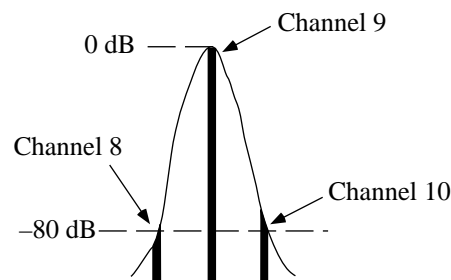# Experiment No. 1. Pre-Lab

Filters and low-pass Filters

By: Prof. Gabriel M. Rebeiz
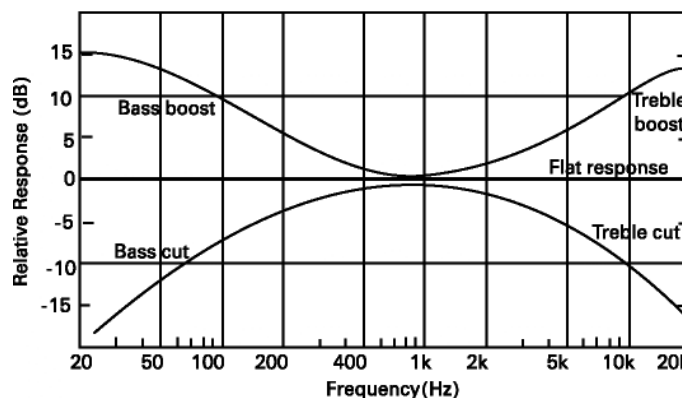The University of Michigan
EECS Dept.
Ann Arbor, Michigan

Filters

Filters, or the selective amplification or attenuation of frequencies, are the most common element in analog electronic circuits. Filters are not only used in audio systems for tone control (EECS 210). Actually, they are used in <u>all</u> systems. Every time you "tune" in a radio/TV station, you are using at least 3 filters (and in high quality systems, 5 filters). The word "tune" means that you "pass" certain frequencies (the radio/TV channel you want) and "attenuate" or "reject" other frequencies. It is very common to have a total rejection of 80 dB for the next FM or TV station which is only 200 KHz (FM) and 5 MHz (TV) away. We will talk about AM and FM modulation and radio receivers later in the lab manual.
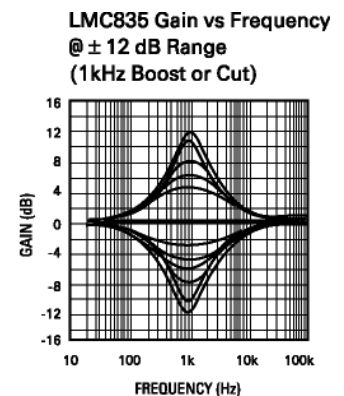

Figure 1: Combined effect of several filters in a TV receiver.

As mentioned in class, filters can be low-pass, high-pass, band-pass or band-reject (Fig. 2a). The tone control filter in Exp. #6 of EECS 210 were LP/Agilent filters depending on the boost/cut setting. Also, midband audio filters (or equalizers) are bandpass/band-reject filters depending on the boost/cut setting (Fig. 2b). Most filters used for radios/TV receivers are of the bandpass type; selecting one channel and rejecting others.
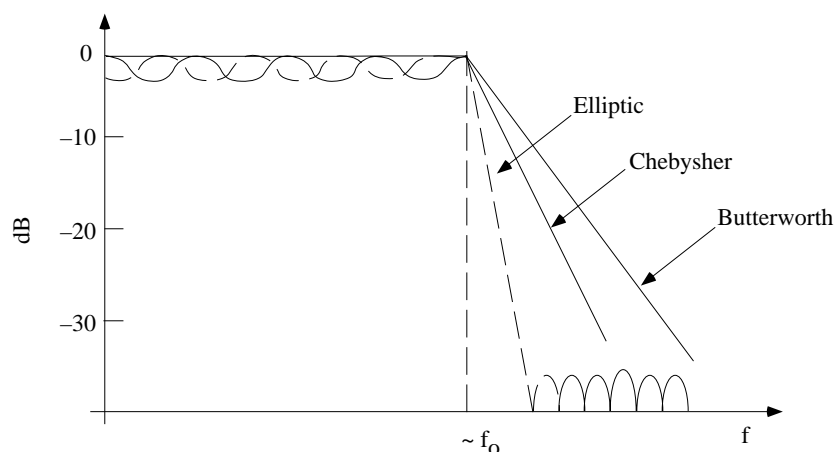
Figure 2: (a) Action of typical bass and treble controls. From *The Science of Sound, Second Edition*, Rossing, p. 444. (b) A midrange filter response. From *Application Specific Analog Products Databook, 1995 Edition*, p. 1-229.

