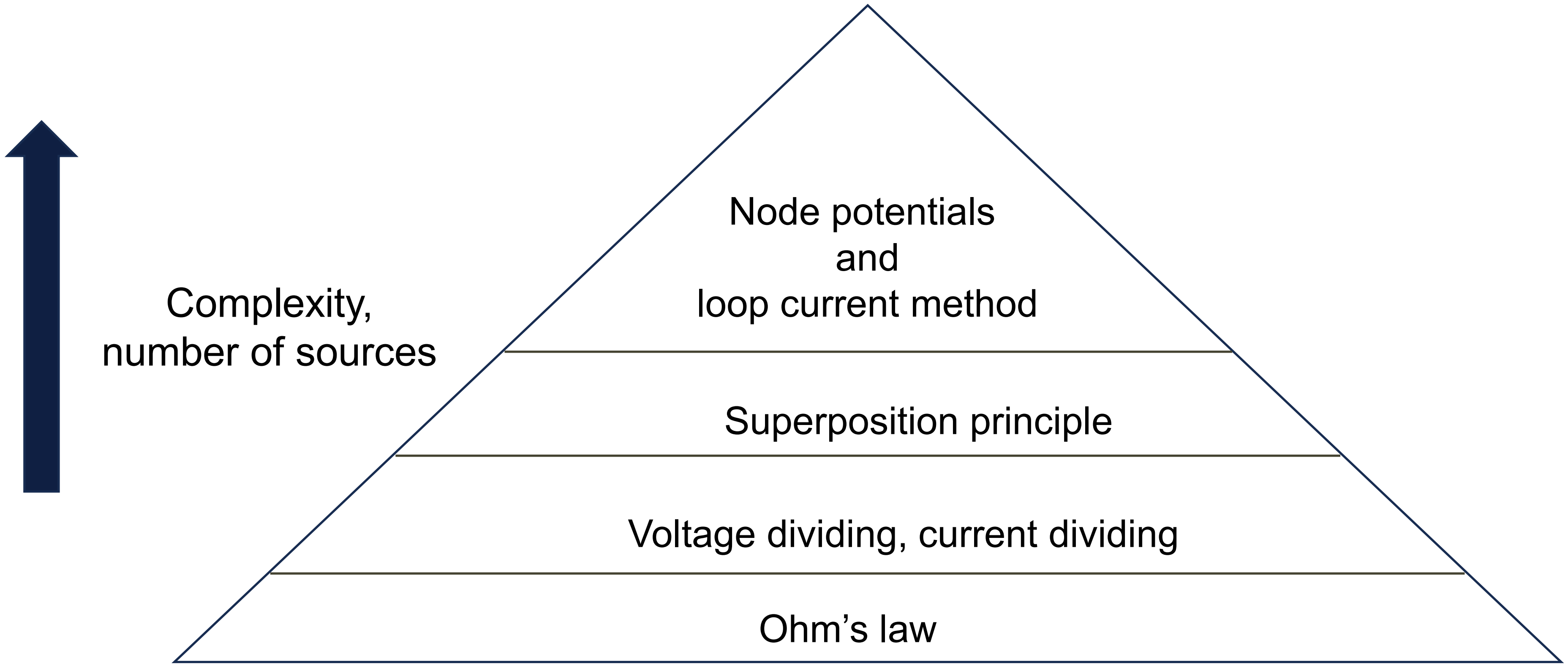
Network calculation methods

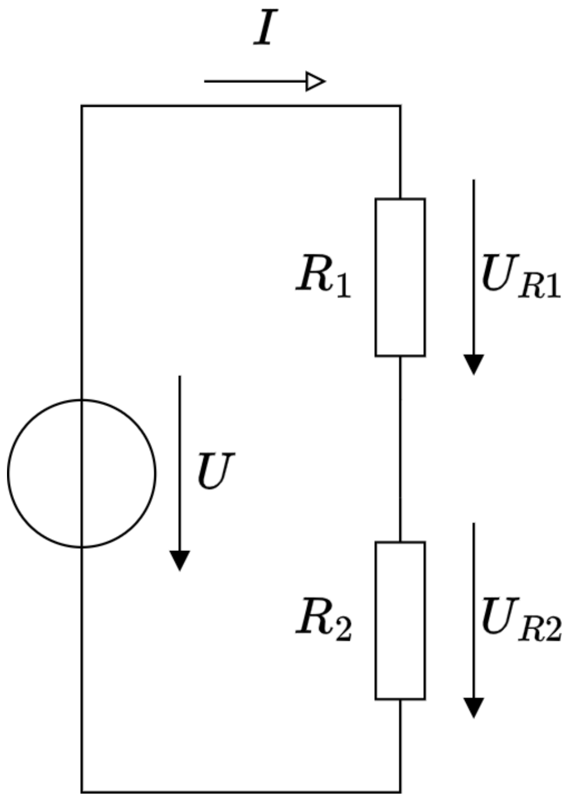
Review

- Overview of network calculation methods
- Voltage dividing
- Current dividing
- Ideal and real sources
- Norton és Thevenin theorem
- Complex network calculation methods: node potential and loop current methods

Circuit solution methods



Voltage dividing



- With the help of a voltage divider, we can produce a LOWER voltage from any given voltage (so $U_{R1} < U$ and $U_{R2} < U$)
- The equvivalent resistivity:

$$R_e = R_1 + R_2$$

- The current (using Ohm"s law):

$$I = \frac{U}{R_e}$$

- The voltage across R_1:

$$U_1 = I \cdot R_1$$

- Replacing the value of current:

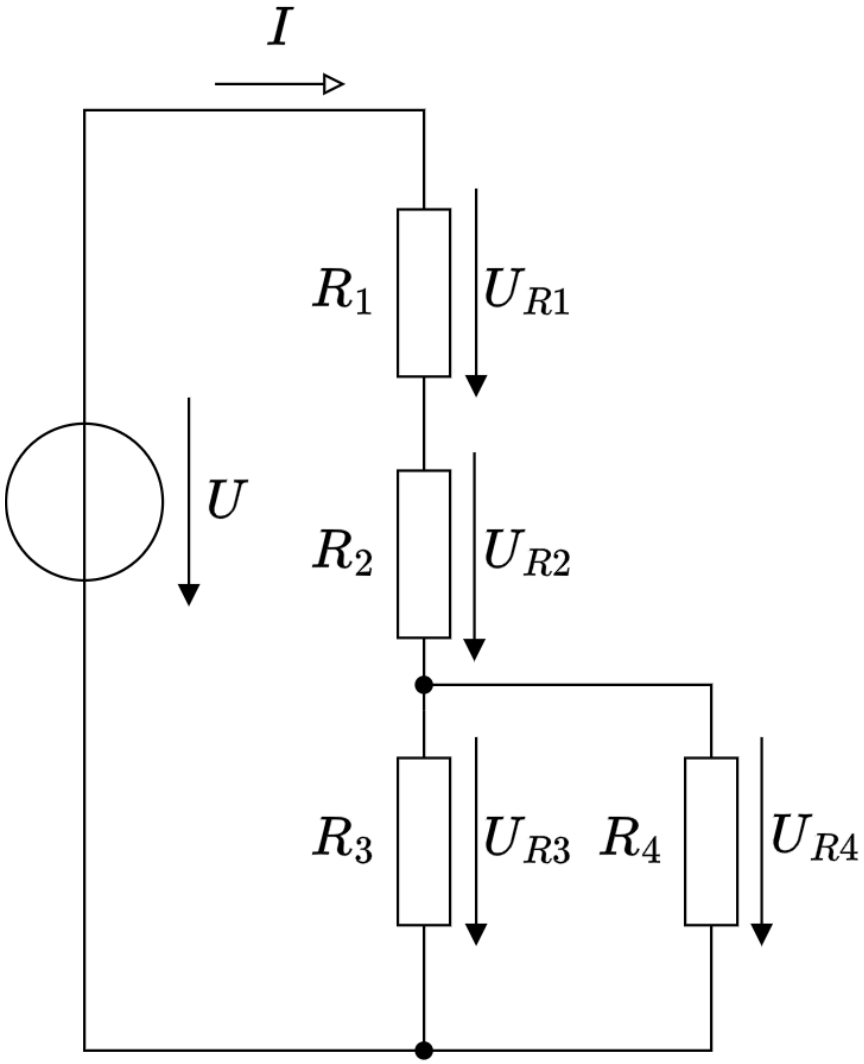
$$U_1 = U\frac{R_1}{R} \quad \text{so} \quad \frac{U_1}{U} = \frac{R_1}{R}$$

- In general:

$$Desired\ resistance\ voltage = Input\ voltage \cdot \frac{Where\ I\ search\ the\ voltage}{Total\ resistance^{*}}$$

*The total resistance always refers to the components to which the voltage to be distributed is connected.

