IRÁNYÍTÁSTECHNIKA - BEVEZETÉS

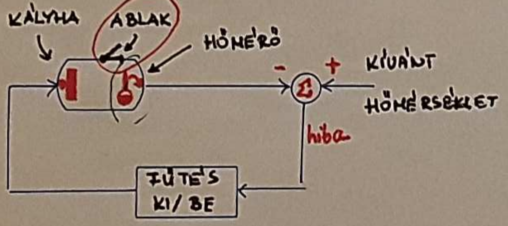
- előszó helyett...
- **IRÁNYÍTÁS** : az irányítás egy folyamatba való beavatkozás valamely cél elérése érdekében

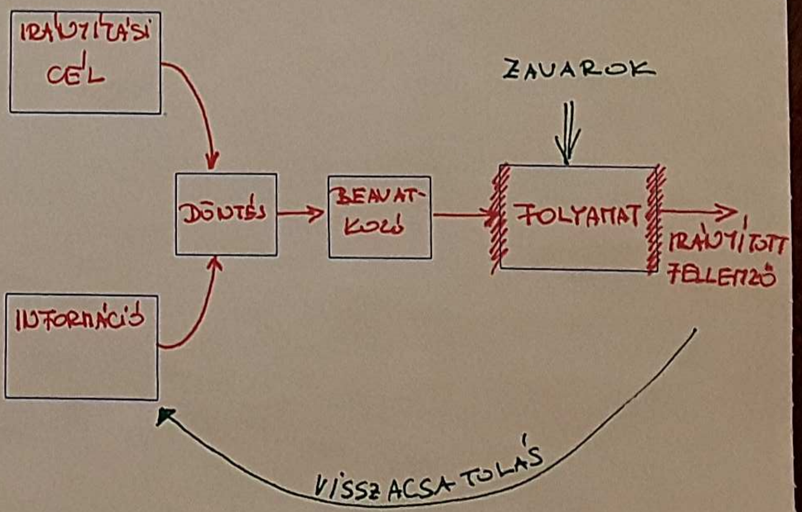
- VEZÉRLÉS
- SZABÁLYOZÁS
  - ÉRTÉKTARTÓ
  - KÖVETŐ

Két bevezető példa :

a.) mosógép időprogram szerinti irányítása : mosás → öblítés → centrifugálás

b.) szoba hőmérsékletének tartása





Bevezető gondolatok

A technika eszközei, a műszaki tudományok eredményei a terhes, unalmas, gépies teendők alól szabadítják fel az embert. Az emberi erőt előbb felváltotta az állati vonóerő, később a gépek, miáltal az izomerőt igénylő feladatok alól az ember szinte teljes mértékben mentesül.

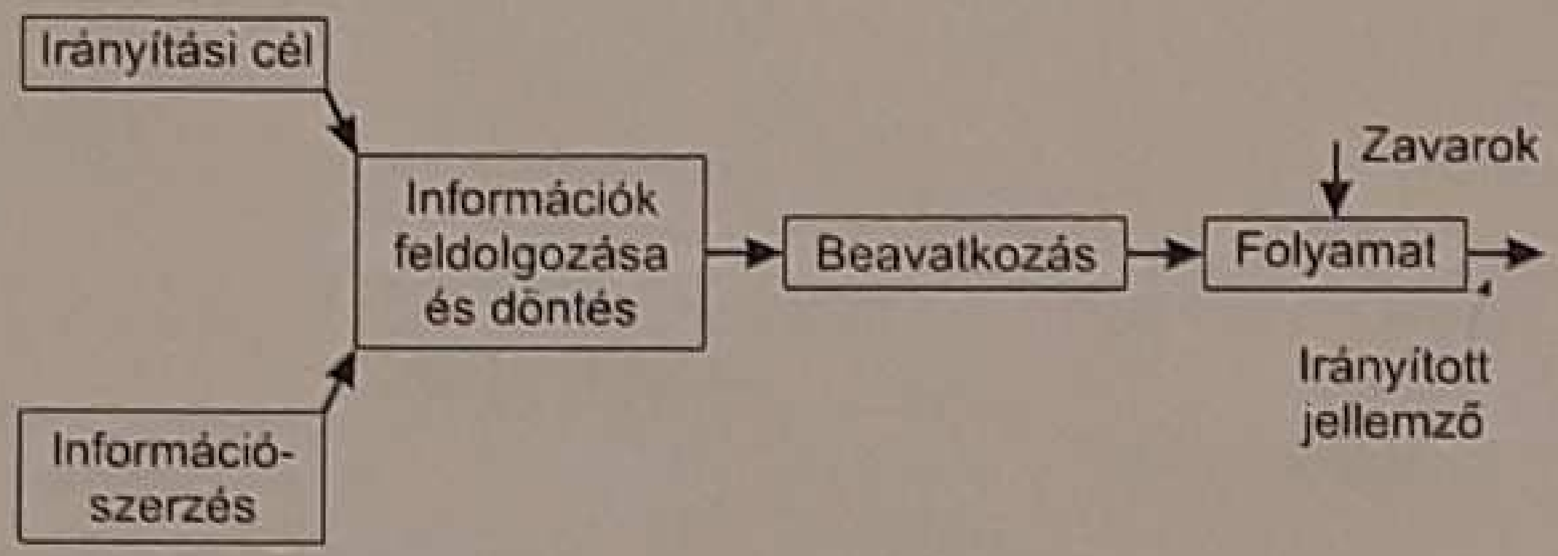
Ezt erősíti tovább az automatizálás, az önműködő berendezések fejlődése, s ennek egy fontos ága az irányítástechnika.

- termelési folyamatok automatizálása
- monoton, unalmas lépések vagy nagy precizitást igénylő lépések robotizálása
- gazdasági növekedés
- szórakoztató elektronikai eszközök
- okos (smart) lakás / épület
- járművek biztonsága
- élő szervezet

.... stb.

Az irányítási rendszer

Az irányítás egy folyamatba való beavatkozás valamely cél elérése érdekében.



Az irányítás során megváltoztatjuk a folyamat valamely közvetlenül befolyásolható jellemzőit, s ezen keresztül hatást gyakorolunk a folyamat más befolyásolni kívánt jellemzőire.

A témakör elhelyezése

- Dinamikus rendszerek modellezése – fizika, matematika;
- Jelek és rendszerek témakörei;
- Érzékelők és beavatkozók;
- Méréstechnika, adatgyűjtés, A/D és D/A átalakítók, erősítők, jelformálók;
- Szoftver, digitális és logikai rendszerek, PLC, mikrovezérlők, beágyazott rendszerek, buszrendszerek, kommunikáció, architektúrák.

Mit használunk matematikából?

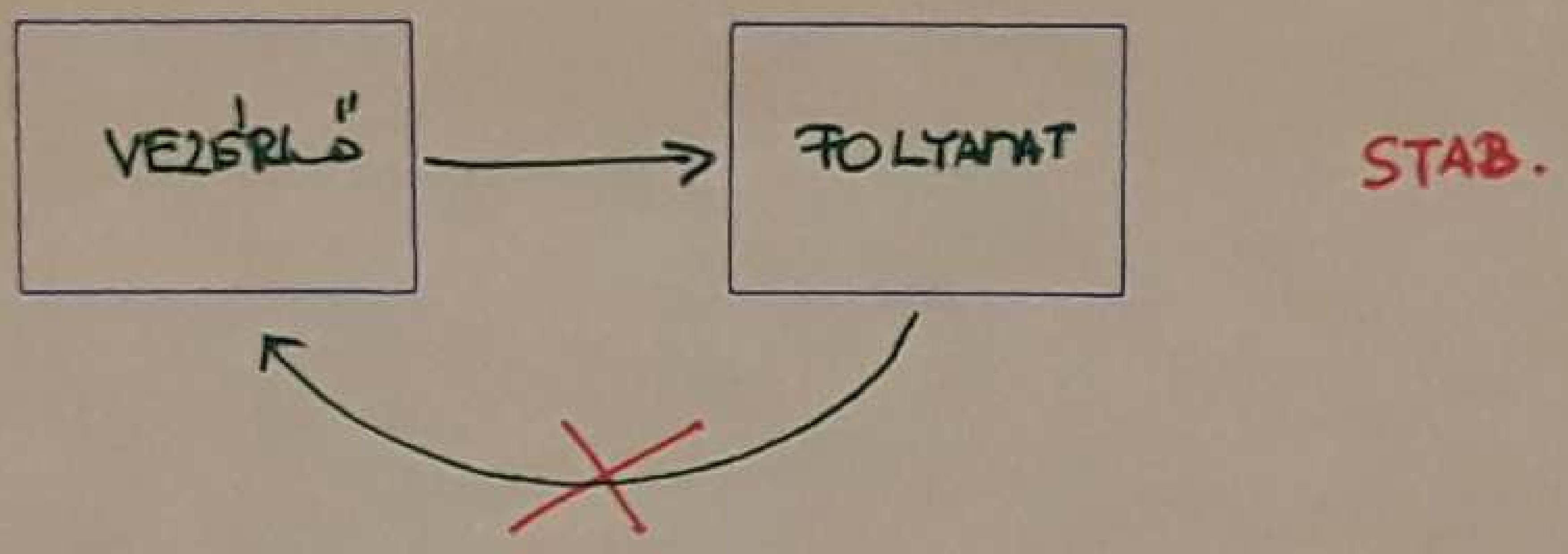
A tárgy nagymértékben épít az alkalmazott matematikára!

Használt fejezetek:

- Differenciálás és integrálás;
- Differenciálegyenletek;
- Függvényábrázolás;
- Komplex számok;
- Mátrixokkal való műveletek.

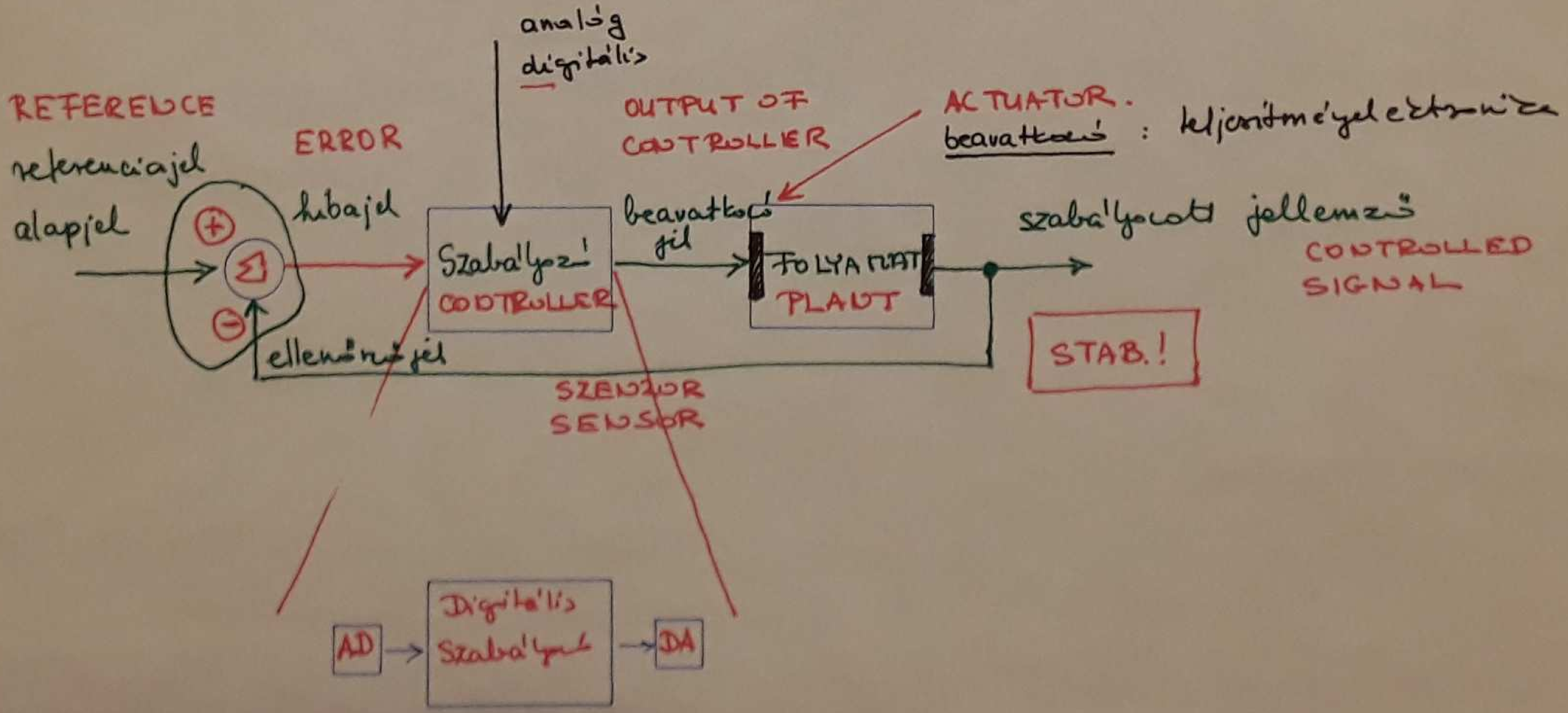
VEZÉRLÉS

NYÍLT LÁNCÚ



ZÁRT LÁNCÚ

SZABÁLYOZÁS

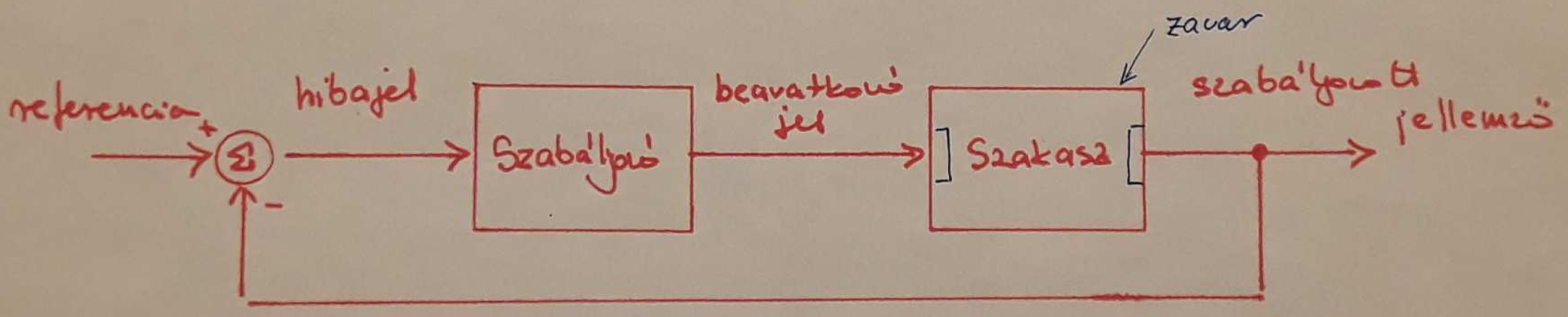


Az irányítási rendszer - Példák

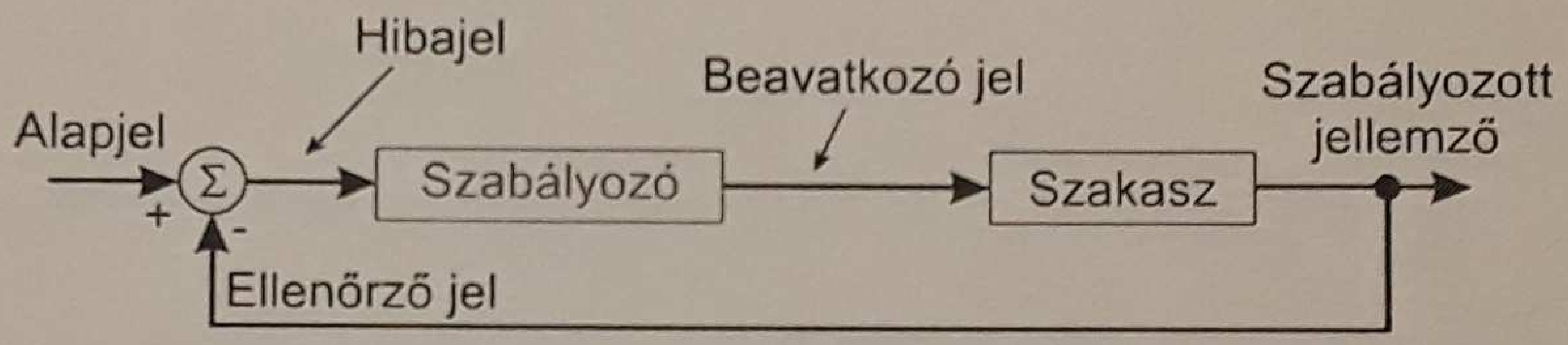
- Szoba hőmérsékletének beállítása (termosztát, légkondícionálás);
- Vízszintszabályozás;
- Tempomat, sebességszabályozás;
- Fordulatszám-szabályozás (pl. motor, ventillátor);
- Követési távolság tartása;
- Automatikus parkolás és a robotpilóta;
- Robotkarok előírt pályán való mozgatása;
- Forgalomirányítás közlekedési lámpákkal;
- Erőművek (pl. ATOM grafitrúd, SZEL lapátszög beállítása);
- Egyensúly járás közben, testhőmérséklet;
- Dialízisgépek, Inzulinszint-, szívritmusszabályozó;
- ...

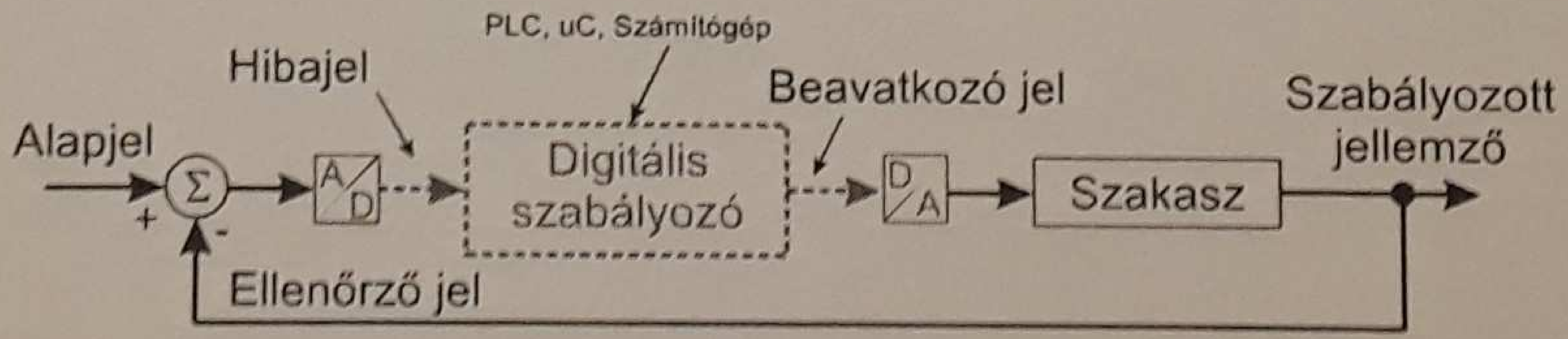
ÉRTÉKTARTÓ

KÖVETŐ



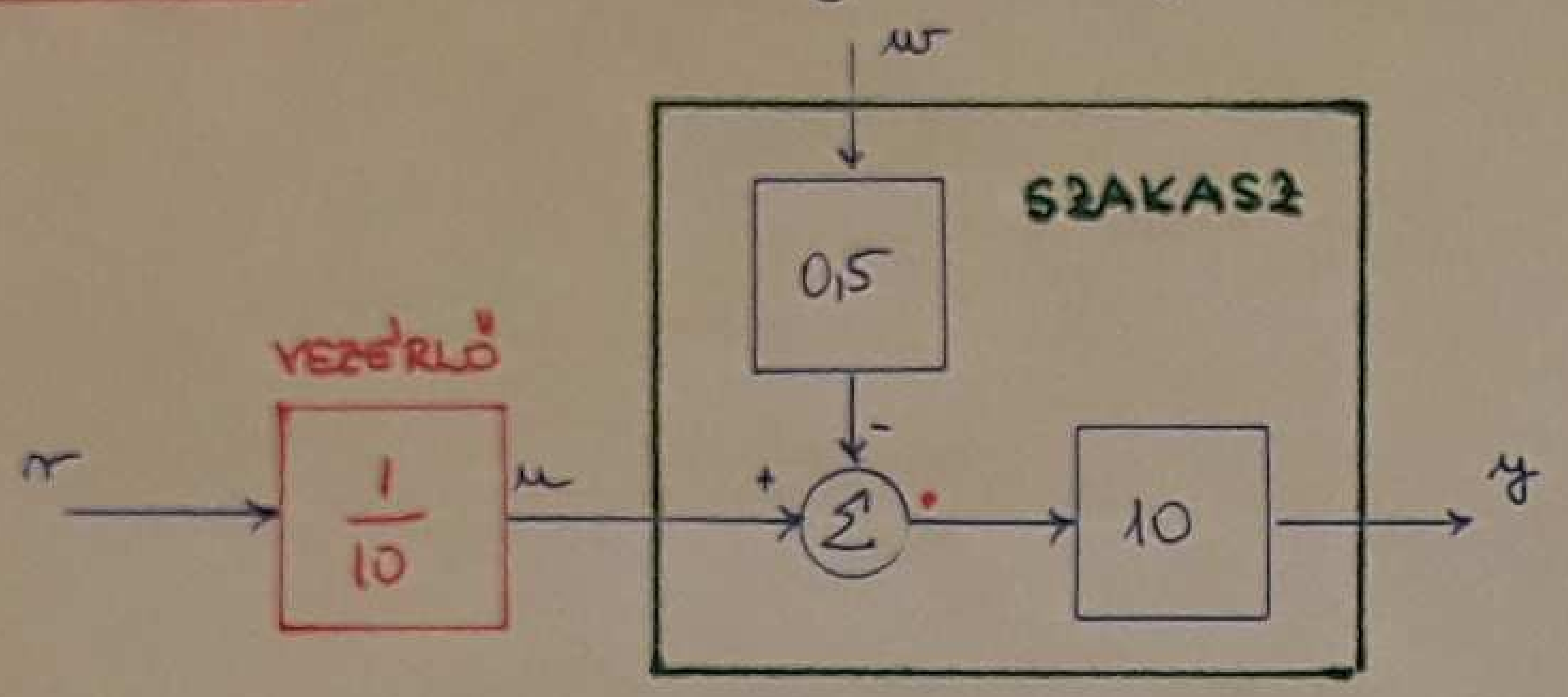
Megvalósítás

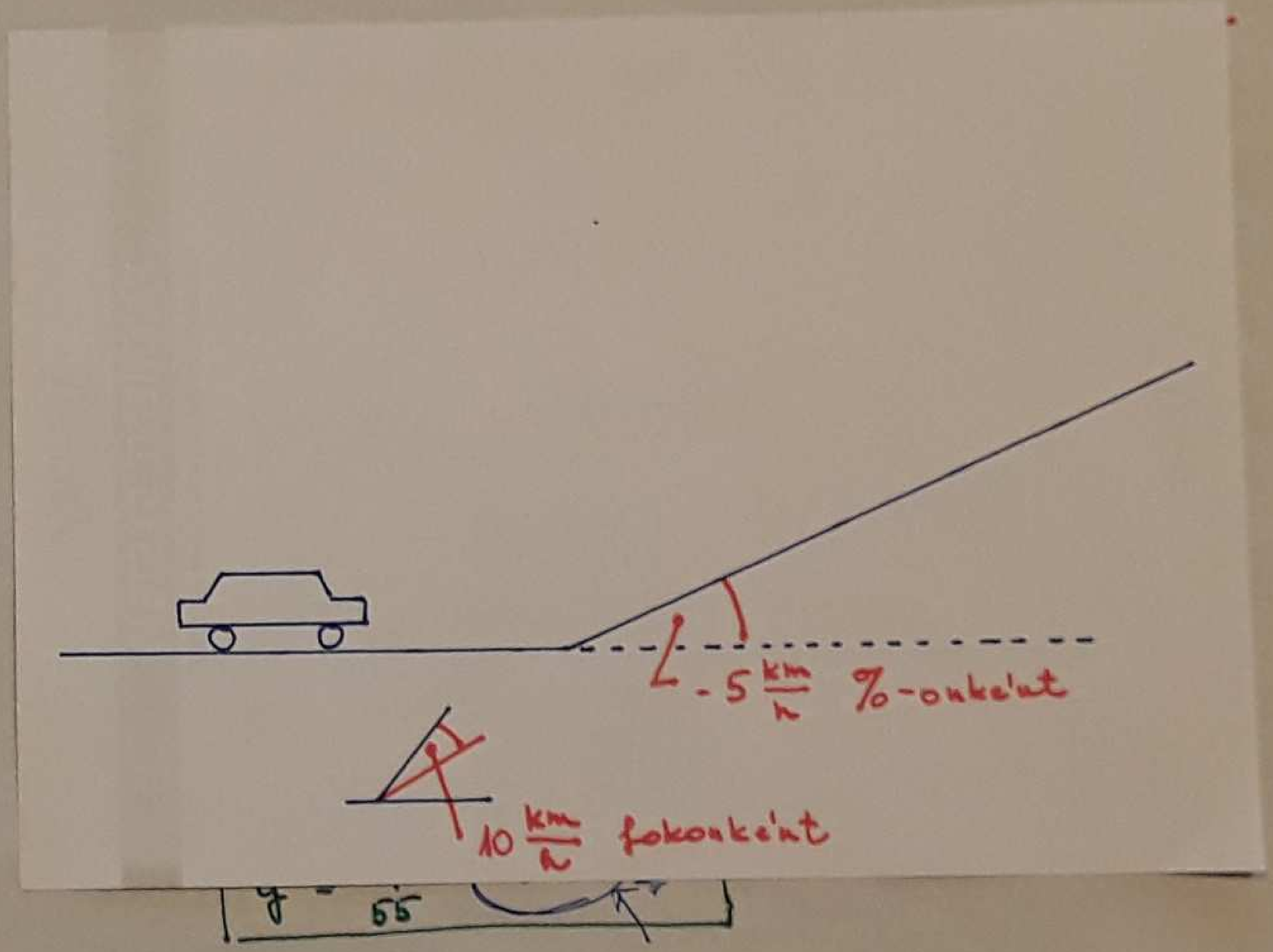


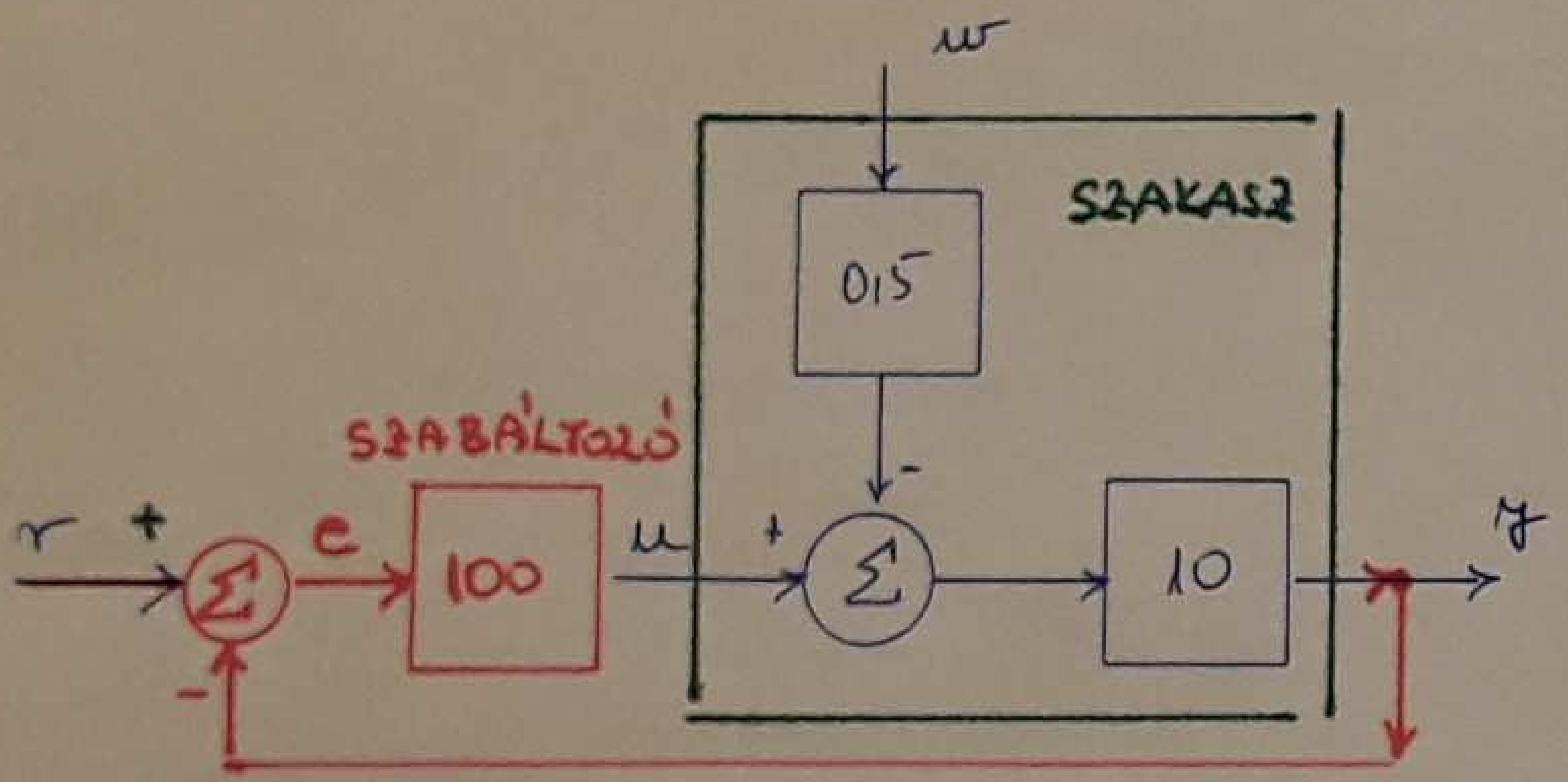


ILLUSZTRÁCIÓ : sebességszabályozás statikus modellje

- Gázpedál 1°-os benyomása 10 km/h sebességet jelent
- 1%-os emelkedő 5 km/h visszaesést jelent
- r = 55 km/h a referencia





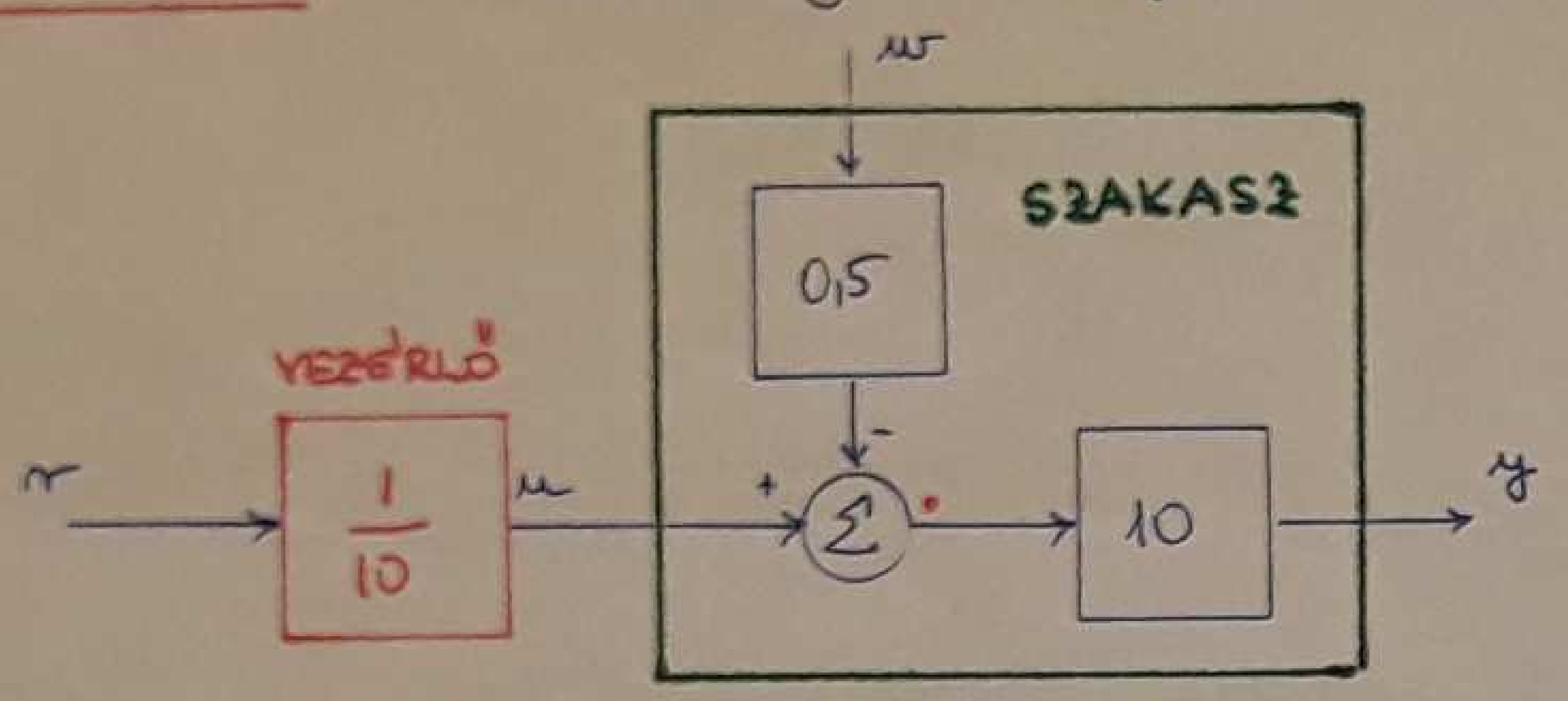


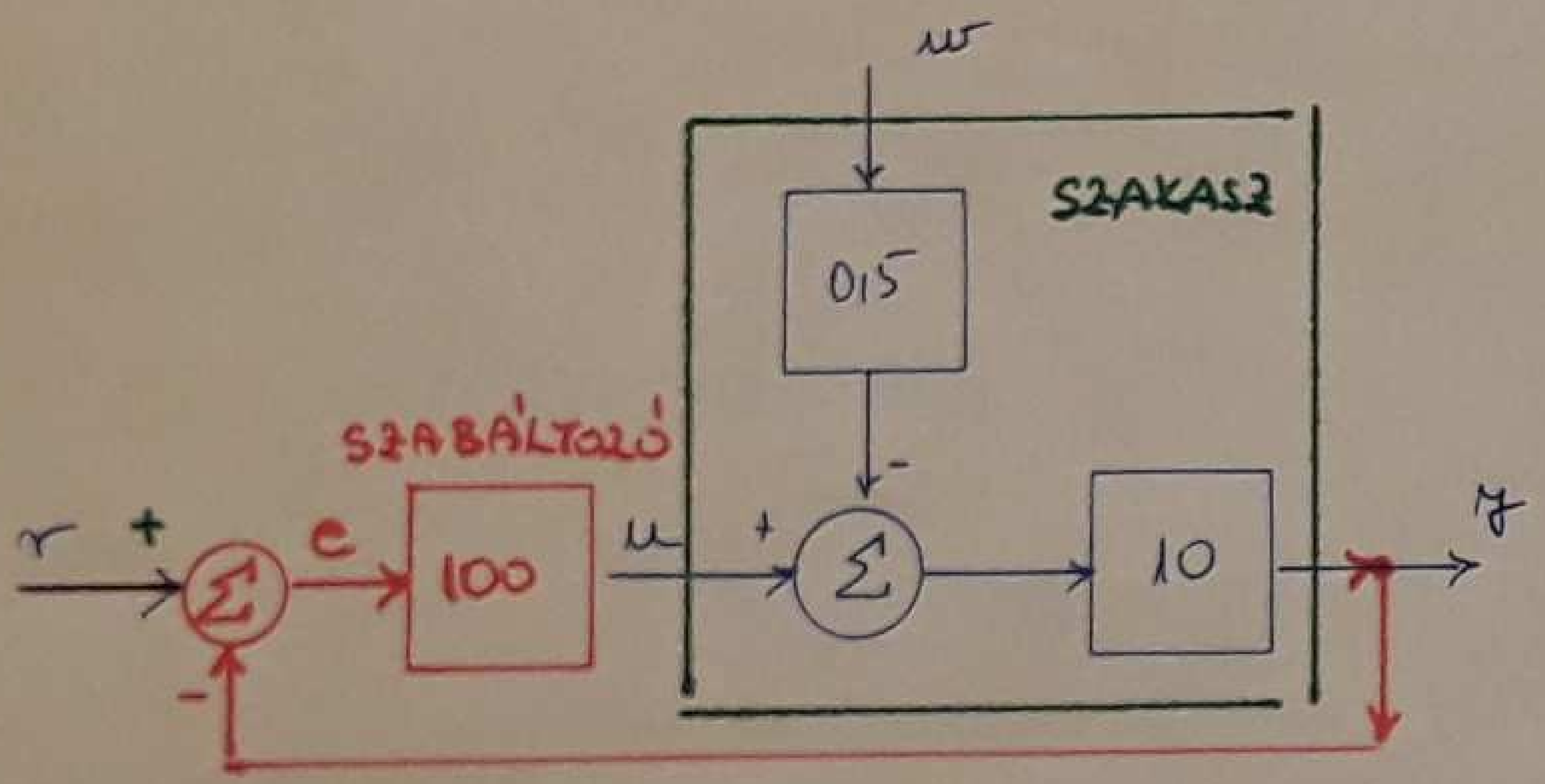
55 - 5w   55-0,0005w

w	vezérlés	szabályozás
0%	55	~55
1%	50	54,995
10%	5	54,95

lehet pontosabb!

ILLUSZTRÁCIÓ sebesség szabályozás statikus modellje





	55 - 5w	55-0,0005w
w	vezérlés	szabályozás
0%	55	~55
1%	50	54,995
10%	5	54,95

lehet pontosabb!

- Gázpedál 1°-os benyomása 10 km/h sebességet jelent
- 1%-os emelkedő 5 km/h visszaesést jelent
- r = 55 km/h a referencia

$$y = 10(u - 0{,}5w) = 10\underset{\substack{\uparrow \\ \frac{r}{10}}}{u} - 5w = r - 5w$$

$$\boxed{y = 55 - 5w}$$

$$y = 10(\underset{\uparrow}{u} - 0{,}5w)$$

$$u = 100e = 100(r - y)$$

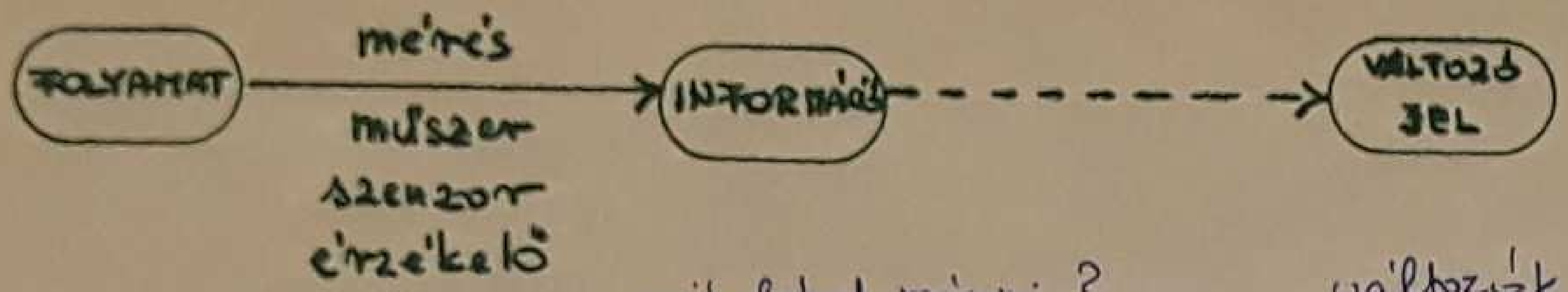
$$y = 10(100(r-y) - 0{,}5w)$$

$$y = 1000r \underline{- 1000y} - 5w$$

$$1001y = 1000r - 5w \quad /: 1001 \approx 1000$$

$$\boxed{y = \underset{55}{r} - 0{,}005w}$$

JELEK



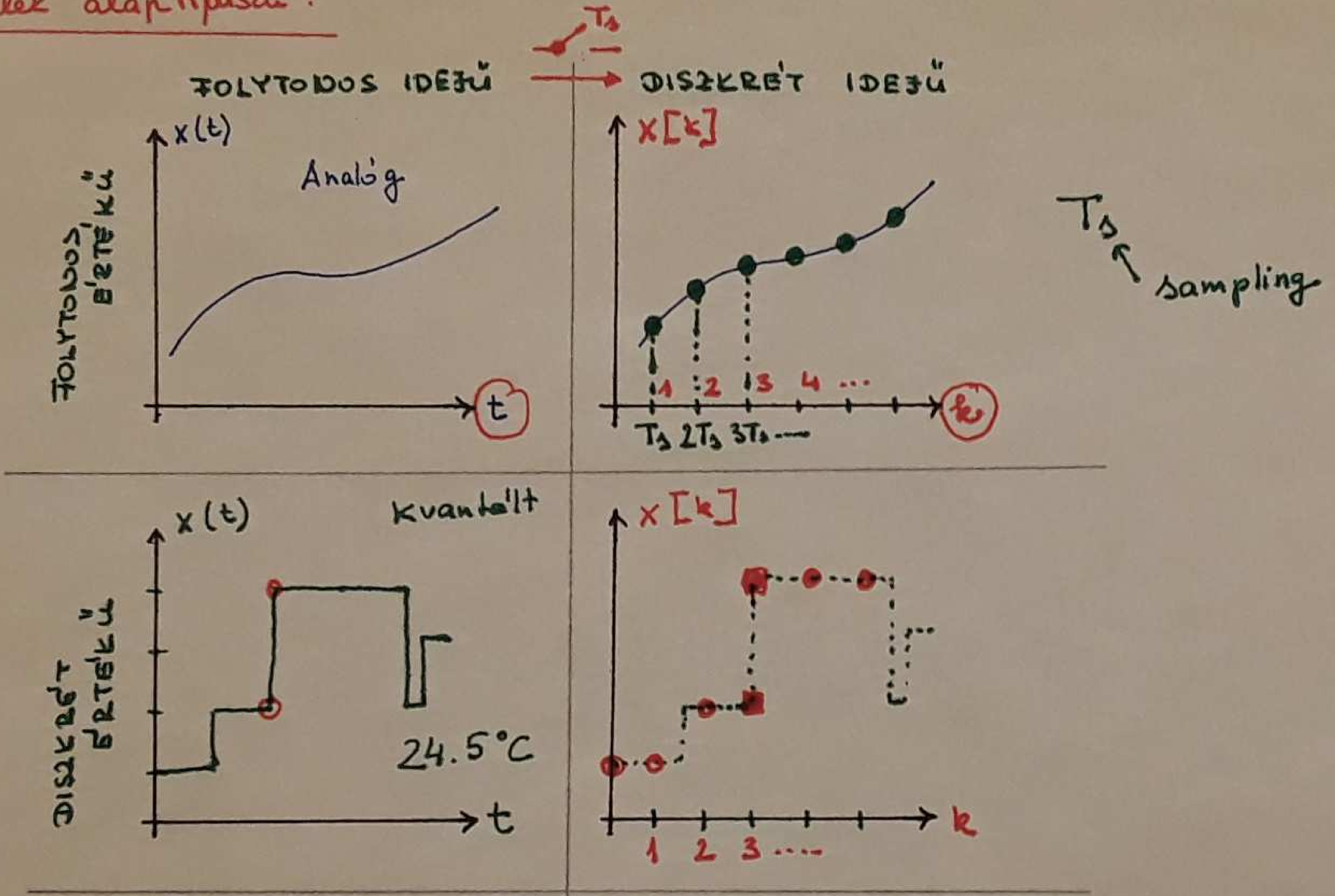
	mit lehet mérni? FIZIKAI MENNYISÉGET	változók A LEÍRÁSRA	(+ mértékegység)
példák:			
haladó jármű	helyzete	$x(\underset{\uparrow}{t})$ $y(\underset{\uparrow}{t})$ $z(\underset{\uparrow}{t})$	120 km; GPS-koordináták
	sebessége	$v(\underset{\uparrow}{t})$	25 km/h
	üzemanyagszint	$V(\underset{\uparrow}{t})$	12 l
	motor fordulata	$n(\underset{\uparrow}{t})$	1000 ford./perc
áramkör	feszültséget	$u(\underset{\uparrow}{t})$	2V
	áramok	$i(\underset{\uparrow}{t})$	3 mA
korsó sör	hőmérséklete	$T(\underset{\uparrow}{t})$	8,2°C
tartály	folyadékszint	$h(\underset{\uparrow}{t})$	0,5 m
kazán	nyomás	$p(\underset{\uparrow}{t})$	1,5 bar
villamosenergia-rendszer	teljesítmény (látszólagos)	$S(\underset{\uparrow}{t})$	2 MVA
⋮ stb.			

IDŐFÜGGVÉNYEK !

A jel a fizikai mennyiség olyan értéke vagy értékváltozása, amely egy egyértelműen hozzárendelt információt hordoz.

$$\boxed{\text{JEL} \equiv \text{VÁLTOZÓ} \equiv \text{IDŐFÜGGVÉNY}}$$

Jelek alaptípusai :



A jel a t idő minden valós értékére értelmezett,

$x = x(t)$

$t \in \mathbb{R}$

$-\infty < t < \infty$

A független változó csak egész értékeket vehet fel,

$x = x[\underset{\uparrow\text{ ütem}}{k}]$

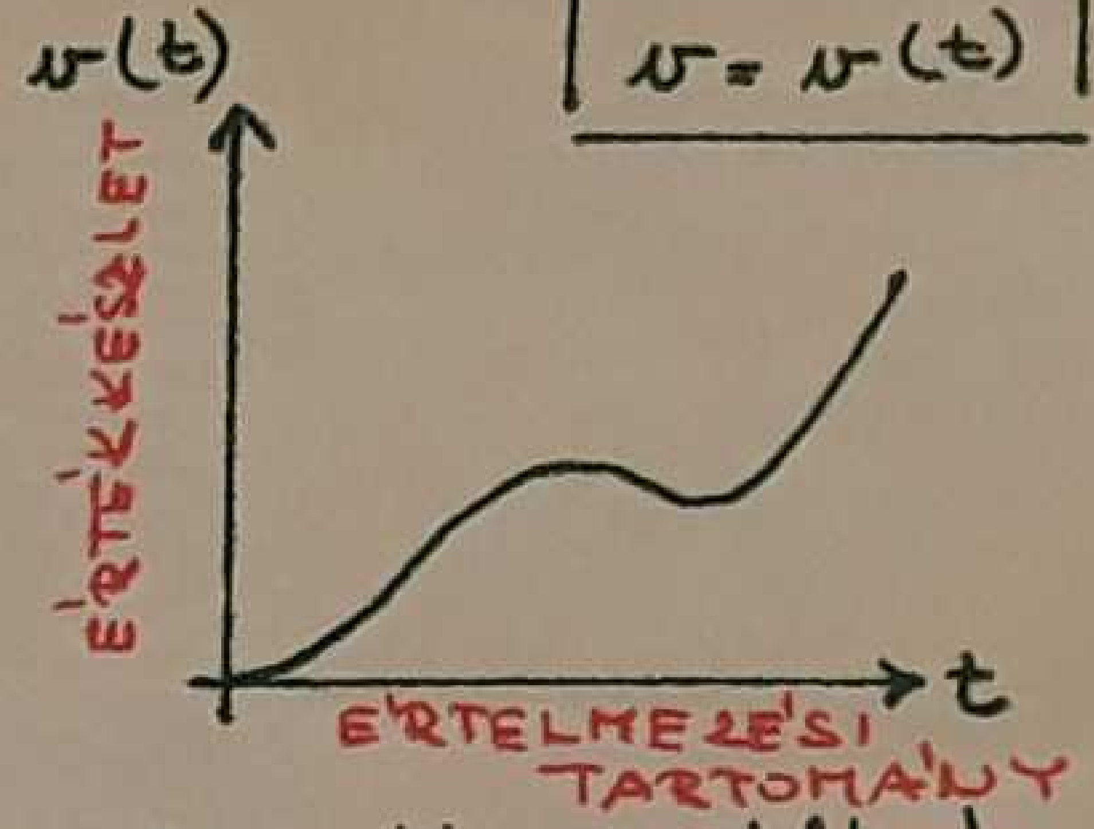
$k \in \mathbb{Z}$

$k \in [-\infty, \dots, -2, -1, 0, 1, \dots]$

A jel időfüggvény !

pl.

$\boxed{v = v(t)}$



egy független változó az idő, jele : t, a függvény argumentumának is nevezik

A jel lehet még :

- determinisztikus
- sztochasztikus

Jelek megadása I.:

$\boxed{T_s = 0{,}1}$

FOLYTONOS IDEJŰ JEL

Képlet

$$x(t) = \begin{cases} 0 & \text{ha } t < 0 \\ 5e^{-2t}, & \text{ha } t \geq 0 \end{cases}$$

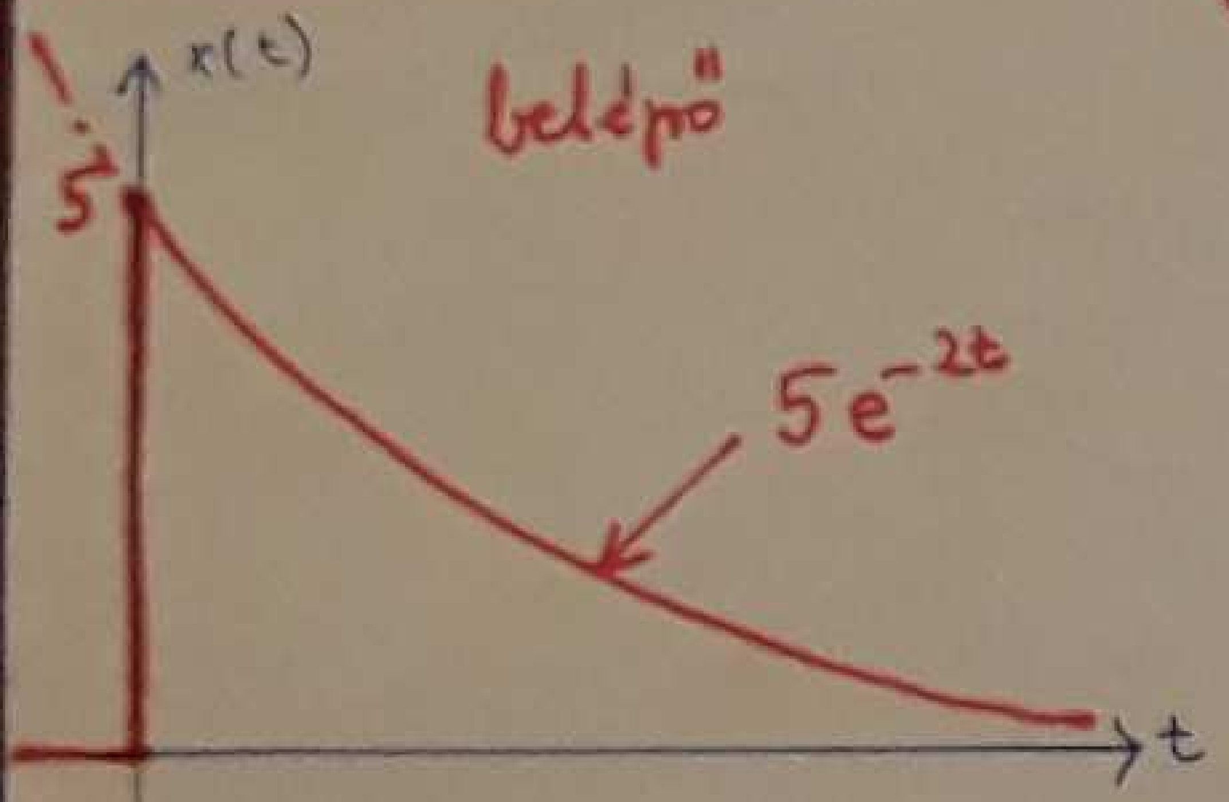
0 ∞

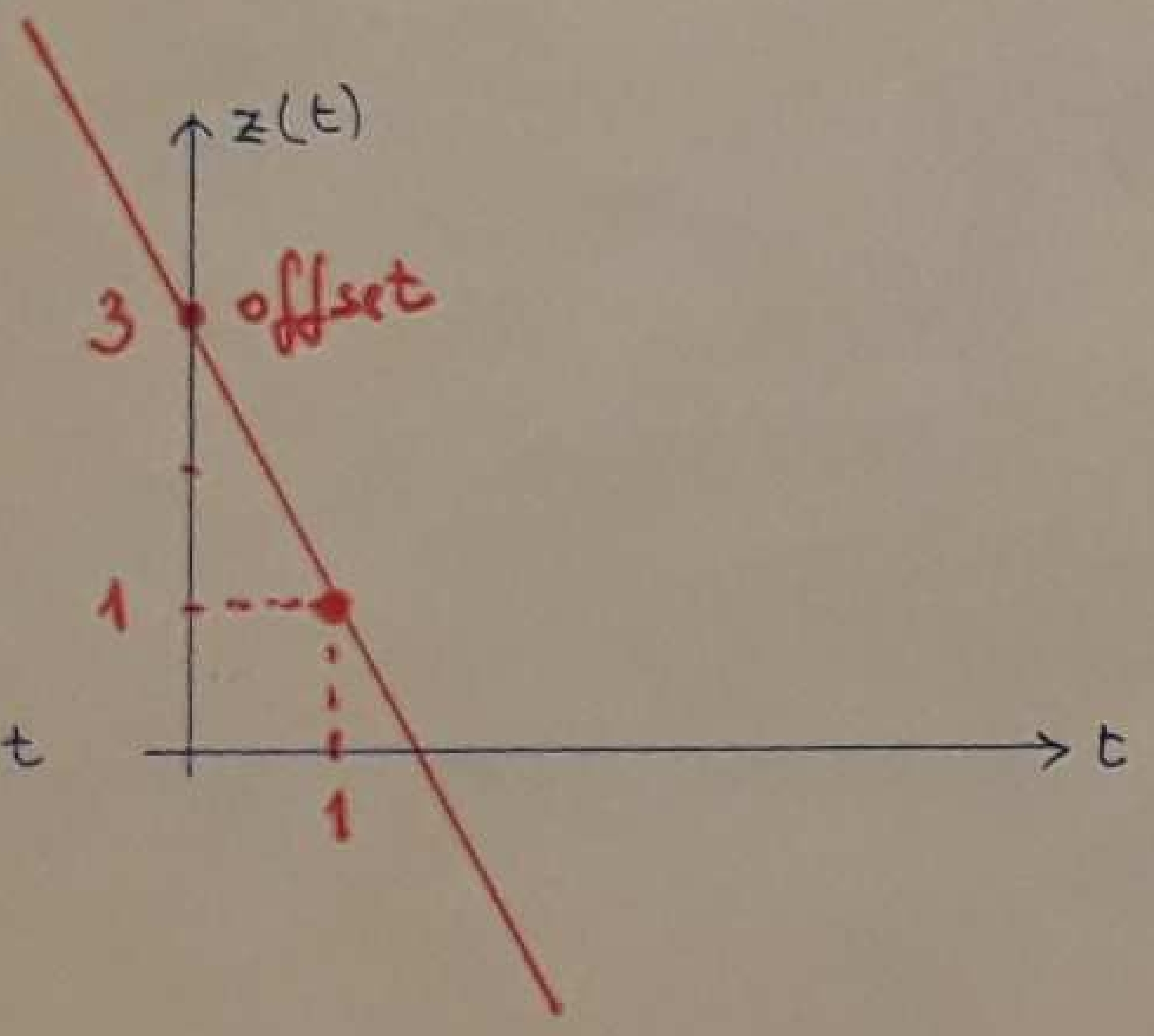
$$y(t) = \underbrace{5}_{} \cos\left(\underbrace{\frac{\pi}{2}}_{\omega} t \overset{\leftarrow}{+} \underbrace{\frac{\pi}{2}}_{\alpha}\right)$$

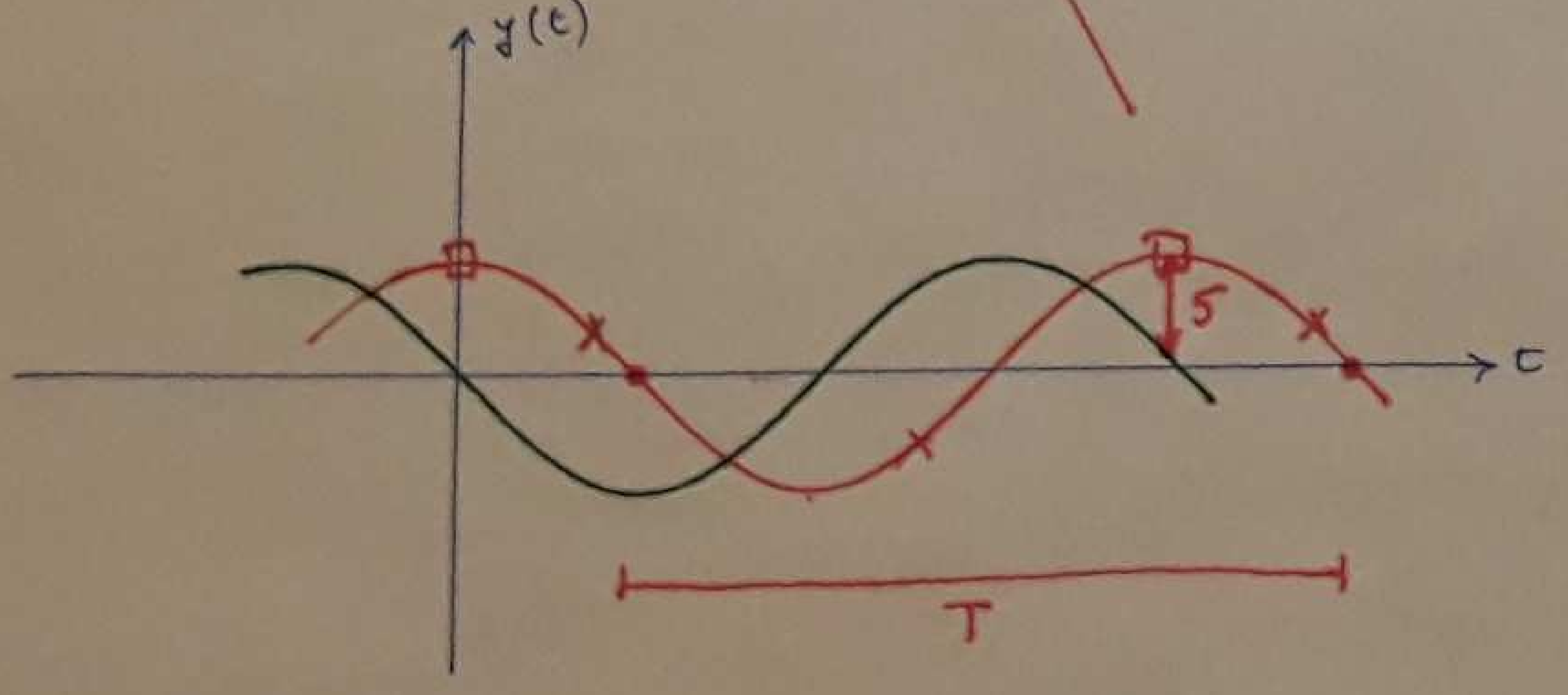
$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$z(t) = 3 - 2t$$

