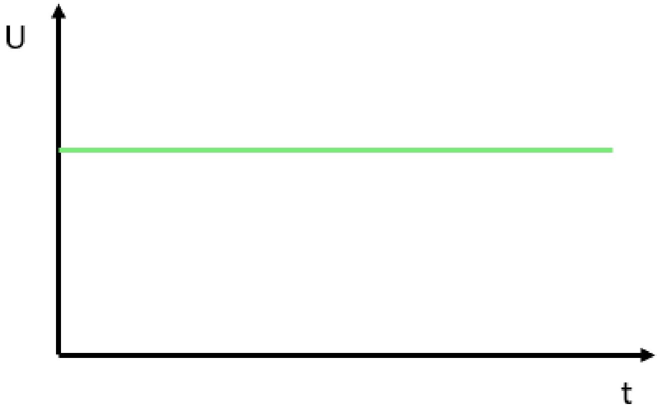
AC circuits

Review

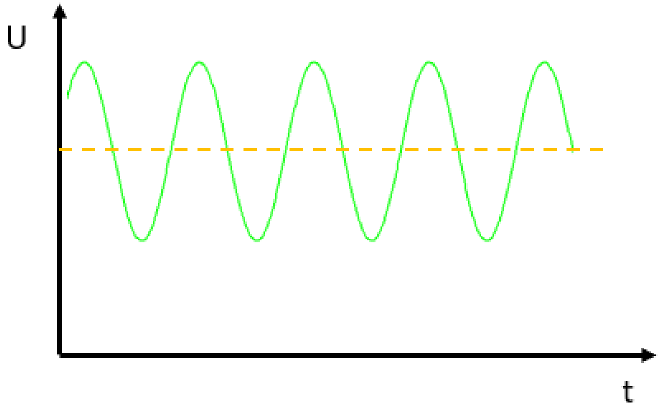
- Basic quantities in alternating current (peak value, rms value, frequency, period, etc.)
- Electric field strength, capacitance
- Capacitors in practice
- Inductances (introduction)
- Turn-on and turn-off phenomena of capacitors and inductors
- Impedance, reactance
- Resistance, capacitors, and inductors in alternating current circuits (capacitive and inductive reactance)
- Bode plot
- Alternating power
- Power factor
- Multiphase systems (chaining, star/delta connection)

Introduction

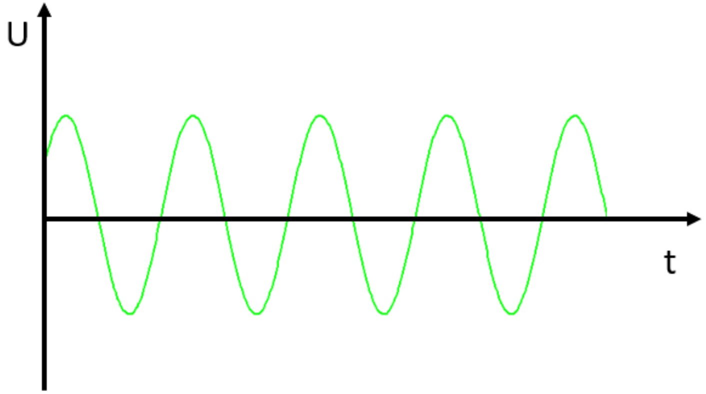
- So far, we have dealt with direct current circuits (DC), in which the voltage remained constant over time. We referred to this as direct voltage, and the current resulting from it as direct current (DC).
- A voltage whose magnitude and direction (polarity) both change is called alternating voltage, and the current resulting from this is called alternating current (AC). This should not be confused with pulsating (periodically varying) direct current!



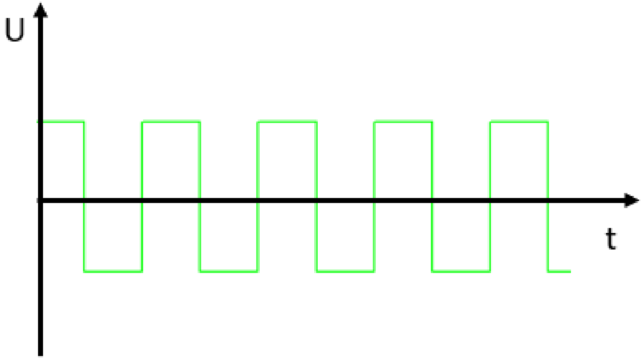
DC voltage



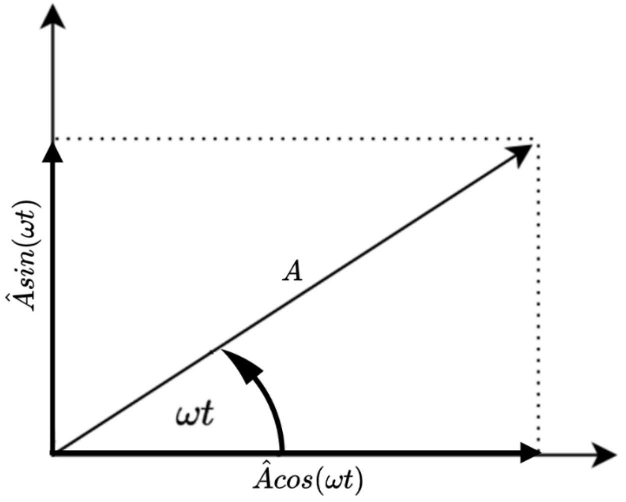
Sinusoidal DC voltage

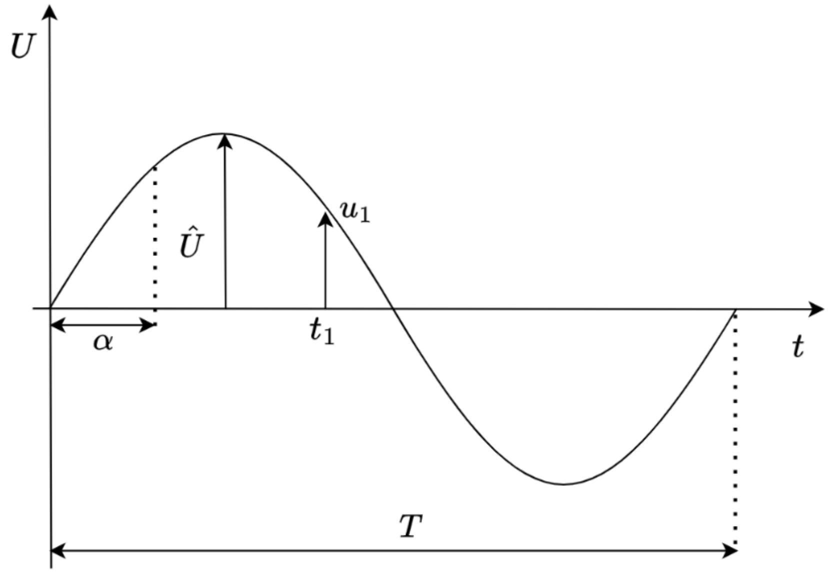


AC voltages



Properties of a sinusoidal signal





- Phase *A* can be visualized as a rotating vector with a direction (angle) and magnitude. These can change at any given moment. The peak of the vector traces a circular path. The value projected onto the vertical axis (y-axis) varies sinusoidally, while the value projected onto the x-axis varies cosinusoidally.

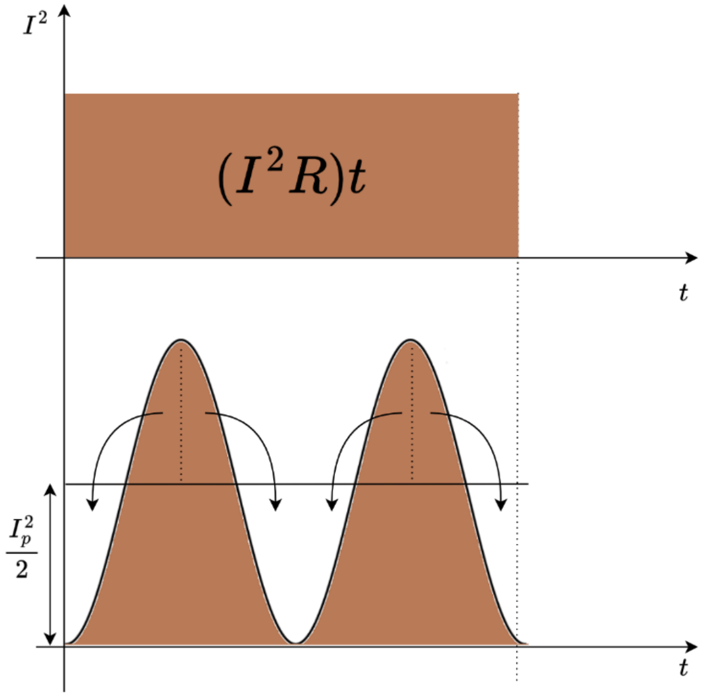
Properties of AC signal:

- Instantaneous value (for sinusoidal signals: $u = \widehat{U}\sin(\omega t - \alpha)$)
  - ω – körfrekvencia (rad/s) (A körfrekvencia egy olyan mennyiség, ami azt mutatja meg, hogy egy rezgés vagy forgás milyen gyorsan halad szögben (radiánban mérve)).
  - α – initial phase (° or rad)
- Frequency (*f*) - Frequency indicates how many times a repeating phenomenon occurring in a second
- Period (*T*) – the time required for a full cycle.
- The relationship between period and frequency:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- Effective value

Effective value



- In the case of DC, the amount of heat (power) generated across a resistor can be calculated using the following formula, given the current and the resistance (here we are analyzing it from the perspective of the current):

$$Q = I^2Rt = (I^2t)R$$

  - This gives the area under the curve (integral), which is proportional to the square of the current
  - Since we are looking for the same thermal quantity in both cases, we can use the fact that we are comparing the areas of the squares of the current over a single period.
  - We can conclude that the effective value is equal to the value of the direct current that performs the same quantity of work (generates the same amount of heat) in the same quantity of time as the alternating current under consideration
  - The general form of the effective value (in terms of voltage):

$$U_{eff} = \sqrt{\frac{1}{T}\int_0^T u^2(t)dt}$$

Effective value

- The power delivered to a single-phase load can be calculated as follows:

$$P = \frac{U^2}{R} = I^2 \cdot R = UI$$

- We can conclude that the effective value is equal to the value of the direct current that performs the same quantity of work (generates the same amount of heat) in the same quantity of time as the alternating current under consideration
- Instantaneous power (given the voltage and resistance):

$$p(t) = \frac{u^2(t)}{R}$$

- Averaged over a period:

$$P = \frac{1}{T}\int_0^T \frac{u^2(t)}{R}dt$$

- Highlighting *R* and equating it to the DC power:

$$\frac{U_{eff}^2}{R} = \frac{1}{R}\frac{1}{T}\int_0^T u^2(t)dt$$

- Simplifying and taking the square root of both sides:

$$U_{eff} = \sqrt{\frac{1}{T}\int_0^T u^2(t)dt}$$

Effective value

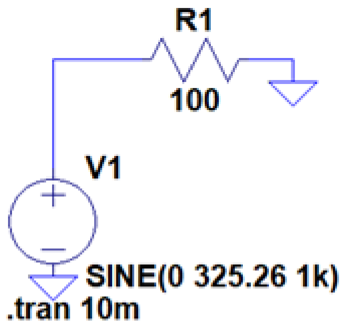
- The rms values of different alternating current signals vary
- In the case of sinusoidal signals, this:

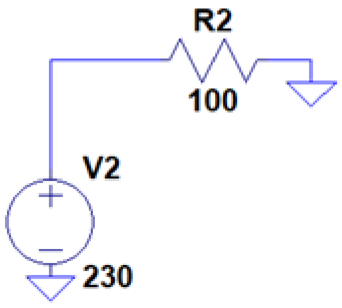
$$U_{eff} = \frac{\hat{U}}{\sqrt{2}}, \qquad or \qquad I_{eff} = \frac{\hat{I}}{\sqrt{2}}$$

- Example:
  - The rms value of the line voltage $U_{eff} = 230V$
  - If we connect a 100Ω resistor to a generator producing such a voltage, the rms value of the current flowing through it is : $i_{eff} = \frac{u_{eff}}{R} = \frac{230V}{100\Omega} = 2{,}3A$
  - The power dissipation: $P = i^2 R = 2{,}3^2 \cdot 100\Omega = 529W$
  - In the case of DC, we are looking for the same amount of heat, so P = 529W, and the resistance is also the same (R = 100Ω), so the current we are looking for is : $I = \sqrt{\frac{P}{R}} = \sqrt{\frac{529W}{100\Omega}} = 2{,}3A$
  - The voltage: $U = IR = 2{,}3A \cdot 100\Omega$=230V

- Let's investigate this with the help of a simulation!