Illustrative example



<u>Details:</u>
U = 36V; R_1 = 2kΩ; R_2 = 2kΩ; R_3 = 600Ω; R_4 = 1.2kΩ

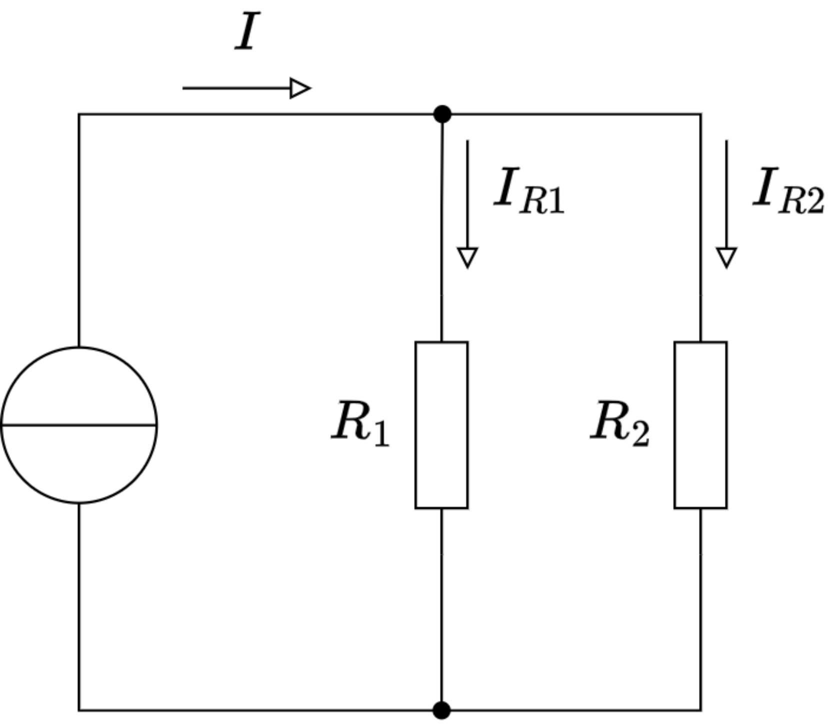
What is the voltage of R_3?

$$U_{R3} = U \cdot \frac{R_3 \otimes R_4}{(R_3 \otimes R_4) + R_1 + R_3} = 40V \cdot \frac{600\Omega \otimes 1{,}2k\Omega}{(600\Omega \otimes 1.2k\Omega) + 2k\Omega + 2k\Omega}$$
$$= 36V \cdot \frac{400\Omega}{4.4k\Omega} = \mathbf{3.27V}$$

What is the voltage of R_2 ?

$$U_{R2} = U \cdot \frac{R_2}{(R_2 \otimes R_4) + R_1 + R_3} = 40V \cdot \frac{2k\Omega}{(600\Omega \otimes 1.2k\Omega) + 2k\Omega + 2k\Omega}$$
$$= 36V \cdot \frac{2k\Omega}{4.4k\Omega} = \mathbf{16.36V}$$

Current dividing



- With the help of a current divider, any current can be divided into SMALLER parts (so $I_{R1} < I$; $I_{R2} < I$)
- In the simplest case, it can be created by connecting two resistors in parallel;
- The currents are distributed proportionally to the conductance: a smaller current flows through the higher resistance;
- The voltage is the same across parallel resistors, therefore :

$$U_1 = I_1 \cdot R_1 \quad \text{és} \quad U_1 = I_2 \cdot R_2$$

- So:

$$I_1 \cdot R_1 = I_2 \cdot R_2$$

- We know, that $\mathrm{I} = I_1 + I_2$ and $I_1 = I - I_2$ and $I_2 = I - I_1$
- Let's substitute the above relationship, for example, in place of I_1:

$$R_1 \cdot (I - I_2) = I_2 \cdot R_2$$

$$R_1 \cdot I - R_1 \cdot I_2 = I_2 \cdot R_2$$

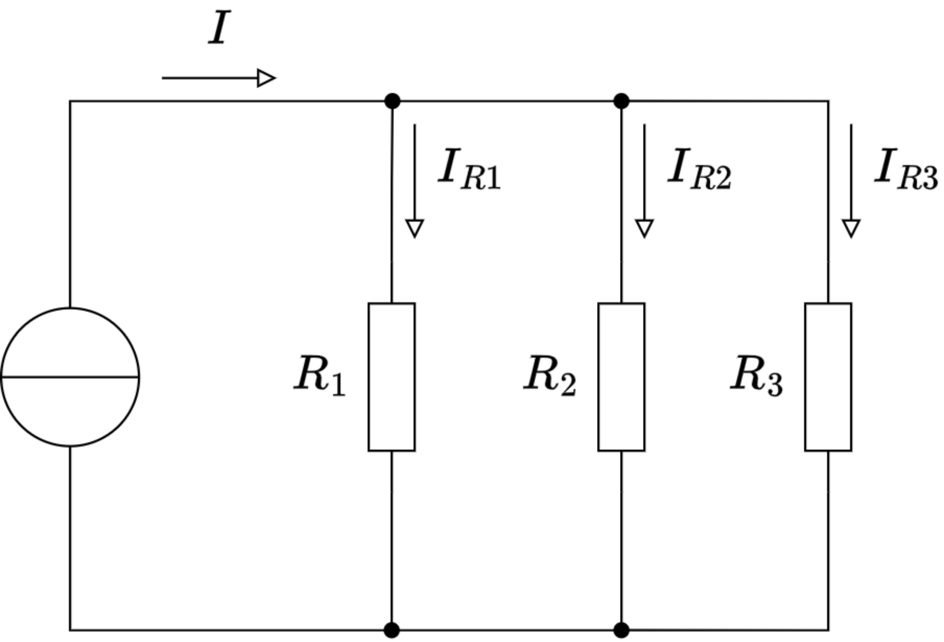
$$R_1 \cdot I = I_2 \cdot (R_1 + R_2)$$

$$I_2 = I \cdot \frac{R_1}{(R_1 + R_2)}$$

In case of multiple resistance (in general):

$$I_{unknown} = I_{in} \cdot \frac{The\ parallel\ sum\ of\ the\ other\ resistances\ (where\ I\ don't\ search\ the\ current)}{Investigated\ resistor\ + The\ parallel\ sum\ of\ the\ other\ resistances}$$

Illustrative example



<u>Details:</u>

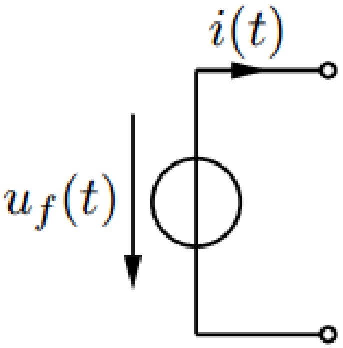
I = 3A; R_1 = 1kΩ; R_2 = 2,5kΩ; R_3 = 750Ω;

What is the value of the current flowing through R3?