Grafikusan







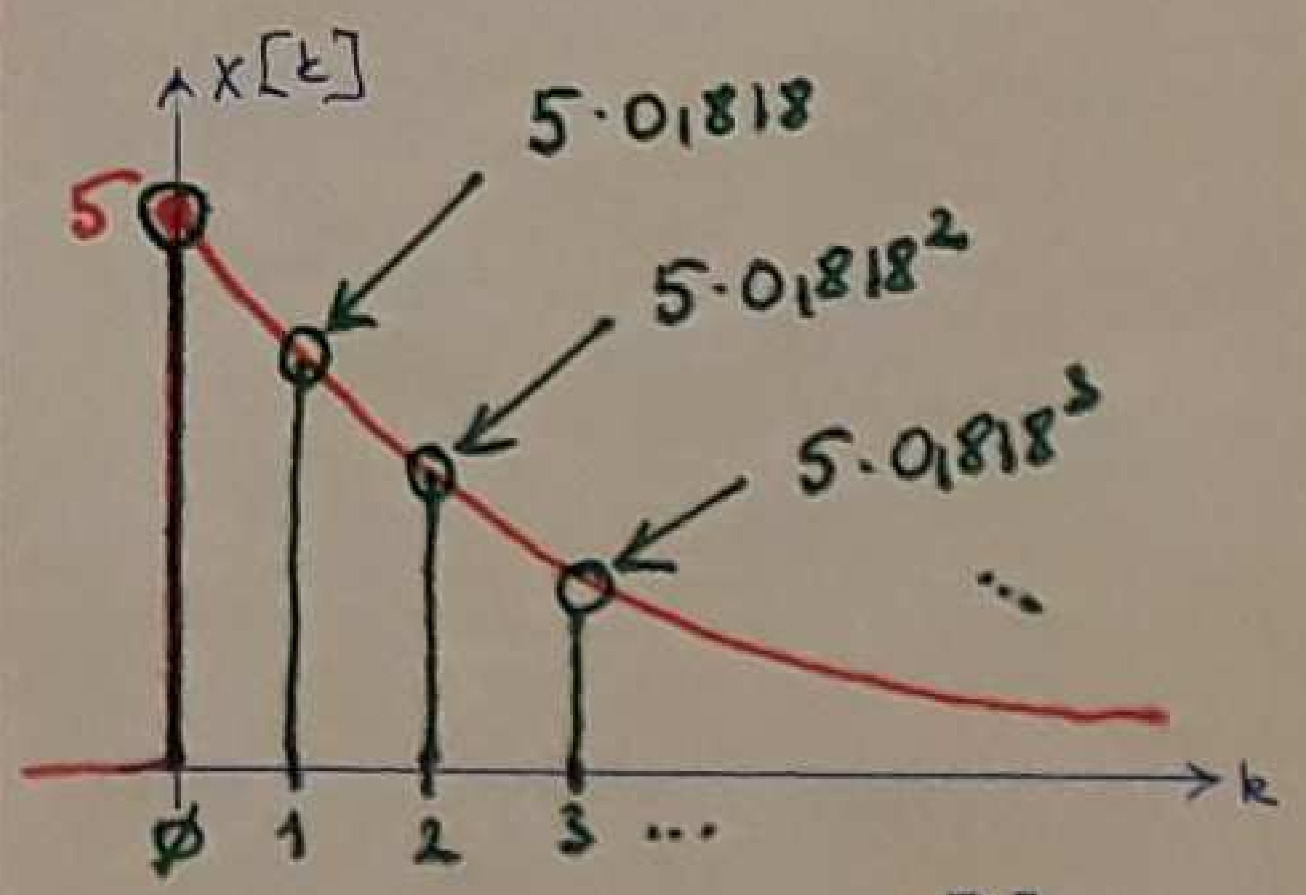
DISZKRÉT IDEJŰ JEL

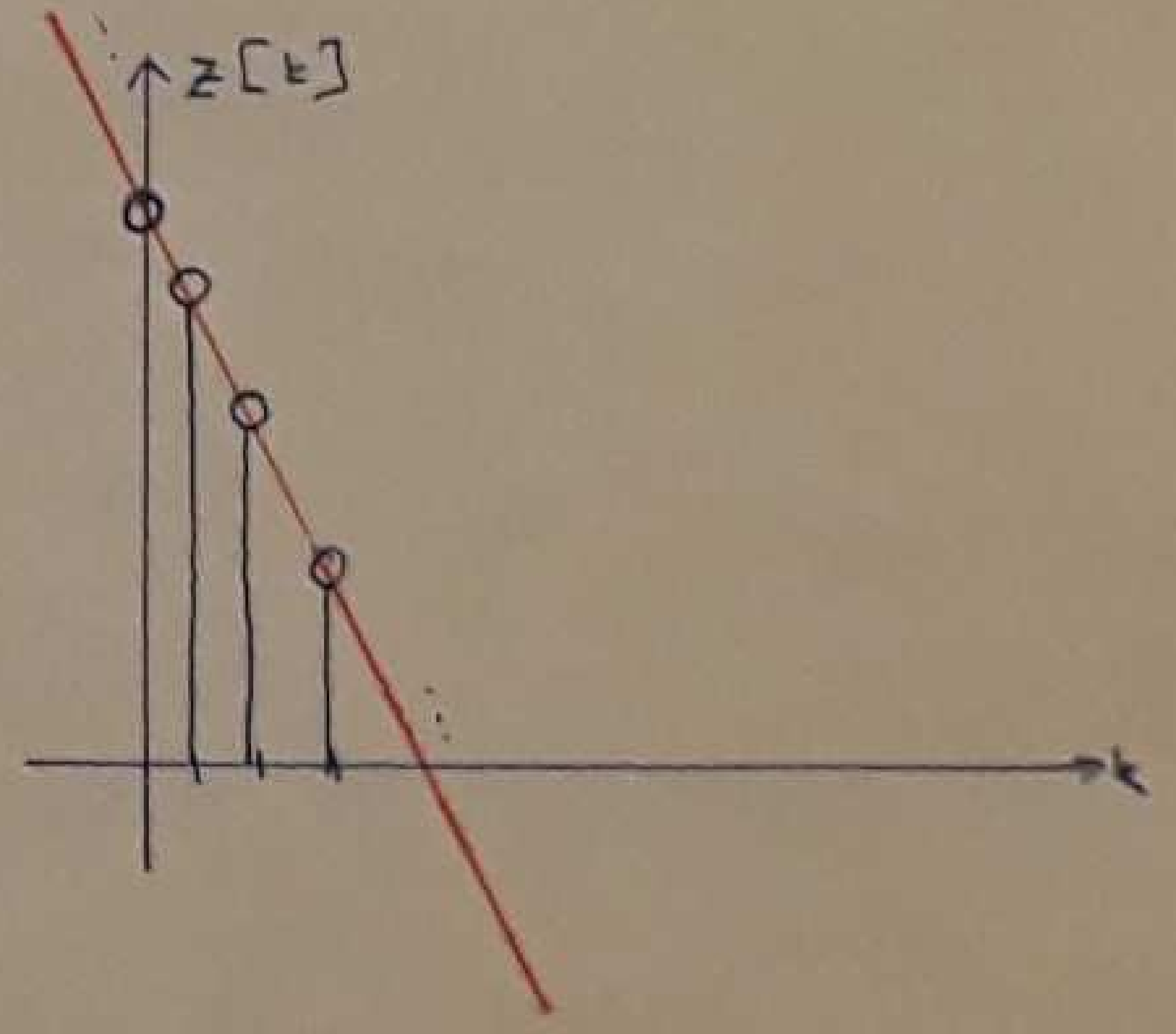
$\boxed{A - D}$

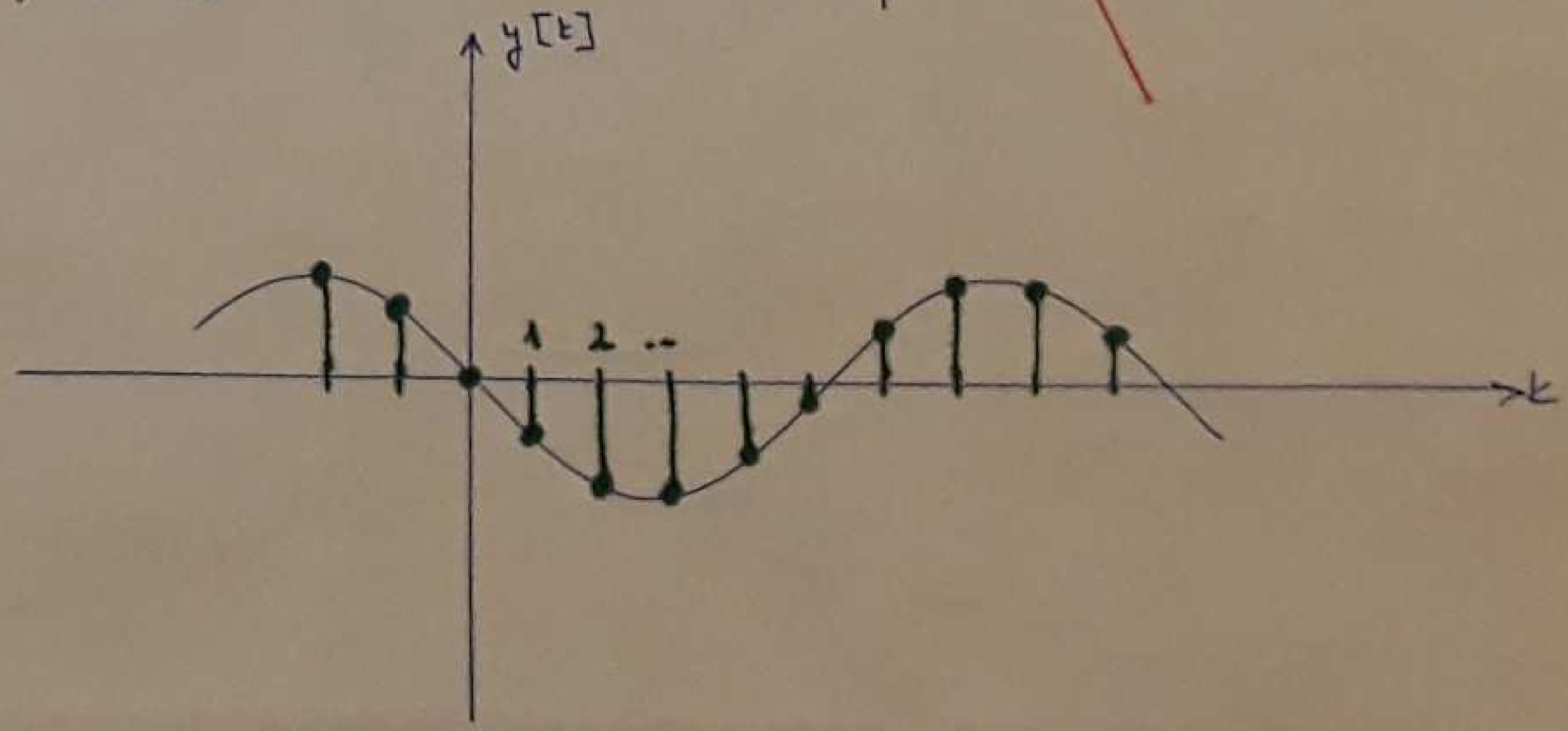
$$x[k] = \begin{cases} 0, & \text{ha } k < 0 \\ 5 \cdot 0{,}1818^k, & \text{ha } k \geq 0 \end{cases}$$

$$y[k] = 5 \cos\left(\frac{\pi}{20} k + \frac{\pi}{2}\right)$$

$$z[k] = 3 - k \cdot 0{,}2$$







Jelek megadása I.:

FOLYTONOS IDEJŰ JEL

Képlet

$$x(t) = \begin{cases} 0 & \text{ha } t < 0 \\ 5e^{-2t}, & \text{ha } t \geq 0 \end{cases}$$

0 ∞

$$y(t) = \underbrace{5}\,\underline{\cos}\left(\underbrace{\frac{\pi}{2}}_{\omega} t \overset{\leftarrow}{+} \underbrace{\frac{\pi}{2}}_{\alpha}\right)$$

$$\omega = \frac{2\pi}{} = 2\pi f$$

$$z(t) = 3 - 2t$$

Grafikusa

$$x(t) = 5e^{-2\overset{\uparrow}{t}}$$

$$t \rightarrow \overset{}{k}\,T_s$$

$$= 5e^{-2\cdot k\cdot 0{,}1}$$

$$= 5\cdot\underbrace{\left(e^{-2\cdot 0{,}1}\right)}_{q\,=\,0{,}818}{}^{k}$$

$$x[k] = 5\cdot 0{,}818^{k}$$

FI DI

$$x(t) = Me^{\lambda t} \longrightarrow x[k] = Mq^{k}$$

$\boxed{T_s = 0{,}1}$

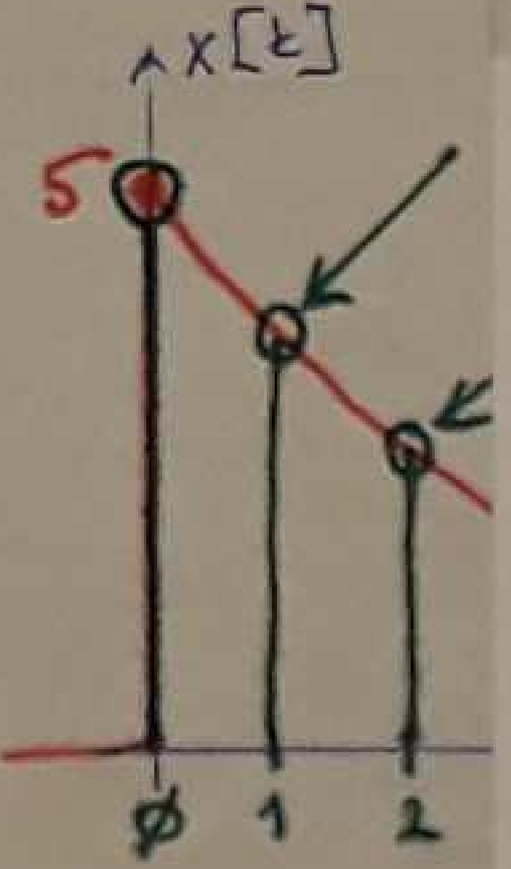
DISZKRÉT IDEJŰ JEL

$\boxed{A - D}$

$$x[k] = \begin{cases} 0, & \text{ha } k < 0 \\ 5\cdot 0{,}818^{k}, & \text{ha } k \geq 0 \end{cases}$$

$$y[k] = 5\cos\left(\frac{\pi}{20}k + \frac{\pi}{2}\right)$$

$$z[k] = 3 - k\cdot 0{,}2$$



$$z(t) = 3 - 2\overset{\uparrow}{t}_{\;k\cdot 0{,}1}$$

$$z[k] = 3 - 2\cdot k\cdot 0{,}1$$

$$= 3 - k\cdot 0{,}2$$

Jelek megadása II:

FOLYTONOS IDEJŰ JEL

Értékek felsorolása

$z(t) = 3-2t$ pl. $t = \{-1, 0, 1, 2, 3\}$ MATLAB SCILAB

$z = \{5, 3, 1, -1, -3\}$

Differenciálegyenlet

Kezdeti feltétel.

$$\frac{dy}{dt} = -2y \qquad \underset{\uparrow}{y(0)} = 5 \qquad y(t) = ?$$

formálisan... y t változók szeparálása

$$\int \frac{dy}{y} = -2\int dt$$

$$\boxed{y(t) = \underset{\uparrow}{M}e^{-2t}}$$

$$\ln y + C_1 = -2(t + C_2)$$

$$\ln y + C_1 = -2t - 2C_2$$

$$\ln y = -2t - 2C_2 - C_1$$

$$y = e^{-2t - 2C_2 - C_1} = e^{-2t} \cdot \boxed{\underset{M}{e^{-2C_2 - C_1}}}$$

$$y(0) = 5 = M\underbrace{e^{-2\cdot 0}}_{1} \Rightarrow M = 5 \qquad y(t) = 5e^{-2t}$$

DISZKRÉT IDEJŰ JEL

$z[k] = 3 - k \cdot 0{,}2$ $\quad k = \{1, 2, 3, 4, 5\}$

$z = \{2{,}8; 2{,}6; 2{,}4; \dots\}$

Rekurzív formula

differencia egyenlet.

$$y[\underset{\uparrow}{k}] = 0{,}2\, y[\underset{\uparrow}{k}-1] - 0{,}5\, y[\underset{\uparrow}{k}-2] \qquad y[-1] = 5,\ y[-2] = 0$$

$$y[0] = 0{,}2\, y[-1] - 0{,}5\, y[-2] = 0{,}2 \cdot 5 - 0{,}5 \cdot 0 = 1$$

$$y[1] = 0{,}2\, y[0] - 0{,}5\, y[-1] = \underbrace{0{,}2 \cdot 1}_{0{,}2} - \underbrace{0{,}5 \cdot 5}_{2{,}5} = -2{,}3$$

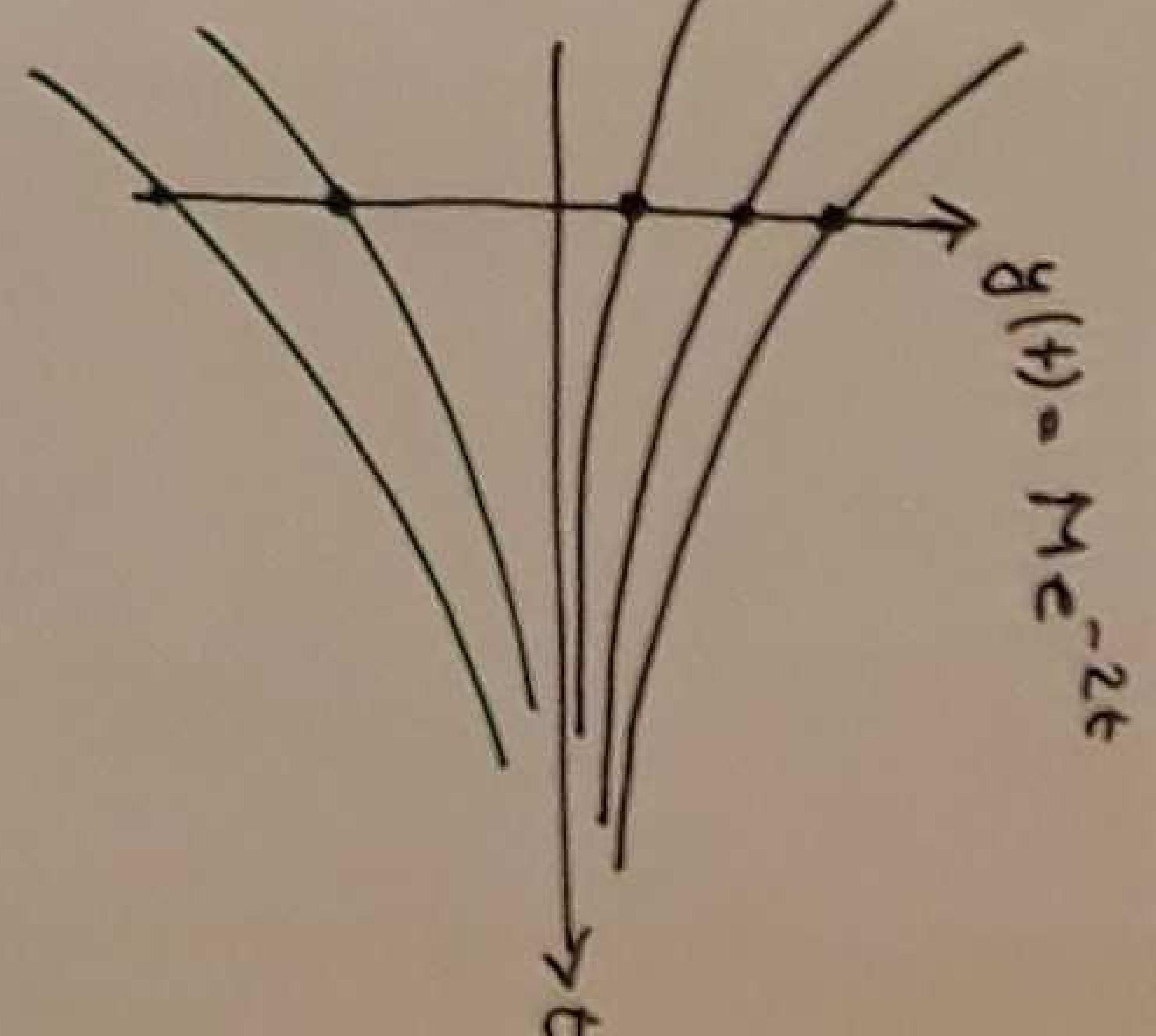
$$y[2] = 0{,}2\, y[1] - 0{,}5\, y[0] = \underbrace{0{,}2 \cdot (-2{,}3)}_{-0{,}46} - \underbrace{0{,}5 \cdot 1}_{0{,}5} = -0{,}96$$

$$y[3] = \dots$$

$$\frac{dy}{dt} = -2 \cdot y$$

$$\frac{d}{dt}\left(5e^{-2t}\right) = -2 \cdot 5e^{-2t}$$

$$5 \cdot e^{-2t}(-2) = -2 \cdot 5e^{-2t}$$



$$y(t) = 5\cos\left(\frac{\pi}{2}\underset{\underset{k\cdot 0{,}1}{\uparrow}}{t} + \frac{\pi}{2}\right)$$

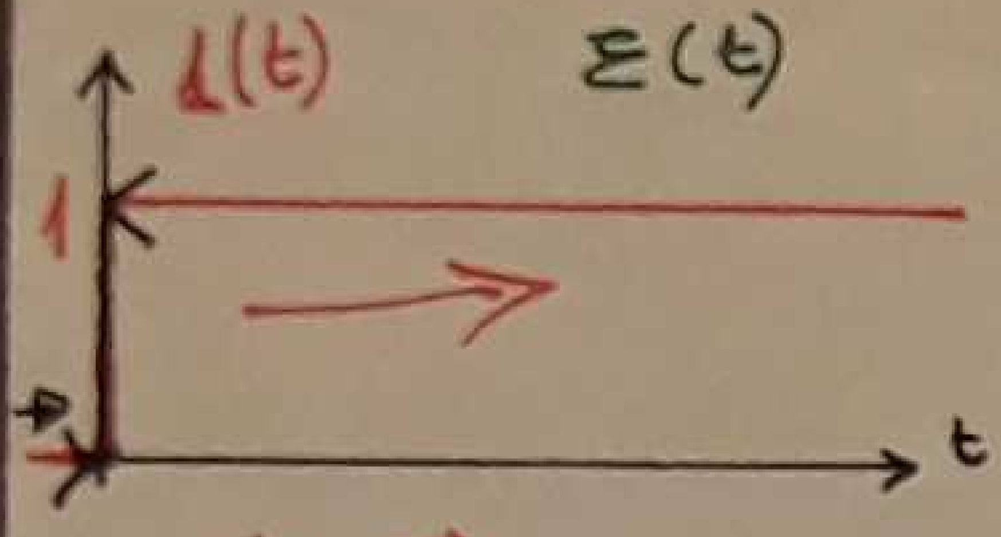
$$= 5\cos\left(\underset{\uparrow}{\frac{\pi}{2}}\, k\cdot \underset{\nearrow}{0{,}1} + \frac{\pi}{2}\right)$$

$$y[k] = 5\cos\left(\underbrace{\frac{\pi}{20}}_{\vartheta = \omega T_s} k + \frac{\pi}{2}\right)$$

Az egységugrásjel

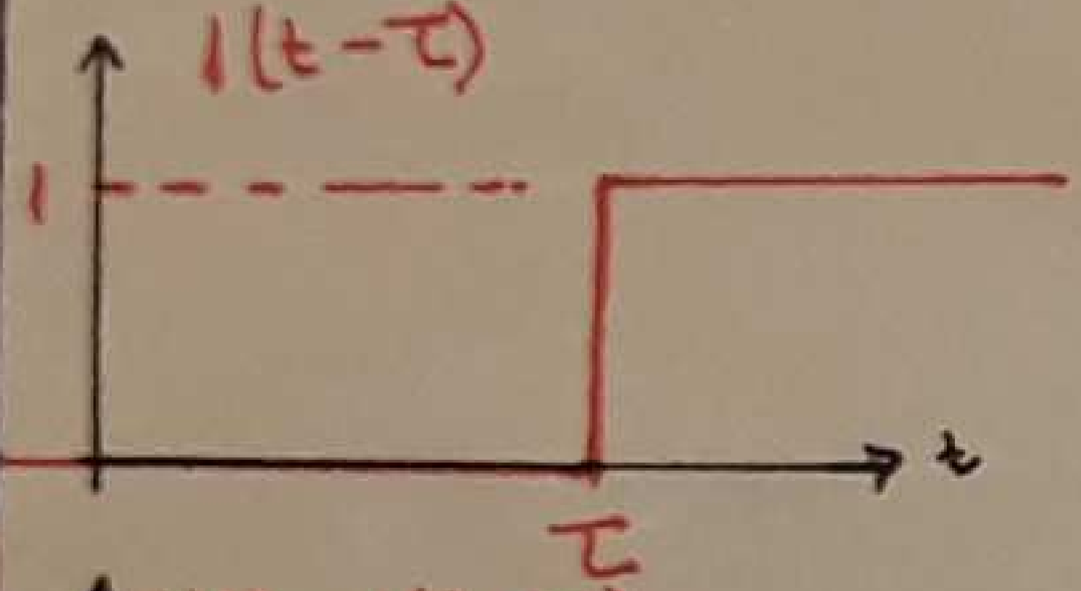
HEAVISIDE

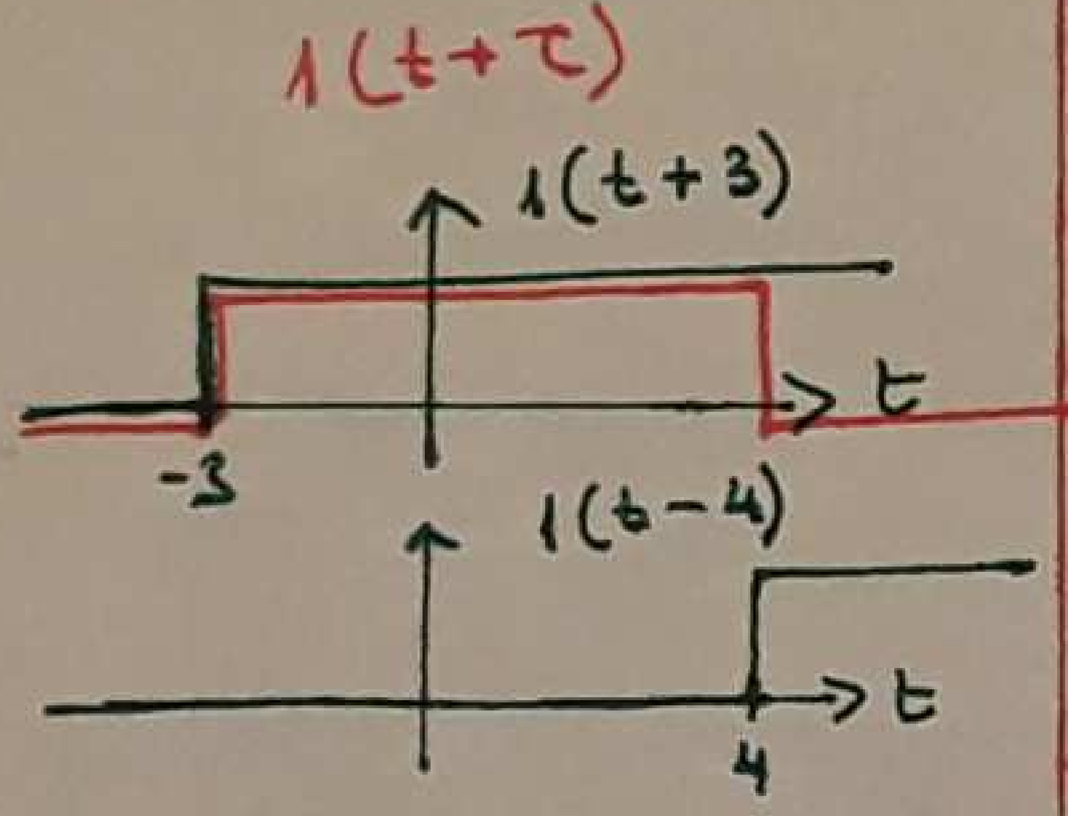
FOLYTONOS IDEJŰ

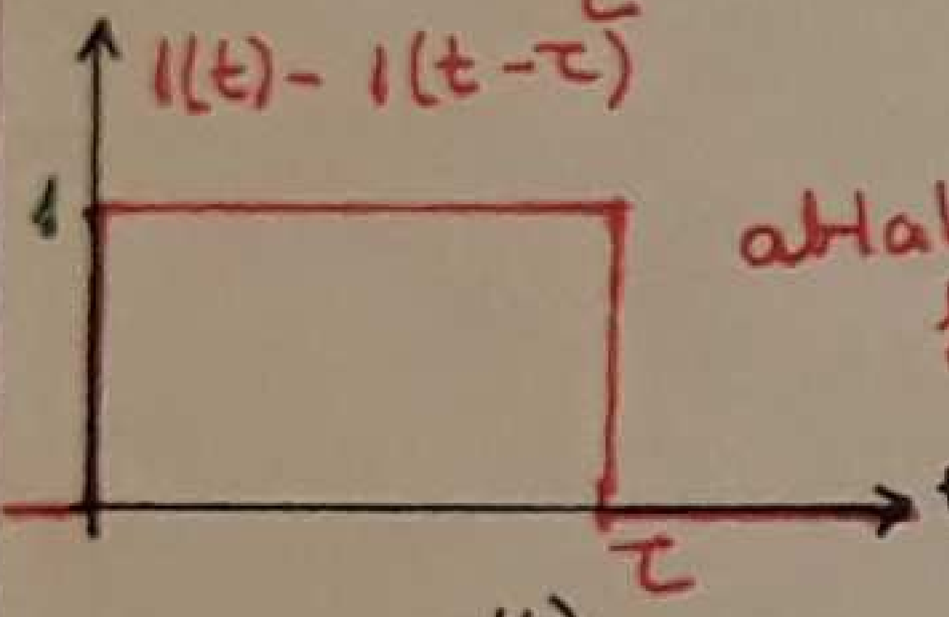


$$\lim_{t\to-\infty} 1(t) = 0 \qquad \lim_{t\to+\infty} 1(t) = 1$$

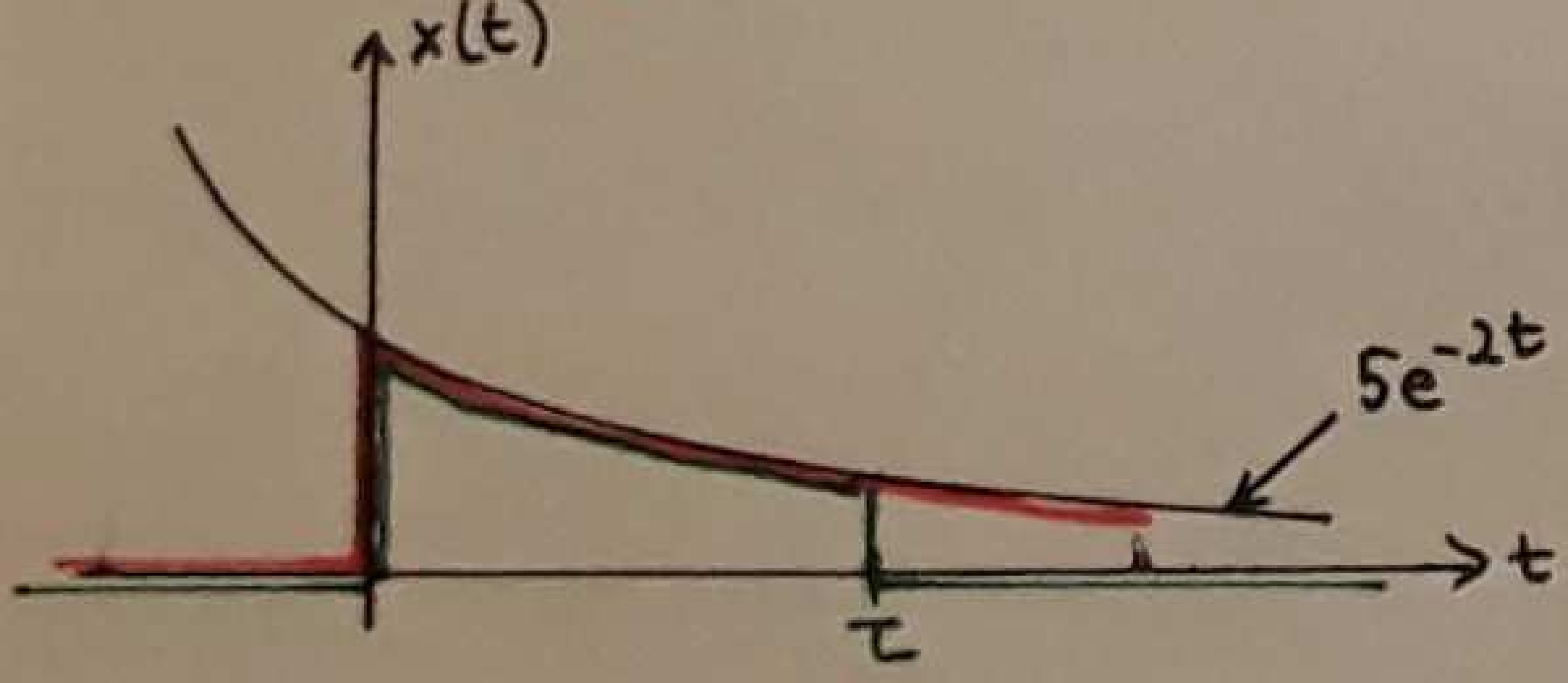
$$1(t) = \begin{cases} 0 & \text{ha } t<0 \\ 1 & \text{ha } t>0 \end{cases}$$







ablakozó függvény



$$x_1(t) = \begin{cases} 0 & \text{ha } t<0 \\ 5e^{-2t} & \text{ha } t \geq 0 \end{cases} \equiv 1(t)\cdot 5e^{-2t}$$

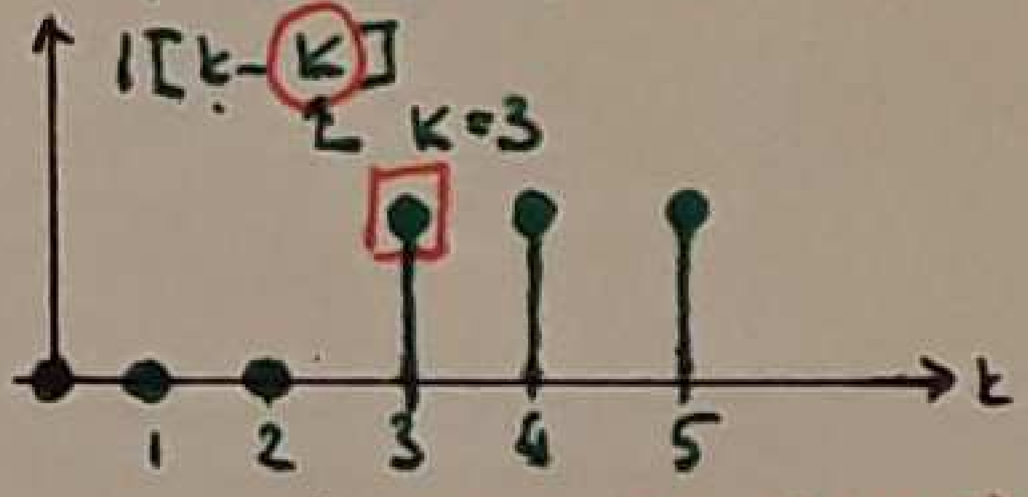
Belépő függvény.

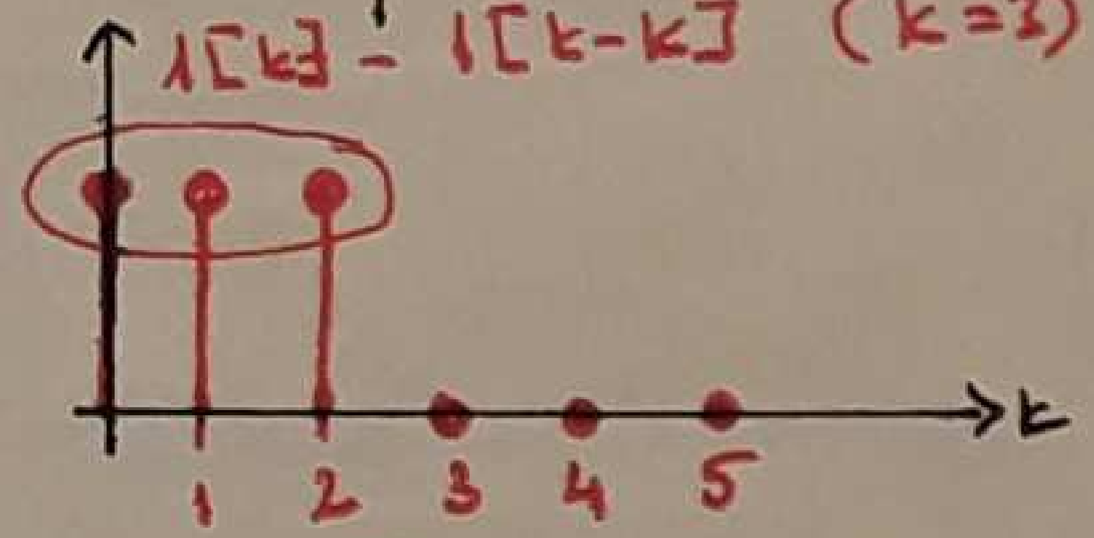
$$x_2(t) = \begin{cases} 0 & \text{ha } t<0 \\ 5e^{-2t} & \text{ha } 0 \leq t \leq \tau \\ 0 & \text{ha } t>\tau \end{cases} = \left[1(t) - 1(t-\tau)\right]\cdot 5e^{-2t}$$

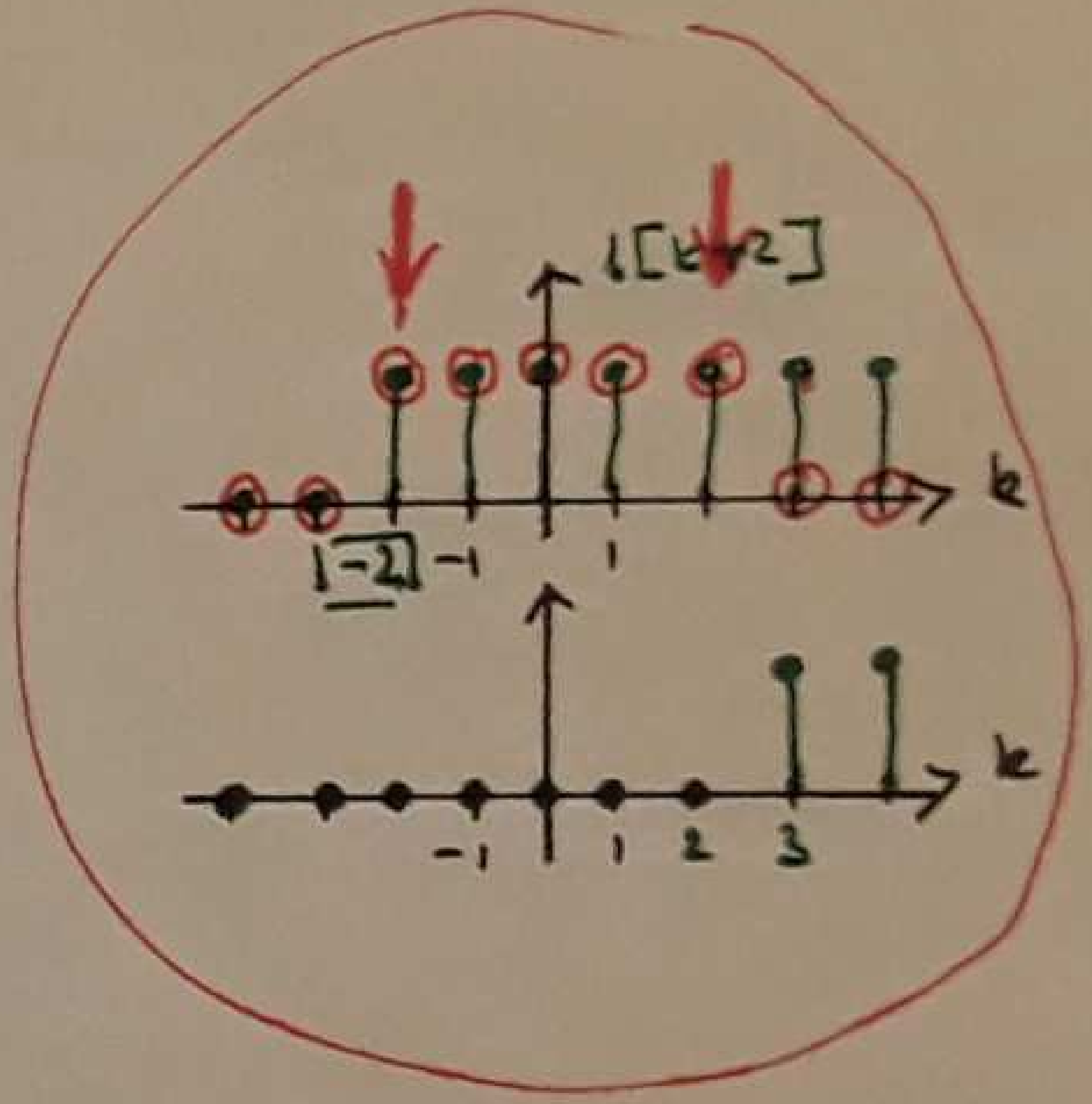
Ablakozó függvény

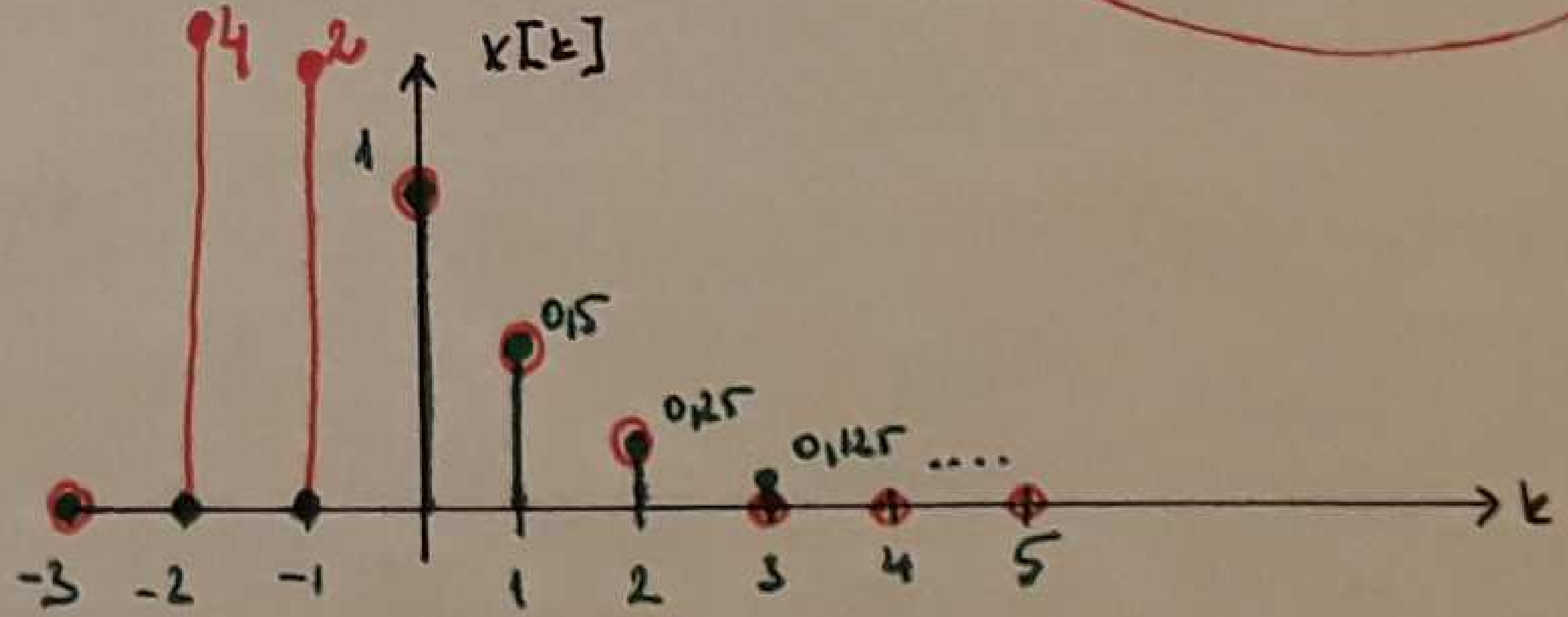
DISZKRÉT IDEJŰ

$$1[k] = \begin{cases} 0 & \text{ha } k<0 \\ 1 & \text{ha } k\geq 0 \end{cases}$$









$$x_1[k] = \boxed{1[k]}\cdot\boxed{0{,}5^k}$$

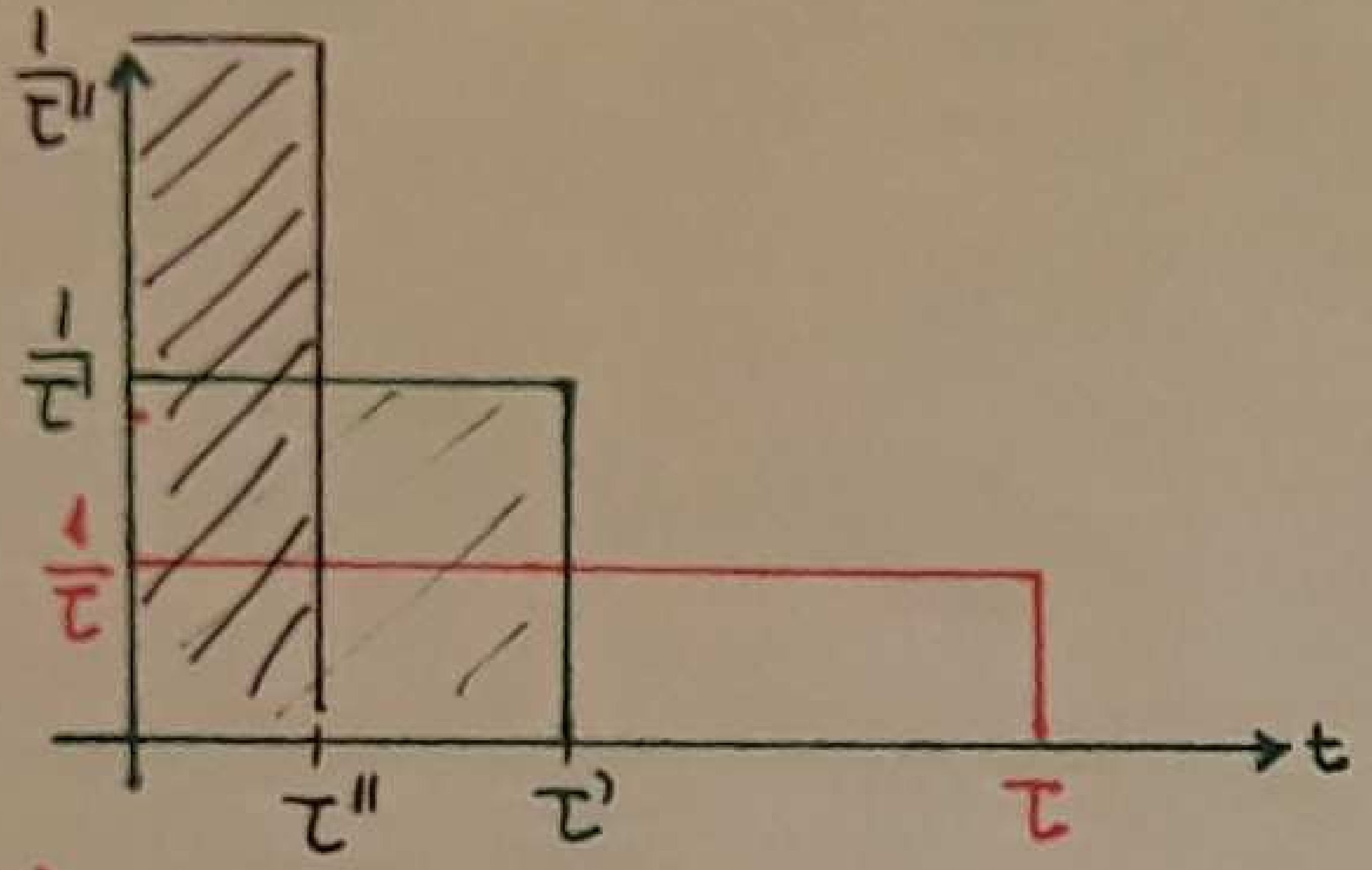
$$x_2[k] = \underbrace{\left\{1[k\boxed{+2}] - 1[k\boxed{-3}]\right\}}_{3-3}\cdot\underbrace{0{,}5^k}$$

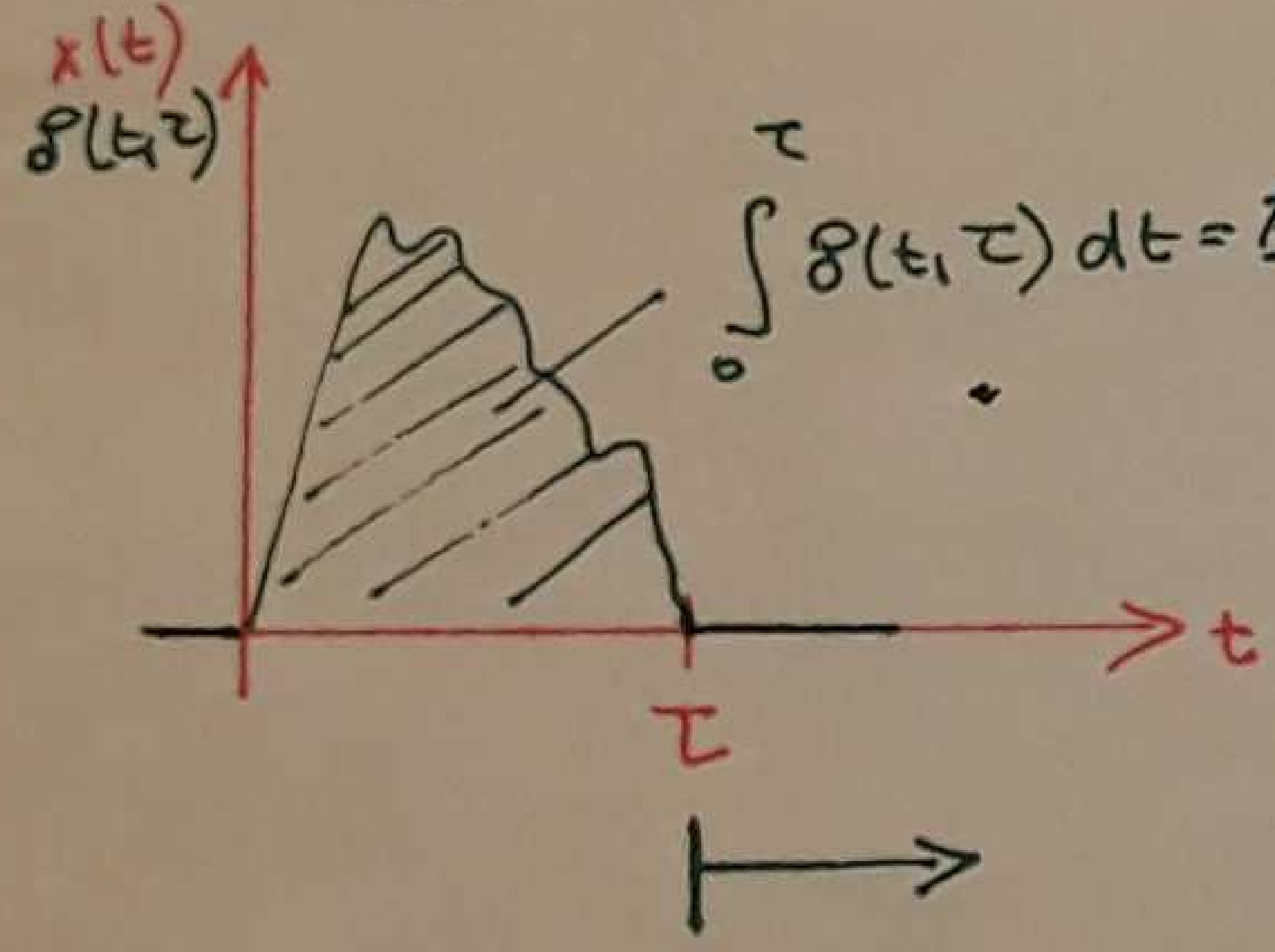
1

A Dirac-impulzus

Paul Dirac.

Egységnyi intenzitású impulzus :





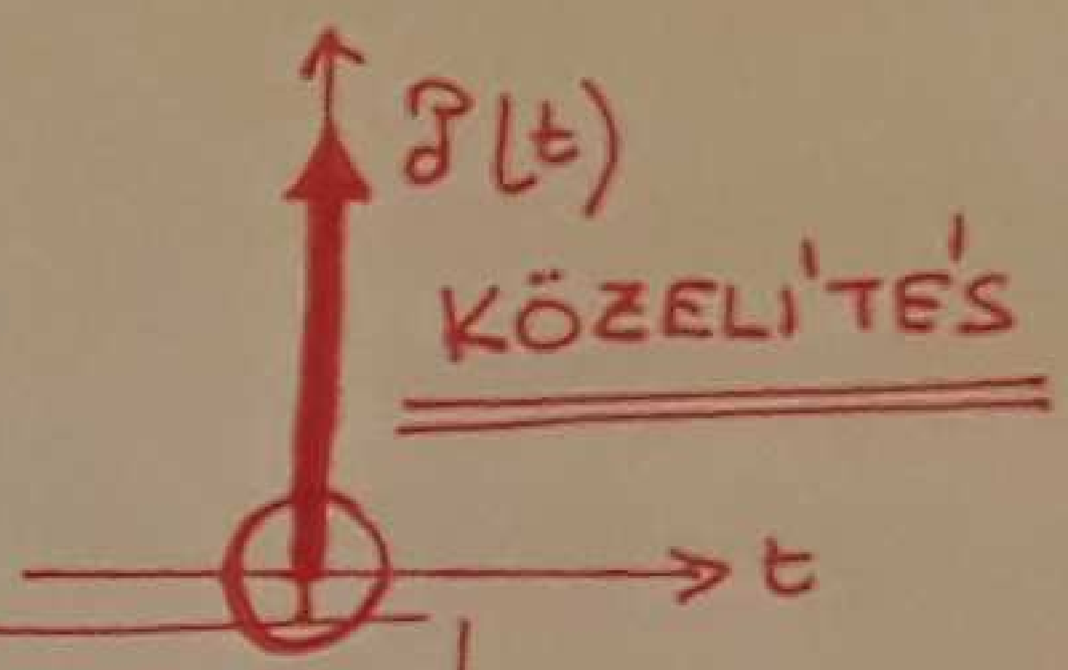
$$\int_0^{\tau} \delta(t,\tau)\,dt = 1$$

δ

$$\delta(t,\tau) = \frac{[1(t) - 1(t-\tau)]}{\tau}$$

$$\boxed{\int_0^{\tau} \underline{\delta(t,\tau)}\,dt = 1}$$

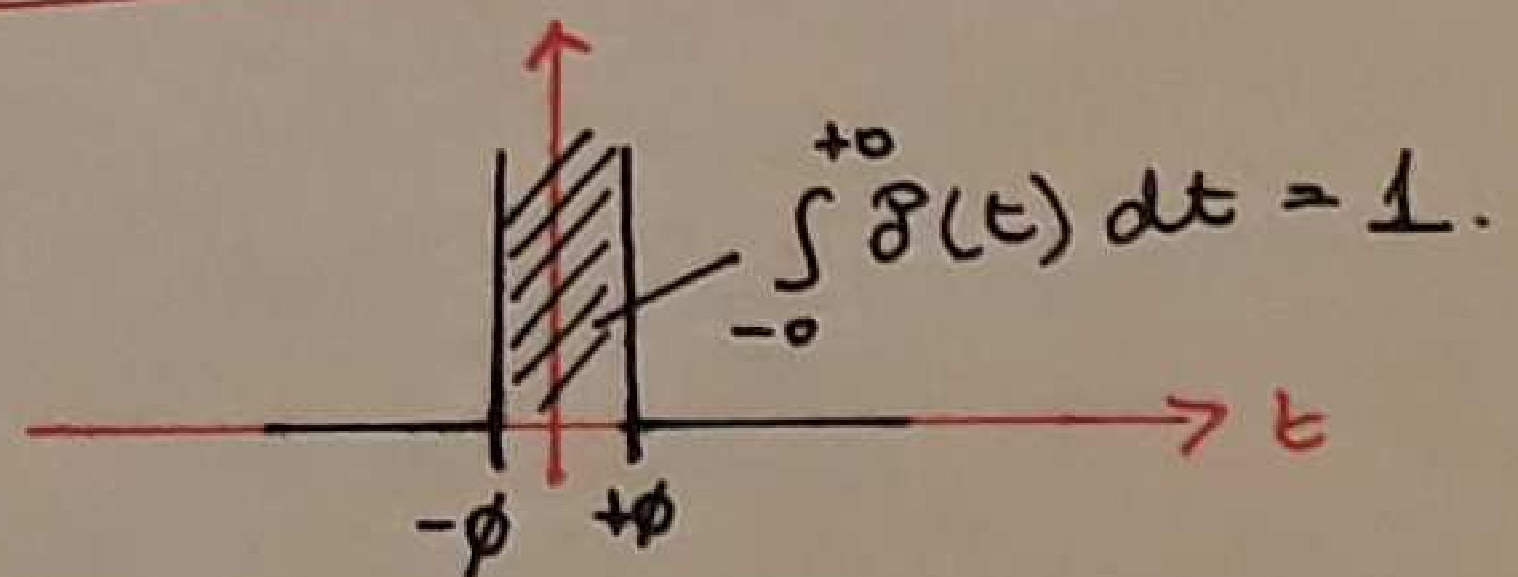
τ ↓ disztribúció



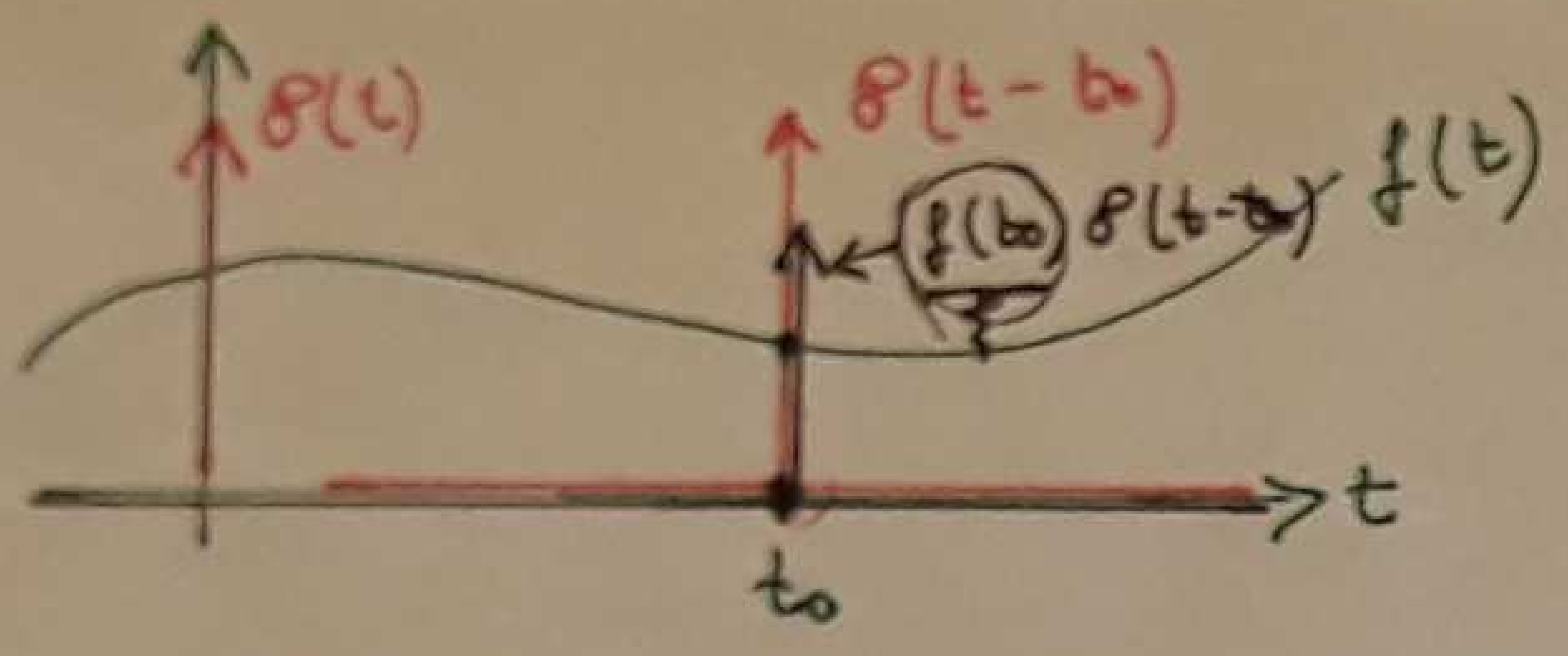
$\tau \rightarrow 0$

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\,dt = 1}$$

$$\boxed{\int_{-0}^{+0} \delta(t)\,dt = 1}$$



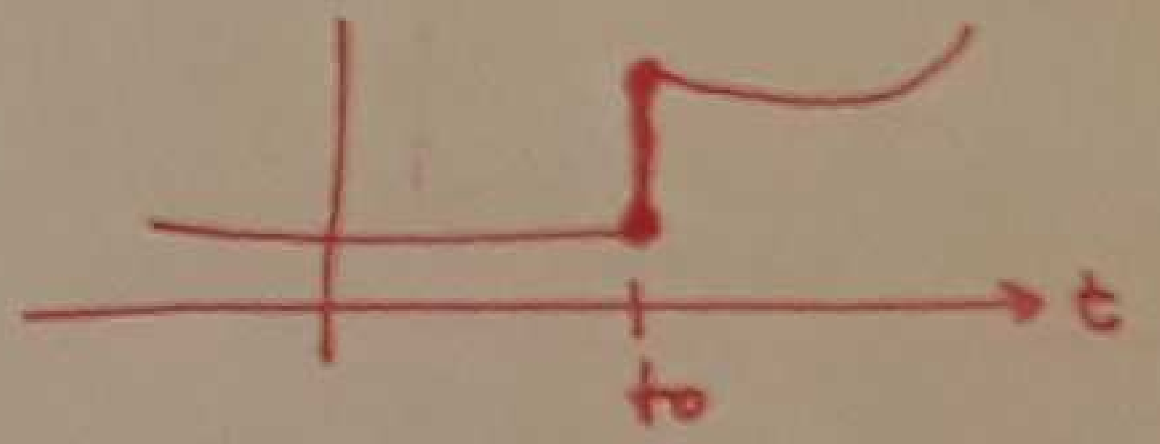
$$\int_{-0}^{+0} \delta(t)\,dt = 1.$$



$$\int_{-\infty}^{\infty} \underline{f(t)\,\delta(t-t_0)}\,dt = \int_{-\infty}^{\infty} \underbrace{f(t_0)}\,\delta(t-t_0)\,dt =$$

$$= f(t_0)\underbrace{\boxed{\int_{-\infty}^{\infty}\delta(t-t_0)\,dt}}_{\equiv 1} = f(t_0)$$

$$\boxed{\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)}$$

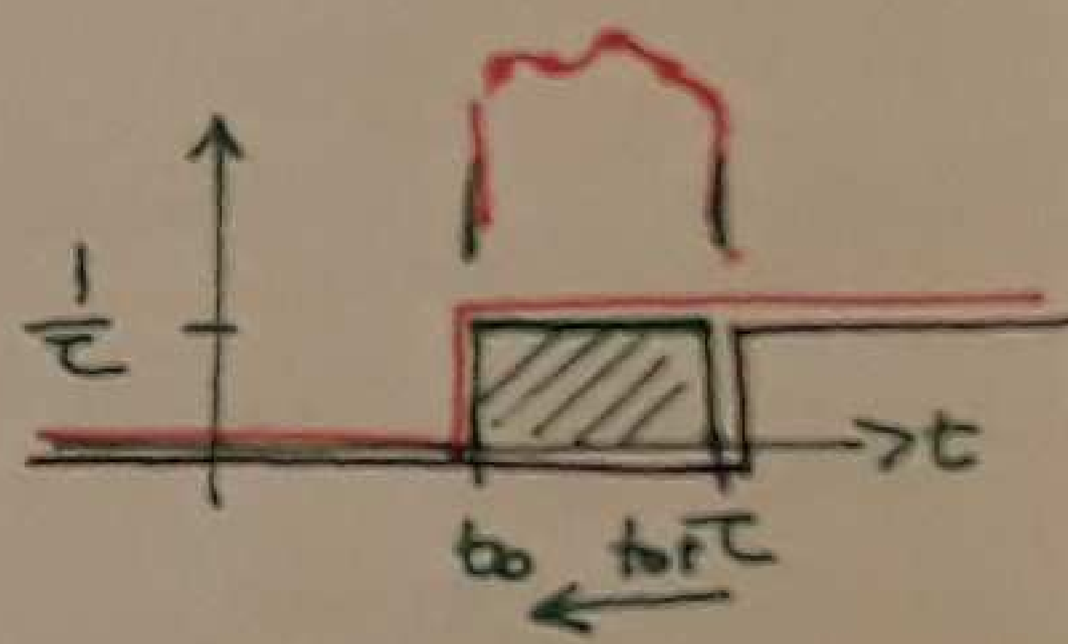


Integrálszámítás középérték tétele

$$\int_{-\infty}^{\infty} f(t)\,\underline{\delta(t-t_0)}\,dt = \lim_{\tau\to 0}\int_{-\infty}^{\infty} f(t)\,\frac{1(t-t_0)-1(t-[t_0+\tau])}{\tau}\,dt =$$

$$= \lim_{\tau\to 0}\int_{t_0}^{t_0+\tau} f(t)\,\frac{1}{\tau}\,dt = \lim_{\tau\to 0}\frac{1}{\tau}\boxed{\int_{t_0}^{t_0+\tau} f(t)\,dt}$$