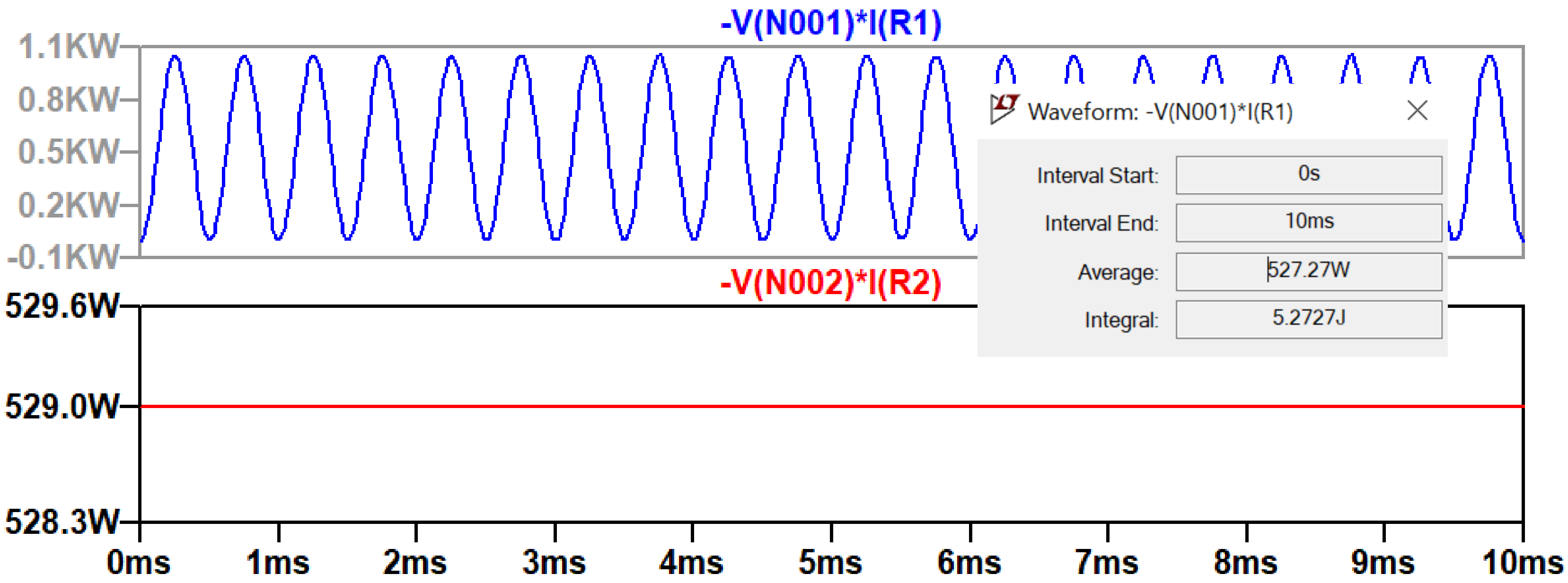
Simulation results (Ltspice)





AC case ***DC case***

(Note: In the LTspice environment, when defining an alternating signal, you can only specify the peak value. That is why the figure on the left shows 325 V (which is the peak value of 230 V).



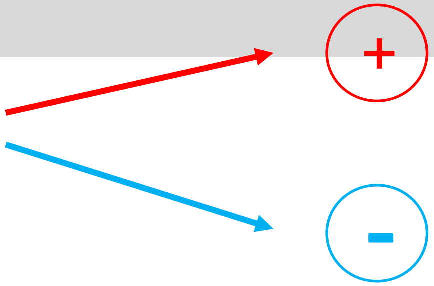
Dynamic elements

- It operates on the basis of an electric field;
- It stores charge;
- Filtering (e.g., voltage ripple),

- It operates on the basis of a magnetic field;
- It can be used to generate or "store" a magnetic field;
- Filtering, suppression (e.g., current ripple)

Electric field

- Electrically charged elementary particles

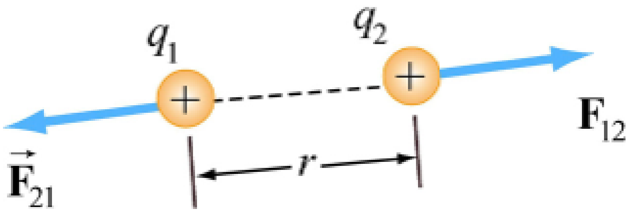


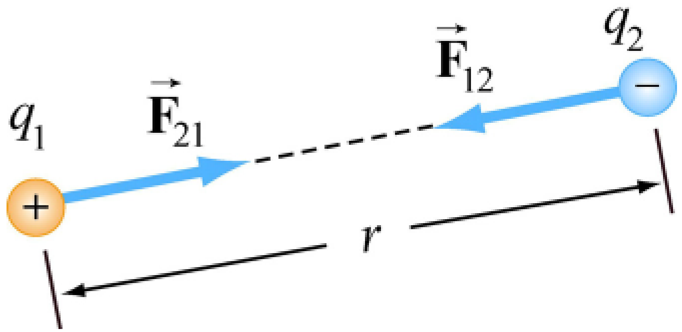
- The smallest "free" unit of charge known in nature is the charge of an electron or a proton, that is $e = 1{,}602 \cdot 10^{-19} C$
- An electric field can be detected experimentally: electroscope
- The charges interact with each other (similar to, for example, magnetic moments in a magnetic field), which manifests itself as a force :
  - Same charges repel each other
  - Different charges attract each others
- The magnitude of the force can be calculated using Coulomb's law:

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2}$$

- Electric field strength is the magnitude of the force acting on a positive test charge, that is:

$$E = \frac{F}{Q}\ [N/C]\ or\ [V/m]$$





Electric field, electric flux

- Let's place a positive charge next to the positive plate (armor)!
- A force of $F = Q \cdot E$ will act on the charge, causing it to move toward the negative plate.
- The work:

$$W = F \cdot s = F \cdot d = Q \cdot E \cdot d$$

- If we know that voltage is the ratio of work done to charge ($U = W/Q$), then the resulting quantity is the electric field strength (*V/m*)

$$E = \frac{U}{d}$$

- The electric field strength can be expressed as the ratio of the flux (the total number of field lines) to the surface area (in the case of a homogeneous electric field):

$$E = \frac{\Psi}{A}$$

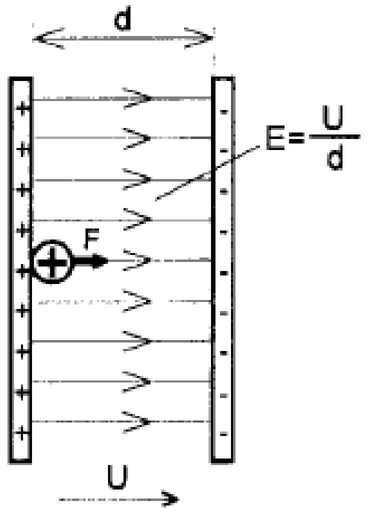
- We know that the electric field strength around a charge Q can be measured at a distance *r* (Coulomb's law):

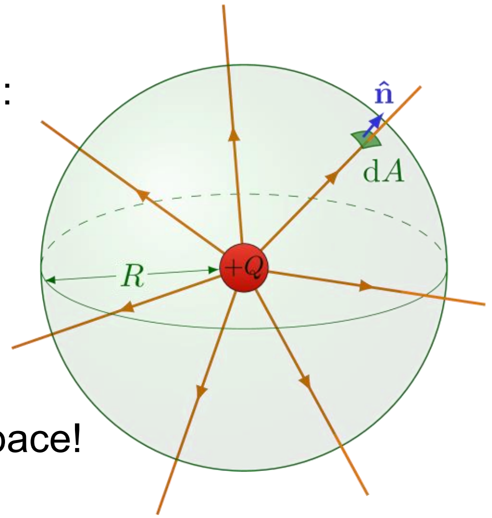
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

- For a sphere ($A = 4 \cdot \pi \cdot r^2$) the "number" of electric field lines, i.e., the flux, is:

$$\Psi = E \cdot A = \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}\right) \cdot (4 \cdot \pi \cdot r^2) = \frac{Q}{\varepsilon_0}$$

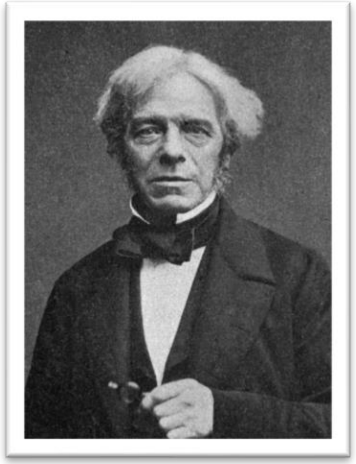
- In other words, the flux (in an electric field) depends on the charge (magnitude) and the material filling the space!





Soruce: https://tikz.net/electric_field_sphere/

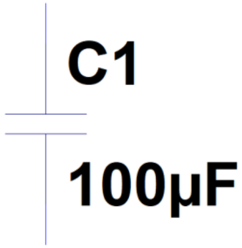
Parallel-plate capacitor, capacitance

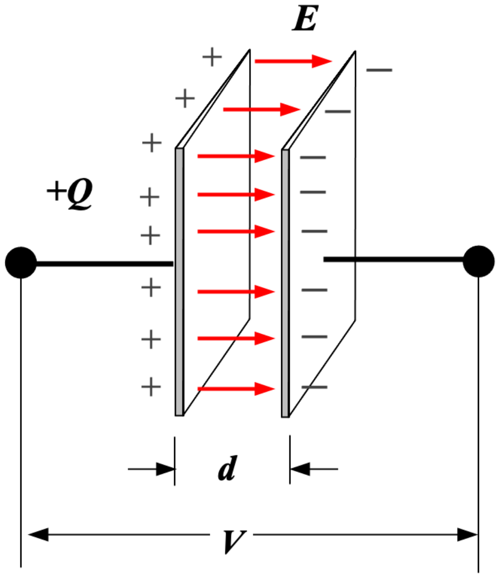


- In electrical circuits, a device capable of storing electric charge is called a capacitor
- The simplest form of this is the parallel-plate capacitor, which consists of two insulated, parallel metal plates (electrodes)
- The plates of a charged capacitor carry equal but opposite charges, and a homogeneous electric field is present between them
- The self parameter of a capacitor is capacitance, denoted by C, with the unit of measurement being the farad (F)
- It can be experimentally verified that the charge applied to the capacitor is directly proportional to the voltage between the plates, that is:

$$C = \frac{Q}{U}$$

- Derivation of the unit of capacitance: $\mathrm{C} = \frac{(As)}{(V)} = (F)\ (Farad)$
- In practice, one farad is a very large capacitance, so we use its fractions: mF, μF, nF;
- The electrical symbol for a capacitor:





Derivation of a parallel-plate capacitor from geometric parameters

- We have determined that the flux (total number of field lines) depends on the charge and the material filling the space (in this case, air/vacuum), that is:

$$\Psi = \frac{Q}{\varepsilon_0}$$

- So:

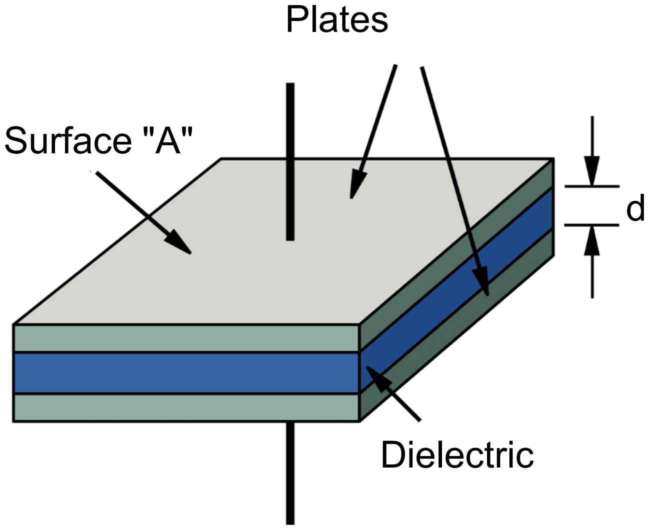
$$E = \frac{\psi}{A} = \frac{Q}{A \cdot \varepsilon_0}$$

- The voltage between the plates (armor):

$$U = E \cdot d = \frac{Q}{A \cdot \varepsilon_0} \cdot d$$

- Hence the storage capacity:

$$C = \frac{Q}{U} = \frac{Q}{\frac{Q}{A\,\varepsilon_0} d} = \varepsilon_0 \frac{A}{d}$$

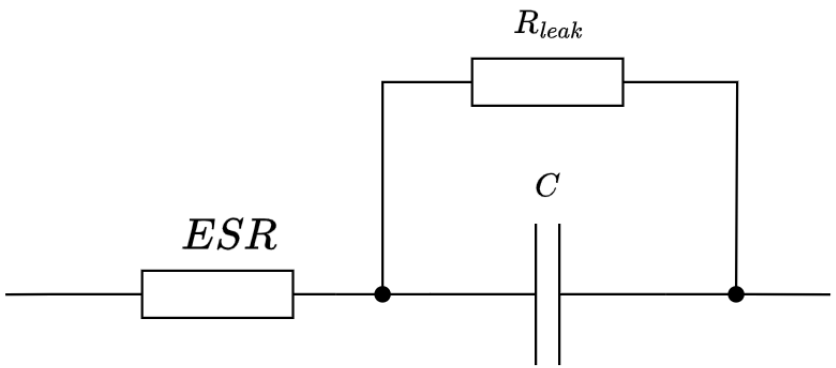


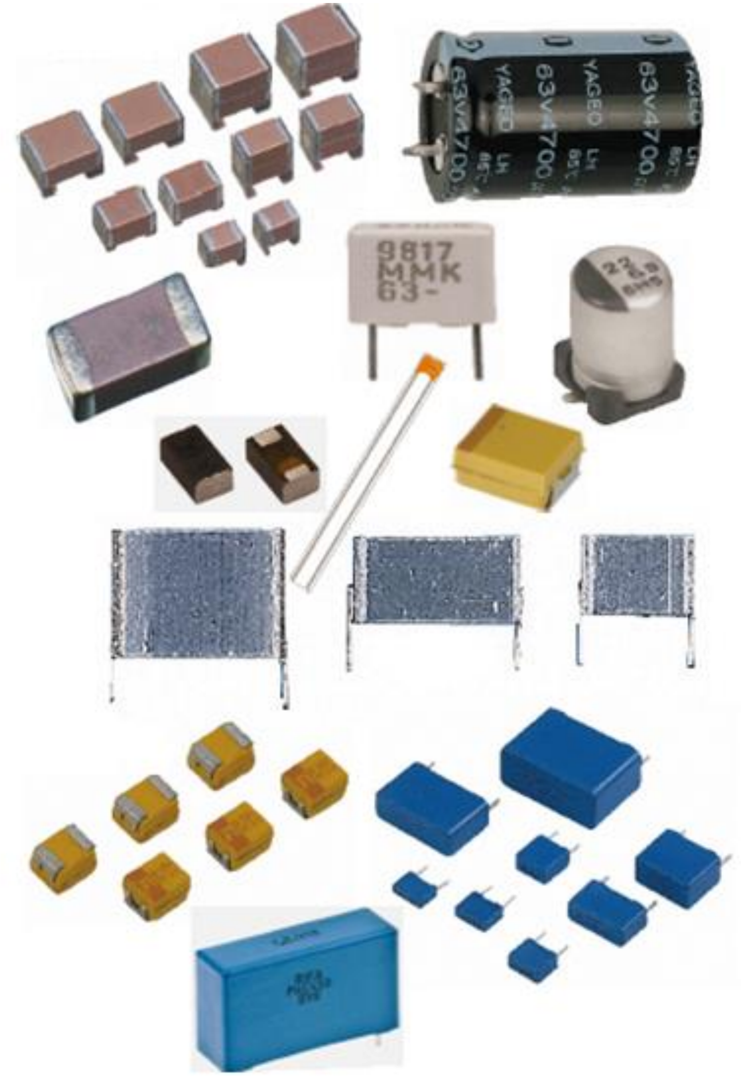
- In the case of real capacitors, the space between the plates is filled not only with a vacuum (air) but also with a dielectric material; thus, the actual capacitance is determined by the geometric parameters:

$$C = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

Capacitors in practice

Real capacitor:

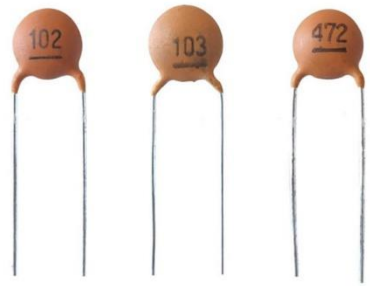




- Important properties to consider when selecting:
- Rated voltage
- Rated capacitance
- Pulse stress
- ESR value (ESR = Equivalent Series Resistance)
- Rleakage = leakage resistance

Electrolytic, film, and ceramic capacitors

- The aluminum electrolytic capacitor (or simply ELKO) belongs, by its construction, to the group of capacitors with liquid electrolyte.
- The electrodes are made of aluminum foil, one of which is coated with an insulating layer (aluminum oxide), and between them there is a separator paper impregnated with liquid electrolyte.
- They are typically used at low frequencies, generally provide high capacitance values (several thousand to tens of thousands of μF), have a cylindrical shape, and are polarity-dependent.
- Their lifetime is more limited due to the electrolyte.
- In film capacitors, as the name suggests, a thin plastic film serves as the dielectric.
- This dielectric film can be made from various plastics (such as PP, PPS, or PET).
They are generally characterized by high insulation resistance, excellent current and pulse handling capability, and good capacitance stability.
- Their voltage range can extend from a few tenths of a volt up to several hundred volts.
- Ceramic capacitors are typically used in high-frequency circuits due to their advantageous properties.
- They are characterized by low ESR values and relatively low capacitance (from a few pF up to several tens of μF).
- Their dielectric is ceramic. Their voltage range is wide, typically from a few volts up to several kilovolts.
- They are usually available in surface-mount form (SMD – Surface-Mount Technology), in small sizes.



Connecting Capacitors

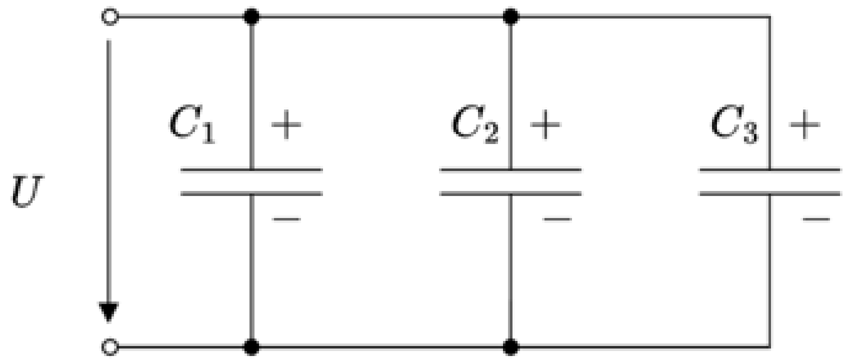
Parallel connection

$Q = UC$

$Q_e = UC_1 + UC_2 + UC_3$

$\frac{Q_e}{U} = C_1 + C_2 + C_3$

$\boldsymbol{C_e = C_1 + C_2 + C_3}$

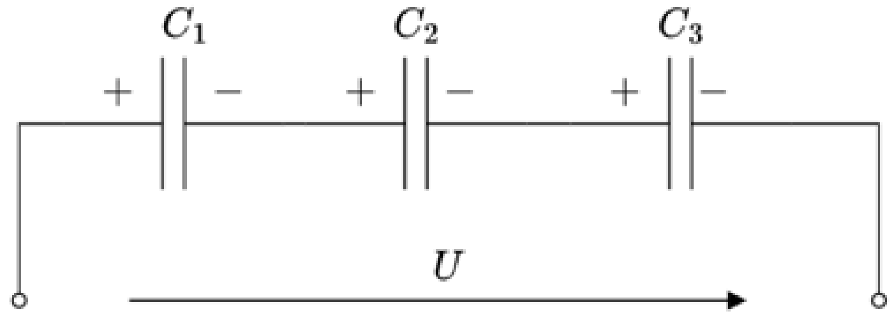


Serial connection

$U = U_1 + U_2 + U_3$

$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

$\boldsymbol{\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$

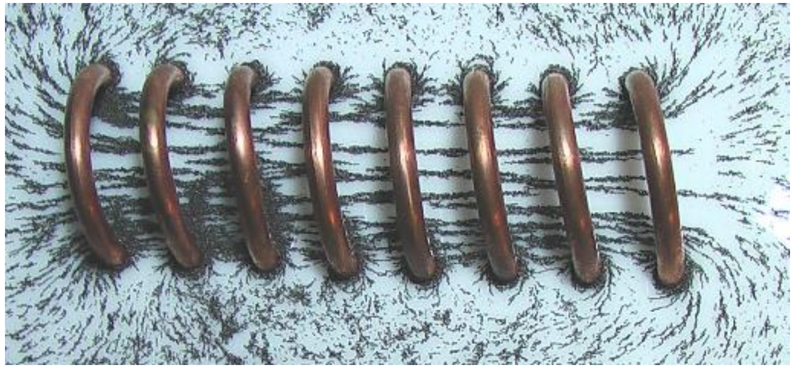


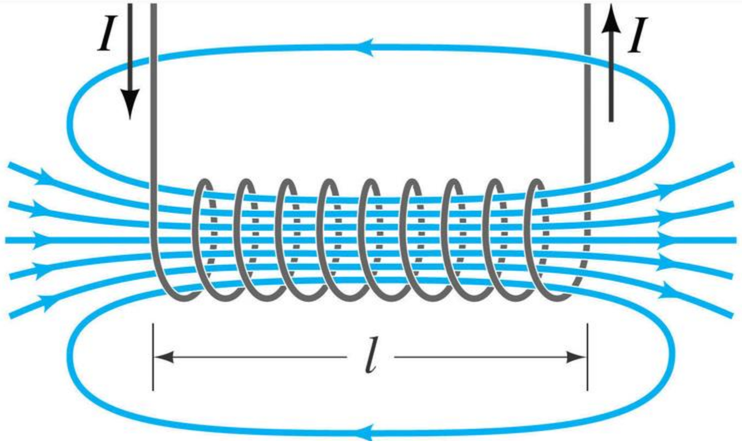
Inductances – Introduction –

- Coils (or inductors) operate based on the magnetic field.
- In the simplest case, a wire is wound onto a bobbin (coil former), turn by turn, placed next to each other as tightly as possible.
  This construction is referred to in the literature as a solenoid coil.
- Thus, the coil is associated with the magnetic field and is used for generating and storing magnetic energy.
- Just as capacitance is the characteristic parameter of capacitors, a coil also has its own parameter, called self-inductance, which is defined by the following relationship:

$$L = \frac{\Psi}{I}$$

- where Ψ is the magnetic flux and *I* is the current flowing through the coil.
- Magnetic flux is the number of magnetic field lines passing through a given surface. In practice, the above relationship indicates the extent to which a current can generate magnetic flux in a given circuit.
- The unit of self-inductance is the henry [H], while the unit of flux is (Vs) or (Wb) (weber).In practice, 1 H is a very large inductance, so fractions of it are used: mH, μH, nH.