Analog filter theory is quite complicated but very elegant. Actually, there are entire books and courses of this subject alone! Filters can be designed to give a maximally-flat response (Butterworth), a small ripple in the passband but sharper attenuation curve (Chebysher) or a very sharp attenuation curve with a small ripple in the passband and stop-band (elliptic).

Figure 3: Low-Pass Filter Prototypes of Butterworth, Chebysher and Elliptic.

The filter response can be designed to have a specific bandwidth (from ~100% to 0.1%) and a certain attenuation curve (or roll-off). The roll-off is determined by the number of poles and zeros in filter function. In a Butterworth (maximally flat response) design, every pole results in $(1/\omega)$ roll-off which is equivalent to -20 dB/decade.

One-pole $\qquad H(s) \propto \dfrac{1}{\omega} \qquad\qquad \omega \gg \omega_O \qquad\qquad$ –20 dB/dec or –6 dB/Oct

Two-pole $\qquad H(s) \propto \left(\dfrac{1}{\omega}\right)^2 \qquad\quad \omega \gg \omega_O \qquad\qquad$ –40 dB/dec or –12 dB/Oct

N-pole $\qquad H(s) \propto \left(\dfrac{1}{\omega}\right)^N \qquad\quad \omega \gg \omega_O \qquad\qquad$ –20xN dB/dec or –6xN dB/Oct

In radio receivers (FM, TV, wireless telephones, ...), it is very typical to result in an effective 10-pole (or higher) bandpass filter having a narrow bandwidth and an elliptic roll-off curve of 100[+] dB/Octave.

Tradeoffs: Filter design is always full of tradeoffs, as are most engineering problems. Typically, the steeper the cutoff slope, the greater the chance for ringing and overshoot. As we'll see in Experiment 1, we can adjust the "Q" of the filter to get faster risetime, but a higher Q also means more ringing and more variability with small changes in component values."
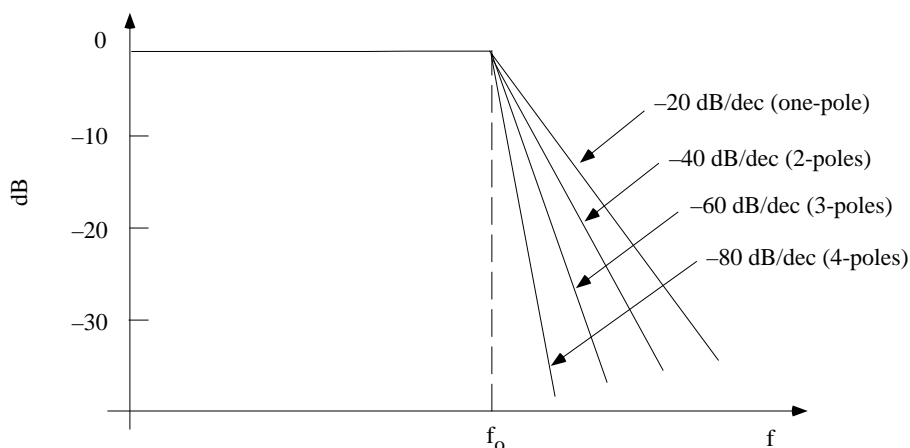
Figure 4: Filter response vs. number of poles.

