Experiment No. 2. Pre-Lab

Frequency Shift Keying (FSK) and Digital Communications

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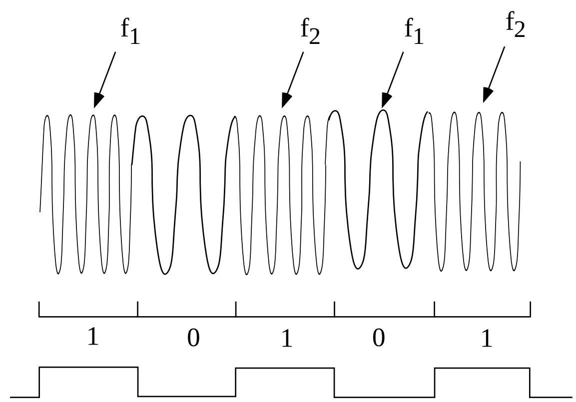
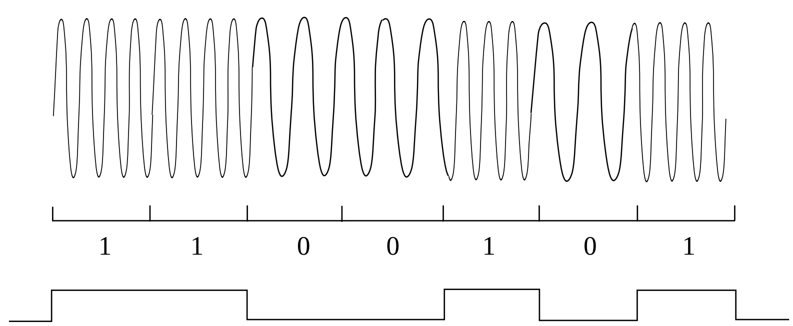
Introduction

One of the easiest and most useful modulation techniques for digital communication is Frequency Shift Keying (FSK). FSK means, quite simply, that one sends two frequencies over the air-waves; one frequency represents a digital "1" and one frequency represents a digital "0". For example, if 101010... is desired, the signal becomes:



If 1100101 is desired, the signal becomes:



The data rate (or the frequency of the square wave or the time of a digital "1" or "0") is called the FSK rate, and is in general several hundred times slower than f_1 or f_2. For example, for f_1 = 10 MHz and f_2 = 10.2 MHz, the FSK rate could be 50 KHz.

Another excellent characteristic of FSK signals is that the transmit power is constant for both the digital "1" and "0" signals. For an antenna impedance of R_{ant}, the transmit power is:

$$P_t = \frac{V_{rms}^2}{R_{ant}} = \frac{Avg\left[\sqrt{2}\,V_{rms}\sin(\omega_1 t)\right]^2}{R_{ant}} =$$

$$\frac{Avg\left[\sqrt{2}\,V_{rms}\sin(\omega_2\, t)\right]^2}{R_{ant}} = \frac{V_{rms}^2}{R_{ant}}\ (indep.\ of\ \ \omega_1\ \ and\ \ \omega_2).$$

There are several questions that may be on your mind about FSK and communications:

1. Why don't we send simply "1" and "0"? If we were in a coaxial cable, we would do just this. However, in transmit/receive systems, we must send electromagnetic waves (which are actually composed of sinusoids) using transmit antennas and capture them using receive antennas.

2. Who regulates the transmit/receive signals? Since all communication systems share the same medium, the "air-waves" are regulated very stringently by the FCC (Federal Communications Commission). This organization regulates the AM, FM, TV, police, hospital, citizen, satellite, wireless telephone, ... bands and sets the modulation technique/spectrum/bandwidth/power/ harmonics/... in each band! In general, the law in the U.S. says that one can receive anything (even police and military communications!) but no one can transmit without getting a license from the FCC which specifies exactly the frequency, power, modulations, etc. of the transmitter. This is done so as to make sure that your transmitter does not interfere with other channels in the area. The license can be virtually free (for example, radio amateurs, etc. ...) or can cost several hundred million dollars for exclusive use (for example, wireless telephone operators in New York city).