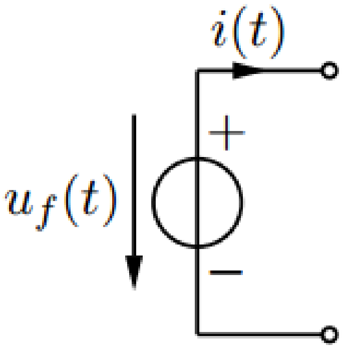
$$I_{R3} = I \cdot \frac{R_1 \otimes R_2}{R_3 + (R_1 \otimes R_2)} = 3A \cdot \frac{1k\Omega \otimes 2.5k\Omega}{(1k\Omega \otimes 2.5k\Omega) + 750\Omega}$$

$$= 3A \cdot \frac{714{,}28\Omega}{714.28\Omega + 750\Omega} = \mathbf{1.463A}$$

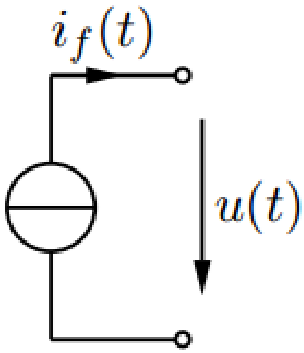
Ideal and real sources – Norton/Thevenin theorem



a. Voltage source

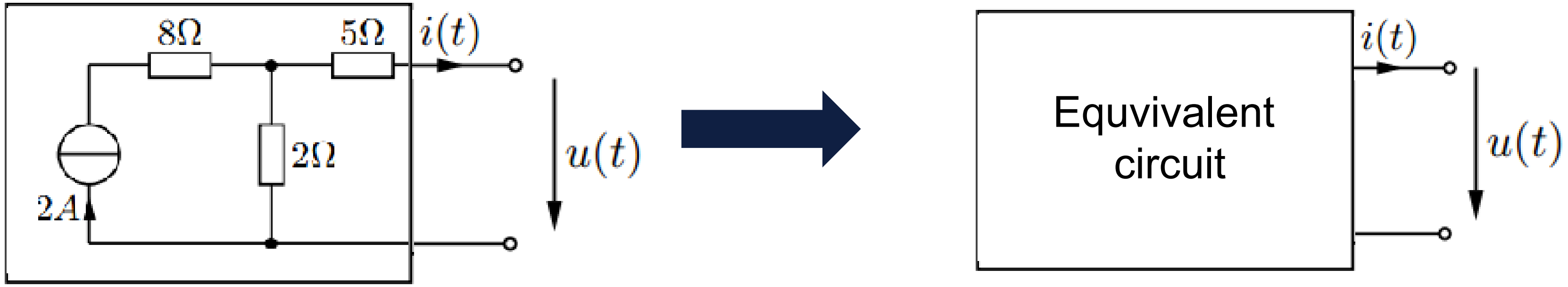


b. Voltage source with poles



c. Current source

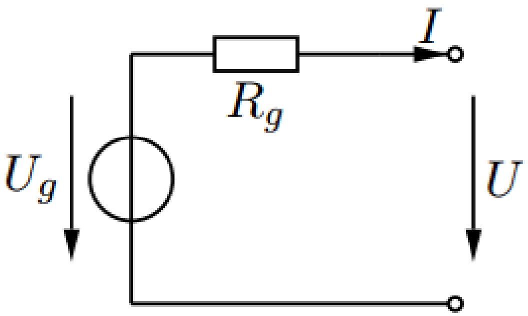
- Ideal sources
- Real sources (ideal source + internal resistance)
- Thevenin's and Norton's theorems state that any two-terminal, linear, resistive electrical network (containing generators and resistors/impedances) can be replaced by a simplified equivalent circuit.



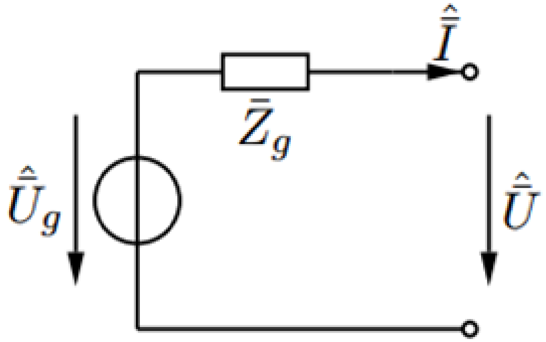
Source: Dr. Kuczmann Miklós – Kovács Gergely: Villamosságtan

Norton and Thevenin source

<u>Thevenin equvivalent circuit</u>: A Thevenin equvivalent circuit consists of a DC voltage source and a resistor connected in series with it.

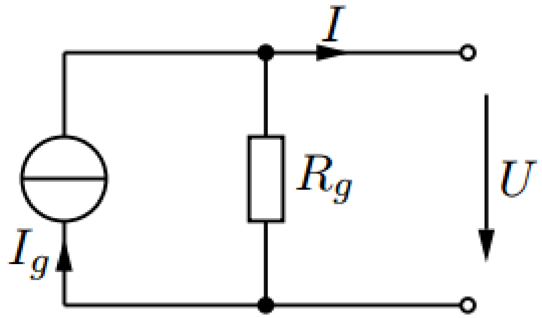


In the case of DC

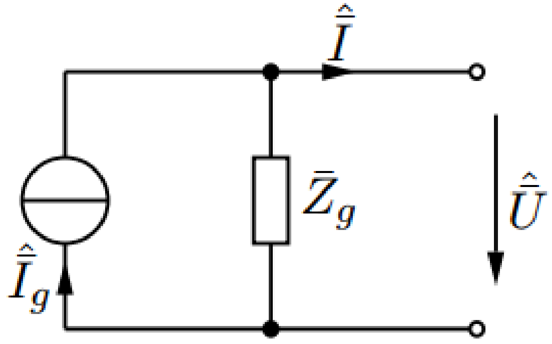


In the case of sine wave

<u>Norton equvivalent circuit</u>: A Norton equvivalent circuit consists of a DC current source and a resistor connected in parallel with it.

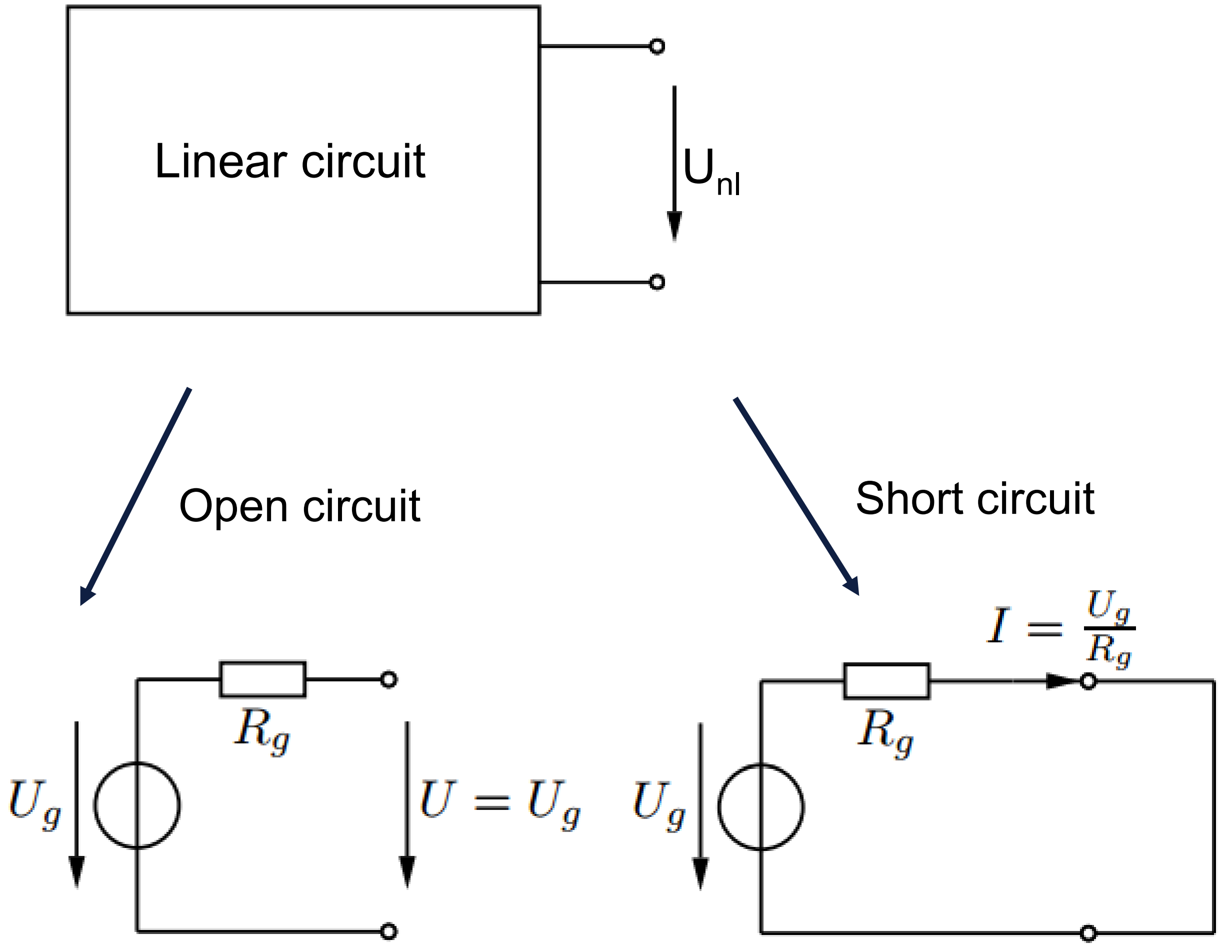


In the case of DC



In the case of sine wave

Characteristic of real soruces (Thevenin type) – illustrative example



Details:

$U_g = 12V$

$R_g = 100\Omega$

Open circuit:

$U_g = U = 12V$

$I = 0\text{A}$

Short circuit:

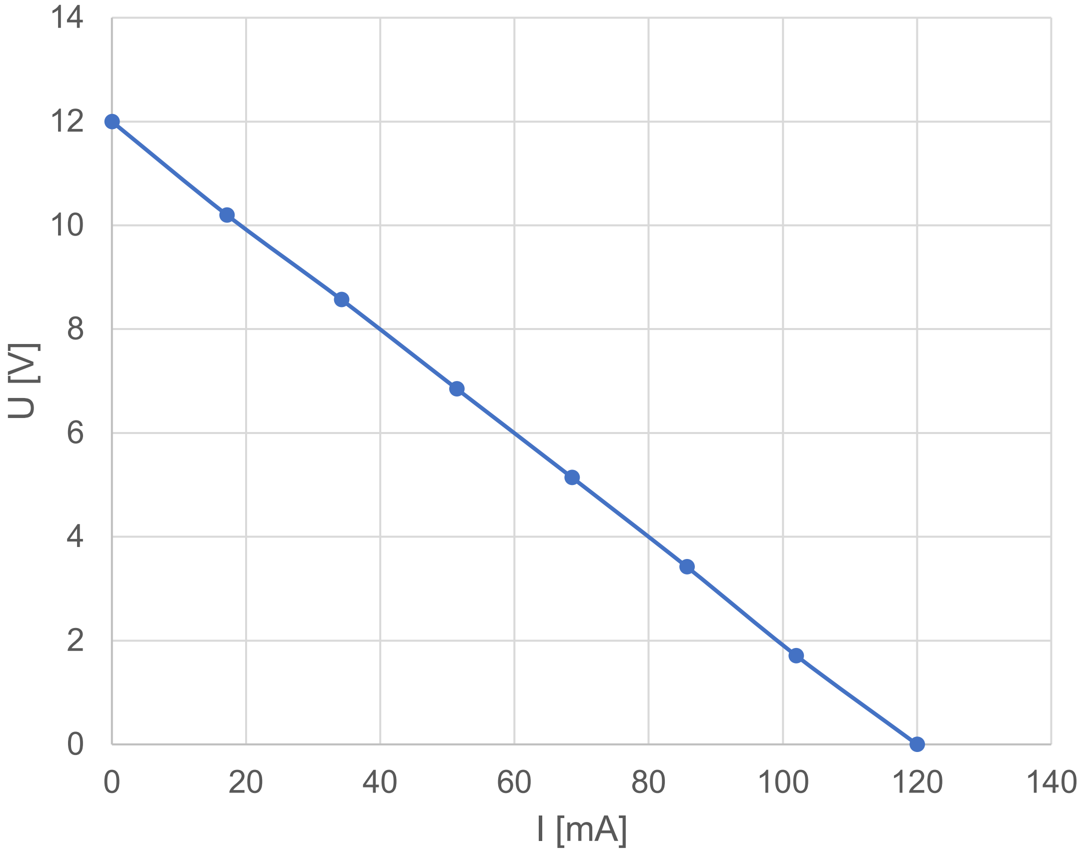
$U_g = 0V$

$$I = \frac{U_g}{R_g} = \frac{12V}{100\Omega} = 0{,}12A$$

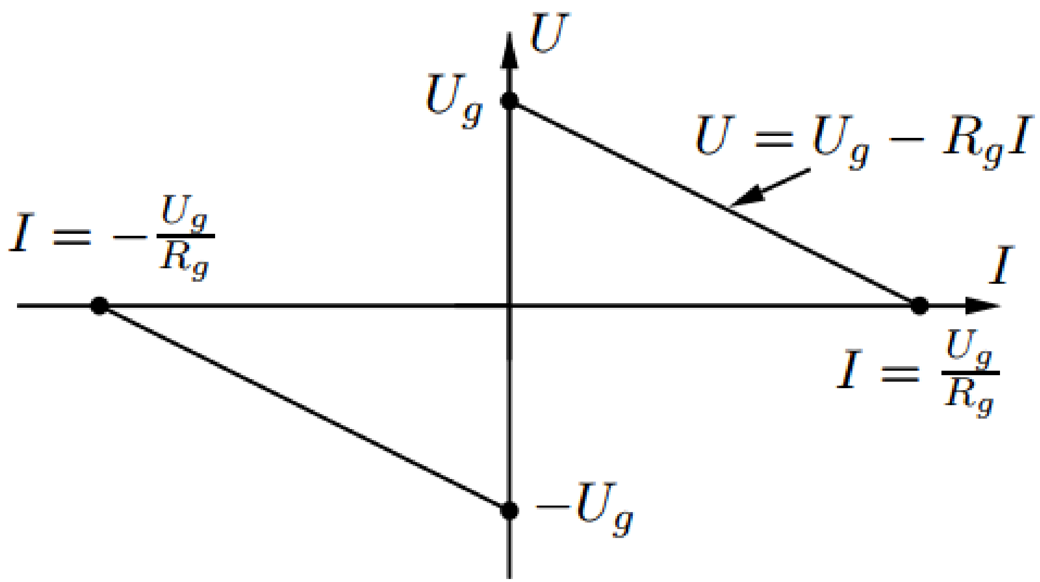
With different loads:

R_{load}	16Ω	40Ω	75Ω	133Ω	250Ω	600Ω
U_{Rload}	1,71V	3,42V	5,14V	6,85V	8,57V	10,2V
I_{Rload}	102mA	85,7mA	68,6mA	51,4mA	34,2mA	17,14mA

Creating a characteristic

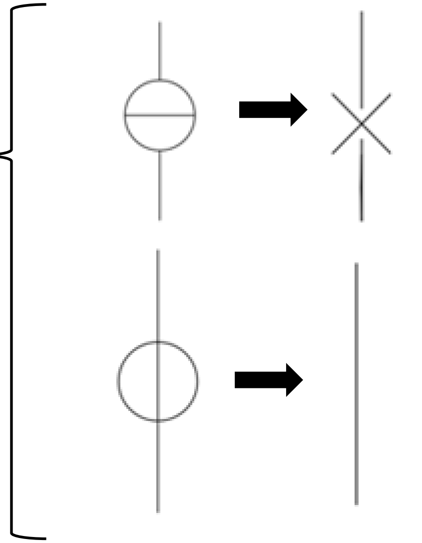


In general form (Thevenin source):

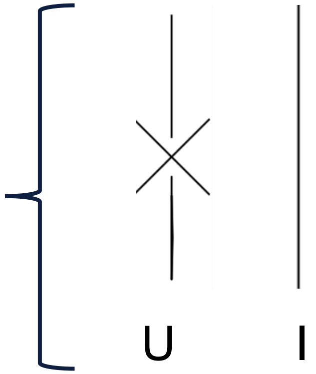


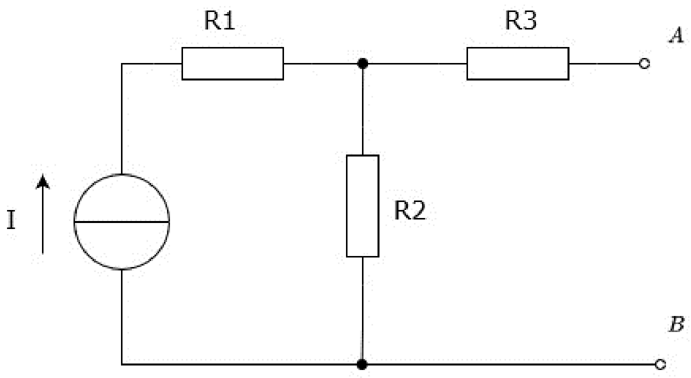
Illustrative example

1. Calculation of the internal resistance
   - Substitution of sources:
     - Current source – open circuit
     - Voltage source – short circuit



2. Calculation of quantities related to output (here AB)
   - If we want to know voltage – open circuit
   - If we want to know current – closed circuit and determine which component(s) will carry the current flowing through the short circuit





$$R_{AB} = R_2 + R_3$$

$$I_{AB} = I_{R3} = I\frac{R_2}{R_3 + R_2}$$

$$U_{AB} = IR_{AB}$$

The internal resistance (R_g) is the same for both equvivalent circuit!