First and Second-Order Circuits in Digital Systems:

The main interest in digital circuits is in the time domain response of pulses. As mentioned in class, the input of a digital circuit is modeled by a parallel RC circuit, and the output of a digital circuit is modeled by a series resistor. The digital connection between gates 1 and 2 is therefore:
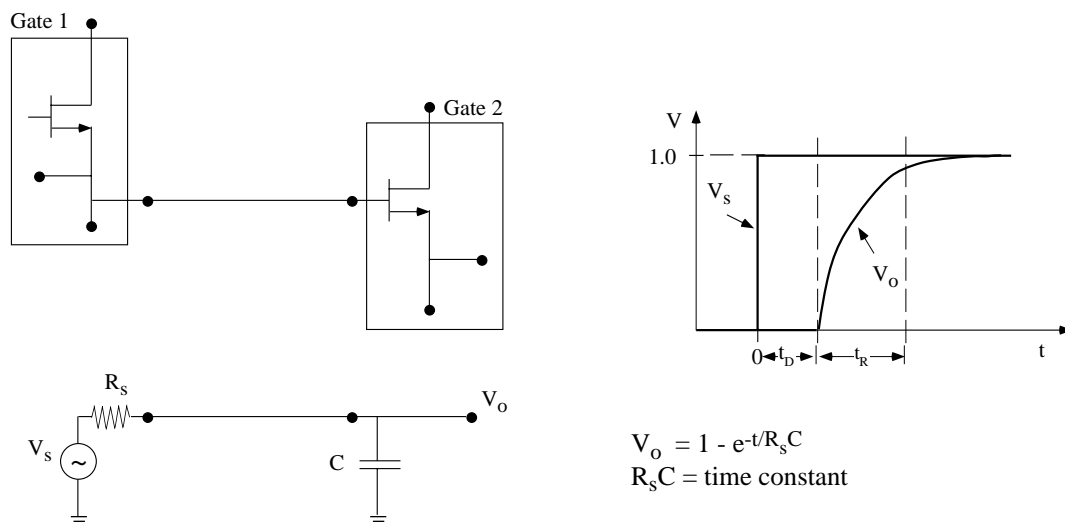

$$V_o = 1 - e^{-t/R_sC}$$
$$R_sC = \text{time constant}$$

Figure 5: Risetime in digital circuits.

Two important things happen here: One is that there is a time delay (called propagation delay, t_D) between the two gates given by the speed of the pulse and the distance between the gates. The speed of the pulse is given approximately by $c/\sqrt{\varepsilon_r}$, where c is the speed of light (3×10^8 m/s) and ε_r is the relative dielectric constant of the material (EECS 230). In fiberglass digital boards with $\varepsilon_r = 4$, the speed of the pulse is around c/2, which is 1.5×10^8 m/s. Inside of a silicon chip ($\varepsilon_r =11.9$), the speed of the pulse is around 0.8×10^7 m/s. This delay needs to be considered when designing very high speed circuits on large digital boards (clock frequency > 200 MHz), but is generally not the limiting factor in 100's MHz clocks.

Another more limiting factor is the risetime of the pulse due to the input capacitance of the digital gate (t_R). For a first-order circuit (Fig. 5), the risetime is given by $1/R_sC$ and is a simple

3

exponential. We see that the gate capacitance of the input transistor (or gate) and the series resistance of the output transistor (or gate) are the most important factor in limiting the risetime! One way to solve this is to build transistors with very small gates (submicron dimensions) and large ($^W/_L$) ratios for low series resistances. The smaller dimensions result in smaller area taken by the transistor which reduces the risetime of the input pulse (for the same resistance), and therefore increases the highest operating speed. In the early 1980's, a 2 µm silicon process was the standard dimension in silicon technology. In 1997, the 0.35 µm silicon process is the norm and some companies have even a 0.16 µm silicon process resulting in GHz speed systems.