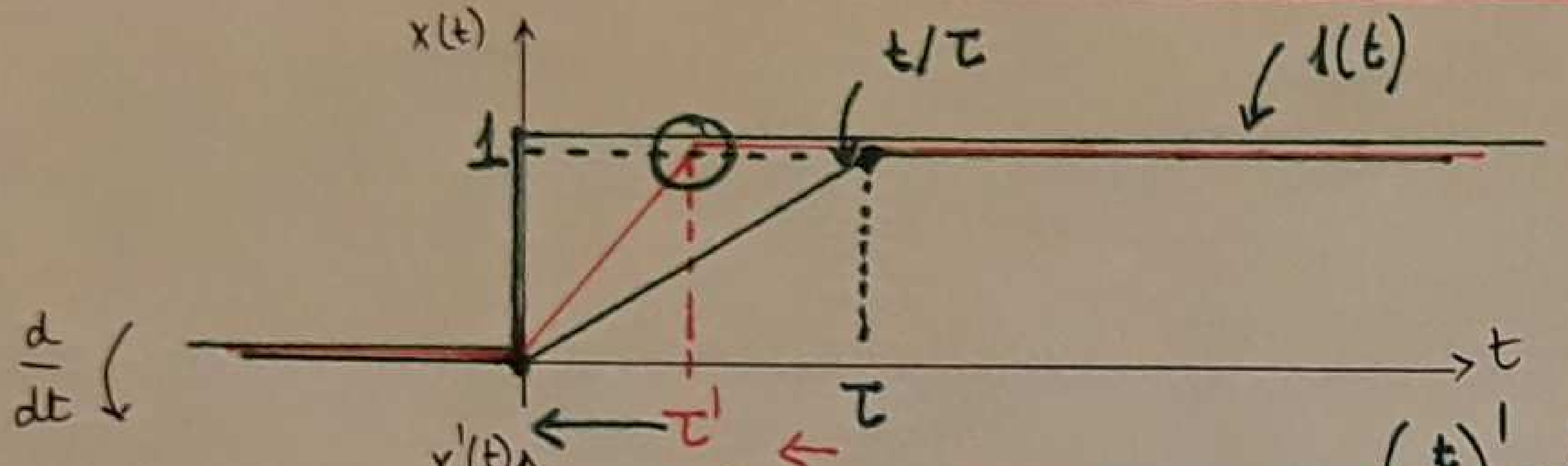
$$= \lim_{\tau\to 0}\cancel{\frac{1}{\tau}}\,\boxed{f(t_0+\xi\tau)\cdot\cancel{\tau}} = \underline{\underline{f(t_0)}}$$

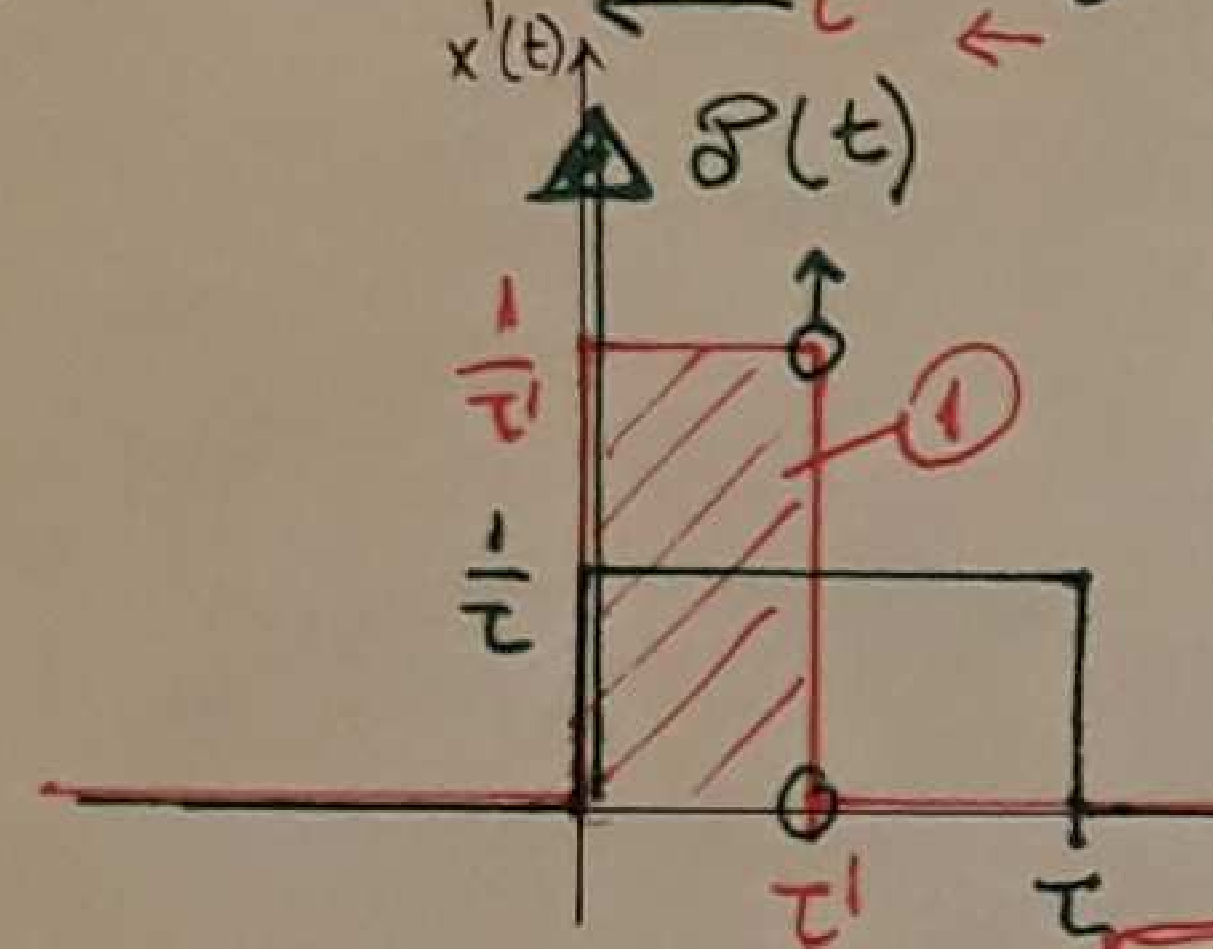


$0 \leq \xi \leq 1$

Az egységugrásjel és az egységimpulzus kapcsolata

FOLYTONOS IDEJŰ





$(\frac{t}{\tau})' = \frac{1}{\tau}$

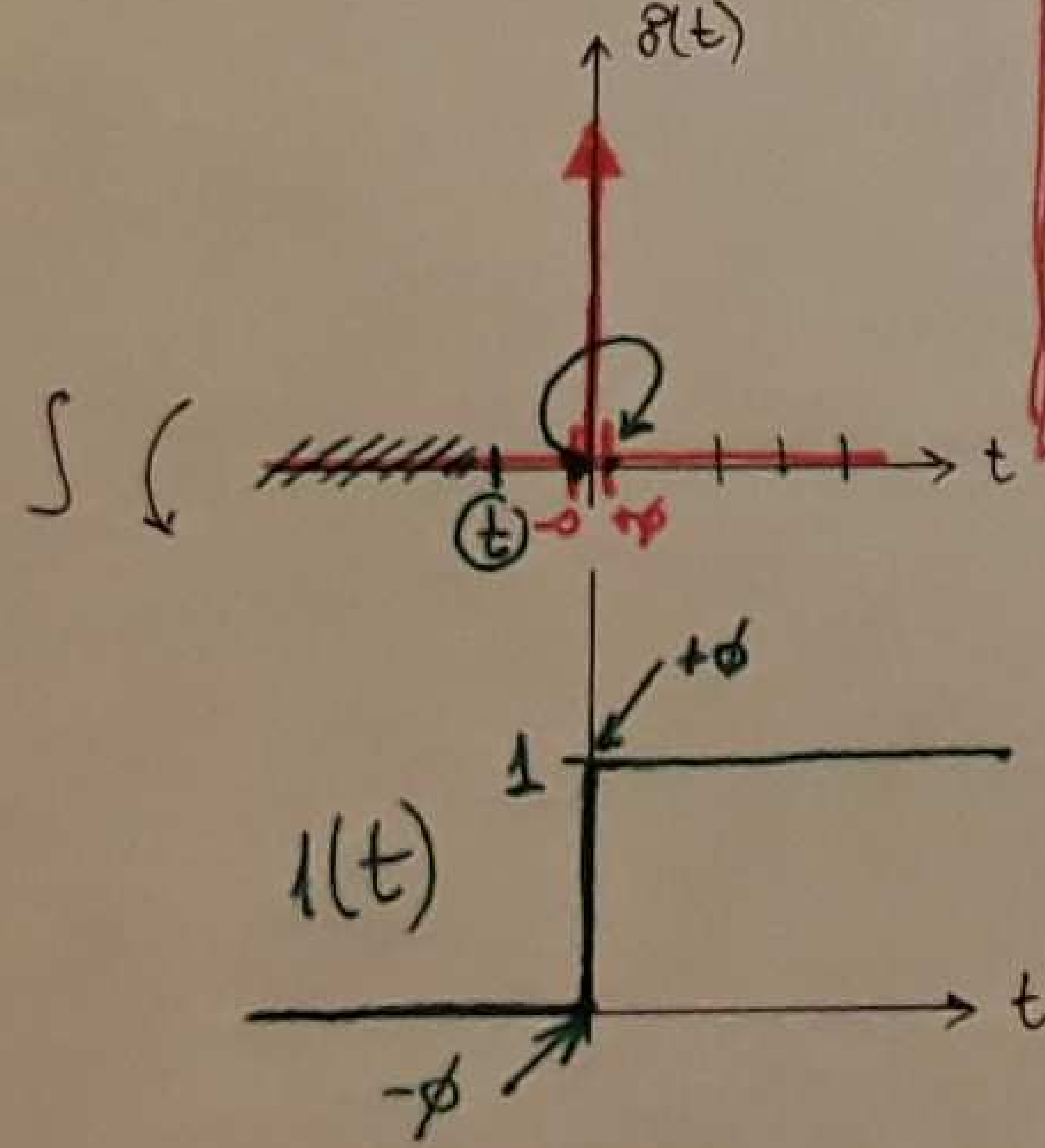
$\frac{1}{\tau'}$

$$\boxed{1'(t) = \delta(t)}$$

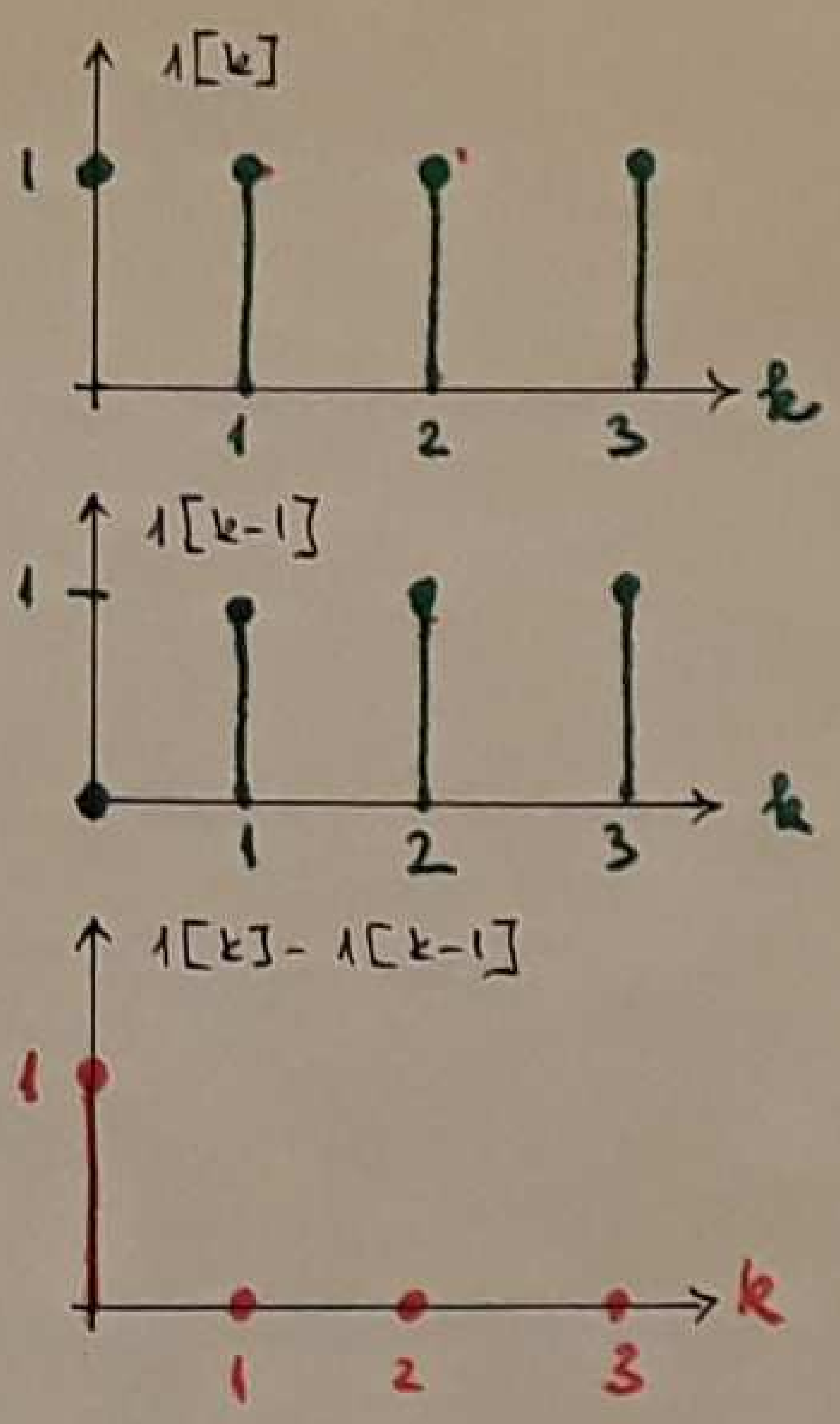
ÁLTALÁNOSÍTOTT DERIVÁLT.

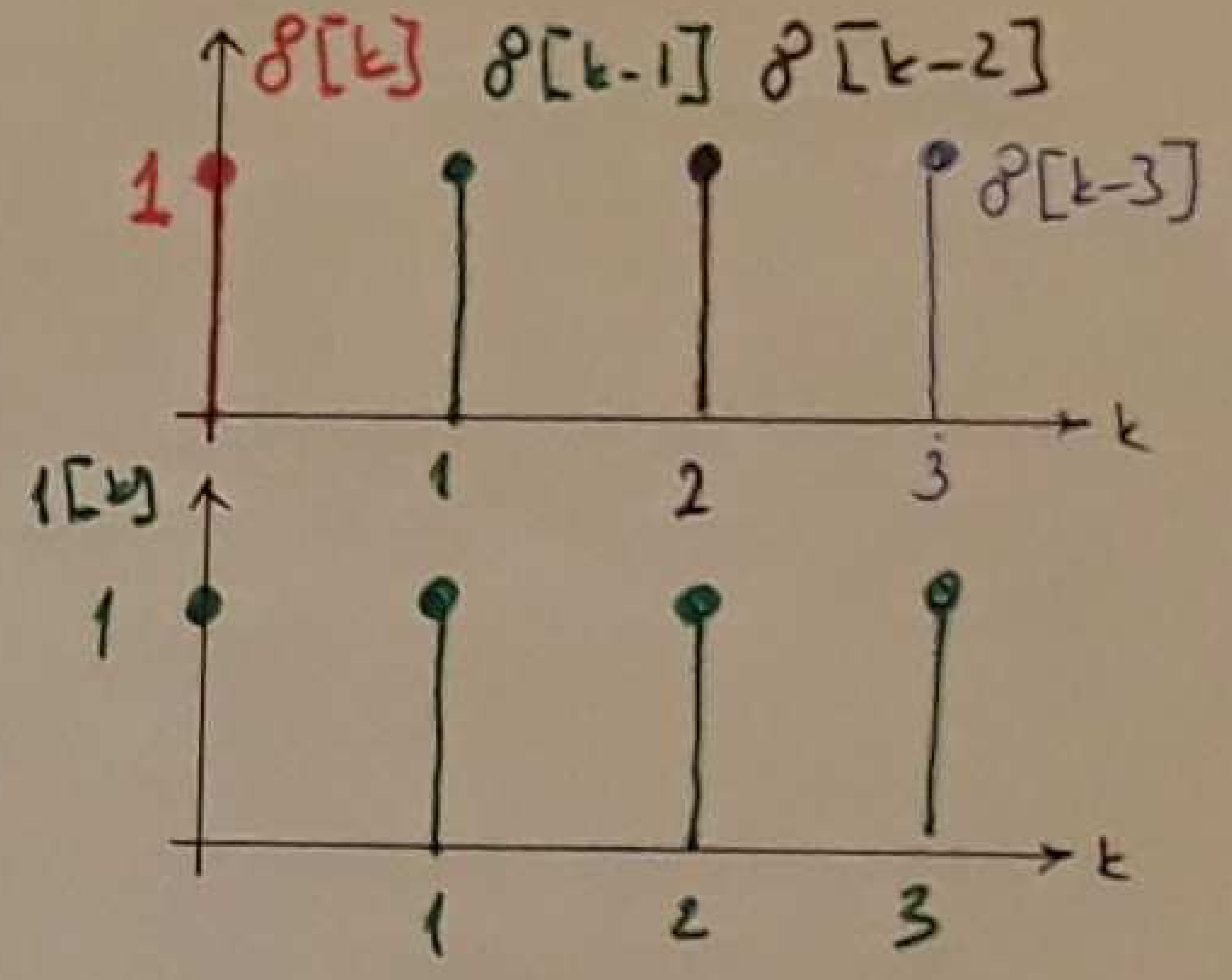
$$\boxed{\int_{-\infty}^{(t)} \delta(\tau)\, d\tau = 1(t)}$$

$$\int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

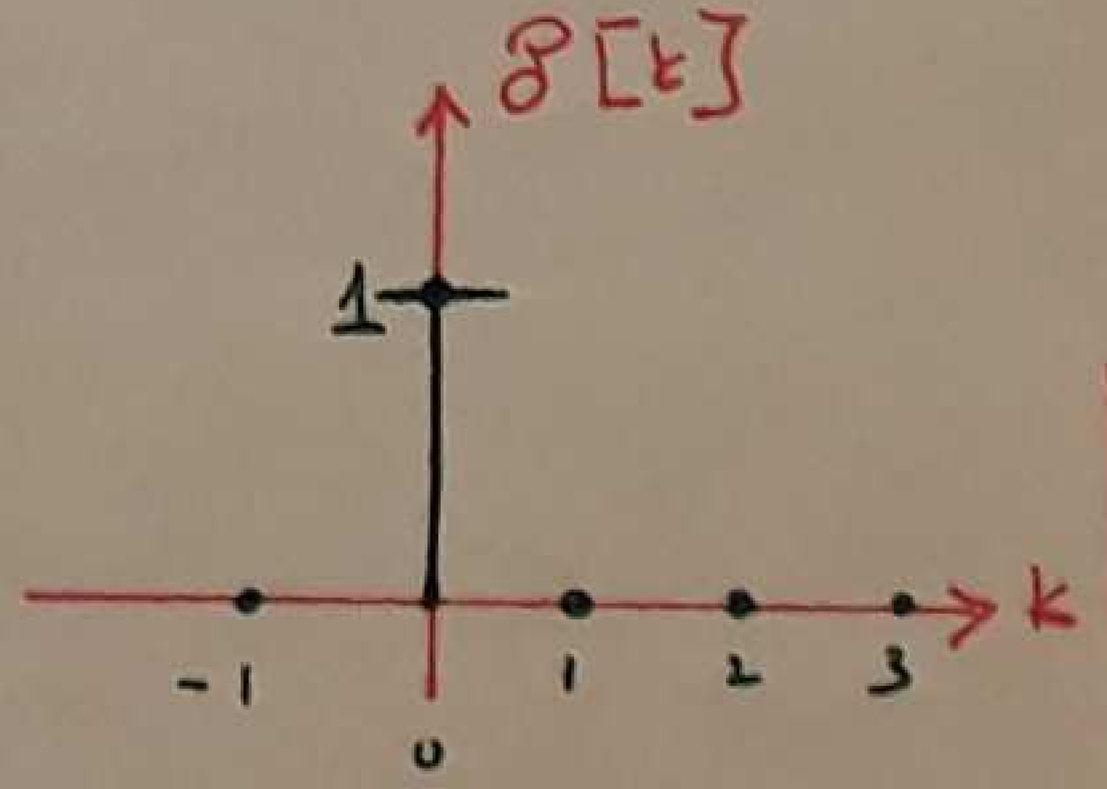


DISZKRÉT IDEJŰ





$$\boxed{\delta[k] = 1[k] - 1[k-1]}$$



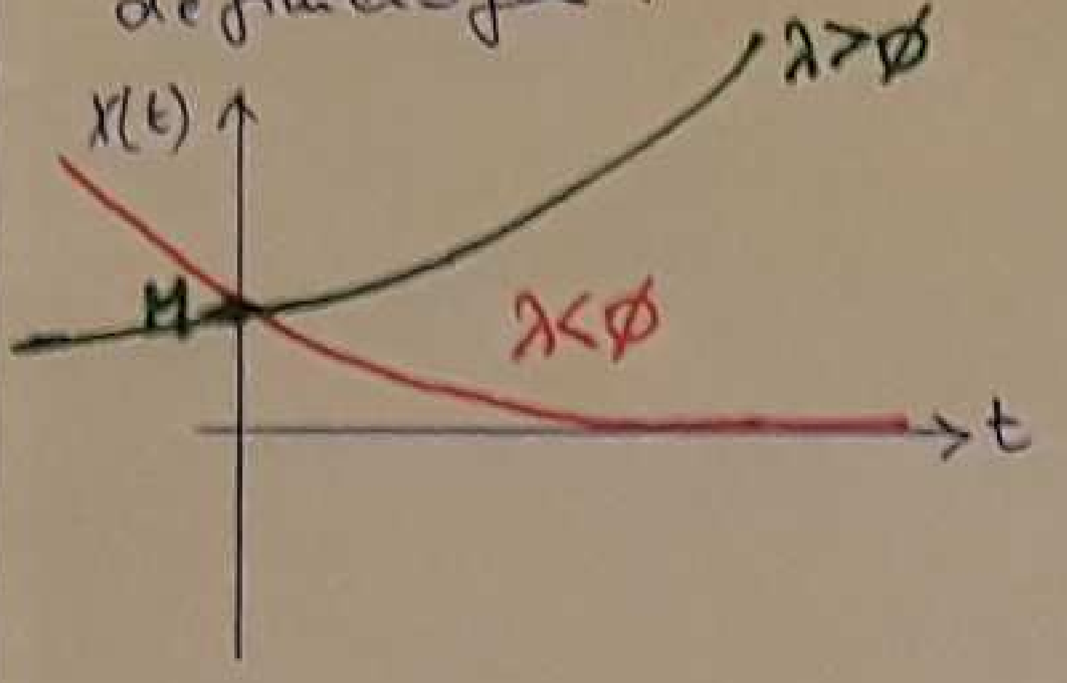
δ[k] + δ[k-1] + δ[k-2]...

⇓

$$\boxed{\sum_{i=0}^{\infty} \delta[k-i] = 1[k]}$$

ANALÓG

Vizsgáljuk meg az $x(t) = Me^{\lambda t}$ exponenciális függvény jellemző adatait! Az időállandó definíciója.



$$x(t) = \underbrace{Me^{\lambda t}}_{\text{TRANZIENS}} \begin{cases} \lambda > 0 \\ \lambda < 0 \end{cases}$$

↗ sajátérték

$(Me^{\lambda t})' = \lambda Me^{\lambda t}$

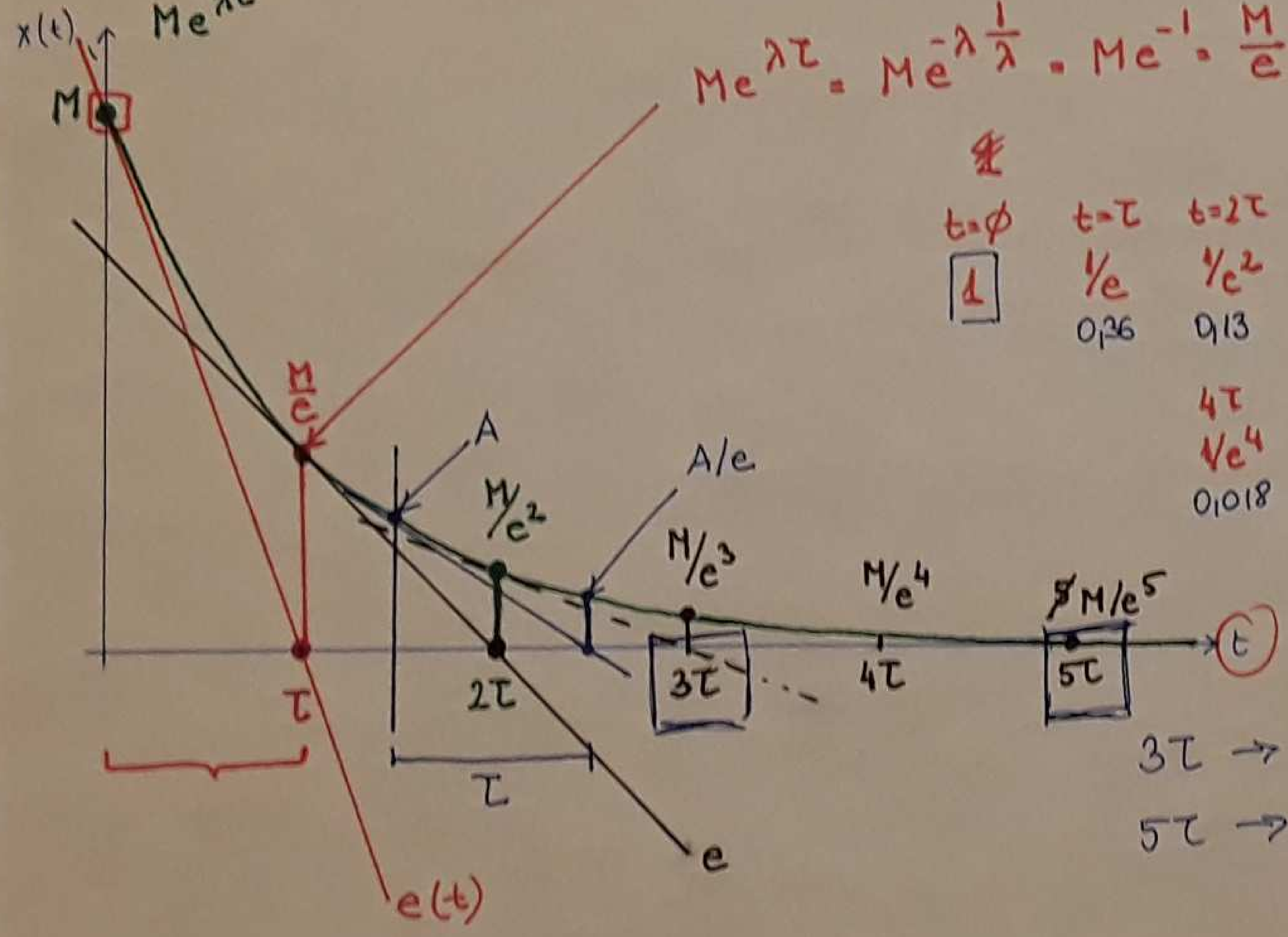
$e = mt + b$

m ↖ λM, b ↖ M

$e(t) = \lambda Mt + M$

$0 = \lambda \cancel{M} \tau + \cancel{M} \rightarrow \boxed{\tau = -\frac{1}{\lambda}}$

$0 = \lambda \tau + 1$



$Me^{\lambda \tau} = Me^{-\lambda \frac{1}{\lambda}} = Me^{-1} = \frac{M}{e}$

~~1~~

$t = 0$	$t = \tau$	$t = 2\tau$	3τ
$\boxed{1}$	$1/e$	$1/e^2$	$1/e^3$
	0,36	0,13	$\boxed{0{,}049}$ <5%

4τ	5τ
$1/e^4$	$1/e^5$
0,018	$\boxed{0{,}006}$ <1%

$3\tau \rightarrow 5\%$

$5\tau \rightarrow 1\%$

$e = mt + b$

$e = m(t - \tau) + b'$

m ↖ $\lambda Me^{-\lambda \frac{1}{\lambda}}$, $\lambda \frac{M}{e}$; b' ↖ $\frac{M}{e}$

$e = \lambda \cancel{\frac{M}{e}} (t - \tau) + \cancel{\frac{M}{e}} = 0$

$\lambda (t - \tau) + 1 = 0$

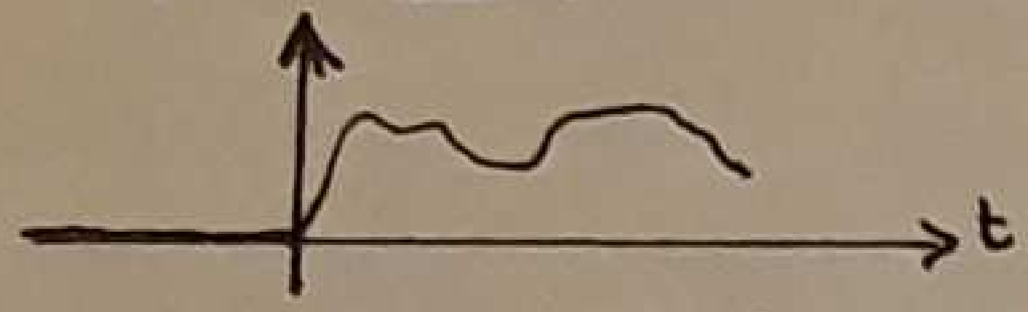
$t - \tau = -\frac{1}{\lambda} = \tau$

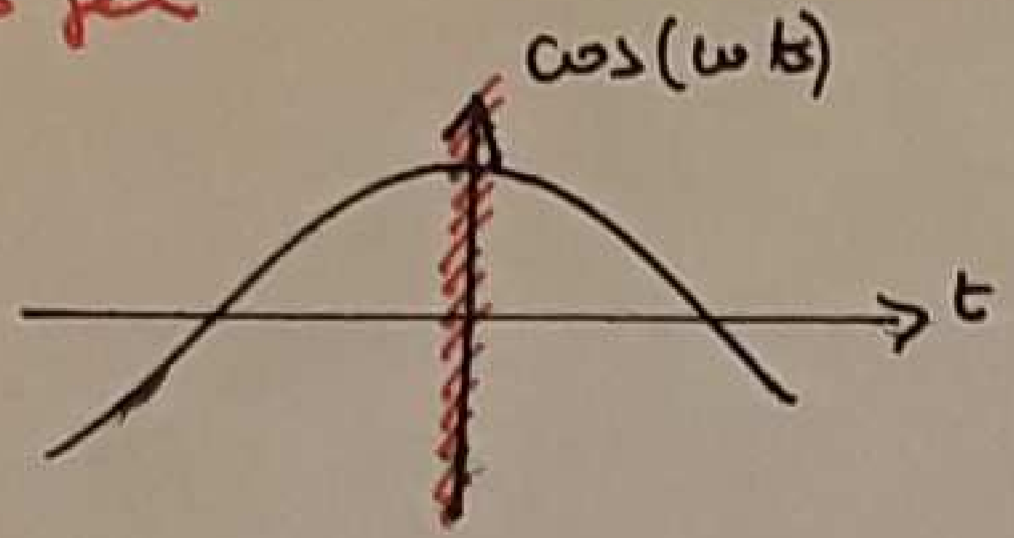
$\boxed{t = 2\tau}$

Jelek osztályozása:

① Belépőjel / nem belépő jel

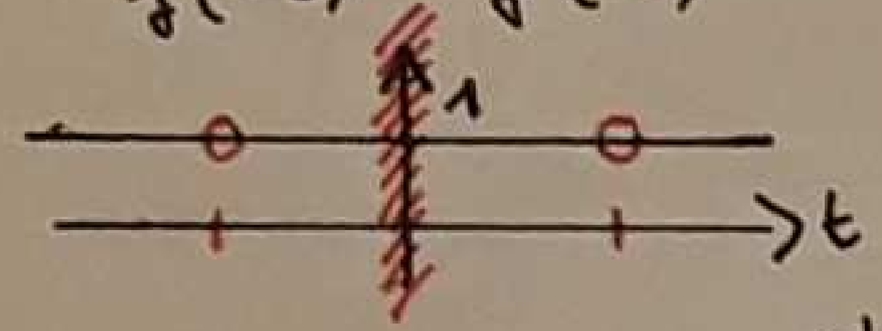
$$f(t) = \underbrace{1(t)}\, g(t)$$



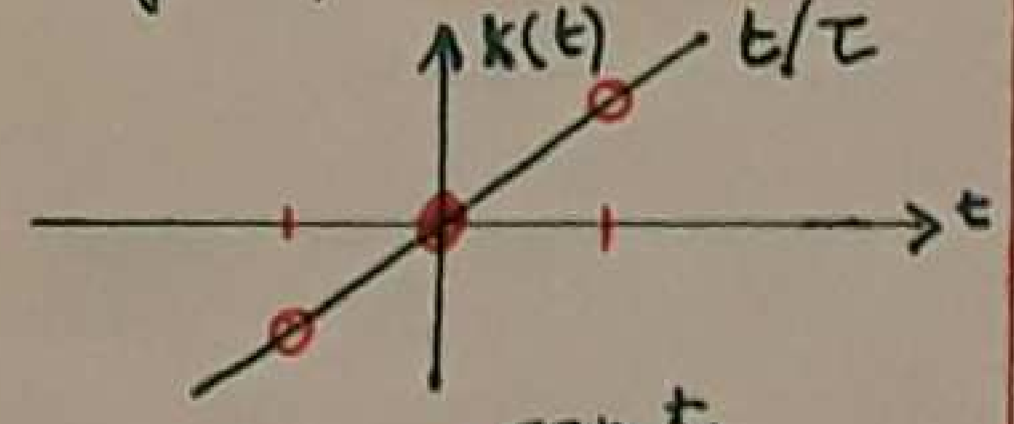


② Páros / Páratlan jel

$$f(-t) = f(t)$$



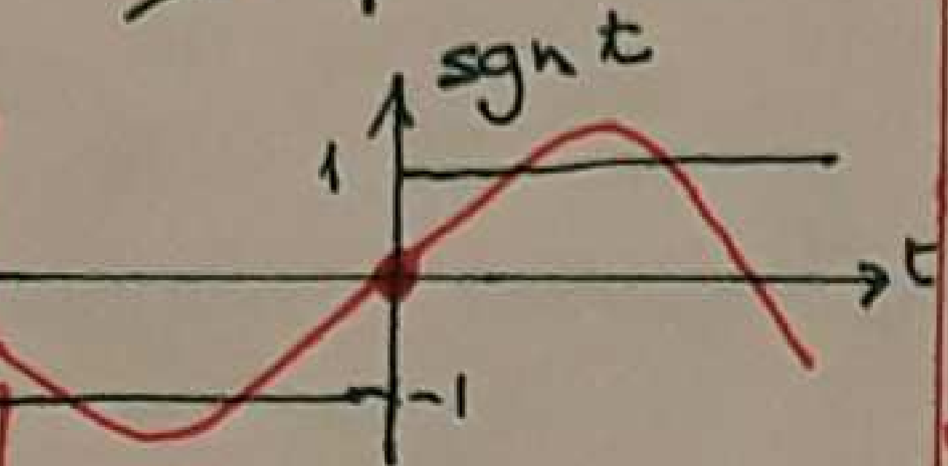
$$f(-t) = -f(t)$$

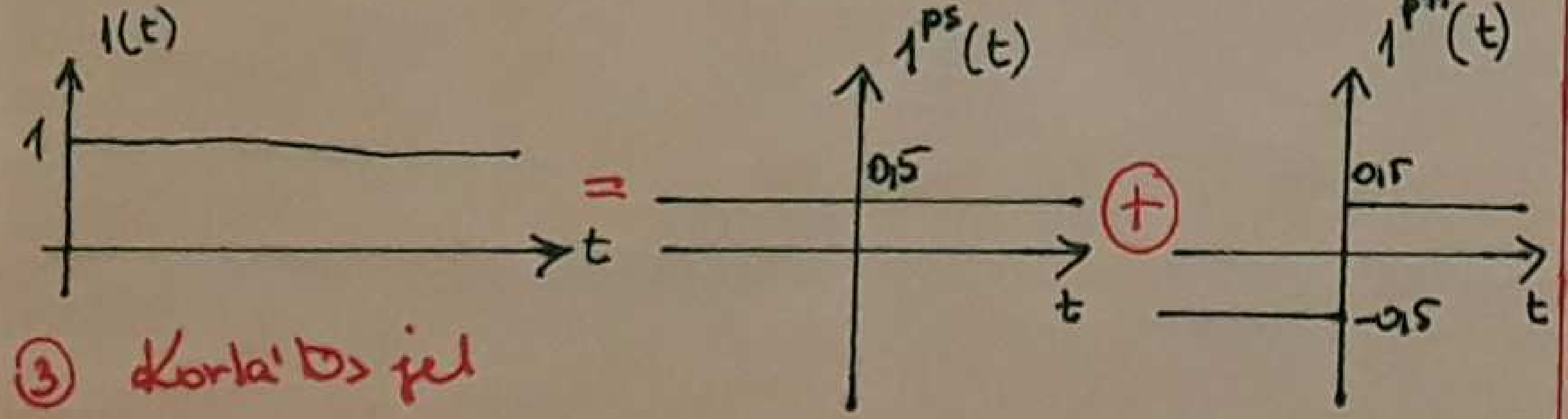


$$x(t) = x^{ps}(t) + x^{ptl}(t)$$

$$x^{ps}(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

$$x^{ptl}(t) = \frac{1}{2}\left[x(t) \ominus x(-t)\right]$$

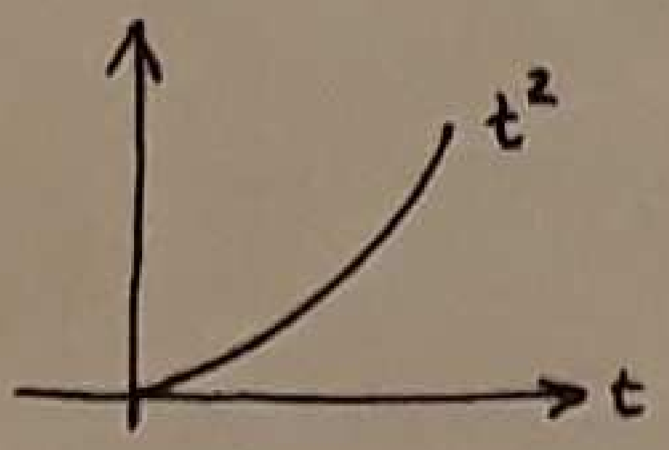




③ Korlátos jel

$$|x(t)| \leq M < \infty$$

$x[k]$ stab.



④ Abszolút integrálható jel

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

⑤ Négyzetesen integrálható jel

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt < \infty$$

energia $\boxed{\text{norma}}$

⑥ Abszolút összegezhető jel

$$\sum_{k=-\infty}^{\infty} |x[k]| < \infty$$

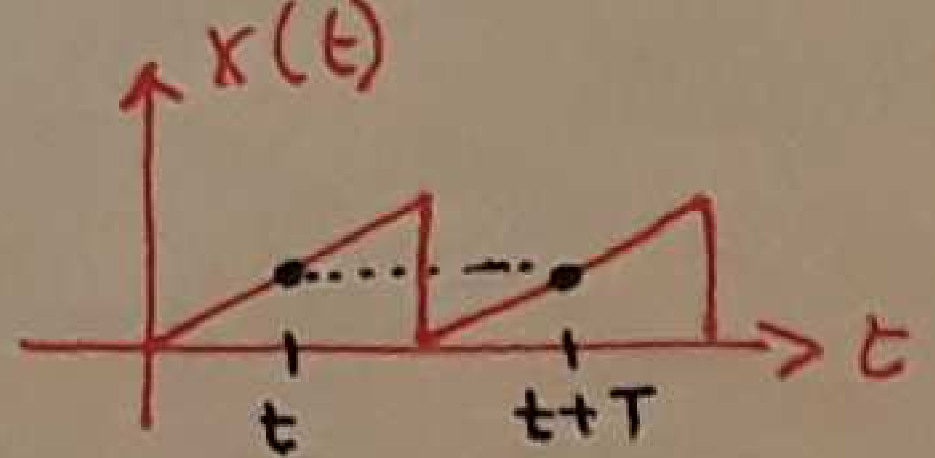
⑦ Négyzetesen összegezhető jel

$$\sum_{k=-\infty}^{\infty} |x[k]|^2 < \infty$$

⑧ Periodikus jel

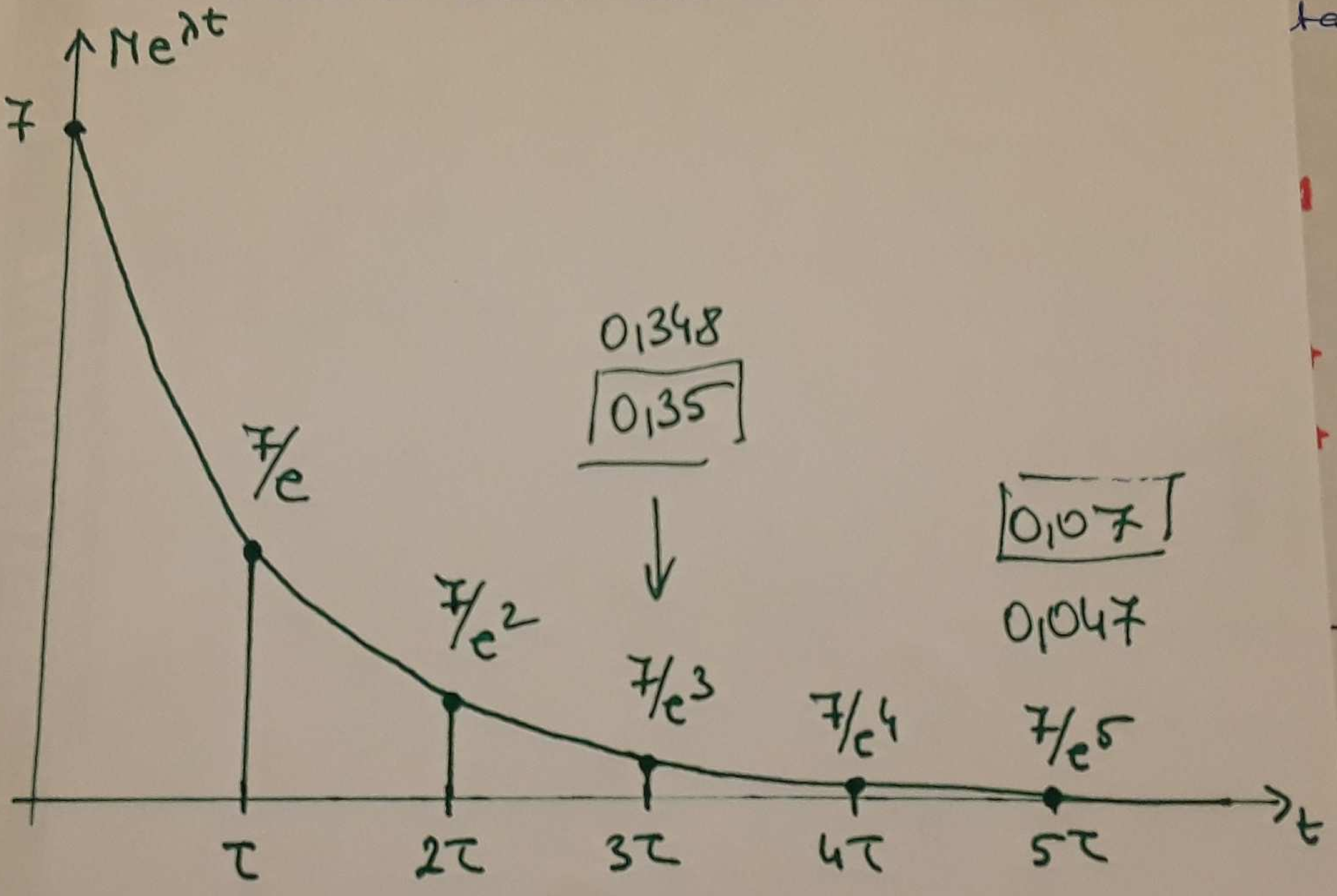
$$x(t \pm \overset{\downarrow}{T}) = x(t)$$

$$x[k \pm K] = x[k]$$



$x(t) = Me^{\textcircled{\lambda} t}$

tant !



$\lambda \frac{M}{e}$

A/e

4τ 5τ

1/e⁴ 1/e⁵

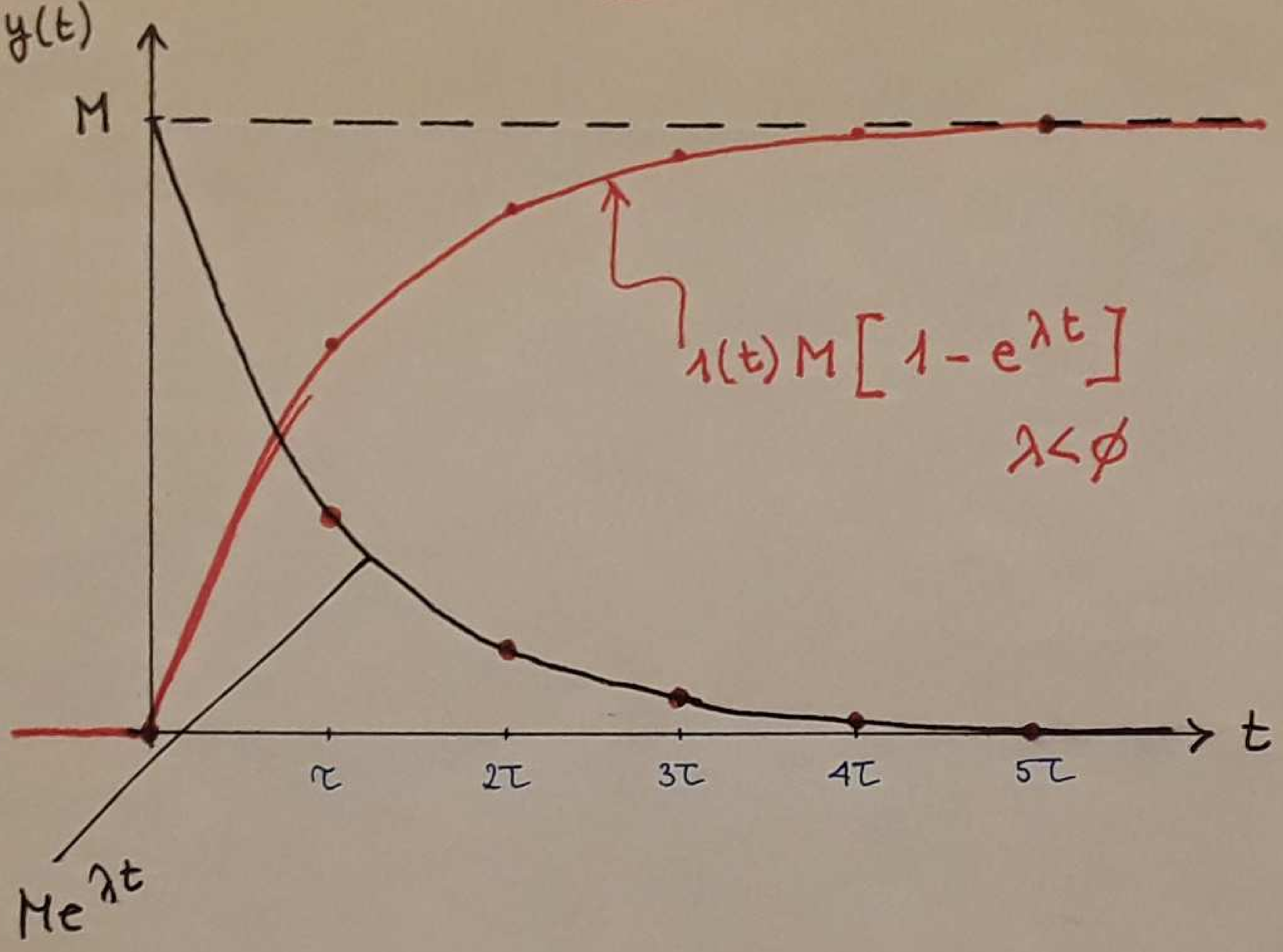
0,018 $\boxed{0,006}$ <1%

$e = \lambda \frac{\cancel{M}}{\cancel{e}} (t-\tau) +$

$1/e^3$ M/e^4 $\cancel{5}M/e^5$

$\lambda(t-\tau) + ($

Vázoljuk fel az $y(t) = 1(t)[1 - e^{\lambda t}]M$ függvényt, $\lambda < 0$ mellett!



$$\underbrace{M - \underbrace{Me^{\lambda t}}_{}}_{}$$

Ábrázoljuk a következő függvényeket !

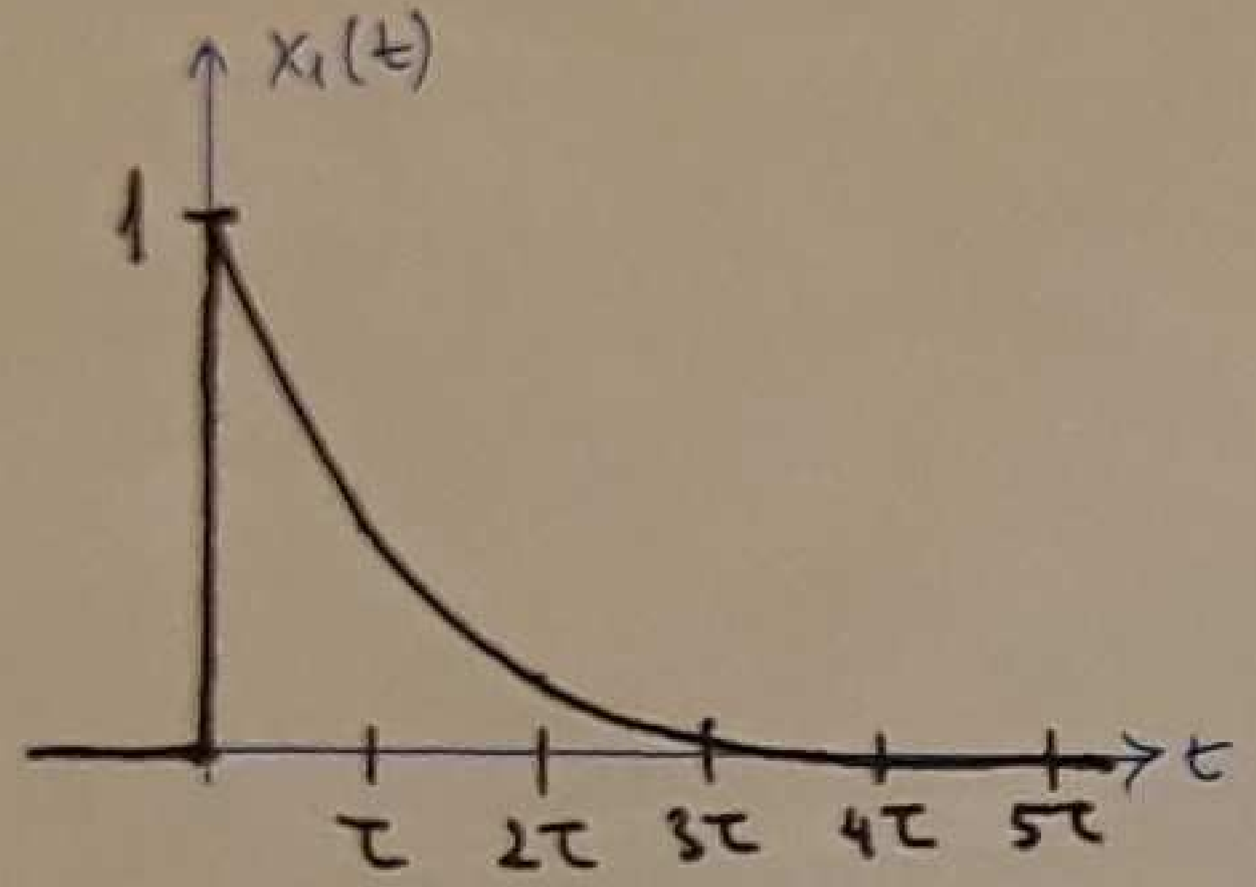
a.) $x_1(t) = 1(t)\, e^{-2t}$

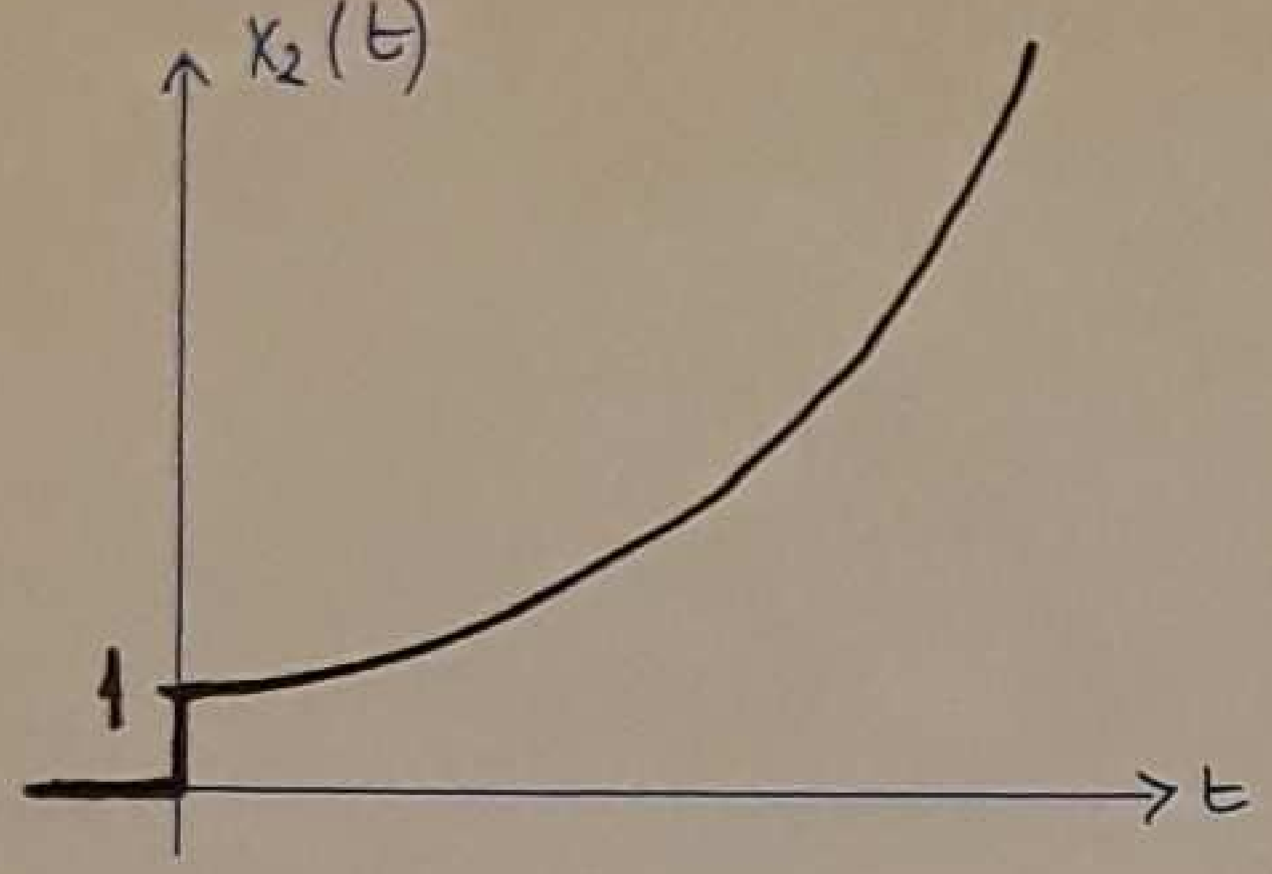
b.) $x_2(t) = 1(t)\, e^{+0,5t}$

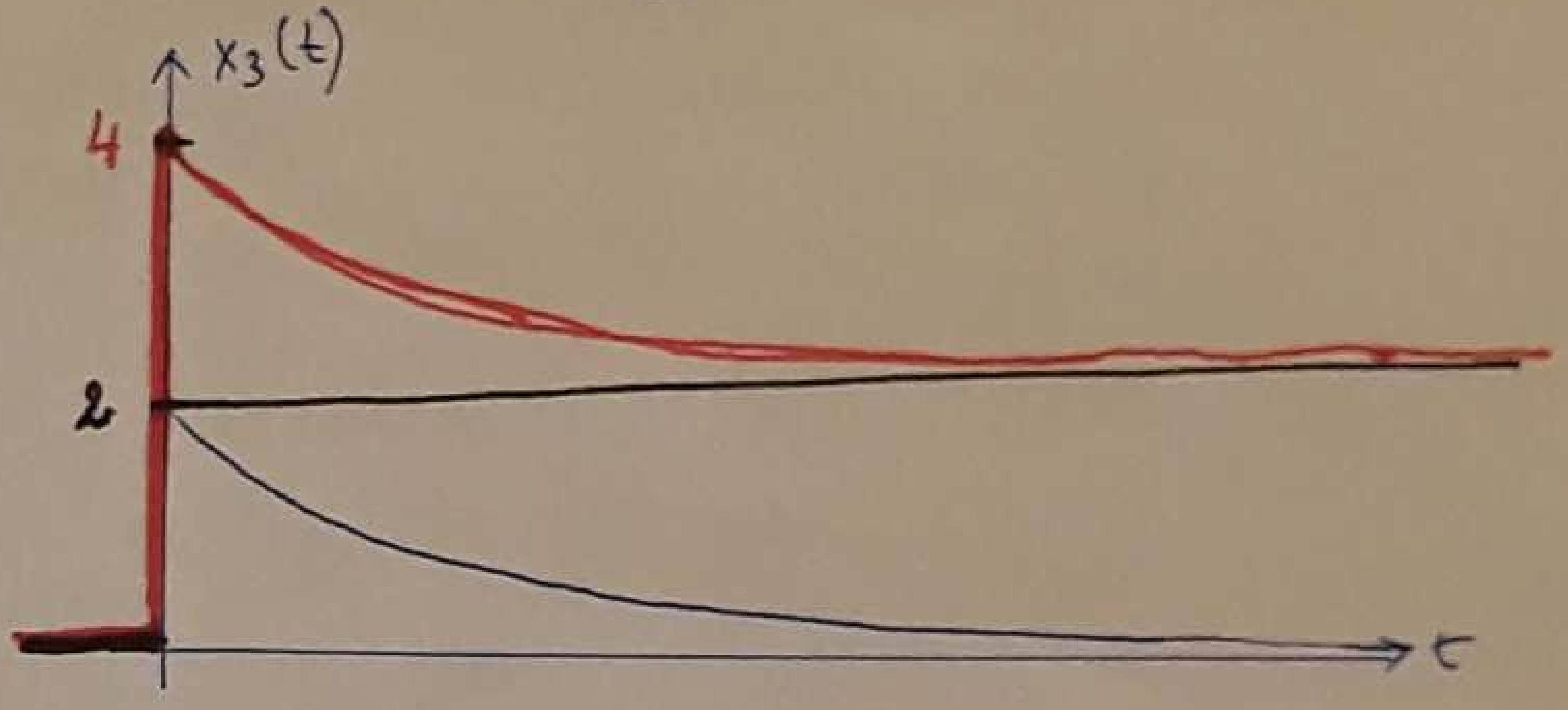
c.) $x_3(t) = 1(t)\left[\boxed{2} + \boxed{2\, e^{-4t}}\right]$

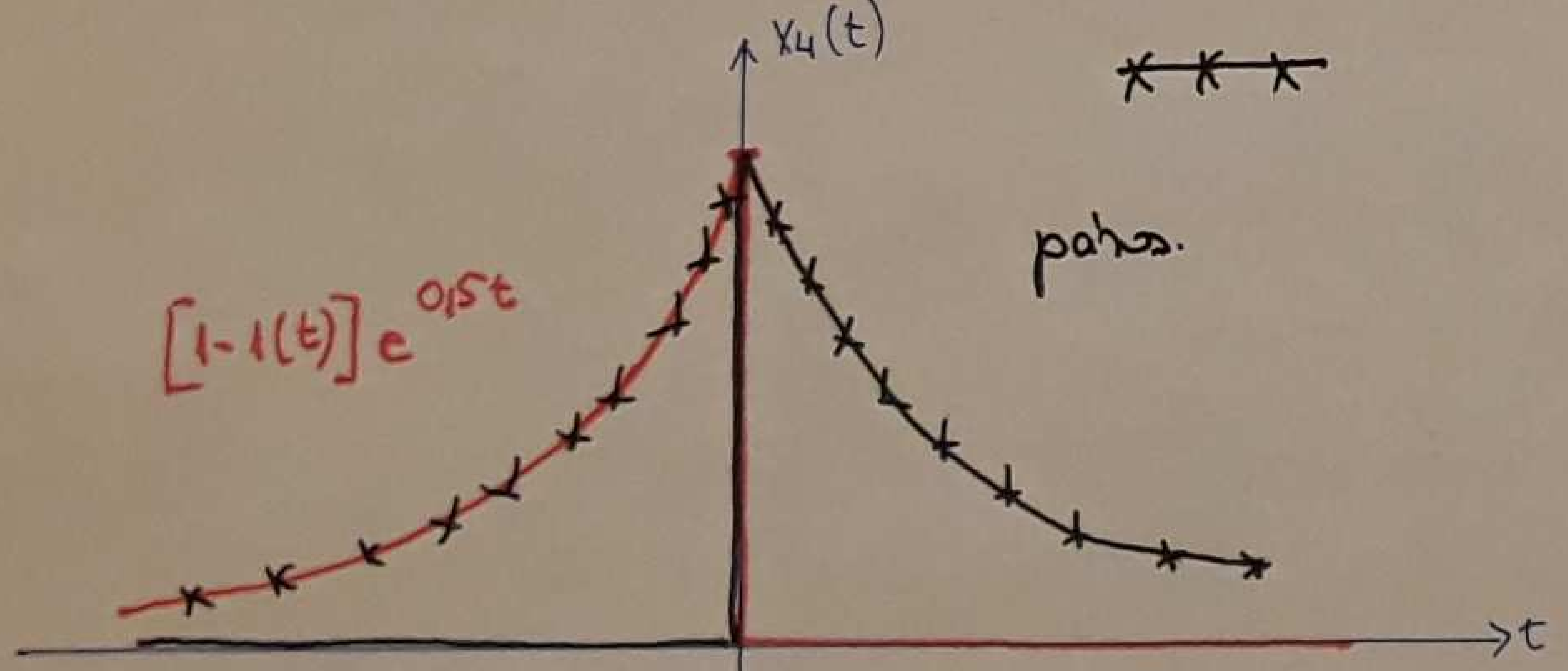
d.) $x_4(t) = \underline{\left[1 - 1(t)\right] e^{+0,5t}} + \underline{1(t)\, e^{-0,5t}}$

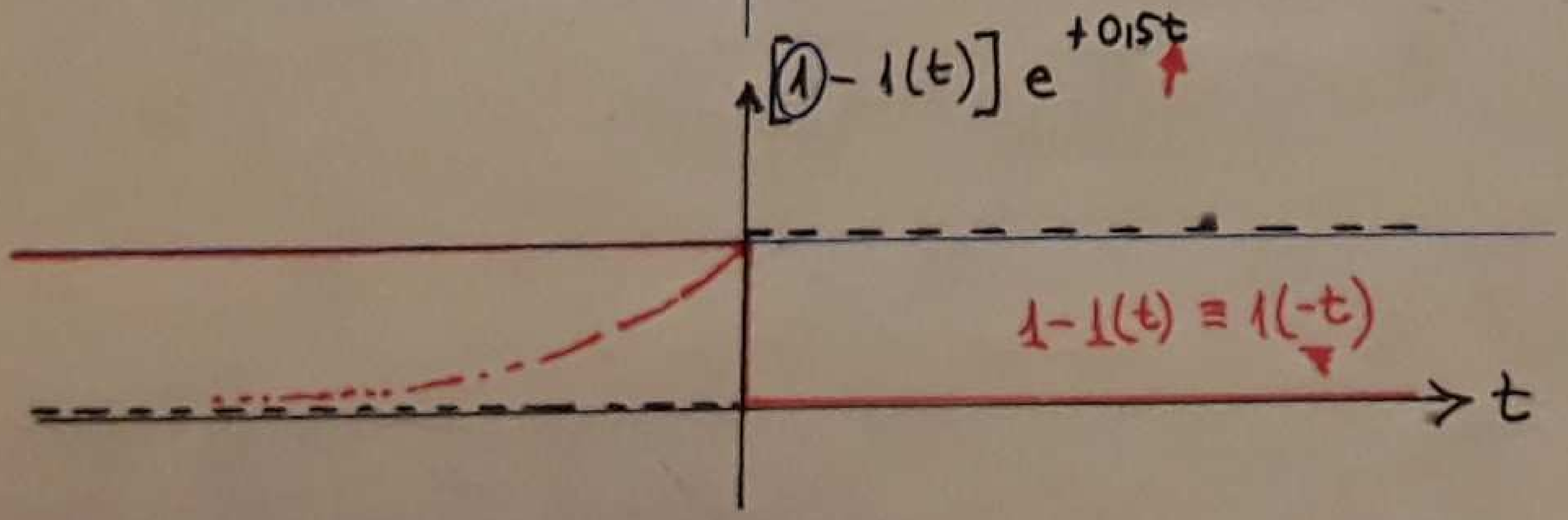
e.) $x_5(t) = 1(t)\, \frac{t}{\boxed{T}} \cdot 3$

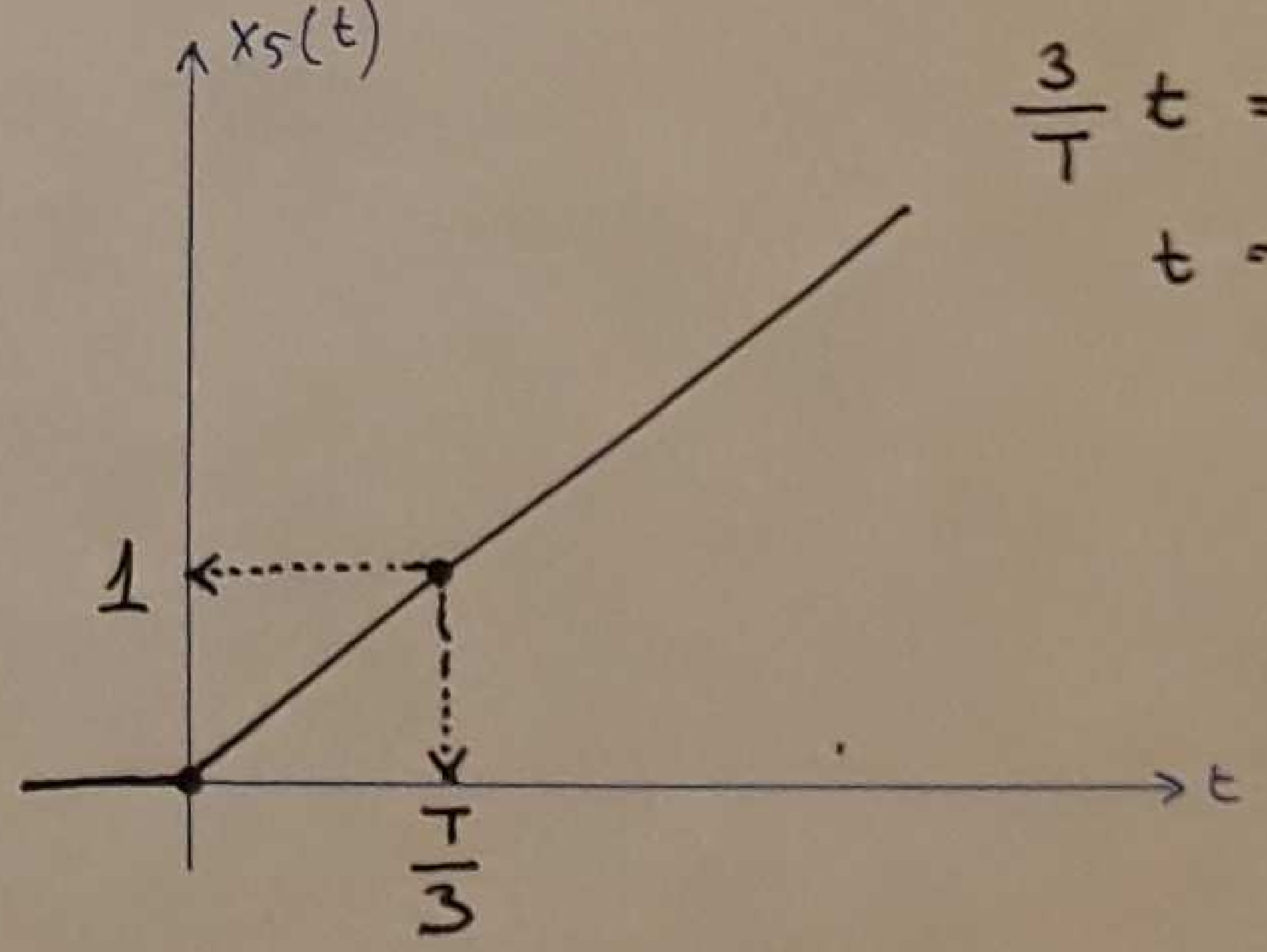












$$\frac{3}{T}\, t = 1$$

$$t = \frac{T}{3}$$

Ábrázolják a következő függvényeket!