Complex circuit solution method – solving steps

Superpositon method

1. Set reference directions
2. Deactivate the sources one by one
3. We examine partial solutions with a sign and summarize them. In the case of a large number of sources, this requires a lot of calculations!

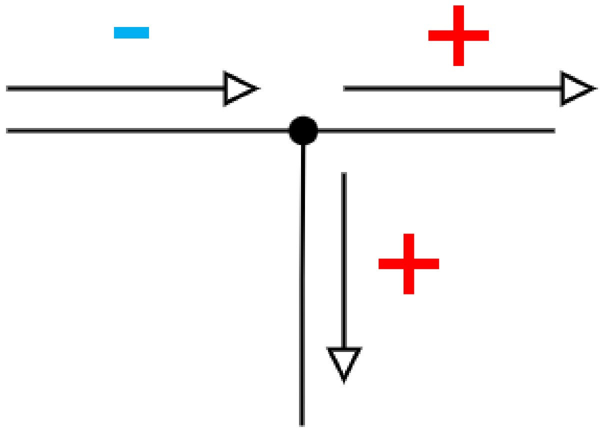
Node potential method:

1. Determine the number of unknown nodes ($n-n_u$)
2. Set reference directions
3. Write the equations

Loop current method:

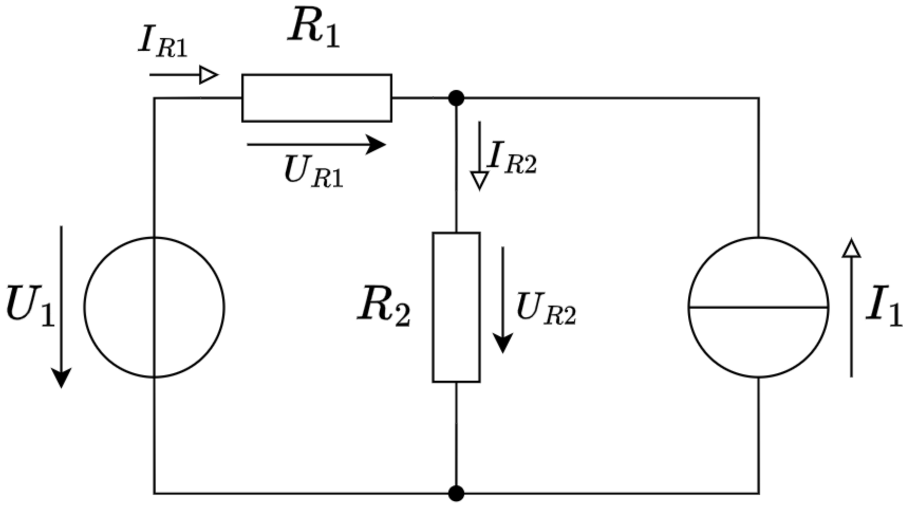
1. Determine the number of unknown loops (l-n+1)
2. Set reference directions
3. Write the equations

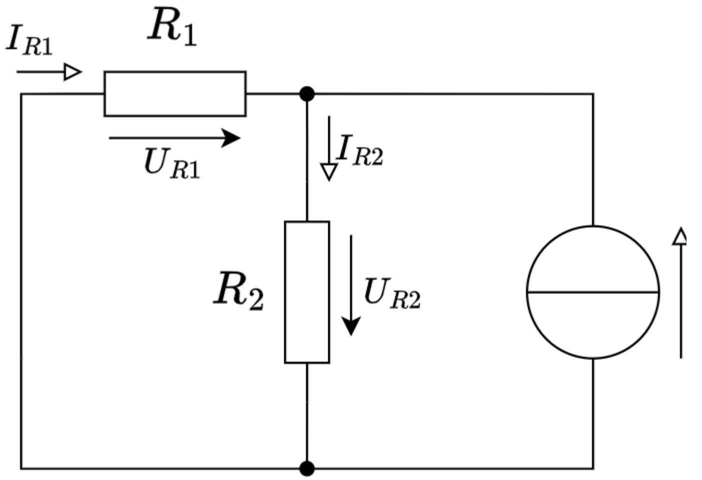
Current source Voltage source

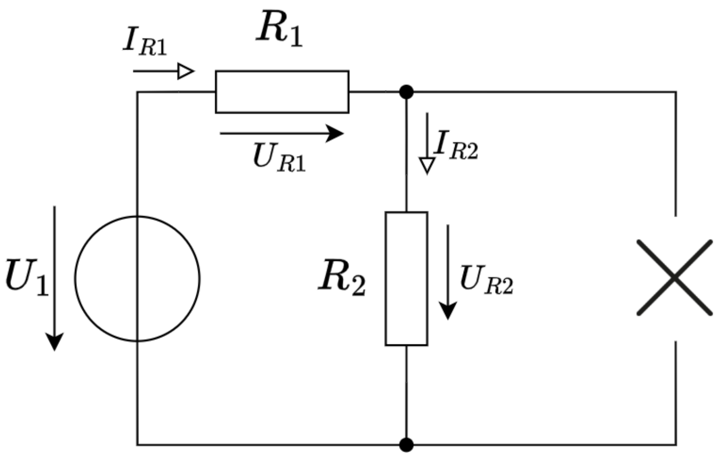


Current flowing into the node has a "-" sign, current flowing out of the node has a "+" sign.

Superpositon method







Solution steps:

1. We remove each source from the circuit one by one, examining their individual effects on the entire circuit.
2. We record the results obtained during each deactivation.
3. At the end, we summarize the results with the correct sign.

Notes:

- The current/voltage direction of the sources is predetermined and fixed! It cannot be changed.
- All other directions can be chosen freely. In the example, we followed the left-to-right, top-to-bottom principle. We will discuss the designation of these directions later
- The sources must be deactivated as many times as there are sources. In this case, twice. This causes complexity in circuits with a large number of sources.

Superposition method – calculation steps

- I.
  - $I_1^1 = I_{I1} \cdot \frac{R_2}{R_1+R_2} = -2A \cdot \frac{5\Omega}{20\Omega+5\Omega} = -0{,}4A$
  - $I_2^1 = I_{I1} \cdot \frac{R_1}{R_1+R_2} = 2A \cdot \frac{20\Omega}{20\Omega+5\Omega} = 1{,}6A$

$$U_{R1}^1 = I_1 R_1 = -0{,}4A \cdot 20\Omega = -8\text{V}$$
$$U_{R2}^1 = I_1 R_2 = 1{,}6A \cdot 5\Omega = 8\text{V}$$

- II.
  - $U_{R1}^2 = U_1 \cdot \frac{R_1}{R_1+R_2} = 20V \frac{20\Omega}{5\Omega+20\Omega} = 16V$
  - $U_{R2}^2 = U_1 \cdot \frac{R_2}{R_1+R_2} = 20V \frac{5\Omega}{5\Omega+20\Omega} = 4V$

$$I_1^2 = \frac{U_{R1}}{R_1} = \frac{16V}{20\Omega} = 0{,}8A$$
$$I_2^2 = \frac{U_{R2}}{R_1} = \frac{4V}{5\Omega} = 0{,}8A$$

In summary: (I. + II):