Sometimes in high-speed digital boards, the input pulse shows some ringing (Fig. 6). This is quite bad since it delays the settling time of the pulse and therefore slows down the digital circuit. This is the response of a second-order circuit and is due to a small inductance which is in series with the input gate capacitance. The inductance can be due to very thin lines on the digital board, or to not-so-good ground connections between the digital chips and the board. It is very hard to measure the value of this inductance but it can be accurately estimated from the oscillation frequency. Also, the Q of the circuit (and therefore the resistance) can be estimated from the level of the first peak. One should always try to maintain a Q of less than 1.5 to keep the overshoot below 25% of the pulse level (which is not that easy in very high speed circuits (f > 300 MHz)).
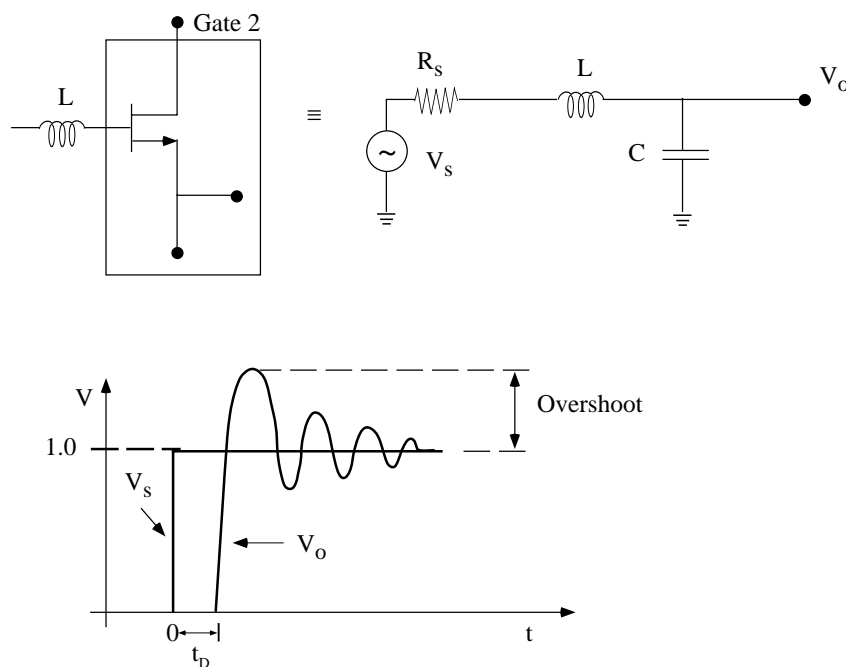

Figure 6: Ringing in digital circuits.

Low-Pass Filters:

A general low-pass filter has a transfer function given by

$$H(s) = \frac{K\omega_o^2}{s^2 + (\omega_o/Q)s + \omega_0^2} \qquad s = j\omega \qquad (1)$$

with a low-frequency ($\omega \rightarrow 0$) gain of K, a "resonant" frequency of ω_O, and a quality factor Q. The frequency response is shown in Figure 1. For Q > 0.5, ω_O is called the "resonant" frequency, while for Q < 0.5, ω_O is called the "mean" frequency, i.e., it is the mean frequency

between two poles ω_1 and ω_2 ($\omega_o = \sqrt{\omega_1 \omega_2}$, see Fig. 1). At $\omega = \omega_o, |H(\omega_o)| = KQ$ and $\angle H(\omega_o) = -90°$. The roll-off for large ω is proportional to $1/\omega^2$ which is -40 dB/decade. It is easy to determine K, ω_o, and Q from the frequency domain measurements as shown in Figure 1.

Determining K, ω_O, and Q from Time-Domain Measurements:

If a step function, Au(t), is impressed on the low-pass filter, the output voltage (see Fig. 2) is (Q > 0.5):