Capacitor Turn-On and Turn-Off Phenomena

- At the moment of switching on, the system is energy-free (U_c = 0V)
- After switching on, according to Kirchhoff's law:

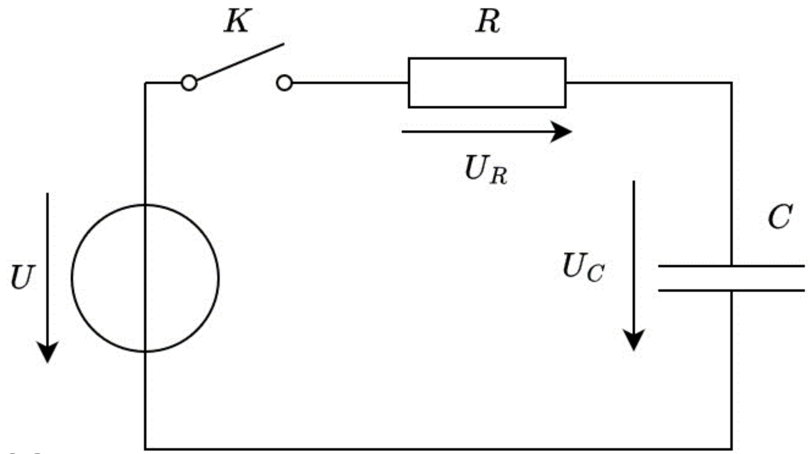
$$U = U_R + U_c$$

- Since at the moment of switching on U_c = 0V, therefore $U = U_R$, hence:

$$I_{max} = \frac{U}{R}$$

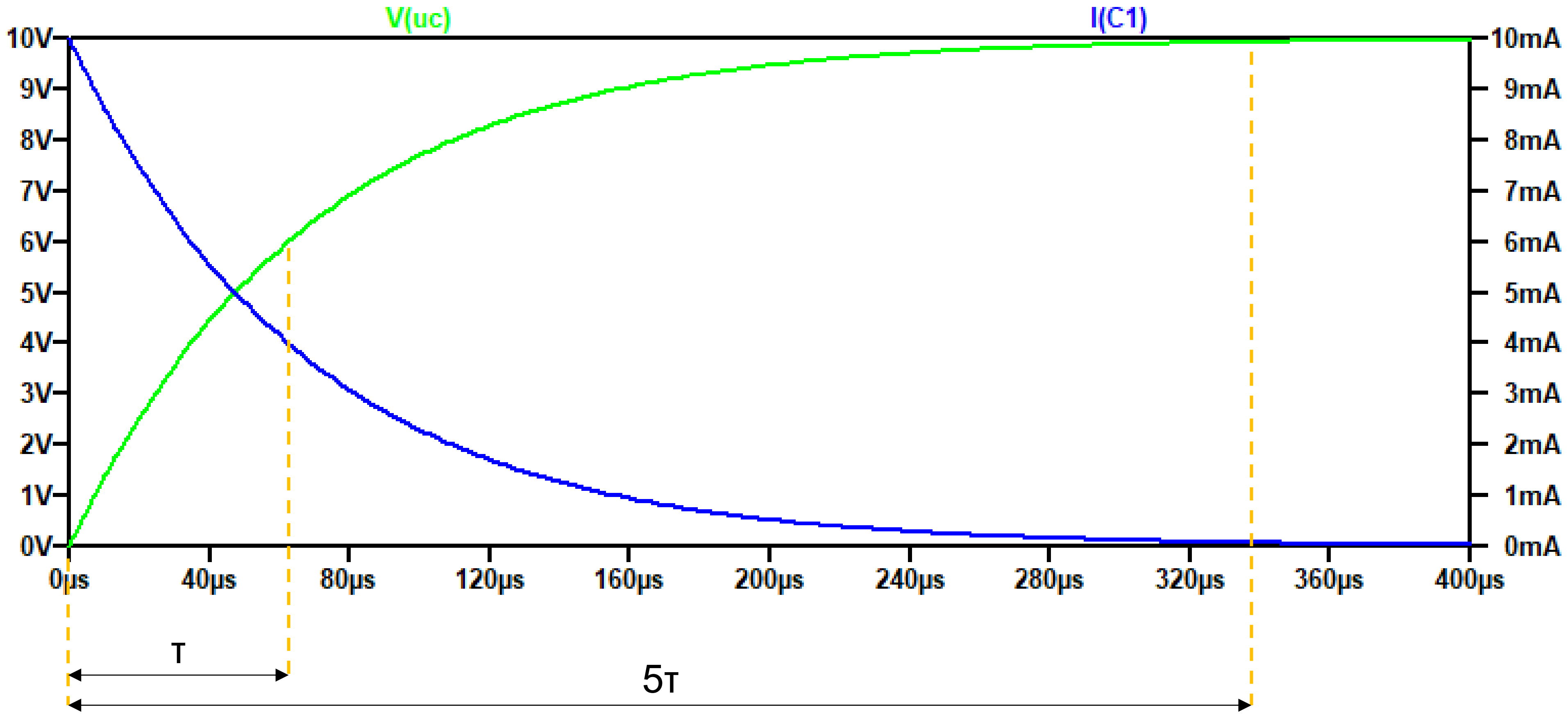
- During the charging (transient process) U_R and I decrease, while U_C increases
- The time constant (τ) of the system is the time required for the capacitor to charge up to 63% of the applied voltage.
- The process is considered complete (fully charged) after a time of 5τ.
- The voltage across the capacitor at any given moment can be calculated using:

$$u_C = U\left(1 - e^{-\frac{t}{\tau}}\right)$$



$\tau = RC$

Time functions



R=1kΩ and C=68nF

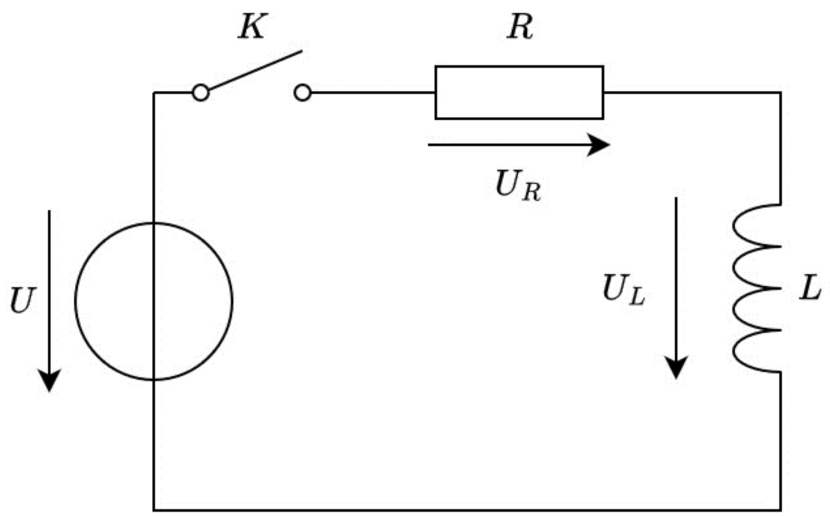
Coil turn-on and turn-off phenomena

- At the moment of switching on, the inductor is **energy-free**, i.e., the current is (I_L = 0A)
- At the time of switching I = 0A, so $U_L = U$
- During the charging (transient) process, $U_R$and I increase, while U_L decreases.
- The time constant of the system (τ) is the value at which the inductor current reaches approximately 63% of its maximum.
- The maximum current in the circuit is:

$$I_{max} = \frac{U}{R}$$

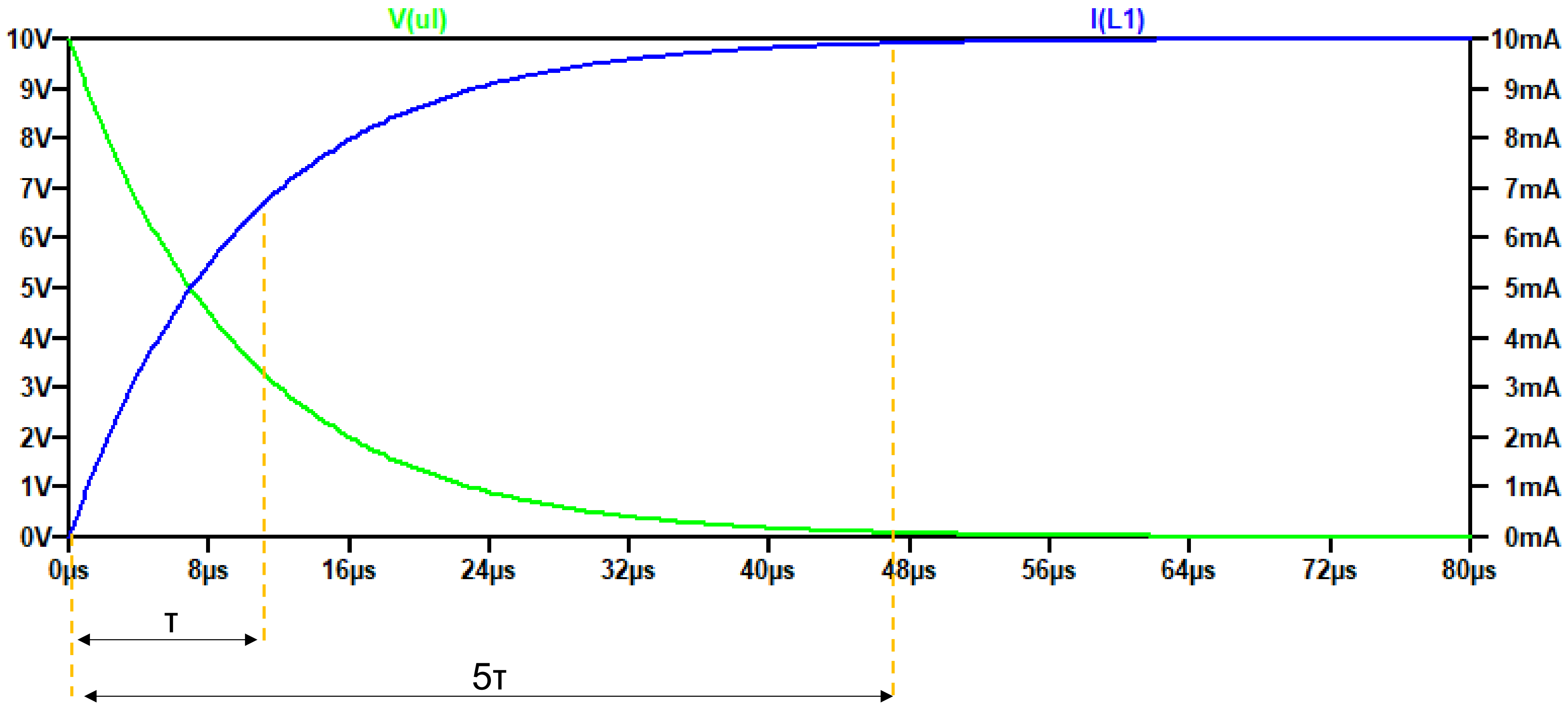
- The process is considered complete after 5τ.
- The current through the inductor at any time can be calculated as:

$$i_L = \frac{U}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$



τ=L/R

Time functions

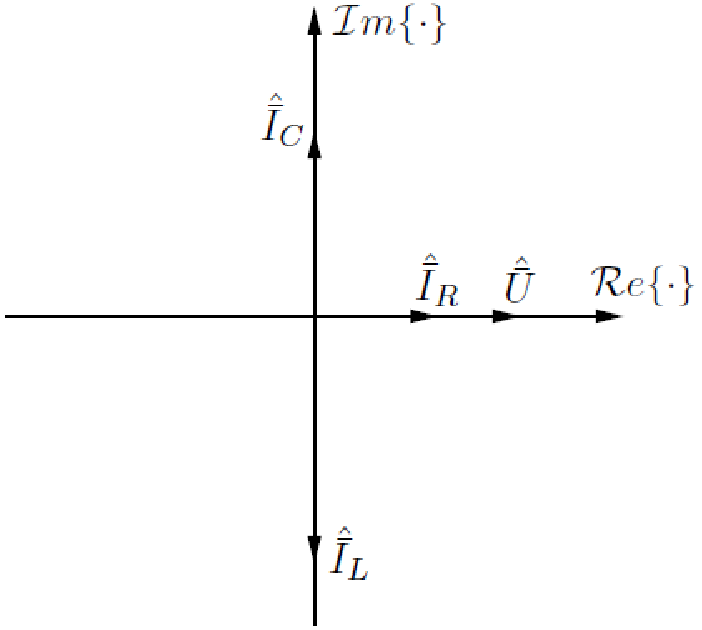


R=1kΩ and L=10mH

Impedance, Reactance – Introduction –

- Previously, we examined that both capacitors and inductors can be described by a parameter with ohmic characteristics, representing a relatively small value that is frequency-independent.
- From an AC (alternating current) perspective, both the capacitor and the inductor have a frequency-dependent property called reactance. Reactance is also measured in [Ω], but its magnitude and phase angle [°] depend on the frequency of the applied signal.
- In AC analysis, we do not talk about resistances, but about impedances.
- Impedance is the "resistance" of a circuit to alternating current.
  It is denoted by Z and measured in [Ω].
- The impedance consists of two parts:
  - Resistance (Ohmic) part – causes "real" (active) power dissipation
  - Reactive part – causes reactive power, storing energy and returning it
- Mathematically:

$$Z = R + jX$$

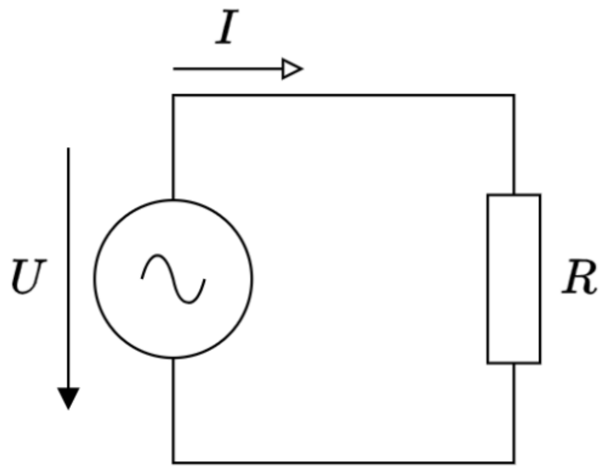


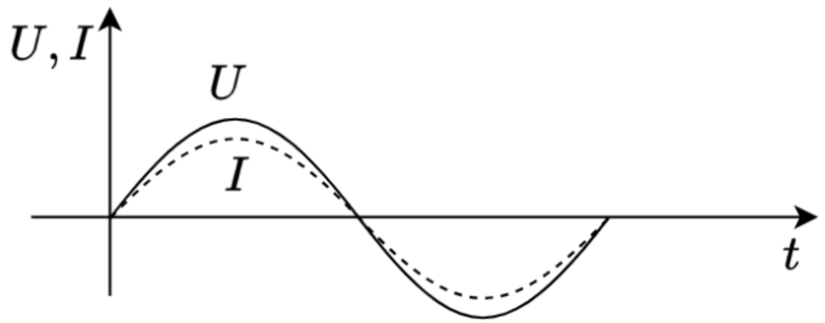
Resistance in an AC circuit

- Let's connect a resistor to a a voltage source of the form $u = \widehat{U}\sin(\omega t)$!
- If the circuit contains only this ohmic load, then the current flowing through the resistor is given by Ohm's law:

$$i = \frac{u}{R} = \frac{\widehat{U}}{R}\sin(\omega t)$$

- It can be observed that the change in current across the resistor over time is also sinusoidal; that is, the voltage and current across the resistor are in phase and correspond to the temporal variation of the excitation. The power dissipated across the resistor is effective (thermal) power.





Inductance in an AC circuit

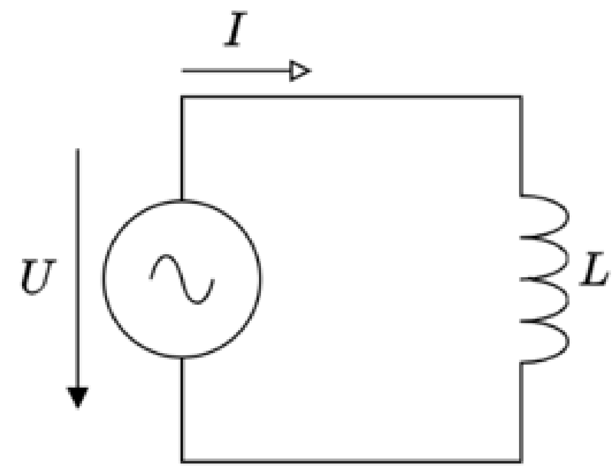
- Let us connect an inductor to an AC generator with a sinusoidal waveform!
- Lenz's law states that if the current in a coil changes over time, an induced voltage is generated that depends on the rate of change of the current and the self-inductance of the coil, that is

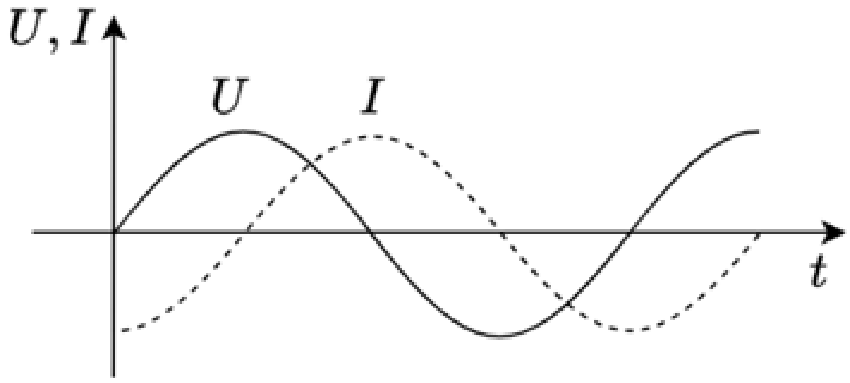
$$u_i(t) = L\frac{di}{dt}$$

- Our source is therefore sinusoidal. To find the time variation of the coil voltage, we substitute the time-dependent function of the generator current into the above formula. Accordingly, the coil voltage is:

$$u_i(t) = L\frac{di}{dt} = L\frac{d\left(\hat{I}_g sin(\omega t)\right)}{dt} = \hat{I}_g Lcos(\omega t)$$

- In other words, the voltage measured across the coil will have a sinusoidal waveform. What does this mean? The current does not follow the generator voltage in phase; it lags behind the voltage by 90°.





Inductive reactance

- Let’s repeat the previous experiment and try to calculate the resistance of the coil (for example, at a discrete frequency)!
- Resistance can be calculated as the ratio of the voltage and current measured across the coil. If we check the result with an ohmmeter, the two resistance values will differ significantly: the value calculated using Ohm’s law will be greater than the measured value! Something else is reducing the current!
- If we repeat the measurement by increasing the frequency of the excitation signal, the resistance value calculated from the measured voltage and current will be higher.
- The explanation for this phenomenon is that the coil also has a frequency-dependent "resistance," which is called inductive reactance.
- Its symbol is X_L, and its unit of measurement—since it is resistance—is the ohm (Ω).
- The magnitude of the inductive reactance is directly proportional to the frequency, that is

$$X_L = \omega L = 2\pi f L.$$

- In other words, the coil behaves as a short circuit at low frequencies and as an open circuit at very high frequencies (due to its infinitely high resistance).

Capacitance in an AC circuit

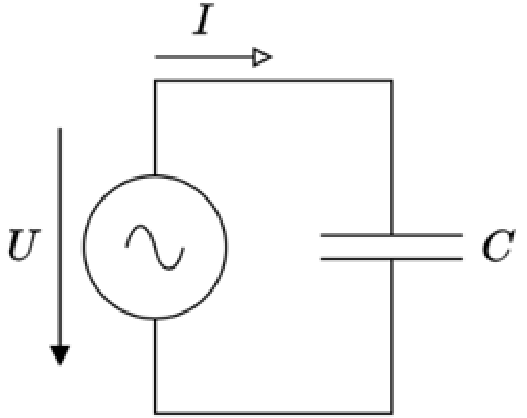
- We follow a similar procedure and way of thinking as we did with the coil.
- Let's connect a capacitor to an AC generator with a sinusoidal waveform!
- The current through a capacitor depends on its capacitance and the rate of change of the voltage across it over time, that is

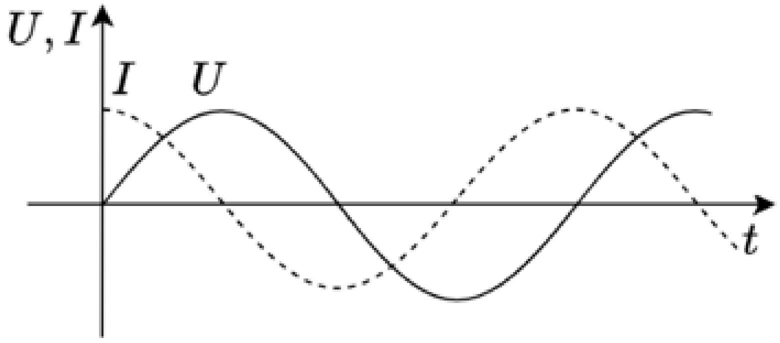
$$i_C(t) = C\frac{du}{dt}$$

- The time-dependent current through the capacitor can be calculated by substituting the time-dependent voltage of the generator into the above formula, that is,

$$i_C(t) = C\frac{du}{dt} = L\frac{d\left(\hat{U}_g sin(\omega t)\right)}{dt} = \hat{U}_g \cdot C \cdot cos(\omega t).$$

- We can conclude that the time-dependent current through the capacitor will follow a cosine waveform, meaning that the capacitor current will lead the voltage by 90°.
- This phase difference is due to the capacitor's ability to store charge: during the first quarter of the period, a charging current flows through the capacitor until the voltage reaches its peak value. As the voltage decreases, a discharging current flows, which reaches its maximum value when the voltage is zero.