3. Why don't we send f_1 for a digital "1" and nothing for a digital "0"? We could conserve on transmit power this way!

   a. Remember that we are transmitting 10's or 100's of Watts in the time of the signal "f_1". This means that it takes a long time (msec!) to switch off a transmitter from "f_1" to "nothing" (digital "0"). Therefore, we cannot use a fast bit rate if "f_1" and "nothing" are sent!

   b. When "nothing" is sent, it does not mean that one receives nothing! The receiver will tune and amplify the noise in the digital "0" area and may treat it as a digital "1". This will lead to a large bit-error rate (BER).

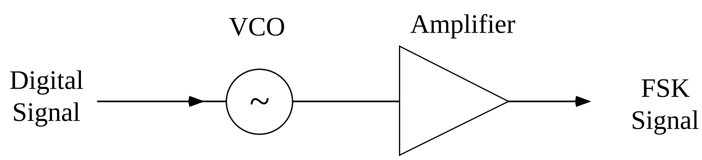
4. What is the optimum f_1 and f_2 vs. the data rate (FSK rate)? This is reserved for communication theory (EECS 455) which predicts that if we want to send a certain data stream with each bit taking a time T_b then the separation between f_1 and f_2 for optimum bandwidth and minimum error is:

$$\Delta f = f_1 - f_2 = \frac{1.43}{T_b}$$ where T_b is the time allocated for a digital "1" or a digital "0".

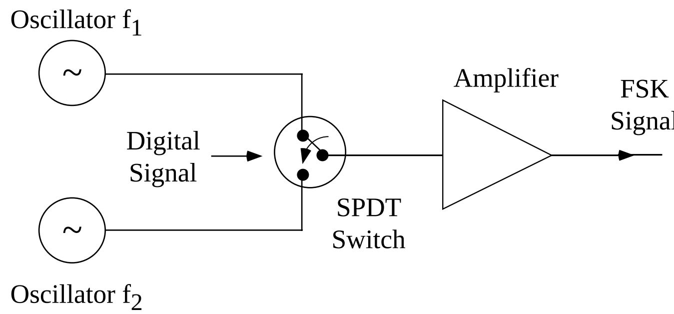
If we have a good detector, then the bit-error rate (BER) is about 10^{-6} for a signal-to-noise ratio of 15 dB at the receiver and 10^{-8} for a signal-to-noise ratio of 20 dB.

Generation of FSK Signals:

The best part of FSK modulation is the ease of signal generation. In the simplest technique, a voltage controlled oscillator (VCO) is used which oscillates at f_1 and f_2 depending on the control "digital" voltage. VCO are very easy to build and are composed of a transistor (or high-frequency op-amp) with a variable capacitor (actually a varactor diode) in the feedback circuit. The digital voltage controls the bias on the diode and hence the oscillation frequency. The VCO output is generally in the mW levels, so an amplifier is needed to boost the signal to the Watt levels.