$$V_o(t) = AK\left[1 + \frac{1}{\sqrt{1-\frac{1}{4Q^2}}} e^{\frac{-\omega_o t}{2Q}} \sin\left(\sqrt{1-\frac{1}{4Q^2}} \, \omega_o t - \tan^{-1}\left(\sqrt{4Q^2-1}\right)\right)\right]u(t)$$
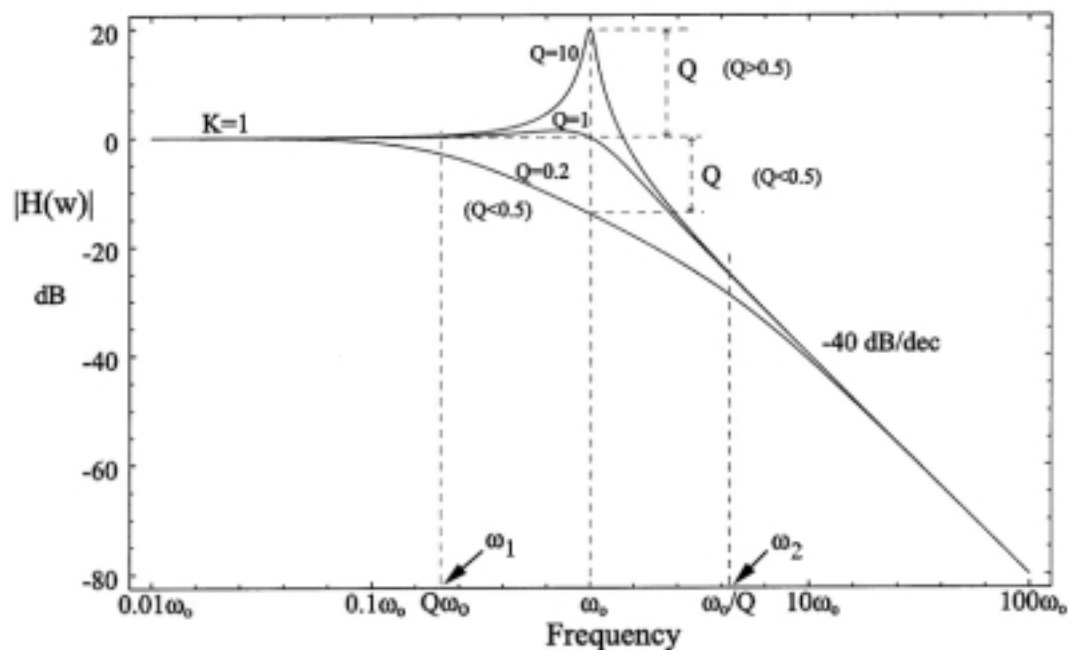
Determining K: The value of K can be measured from the final value of the response and knowledge of A.

$$K = \frac{V_o(t\to\infty)}{A} = \frac{V_f}{A}$$

Determining ω_O: The oscillation frequency of the decaying sinusoid is

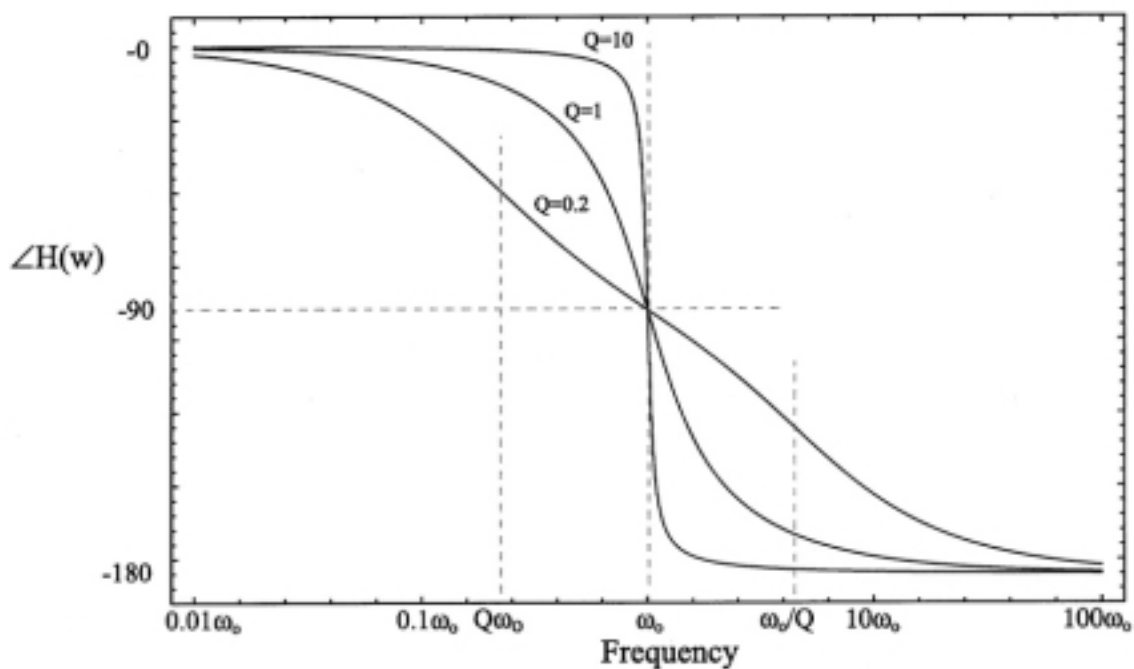$$\omega_R = 2\pi f_R = 2\pi/t_R = \omega_o \sqrt{1-\frac{1}{4Q^2}}$$

