Prove that, at resonance ($\omega = \omega_0$), $|H(\omega_0)| = K$ and the phase of $H(\omega_0)$ is equal to 0° .



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Lab Report Assignment

1.
 - a. Draw the filter circuit and neatly summarize all your measured data for the low-Q and high-Q cases. Putting the data in a table form will result in an excellent presentation and you should do it. Comment on your measured vs. calculated values for K, f_0 , Q.
 - b. Plot on the same Bode Plot (dB, $\log f$) the filter response for $Q \sim 1$ and $Q \sim 18$. The vertical scale should be around +20 dB to -40 dB and the horizontal scale should be from around 500 Hz to 100 KHz (depending on your measured values). Clearly label the resonant frequency (f_0), the gain at resonance (K) and the bandwidth (Δf) of the filters.
2. For the high-Q filter, calculate f_0 and Q from time-domain measurements. For the Q calculations, in one case use two peaks which are close to each other, and in another case, use two peaks which are farthest away from each other. Compare the results with the frequency domain measurements comment.
3.
 - a. For a square-wave of $V_{pk} = 1V$ ($V_{ppk} = 2V$) and $f = f_0$ (~ 8.5 KHz), calculate the fundamental, third, fifth and seventh harmonic levels in Volts and dB before and after it passes by the high-Q filter. Compare with frequency domain measurements for V_i and V_o .
 - b. Calculate the THD (total harmonic distortion) of the output sinusoidal waveform. Do you consider this waveform to be a "clean" sinusoid?
4.
 - a. Using the formulas given in the experiment, design a filter with $Q = 40$, $K = 100$ and $f_0 = 8.5$ KHz. Choose $C = 2.2nF$. What is the main difference in resistances with the design of $Q = 18$, $K = 10$?
 - b. What component(s) would you change in the $Q = 18$, $K = 10$ filter to get an $f_0 \sim 1.8$ KHz? What will happen to Q and K for the new 1.8 KHz filter?
 - c. Drawing of the filter circuit above with the calculated values for R's and C's. Circle the components that:
 - 1) determine the bandwidth,
 - 2) determine Q,
 - 3) determine ω_0 .
5.
 - a. A wideband (200 KHz) FSK signal is composed of a 10 MHz signal (f_1) and a 10.2 MHz signal (f_2). Determine the optimum time T_b for a digital bit (1 or 0).
 - b. Determine the minimum Q of the filter needed so as to pass the 10 MHz signal and attenuate the 10.2 MHz by at least 20 dB. (Hint: Q is very large, $Q > 100$). Either calculate Q analytically (and show it in detail) or include an expanded view of a Matlab plot (9.6-10.4 MHz) showing the attenuation at 10.2 MHz and the calculated Q.