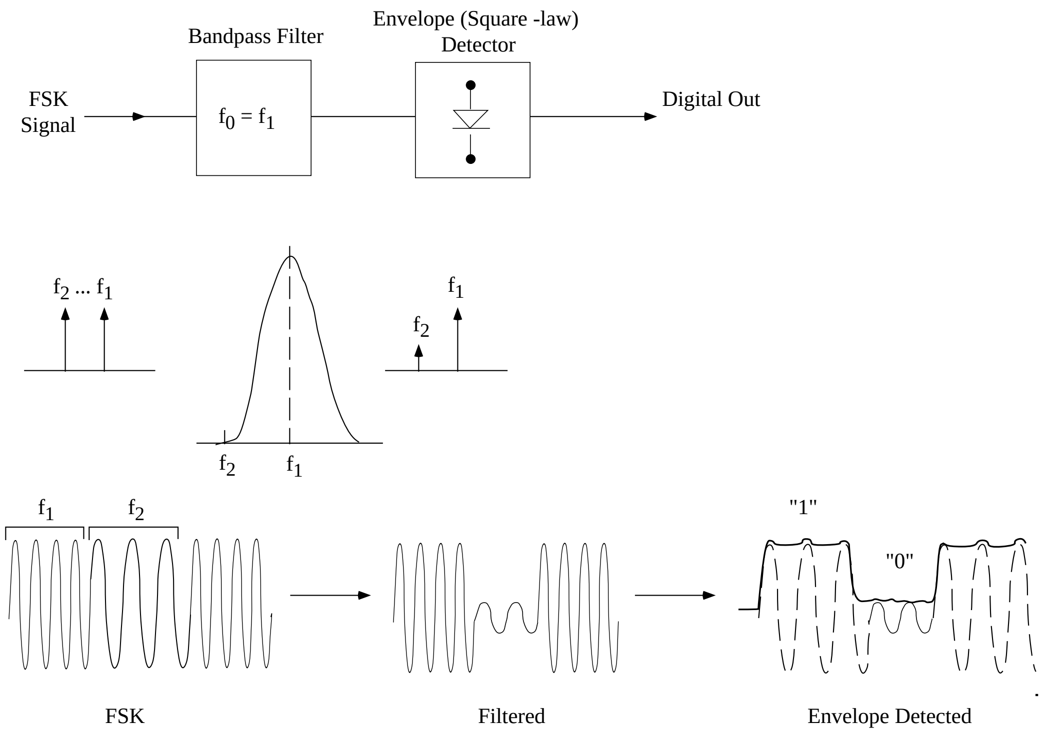


A low cost VCO takes a certain time to switch from f_1 to f_2, typically around 1 µsec, and therefore this technique is not suitable for very high data rate (MHz rates) signals. In this case, two oscillators are used, one at f_1 and one at f_2, and the digital signal controls a single pole double throw (SPDT) switch which selects either f_1 or f_2. It is common to have a switching time of nanoseconds and therefore this technique can handle 10's of MHz of data rates.



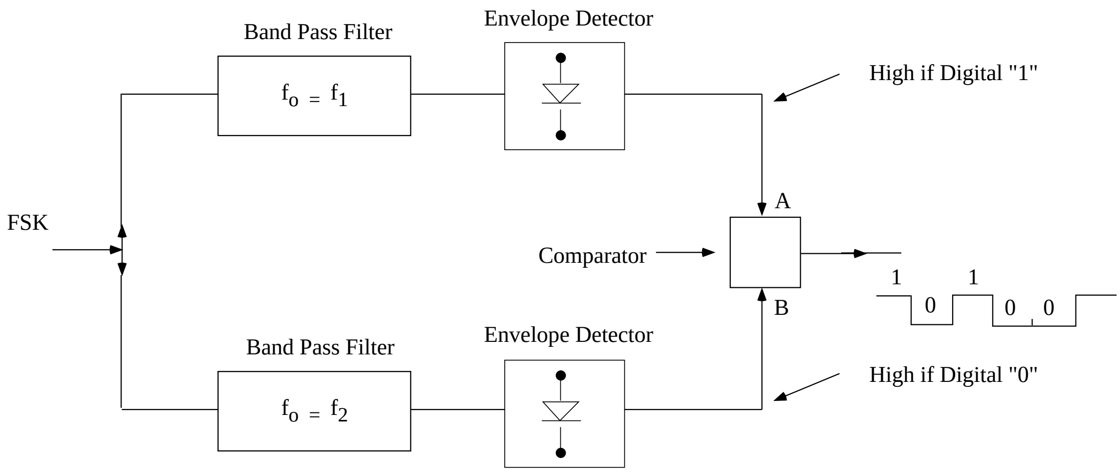
Detection of FSK Signals:

1. Single Filter: A single bandpass filter is used which is tuned to f_1 and attenuates f_2 by +30 dB. After the filter, the signal is passed through an envelope detector to smooth out the waveform (this is Experiment #4). The resulting signal is the digital waveform.

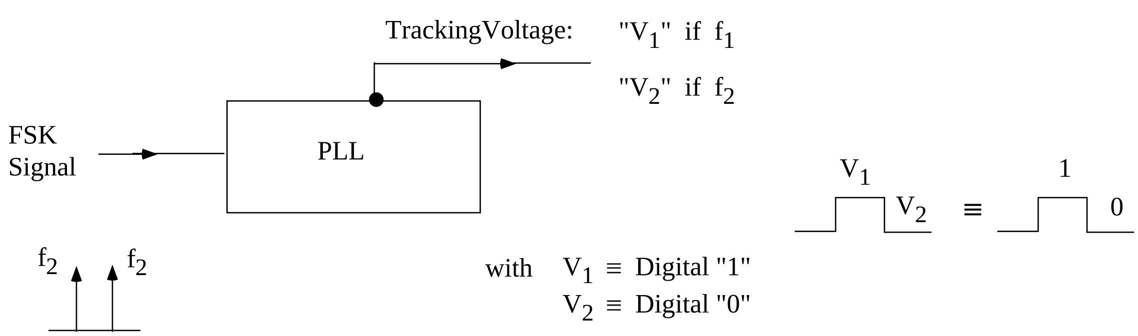


The main disadvantage of this technique is that noise peaks can affect the digital "0" area and it can be seen as an "1" resulting in a high BER (bit-error rate).