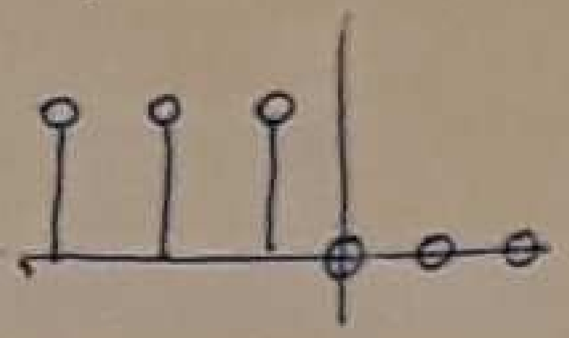
a.) $x_1[k] = 1[k] \cdot \boxed{0{,}5}^k$

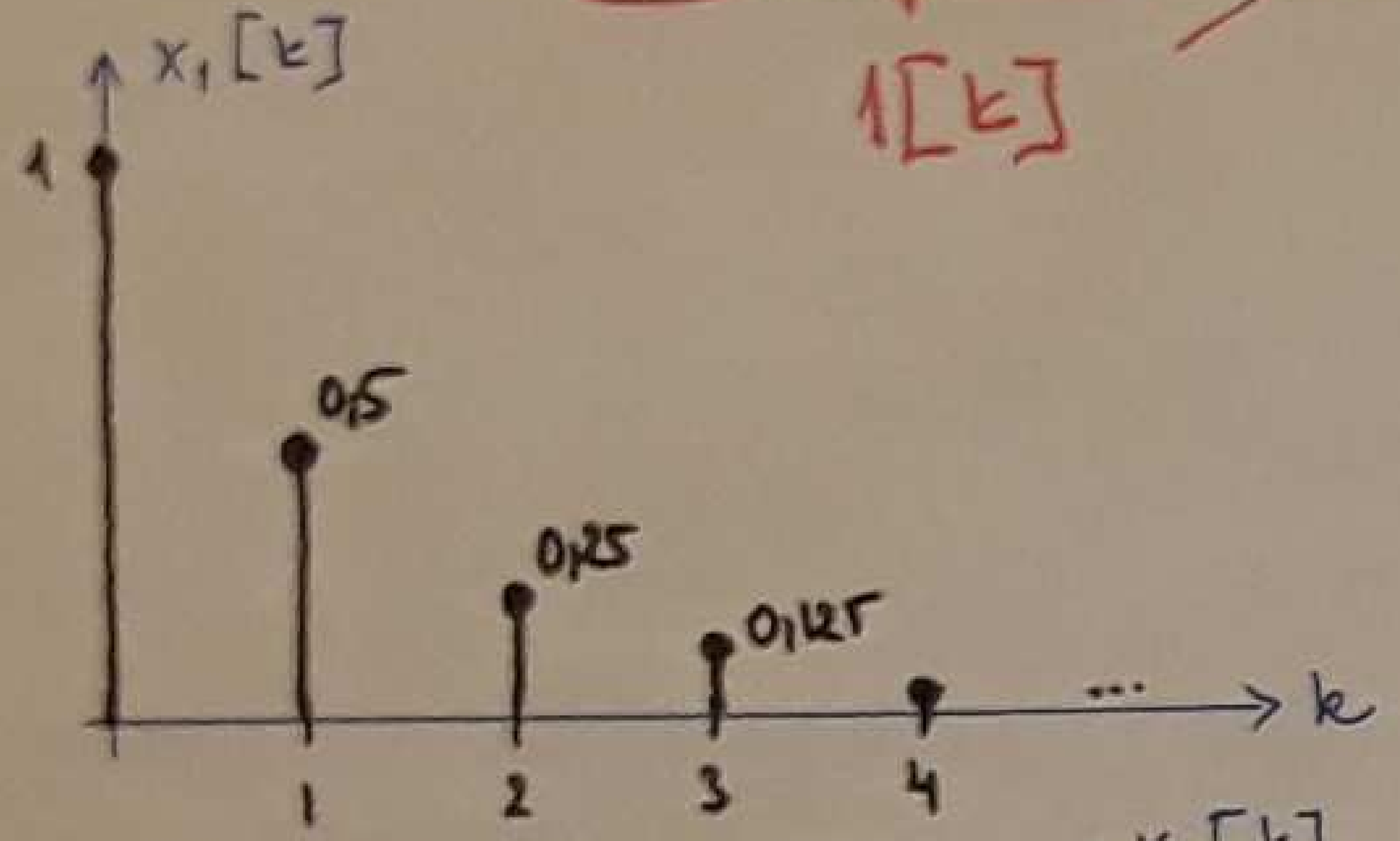
b.) $x_2[k] = 1[k] \cdot \boxed{2}^k$

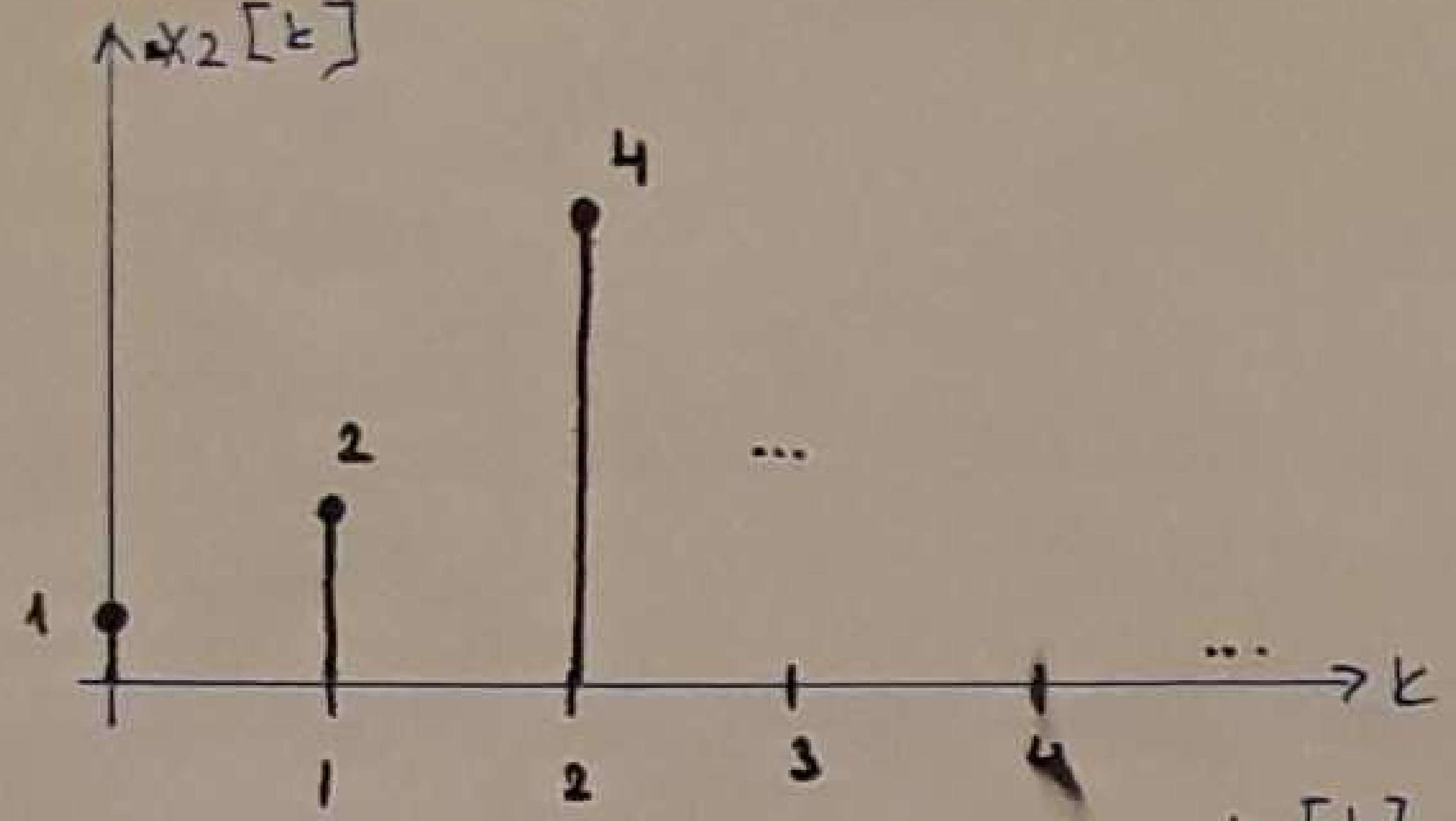
c.) $x_3[k] = \boxed{1} \boxed{-} 1[k]$

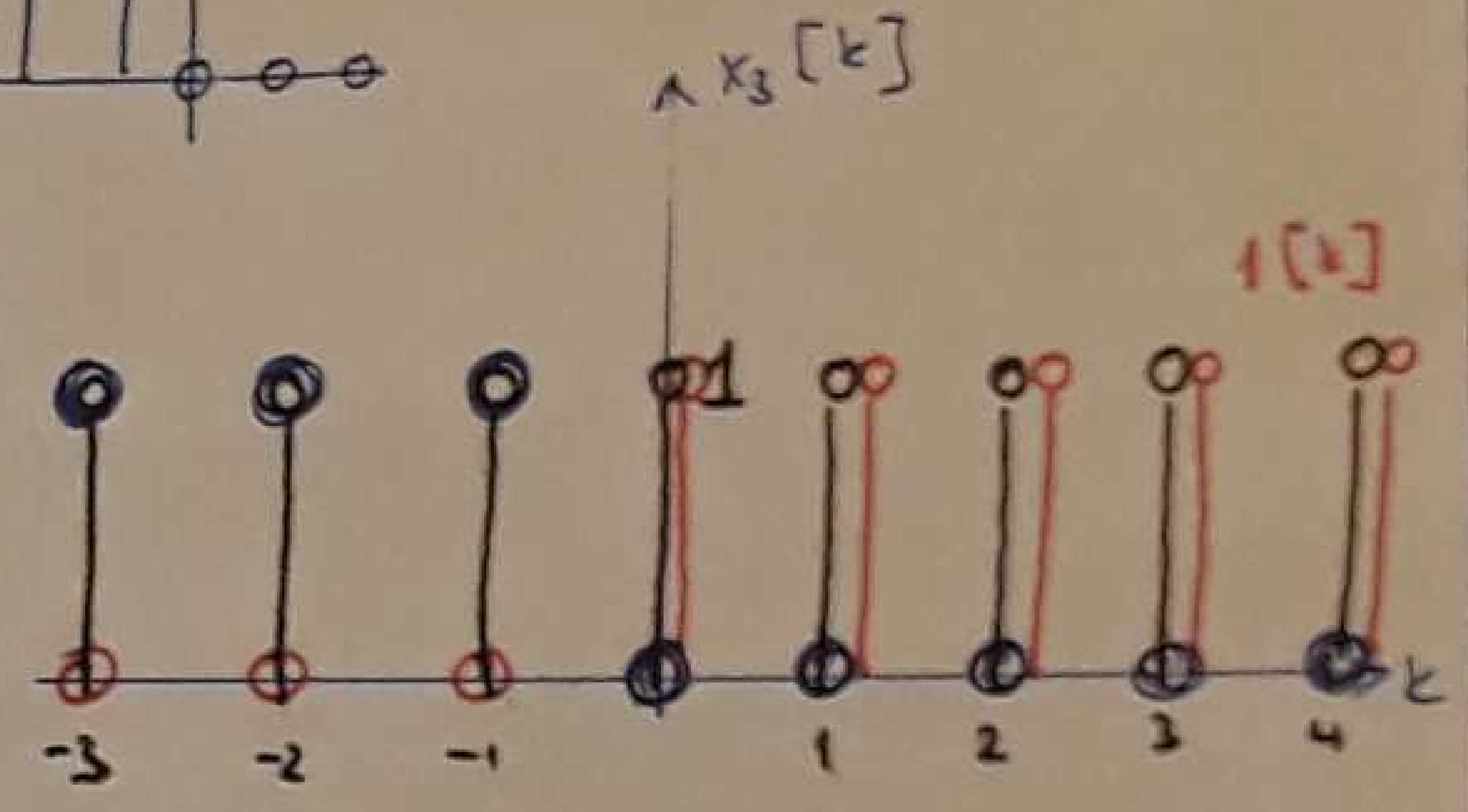
d.) $x_4[k] = \underbrace{\{1 - 1[-k-1]\}}_{1[k]} \boxed{2^k}$

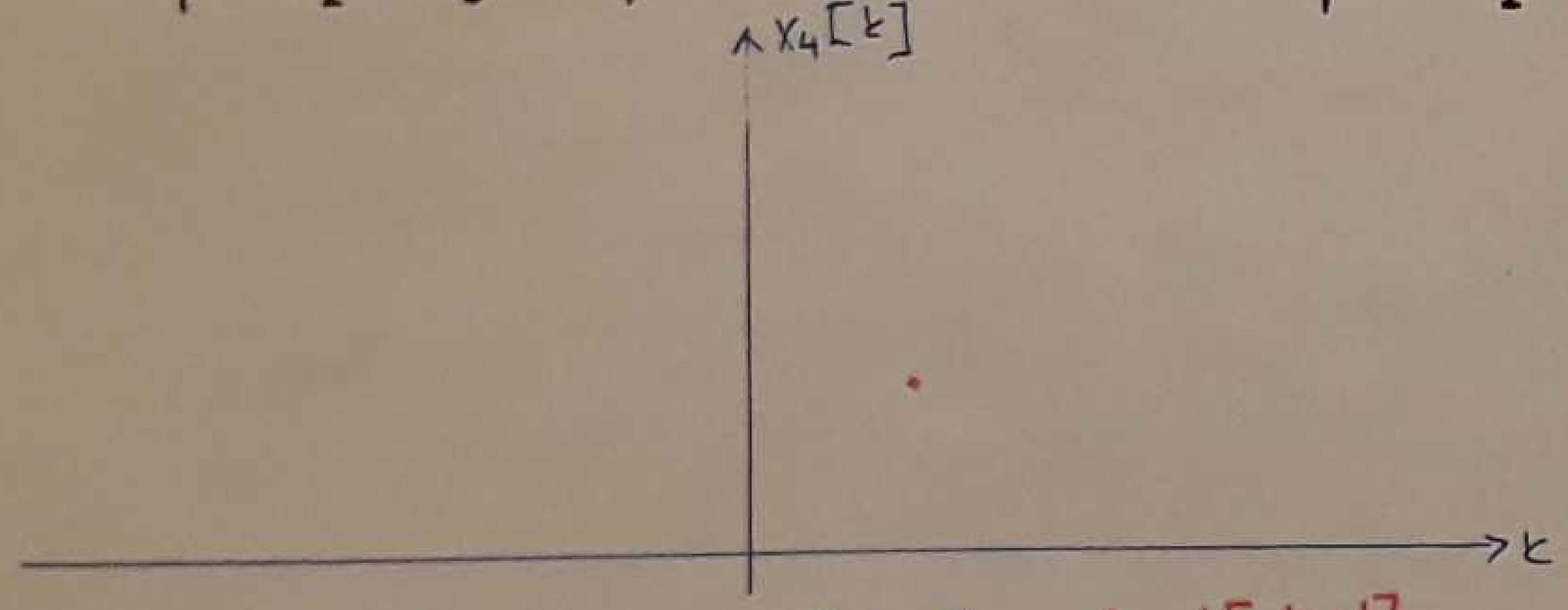
e.) $x_5[k] = \underbrace{\{1[k] - 1[k-5]\}}(2 \cdot k)$

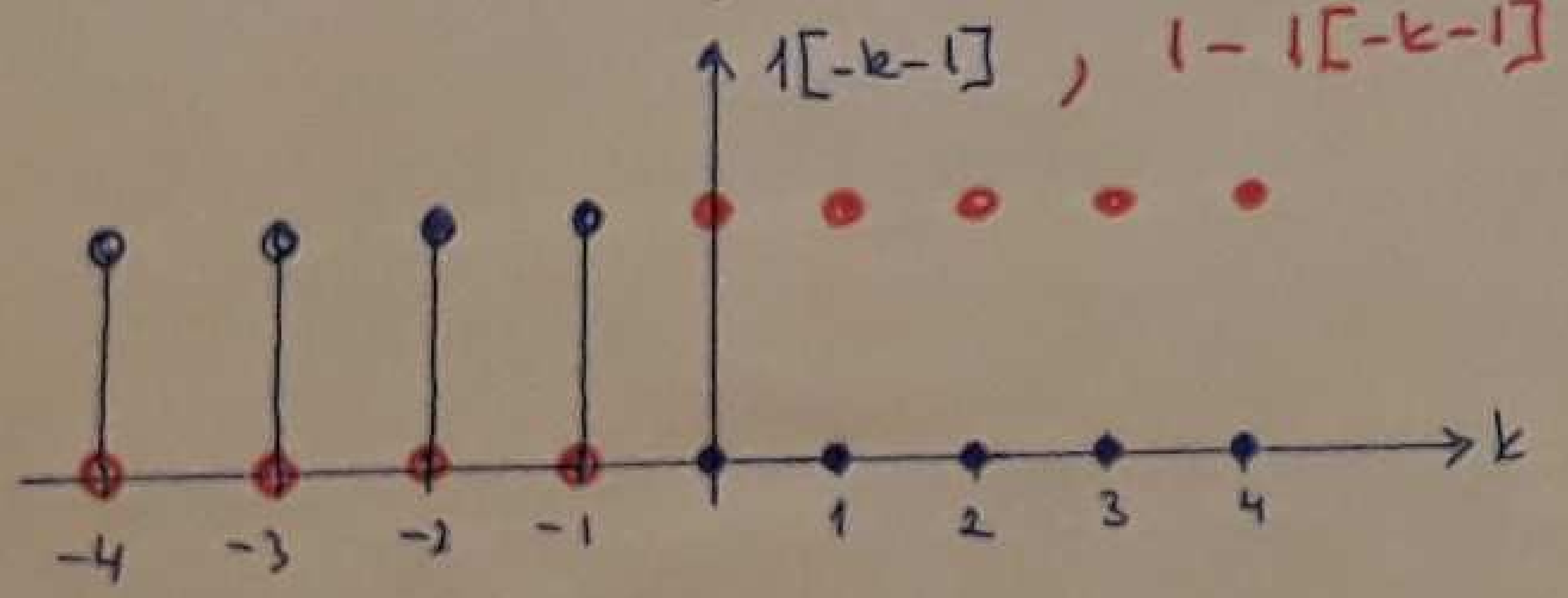




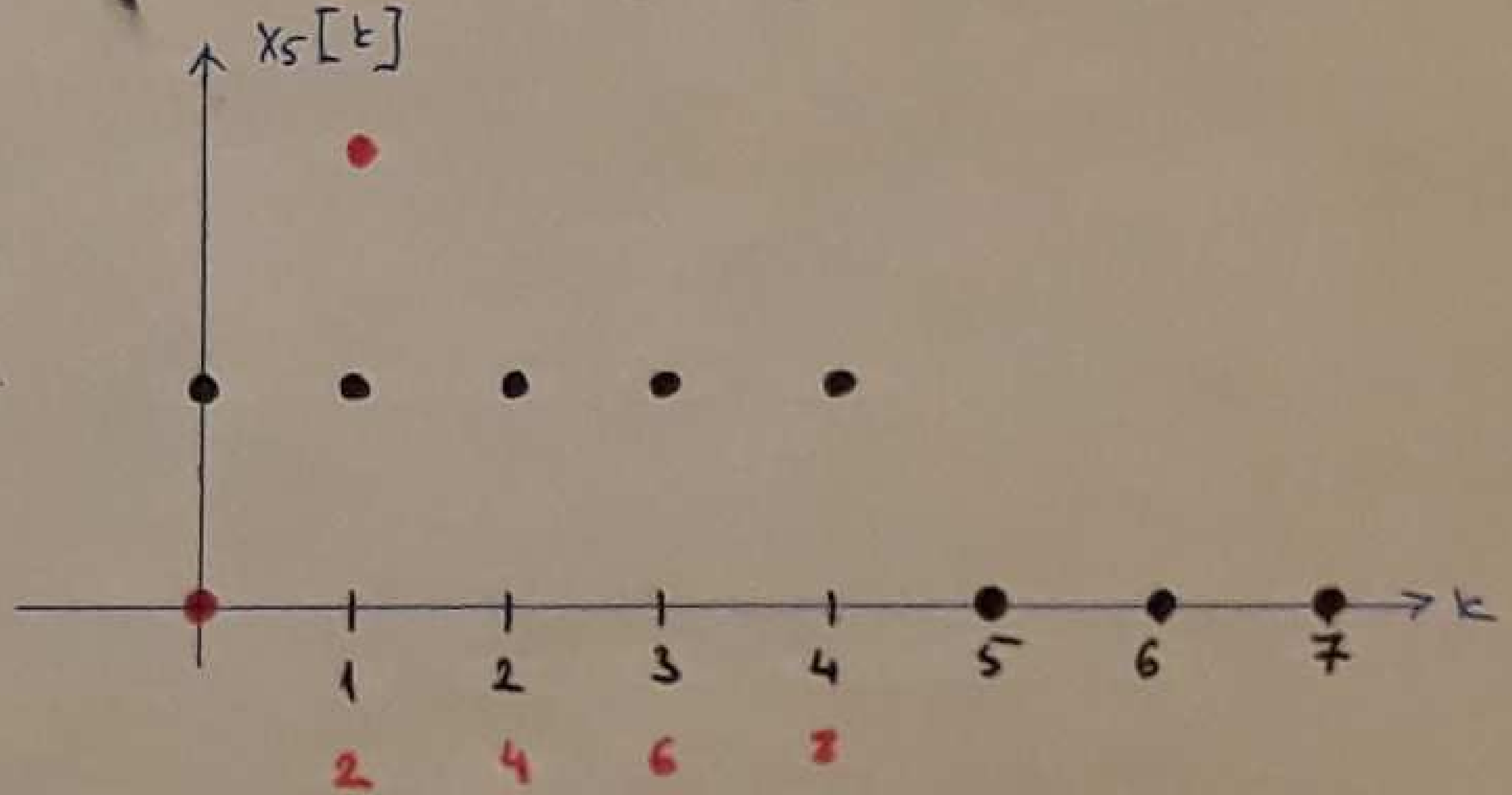


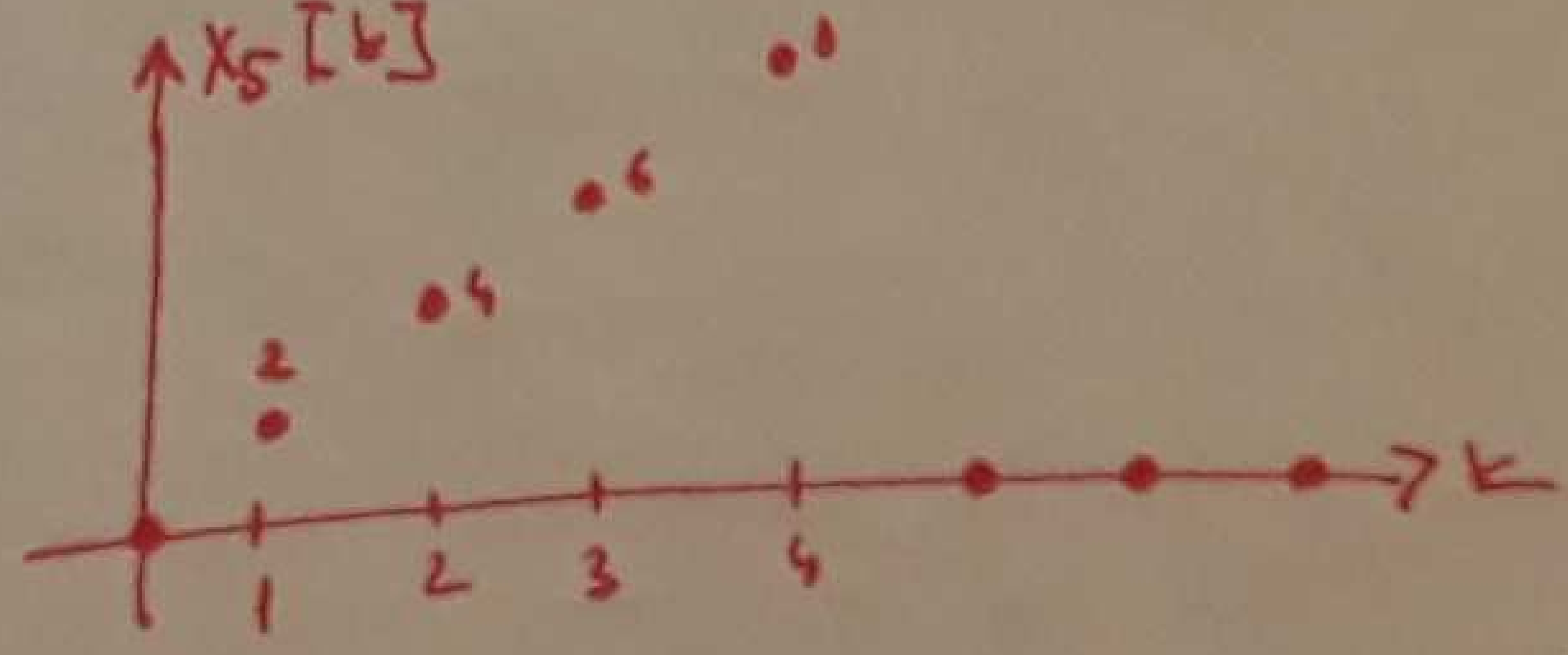






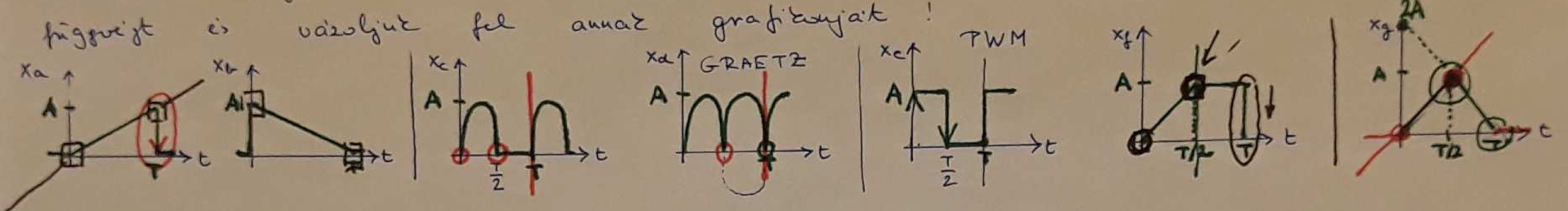
$1[k] = 1 - 1[-k-1]$





$Me^{\lambda t} \rightarrow M q^k$

Határozzuk meg az alábbi függvények képletét, határozzuk meg a derivált függvényt és vázoljuk fel annak grafikonjait!

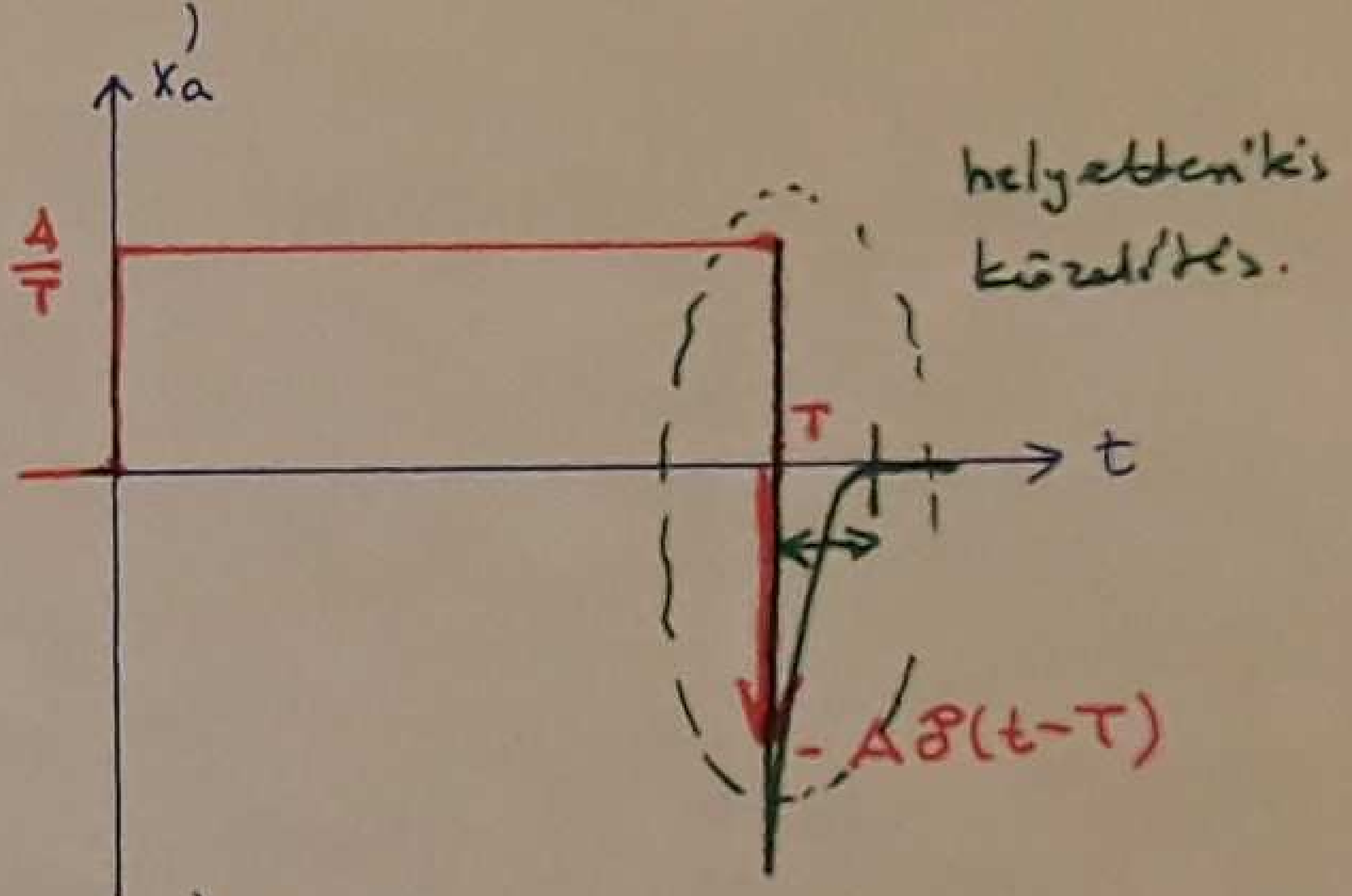


$$x_a(t) = mt + b$$
$$0 = m\cdot 0 + b \rightarrow \boxed{b = 0}$$
$$A = mT + \cancel{b} \rightarrow \boxed{m = \frac{A}{T}}$$
$$x_a = \overset{u}{\frac{A}{T}t}\,\overset{v}{\left[1(t) - 1(t-T)\right]}$$

$$(uv)' = u'v + uv'$$

$$x_a' = \frac{A}{T}\left[1(t) - 1(t-T)\right] + A\frac{t}{T}\left[\delta(t) - \delta(t-T)\right]$$
$$\underbrace{A\frac{t}{T}\delta(t)}_{0} - \underbrace{\frac{A}{T}t\,\delta(t-T)}_{-A}$$

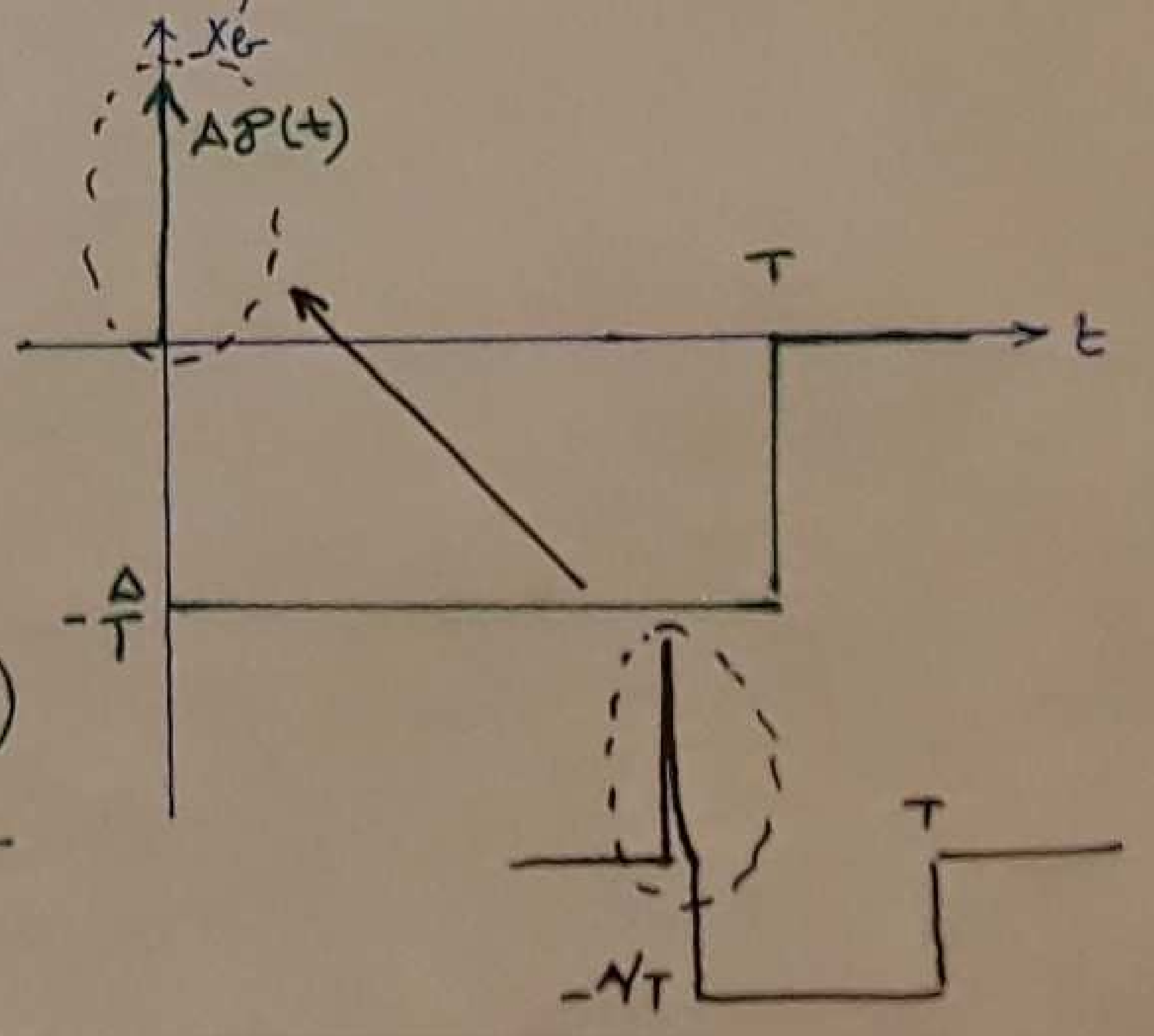
$$x_a' = \frac{A}{T}\left[1(t) - 1(t-T)\right] - A\delta(t-T)$$



$$x_b(t) = mt + b$$
$$A = m\cdot 0 + b \rightarrow \boxed{b = A}$$
$$0 = mT + b \rightarrow \boxed{m = -\frac{A}{T}}$$
$$x_b = \underbrace{\left(-\frac{A}{T}t + A\right)}_{u}\underbrace{\left[1(t) - 1(t-T)\right]}_{v}$$

$$x_b' = -\frac{A}{T}\left[1(t) - 1(t-T)\right] + A\delta(t)$$

$$x_b' = -\frac{A}{T}\left[1(t) - 1(t-T)\right] + \left(-\frac{A}{T}t + A\right)\left[\delta(t) - \delta(t-T)\right]$$
$$\underbrace{\left(-\frac{A}{T}t + A\right)\delta(t)}_{A\delta(t)} - \underbrace{\left(-\frac{A}{T}\underset{T}{t} + A\right)\delta(t-T)}_{0}$$



$$\omega = 2\pi / T$$

$$x_c(t) = \underbrace{\left[1(t) - 1\left(t - \tfrac{T}{2}\right)\right]}_{u} \underbrace{A \sin \omega t}_{v}$$

$$(uv)' = u'v + uv'$$

$$(A \sin \omega t)' = A\omega \cos \omega t$$

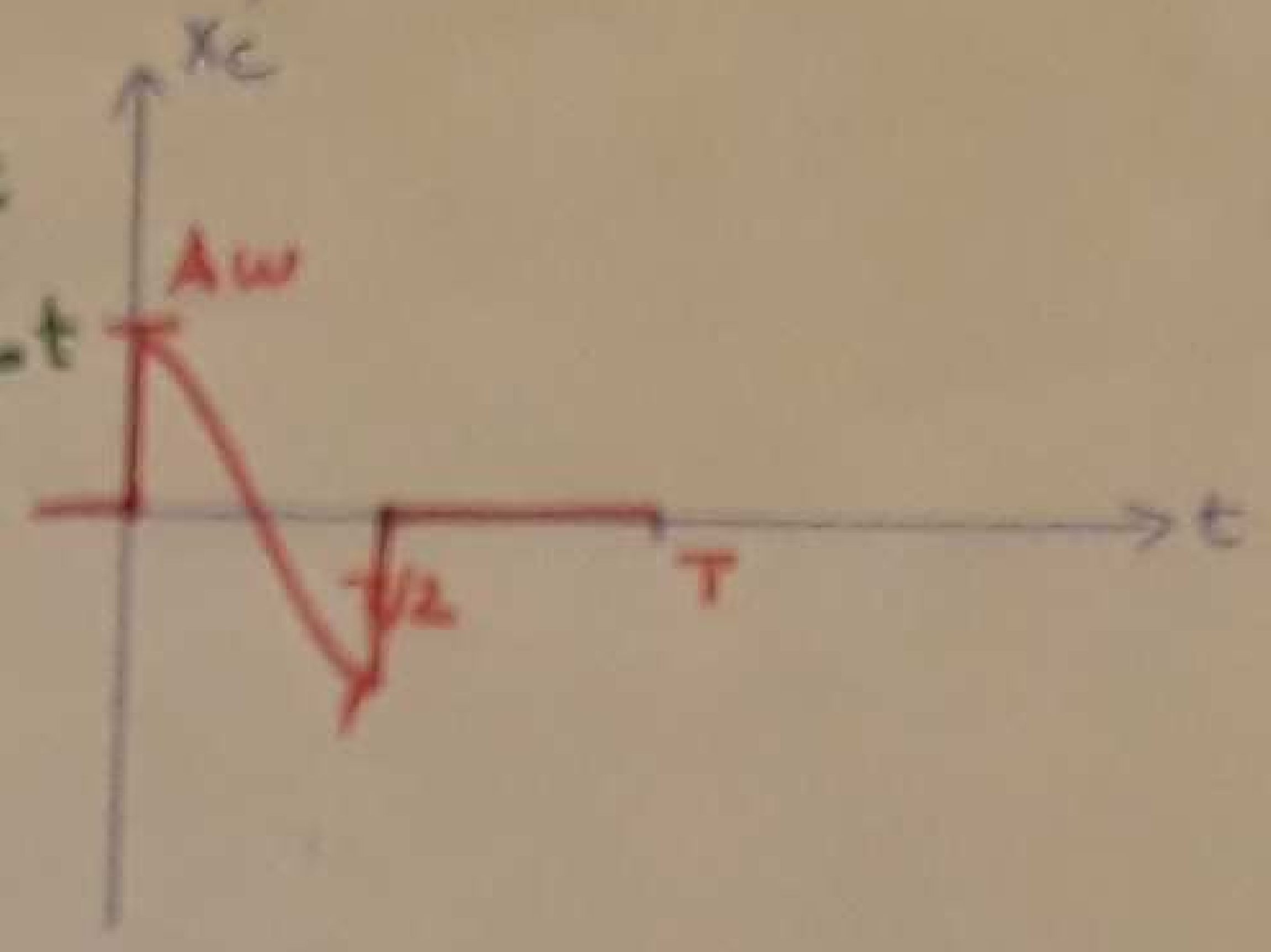
$$(A \cos \omega t)' = -A\omega \sin \omega t$$

$$x_c' = \left[\underline{\delta(t)} - \delta(t - T/2)\right] A \sin \omega t \;+$$

$$\cancel{\delta(t) A \sin \omega t} - \cancel{\delta(t - T/2) A \sin \omega t}$$

$$+ \left[1(t) - 1(t - T/2)\right] A\omega \cos \omega t$$

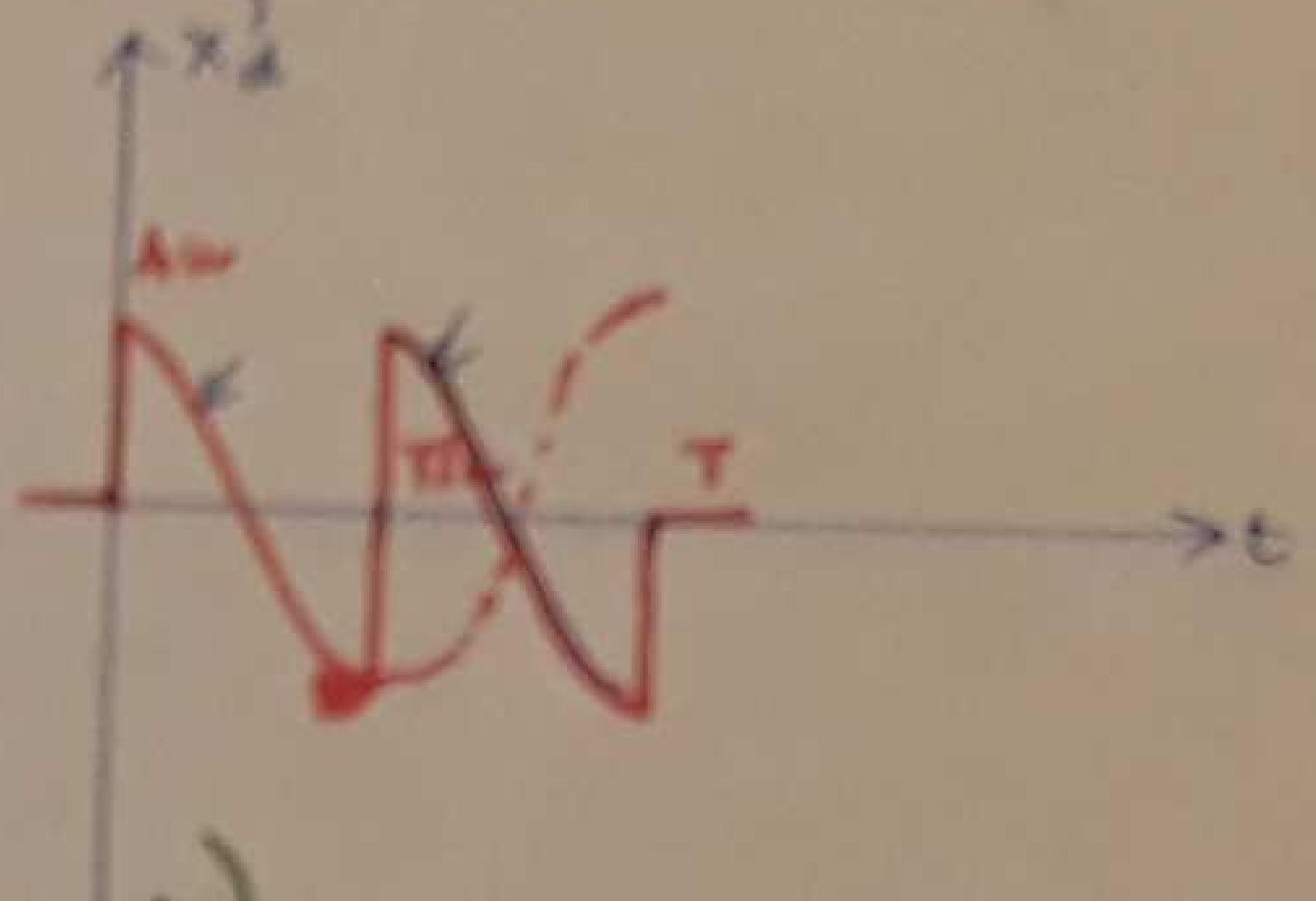
$$x_c' = \left[1(t) - 1(t - T/2)\right] A\omega \cos \omega t .$$



$$x_d(t) = \underbrace{\left[1(t) - 1(t - T/2)\right] A \sin \omega t}_{\downarrow \text{ IDEM.}} + \overbrace{\underbrace{\left[1(t - T/2) - 1(t - T)\right]}_{u} (-A \sin \omega t)}^{v}$$

$$x_d' = \left[1(t) - 1(t - T/2)\right] A\omega \cos \omega t + \left[1(t - T/2) - 1(t - T)\right] (\pm A\omega \cos \omega t)$$

$$\underbrace{\left[\delta(t - T/2) - \delta(t - T)\right](-A \sin \omega t)}_{\emptyset} + \left[1(t - T/2) - 1(t - T)\right](-A\omega \cos \omega t)$$

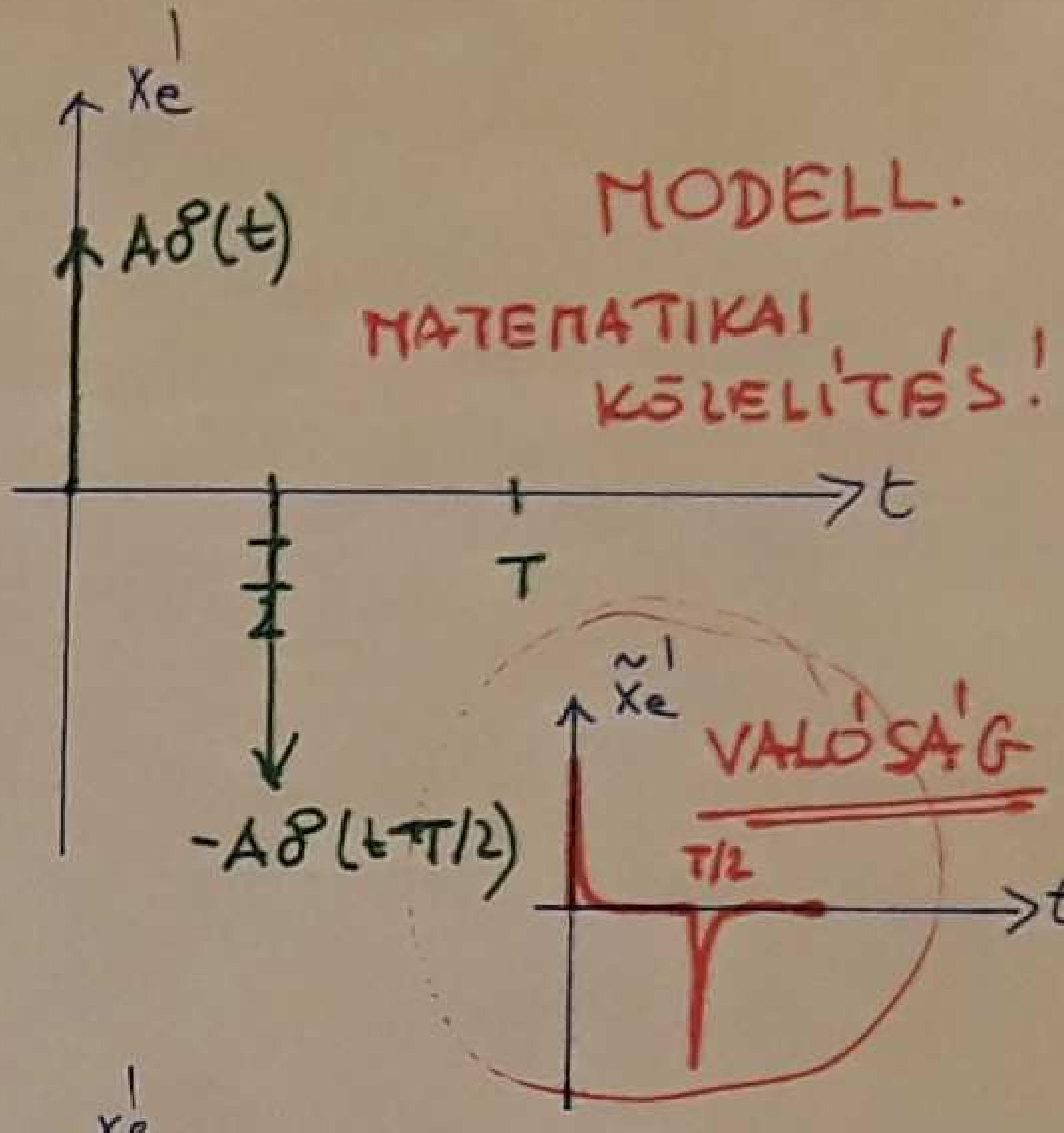


pulse width modulation - PWM

$x_e(t) = [1(t) - 1(t - T/2)]\,A$

$(uv)' = u'v + uv'$

$x_e' = [\delta(t) - \delta(t - T/2)]\,A$



$x_f = [\overset{u}{1(t) - 1(t - T/2)}]\,(\overset{v}{\frac{2A}{T}t})$

$+ [1(t - T/2) - 1(t - T)]\,A$

$(uv)' = u'v + uv'$

$mt + b$

$0 = m \cdot 0 + b \rightarrow \underline{b = 0}$

$A = m\,\frac{T}{2} + b$

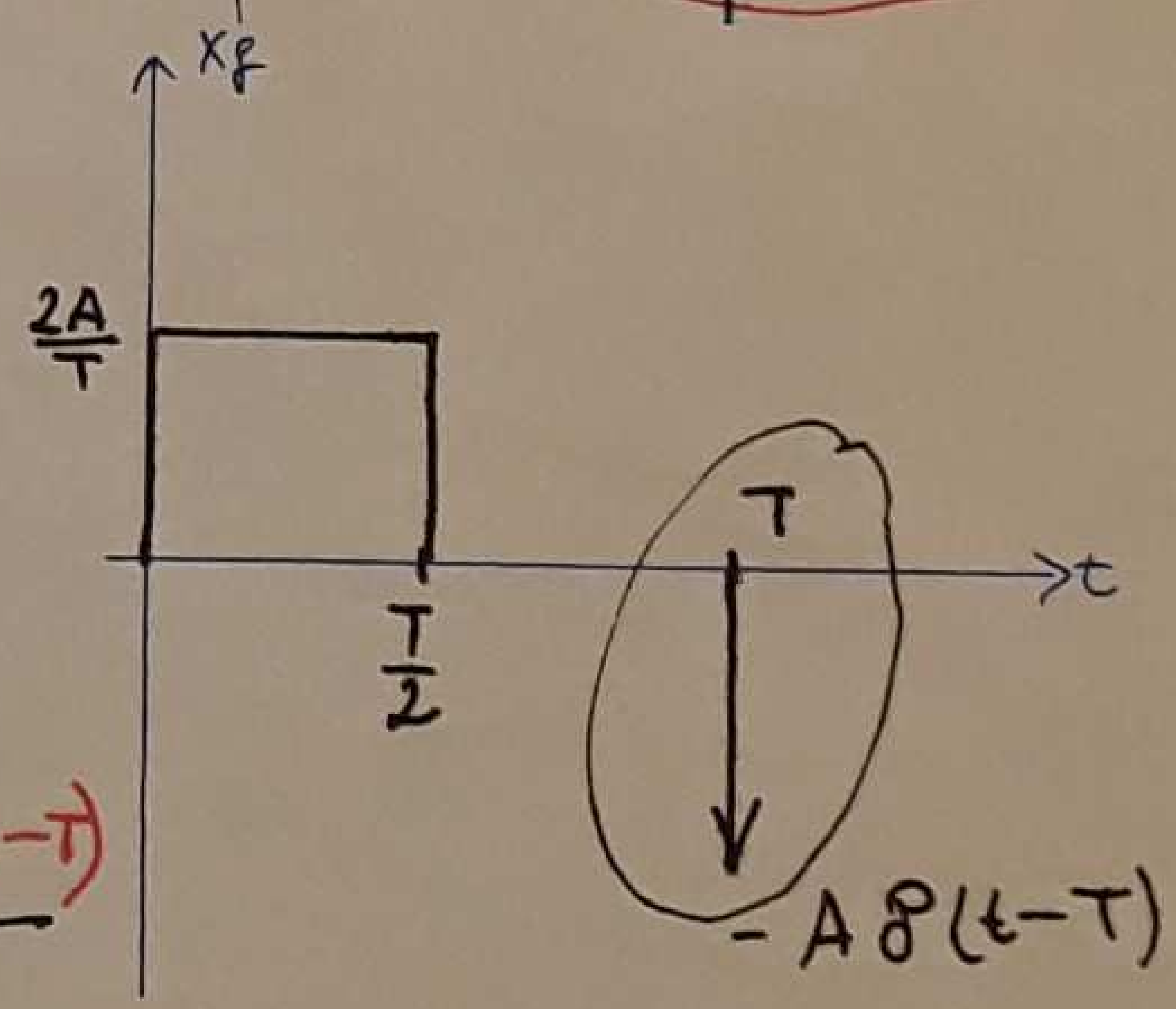
$\hookrightarrow m = \frac{2A}{T}$

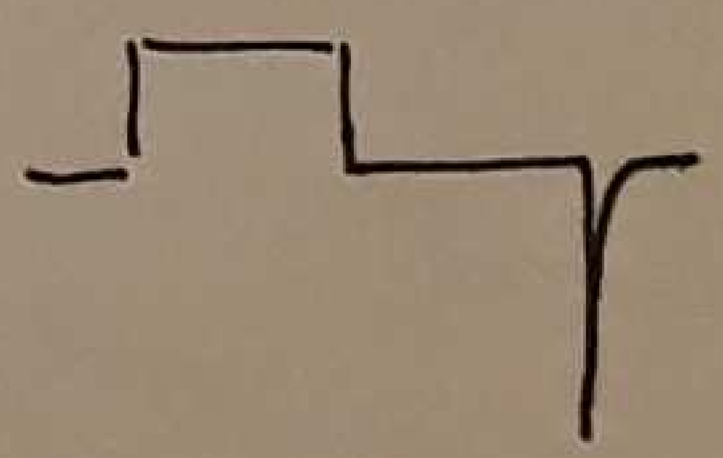
$x_f' = [\underline{\delta(t)} - \delta(t - T/2)]\,\underline{\frac{2A}{T}t} + [1(t) - 1(t - T/2)]\,\frac{2A}{T} +$

$\cancel{\left(\frac{2A}{T}\right)t\,\delta(t)} - \frac{2A}{T}t\,\delta(t - T/2) = -\frac{2A}{T}\frac{T}{2}\,\delta(t - T/2) = -A\,\delta(t - T/2)$

$+ [\delta(t - T/2) - \delta(t - T)]\,\underline{A} + [1(t - T/2) - \cancel{1(t - T)]\,0} =$