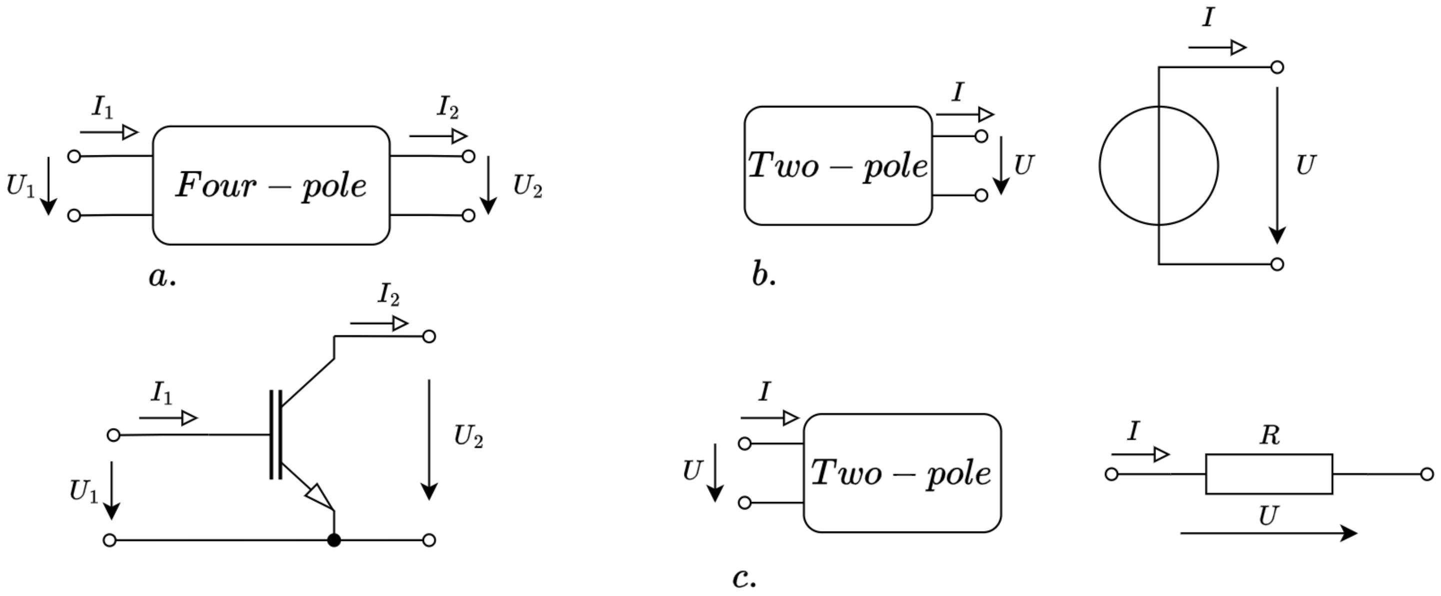
Capacitive reactance

- Just as with an inductor, the ratio U/I will not yield the same value as a resistance meter when a capacitor is connected to an AC circuit.
- The reactance of a capacitor does not follow a linear relationship, but rather a function of the form $1/x$, that is,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

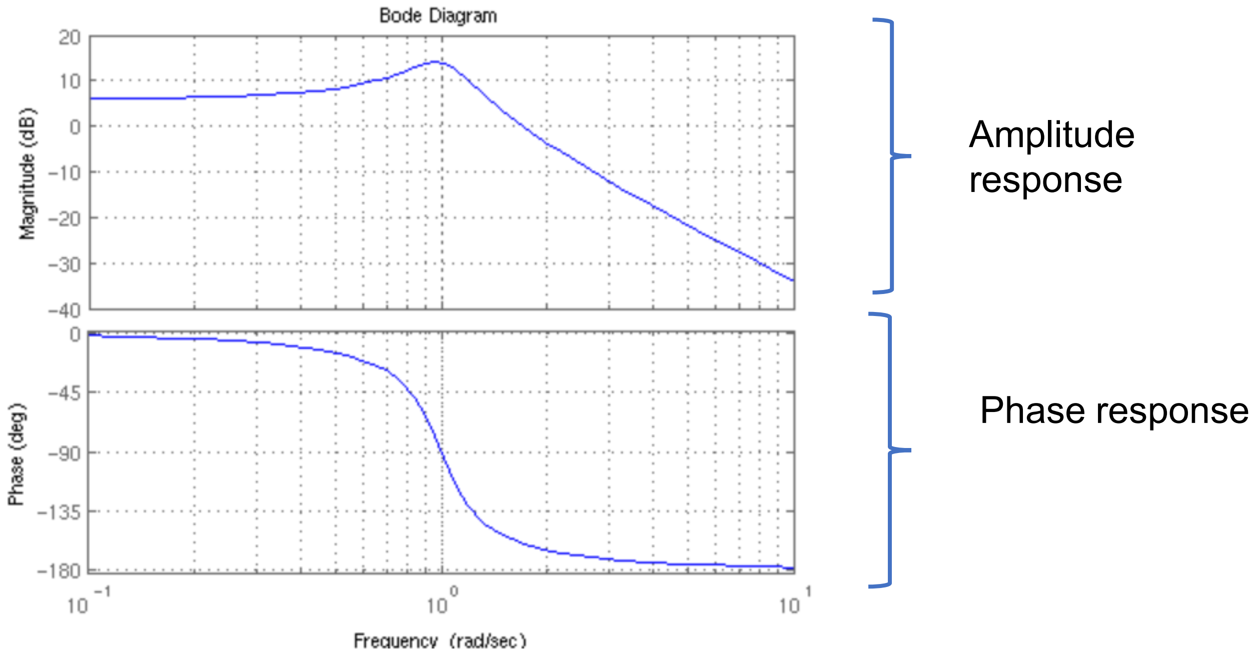
Transfer Characteristics – Introduction

- Systems can be classified as follows:
  - Based on the ratio of output to input energy
  - By linearity
  - Based on electrical poles, etc.
- Based on the latter, the systems can be modeled as two-pole and four-pole systems
- We can define so-called system description functions, such as the transfer function, which describes the relationship between the output and the input. That is, U_2/U_1



Bode plot

- Transfer characteristics are typically plotted as a function of frequency; in other words, we examine how our circuit behaves within a given frequency range.
- The impedance of a circuit containing reactive components is frequency-dependent, so the magnitude of the current flowing through it and the voltage drop, as well as the phase angle between the two, can vary. Therefore, when plotting these values, we must examine both the amplitude (peak value) and the phase angle.
- The Bode plot shows the amplitude and phase responses in a separate graph.
- The horizontal scale is logarithmic, allowing us to plot data over a wider range.

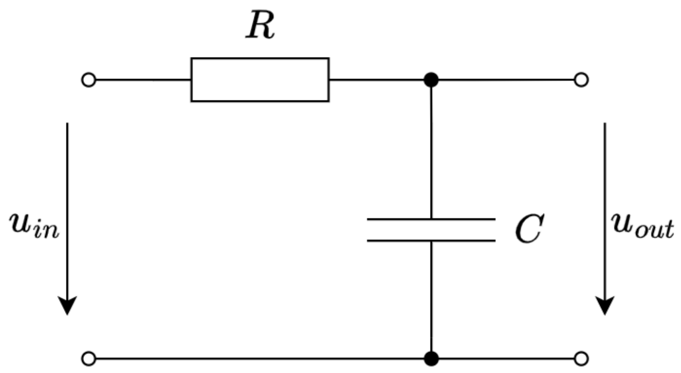


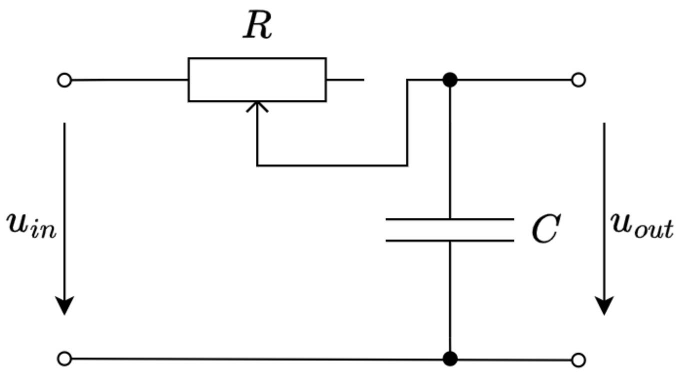
Complex AC Circuits – An Illustrative Example-

- The transfer function describes the behavior of the system; in practice, we examine how the system responds to a given excitation:

$$H = \frac{u_{out}}{u_{in}}$$

- If we examine the phase and amplitude values at a finite number of frequency points, we obtain the transfer function
- Let's take a series RC circuit as an example! Let R be 1kΩ and C be 100nF. Let's examine it at a few frequency points! Let the input voltage have an amplitude of 1V!



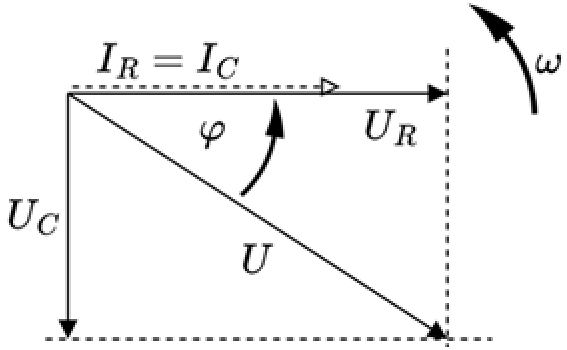


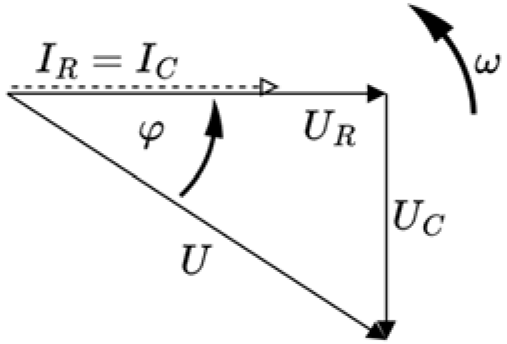
Calculation principle

- In DC circuits, the equivalent resistance in a series connection is the mathematical sum of the resistances that make up the circuit. It can also be established that the current will be the same, while the voltages across the components will differ (except in the case where the resistances are equal), that is

$$\begin{cases} R_e = R_1 + R_2 + \cdots R_n \\ I_e = I_{R1} = I_{R2} + \cdots I_{Rn} \\ U_{R1} \neq U_{R2} + \cdots U_{Rn} \end{cases}$$

- In the case of a network containing reactive elements, this happens differently!
- The basic principle still holds: in a series circuit, the current has the same value through all components, but a quadratic relationship will develop between the voltages. Why?
- The voltage and current across the resistor are in phase; there is no phase lag (or lead) between them.
- There is a 90° phase shift between the current and voltage across the capacitor; the current lags behind the voltage
- There is a quadratic relationship between the voltages!