2. Matched Filters: Two bandpass filters are used. One is tuned to f_1 and one is tuned to f_2. Each filter attenuates the other frequency by 30 dB or more. If the output of filter 1 (f_1) is high, then a digital "1" is being received. If the output of filter 2 (f_2) is high, then a digital "0" is being received. The comparator looks at both signals and outputs "1" if input A is high and "0" if input B is high. This can be easily done using op-amps or digital gates.



3. Phase Locked Loop (PLL): This is the modern and best way to detect FSK signals. A (PLL) is actually a voltage controlled oscillator (VCO) which tracks a signal in frequency and phase. The PLL outputs a tracking DC signal (V_O) which is a direct indication of the input frequency. For example, V_O can be 1V for f_2 = 10 MHz and 1.5 V for f_2 = 10.2 MHz. The output voltage is then sent to an op-amp comparator which turns it into a digital "1" or "0". The FSK signal is therefore fed to a PLL and the tracking signal is monitored to get the digital information.



In a real communications system, some error correction is always applied. A bit error rate of 10^{-6} is probably ok for your personal e-mail, but totally unacceptable for a bank.

Band-Pass Filters:

A general second-order bandpass filter has a transfer function given by:

$$H(s) = \frac{K(\omega_o/Q)s}{s^2 + (\omega_o/Q)s + \omega_o^2} \qquad s = j\omega \qquad (1)$$

with a gain of |K|, a "resonant" frequency of ω_O, and a quality factor Q. The frequency response is shown in Figure 1. The quality factor Q determines the 3-dB bandwidth ($\Delta\omega$) of the filter, and for Q > 1, $\Delta\omega = \omega_O/Q$. For Q > 0.5, ω_O is called the "resonant" frequency, while for Q < 0.5, ω_o is called the "mean" frequency, i.e., the mean frequency between two poles ω_1 and $\omega_2 \left(\omega_o = \sqrt{\omega_1 \omega_2}, \text{see Fig.1}\right)$. At ω - ω_o, $|H(\omega_o)|$ = K, independent of Q, and $\angle H(\omega_o) = 0$. The roll-off at $\omega \ll \omega_o$ and $\omega \gg \omega_o$ is proportional to 1/ω and is -20 dB/dec. It is easy to determine K, ω_o, and Q from the frequency domain measurements.

Determining K, ω_o, and Q from Time-Domain Measurements:

If a step function, Au(t), is impressed on the bandpass filter, the output voltage is:

$$V_o(t) = \frac{2KA}{\sqrt{4Q^2 - 1}} e^{\frac{-\omega_o t}{2Q}} \sin\left(\sqrt{1 - \frac{1}{4Q^2}}\,\omega_o t\right) u(t) \qquad (2)$$

Determining K:

Unfortunately, the final value of the output voltage is zero $\left(V_o(t \to \infty) = 0\right)$, since the bandpass filter cannot pass the DC-component of the step-function. However, we can calculate the first peak which occurs for $\sin\left(\sqrt{1 - \frac{1}{4Q^2}}\,\omega_o t_p\right) = 1$. For Q > 3, this is equivalent to sin $(\omega_o t_p)$ = 1, and results in $\omega_o t_p = \pi/2$. Substituting this into (2), we find that the first peak value is:

$$V_{p_1} = \frac{2AK}{\sqrt{4Q^2 - 1}} e^{\frac{-\pi/2}{2Q}} \qquad (3)$$

And again, for Q > 3;

$$V_{p_1} \approx \frac{AK}{Q} e^{\frac{-\pi/2}{2Q}} \quad \text{with less than 1.5\% error.} \qquad (4)$$