$= \cancel{-A\,\delta(t - T/2)} + [1(t) - 1(t - T/2)]\,\frac{2A}{T} + \cancel{A\,\delta(t - T/2)}\,\underline{- A\,\delta(t - T)}$





$$x_g(t) = \left[1(t) - \overset{u}{1}(t - T/2)\right]\left(\overset{v}{\frac{2A}{T}}t\right) +$$

$$+ \left[1(t-T/2) - 1(t-T)\right]\left(-\frac{2A}{T}t + 2A\right)$$

$(uv)' = u'v + uv'$

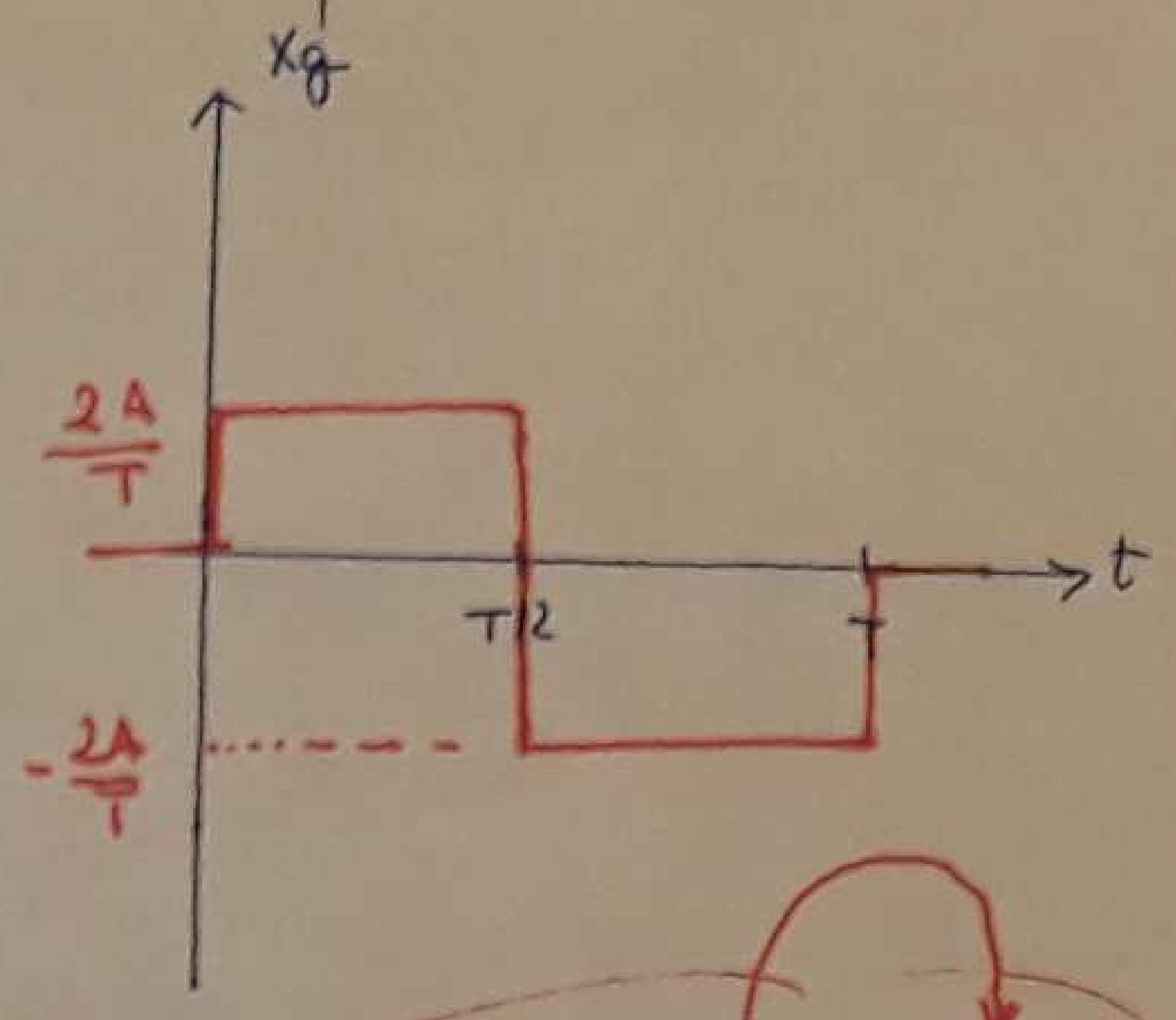
$$mt + b \; : \; \left.\begin{matrix} 0 = m0 + b \\ A = m\cdot\frac{T}{2} + b \end{matrix}\right\} \quad \left.\begin{matrix} b = 0 \\ m = \frac{2A}{T} \end{matrix}\right\} \quad \frac{2A}{T}t$$

$$mt + b \; : \; \left.\begin{matrix} (1)\; A = m\frac{T}{2} + b \\ (2)\; \underline{0 = mT + b} \end{matrix}\right\}$$

$1-2 \quad A = -m\frac{T}{2} \rightarrow m = -\frac{2A}{T}$

$b = -mT = +\frac{2A}{T}T = 2A$

$-\frac{2A}{T}t + 2A$



$$x_g' = \left[\underline{\delta(t)} - \delta(t-T/2)\right]\frac{2A}{T}t + \left[1(t) - 1(t-T/2)\right]\frac{2A}{T} +$$

$$-\delta(t-T/2)\frac{2A}{T}\frac{T}{2} = \cancel{-\delta(t-T/2)A}$$

$$+ \underbrace{\left[\delta(t-T/2) - \delta(t-T)\right]\left(-\frac{2A}{T}t + 2A\right)}_{\cancel{A\delta(t-T/2)} + \; +0\,\delta(t-T)} + ~~[1(t) - 1(t-T~~$$

$$\left[1(t-T/2) - 1(t-T)\right]\left(-\frac{2A}{T}\right)$$

$-\frac{2A}{T}\frac{T}{2} + 2A$

$-A + 2A = A$

$-\frac{2A}{T}T + 2A = 0$

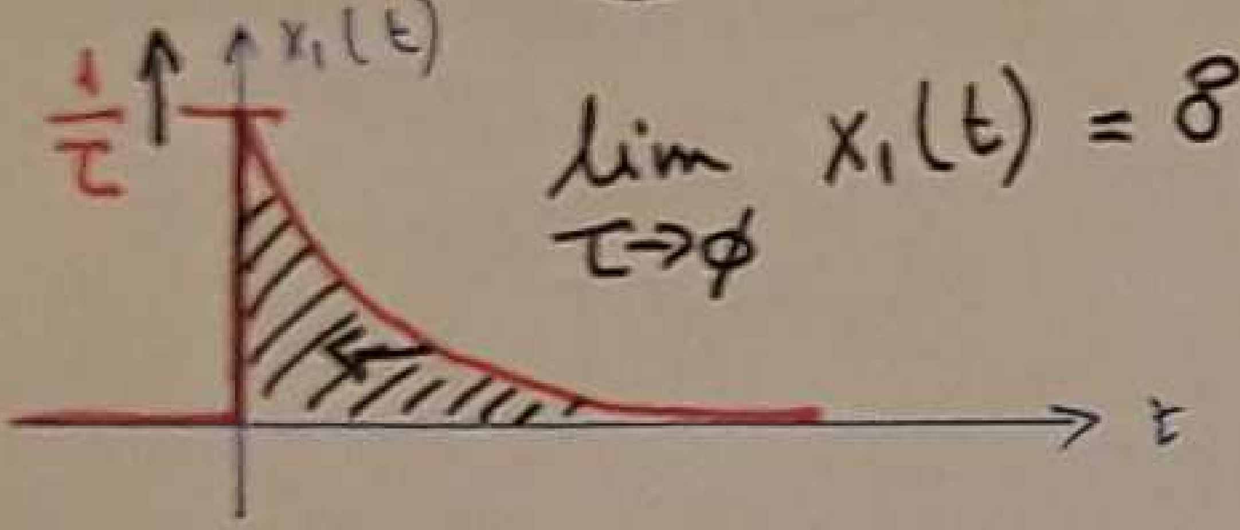
$\delta(t-T)\,f(t) =$

$= \underline{\delta(t-T)\,f(T)}$

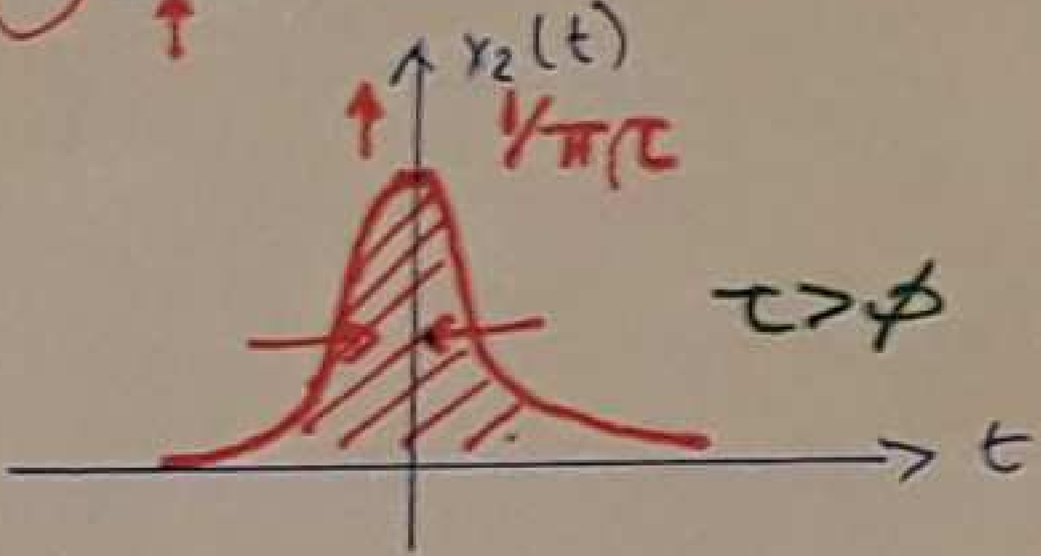
$$x_g'(t) = \left[1(t) - 1(t-T/2)\right]\frac{2A}{T} +$$
$$\left[1(t-T/2) - 1(t-T)\right]\left(-\frac{2A}{T}\right)$$

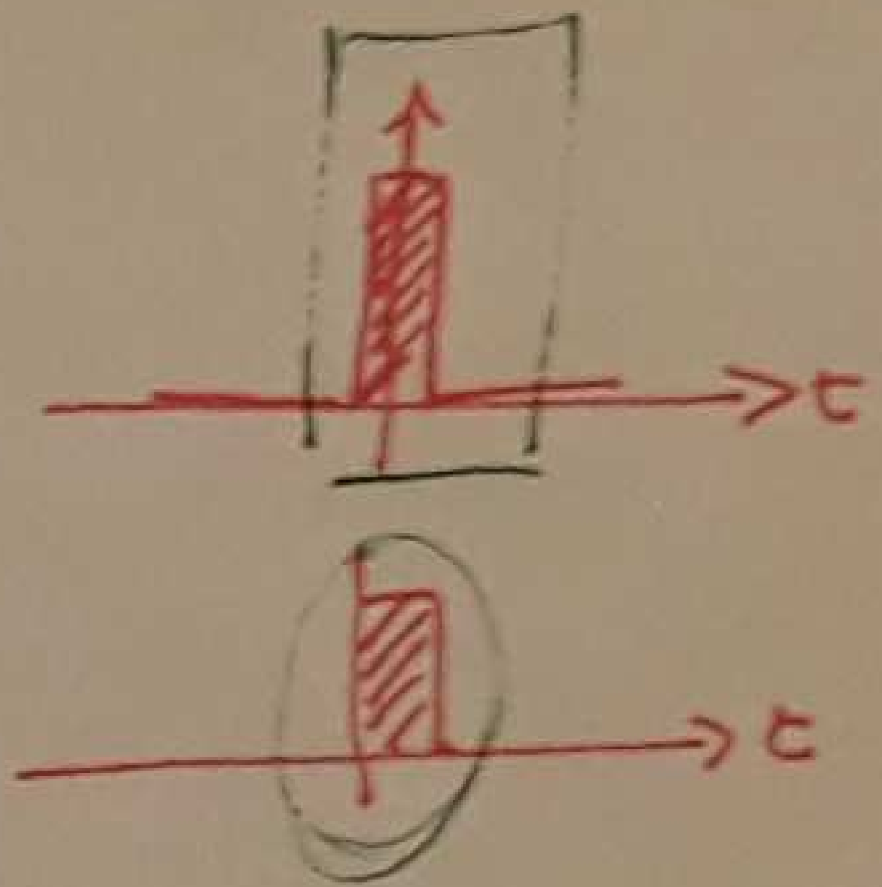
Igazolják, hogy az alábbi függvények $\tau \to 0$ mellett a Dirac-féle delta függvénybe mennek át!

$x_1(t) = \frac{1}{\tau} 1(t)\, e^{-t/\tau}$ (belépő) ; $x_2(t) = \frac{\tau}{\pi(t^2+\tau^2)}$ (nem belépő $\boxed{\text{páros}}$)



$$\lim_{\tau \to 0} x_1(t) = \delta(t)$$





Impulzus.

$$\int_{-\infty}^{\infty} x_1(t)\,dt \overset{?}{=} 1.$$

$$\int_0^{\infty} \frac{1}{\tau} e^{-t/\tau} dt = \frac{1}{\tau}\left[\frac{e^{-t/\tau}}{-1/\tau}\right]_0^{\infty} =$$

$$= \cancel{\frac{1}{\tau}} \frac{0-1}{-\cancel{\frac{1}{\tau}}}$$

$$= 1$$

$$\left(e^{-t/\tau}\right)' = e^{-t/\tau}\left(-\frac{1}{\tau}\right)$$

Impulzus.

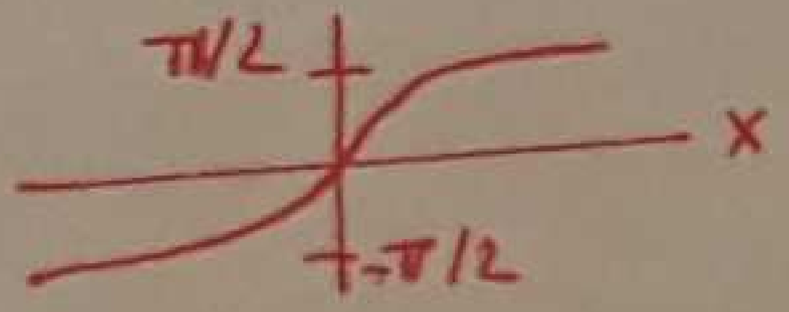
$$\int_{-\infty}^{\infty} x_2(t)\,dt \overset{?}{=} 1$$

$$\Longrightarrow \int \frac{1}{a^2+x^2}dx = \frac{1}{a} \operatorname{arctg} \frac{x}{a}$$

$a > 0$

$$\int \frac{1}{\tau^2+t^2} dt = \frac{1}{\tau} \operatorname{arctg} \frac{t}{\tau}$$

$$\frac{\tau}{\pi}\int_{-\infty}^{\infty} \frac{1}{t^2+\tau^2} dt = \frac{\cancel{\tau}}{\pi} \cancel{\frac{1}{\tau}} \underbrace{\left[\operatorname{arctg} \frac{t}{\tau}\right]_{-\infty}^{\infty}}_{\frac{\pi}{2}+\frac{\pi}{2} = \pi} =$$



$$= \frac{1}{\pi} \cdot \pi = 1.$$

Periodikusak az alábbi függvények ?

$y_a[k] = 3\cos\left(\underbrace{\frac{3\pi}{7}}k\right)$ $y_b[k] = 2\cos\left(\underbrace{\sqrt{2}\,\pi}\,k\right)$ $y_c[k] = 5\cos\left(\underbrace{2}k\right)$

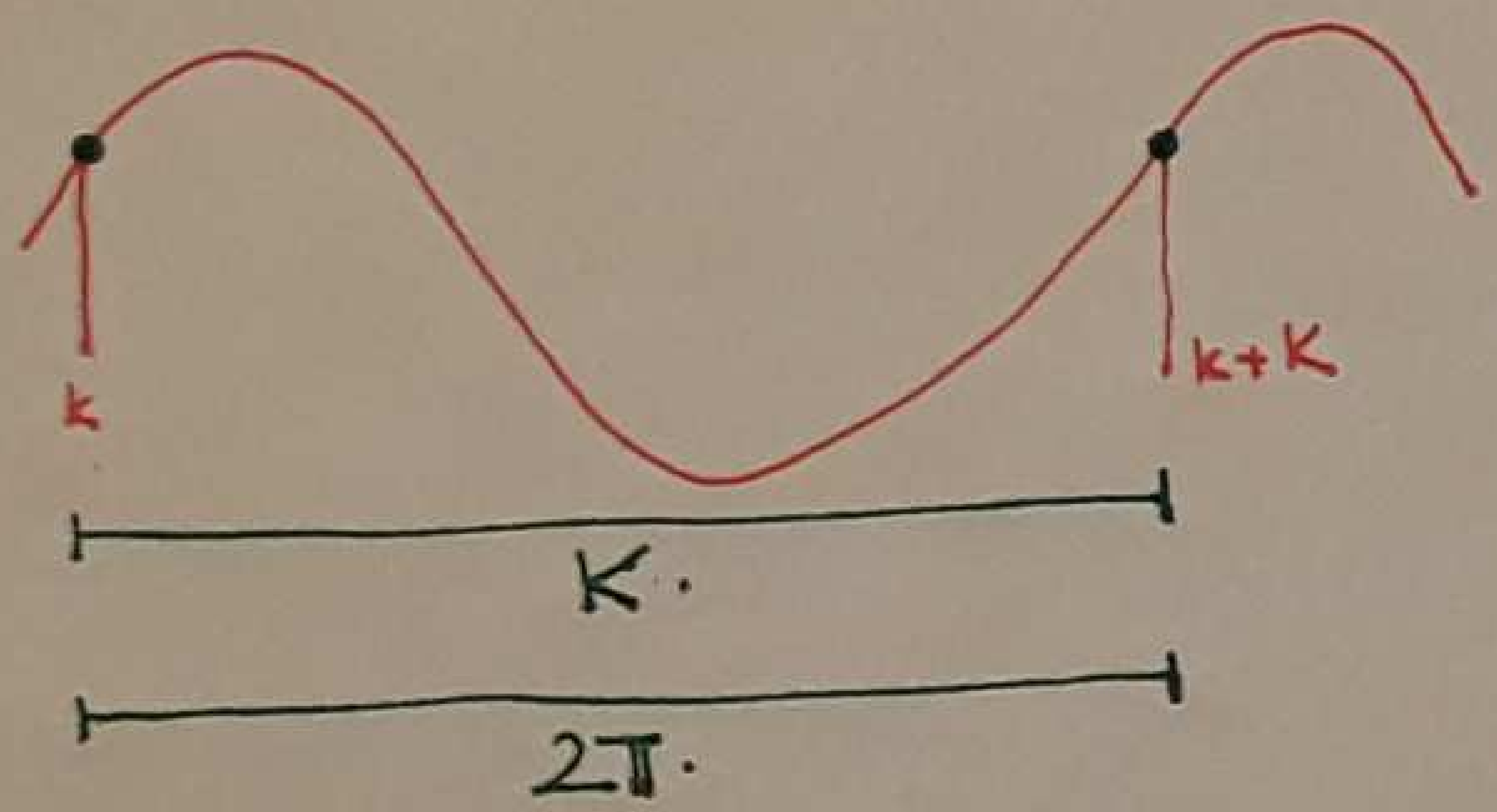
$$\cos\omega t \xrightarrow{t \to kT_s} \cos\omega kT_s = \cos k\underbrace{\omega T_s}_{\vartheta} = \underline{\cos k\vartheta}$$

ϑ

$$\vartheta = \omega T_s = \frac{2\pi}{T}T_s = 2\pi\frac{T_s}{T}$$

$$\frac{\vartheta}{2\pi} = \frac{T_s}{T} \in \mathbb{Q} \quad \text{RACIONÁLIS.}$$

$$\boxed{\frac{\vartheta}{2\pi} = \frac{M}{N}} \quad M, N \in \mathbb{Z}, \; N \neq 0$$



$$\rightarrow \frac{3\pi}{7}(k+K) = \frac{3\pi}{7}k + 2\pi$$

$$\cancel{\frac{3\pi}{7}k} + \frac{3\cancel{\pi}}{7}K = \cancel{\frac{3\pi}{7}k} + 2\cancel{\pi}$$

$$\underline{\underline{K = \frac{14}{3}}}$$

periodikus

$$\rightarrow \frac{\vartheta}{2\pi} = \frac{\frac{3\pi}{7}}{2\pi} = \frac{\frac{3}{7}}{2} = \frac{3}{14} \checkmark$$

$$\sqrt{2}\,\pi(k+K) = \sqrt{2}\,\pi k + 2\pi$$

$$\cancel{\sqrt{2}\,\pi k} + \sqrt{2}\,\cancel{\pi}K = \cancel{\sqrt{2}\,\pi k} + 2\cancel{\pi}$$

$$K = \frac{2}{\sqrt{2}}$$

nem periodikus

$$\frac{\vartheta}{2\pi} = \frac{\sqrt{2}\,\pi}{2\pi} = \frac{\sqrt{2}}{2} \times$$

$$2(k+K) = 2k + 2\pi$$

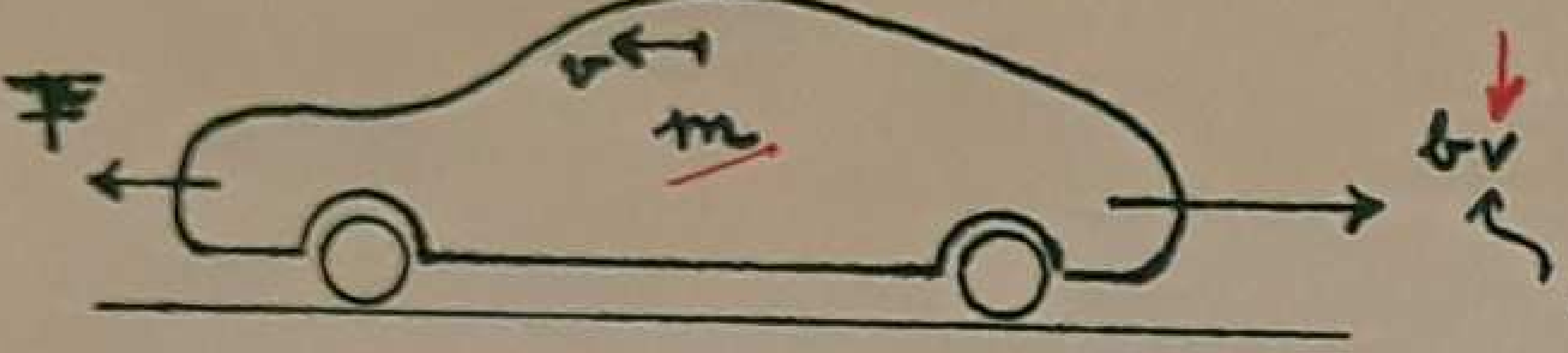
$$\cancel{2k} + 2K = \cancel{2k} + 2\pi$$

$$K = \pi$$

nemperiodikus

$$\frac{\vartheta}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi} \times$$

Írjuk fel az alábbi, egyszerűsített járműmodell dinamikáját leíró egyenleteket!



súrlódás, menetellenállás

$ma = F - bv$ **NEWTON** $\quad x \quad \dot{x} = \frac{dx}{dt} \quad \ddot{x} = \frac{dv}{dt} = a$

v

III. Euler-Lagrange:

$$\cancel{\frac{d}{dt}\frac{\partial K}{\partial y}}$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{x}} - \frac{\partial K}{\partial x} + \frac{\partial P}{\partial x} + \frac{\partial R}{\partial \dot{x}} = \tau$$

$K = \frac{1}{2} m \dot{x}^2$

$P = 0$

$R = \frac{1}{2} b \dot{x}^2$

$$\boxed{m\ddot{x} + b\dot{x} = F}$$

$m\dot{x} = F - b\dot{x}$

$$\boxed{m\ddot{x} = F - b\dot{x}}$$

$\to m\dot{v} = F - bv$

(I) $$\boxed{\dot{v} = \left(-\frac{b}{m}\right) v + \left(\frac{1}{m}\right) F}$$

bemenet

$$\underline{\dot{v} = A v + B F}$$

$m\ddot{x} = -b\dot{x} + F \quad \to \quad \ddot{x} = -\frac{b}{m}\dot{x} + \frac{1}{m} F$

$x_2 = \dot{x} \quad / \frac{d}{dt} \quad \dot{x}_2 = \ddot{x}$

$x_1 = x \quad / \frac{d}{dt} \quad \dot{x}_1 = \dot{x} = x_2$

$$\begin{cases} \dot{x}_2 = -\frac{b}{m} x_2 + \frac{1}{m} F \\ \dot{x}_1 = x_2 \end{cases}$$

$x_1 = x \longrightarrow$ elmozdulás

$x_2 = \dot{x} \longrightarrow$ sebesség

$\Downarrow$ (II)

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{\underline{x}}} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{\underline{b}} F$$

$\underline{\underline{B}} \; \underline{F}$

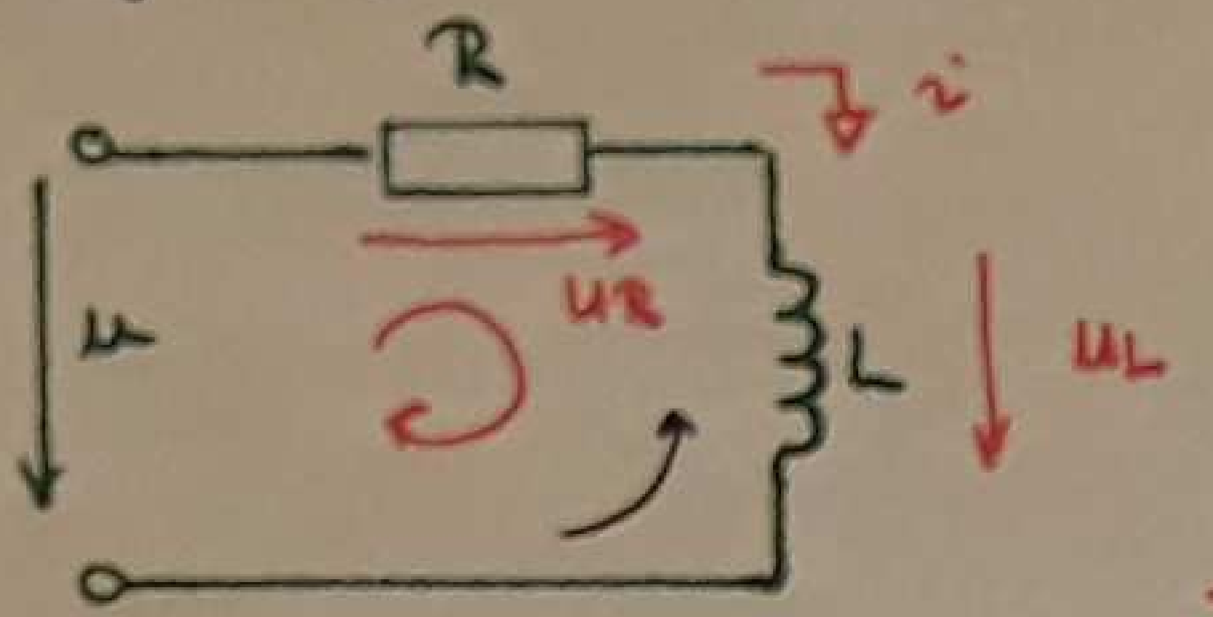
ANALÓGIÁK.

fizika	elmozdulás erő hatására hőmérséklet változása kondenzátor feltöltődése
kémia	vegyipari folyamatok
műszaki	robotok mozgása járművek
gazdasági	pénzbefektetéshez tartozó árukészlet
stb ⋮	

ANALÓG PROBLÉMÁK
(matematikailag hasonló)

Differenciálegyenlet
- közönséges - koncentrált paraméterű (pontszerű)
- parciális - elosztott paraméterű

Írjuk fel a soros RL-kör differenciálegyenletét!



$$\dot{v} = -\frac{k}{m} v + \frac{1}{m} F$$

$$\Rightarrow \dot{u}_C = -\frac{1}{RC} u_C + \frac{1}{RC} u$$

$$\rightarrow \dot{q} = -\frac{1}{RC} q + \frac{1}{R} u$$

$$+u_R + u_L - u = 0 \longleftrightarrow -u_L - u_R + u = 0$$

KIRCHHOFF

$$R i_L + \underline{L \dot{i}_L} - u = 0$$

$$\Rightarrow \boxed{\dot{i}_L = -\frac{R}{L} i_L + \frac{1}{L} u}$$

$$u_R = R i_R$$

$$\boxed{u_L = L \dot{i}_L} \Leftarrow$$

$$L = \frac{\Psi}{i}$$

$$L i = \Psi \quad / \frac{d}{dt}$$

$$L \dot{i} = \underbrace{\dot{\Psi} = u_L}$$

$$u_L = L \dot{i}_L$$

$$\underline{\dot{x} = Ax + Bu}$$

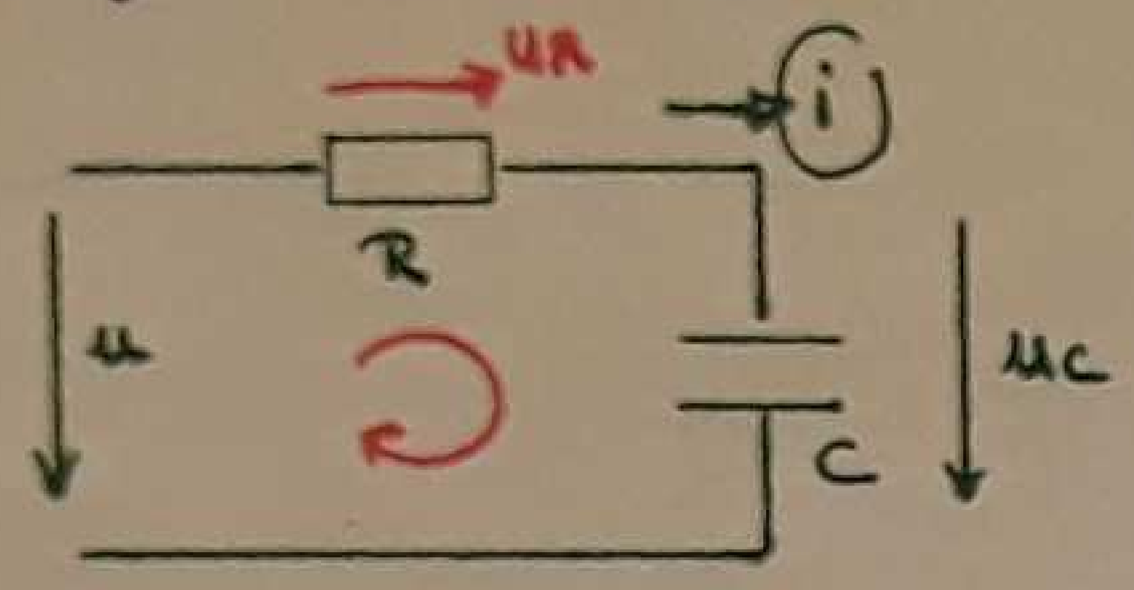
ANALÓGIA

állapotváltozó

gerjesztés

Írjuk fel egy soros RC-kör viselkedését leíró egyenletet !

$\dot{v} = -\frac{b}{m} v + \frac{1}{m} F.$



KIRCHHOFF.

huroktörvény.

$$\boxed{C \dot{u}_C = i_C.}$$

$$\boxed{u_R = R\, i_R}$$ OHM

$$↻ \; + u_R + u_C - u = 0$$

$$Ri + u_C - u = 0$$

$$RC \dot{u}_C + u_C - u = 0$$

$$\Rightarrow \boxed{\dot{u}_C = -\frac{1}{RC} u_C + \frac{1}{RC} u}$$

$$Ri + u_C - u = 0$$

$$R\dot{q} + \frac{1}{C} q - u = 0$$

$$\boxed{\dot{q} = -\frac{1}{RC} q + \frac{1}{R} u}$$

$i = \frac{dq}{dt}$

$$\boxed{C = \frac{q}{u}}$$

$u = \frac{1}{C} q$

$C = \frac{q}{u}$

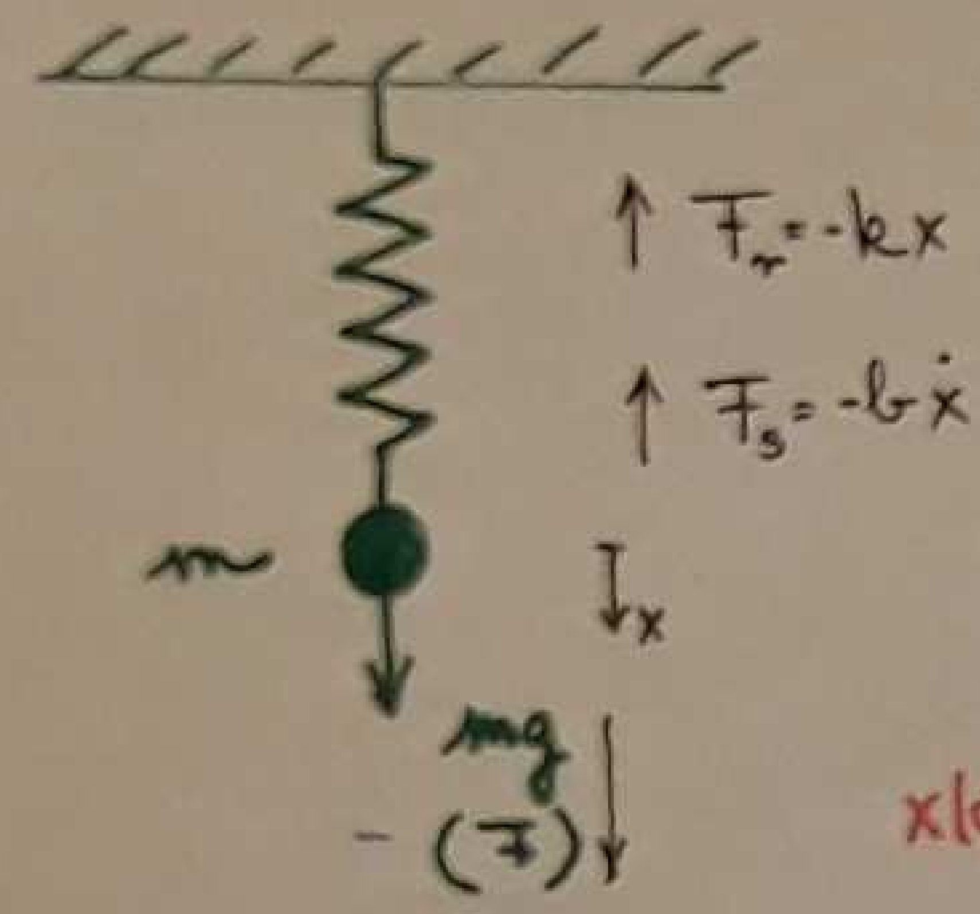
$Cu = q \quad / \frac{d}{dt}$

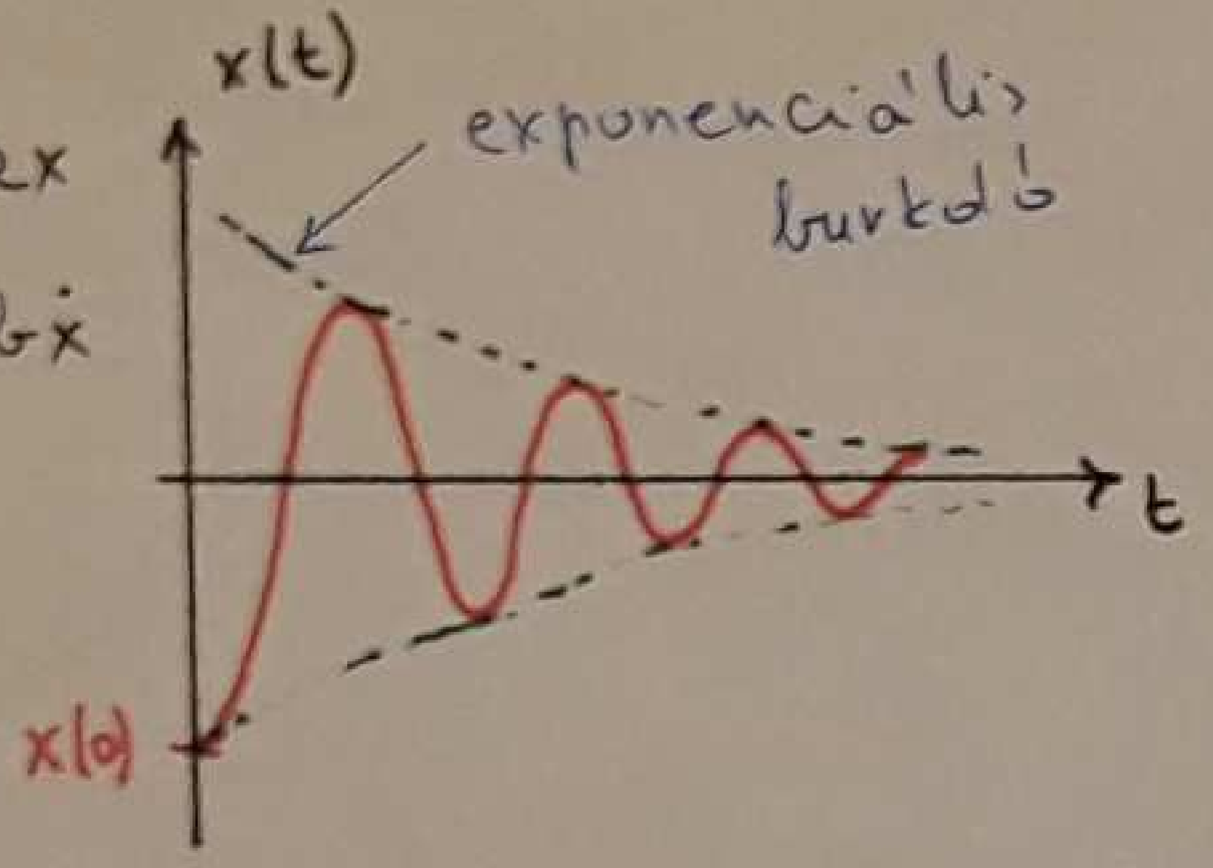
$C\dot{u} = \dot{q}$

$$\boxed{C \dot{u}_C = i_C}$$

Karakterisztika. $\underline{bv}$

Írjuk fel az egyik végén rögzített rugóra függesztett test rezgőmozgását leíró differenciálegyenletet!





$$m\ddot{x} = \underbrace{F - b\dot{x} - kx}$$

RENDSZEREGYENLET

$$\ddot{x} = -\frac{b}{m}\dot{x} - \frac{k}{m}x + \frac{1}{m}F. \Leftarrow$$

$$\boxed{\begin{aligned} x_2 &= \dot{x}. \\ x_1 &= x. \end{aligned}} \quad \begin{aligned} &/\tfrac{d}{dt} \\ &/\tfrac{d}{dt} \end{aligned} \quad \Rightarrow \underline{\dot{x}_1 = \dot{x}} = x_2$$

$$\rightarrow \begin{cases} \dot{x}_2 = -\dfrac{b}{m}x_2 - \dfrac{k}{m}x_1 + \dfrac{1}{m}F \\ \dot{x}_1 = x_2 \end{cases}$$

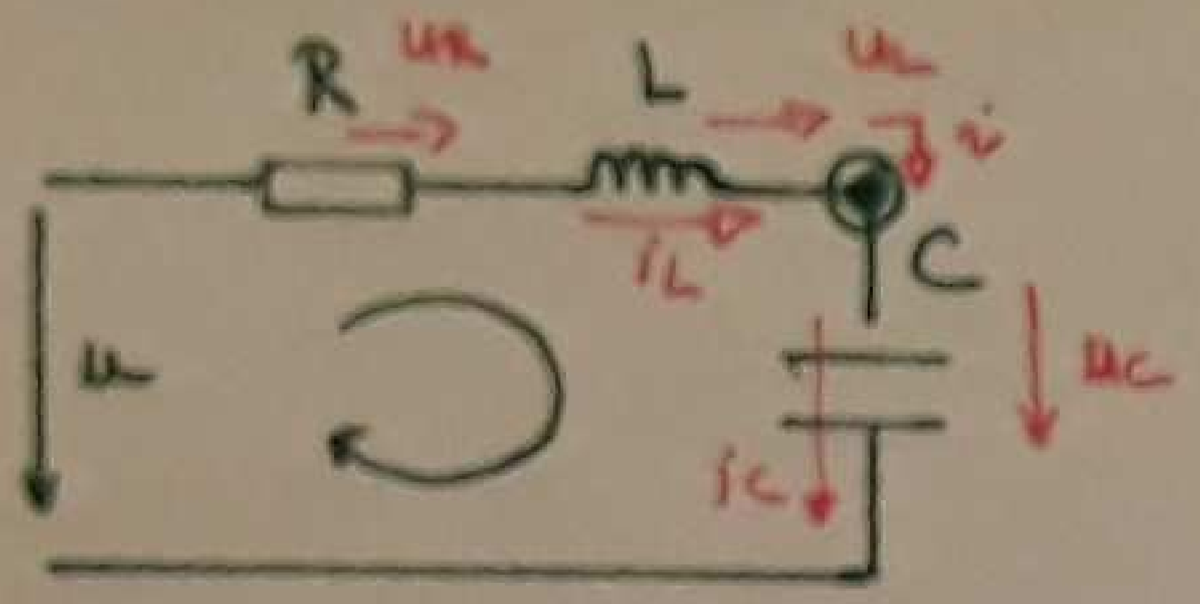
ÁLLAPOTVÁLTOZÓS LEÍRÁS

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F \Rightarrow \boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \frac{1}{m}F \end{aligned}}$$

$$\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{B}F$$

$$\underline{B} \uparrow \underline{b}$$

Írjuk fel a soros rezgőkör viselkedését leíró differenciálegyenletet!



$$u_R + u_L + u_C - u = 0$$

$$Ri + L\dot{i} + u_C - u = 0$$

$$\boxed{\dot{i}_C - \frac{R}{L} i - \frac{1}{L} u_C + \frac{1}{L} u}$$

$$-i_L + i_C = 0$$

$$-i + C\dot{u}_C = 0$$

$$\boxed{\dot{u}_C = \frac{1}{C} i}$$

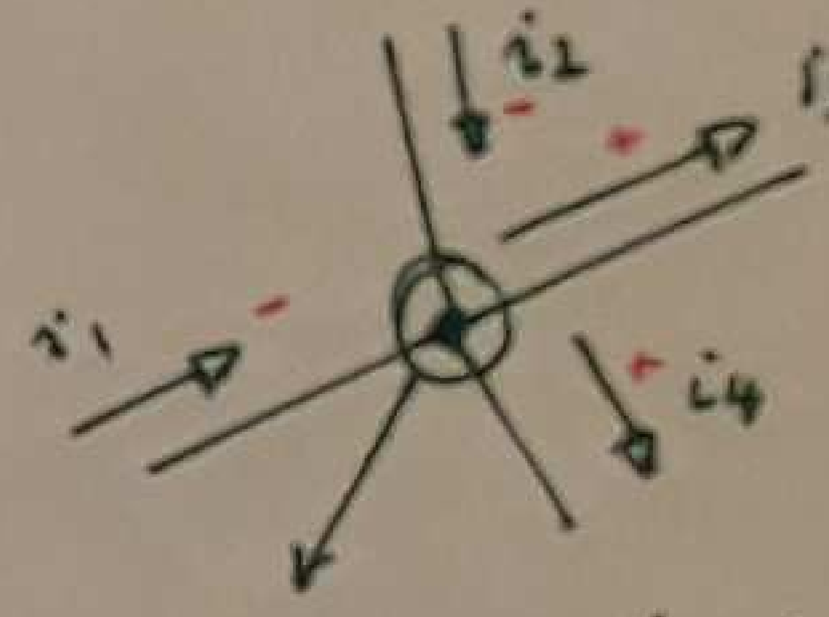
$$\Downarrow$$

Kirchhoff.

$u_R = Ri$

$u_L = L\dot{i}_L$ $\boxed{i_L}$

$i_C = C\dot{u}_C$ $\boxed{u_C}$



$$-i_1 - i_2 + i_3 + i_4 = 0$$

$$\underbrace{\begin{bmatrix}\dot{u}_C \\ \dot{i}_L\end{bmatrix}}_{\dot{\underline{x}}} = \underbrace{\begin{bmatrix}0 & 1/C \\ -\frac{1}{L} & -\frac{R}{L}\end{bmatrix}}_{\underline{\underline{A}}} \cdot \underbrace{\begin{bmatrix}u_C \\ i_L\end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix}0 \\ \frac{1}{L}\end{bmatrix}}_{\underline{b}} u$$

$$\dot{\underline{x}} = \underline{\underline{A}} \cdot \underline{x} + \underline{b}\, u \leftarrow$$

u_C

i_L

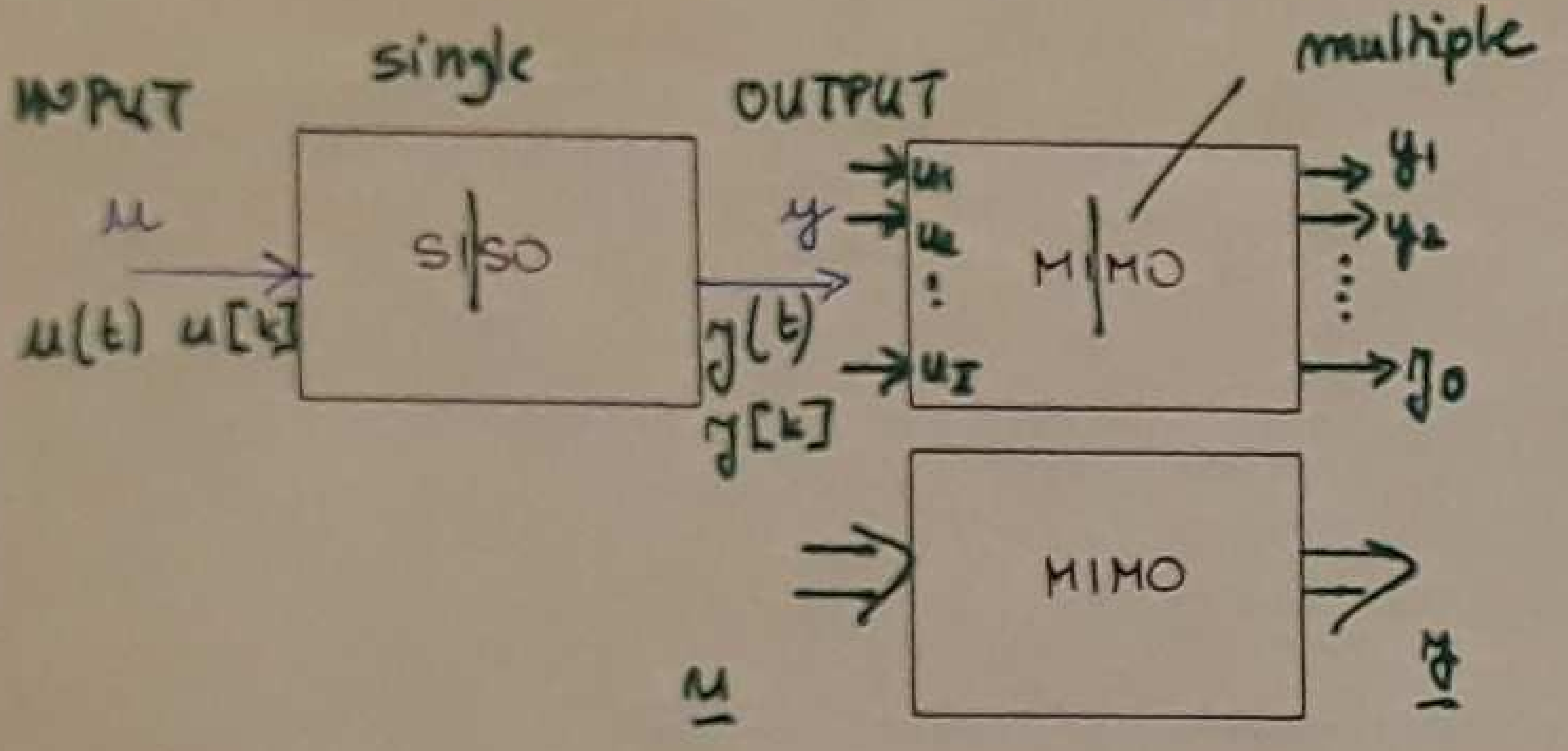
RENDSZEREK

2 megfogalmazás !

A rendszer egy valóságos objektum egy egyszerűsített leírása, modellje.

létező vagy megvalósítandó

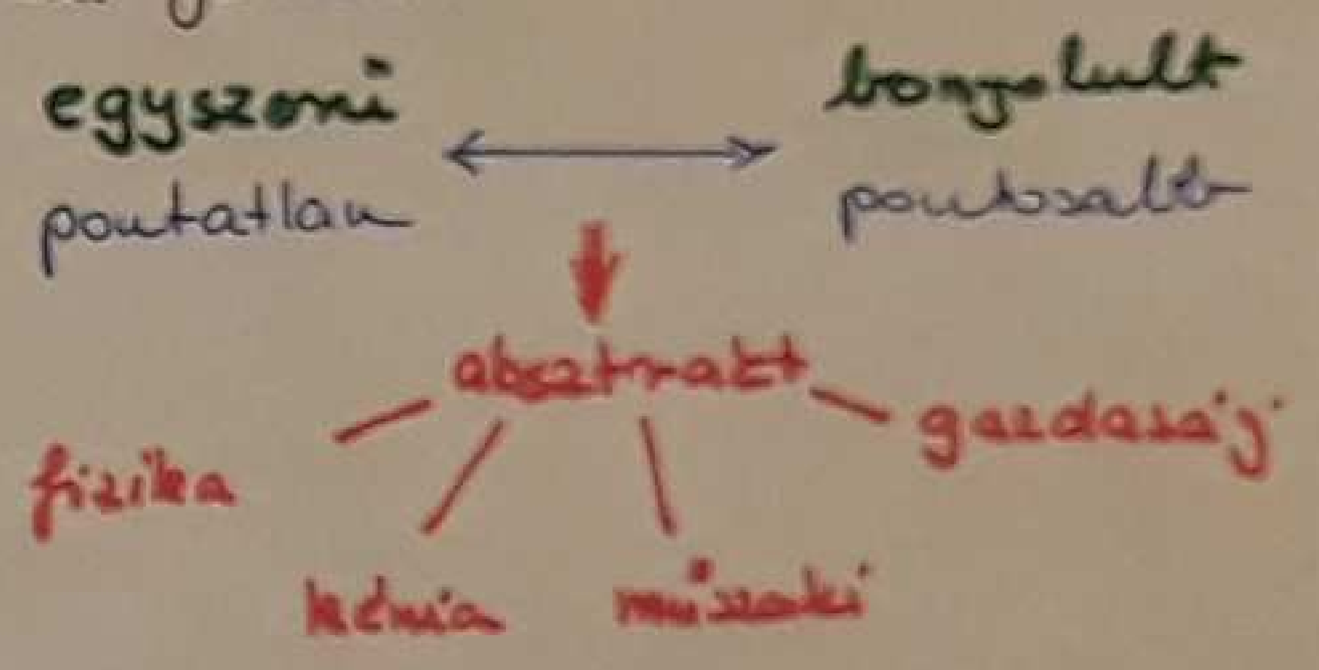
matematikai formula



SIMO
MISO

$$\underline{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_I \end{bmatrix} \qquad \underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_O \end{bmatrix}$$

$\underline{u}$ $\underline{\underline{A}}$

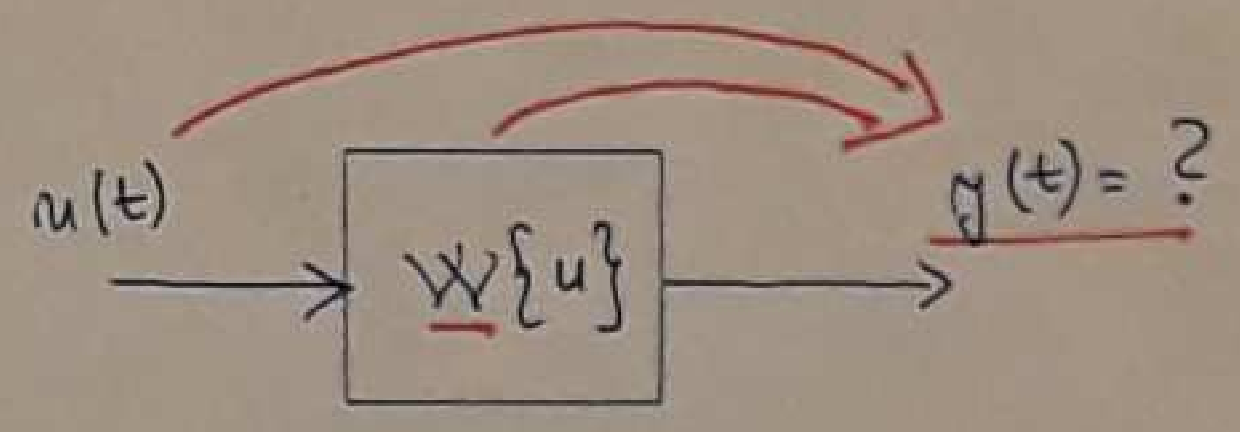


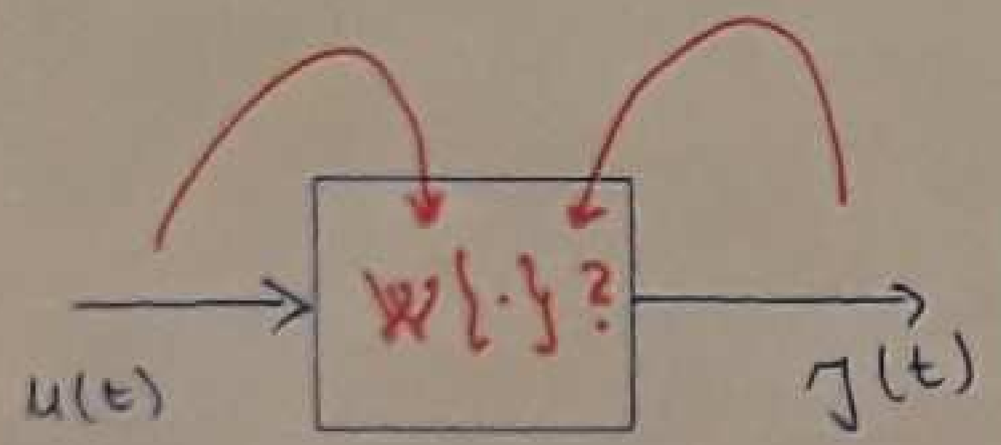
A rendszer egy transzformáció, ami a gerjesztéshez választ rendel : bemenet- kimenet kapcsolat.

Rendszeranalízis:

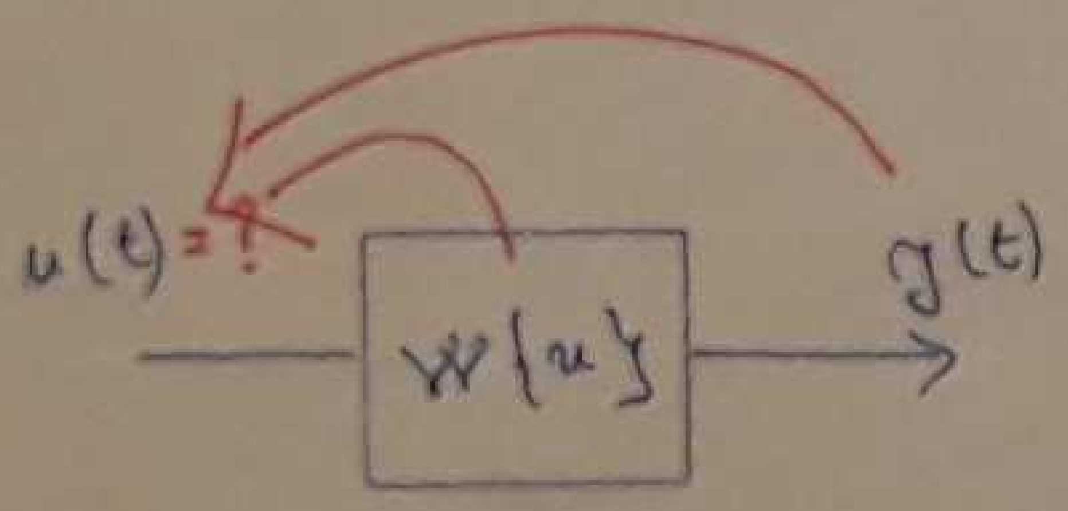
Rendszeridentifikáció:

Rendszerszintézis:





fehér - white box
szürke - grey box
fekete - black box (neurális)

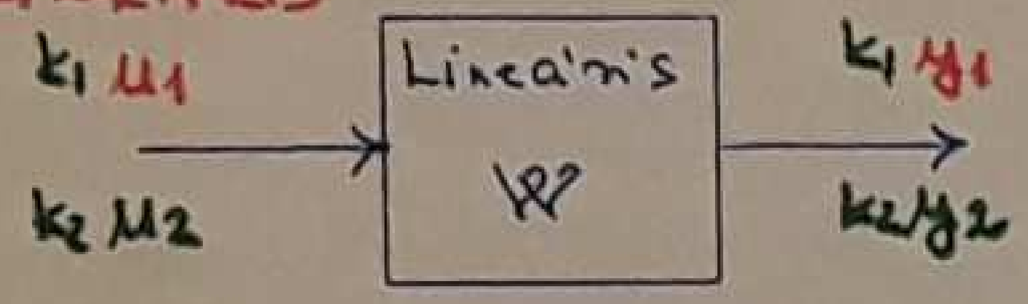


$y(t) = \underline{\underline{\underline{W}}}\{ u(t) \}$

- időtartomány
- frekvenciatartomány
- komplex frekvenciatartomány

Rendszerek osztályozása

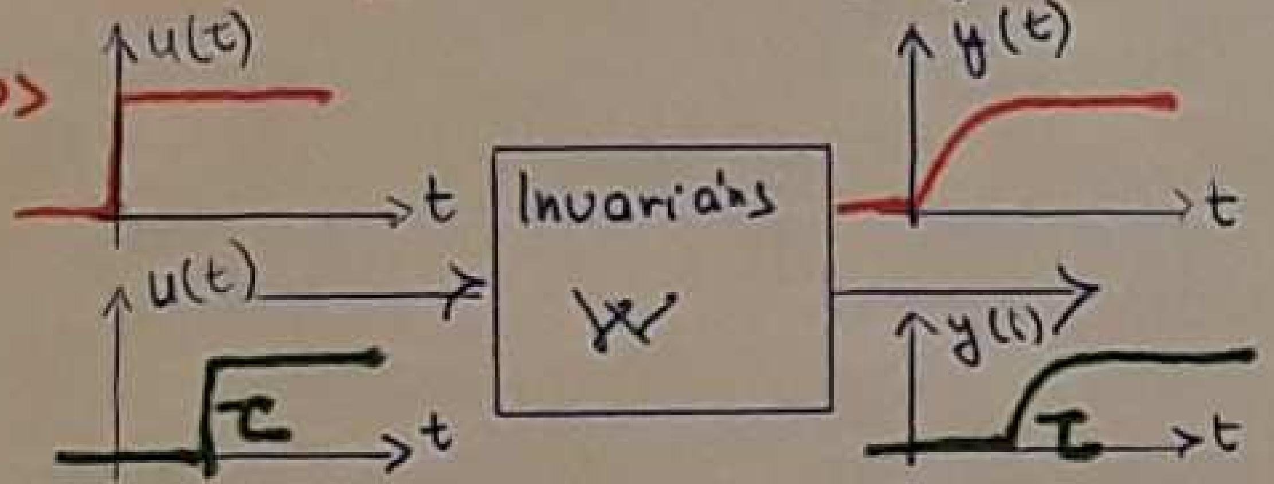
- SISO, MIMO, SIMO, MISO
- determinisztikus gerjesztésű, determinisztikus válaszú
  sztochasztikus gerjesztésű, sztochasztikus válaszú
- folytonos idejű ~~válaszú~~ gerjesztésű, folytonos idejű ~~gerjesztésű~~ válaszú
  diszkrét idejű gerjesztésű, diszkrét idejű válaszú
  A/D ; D/A - átalakító
- LINEÁRIS ↕ NEMLINEÁRIS : a $\boxed{\mathcal{W}}$ operátor lineáris, érvényes rá a szuperpozíció elve



$$k_1 u_1 \underset{\uparrow}{\pm} k_2 u_2 \longrightarrow k_1 y_1 \underset{\uparrow}{\pm} k_2 y_2$$

- „kis" változás → lineáris
- „nagy" változás → esetleg pl. valami eltérő

- INVARIÁNS ↕ VARIÁNS : a gerjesztés időbeli eltolása a válaszban ugyanekkora eltolást okoz

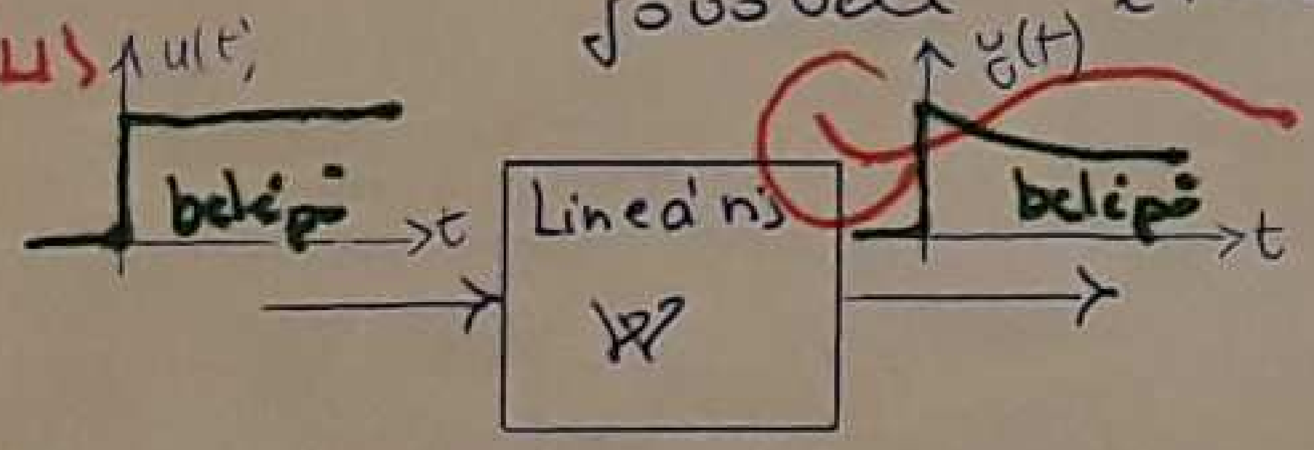


$$\mathcal{W}\{u(t)\} = y(t)$$

$$\mathcal{W}\{u(t-\tau)\} = y(t)\Big|_{t \to t-\tau}$$

- öregedés
- hőmérséklet megváltozása

- KAUZÁLIS ↕ AKAUZÁLIS : a válasz $t = t_1$ időpillanatban ($k = k_1$ ütemben) nem függ a gerjesztés jövőbeli értékeitől.