Calculation results

- We know that the circuit is a frequency-dependent voltage divider, so (recalling the DC voltage divider):

$$H = \frac{u_{ki}}{u_{be}} = \frac{X_C}{Z}$$

- X_C can be calculated:

$$X_c = \frac{1}{2\pi f C}$$

- The logarithm of the transfer function in dB:

$$h = 20\log(H)$$

- Let's examine this in detail at 1 kHz :

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi 100Hz 100nF} = 1{,}5915k\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{1^2 k\Omega + 15{,}9^2 k\Omega} = 1{,}87k\Omega$$

$$H = \frac{u_{out}}{u_{in}} = \frac{X_C}{Z} = \frac{1{,}5915k\Omega}{1{,}87k\Omega} = 0{,}846$$

$$h = 20\log(H) = 20\log(0{,}61414) = -1{,}44dB$$

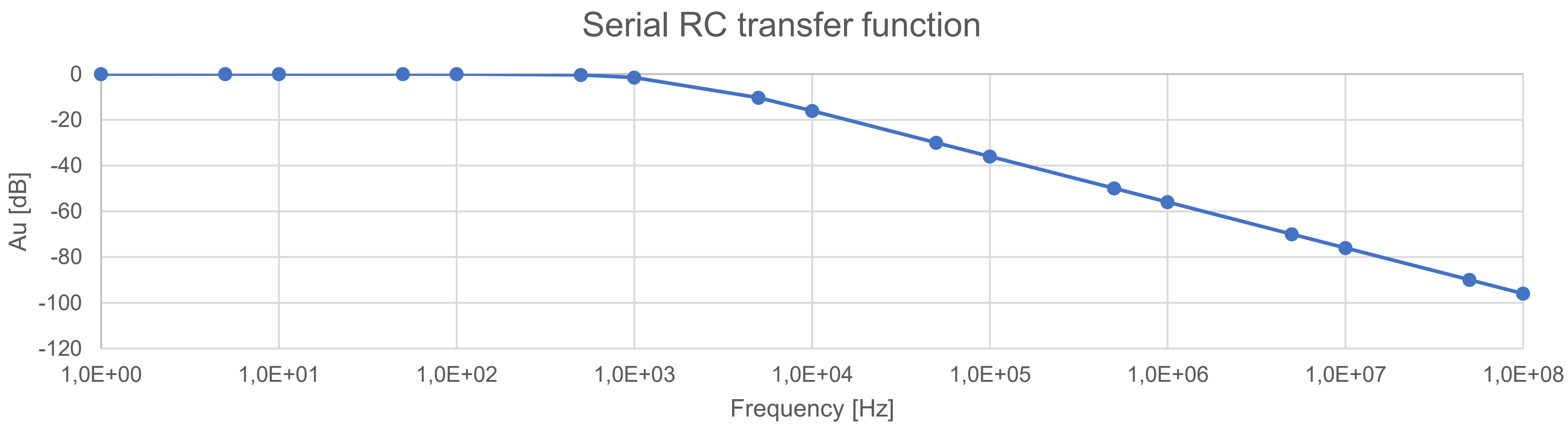
$$\varphi = -\tan^{-1}\left(\frac{R}{X_C}\right) = -\tan^{-1}\left(\frac{1k\Omega}{1{,}5915k\Omega}\right) = -32{,}2°$$

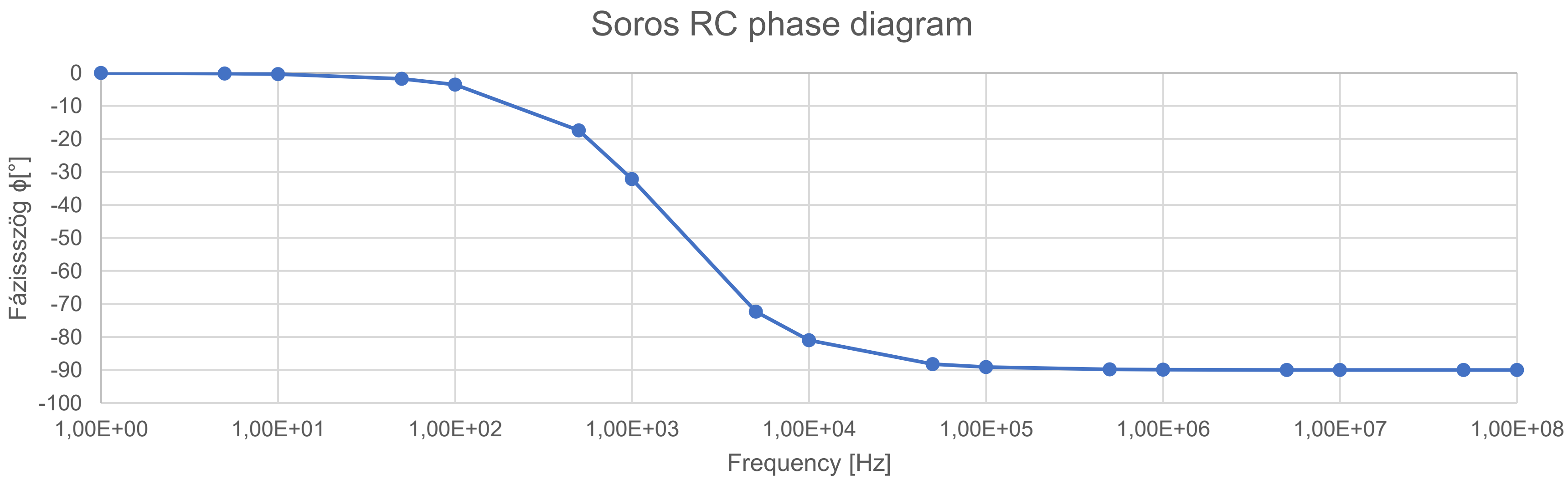
Calculated values at a few additional discrete frequencies

R	1000						
C	1,00E-07						
	f [Hz]		Xc [Ω]	Z [Ω]	H	H [dB]	φ [°]
	1		1,59E+06	1591597	1,00E+00	-1,7E-06	-3,60E-02
	5		3,18E+05	318320,8	1,00E+00	-4,3E-05	-1,80E-01
	10		1,59E+05	159162,8	1,00E+00	-0,00017	-3,60E-01
	50		3,18E+04	31847,63	1,00E+00	-0,00428	-1,80E+00
	100		1,59E+04	15947,35	9,98E-01	-0,01711	-3,60E+00
	500		3,18E+03	3336,572	9,54E-01	-0,40875	-1,74E+01
	1000		1,59E+03	1879,675	8,47E-01	-1,445	-3,21E+01
	5000		3,18E+02	1049,441	3,03E-01	-10,3619	-7,23E+01
	10000		1,59E+02	1012,587	1,57E-01	-16,072	-8,10E+01
	50000		3,18E+01	1000,507	3,18E-02	-29,9471	-8,82E+01
	100000		1,59E+01	1000,127	1,59E-02	-35,9644	-8,91E+01
	500000		3,18E+00	1000,005	3,18E-03	-49,9428	-8,98E+01
	1000000		1,59E+00	1000,001	1,59E-03	-55,9634	-8,99E+01
	5000000		3,18E-01	1000	3,18E-04	-69,9427	-9,00E+01
	10000000		1,59E-01	1000	1,59E-04	-75,9633	-9,00E+01
	50000000		3,18E-02	1000	3,18E-05	-89,9427	-9,00E+01
	100000000		1,59E-02	1000	1,59E-05	-95,9633	-9,00E+01

Note: The calculation shows that the reactance of the capacitor decreases as the frequency increases.

Plotting results as a function of frequency (Bode plot)





AC Power

- In the case of DC, power is equal to the product of current and voltage, that is $P = U \cdot I$
- To determine AC power, the instantaneous values of the corresponding current and voltage are generally multiplied together, that is,

$$p(t) = u(t)i(t)$$

- In a resistive load, there is no phase difference between the current and voltage signals. In other words, if the load is purely resistive, the resulting power is real, and the power curve lies above the time axis (*t*). The symbol for active power is P, and its unit is [*W*]. Active power can be calculated as follows:

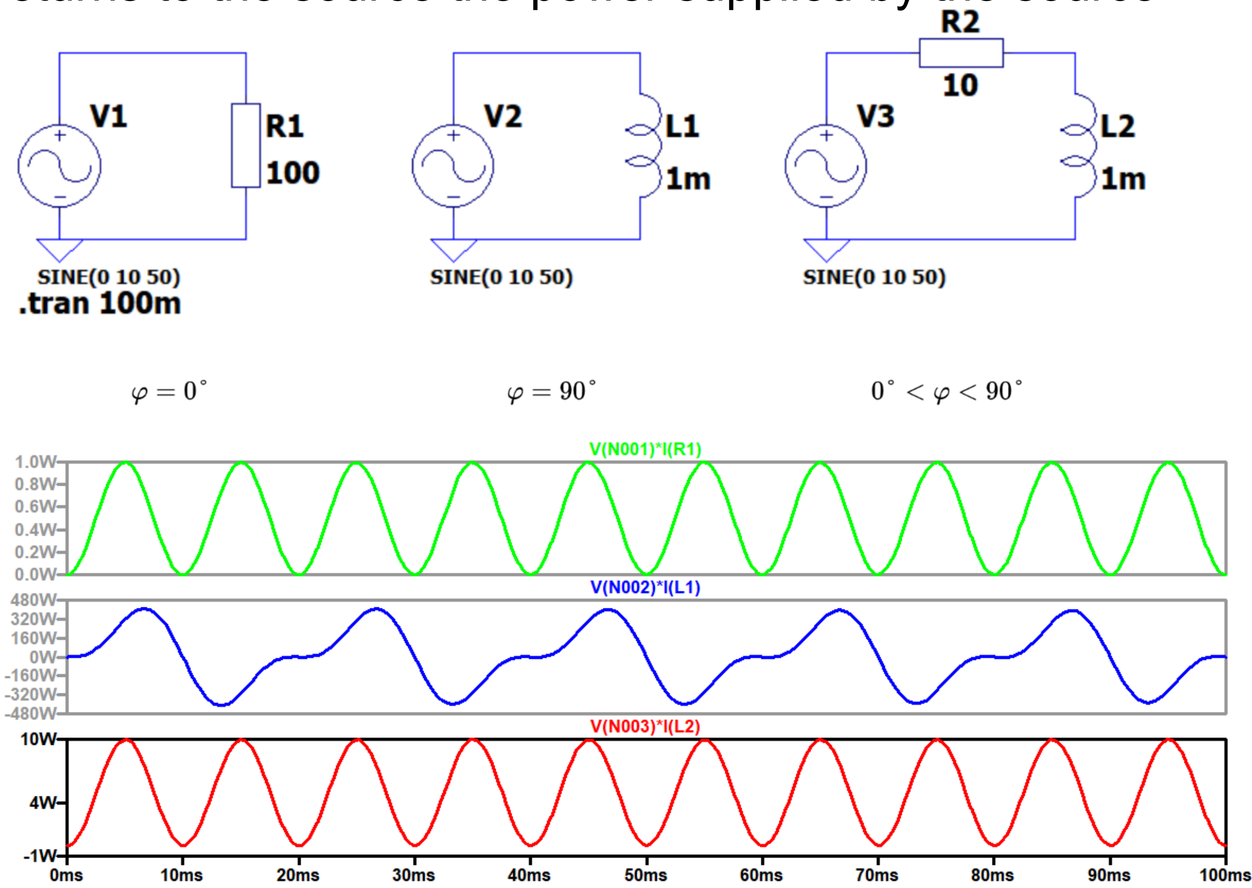
$$P = UI\cos(\varphi)$$

- where U and I are RMS values, and $\cos(\varphi)$is the so-called power factor, i.e., the cosine of the phase difference between the current and the voltage.
- Let us examine the above relationship for the case when the load is resistive. There is no phase difference between the current and the voltage: substituting into the above formula, $\cos(0°) = 1$,thus our reasoning is correct, since only the product $U \cdot I$ remains in the formula..