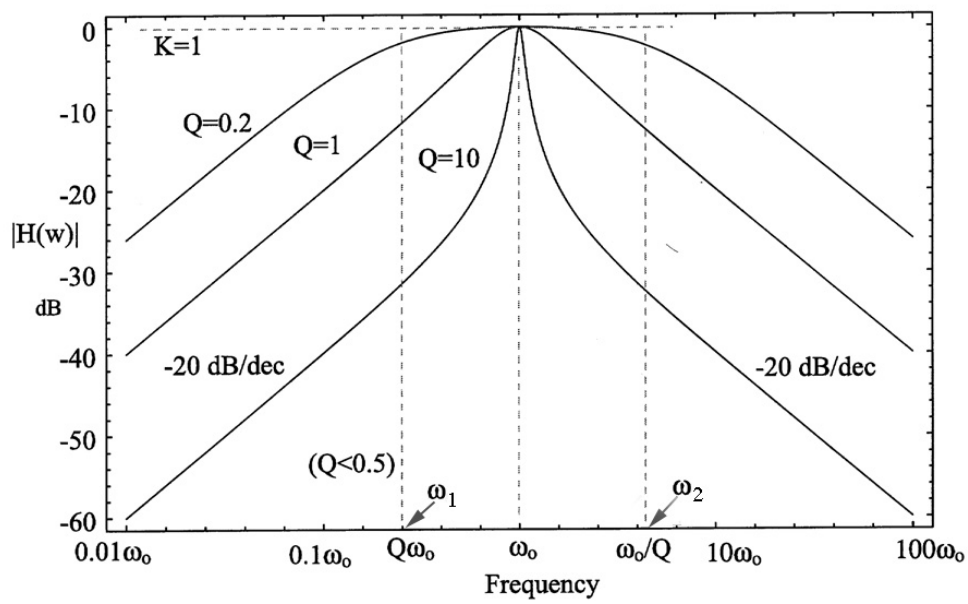


Figure 1a. Band-pass frequency response, magnitude.

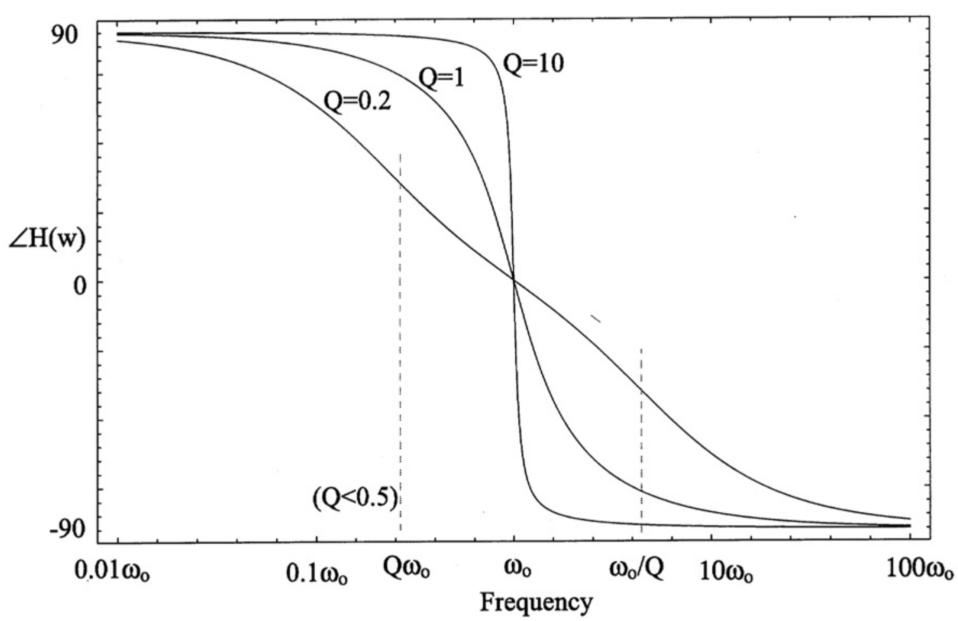


Figure 1b. Band-pass frequency response, phase.

Determining ω_o:

The oscillation frequency of the decaying sinusoid is

$$\omega_R = 2\pi / t_R = \omega_o \sqrt{1 - \frac{1}{4Q^2}} \qquad (5)$$

For Q > 3, $\omega_R \simeq \omega_o$ with less than 1.5% error.