- fizikai rendszer mindig kauzális ↔ megvalósítható
- jóslás - predikció

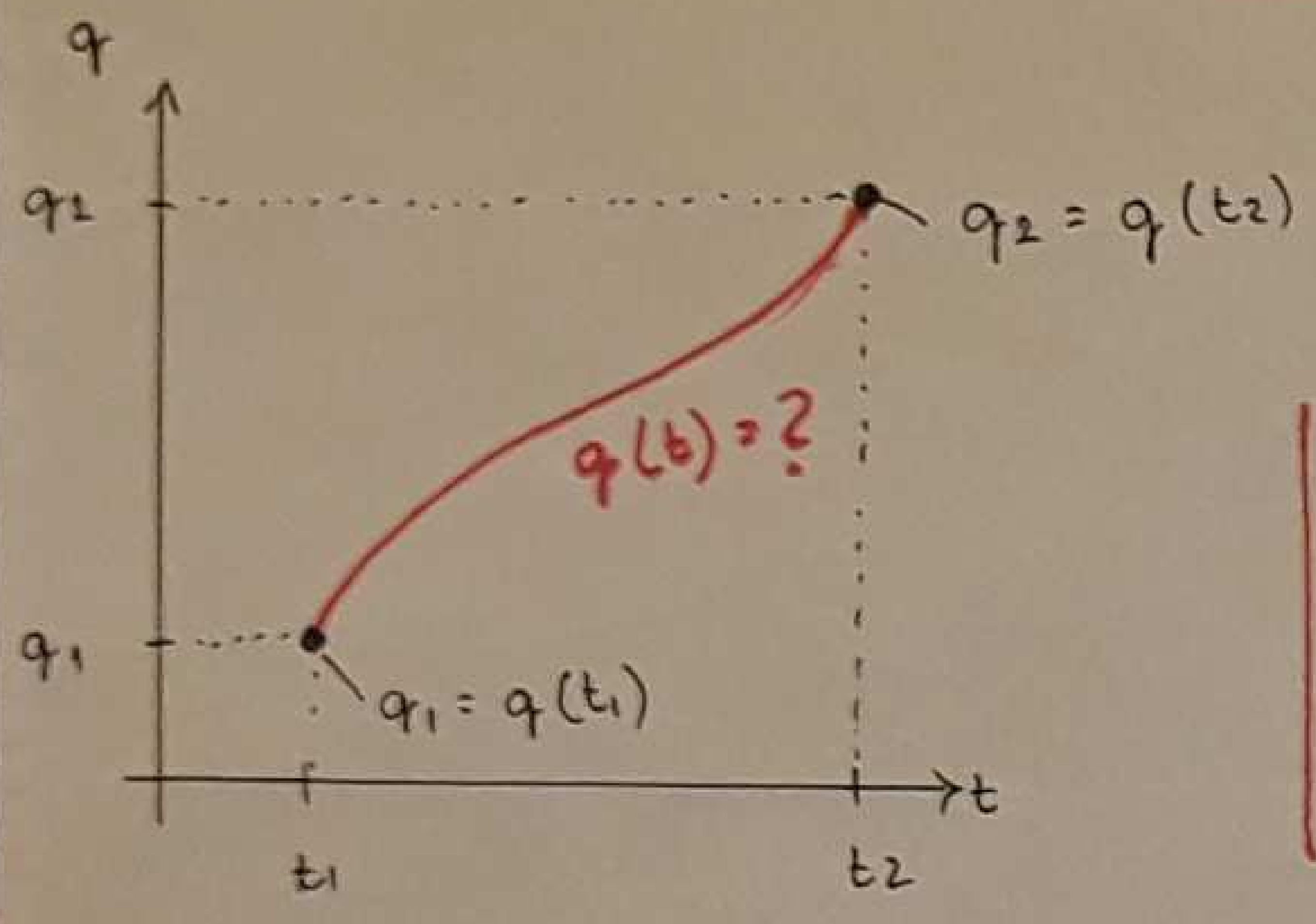
- STABILIS ↕ LABILIS : korlátos gerjesztésre korlátos válasszal felel (BIBO).

Gerjesztés - válasz

bounded

EULER - LAGRANGE - MODELLEZÉS

\- legkisebb hatás elve -



általános koordináta

$\underline{q} = [q_1, q_2, \dots]^T$

általános sebesség

$\underline{\dot{q}} = [\dot{q}_1, \dot{q}_2, \dots]^T$

$q_i = q_i(t)$

mechanikai állapot

HATÁSFÜGGVÉNY:

$$\boxed{\int_{t_1}^{t_2} L(\underline{q}, \underline{\dot{q}}, t)\, dt \longrightarrow \text{min.}}$$

↑ Lagrange-függvény

$$\boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \qquad i = 1, 2, \dots}$$

Euler-Lagrange-egyenlet.

(Simonyi Károly
A fizika kultúrtörténete)

$L = K - P$

potenciális energia $P(q)$

kinetikus energia : $K(\underline{q}, \dot{q})$

$$\frac{d}{dt}\frac{\partial}{\partial \dot{q}_k}(K - \cancel{P}) - \frac{\partial}{\partial q}(K - P) = 0$$

$$\boxed{\frac{d}{dt}\frac{\partial K}{\partial \dot{q}} - \frac{\partial K}{\partial q} + \frac{\partial P}{\partial q} = 0}$$

Rayleigh

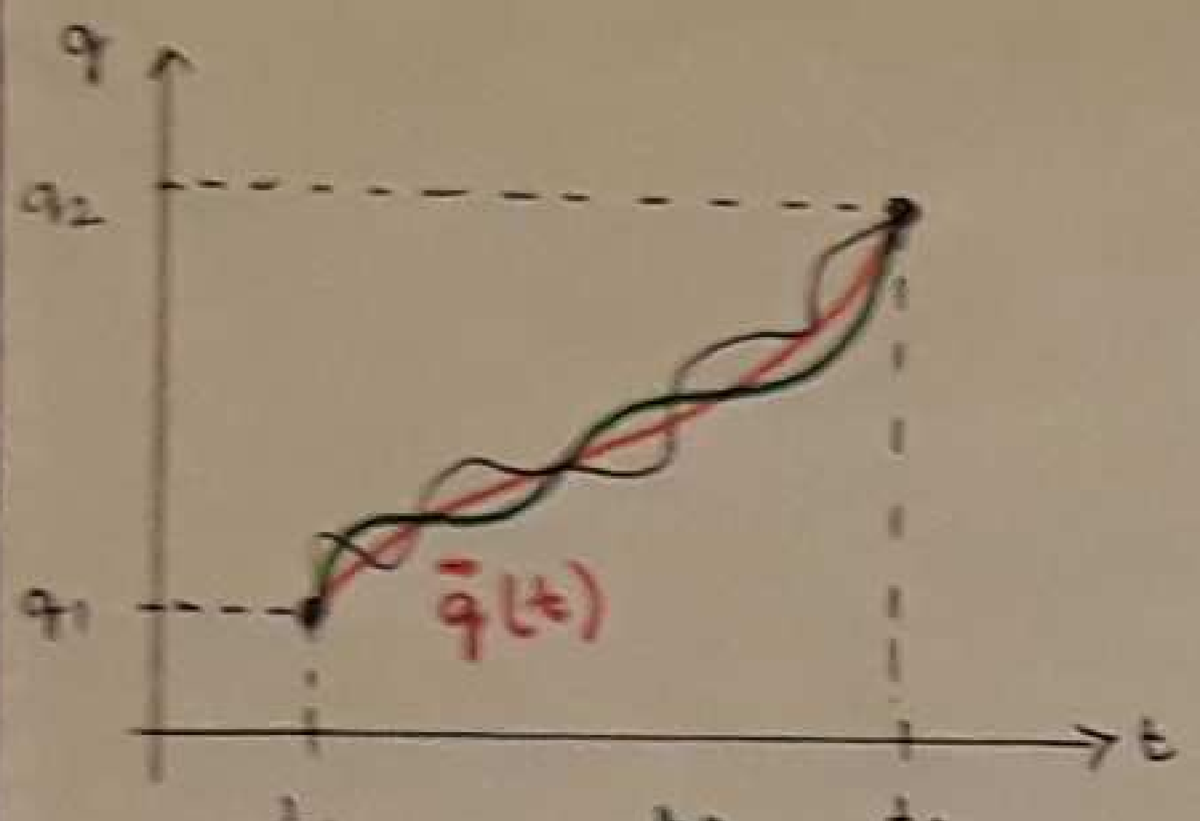
$\frac{1}{2} b\dot{q}^2$

$b\dot{q}$

$$\boxed{\frac{d}{dt}\frac{\partial K}{\partial \underline{\dot{q}}} - \frac{\partial K}{\partial \underline{q}} + \frac{\partial P}{\partial \underline{q}} + \frac{\partial R}{\partial \underline{\dot{q}}} = \underline{\tau}}$$

⇓ ⇑

MOZGÁSEGYENLET



EULER- LAGRANGE- EGYENLET BIZONYÍTÁSA.

$$\boxed{\begin{array}{l} \bar{q}(t): \displaystyle\int_{t_1}^{t_2} L(\underset{\uparrow}{q}, \underset{\uparrow}{\dot{q}}, t)\, dt \longrightarrow \min \\ \\ \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{q}} - \dfrac{\partial L}{\partial q} = 0 \end{array}}$$

$$q(t) = \bar{q}(t) + \underset{\xi}{\varepsilon}\,\eta(t)$$

$$\eta(t_1) = 0$$

$$\eta(t_2) = 0$$

$$\varepsilon = 0$$

$$\frac{dq}{d\varepsilon} = \frac{d\bar{q}}{d\varepsilon} + \frac{d}{d\varepsilon}(\varepsilon\eta)$$

$$= \eta$$

$$I(\varepsilon) = \int_{t_1}^{t_2} L(\underbrace{\bar{q} + \varepsilon\eta}_{q}, \underbrace{\dot{\bar{q}} + \varepsilon\dot{\eta}}_{\dot{q}}, t)\, dt \rightsquigarrow \cancel{\frac{\partial L}{\partial t}\frac{dt}{d\varepsilon}}$$

$$\frac{\partial I}{\partial \varepsilon} = \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q}\underbrace{\frac{dq}{d\varepsilon}}_{\eta} + \frac{\partial L}{\partial \dot{q}}\underbrace{\frac{d\dot{q}}{d\varepsilon}}_{\dot{\eta}}\right) dt =$$

$$= \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q}\eta + \underbrace{\overset{v}{\frac{\partial L}{\partial \dot{q}}}\overset{u'}{\dot{\eta}}}\right) dt =$$

$$\Rightarrow \int u'v = uv - \int uv'$$

$$\int_{t_1}^{t_2} \dot{\eta}\frac{\partial L}{\partial \dot{q}}\, dt = \underbrace{\left[\eta\frac{\partial L}{\partial \dot{q}}\right]_{t_1}^{t_2}}_{0} - \underline{\underline{\int_{t_1}^{t_2} \eta\frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\, dt}}$$

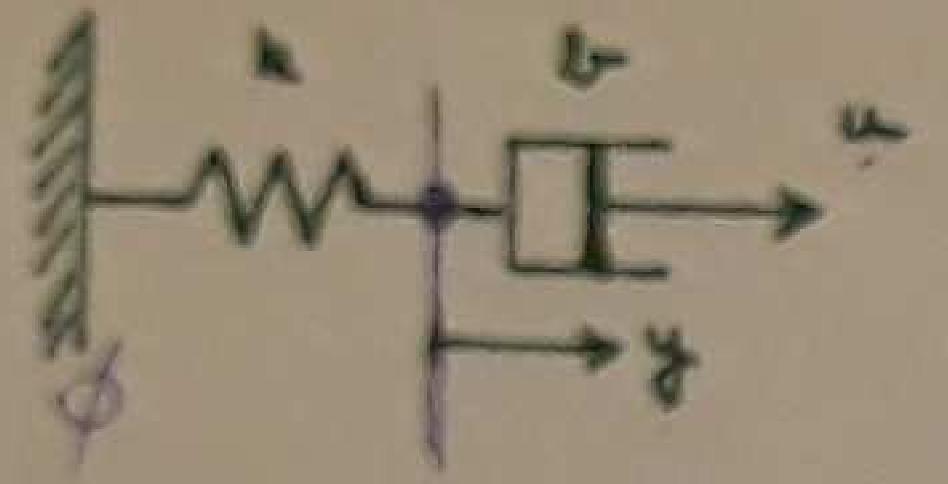
$$= \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q}\eta - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\eta\right) dt$$

$$= \int_{t_1}^{t_2} \eta \cdot \underbrace{\left(\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\right)}_{0} dt = 0 \quad \Rightarrow \quad \boxed{\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0}$$

Euler-Lagrange-egyenlet

⟹ mozgásegyenlet.

Írjuk fel az alábbi rendszer modelljét (differenciálegyenletét)!



$$\underbrace{\frac{d}{dt}\frac{\partial K}{\partial \dot{y}} - \frac{\partial K}{\partial y}} + \frac{\partial P}{\partial y} + \frac{\partial R}{\partial \dot{y}} = \tau$$

$K = 0$

$P = \frac{1}{2} k y^2 \qquad \frac{\partial P}{\partial y} = k y$

$R = \frac{1}{2} b (\dot{u} - \dot{y})^2 \qquad \frac{\partial R}{\partial \dot{y}} = b(\dot{u} - \dot{y})(-1)$

$k y - b(\dot{u} - \dot{y}) = 0$

$k y - b \dot{u} + b \dot{y} = 0$

$\dot{y} = -\frac{k}{b} y + \textcircled{$\dot{u}$} \quad / \int$

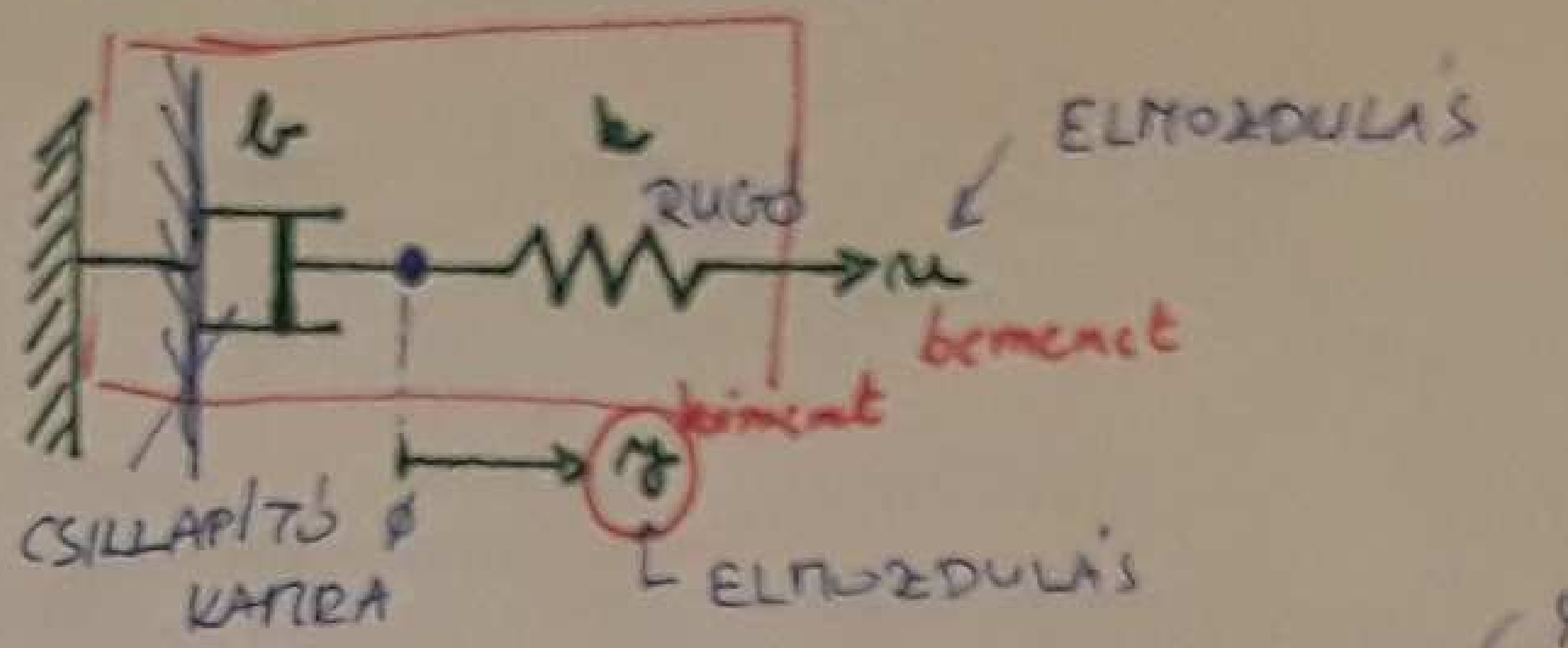
TRŪKK !

$y = \underbrace{-\int \frac{k}{b} y \, dt}_{\textcircled{x}} + u \Rightarrow \boxed{y = x + u}$ ✓

$x = -\frac{k}{b} \int y \, dt \quad / \frac{d}{dt}$

$\dot{x} = -\frac{k}{b} y \Rightarrow \boxed{\dot{x} = -\frac{k}{b} x - \frac{k}{b} u}$

$$\boxed{\begin{aligned} \dot{x} &= -\frac{k}{b} x - \frac{k}{b} u \\ y &= x + u \end{aligned}}$$



$$\underbrace{\frac{d}{dt}\frac{\partial K}{\partial \dot{y}} - \frac{\partial K}{\partial y}}_{0} + \frac{\partial P}{\partial y} + \frac{\partial R}{\partial \dot{y}} = \overset{0}{\tau}$$

$K = 0$

$P = \frac{1}{2} k (u - y)^2 \qquad \frac{\partial P}{\partial y} = k(u - y) \cdot (-1)$

$R = \frac{1}{2} b \dot{y}^2 \qquad \frac{\partial R}{\partial \dot{y}} = b \dot{y}$

$\frac{1}{2} m v^2$

$m g h$

$\frac{1}{2} k x^2$

$-k(u - y) + b \dot{y} = 0$

$-k \textcircled{u} + k y + b \dot{y} = 0$

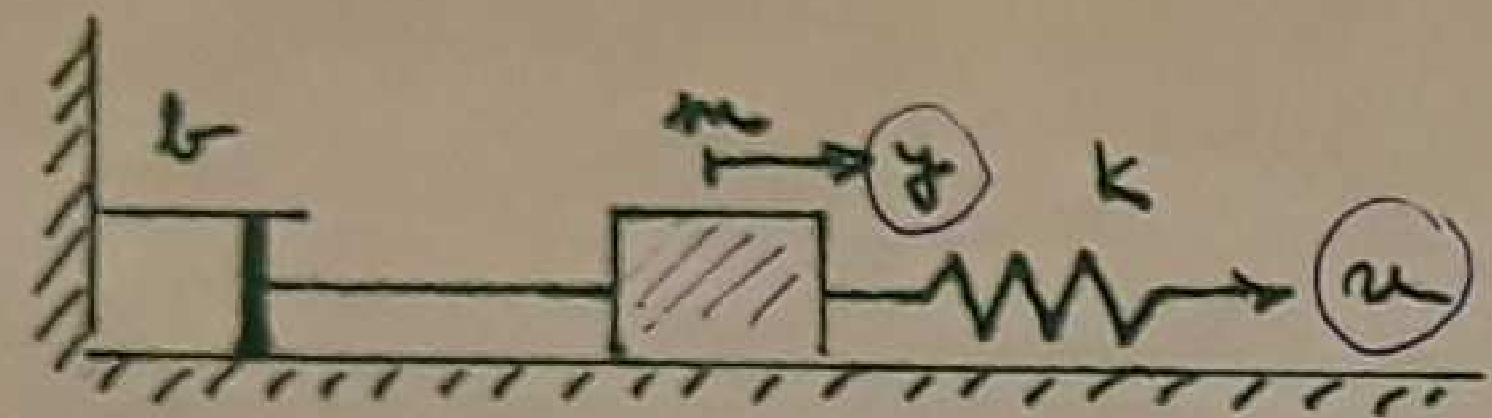
$\dot{y} = -\frac{k}{b} y + \frac{k}{b} u$ ✓

$y \rightarrow x$!

$$\boxed{\begin{aligned} \dot{x} &= \underbrace{-\frac{k}{b}} x + \underbrace{\frac{k}{b}} u \\ y &= x \end{aligned}}$$

$$\boxed{\begin{aligned} \dot{x} &= A x + b \textcircled{u} \\ y &= c x + D \textcircled{u} \end{aligned}}$$

Írjuk fel az alábbi rendszer differenciálegyenletét!



$$\underline{\frac{d}{dt}\frac{\partial K}{\partial \dot y}} - \cancel{\frac{\partial K}{\partial y}} + \frac{\partial P}{\partial y} + \frac{\partial R}{\partial \dot y} = \overset{\swarrow 0}{\tau}$$

$$K = \frac{1}{2} m \dot y^2 \qquad \frac{\partial K}{\partial \dot y} = m\dot y \qquad \frac{d}{dt}\frac{\partial K}{\partial \dot y} = \frac{d}{dt}(m\dot y) = m\ddot y \qquad \frac{\partial K}{\partial y} = 0$$

$$P = \frac{1}{2} k (u-y)^2 \qquad \frac{\partial P}{\partial y} = -k(u-y)$$

$$R = \frac{1}{2} b \dot y^2 \qquad \frac{\partial R}{\partial \dot y} = b\dot y$$

$$m\ddot y - k(u-y) + b\dot y = 0$$

$$m\ddot y - ku + ky + b\dot y = 0$$

$$\boxed{m\ddot y + b\dot y + ky - ku = 0}$$

Rendszeregyenlet

$$\boxed{\begin{aligned} \dot x_1 &= x_2 \\ \dot x_2 &= -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \frac{k}{m}u \\ y &= x_1 \end{aligned}}$$

Állapotváltozós leírás.

$$m\ddot y + \underline{b\dot y} + \underline{ky} - ku = 0$$

$$\ddot y = -\frac{b}{m}\dot y - \frac{k}{m}y + \frac{k}{m}u$$

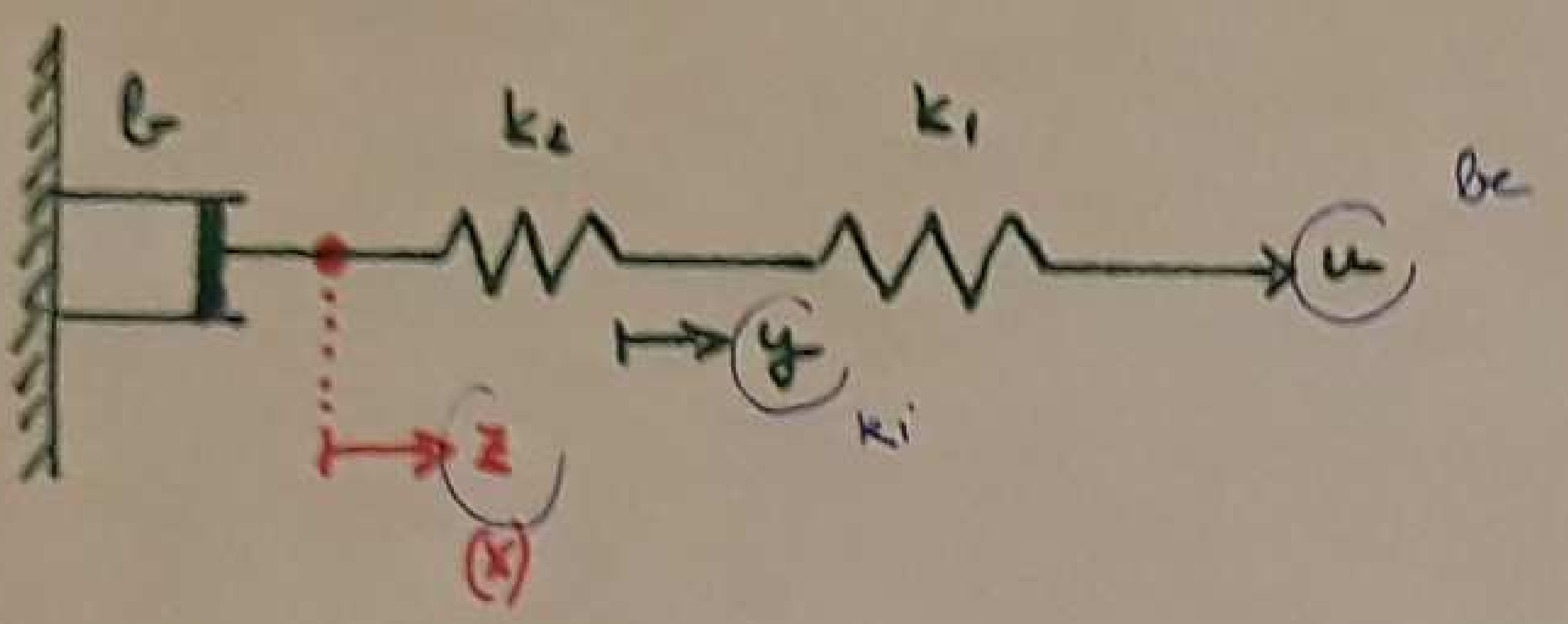
hf. $\dot y = x_1$

$y = x_2$

$$\boxed{\begin{aligned} \dot y &= x_2 \\ y &= x_1 \end{aligned}}$$

$$\begin{cases} \dot x_2 = -\frac{b}{m}x_2 - \frac{k}{m}x_1 + \frac{k}{m}u \\ \dot x_1 = x_2 \\ y = x_1 \end{cases}$$

Írjuk fel az alábbi lengőrendszer modelljét!



$$\underline{q} = \begin{bmatrix} y \\ z \end{bmatrix} \quad q_i \quad i = 1,2 \qquad \boxed{\begin{matrix} q_1 = y \\ q_2 = z \end{matrix}}$$

$$\cancel{\frac{d}{dt}\frac{\partial K}{\partial \dot{q}_i} - \frac{\partial K}{\partial q_i}} + \frac{\partial P}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = \overset{0}{\tau_i} \qquad i = 1,2$$

$$K = 0$$

$$P = \frac{1}{2}k_1(u-y)^2 + \frac{1}{2}k_2(y-z)^2$$

$$R = \frac{1}{2}b\dot{z}^2$$

$i=1; y.)$ $\frac{\partial P}{\partial y} = -k_1(u-y) + k_2(y-z)$ $\qquad \frac{\partial R}{\partial \dot{y}} = 0$

$i=2; z.)$ $\frac{\partial P}{\partial z} = -k_2(y-z)$ $\qquad \frac{\partial R}{\partial \dot{z}} = b\dot{z}$

$$\left.\begin{aligned} -k_1(u-y) + k_2(y-z) &= 0 \\ -k_2(y-z) + b\dot{z} &= 0 \end{aligned}\right\} \longrightarrow -k_1u + \underline{k_1y + k_2y} - \underline{k_2z} = 0 \qquad \textcircled{z}$$

$$\boxed{y = \frac{k_2}{k_1+k_2}z + \frac{k_1}{k_1+k_2}u} \iff y = Cz + Du$$

$$\dot{z} = Az + Bu$$

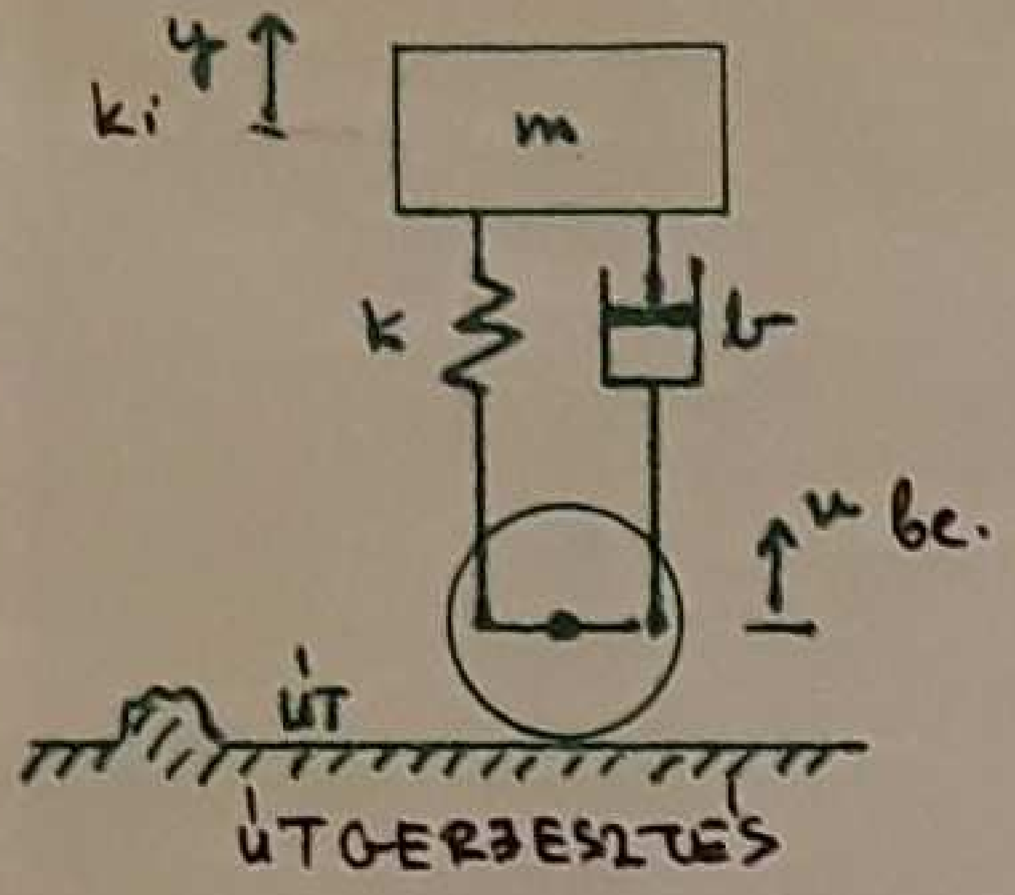
$$-k_2y + k_2z + b\dot{z} = 0$$

$$-k_2\left(\frac{k_2}{k_1+k_2}z + \frac{k_1}{k_1+k_2}u\right) + \underline{\underline{k_2z}} + b\dot{z} = 0$$

$$\underbrace{\left(k_2 - \frac{k_2^2}{k_1+k_2}\right)}_{\frac{k_1k_2 + \cancel{k_2^2} - \cancel{k_2^2}}{k_1+k_2}}z - \frac{k_1k_2}{k_1+k_2}u + \underline{b\dot{z}} = 0 \longrightarrow$$

$$\boxed{\begin{aligned} \dot{z} &= -\frac{k_1k_2}{b(k_1+k_2)}z + \frac{k_1k_2}{b(k_1+k_2)}u \\ y &= \frac{k_2}{k_1+k_2}z + \frac{k_1}{k_1+k_2}u \end{aligned}}$$

Írjuk fel az alábbi (egyszerűsített gépjármű-felfüggesztési) modell differenciálegyenletét!



$$\underbrace{\frac{d}{dt}\frac{\partial K}{\partial \dot{y}}} - \frac{\partial K}{\partial y} + \frac{\partial P}{\partial y} + \frac{\partial R}{\partial \dot{y}} = \underset{\nwarrow 0}{\Sigma}$$

$$K = \frac{1}{2} m \dot{y}^2$$

$$P = mgy + \frac{1}{2} k (y-u)^2$$

$$R = \frac{1}{2} b (\dot{y} - \dot{u})^2$$

$$\frac{\partial K}{\partial \dot{y}} = m\dot{y} \qquad \frac{d}{dt}\frac{\partial K}{\partial \dot{y}} = \frac{d}{dt}(m\dot{y}) = \underline{m\ddot{y}} \qquad \frac{\partial K}{\partial y} = 0$$

$$\frac{\partial P}{\partial y} = mg + k(y-u)$$

$$\frac{\partial R}{\partial \dot{y}} = b(\dot{y} - \dot{u})$$

$$m\ddot{y} + mg + k(y-u) + b(\dot{y} - \dot{u}) = 0$$

$$m\ddot{y} + mg + ky - ku + b\dot{y} - b\dot{u} = 0 \quad /:m$$

$$\ddot{y} = -\frac{b}{m}\dot{y} - \frac{k}{m}y + \frac{b}{m}\dot{u} + \frac{k}{m}u - g \quad /\int$$

$$\dot{y} = -\frac{b}{m}y \underbrace{- \frac{k}{m}\int y\,dt} + \frac{b}{m}u + \underbrace{\frac{k}{m}\int u\,dt} \underbrace{- g\int dt}$$

$$\dot{y} = -\frac{b}{m}y + \frac{b}{m}u + x_1$$

$$\boxed{y = x_2}$$

$$\boxed{\dot{x}_2 = -\frac{b}{m}x_2 + \frac{b}{m}u + x_1}$$

$$\Rightarrow x_1 = -\frac{k}{m}\int y\,dt + \frac{k}{m}\int u\,dt - g\int dt \quad /\frac{d}{dt}$$

$$\dot{x}_1 = -\frac{k}{m}y + \frac{k}{m}u - g$$

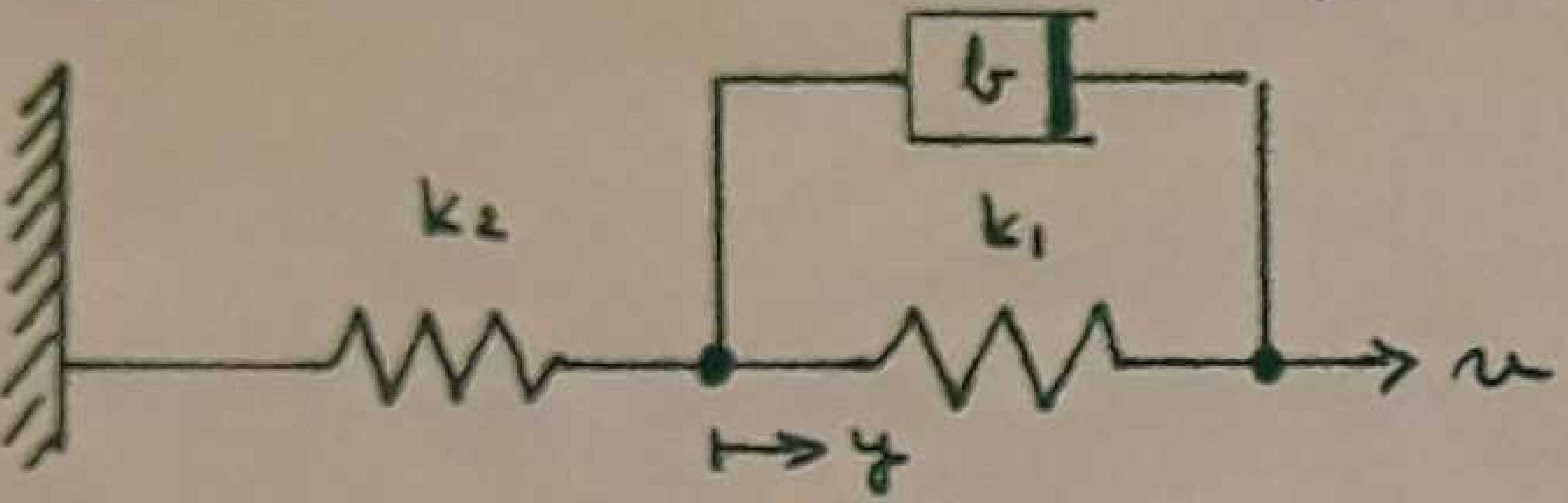
$$\boxed{\dot{x}_1 = -\frac{k}{m}x_2 + \frac{k}{m}u - g}$$

gerjesztés konstans!

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{\underline{x}}} = \underbrace{\begin{bmatrix} 0 & -\frac{k}{m} \\ 1 & -\frac{b}{m} \end{bmatrix}}_{\underline{\underline{A}}} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} \frac{k}{m} & -g \\ \frac{b}{m} & 0 \end{bmatrix}}_{\underline{\underline{B}}} \underbrace{\begin{bmatrix} u \\ 1 \end{bmatrix}}_{\underline{u}'}$$

$$y = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{\underline{c}^T} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{\underline{D}^T} \cdot \underbrace{\begin{bmatrix} u' \\ 1 \end{bmatrix}}_{\underline{u}'}$$

Írjuk fel a rendszer differenciálegyenletét!



$$\underbrace{\frac{d}{dt}\frac{\partial K}{\partial \dot{y}} - \frac{\partial K}{\partial y}}_{0} + \frac{\partial P}{\partial y} + \frac{\partial R}{\partial \dot{y}} = \tau$$

$$P = \frac{1}{2} k_1 (u-y)^2 + \frac{1}{2} k_2 y^2 \qquad \frac{\partial P}{\partial y} = -k_1(u-y) + k_2 y$$

$$R = \frac{1}{2} b (\dot{u}-\dot{y})^2 \qquad \frac{\partial R}{\partial \dot{y}} = -b(\dot{u}-\dot{y})$$

$$-k_1(u-y) + k_2 y - b(\dot{u}-\dot{y}) = 0$$

$$-k_1 u + \underbrace{k_1 y + k_2 y} - b\dot{u} \underbrace{+ b\dot{y}} = 0$$

$$\underbrace{\dot{y} = -\frac{k_1+k_2}{b} y + \boxed{\dot{u}} + \frac{k_1}{b} u} \quad / \int$$

$$y = \underbrace{-\frac{k_1+k_2}{b}\int y\,dt} + u + \underbrace{\frac{k_1}{b}\int u\,dt} \longrightarrow x = -\frac{k_1+k_2}{b}\int y\,dt + \frac{k_1}{b}\int u\,dt$$

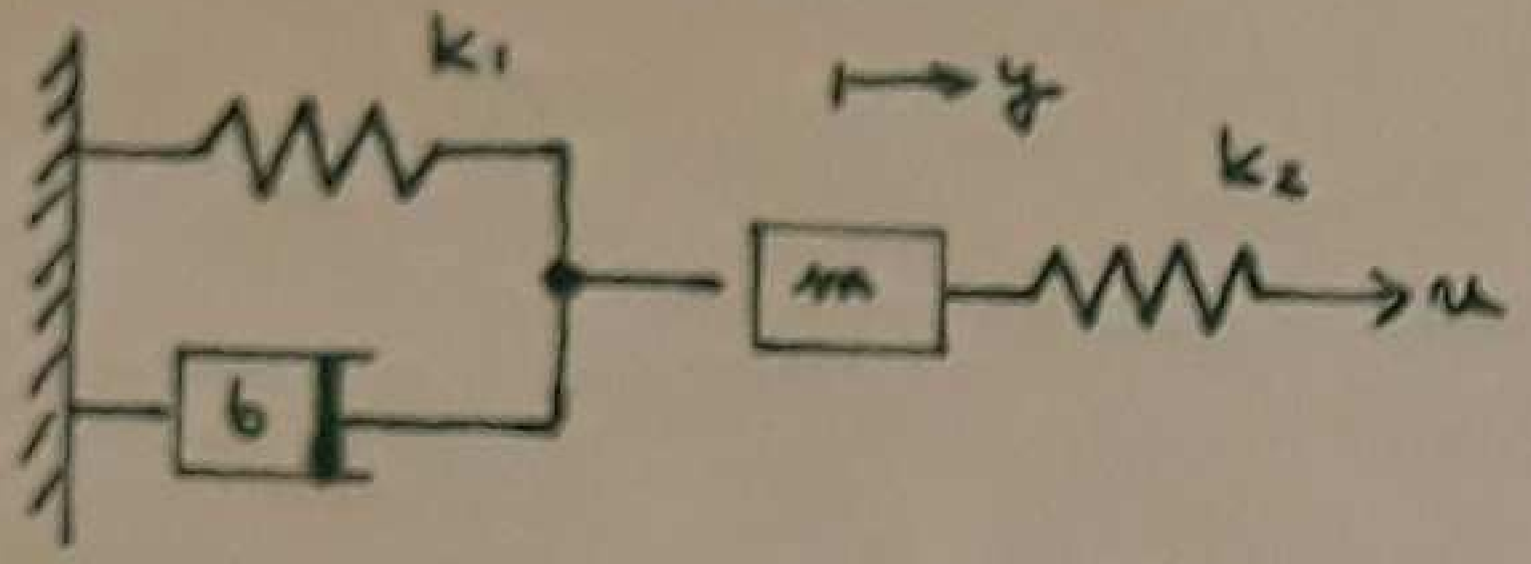
$$\dot{x} = -\frac{k_1+k_2}{b} y + \frac{k_1}{b} u$$

$$\boxed{y = x + u}$$

$$\dot{x} = -\frac{k_1+k_2}{b} x - \frac{\cancel{k_1}+k_2}{b} u + \frac{\cancel{k_1}}{b} u$$

$$\boxed{\begin{aligned} \dot{x} &= \underbrace{-\frac{k_1+k_2}{b}}_{A} x \underbrace{- \frac{k_2}{b}}_{B} u \\ y &= \underbrace{\qquad}_{C=1} x + \underbrace{\qquad}_{D=1} u \end{aligned}}$$

Írjuk fel az alábbi mechanikai rendszer modelljét!



$$\frac{d}{dt}\frac{\partial K}{\partial \dot{y}} - \cancel{\frac{\partial K}{\partial y}} + \frac{\partial P}{\partial y} + \frac{\partial R}{\partial \dot{y}} = \tau$$

$$K = \frac{1}{2} m \dot{y}^2$$

$$P = \frac{1}{2} k_1 y^2 + \frac{1}{2} k_2 (u-y)^2$$

$$R = \frac{1}{2} b \dot{y}^2$$

$$\frac{\partial K}{\partial \dot{y}} = m\dot{y} \qquad \frac{d}{dt}\frac{\partial K}{\partial \dot{y}} = \underline{m\ddot{y}}$$

$$\frac{\partial P}{\partial y} = \underline{k_1 y - k_2}(u-y)$$

$$\frac{\partial R}{\partial \dot{y}} = \underline{b\dot{y}}$$

$$m\ddot{y} + \underline{k_1 y} - k_2(u-\underline{y}) + b\dot{y} = 0$$

$$m\ddot{y} + b\dot{y} + (k_1+k_2)y - k_2 u = 0$$

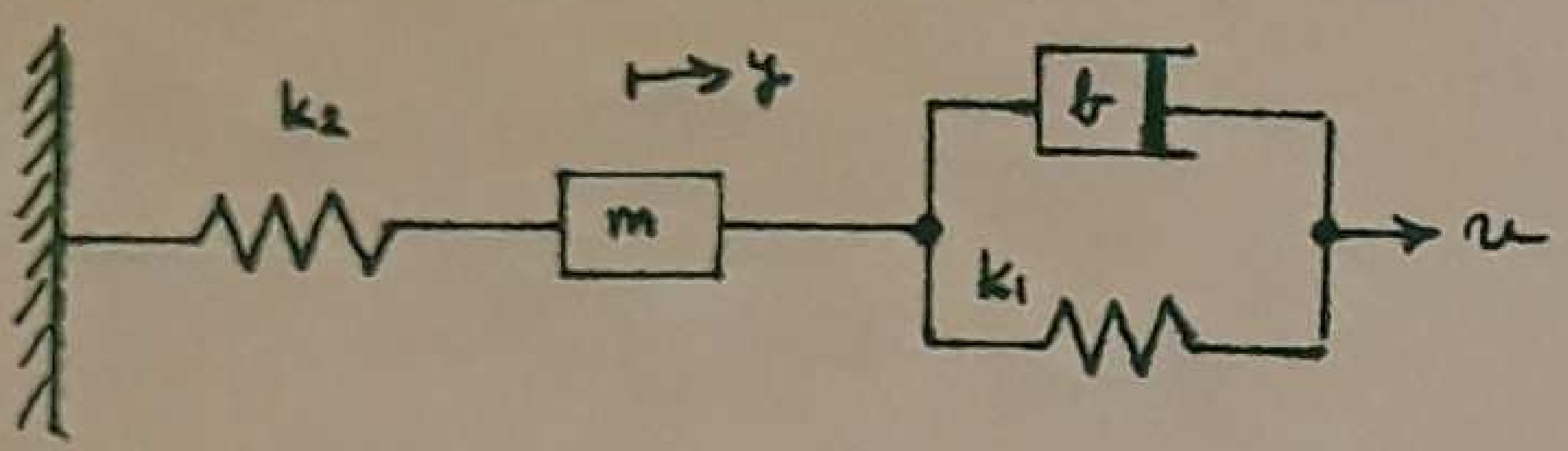
$$\ddot{y} = -\frac{b}{m}\dot{y} - \frac{k_1+k_2}{m}y + \frac{k_2}{m}u$$

$$\left|\begin{aligned} \dot{y} &= x_2 \\ y &= x_1 \end{aligned}\right| \rightarrow \begin{cases} \dot{x}_2 = -\frac{b}{m}x_2 - \frac{k_1+k_2}{m}x_1 + \frac{k_2}{m}u \\ \dot{x}_1 = x_2 \\ y = x_1 \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_1+k_2}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_2}{m} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0\,u.$$

Írjuk fel az alábbi mechanikai rendszer differenciálegyenletét !



$$\underbrace{\frac{d}{dt}\frac{\partial K}{\partial \dot{y}}} - \cancel{\frac{\partial K}{\partial y}} + \frac{\partial P}{\partial y} + \frac{\partial R}{\partial \dot{y}} = \tau .$$

$$K = \frac{1}{2} m \dot{y}^2$$

$$P = \frac{1}{2} k_1 (u-y)^2 + \frac{1}{2} k_2 y^2$$

$$R = \frac{1}{2} b (\dot{u} - \dot{y})^2$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{y}} = m\ddot{y}$$

$$\frac{\partial P}{\partial y} = -k_1 (u-y) + k_2 y$$

$$\frac{\partial R}{\partial \dot{y}} = -b(\dot{u} - \dot{y})$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{k_1+k_2}{m} \\ 1 & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{k_1}{m} \\ \frac{b}{m} \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0\, u .$$

$$m\ddot{y} - k_1(u-y) + k_2 y - b(\dot{u} - \dot{y}) = 0$$

$$\underline{m\ddot{y}} - k_1 u + \underline{k_1 y + k_2 y} - b\dot{u} + \underline{b\dot{y}} = 0$$

$$m\ddot{y} + b\dot{y} + (k_1 + k_2) y = b\dot{u} + k_1 u$$

$$\ddot{y} = -\frac{b}{m}\dot{y} - \frac{k_1+k_2}{m} y + \frac{b}{m}\boxed{\dot{u}} + \frac{k_1}{m} u \quad /\int$$

$$\dot{y} = -\frac{b}{m} y \underbrace{- \frac{k_1+k_2}{m}\int y\,dt} + \frac{b}{m} u + \underbrace{\frac{k_1}{m}\int u\,dt}$$

$$\dot{y} = -\frac{b}{m} y + \frac{b}{m} u + x_1$$

$$\boxed{y = x_2}$$

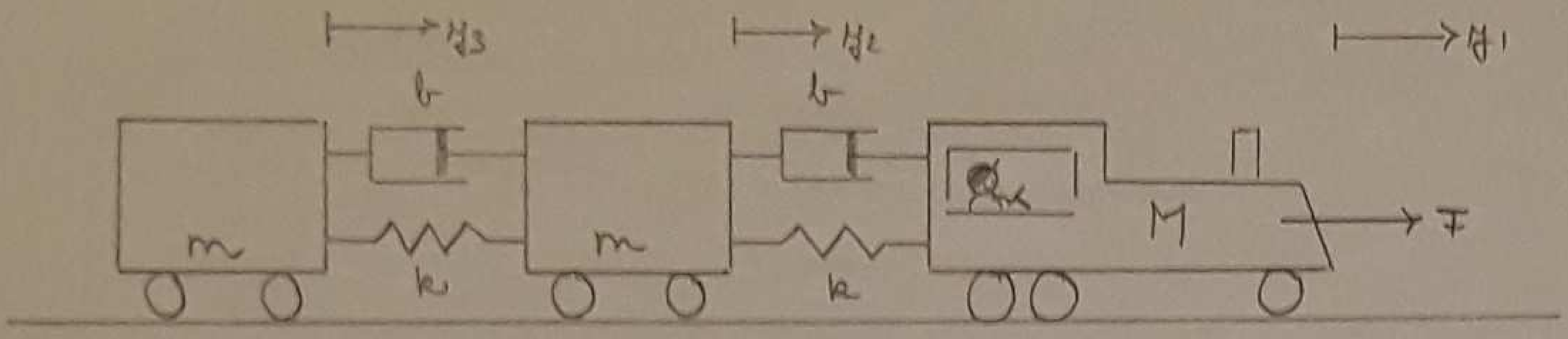
$$\boxed{\dot{x}_2 = -\frac{b}{m} x_2 + \frac{b}{m} u + x_1}$$

$$\rightarrow x_1 = -\frac{k_1+k_2}{m}\int y\,dt + \frac{k_1}{m}\int u\,dt \quad /\frac{d}{dt}$$

$$\dot{x}_1 = -\frac{k_1+k_2}{m} \underset{\substack{\uparrow \\ x_2}}{y} + \frac{k_1}{m} u$$

$$\boxed{\dot{x}_1 = -\frac{k_1+k_2}{m} x_2 + \frac{k_1}{m} u}$$

Írjuk fel a vonati szerelvény mechanizmus modelljét!



$$K = \frac{1}{2} M \dot{y}_1^2 + \frac{1}{2} m \dot{y}_2^2 + \frac{1}{2} m \dot{y}_3^2$$

$$P = \frac{1}{2} k (y_1 - y_2)^2 + \frac{1}{2} k (y_2 - y_3)^2$$

$$R = \frac{1}{2} b (\dot{y}_1 - \dot{y}_2)^2 + \frac{1}{2} b (\dot{y}_2 - \dot{y}_3)^2$$

$$\frac{d}{dt} \frac{\partial K}{\partial \dot{y}_i} - \frac{\partial K}{\partial y_i} + \frac{\partial P}{\partial y_i} + \frac{\partial R}{\partial \dot{y}_i} = \tau_i$$

$$\begin{cases} M\ddot{y}_1 + k(y_1 - y_2) + b(\dot{y}_1 - \dot{y}_2) = F \\ m\ddot{y}_2 - k(y_1 - y_2) + k(y_2 - y_3) - b(\dot{y}_1 - \dot{y}_2) + b(\dot{y}_2 - \dot{y}_3) = 0 \\ m\ddot{y}_3 - k(y_2 - y_3) - b(\dot{y}_2 - \dot{y}_3) = 0 \end{cases}$$

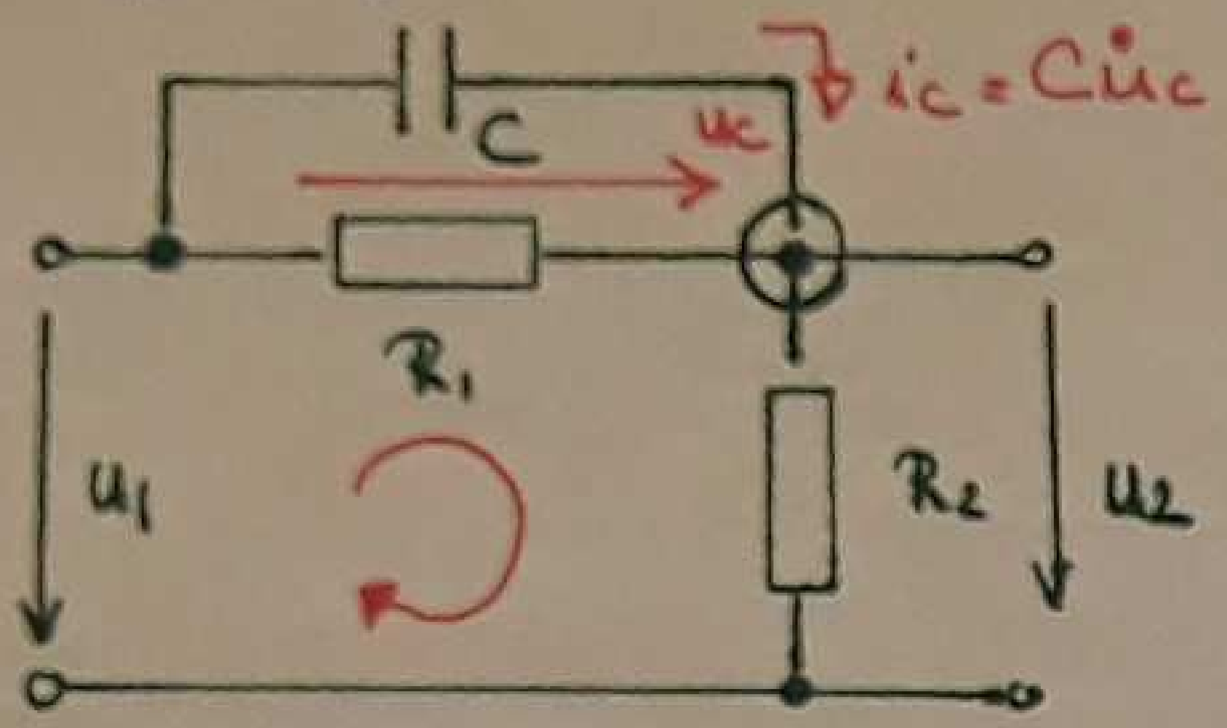
Írjuk fel az alábbi villamos hálózat differenciálegyenletét!

ANALÓG SZÁMÍTÓGÉP

Kirchhoff.

hurok

csomóponti



↻ $u_C + u_2 - u_1 = 0 \longrightarrow \boxed{\underset{(y)}{u_2} = -\underset{(x)}{u_C} + \underset{(u)}{u_1}}$

⊕ $-i_C - i_{R1} + i_{R2} = 0$

$$-C\dot{u}_C - \frac{u_C}{R_1} + \frac{u_2}{R_2} = 0$$

$$-C\dot{u}_C - \frac{u_C}{R_1} + \frac{-u_C + u_1}{R_2} = 0$$

$$-C\dot{u}_C \underbrace{- \frac{u_C}{R_1} - \frac{u_C}{R_2}}_{-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)u_C} + \frac{u_1}{R_2} = 0$$

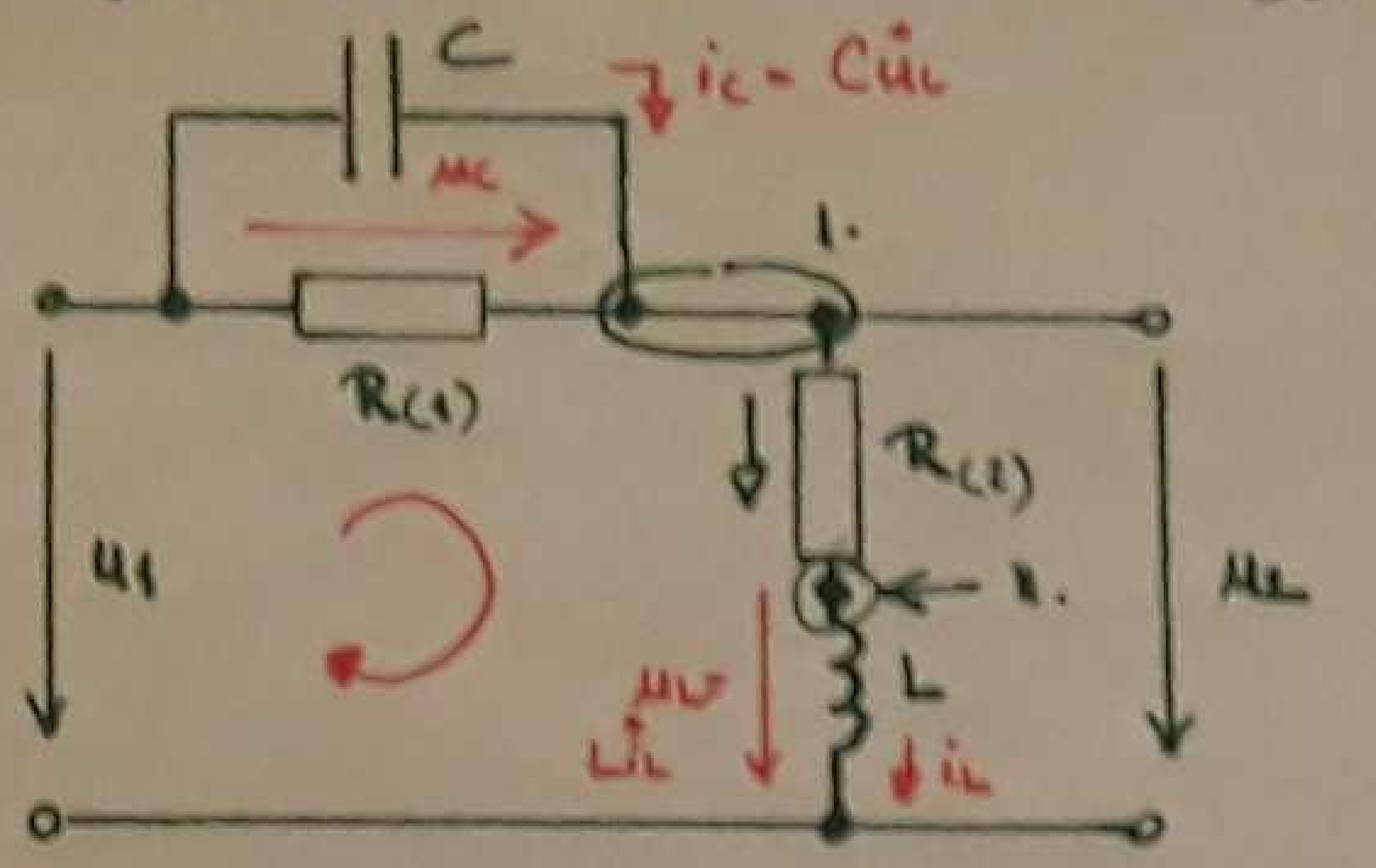
$$-\frac{R_1 + R_2}{R_1 R_2}$$

$$-C\dot{u}_C - \frac{R_1 + R_2}{R_1 R_2}u_C + \frac{u_1}{R_2} = 0$$

$$\boxed{\underset{\dot{x}}{\dot{u}_C} = -\frac{R_1 + R_2}{CR_1R_2}\underset{x}{u_C} + \frac{1}{R_2C}\underset{u}{u_1}}$$

$$\boxed{\begin{aligned} \underset{\dot{u}_C}{\dot{x}} &= -\frac{R_1 + R_2}{CR_1R_2}\underset{u_C}{x} + \frac{1}{R_2C}\underset{u_1}{u} \\ \underset{u_2}{y} &= -\underset{u_C}{x} + \underset{u_1}{u} \end{aligned}}$$

Írjuk fel az alábbi négypólus differenciálegyenletét !