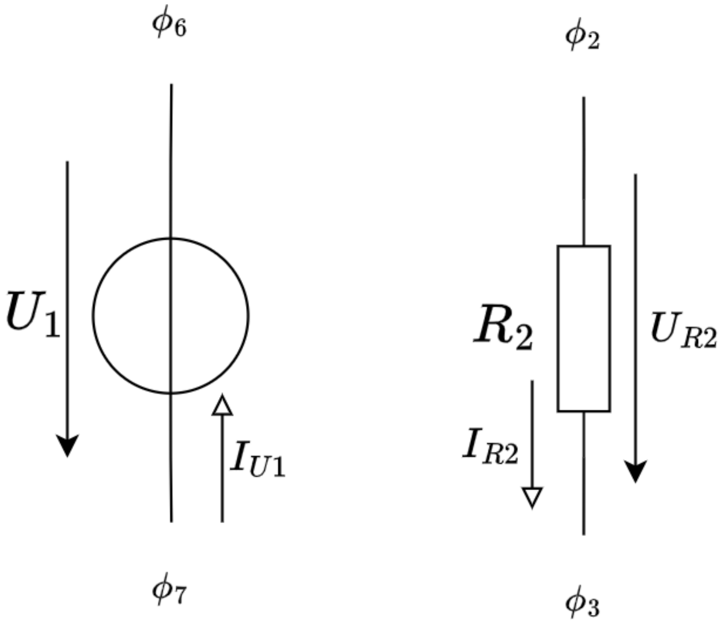
$$U_{R1} = U_{R1}^1 + U_{R1}^2 = -8V + 16V = 8V$$
$$U_{R2} = U_{R2}^1 + U_{R2}^2 = 8V + 4V = 12V$$

$$I_1 = I_1^1 + I_1^2 = -0{,}4A + 0{,}8A = 0{,}4A$$
$$I_2 = I_2^1 + I_2^2 = 1{,}6A + 0{,}8A = 2{,}4A$$

Component	U[V]	I[A]	P [W]
R1	8	0,4	3,2
R2	12	2,4	28,8
V1	20	I_1=0,4	-8
I1	12	-2	-24
Total:			**0**

The result needs to be corrected.

Node potential method -overview-



- Voltage is defined as: The difference in potential between two points. In most cases, we always refer to the positive potential, i.e., I subtract the lower potential from the higher potential (the voltage arrow points from "positive" to "negative").
- For example, the voltage of the voltage source:

$$U_1 = \Phi_6 - \Phi_7$$

- Or, for example, if I want to define the current of R_2, then:

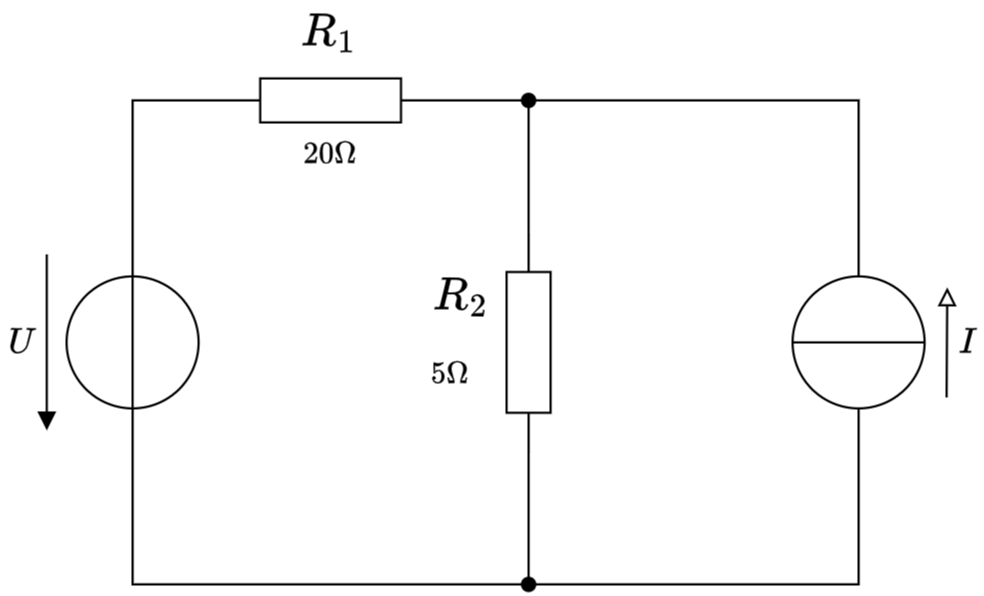
$$I_{R2} = \frac{U_{R2}}{R_2} = \frac{\Phi_2 - \Phi_3}{R_2}$$

- If I look at it from point Φ_3 :

$$I_{R2} = -\frac{U_{R2}}{R_2} = -\frac{\Phi_2 - \Phi_3}{R_3}$$

- In equations, the current of voltage sources (and the voltage of current sources) is usually unknown.
- In most cases, "generators" behave as sources, i.e., the voltage across them and the current flowing through them are in opposite directions.

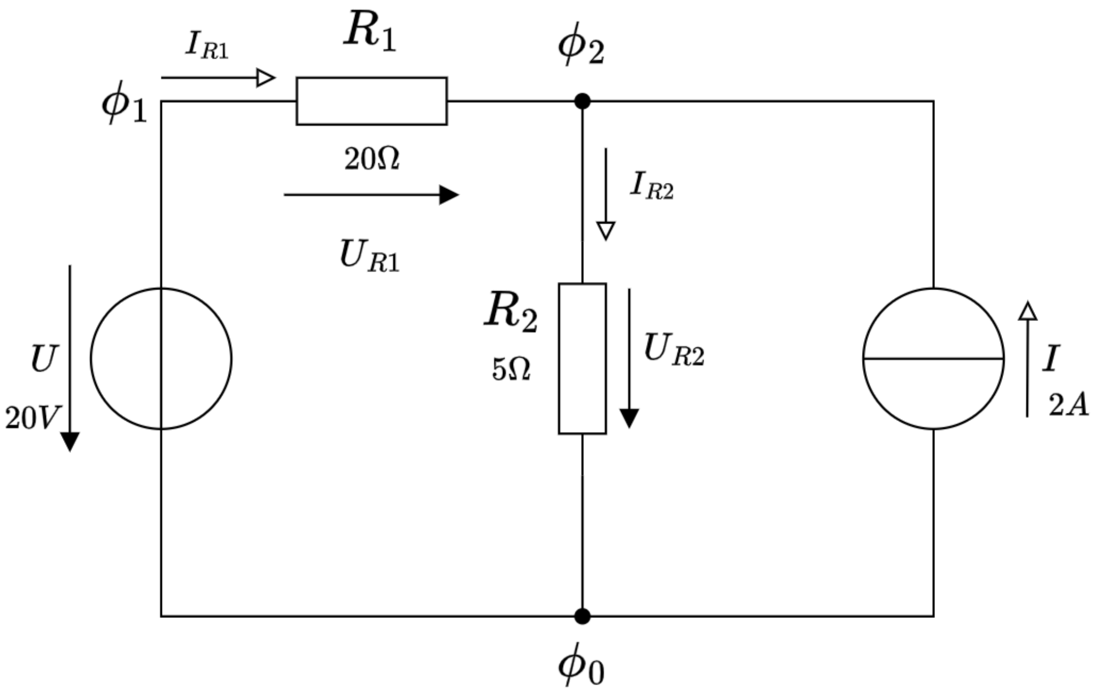
Illustrative example – solution steps



1. We begin the task by identifying unknown nodes.
2. In all cases, a reference point (GND or ground point) is required for reference. This reduces the number of unknown nodes. It is recommended to choose the negative pole of the voltage generator (or one of the voltage generators, if there are several) for this purpose, so that the other pole of the generator provides a known potential.
3. We write down the circuit equations (Kirchhoff equations) for the unknown nodes and then solve them.

The reference directions of the sources are fixed, the rest are arbitrary!Currents flowing into the node are marked with a "-" sign, and currents flowing out are marked with a "+" sign!