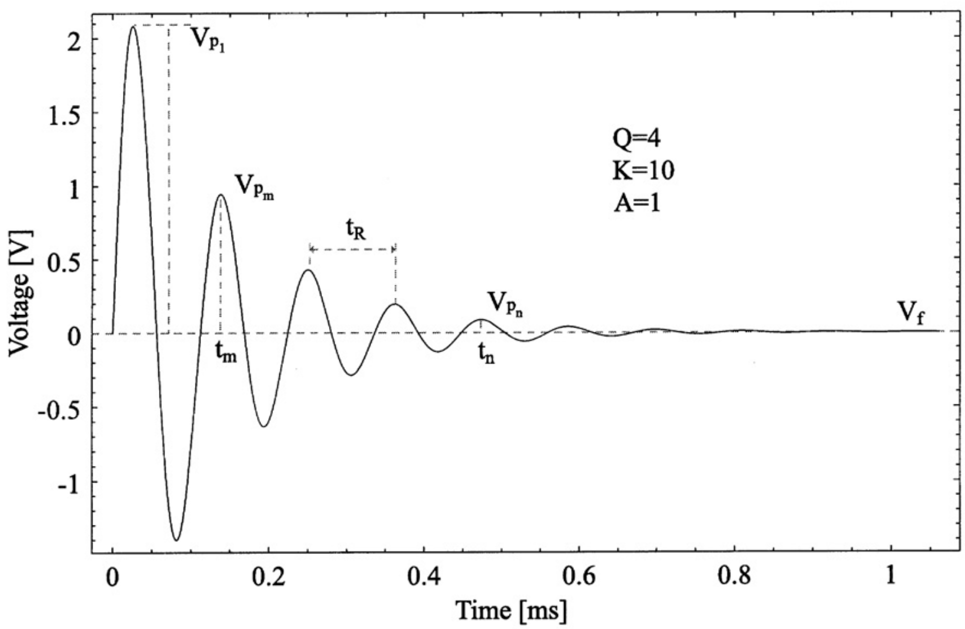


Figure 2. Band-pass time domain response.

Determining Q:

The same approach as Method 2 in Lab #1 is used (Decaying Peaks Approach). To repeat,

$$Q = \frac{\frac{1}{2}\omega_o \Delta t}{In\left(\frac{V_{pm}}{V_{pn}}\right)} \qquad (6)$$

where $\Delta t = t_n - t_m$ and $V_f = 0$ (band-pass filter). Knowing Q, we can determine K from equations (3) and (4). As usual, for large Q, you need to take peaks which are far away from each other so as to minimize the effect of the measurement error.

What if 0.5 < Q < 3?:

In this case, the natural resonant frequency (ω_O) is different from the resonant frequency (ω_R), since $\omega_R = \omega_o \sqrt{1 - \frac{1}{4Q^2}}$.

Q is determined using equation (6) but care must be taken to find the exact value and times of the peaks (which could be very broad). To avoid a complex equation between ω_O and Q, an iterative approach is used:

> Take a good guess at $\omega_o \left(\omega_o \cong \omega_R \text{ at the beginning}\right)$ and calculate Q using equation (6). Use this value of Q to calculate a new ω_O using equation (5), then a new Q using equation (6). After 2 to 3 iterations, the exact values of Q and ω_O are found.

To determine K, we have at the first peak, $\sin\left(\sqrt{1 - \frac{1}{4Q^2}}\,\omega_o t_p\right) = 1$ and

$$\omega_o t_p = \frac{\pi/2}{\sqrt{1 - \frac{1}{4Q^2}}}$$

$$V_{p_1} = \frac{2AK}{\sqrt{4Q^2 - 1}} e^{\frac{-\pi/2}{2Q\sqrt{1 - 1/4Q^2}}} \qquad (7)$$

Equation (7) can be used with the known values of ω_O and Q to calculate AK (or K, knowing the input step function A).

What if Q < 0.5?:

For Q < 0.5, the step response is dominated by the first pole-zero at $\omega_1 = \omega_o Q$, and the fall-time (not the rise-time) of the step response can be determined by ω_1 (problem 3 in pre-lab). We cannot really determine Q and ω_2 from time domain measurements alone, but this is not important. The system performance is limited by ω_1, and we know how to find its value!