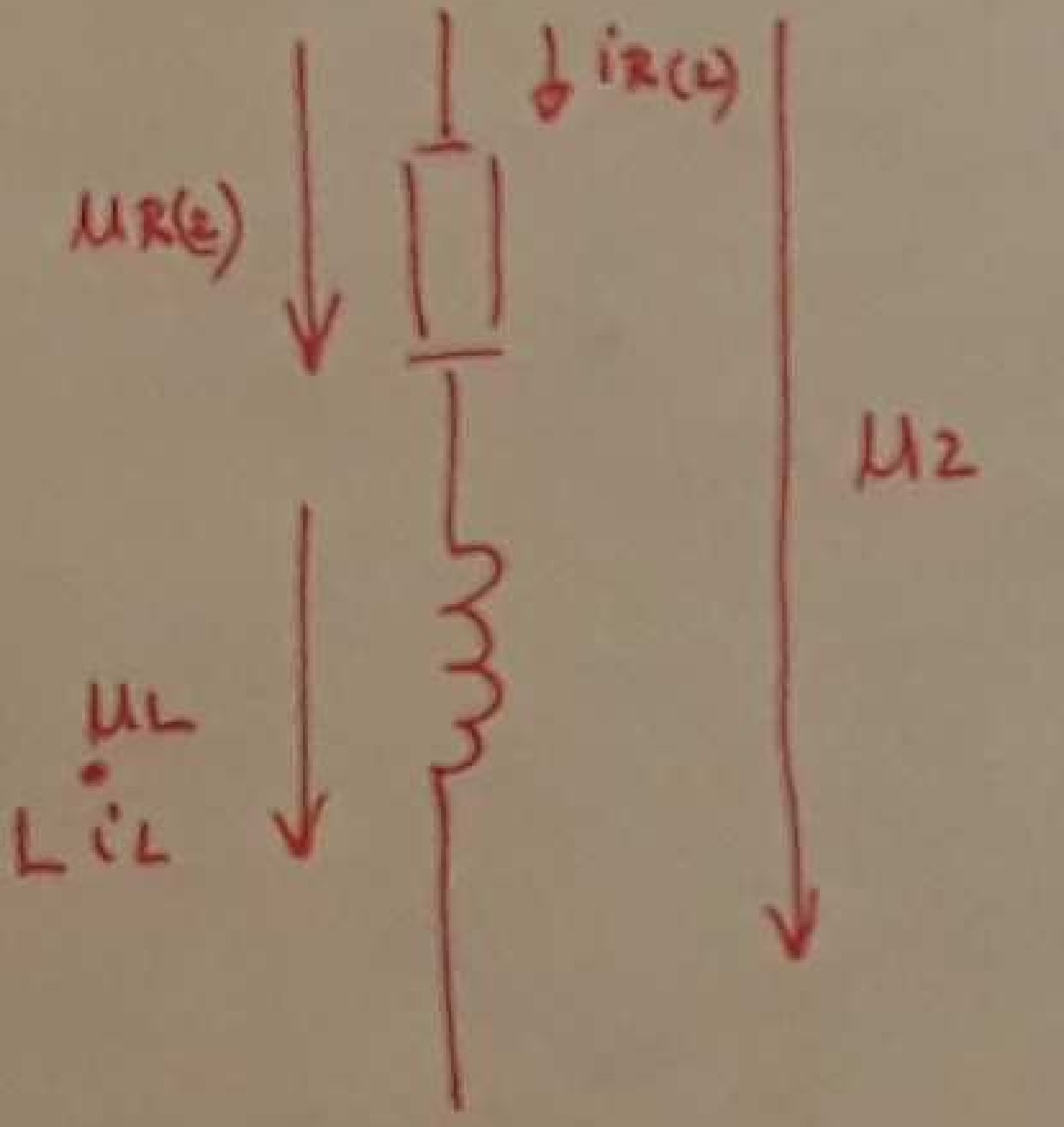


$$u_c + u_2 - u_1 = 0 \longrightarrow \boxed{\overset{(y)}{u_2} = -\overset{-x_1}{u_c} + \overset{+u}{u_1}}$$

① $-i_c - i_{R_1} + i_{R_2} = 0$

$$-C\dot{u}_c - \frac{u_c}{R} + \overset{!}{i_L} = 0 \longrightarrow \boxed{\underset{\dot{x}_1}{\dot{u}_c} = -\frac{1}{RC}\underset{x_1}{u_c} + \frac{1}{C}\underset{x_2}{i_L}}$$

II. $-\underline{i_{R_2}} + i_L = 0$



$$i_{R(2)} = \frac{u_{R(2)}}{R} = \frac{u_2 - L\dot{i}_L}{R}$$

$$u_2 = u_{R(2)} + u_L$$

$$u_{R(2)} = u_2 - u_L$$

$$= u_2 - L\dot{i}_L$$

$$-\frac{u_2 - L\dot{i}_L}{R} + i_L = 0$$

$$-\frac{-u_c + u_1 - L\dot{i}_L}{R} + i_L = 0 \quad /R$$

$$\underline{u_c} - u_1 + L\dot{i}_L + \underline{R}i_L = 0$$

$$\boxed{\underset{\dot{x}_2}{\dot{i}_L} = -\frac{1}{L}\underset{x_1}{u_c} - \frac{R}{L}\underset{x_2}{i_L} + \frac{1}{L}\underset{u}{u_1}}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u \qquad y = \begin{bmatrix} -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 1u$$

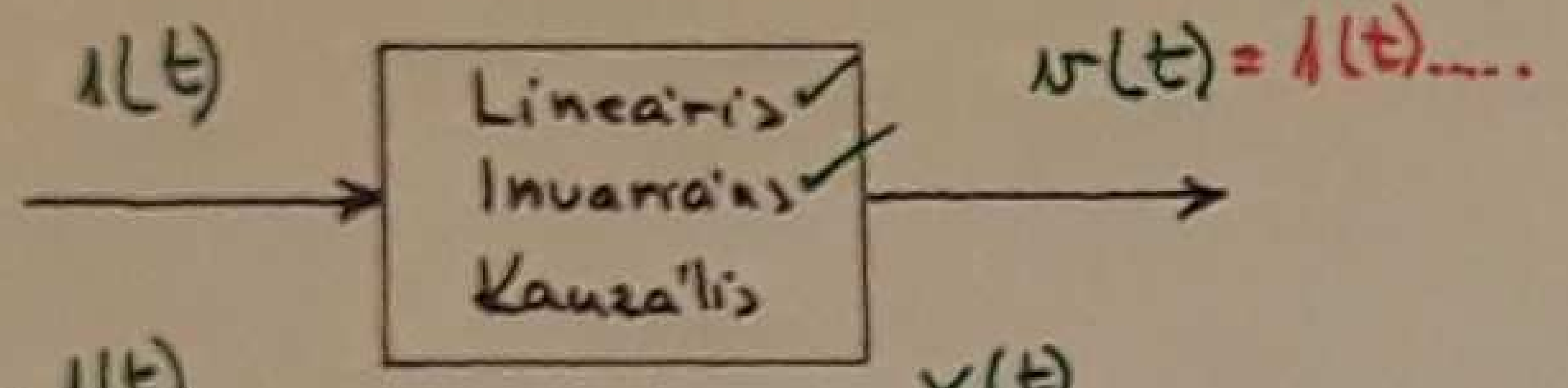
FOLYTONOS IDEJŰ RENDSZEREK ANALÍZISE AZ IDŐTARTOMÁNYBAN

Ⓘ

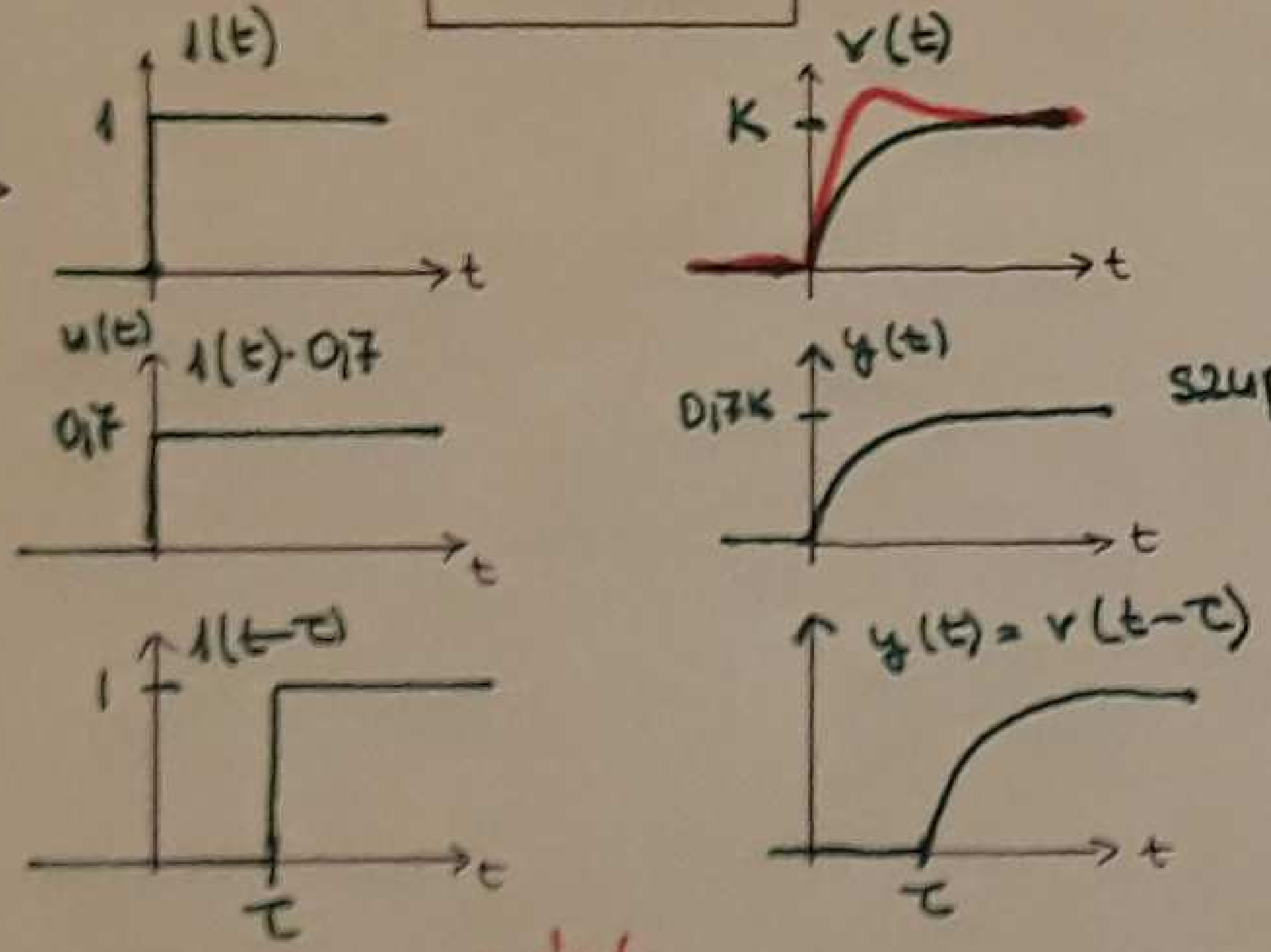
UGRÁSVÁLASZ

$\underbrace{\text{UGRÁS}}_{1(t)}$

DEF. →



JELLEG →



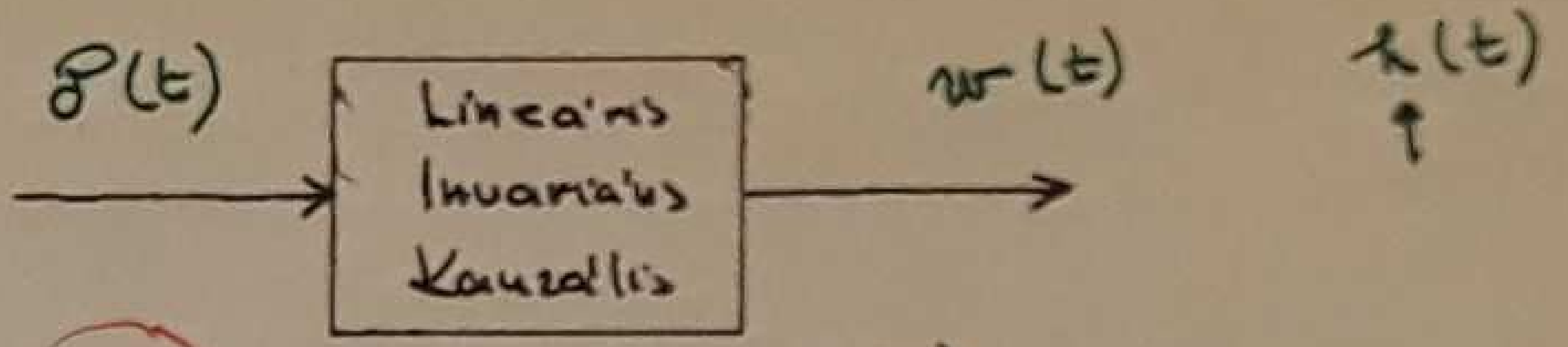
szuperpozíció

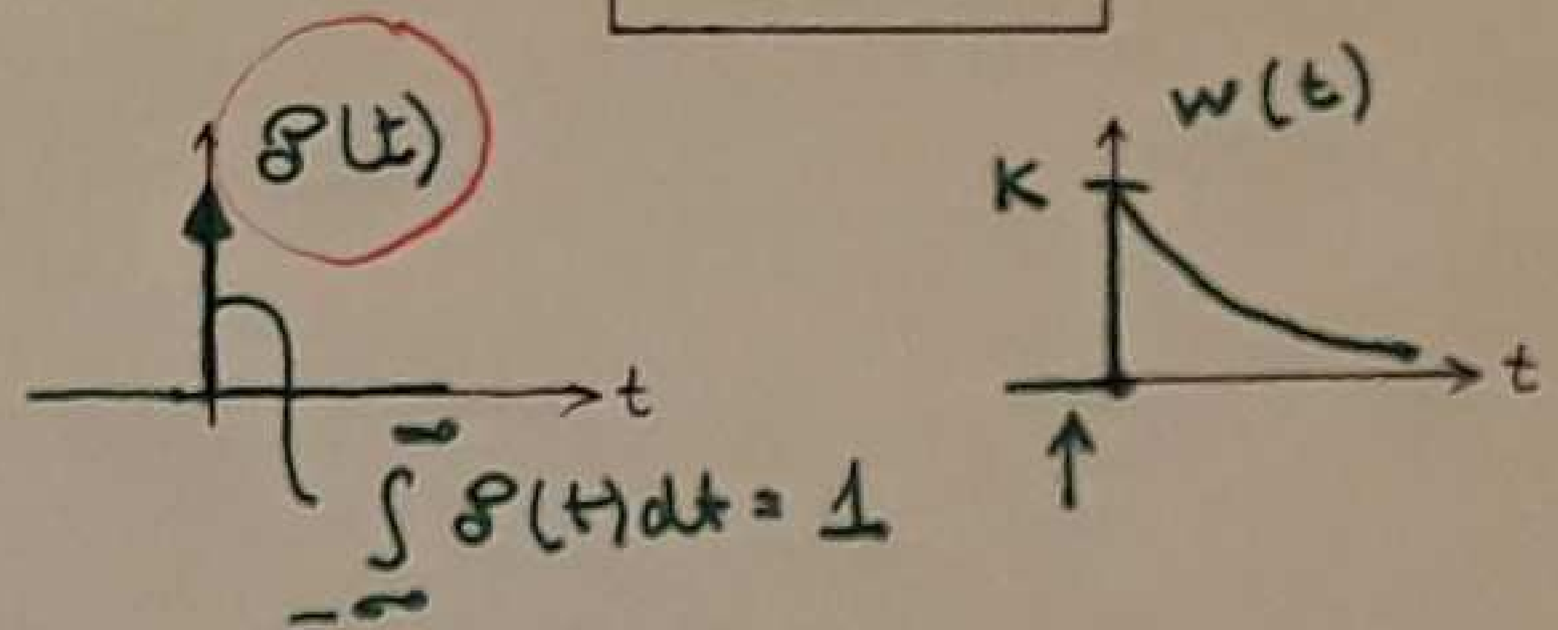
mérés

Példa: $v(t) = \underbrace{1(t)}\underbrace{[1-e^{-2t}]\cdot 0{,}5}$

Identifikáció

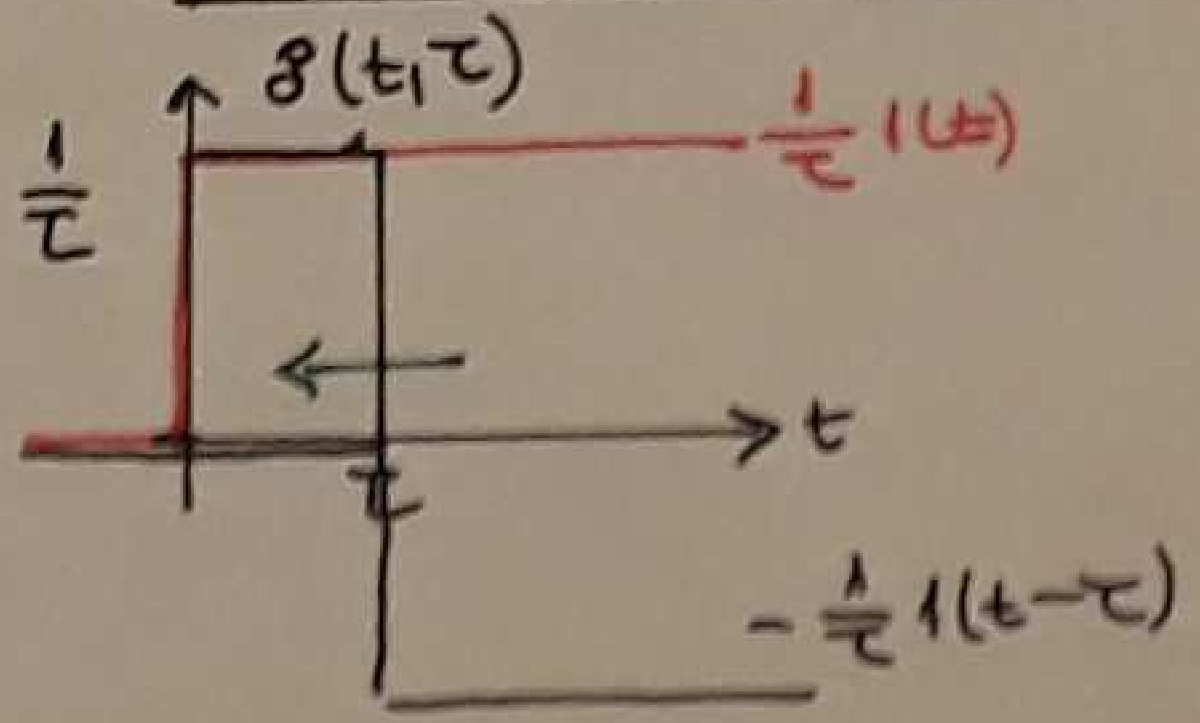
IMPULZUSVÁLASZ → konvolúció





$$\int_{-\infty}^{\infty}\delta(t)\,dt = 1$$

v(t) és w(t) kapcsolata.



$$\delta(t,\tau) = \frac{1(t)-1(t-\tau)}{\tau} = \delta(t)$$

$$\frac{1}{\tau}1(t) - \frac{1}{\tau}1(t-\tau)$$

$$\frac{1}{\tau}v(t) - \frac{1}{\tau}v(t-\tau)$$

$$\lim_{\tau\to 0}\frac{v(t)-v(t-\tau)}{\tau} = w(t)$$

$$\boxed{\begin{aligned}\delta(t) &= 1'(t)\\ w(t) &= v'(t)\end{aligned}}$$

$w(t) = \underbrace{1(t)}\underbrace{e^{-2t}}$ $(+D\delta(t))$

$v(t) = 1(t)\ldots\ldots$

$\delta(t)$ → Lineáris Invariáns → $w(t)$

I. [UGRÁSVÁLASZ
IMPULZUSVÁLASZ - KONVOLÚCIÓ

II. [RENDSZEREGYENLET

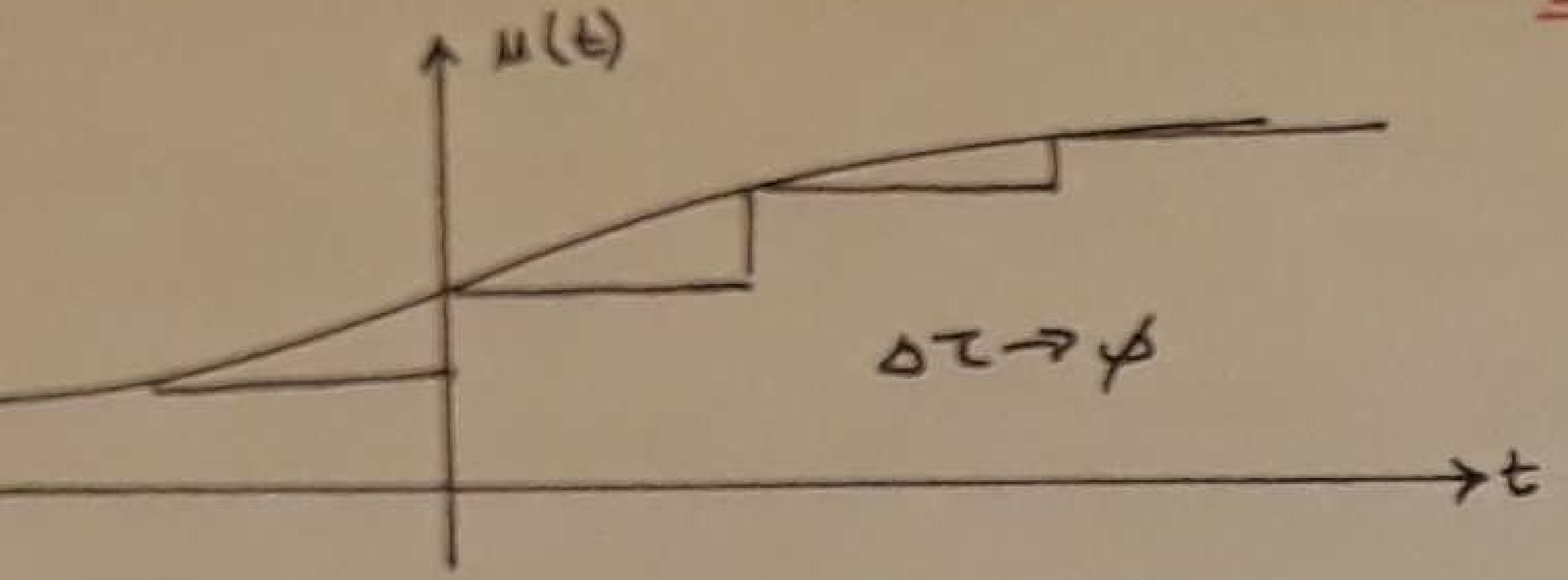
III. [ÁLLAPOTVÁLTOZÓS LEÍRÁS

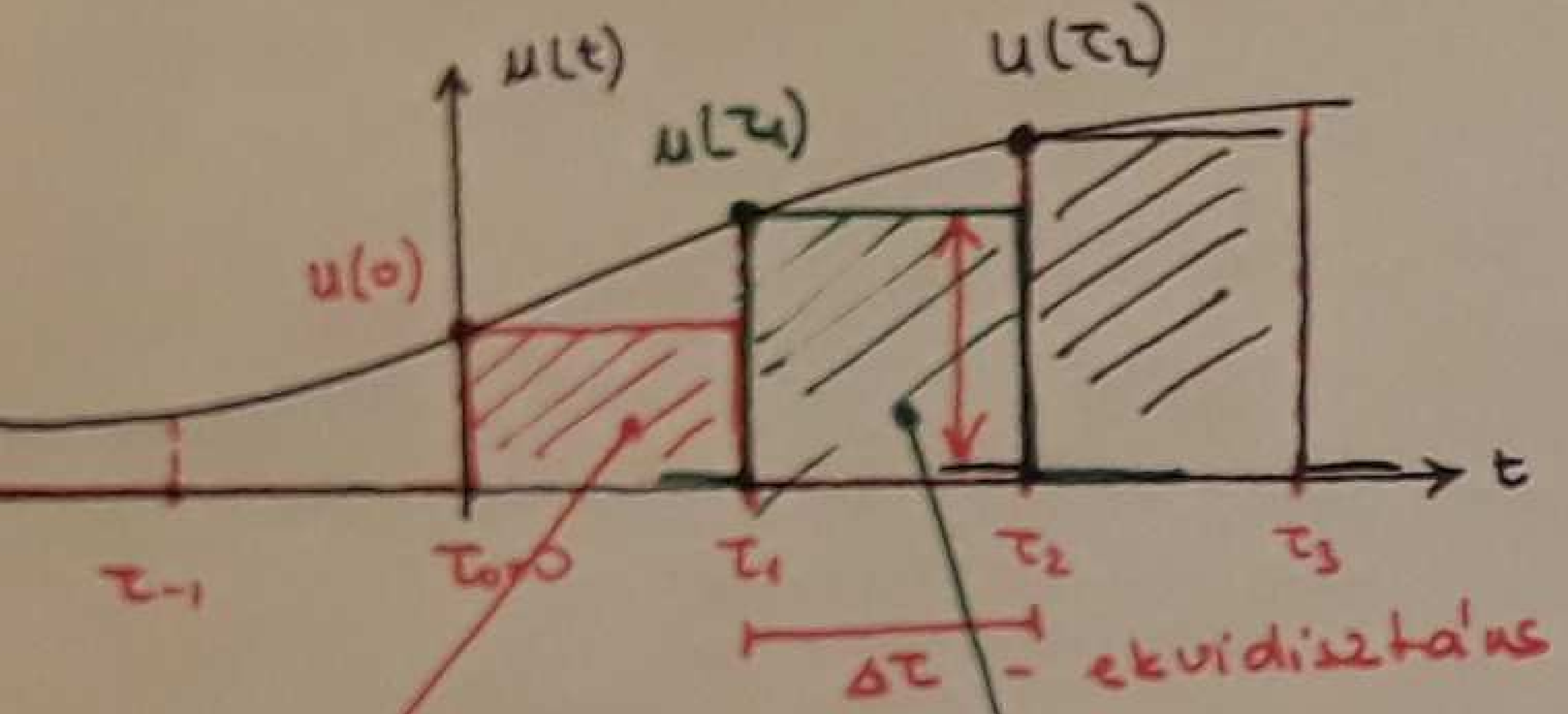
STABILITÁS

Rendszerjellemző függvények

$$w(t) = \underbrace{1(t)}\,\underbrace{e^{-2t}} \left(+ D\,\delta(t)\right)$$

KONVOLÚCIÓ





$u(0)[1(t) - 1(t-\tau_1)]$

$\underline{u(0)}[1(t) - 1(t-\Delta\tau)]$

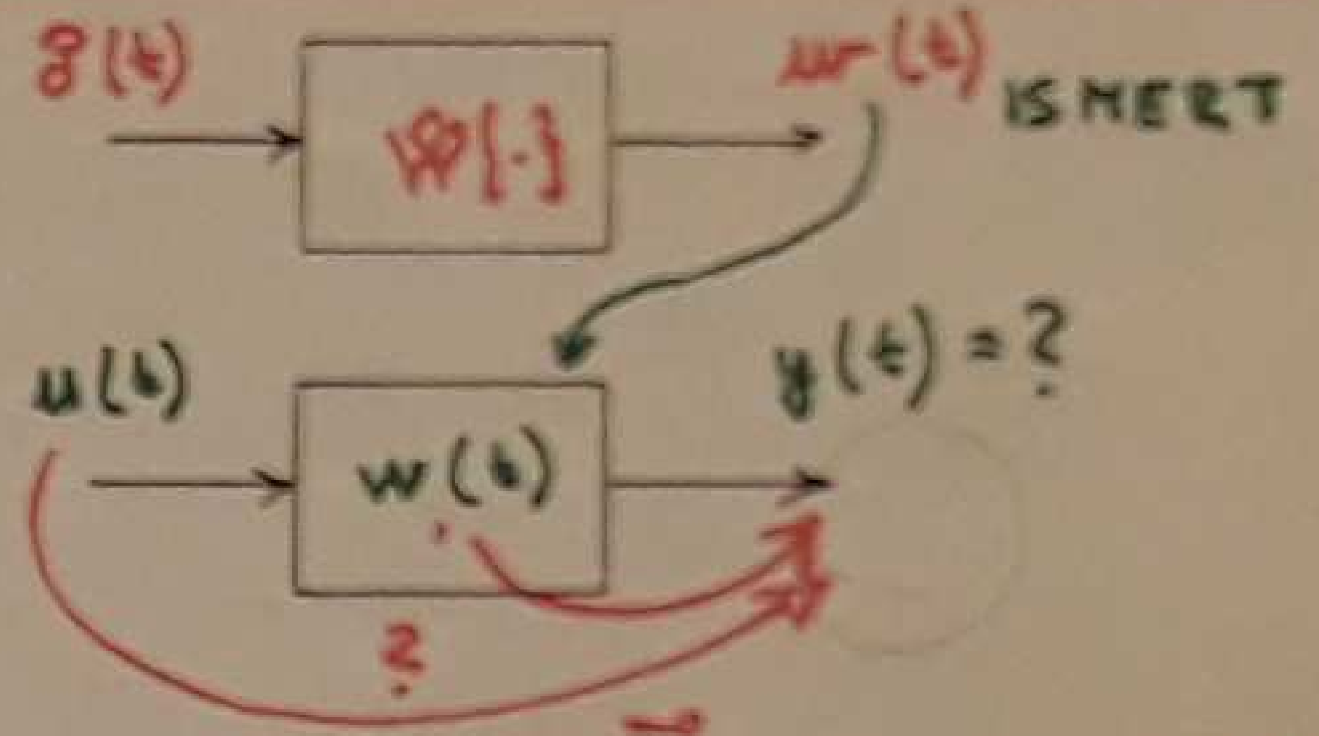
$u(\tau_1)[1(t-\tau_1) - 1(t-\tau_2)]$

$\underline{u(\tau_1)}[1(t-\underline{\tau_1}) - 1(t-[\underline{\tau_1}-\Delta\tau])$

$$u(t) \approx \sum_{i=-\infty}^{\infty} u(\tau_i)\left[1(t-\underset{\uparrow}{\tau_i}) - 1(t-[\underset{\uparrow}{\tau_i}-\Delta\tau])\right] \underbrace{\frac{\Delta\tau}{\Delta\tau}}_{1}$$

$$= \sum_{i=-\infty}^{\infty} u(\tau_i) \underbrace{\frac{1(t-\underline{\tau_i}) - 1(t-[\underline{\tau_i}-\Delta\tau])}{\Delta\tau \to 0}}_{\substack{\Delta\tau\to 0 \\ \delta(t-\tau)}} \underset{\to 0}{\Delta\tau}$$

A válasz számítása az impulzusválasz ismeretében.



$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau.$$

~~$u(\tau)$~~

$\delta(t) \longrightarrow w(t) \longleftarrow$

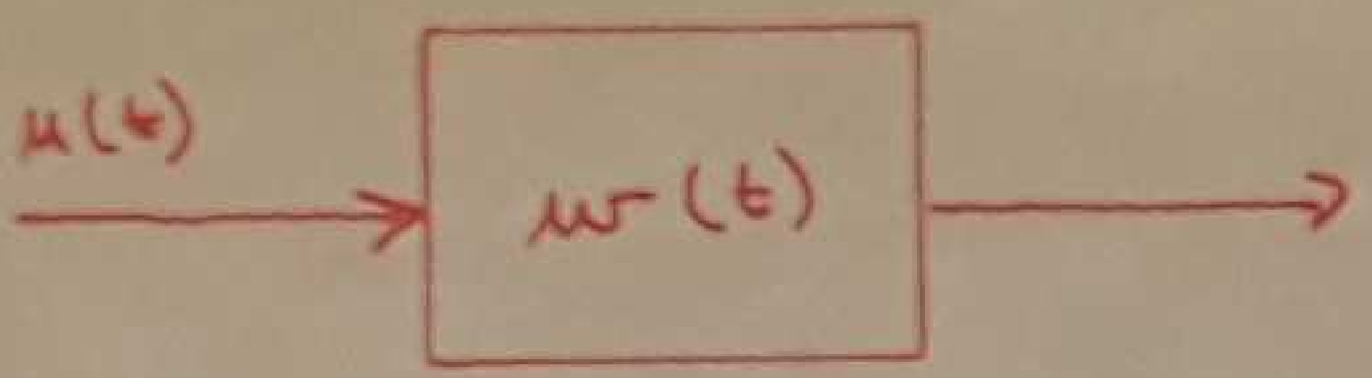
$\delta(t-\tau) \longrightarrow w(t-\tau) \longleftarrow$

$$\int_{-\infty}^{\infty} u(\tau)\,\delta(t-\tau)\,\underset{d\tau}{\curlywedge} \rightarrow \int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau$$

$$\Rightarrow \boxed{\underline{u(t)} = \int_{-\infty}^{\infty} \underline{u(\tau)\,\underline{\delta(t-\tau)}}\, d\tau}$$

$$\boxed{y(t) = \int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau}$$

súlyfüggvénytétel



$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau .$$

$$y(t) = u(t) \underset{\llcorner\lrcorner}{*} w(t) = w(t) * u(t)$$

Kommutatív.

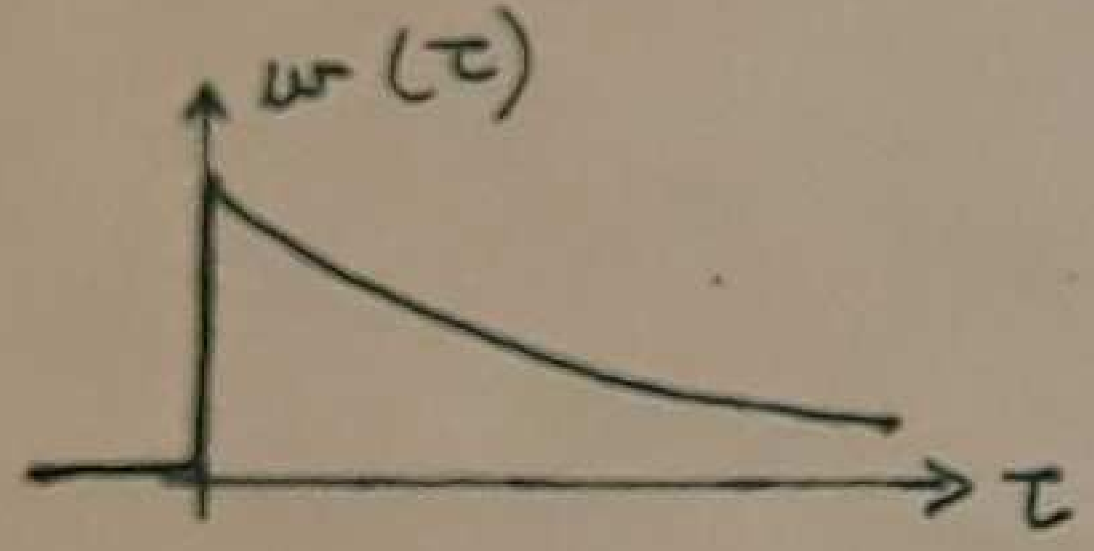
1.) $u(t)$ belépő : $u(t) = \underline{1(t)} f(t) \longrightarrow y(t) = \int_{\textcircled{0}}^{\infty} \underline{u(\tau)}\, \underbrace{w(t-\tau)}\, d\tau .$

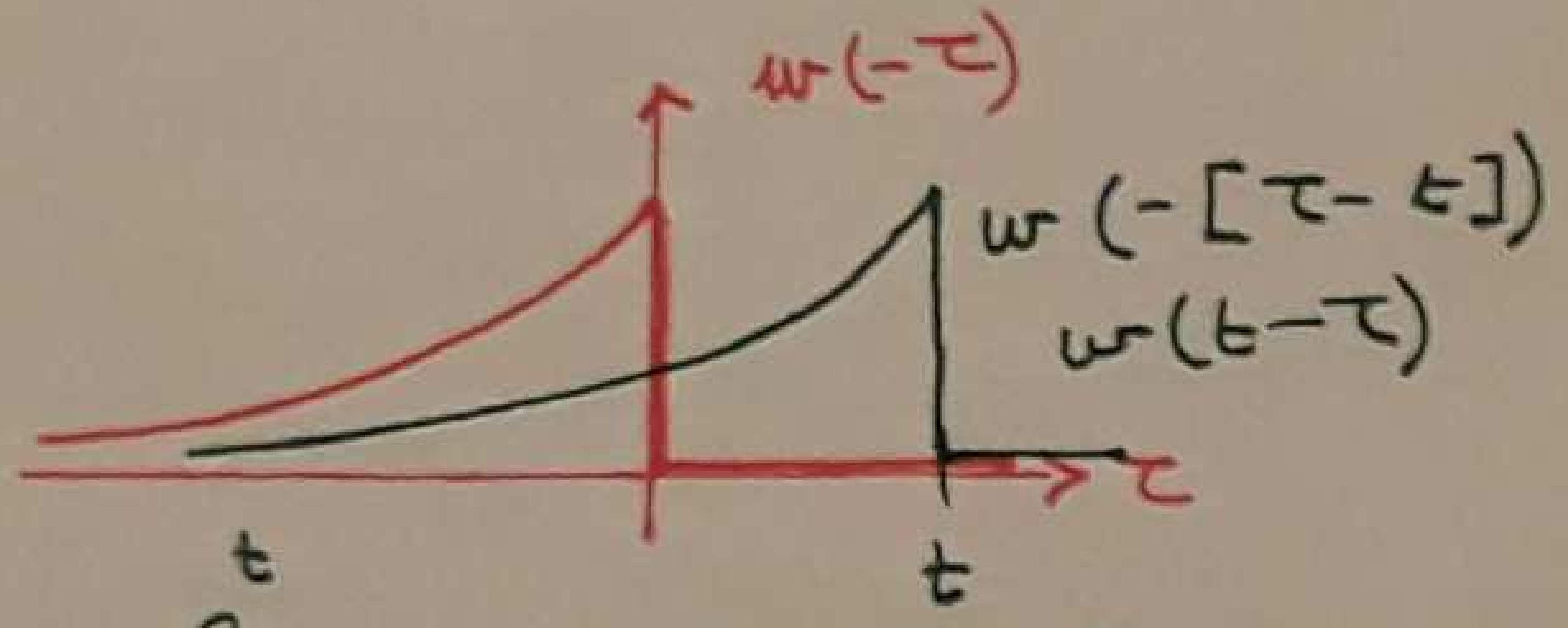
2.) $w(t)$ belépő : $w(t) = \underline{1(t)}\, g(t)$

$w(t-\tau)$

$w(-\tau+t)$

$w(-[\tau - \overrightarrow{t}])$





$$y(t) = \int_{-\infty}^{\textcircled{t}} u(\tau)\, \underline{w(t-\tau)}\, d\tau$$

3.) $u(t), w(t)$ belépő : $y(t) = \int_{0}^{t} u(\tau)\, w(t-\tau)\, d\tau$

$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, \underbrace{w(t-\tau)}_{\alpha}\, d\tau = \int_{-\infty}^{\infty} w(\tau)\, u(t-\tau)\, d\tau$$

$\alpha = t-\tau$

$\tau = t-\alpha$

$d\tau = -d\alpha$

$$\int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau = -\int_{\infty}^{-\infty} u(t-\alpha)\, w(\alpha)\, d\alpha = \int_{-\infty}^{\infty} w(\alpha)\, u(t-\alpha)\, d\alpha$$

Asszociatív

$a(t) * [b(t) * c(t)] =$

$[a(t) * b(t)] * c(t).$

Disztributív:

$[a+b] * c =$

$a*c + b*c$

SPEC.: $u(t) = 1(t) \Rightarrow y(t) = \boxed{v(t)} = \int_{-\infty}^{\textcircled{\infty}} w(\tau)\, 1(t-\tau)\, d\tau = \boxed{\int_{-\infty}^{t} w(\tau)\, d\tau} \Leftrightarrow \boxed{w(t) = v'(t)}$

A DUHAMEL - TÉTEL

$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau \qquad \Longleftarrow$$

$u(t)$ = belépő

$$y(t) = u(t)\, v(0) + \int_{0}^{t} u(\tau)\, \frac{d\, v(t-\tau)}{dt}\, d\tau \qquad \Longleftarrow$$

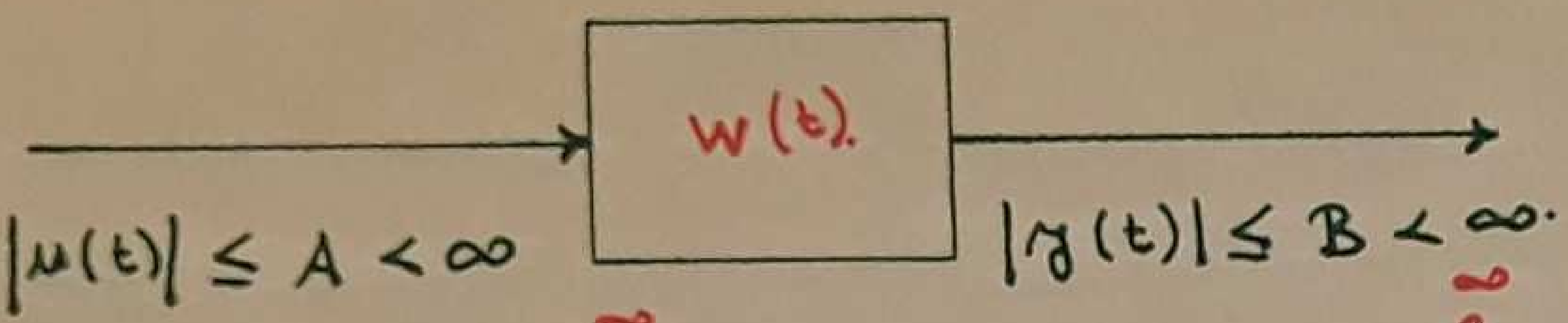
$v(-\infty)$

$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, \frac{d}{dt}\, v(t-\tau)\, d\tau$$

Gerjesztés - válasz stabilitás.

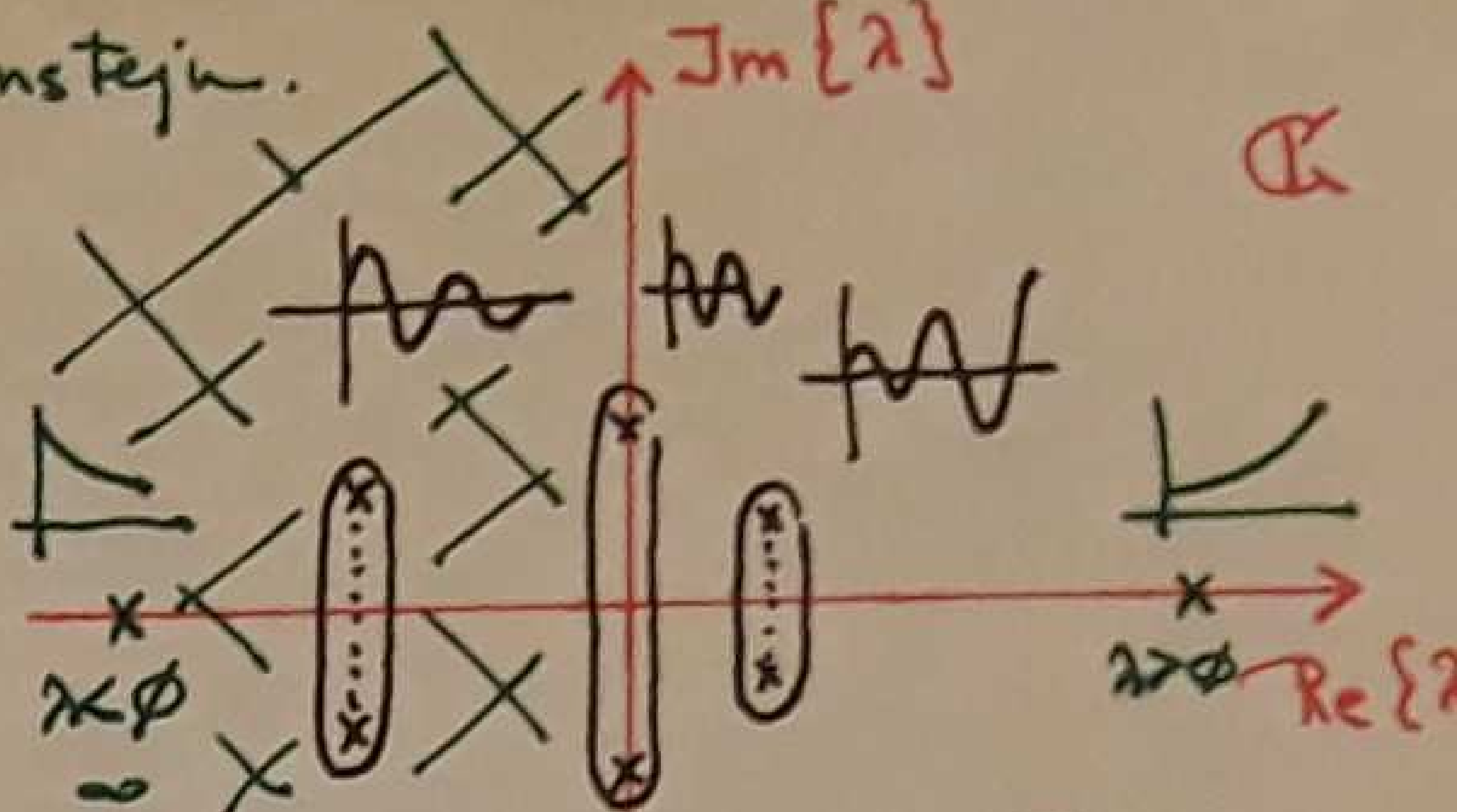
w(t) - G.V.

Bronstejn.



$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau = \int_{-\infty}^{\infty} w(\underset{\uparrow}{\tau})\, u(t-\tau)\, d\tau$$

$$|y(t)| = \left| \int_{-\infty}^{\infty} w(\tau)\, u(t-\tau)\, d\tau \right| \le \int_{-\infty}^{\infty} |w(\tau)\, u(t-\tau)|\, d\tau \le \int_{-\infty}^{\infty} |w(\tau)| \cdot |u(t-\tau)|\, d\tau$$



$$\boxed{\mathrm{Re}\{\lambda\} < 0}$$

$$|y(t)| \le \int_{-\infty}^{\infty} |w(\tau)| \cdot \underbrace{|u(t-\tau)|}_{|u(t-\tau)| \le A}\, d\tau$$

$$|y(t)| \le A \underbrace{\int_{-\infty}^{\infty} |w(\tau)|\, d\tau}_{} < \infty \Rightarrow \boxed{\int_{-\infty}^{\infty} |w(t)|\, dt < \infty} \quad \boxed{\int_{0}^{\infty} |w(t)|\, dt < \infty}$$

λ>0

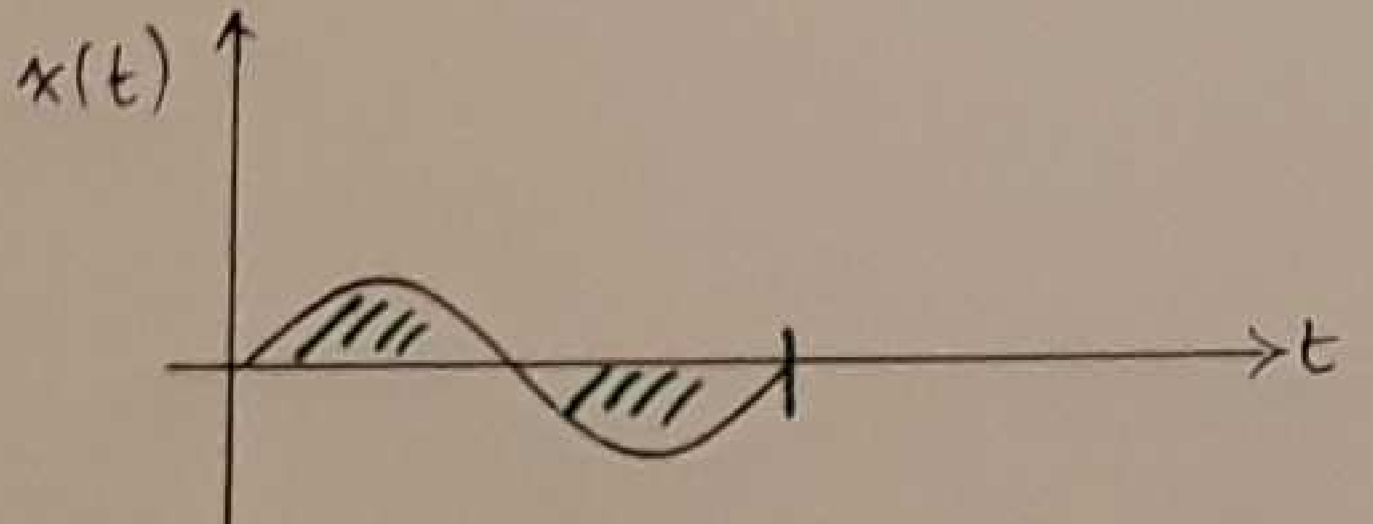
$e^{\lambda t}$ $\quad te^{\lambda t}$

$t^2 e^{\lambda t}$

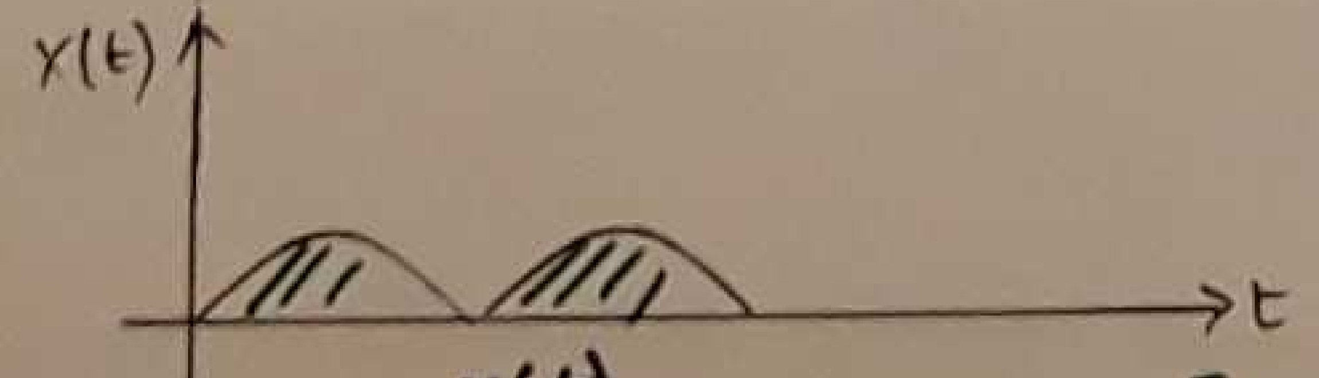
$\vdots$

$t^n e^{\lambda t}$.

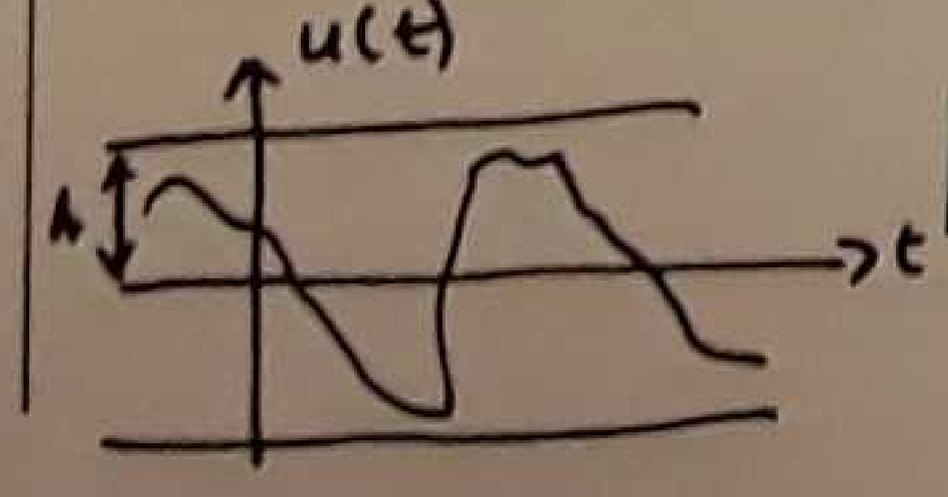
$$\boxed{\lim_{t \to \infty} w(t) = 0}$$



$$\int_0^T x(t)\, dt = 0$$



$$\int_0^T |x(t)|\, dt$$



$$\left| \int_0^T x(t)\, dt \right| \le \int_0^T |x(t)|\, dt.$$

Egy rendszer impulzusválasza $w(t) = \underline{1(t)}\, 5e^{\textcircled{-2t}}$. Határozzuk meg a válaszjelet konvolúcióval, ha $u(t) = \underline{1(t)}$, $u(t) = 1(t)\,e^{-3t}$, $u(t) = 1(t)\,e^{-2t}$, $u(t) = \underline{2\delta(t)} + 1(t)\,e^{-2t}$, $u(t) = 1(t)\cos\omega t$, $u(t) = [1(t) - 1(t-\tau)]/\tau$, $u(t) = \cos\omega t$.

$$1(t) \longrightarrow \boxed{w(t)} \longrightarrow y(t) = v(t)$$

$\cancel{w}$ $\underline{w = v'}$

$$y(t) = \int_{\overset{}{-\infty}\,0}^{\infty\,t} \underline{u(\tau)}\,\underline{w(t-\tau)}\,d\tau = \int_{\cancel{-\infty}\,0}^{\cancel{\infty}\,t} \underline{w(\tau)}\,\underline{u(t-\tau)}\,d\tau.$$

$(t > 0)$

$$\int_0^t \underbrace{1}\cdot 5e^{-2(t-\tau)}\,d\tau = \int_0^t \underbrace{5e^{-2t}}\,e^{2\tau}\,d\tau = 5e^{-2t}\int_0^t e^{2\tau}\,d\tau = 5e^{-2t}\left[\frac{e^{2\tau}}{2}\right]_0^t =$$

$$= 5e^{-2t}\,\frac{e^{2t}-1}{2} = \underline{\underline{2{,}5\,(1 - e^{-2t})}}\;1(t).$$

$$\int_0^t 5e^{-2\tau}\cdot 1\,d\tau = 5\left[\frac{e^{-2\tau}}{-2}\right]_0^t = -2{,}5\,(e^{-2t} - 1) = \underline{\underline{2{,}5\,(1 - e^{-2t})\;1(t)}}$$

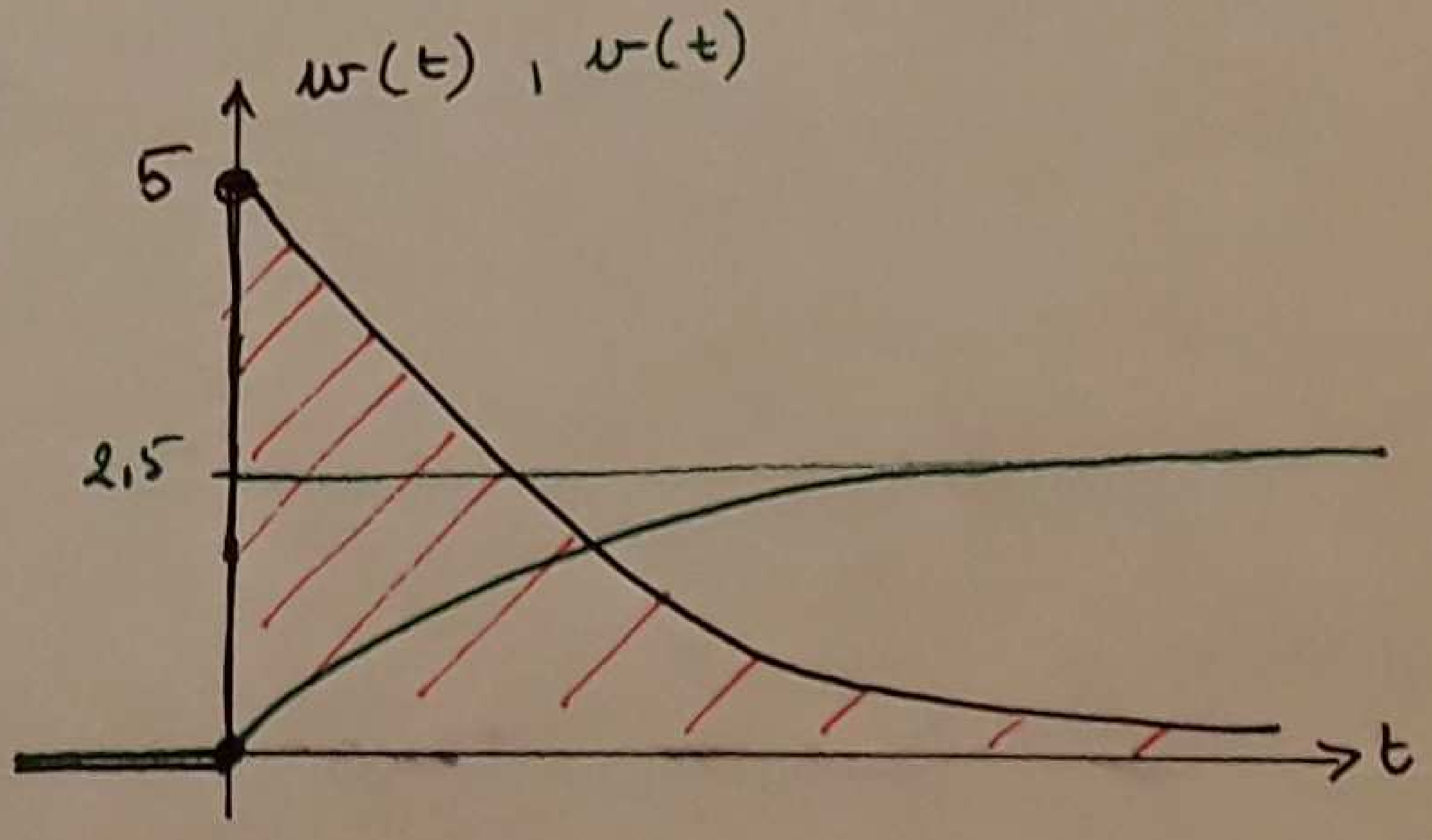
$(ab)' = a'b + ab'$

ell.: $v(t) = 2{,}5\cdot \underbrace{1(t)}\,\underbrace{(1 - e^{-2t})}$

$v' = 2{,}5\cdot \underline{\delta(t)}\,\underline{(1 - e^{-2t})}^{\swarrow 0} +$

$+ 2{,}5\cdot 1(t)\,(+2e^{-2t})$

$v' = 5\cdot 1(t)\,e^{-2t} = w.$



$$\left.\begin{aligned} w(t) &= 1(t)\,5\,e^{-2t} \\ u(t) &= 1(t)\,e^{-3t} \end{aligned}\right\} \; y(t) = 1(t)\,5\left(e^{-2t} - e^{-3t}\right).$$

$$y(t) = \int_0^t u(\tau)\,w(t-\tau)\,d\tau = \int_0^t e^{-3\tau}\cdot 5\,\underset{e^{-2t}\cdot e^{2\tau}}{e^{-2(t-\tau)}}\,d\tau = 5e^{-2t}\int_0^t e^{-\tau}\,d\tau = 5e^{-2t}\left[\frac{e^{-\tau}}{-1}\right]_0^t =$$

$$= -5\,e^{-2t}\left(e^{-t} - 1\right) = \left[5e^{-2t} - \underline{\underline{5e^{-3t}}}\right]1(t).$$

$$y(t) = \int_0^t w(\tau)\,u(t-\tau)\,d\tau = \int_0^t 5e^{-2\tau}\cdot \underset{e^{-3t}\cdot e^{3\tau}}{e^{-3(t-\tau)}}\,d\tau = 5e^{-3t}\int_0^t e^{\tau}d\tau = 5e^{-3t}\left[e^{\tau}\right]_0^t =$$

$$= 5e^{-3t}\left(e^{t} - 1\right) = 5\left(e^{-2t} - \underline{\underline{e^{-3t}\right)1(t)}}$$

$$\left.\begin{aligned} w(t) &= 1(t)\,5\,e^{-2t} \\ u(t) &= 1(t)\,e^{-2t} \end{aligned}\right\}$$

$$y(t) = \int_0^t u(\tau)\,w(t-\tau)\,d\tau = \int_0^t \cancel{e^{-2\tau}} \cdot \underline{5}\,e^{-2(t-\tau)}\,d\tau = 5e^{-2t}\underbrace{\int_0^t dt}_{t} = \underline{\underline{5\enclose{circle}{t}e^{-2t}\,1(t)}}$$

$$\underline{e}^{-2t} \cdot \cancel{e^{2\tau}}$$

$$y(t) = \int_0^t w(\tau)\,u(t-\tau)\,d\tau = \int_0^t 5e^{-2\tau} \cdot e^{-2(t-\tau)}\,d\tau =$$

- " - .

$$2\delta(t) \longrightarrow \boxed{} \longrightarrow 2w(t)$$

$$\left.\begin{aligned} w(t) &= 1(t)\,5\,e^{-2t} \\ u(t) &= \underbrace{2\delta(t)} + \underline{1(t)\,e^{-2t}} \end{aligned}\right\}$$

$$y(t) = \int_0^t u(\tau)\,w(t-\tau)\,d\tau = \int_0^t \left[2\delta(\tau) + e^{-2\tau}\right] \cdot 5e^{-2(t-\tau)}\,d\tau =$$

$$= \int_0^t \underbrace{2\delta(\tau)}_{\tau=0} \cdot 5e^{-2(t-\underset{\tau=0}{\overset{\uparrow}{\tau}})}\,d\tau + \int_0^t \cancel{e^{-2\tau}} \cdot 5\,\underset{e^{-2t}\cdot\cancel{e^{2\tau}}}{e^{-2(t-\tau)}}\,d\tau$$

$$= \int_0^t \underline{10}\,\delta(\tau)\,\underline{e^{-2t}}\,d\tau + 5e^{-2t}\underbrace{\int_0^t d\tau}_{t}$$

$$= 10e^{-2t}\underbrace{\int_{-0}^{t}\delta(\tau)\,d\tau}_{1(t)}$$

$$= 10 \cdot 1(t) \cdot e^{-2t} + \underline{\underline{\overset{\downarrow}{5t} \cdot 1(t)\,e^{-2t}}}$$

$w(t) = 1(t)\, 5 e^{-2t}$

$u(t) = 1(t) \cos\omega t$

$$y(t) = \int_0^t u(\tau)\, w(t-\tau)\, d\tau = \int_0^t \cos\omega\tau \cdot 5 \underset{e^{-2t}\cdot e^{2\tau}}{e^{-2(t-\tau)}}\, d\tau .$$

$$= 5e^{-2t} \int_0^t \frac{e^{j\omega\tau} + e^{-j\omega\tau}}{2} \cdot e^{2\tau}\, d\tau =$$

$$= 2{,}5\, e^{-2t} \int_0^t \left[e^{(2+j\omega)\tau} + e^{(2-j\omega)\tau} \right] d\tau =$$

$$= 2{,}5\, e^{-2t} \left[\frac{e^{(2+j\omega)\tau}}{2+j\omega} + \frac{e^{(2-j\omega)\tau}}{2-j\omega} \right]_0^t =$$

$$= 2{,}5\, e^{-2t} \left[\frac{e^{(2+j\omega)t} - 1}{2+j\omega} + \frac{e^{(2-j\omega)t} - 1}{2-j\omega} \right] =$$

$$= 2{,}5\, e^{-2t} \left[\frac{e^{(2+j\omega)t}}{\underline{2+j\omega}} - \frac{1}{\underline{2+j\omega}} + \frac{e^{(2-j\omega)t}}{\underline{2-j\omega}} - \frac{1}{\underline{2-j\omega}} \right] =$$

$$= \frac{2{,}5}{4+\omega^2} \cancel{e^{-2t}} \left[(2-j\omega)\, e^{(\cancel{2}+j\omega)t} + (2+j\omega)\, e^{(\cancel{2}-j\omega)t} \overset{-4e^{-2t}}{\underbrace{-(2-j\omega) - (2+j\omega)}_{-2+j\omega-2-j\omega\,=\,-4}} \right]$$

$$= \frac{2{,}5}{4+\omega^2} \left[2\underline{e^{j\omega t}} \underline{-j\omega\, e^{j\omega t}} + 2\underline{e^{-j\omega t}} \underline{+j\omega\, e^{-j\omega t}} - 4e^{-2t} \right].$$

$$= \frac{2{,}5}{4+\omega^2} \left[2 \boxed{\frac{e^{j\omega t} + e^{-j\omega t}}{\underset{\cos\omega t}{2}}}\, 2 - j\omega \boxed{\frac{e^{j\omega t} - e^{-j\omega t}}{\underset{\sin\omega t}{2j}}}\, 2j - 4e^{-2t} \right] = \frac{2{,}5}{4+\omega^2} \left[4\cos\omega t + 2\omega \sin\omega t - 4e^{-2t} \right]$$

$$\boxed{\cos\omega\tau = \frac{e^{j\omega\tau} + e^{-j\omega\tau}}{2}}$$

$\cos\omega\tau + j\sin\omega\tau +$

$\cos\omega\tau - j\sin\omega\tau =$

$\frac{2\cos\omega\tau}{2} = \cos\omega\tau$

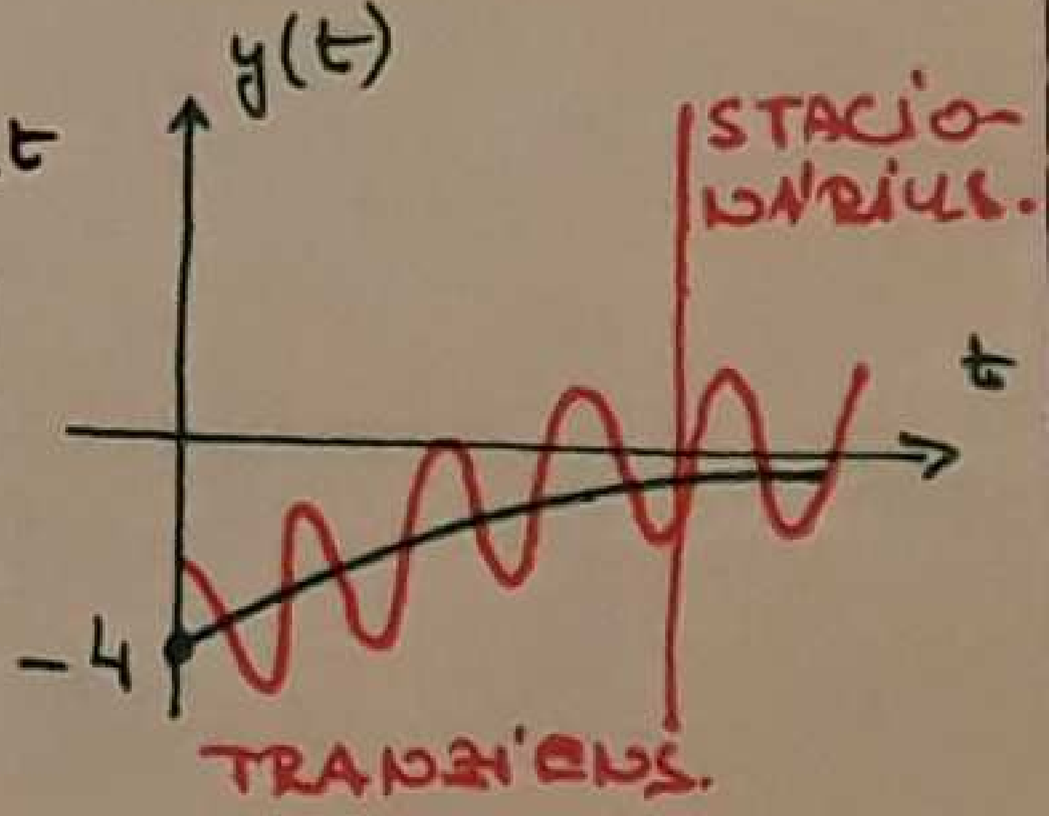
$(2+j\omega)(2-j\omega) = 4+\omega^2$!