Illustrative example – calulation steps



Observations:

- The circuits contains n = 3 i.e., the number of unknown nodes $r = n - 1 = 3 - 1 = 2$
- Of the potentials Φ_0 is the common (reference) node, to which everything else is compared.
- Accordingly, Φ_1 can be considered known, with a value of 20V (since it is located between the common reference point and Φ_1).
- Only one unknown potential remains, Φ_2. Three components are connected to this potential, so they form the node equations for this node. That is,

$$\Phi_2{:})\ -2 - \frac{20 - \Phi_2}{20} + \frac{\Phi_2 - 0}{5} = 0$$

$$\Phi_2{:})\ -20 \cdot 2 - \frac{20 - \Phi_2}{20} + \frac{4(\Phi_2 - 0)}{20} = 0$$

$$\Phi_2{:})\ -40 - 20 + \Phi_2 + 4\Phi_2 = 0$$

$$\Phi_2{:})\ 5\Phi_2 = 60$$

$$\Phi_2{:})\ \Phi_2 = 12$$

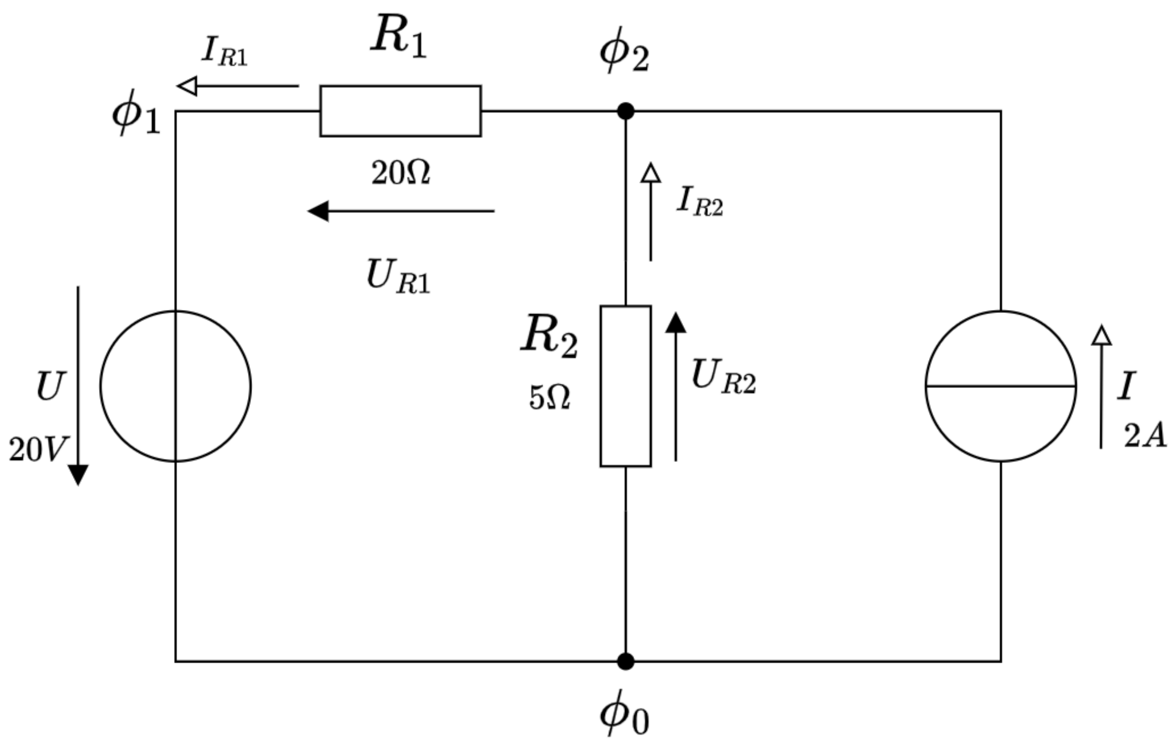
Results

Component	U[V]	I[A]	P|W]
R_1	$\Phi_1 - \Phi_2 = 8$	0,4	3,2
R_2	$\Phi_2 - \Phi_0 = 12$	2,4	28,8
U	20	$-I_{R1}$=-0,4	-8
I	U_{R2}=12	2	-24
Total:			0

The generator current is equal to I_{R1}. Since I_{R1} is the output current relative to Φ_1, the current flowing through the generator must be equal to Φ_1. This gives a conflicting reference direction.

The voltage and current of the power source vary according to the reference direction, so the result must be corrected!

Solving the example with different reference direction



In this case, we measured U_{R1} and U_{R2} differently, but keeping in mind that the current and voltage in them are in the same direction!

The equation of the unknown node:

$$\Phi_2{:})\ -2+\frac{\Phi_2-20}{20}-\frac{0-\Phi_2}{5}=0$$

$$\Phi_2{:})\ -20\cdot 2+\frac{\Phi_2-20}{20}-\frac{4(0-\Phi_2)}{20}=0$$

$$\Phi_2{:})\ -40-20+\Phi_2+4\Phi_2=0$$

$$\Phi_2{:})\ 5\Phi_2=60$$

$$\Phi_2{:})\ \Phi_2=12$$

We got the same result! So the reference direction does not influence the final result!

Calculation results

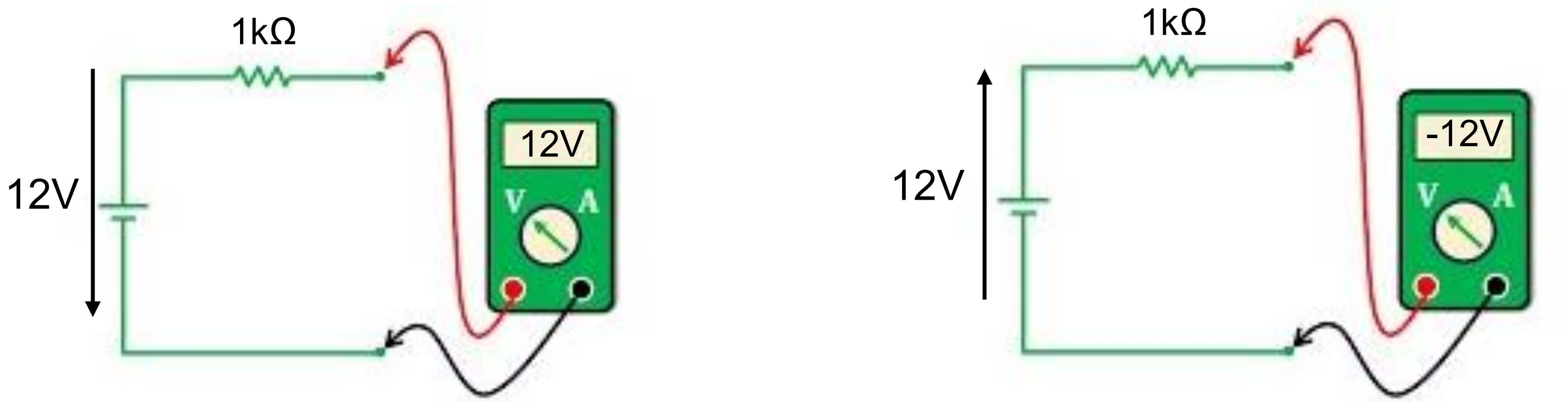
Component	U[V]	I[A]	P|W]
R_1	$\Phi_2 - \Phi_1 = -8$	-0,4	3,2
R_2	$\Phi_0 - \Phi_2 = -12$	-2,4	28,8
U	20	I_{R1}=-0,4	-8
I	U_{R2}=-12	2	-24
Total:			0

The current in the generator is equal to I_{R1}. Since the current I_{R1} flowing into Φ_1, the current flowing through the generator must be the current flowing out of Φ_1. There is no conflicting reference direction!

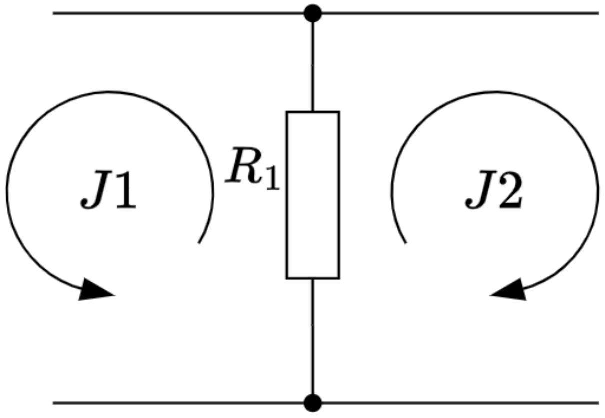
The voltage and current of the power source do not conflict with the reference values, so no correction is necessary!

How do I set the reference directions?