Power ratings for reactive components

- When connected to an AC network, a capacitor and a coil behave as reactors; that is, there is a 90° phase shift between the current and voltage flowing through them (the current lags behind the voltage in the coil, while it leads the voltage in the capacitor).
- The instantaneous power value is zero: the reactance returns to the source the power supplied by the source during one half-cycle.
- In practice, real-world loads are a combination of ohmic, inductive, and capacitive loads, meaning that the phase angle lies between 0 and 90 degrees
- Reactive power is symmetrical about the time axis (t). The symbol for reactive power is Q, and its unit is (VAR) (volt-ampere reactive). The reactive power can be calcualted:

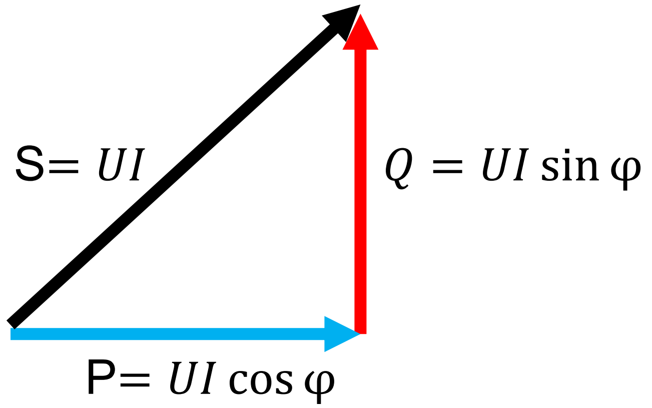
$$Q = UI\sin(\varphi)\,.$$

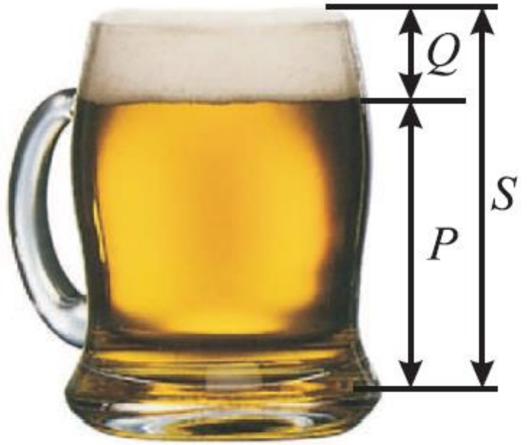


Relationships between AC power

- Active power (P) and reactive power (Q) together provide apparent power, which is denoted by S and measured in (VA) (volt-amperes). The relationship between these power components is as follows

Power	Sign	Calculation	Unit
Apparent	S	$S = UI$	VA
Active	P	$P = UI\cos(\varphi)$	W
Reactive	Q	$Q = UI\sin(\varphi)$	VAr





Source: Dr. Kuczmann Miklós – Kovács Gergely: Villamosságtan

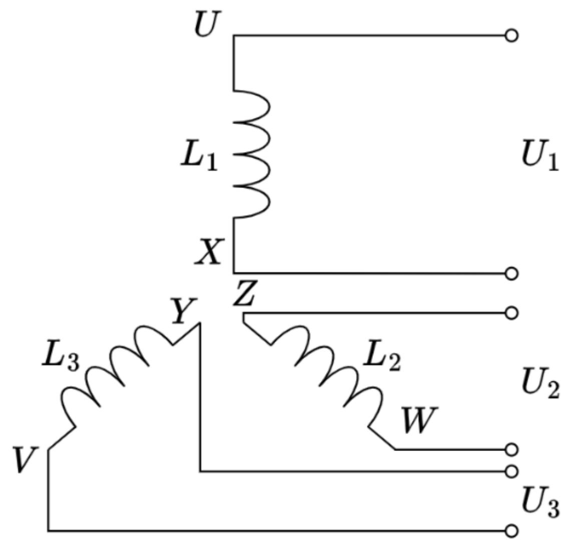
Power factor- $\cos(\varphi)$

- In the case of active power, in addition to the product $U \cdot I$, another term also appears: $cos(\varphi)$.
- This is the so-called power factor, which indicates the ratio between active (P) and reactive (Q) power. The larger this value, the greater the amount of active power.
- One extreme case is when $cos(\varphi) = 1$,meaning that only active power is present.
- Why is it undesirable to have a low $cos(\varphi)$and high reactive power? For example, in the case of an electric motor, the useful power is taken from the grid and converted into rotational motion (this is the shaft power).
- Since electrical rotating machines are inductive, reactive power is also required. This is drawn from the grid but then returned to it, meaning it cannot be directly utilized.
- From a practical point of view, reactive energy causes additional losses. It must be transmitted just like active energy, but it unnecessarily loads the electrical power network.
- When paying an electricity bill, reactive energy must also be paid for. Therefore, it is desirable to have the power factor as high as possible, ideally equal to 1.
- To address this issue, so-called power factor correction (or phase compensation) must be applied, either by passive or active (PFC) methods.

Multphase system

- A system is called multiphase when multiple phases are available simultaneously, shifted in time relative to each other.
- In single-phase systems, we considered a single conductor loop rotating in a homogeneous magnetic field.
- We established that if the rotational speed is constant, a sinusoidally varying voltage is induced over time at the terminals of the loop.
- What happens if, instead of one conductor loop, we rotate three loops in the same homogeneous magnetic field, each shifted by 120° relative to the others?
- The result is induced voltages with identical frequency (period) and peak value, but shifted in time by 120°.

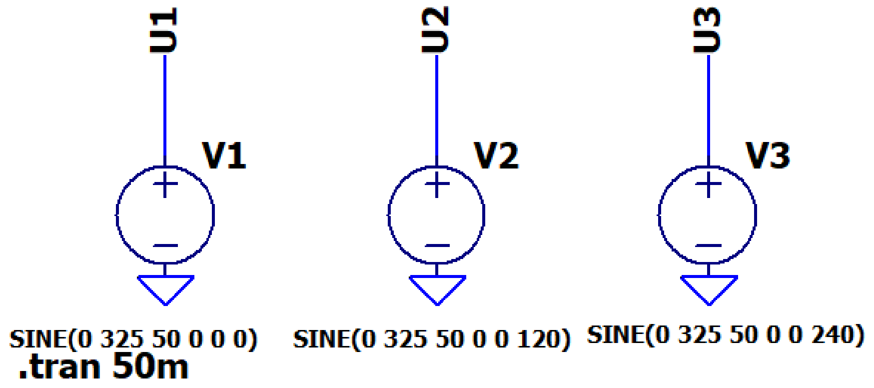
- The result is an induced voltage with the same frequency (period) and peak value, shifted by 120° in time. In a three-phase network, the individual windings are typically designated L1, L2, and L3 (in accordance with the MSZ EN 60617 / IEC 60617 standard). These are the phase windings
- Because of the three-phase system, each phase winding has two ends.
- The starts of the phase windings are labeled U, V, and W, while their ends are labeled X, Y, and Z.
- Accordingly, this configuration results in 6 wires.

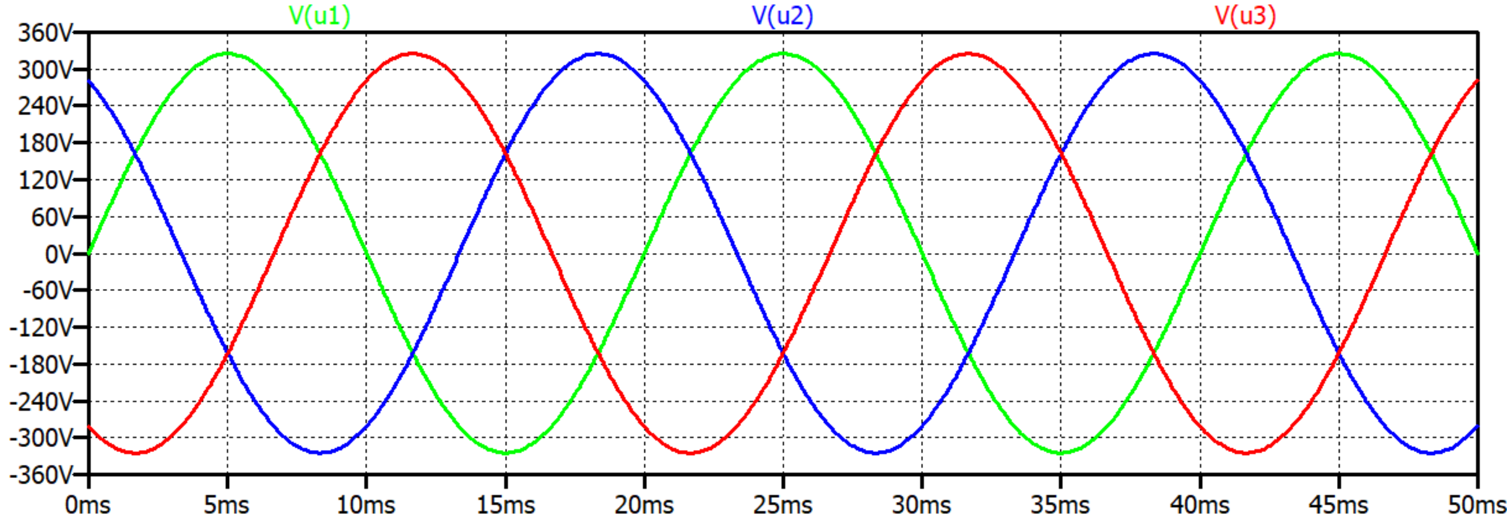


Interconnection of multiphase systems

- It is important to note that, regardless of the specific moment in time chosen, the sum of the voltages will be zero, that is:

$$\sum U = 0$$





- Summing the voltages at $t = 5ms$:

$$U_1 + U_2 + U_3 = 325V + (-163{,}69V) + (-161{,}281V) \cong 0$$

Number of wires

- The voltages of the generator are of equal magnitude (in Hungary, $\widehat{U} \cong 325V$) and the phase angle between them is 120°.
- In general multi-phase systems, the phase angle (φ) is given by

$$\varphi = \frac{2\pi}{n} = \frac{360°}{n}$$

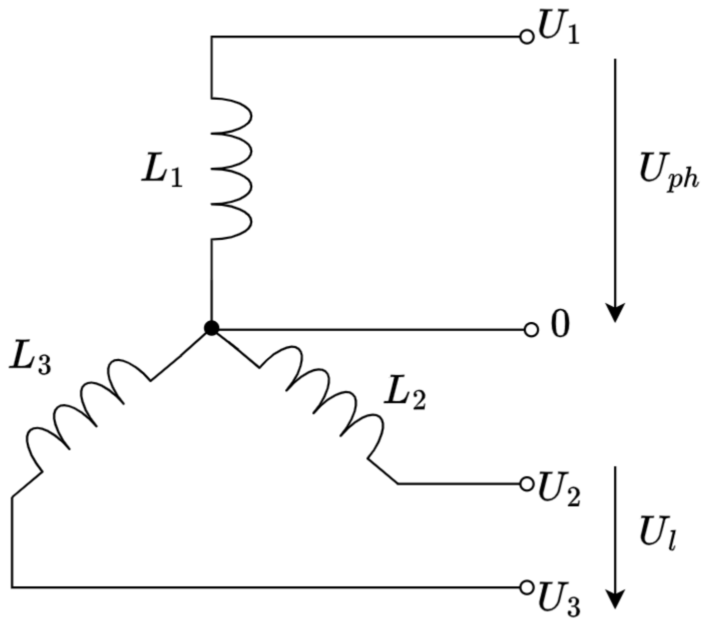
  where n denotes the number of phases in the system

- It can also be observed that, for a system with n phases, the number of wires is 2n (since each phase winding has two terminals).
- The number of wires can be reduced to (n+1) or n by connecting the phase windings. The two most important connection methods are star and delta (or triangle) connections.

Star Connection

- In a star connection, the ends of the three-phase windings are connected at a single point, which is called the star point (hence the name).
- The star point is often grounded, which is referred to as the neutral conductor. As a result, we measure phase voltage between the terminal of any phase winding (U, V, and W) and the star point, and line voltage between any two-phase windings (UV, UW, VW).
- In Hungary, the phase voltage is 230 V, while the line voltage is 400 V.
- The line voltage is √3 times the phase voltage, that is

$$U_l = \sqrt{3} U_{ph}.$$



Delta connection

- In a delta or three-phase connection, the individual phases are connected in series. The line voltages (the voltage between two phases) can be measured at the common connection points of the windings.
- In a delta connection, the line voltage and the phase voltage are equal. The line current is the vector sum of the currents in two adjacent phase conductors, that is,

$$I_L = \sqrt{3} I_{ph}.$$