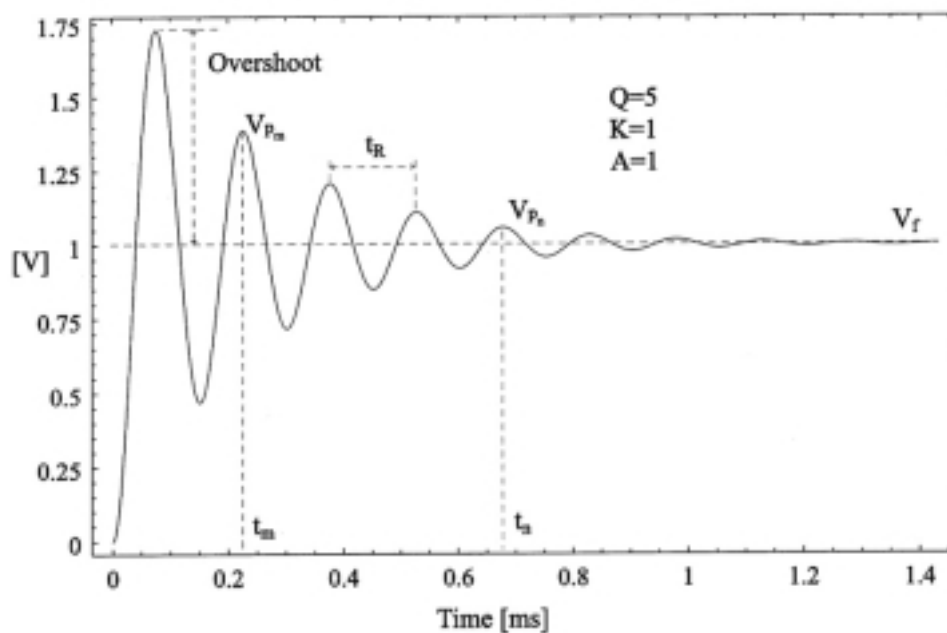For $Q>3, \omega_R \cong \omega_o$ with less than 1.5% error.

Figure 1a. Low-pass frequency response, magnitude vs. frequency. This is called a Bode-Plot.

Figure 1b. Low-pass frequency response, phase vs. frequency.

Figure 2. Time domain step response of a low-pass filter with K = 1, Q = 5 and A = 1.
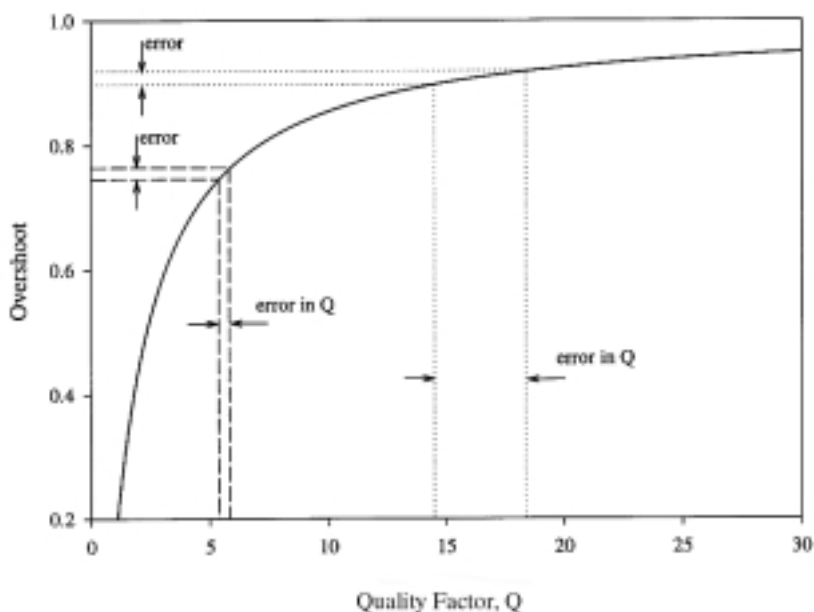
Determining Q:
Method 1: The Overshoot Approach:

The first, and highest peak occurs at t = t_1 when $dV_O/dt = 0$. To find the peak overshoot value, first find t_1 and replace it in equation (2) above. When all is done, we find:

$$V_p = K\left(1+e^{\frac{-\pi}{\sqrt{4Q^2-1}}}\right)u(t)$$

The overshoot value is defined as $V_p - V_O(t \rightarrow \infty)$ and is:

$$Overshoot = V_p - V_f = AKe^{\frac{-\pi}{\sqrt{4Q^2-1}}}$$

$$= e^{\frac{-\pi}{\sqrt{4Q^2-1}}} \qquad for \ K=1, \ A=1.$$

Figure 3. Overshoot vs. Q. Notice the error in Q for a small measurement error when Q > 8.

Figure 3 shows the value of the overshoot vs. Q for K = 1, A = 1. Notice that for Q > 5, any measurement error in the value of the overshoot voltage results in a large error in the determination of Q. For this reason, this method is accurate for 0.5 < Q < 5 and is risky for Q > 5.

There are also other reasons why this method should be used with caution in high Q systems. The op-amp could be *slew-rate* limited (the output voltage will not change fast enough to arrive to the first peak). This is especially true in high frequency filters (f_O > 500 kHz) where the op-amp bandwidth might limit the total voltage swing. Also, for small-signal op-amps, it could be in a current-limiting mode, and therefore will not be able to drive the necessary currents in the capacitors in the circuit. Since i = C $dV_O(t)/dt$, this will limit the slope of the voltage and result in reduced first peak. Again, this may occur for high frequency filters (dt is very small). Therefore, use this method with caution for high-Q filters and at high frequencies!

Method 2: The Decaying Peaks Approach:

A much better way to determine Q in high-Q filters is from the decay values of the sinusoidal waveform peaks. First, measure the values of several peaks and their corresponding times $(V_{p1}, t_1; V_{p2}, t_2; ...)$. see Fig. 2. The peak values occur when the sinusoid in equation (2) is equal to 1, so it can be removed from the analysis. The Q can then be calculated from the peak values and times of two peaks (m, n) at times (t_m, t_n), where $0 < t_m < t_n$ (one of the peaks could very well be the first peak).

At t_m,
$$Overshoot(m) = V_{p_m} - V_f = AK \frac{e^{\frac{-\omega_o t_m}{2Q}}}{\sqrt{1 - \frac{1}{4Q^2}}}$$

At t_n,
$$Overshoot(n) = V_{p_n} - V_f = AK \frac{e^{\frac{-\omega_o t_n}{2Q}}}{\sqrt{1 - \frac{1}{4Q^2}}}$$

Dividing;
$$\frac{V_{p_n} - V_f}{V_{p_m} - V_f} = \frac{e^{\frac{-\omega_o t_n}{2Q}}}{e^{\frac{-\omega_o t_m}{2Q}}}$$

and
$$Q = \frac{\frac{1}{2}\omega_o \Delta t}{\ell n\left(\frac{V_{p_m} - V_f}{V_{p_n} - V_f}\right)} \qquad \Delta t = t_n - t_m$$

The two peaks (m, n) should be chosen carefully. One one side, they should be spaced far apart to minimize errors, but one the other side, the peaks must be well defined above the final value. For example, in Fig. 3, the peaks after t = 0.9 ms should not be used since they are very small and not well defined. Therefore, this method is not very accurate for small Q filters (Q < 2) where all the peaks (after the first one) are poorly defined.

Revisiting ω_o for 0.5 < Q < 2:

In this case, the natural resonant frequency (ω_R) is different from the resonant frequency (ω_o). From equation (2), we have $\omega_R = \omega_o \sqrt{1 - \frac{1}{4Q^2}}$. The best way to solve it is to first determine Q from the overshoot method with $\omega_R \cong \omega_o$, and then use Q to determine ω_o from the measurement of ω_R.

What if Q < 0.5?:

For Q < 0.5, the frequency response is similar to a single pole system with $\omega_1 \cong \omega_o Q$, (Fig. 1). The risetime of the step response can be used to determine ω_1 accurately (problem 3 in pre-lab). The final value of the output voltage is still AK and this can be used to determine K. We cannot really determine Q and ω_2 from the time domain measurements *alone*, but this is not important. The system performance is limited by ω_1, and we know how to find its value!