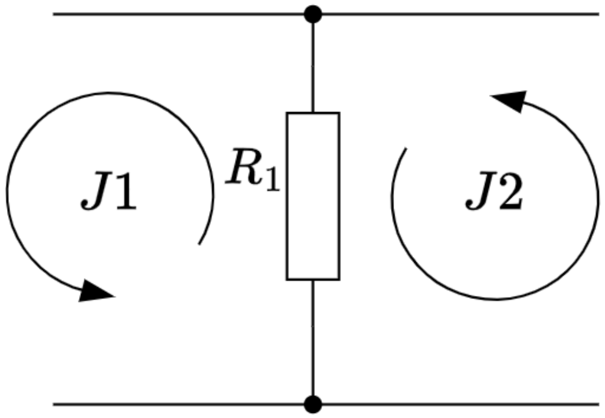
- According to Kirchhoff's node law, currents flowing into a node are considered negative ("-"), while those flowing out are considered positive ("+")
- As for the loop law, the direction of current flow around the loop determines the positive direction; voltages in the same direction as the current are positive ("+"), while voltages in the opposite direction are negative ("-")
- In reality, voltage flows from the higher potential to the lower one, and current also flows from the more positive point to the more negative one
- The choice of reference direction is flexible; we can follow the "left-to-right, top-to-bottom" principle or the "right-to-left, bottom-to-top" principle. Note that in both cases, we draw a loop
- What does this mean in practice? If you measure an unknown current or voltage with a Deprez meter and the pointer moves in the positive (rightward) direction, the reference direction matches the real direction. If it deflects in the opposite direction, the reference direction is opposite to the real direction. In the case of a digital multimeter, the instrument indicates the opposite direction with a minus "-" sign.



Loop Current Method



a.



b.

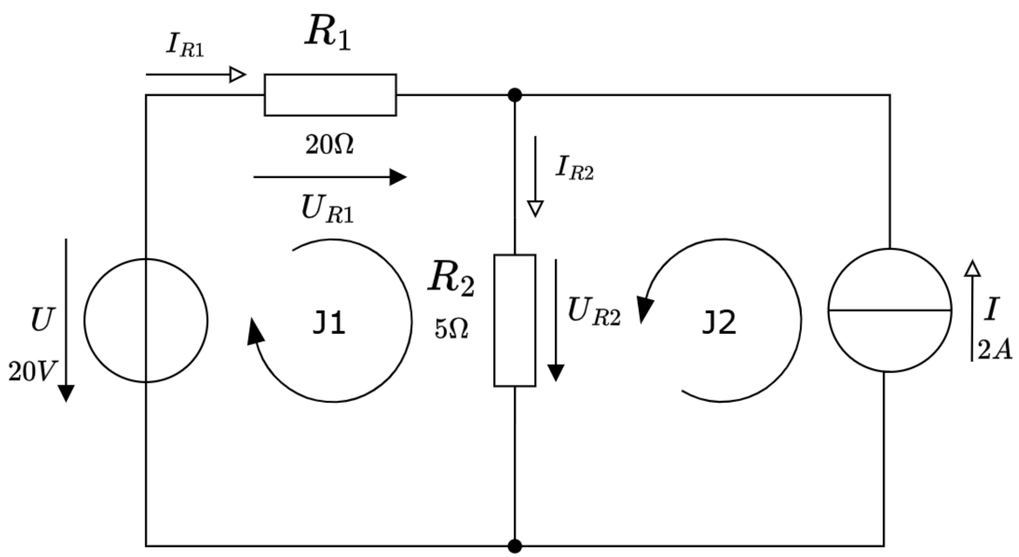
- After identifying the independent loops, we assign a loop current to each loop and write down Kirchhoff's loop law for each loop
- How do we know the number of unknown loops? The number of unknown loops can be calculated based on the number of components and nodes:

$$l = b - n + 1,$$

Key findings:

- The current generator always defines a loop whose direction is fixed and can be considered known, its direction cannot be changed
- The direction of the non-fixed loops can be random, clockwise, or counterclockwise
- In the case of a more complex network, loops must always be configured so that every component is connected to at least one loop
- Resistances are always included in the equation with a positive sign, unless multiple loops are influencing it
- If two loops are influencing a component, we always subtract the value of the other loop from the loop being analyzed:
  - In case a, both loops approach R_1 from the same direction: $R_3(J1 + J2)$
  - b. For example, if we examine loop J1: $R_3(J1 - J2)$, if we examine loop J2: $R_3(J2 - J1)$

Illustrative example



- Let's determine the number of unknown loops:

$$l = b - n + 1 = 4 - 3 + 1 = 2$$

- Of these, the loop designated by the current generator can be considered known, so one loop remains unknown
- As the first step in the solution, let’s identify the directions of the voltages for each component (here we followed the convention of left to right and top to bottom). They do not play a role in the solution itself, but they will be important when summarizing the results
- Next, let’s select the J1 loop, for example, in a clockwise direction. The loop current of J2 (current generator) is fixed, and it flows counterclockwise.
- The next step is to calculate the sum of the voltages across the components in the unknown loop, based on Kirchhoff’s law, i.e., the equation for loop J1:

$$J1:)\ -20 + 20J1 + 5(J1 + J2) = 0$$

$$J1:)\ -20 + 25J1 + 5 \cdot 2 = 0$$

$$J1:)\ 25J1 = 10$$

$$J1:)\ J1 = 0{,}4$$

Calculation results

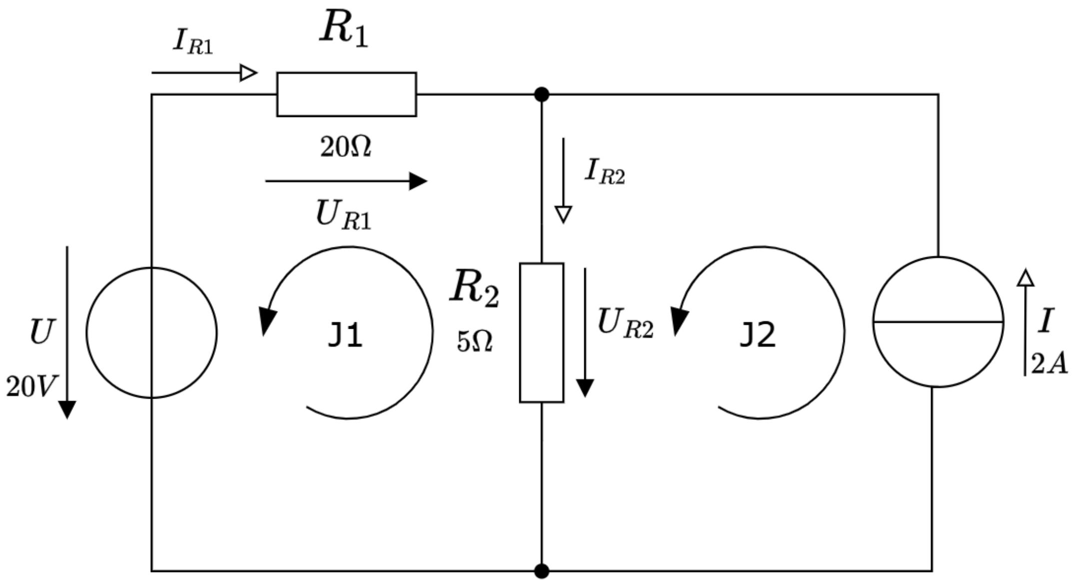
Component	U[V]	I[A]	P|W]
R_1	8	J1=0,4	3,2
R_2	12	J1+J2=2,4	28,8
U	20	-J1=-0,4	-8
I	U_{R2}=12	2	-24
Total:			0

The chosen reference direction (top to bottom) defines the positive direction. Since both loops are similar, we add J1 and J2 together.

The direction of the generator's voltage is opposite to the direction of the loop, so it must be considered as negative.

The voltage and current of the voltage source vary in opposite directions relative to the reference direction, so the result must be corrected!

Solving the example with different reference direction



Equation for a J2 loop:

$$J1{:})\,20 + 20J1 + 5(J1 - J2) = 0$$

$$J1{:})\,20 + 25J1 - 5 \cdot 2 = 0$$

$$J1{:})\; 25J1 = -10$$

$$J1{:})\; J1 = -0{,}4$$

Since we write the equation for J1, it provides the reference direction. The direction of J2 is opposite, so in the case of resistor R2, we consider J2 with a negative sign!
Recall that in the previous case, the direction of J1 was clockwise; J2 was in the same direction, so we considered J2 with a positive sign.

Calulation results

Component	U[V]	I[A]	P|W]
R_1	-8	J1=-0,4	3,2
R_2	12	J2-J1=2,4	28,8
U	20	J1=-0,4	-8
I	U_{R2}=12	2	-24
Total:			0

The chosen reference direction (top to bottom) defines the positive direction. Since the J2 loop is equivalent to this, it becomes the dominant one.

$$J2 - J1 = 2 - (-0{,}4) = 2{,}4$$

The voltage and current of the power source are still in opposite directions relative to the reference direction, so the result must be corrected!

The direction of the generator's voltage is now the same, so its current is given by J1.