$(a+b)(a-b) = a^2 - b^2$

$$\boxed{\sin\omega\tau = \frac{e^{j\omega\tau} - e^{-j\omega\tau}}{2j}}$$

$\cancel{\cos\omega\tau} + j\sin\omega\tau -$

$\cancel{\cos\omega\tau} + j\sin\omega\tau = j2\sin\omega\tau$



$$\left.\begin{aligned} w(t) &= 1(t)\cdot 5\,\underline{e^{-2t}} \\ u(t) &= \cos\omega t \end{aligned}\right\} \leftarrow \text{NEM BELÉPŐ.}$$

$$\cos\omega\tau = \frac{e^{j\omega\tau}+e^{-j\omega\tau}}{2}.$$

$$\sin\omega\tau = \frac{e^{j\omega\tau}-e^{-j\omega\tau}}{2j}.$$

$$y(t) = \int_{-\infty}^{\infty\ t} \underline{u}(\tau)\,\underline{w}(t-\tau)\,d\tau = \int_{-\infty}^{t} u(\tau)\,w(t-\tau)\,d\tau.$$

$$= \int_{-\infty}^{t} \frac{e^{j\omega\tau}+e^{-j\omega\tau}}{2}\cdot 5e^{-2(t-\tau)}\,d\tau = 2{,}5\,e^{-2t}\int_{-\infty}^{t}\left(e^{j\omega\tau}+e^{-j\omega\tau}\right)\cdot e^{2\tau}\,d\tau =$$

$$(2+j\omega)(2-j\omega) = 4+\omega^2$$

$$= 2{,}5\,e^{-2t}\int_{-\infty}^{t}\left[e^{(2+j\omega)\tau}+e^{(2-j\omega)\tau}\right]d\tau$$

$$= 2{,}5\,e^{-2t}\left[\frac{e^{(2+j\omega)\tau}}{2+j\omega}+\frac{e^{(2-j\omega)\tau}}{2-j\omega}\right]_{\boxed{-\infty}}^{t}$$

$$= 2{,}5\,e^{-2t}\left(\frac{e^{(2+j\omega)t}}{2+j\omega}+\frac{e^{(2-j\omega)t}}{2-j\omega}\right) = \frac{2{,}5}{4+\omega^2}\,\cancel{e^{-2t}}\left(\cancel{e^{2t}}\,e^{j\omega t}(2-j\omega)+\cancel{e^{2t}}\,e^{-j\omega t}(2+j\omega)\right)$$

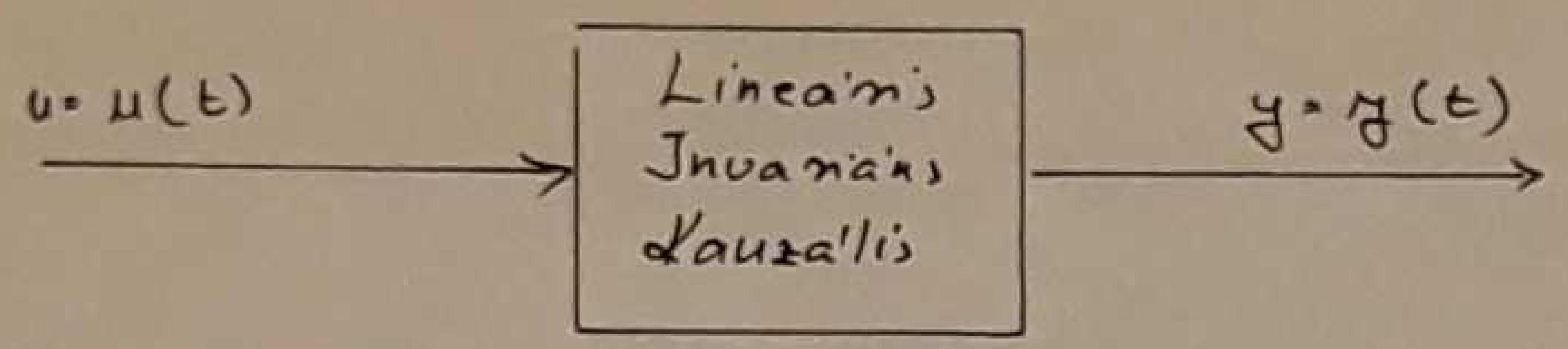
$$= \frac{2{,}5}{4+\omega^2}\left[\underline{2e^{j\omega t}}\ \underline{-j\omega e^{j\omega t}} + \underline{2e^{-j\omega t}} + \underline{j\omega e^{-j\omega t}}\right]$$

$$= \frac{2{,}5}{4+\omega^2}\left[2\,\underbrace{\frac{e^{j\omega t}+e^{-j\omega t}}{2}}_{\cos\omega t}\,2 - j\omega\,\underbrace{\frac{e^{j\omega t}-e^{-j\omega t}}{2j}}_{\sin\omega t}\,2j\right]$$

$$= \frac{2{,}5}{4+\omega^2}\left[4\cos\omega t + 2\omega\sin\omega t\right]$$

FOLYTONOS IDEJŰ RENDSZEREK ANALÍZISE AZ IDŐTARTOMÁNYBAN ⓘ

RENDSZEREGYENLET



$$\boxed{1\cdot y^{(n)} + \sum_{i=1}^{n} a_i\, y^{(n-i)} = \sum_{i=0}^{n} b_i\, u^{(n-i)}}$$

közönséges t

homogén / inhomogén

lineáris

állandó együtthatós

n-edrendű

differenciálegyenlet

$$m\ddot{x} = F - b\dot{x} \qquad (y=x)\ (u=F)$$

$$m\ddot{x} + b\dot{x} = F$$

$$\ddot{x} + \underline{\underline{\frac{b}{m}\dot{x}}} = \frac{1}{m}F$$

$$m\dot{v} + bv = F$$

$$\dot{v} + \underline{\underline{\frac{b}{m}v}} = \frac{1}{m}F$$

$$\dot{u}_C + \underline{\underline{\frac{1}{RC}u_C}} = \frac{1}{RC}u$$

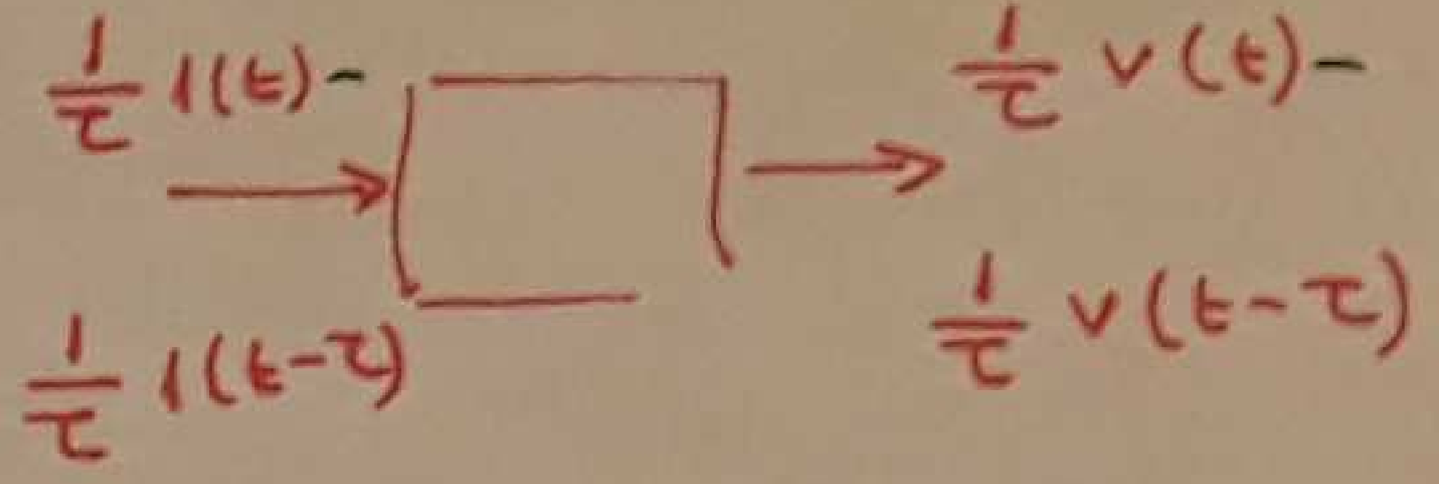
$$m\ddot{y} + b\dot{y} + ky = ku$$

$$\ddot{y} + \frac{b}{m}\dot{y} + \underline{\underline{\frac{k}{m}y}} = \frac{k}{m}u$$

$\Longrightarrow$ állapotváltozós leírás.

$$\left.\begin{aligned} w(t) &= 1(t)\,5e^{-2t} \\ v(t) &= 2{,}5\cdot 1(t)\cdot(1-e^{-2t}) \\ u(t) &= \frac{1(t)-1(t-\tau)}{\tau}\ ;\ \tau\to 0 \end{aligned}\right\}$$

$$u(t) = \frac{1}{\tau}1(t) - \frac{1}{\tau}1(t-\tau)$$



$$y(t) = \frac{v(t)-v(t-\tau)}{\tau} \qquad \lim_{\tau\to 0}\frac{v(t)-v(t-\tau)}{\tau} = \dot v(t) = w(t).$$

$$y(t) = \frac{2{,}5\cdot 1(t)(1-e^{-2t})}{\tau} - \frac{2{,}5\cdot 1(t-\tau)(1-e^{-2(t-\tau)})}{\tau}$$

$$= \underline{\frac{2{,}5\cdot 1(t)}{\tau}} - \frac{2{,}5\cdot 1(t)e^{-2t}}{\tau} - \underline{\frac{2{,}5\cdot 1(t-\tau)}{\tau}} + \frac{2{,}5\cdot 1(t-\tau)e^{-2(t-\tau)}}{\tau}$$

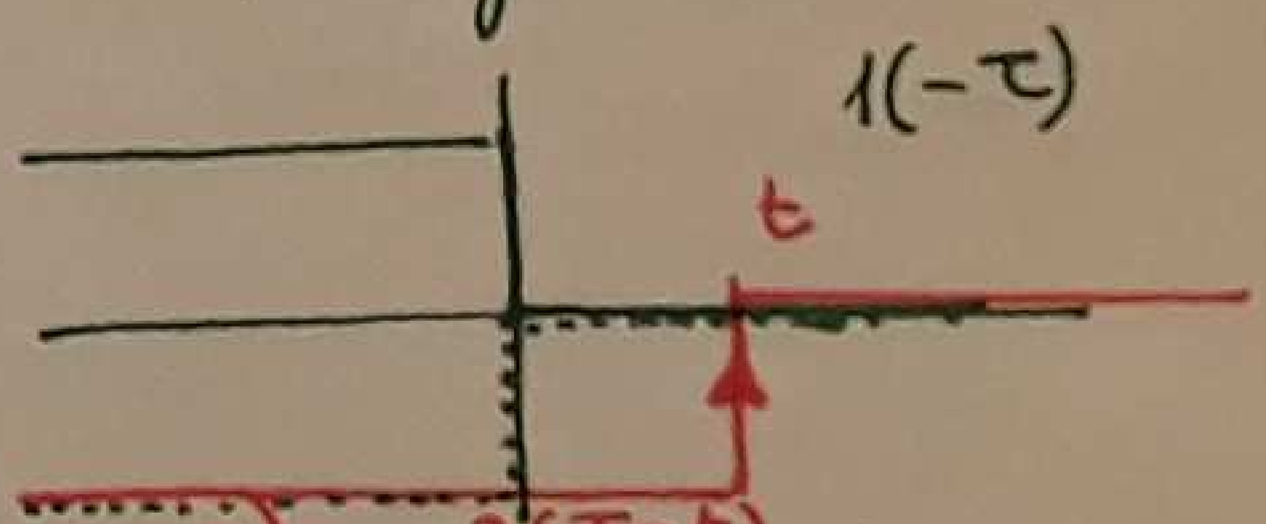
$$= 2{,}5\cdot\frac{1(t)-1(t-\tau)}{\tau} \qquad -2{,}5\,\frac{\cancel{1(t)e^{-2t}}\ \boxed{-e^{-2(t-\tau)}1(t-\tau)}}{\tau}$$

$\tau\to 0$

$$= \cancel{2{,}5\delta(t)} - \cancel{2{,}5\delta(t)} + 1(t)5e^{-2t}$$

$$= 1(t)5e^{-2t} = w(t)$$

L'Hospital. τ!



$$-1(t-\tau)e^{-2(t-\tau)}$$

$$\underline{-1(-[\tau-t])}\,e^{2(\tau-t)}$$

$$+\delta(-[\tau-t])\underbrace{e^{2(\tau-t)}}_{1} - 1(-[\tau-t])\cdot e^{2(\tau-t)}\cdot 2$$

$$\delta(t-\tau) - 1(t-\tau)2e^{-2(t-\tau)}$$

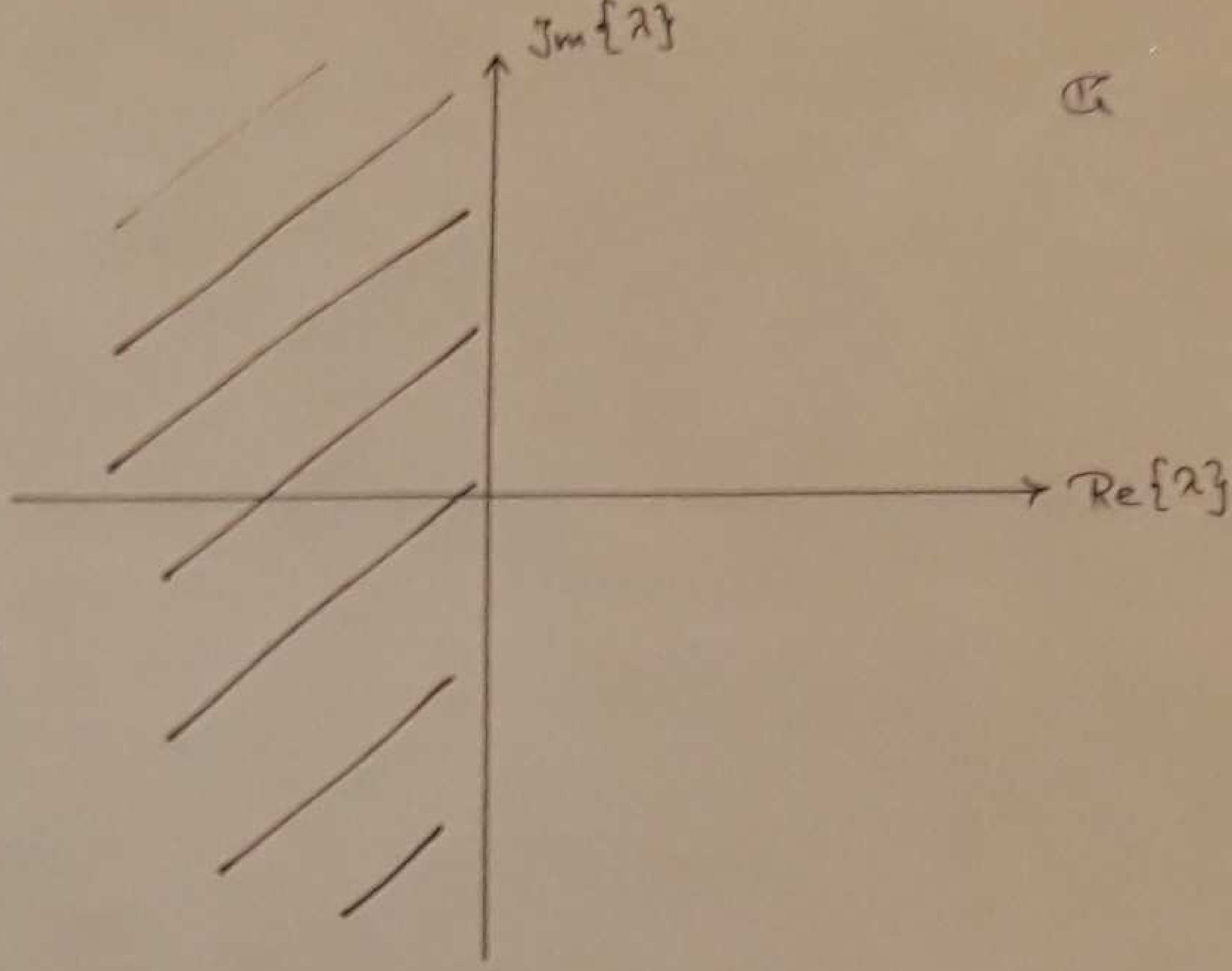
$$-2{,}5\delta(t-\tau) + 1(t-\tau)5e^{-2(t-\tau)} \qquad \underline{\tau\to 0}$$

$$-2{,}5\delta(t) + 1(t)5e^{-2t}.$$

Aszimptotikus stabilitás

$$\boxed{\lim_{t\to\infty} \underline{x}(t) = \underline{0}}$$

$$\boxed{\underline{x}(0)}$$



$$e^{\underline{\underline{A}}t} \to 0$$

$\underline{\underline{A}} \to |\lambda \underline{\underline{E}} - \underline{\underline{A}}| = 0$ karakterisztikus egyenlet

$$\lambda_1 \quad \lambda_2 \ldots \quad \lambda_n$$

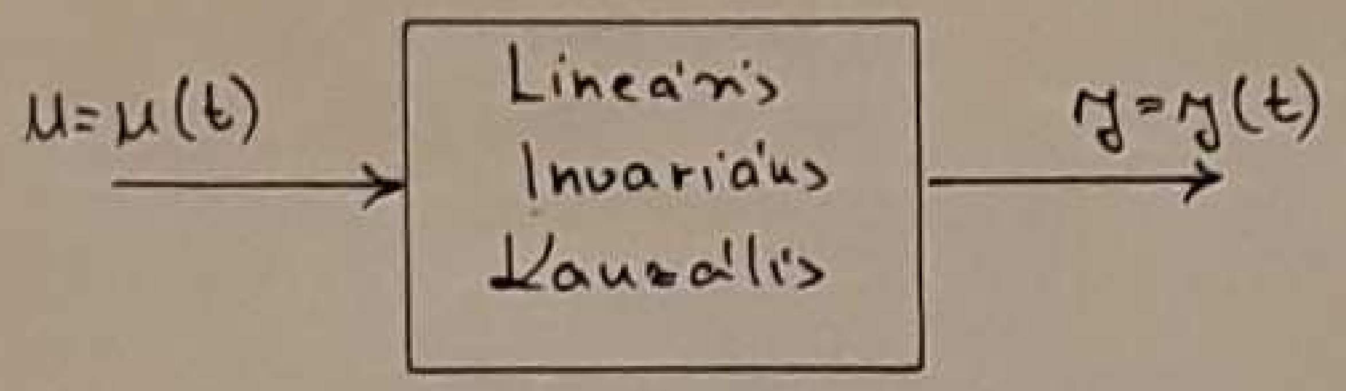
$$e^{\lambda_1 t} \quad e^{\lambda_2 t} \ldots \quad e^{\lambda_n t}$$

$\mathrm{Re}\{\lambda_i\} < 0 \qquad i = 1 \ldots n$

FOLYTONOS IDEJŰ RENDSZEREK ANALÍZISE AZ IDŐTARTOMÁNYBAN

Ⓘ III.

ÁLLAPOTVÁLTOZÓS LEÍRÁS



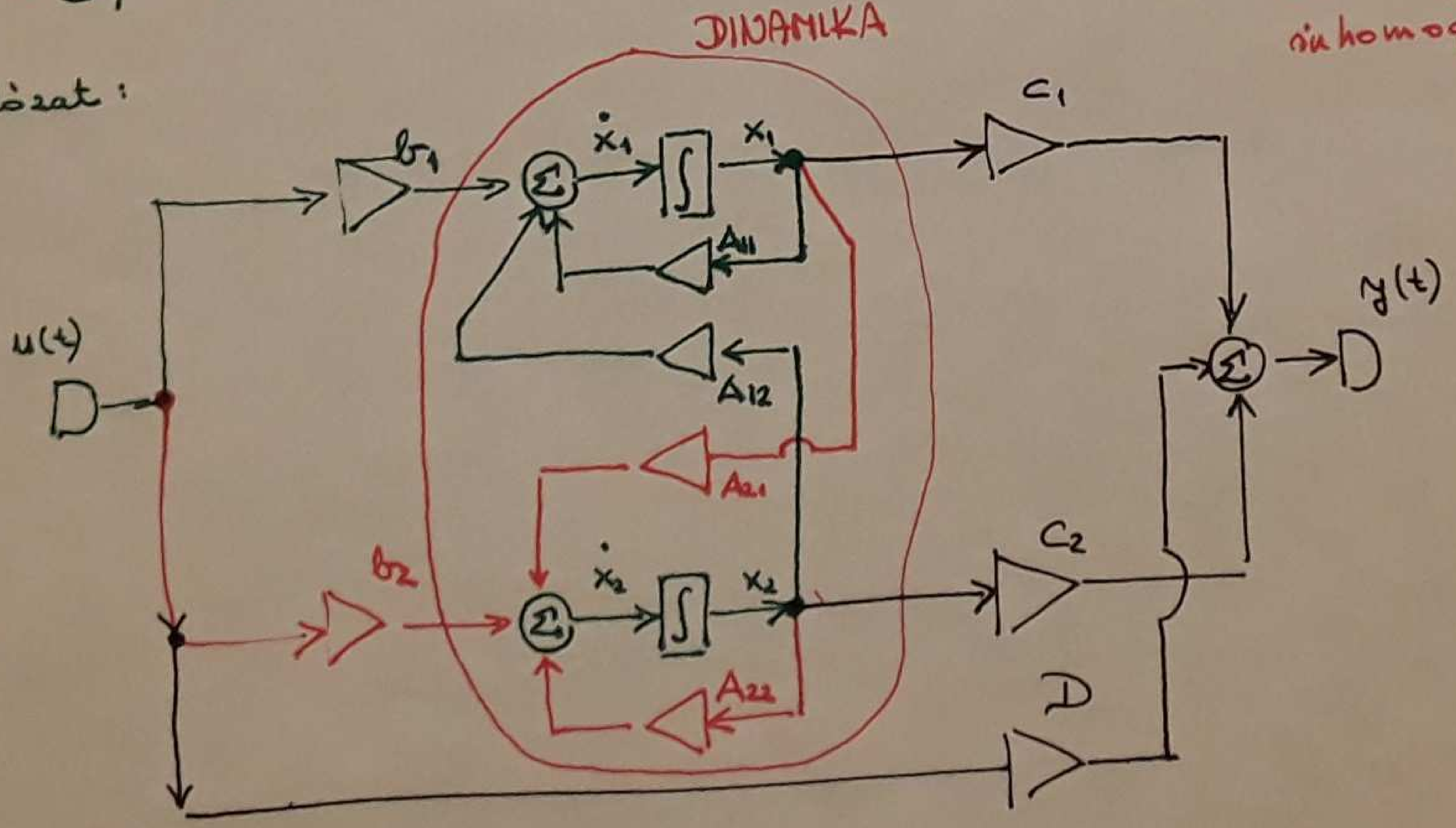
$$\left.\begin{aligned}\dot{\underline{x}} &= \boxed{\underline{\underline{A}}}\,\underline{x} + \underline{b}\,u \\ y &= \underline{c}^T\underline{x} + D\,u\end{aligned}\right] \qquad \begin{aligned}\dot{\underline{x}} &= \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} \\ \underline{y} &= \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}\end{aligned}$$

$$\begin{aligned}\dot{x}_1 &= A_{11}x_1 + A_{12}x_2 + b_1 u \\ \dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + b_2 u \\ y &= c_1 \underline{x_1} + c_2 \underline{x_2} + D\underline{u}\end{aligned} \Rightarrow \begin{matrix}x_1(t)\\x_2(t)\end{matrix} \quad \underline{\underline{A}} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix} \quad \underline{b} = \begin{bmatrix}b_1\\ b_2\end{bmatrix}$$

$$\underline{c}^T = [c_1 \quad c_2] \qquad D$$

lineáris, állandó együtthatós
első ~~fokú~~ rendű deriváltakból
álló egyenletrendszer.
inhomogén, közönséges.

Jelfolyam hálózat:



Lagrange - mátrix

Hermit - mátrix

$$e^{\underline{\underline{A}}t}$$

A MUNKAPONTI LINEARIZÁLÁS

$$\boxed{\begin{aligned}\dot{\underline{x}} &= \underline{f}(\underline{x}, u)\\ y &= g(\underline{x}, u)\end{aligned}} \longrightarrow \boxed{\begin{aligned}\dot{\underline{x}} &= \underline{\underline{A}}\,\underline{x} + \underline{b}\,u\\ y &= \underline{c}^T\underline{x} + D\,u\end{aligned}}$$

$\underline{x} = \bar{\underline{x}} + \tilde{\underline{x}}$ $\quad u = \bar{u} + \tilde{u}$

$y = \bar{y} + \tilde{y}$

egyensúlyi állapotban: $\dot{\underline{x}} = \underline{\emptyset} \longrightarrow \boxed{\underline{0} = \underline{f}(\bar{\underline{x}}, \bar{u})}$

$$\dot{\underline{x}} = \cancel{\dot{\bar{\underline{x}}}} + \dot{\tilde{\underline{x}}} = \underline{f}(\bar{\underline{x}} + \tilde{\underline{x}}, \bar{u} + \tilde{u}) \cong \cancel{\underbrace{\underline{f}(\bar{\underline{x}}, \bar{u})}_{\emptyset}} + \left.\frac{\partial \underline{f}(\underline{x}, u)}{\partial \underline{x}^T}\right|_{\bar{\underline{x}}, \bar{u}} \tilde{\underline{x}} + \left.\frac{\partial \underline{f}(\underline{x}, u)}{\partial u}\right|_{\bar{\underline{x}}, \bar{u}} \tilde{u}$$

$$y = \cancel{\bar{y}} + \tilde{y} = g(\bar{\underline{x}} + \tilde{\underline{x}}, \bar{u} + \tilde{u}) \cong \cancel{\underbrace{g(\bar{\underline{x}}, \bar{u})}_{\bar{y}}} + \left.\frac{\partial g(\underline{x}, u)}{\partial \underline{x}^T}\right|_{\bar{\underline{x}}, \bar{u}} \tilde{\underline{x}} + \left.\frac{\partial g(\underline{x}, u)}{\partial u}\right|_{\bar{\underline{x}}, \bar{u}} \tilde{u}$$

$$\dot{\tilde{\underline{x}}} = \overset{\underline{\underline{A}}}{\boxed{\left.\frac{\partial \underline{f}(\underline{x}, u)}{\partial \underline{x}^T}\right|_{\bar{\underline{x}}, \bar{u}}}} \tilde{\underline{x}} + \overset{\underline{b}}{\boxed{\left.\frac{\partial \underline{f}(\underline{x}, u)}{\partial u}\right|_{\bar{\underline{x}}, \bar{u}}}} \tilde{u}$$

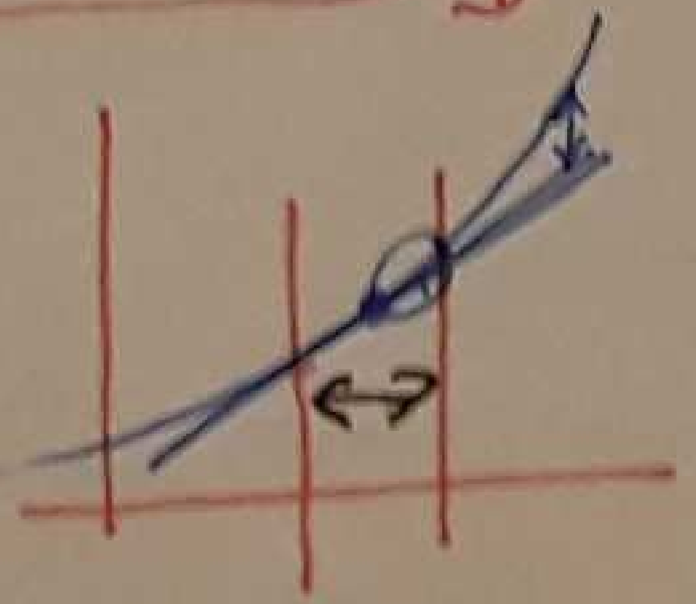
$$\tilde{y} = \underset{\underline{c}^T}{\boxed{\left.\frac{\partial g(\underline{x}, u)}{\partial \underline{x}^T}\right|_{\bar{\underline{x}}, \bar{u}}}} \tilde{\underline{x}} + \underset{D}{\boxed{\left.\frac{\partial g(\underline{x}, u)}{\partial u}\right|_{\bar{\underline{x}}, \bar{u}}}} \tilde{u}$$

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}\,u$$
$$y = \underline{c}^T\underline{x} + D\,u$$

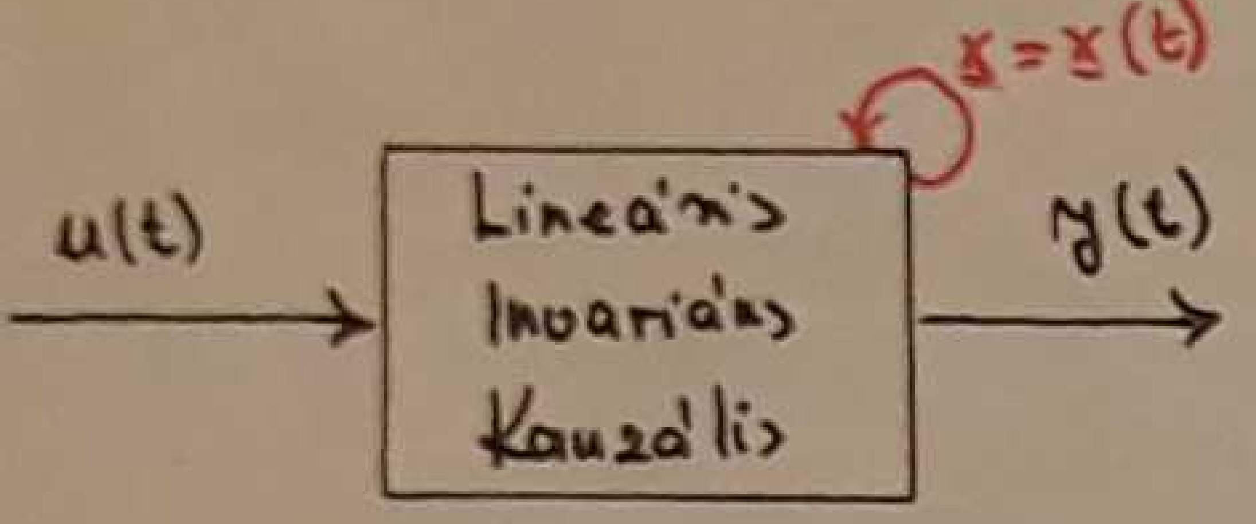
$\underline{f} = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$ $\quad g \in$

$$\underline{\underline{A}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{\bar{\underline{x}}\,\bar{u}} \qquad \underline{b} = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix}_{\bar{\underline{x}}\,\bar{u}}$$

$$\underline{c}^T = \begin{bmatrix} \frac{\partial g}{\partial x_1} & \frac{\partial g}{\partial x_2} \end{bmatrix}_{\bar{\underline{x}}\,\bar{u}} \qquad D = \left.\frac{\partial g}{\partial u}\right|_{\bar{\underline{x}}\,\bar{u}}$$



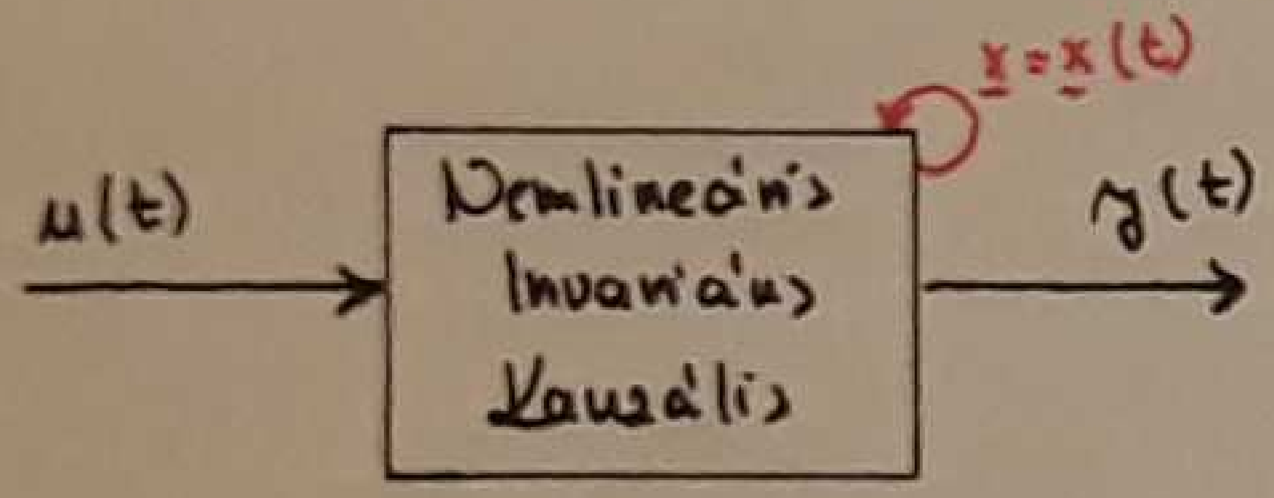
AZ ÁLLAPOTVÁLTOZÓS LEÍRÁS ÁLTALÁNOSABB ALAKJAI



$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}\,u \qquad \dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$$

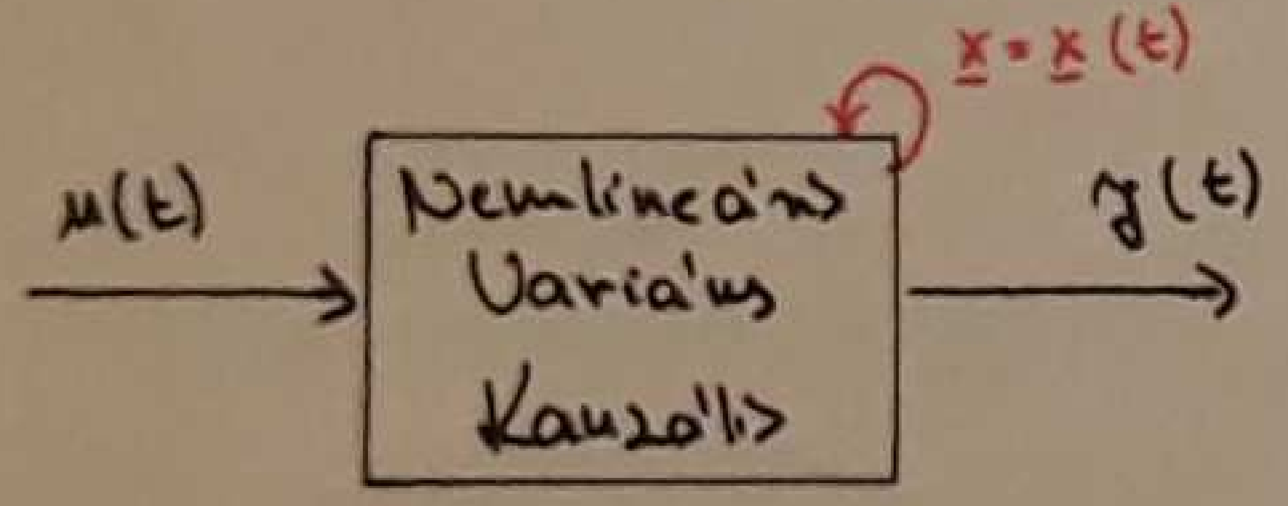
$$y = \underline{c}^T\underline{x} + D\,u \qquad \underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$$

Munkaponti linearizálás



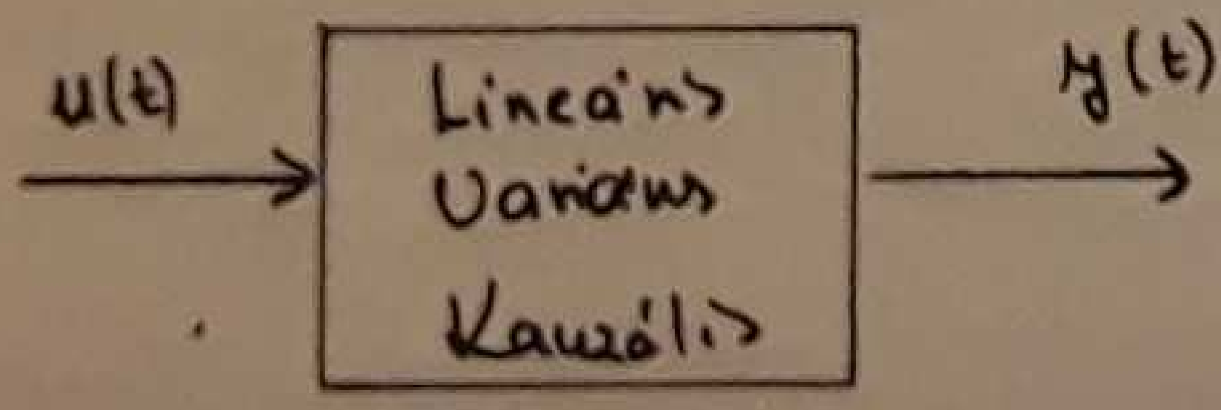
$$\dot{\underline{x}} = \underline{f}(\underline{x}, u) \qquad \dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u})$$

$$y = g(\underline{x}, u) \qquad \underline{y} = \underline{g}(\underline{x}, \underline{u})$$



$$\dot{\underline{x}} = \underline{f}(\underline{x}, u, t) \qquad \dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, t)$$

$$y = g(\underline{x}, u, t) \qquad \underline{y} = \underline{g}(\underline{x}, \underline{u}, t)$$



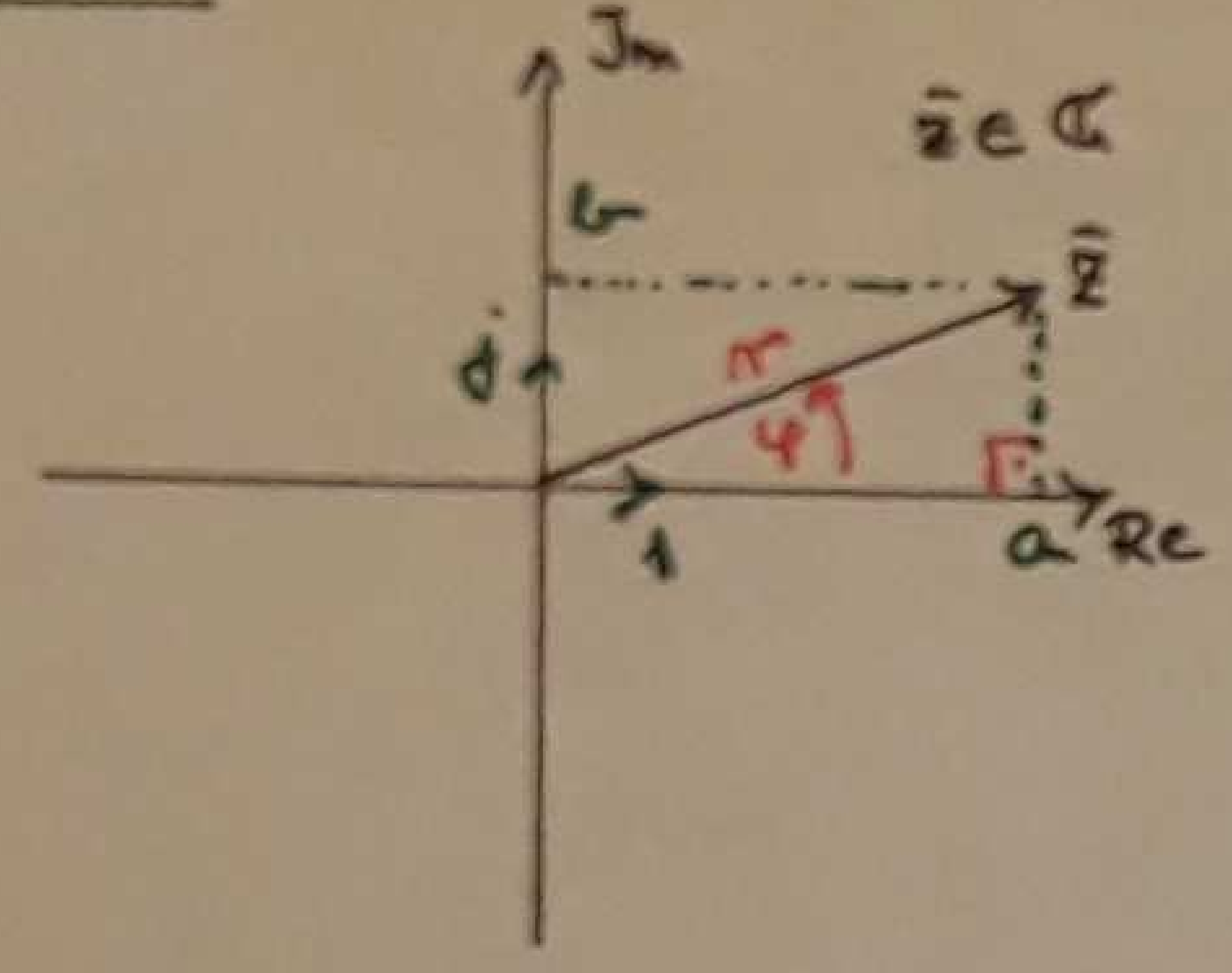
$$\dot{\underline{x}} = \underline{\underline{A}}(t)\,\underline{x} + \underline{b}(t)\,u \qquad \dot{\underline{x}} = \underline{\underline{A}}(t)\,\underline{x} + \underline{\underline{B}}(t)\,\underline{u}$$

$$y = \underline{c}^T(t)\,\underline{x} + D(t)\,u \qquad \underline{y} = \underline{\underline{C}}(t)\,\underline{x} + \underline{\underline{D}}(t)\,\underline{u}$$

KOMPLEX SZÁMOK - RÖVID ÖSSZEFOGLALÓ

$j^2 = -1 \quad j = \sqrt{-1}$ (i)

FAZOR



① algebrai alak: $\bar{z} = a + jb$

(±) $\bar{z}_1 \pm \bar{z}_2 = (a_1 + jb_1) \pm (a_2 + jb_2) = (a_1 \pm a_2) + j(b_1 \pm b_2).$

② trigonometrikus alak:

$r = |\bar{z}| = \sqrt{a^2 + b^2} \qquad a = r\cos\varphi \qquad b = r\sin\varphi$

$\varphi = \operatorname{arctg}\frac{b}{a} \qquad \bar{z} = r\cos\varphi + jr\sin\varphi = r\,\underline{(\cos\varphi + j\sin\varphi)}$

③ Euler-alak:

(i) $\boxed{\bar{z} = r\underline{e^{j\varphi}}}$

$\bar{z}_1 \cdot \bar{z}_2 = r_1 e^{j\varphi_1} \cdot r_2 e^{j\varphi_2} = \underline{r_1 r_2}\, e^{j\underline{(\varphi_1 + \varphi_2)}}$

$\frac{\bar{z}_1}{\bar{z}_2} = \frac{r_1 e^{j\varphi_1}}{r_2 e^{j\varphi_2}} = \frac{r_1}{r_2} e^{j(\underline{\varphi_1 - \varphi_2})}.$

$\cos\varphi = 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \frac{\varphi^6}{6!} \dots$

$\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \frac{\varphi^7}{7!} \dots$

$\leftarrow j\varphi$

$e^{x} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \dots$

$\Rightarrow e^{j\varphi} = 1 + \frac{j\varphi}{1!} + \frac{(j\varphi)^2}{2!} + \frac{(j\varphi)^3}{3!} \dots$

$= \underline{1} + j\frac{\varphi}{1!} - \underline{\frac{\varphi^2}{2!}} - j\frac{\varphi^3}{3!} \dots$

$= \cos\varphi + j\sin\varphi$

$j^2 = -1$

$jjj = -j$

$\boxed{e^{j\varphi} = \cos\varphi + j\sin\varphi}$

$\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2} \Rightarrow \frac{\cos\varphi + \cancel{j\sin\varphi} + \cos\varphi - \cancel{j\sin\varphi}}{2} = \cos\varphi.$

$\sin\varphi = \frac{e^{j\varphi} - e^{-j\varphi}}{2j} \Rightarrow \frac{\cancel{\cos\varphi} + j\sin\varphi - \cancel{\cos\varphi} + j\sin\varphi}{2j} = \sin\varphi.$

$|e^{j\varphi}| = \sqrt{\cos^2\varphi + \sin^2\varphi} = 1$

AZ ÁLLAPOTEGYENLET MEGOLDÁSÁNAK FORMULÁJA

$$\dot{x} = \lambda x \longrightarrow x(t) = Me^{\lambda t} \qquad M = x(0) \qquad \underline{x}(+0) = \underline{x}(-0)$$

$$e^{\lambda t} = 1 + \frac{t}{1!}\lambda + \frac{t^2}{2!}\lambda^2 + \frac{t^3}{3!}\lambda^3 + \dots + \frac{t^n}{n!}\lambda^n + \dots$$

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} \longrightarrow \underline{x}(t) = e^{\underline{\underline{A}}t}\underline{M} = e^{\underline{\underline{A}}t}\underline{x}(-0)$$

$$e^{\underline{\underline{A}}t} = \underline{\underline{E}} + \frac{t}{1!}\underline{\underline{A}} + \frac{t^2}{2!}\underline{\underline{A}}^2 + \frac{t^3}{3!}\underline{\underline{A}}^3 + \dots + \frac{t^n}{n!}\underline{\underline{A}}^n + \dots$$

/ Lagrange
\ Hermite

⇔ Laplace-t.

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}\,u \longrightarrow \boxed{\underline{x}(t) = e^{\underline{\underline{A}}t}\underline{x}(-0) + \int_0^t \underbrace{e^{\underline{\underline{A}}(t-\tau)}\underline{b}}_{\underline{w}_x(t-\tau)}\,\underbrace{u(\tau)}_{u(\tau)}\,d\tau}$$

$$\boxed{\underline{w}_x(t) = 1(t)\,e^{\underline{\underline{A}}t}\underline{b}}$$

$$y(t) = \underline{c}^T\underline{x} + Du \rightarrow y(t) = \underbrace{\underline{c}^T e^{\underline{\underline{A}}t}\underline{x}(-0)}_{\emptyset} + \int_0^t \underline{\underline{c}^T e^{\underline{\underline{A}}(t-\tau)}\underline{b}}\,\underbrace{u(\tau)}_{\delta(\tau)}\,d\tau + \underbrace{Du(t)}_{D\delta(t)}$$

$$\int_0^t \boxed{\underline{c}^T e^{\underline{\underline{A}}t}\underline{b}}\,\delta(\tau)\,d\tau = \underline{c}^T e^{\underline{\underline{A}}t}\underline{b}\underbrace{\int_0^t \delta(\tau)\,d\tau}_{1}$$

$$\underline{1(t)\,\underline{c}^T e^{\underline{\underline{A}}t}\underline{b}}$$

$$w(t) = 1(t)\,\cancel{\underline{c}^T} e^{\underline{\underline{A}}t}\underline{b} + \cancel{D}\delta(t)$$

Az impulzusválasz:

$$\begin{pmatrix} \underline{\underline{A}} & \underline{b} \\ \underline{c}^T & D \end{pmatrix} \rightarrow \boxed{w(t) = 1(t)\,\underline{c}^T e^{\underline{\underline{A}}t}\underline{b} + D\delta(t)}$$

① SZINUSZOS JELRE ADOTT VÁLASZ

② PERIODIKUS JELRE ADOTT VÁLASZ

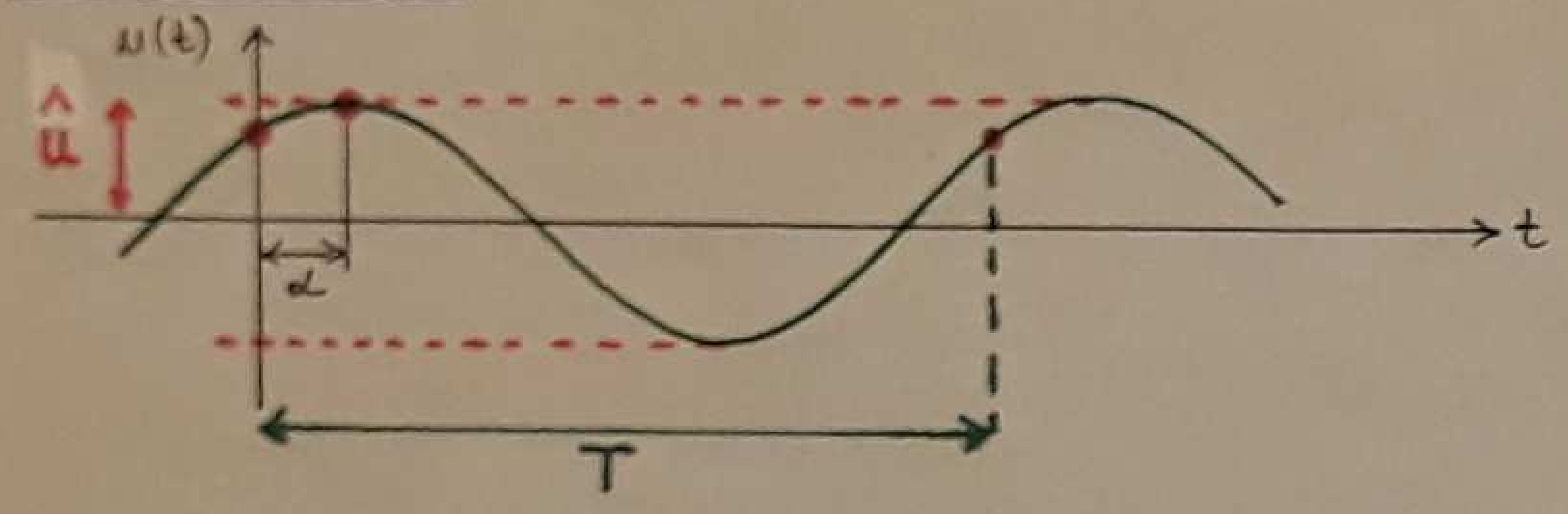
FOURIER-SOR

③ FOURIER-TRANSZFORMÁCIÓ

FOLYTONOS IDEJŰ RENDSZEREK ANALÍZISE A FREKVENCIATARTOMÁNYBAN (I)

SZINUSZOS JELRE ADOTT VÁLASZ

Szinuszos jel



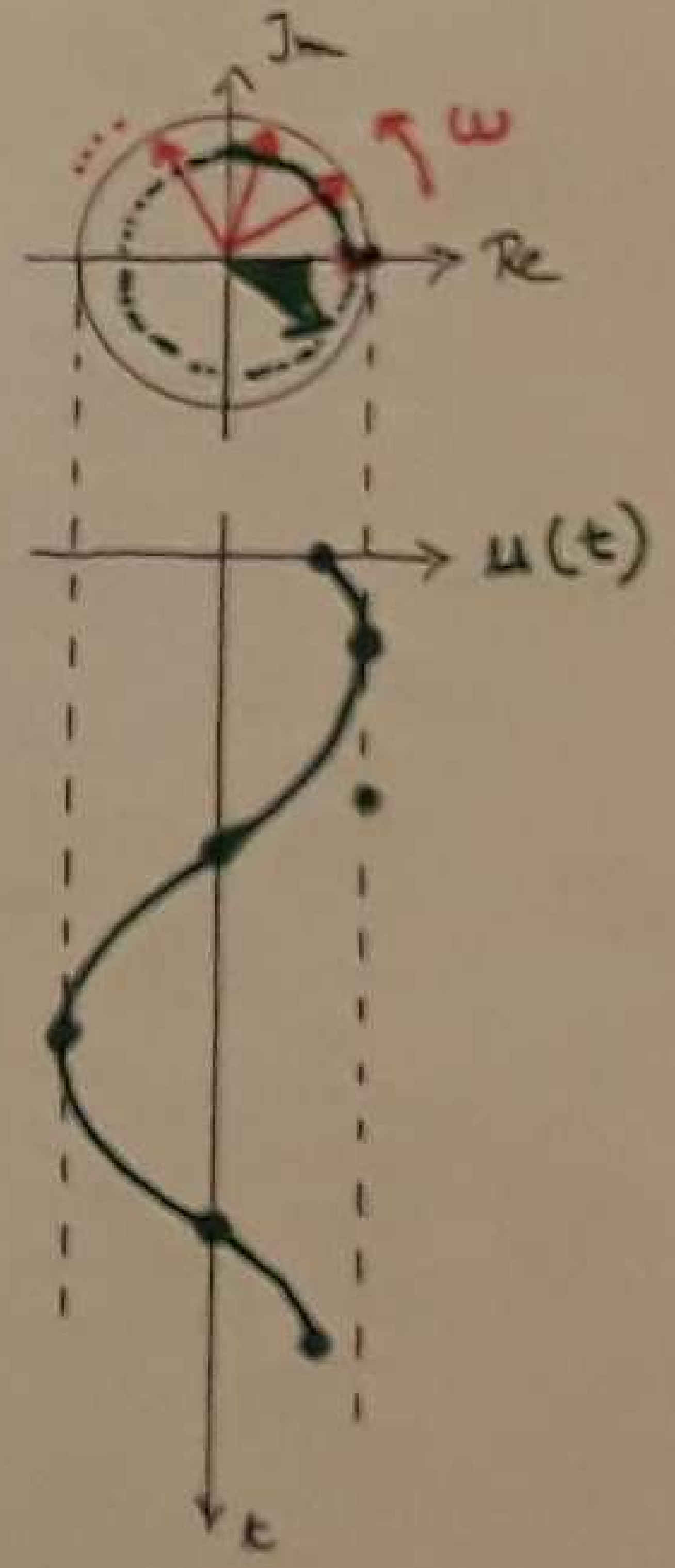
$\hat{U}$

T

α

$f = \frac{1}{T} \quad \omega = \frac{2\pi}{T} = 2\pi f$

$$\boxed{u(t) = \hat{U} \boxed{\cos}(\omega t + \alpha)}$$



Szinuszos jel komplex leírása

$$u(t) = \hat{U} \underline{\cos}(\omega t + \alpha) = \mathrm{Re}\left\{ \underline{\hat{U} \cos(\omega t + \alpha)} + j \hat{U} \sin(\omega t + \alpha) \right\} =$$

$$= \mathrm{Re}\left\{ \hat{U} e^{j(\omega t + \alpha)} \right\} = \mathrm{Re}\left\{ \hat{U} \boxed{e^{j\omega t}} e^{j\alpha} \right\}$$

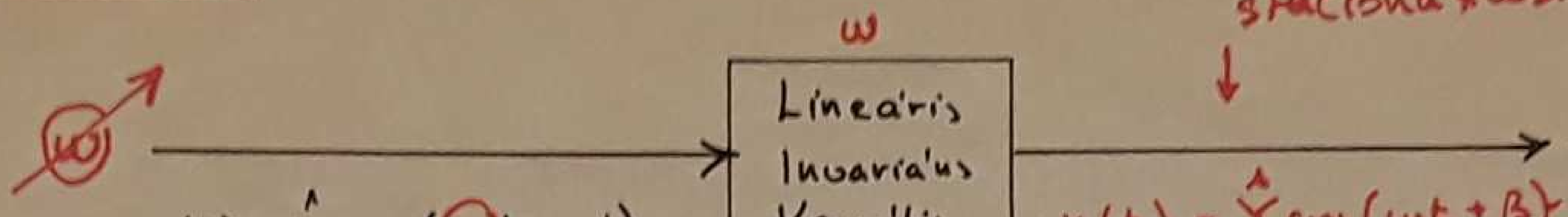
$$\boxed{\underline{\hat{U}} = \hat{U} e^{j\alpha}}$$

Komplex csúcsérték

$$u(t) = \mathrm{Re}\left\{ \underbrace{\hat{U} e^{j\alpha}} e^{j\omega t} \right\} = \mathrm{Re}\left\{ \underline{\hat{U}} \underbrace{e^{j\omega t}} \right\}.$$

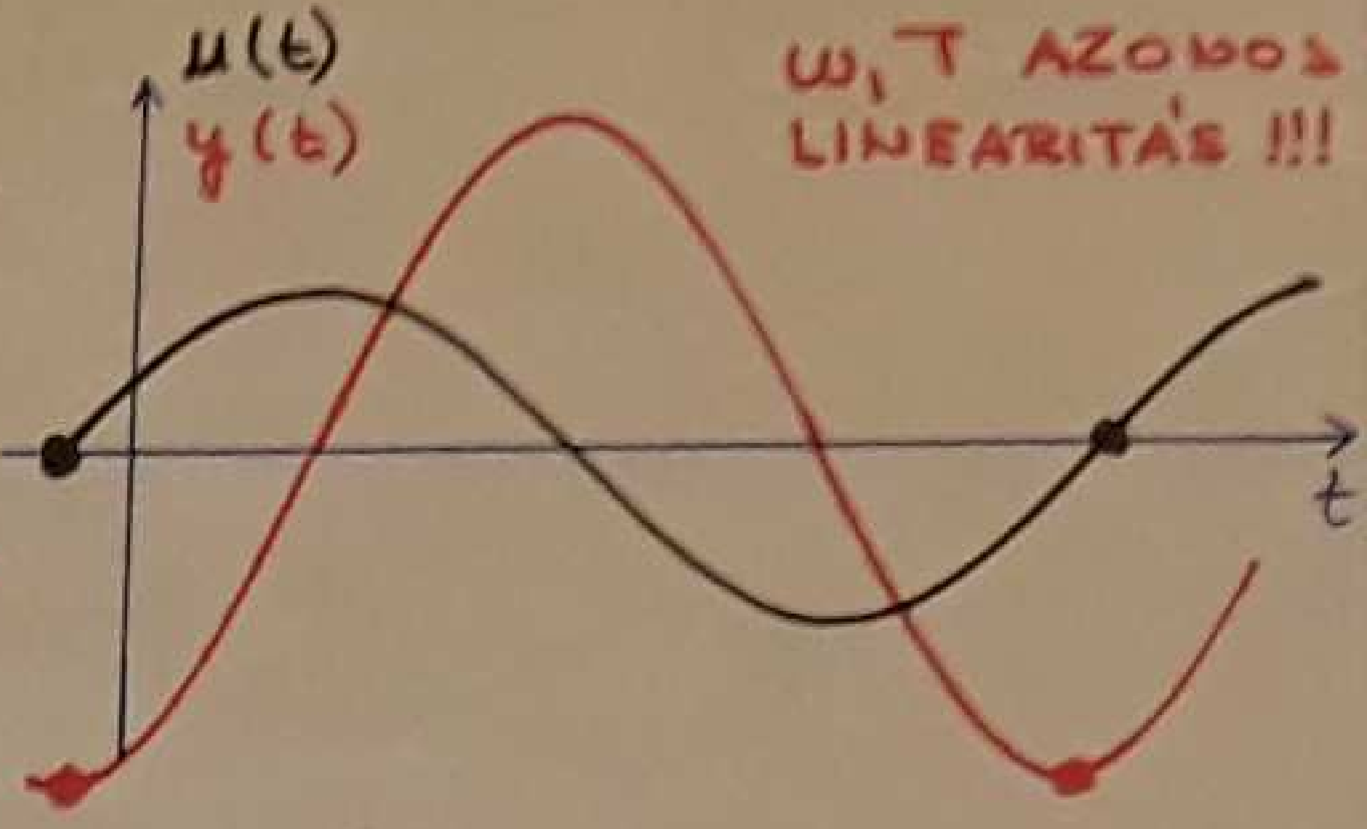
$$\left| e^{j\omega t} \right| = 1.$$

Átviteli karakterisztika



$u(t) = \hat{U}\cos(\omega t + \alpha)$ → $y(t) = \hat{Y}\cos(\omega t + \beta)$

ww.



ω, T AZONOS
LINEARITÁS !!!

0........

$\hat{U} = 1$

$\alpha = 0°$

$$\hat{\bar{U}} = \hat{U}e^{j\alpha} = 1e^{j0°} = 1 \qquad \hat{\bar{Y}} = \hat{Y}e^{j\beta}$$

$$\frac{\hat{\bar{Y}}}{\hat{\bar{U}}} = \hat{\bar{Y}} = \hat{Y}e^{j\beta}$$

$\hat{Y}(\omega)$ $\beta(\omega)$

$$\boxed{\underline{W}(j\omega) = \frac{\hat{\bar{Y}}}{\hat{\bar{U}}}} = \underbrace{K(\omega)}_{} e^{j\Phi(\omega)}$$

fáziskarakterisztika.

amplitúdó karakterisztika

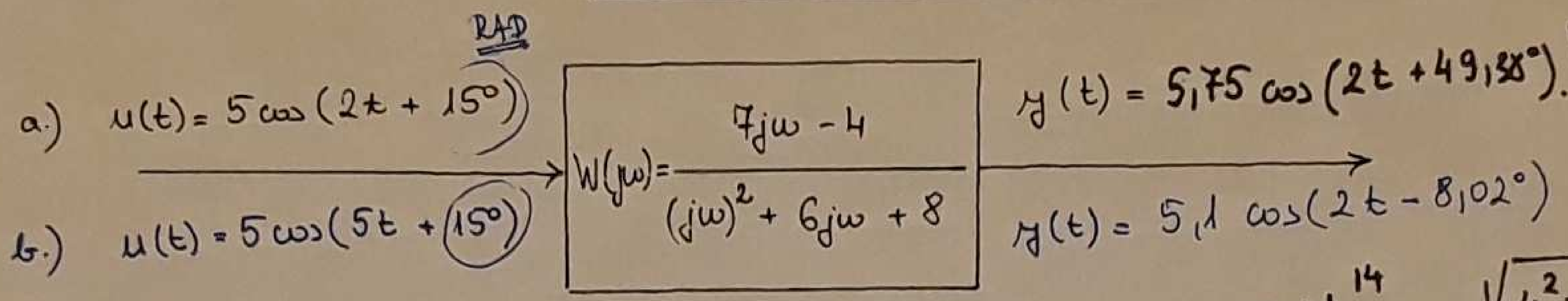
$$\underbrace{\hat{\bar{Y}} = W(j\omega)\hat{\bar{U}}}_{} = \hat{Y}e^{j\beta}$$

$$= K(\omega)e^{j\Phi(\omega)} \cdot \hat{U}e^{j\alpha} = \underbrace{K(\omega)\hat{U}}_{\hat{Y}} e^{j\underbrace{(\Phi(\omega)+\alpha)}_{\beta}}.$$

$$K(\omega) = \frac{\hat{Y}}{\hat{U}}$$

$\Phi(\omega)$ β α

Határozzuk meg az alábbi rendszer <u>válaszjelét stacionárius állapotban</u> !



a.) $u(t) = 5\cos(2t + \overset{\text{RAD}}{15°})$

b.) $u(t) = 5\cos(5t + 15°)$

$$W(j\omega) = \frac{7j\omega - 4}{(j\omega)^2 + 6j\omega + 8}$$

$y(t) = 5{,}75 \cos(2t + 49{,}38°)$.

$y(t) = 5{,}1 \cos(2t - 8{,}02°)$

$y(t) = \underset{Me^{\lambda t}}{y_{tr}(t)} + y_{st}(t) \Longleftarrow$

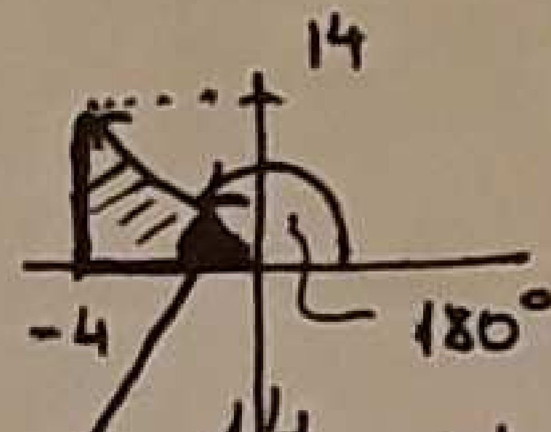
a.) $\hat{u} = 5e^{j15°}$; $\omega = 2$

$$W(j2) = \frac{7\cdot j2 - 4}{\underbrace{(j\cdot 2)^2}_{-4} + 6\cdot j2 + \underline{8}} = \frac{-4 + j14}{4 + j12} = \frac{14{,}56\, e^{j105{,}95°}}{12{,}65\, e^{j71{,}57°}} =$$

$$= \underline{\underline{1{,}15\, e^{j34{,}38°}}}$$

$$\hat{Y} = \bar{W}\big|_{\omega=2} \cdot \hat{u} = 1{,}15\, e^{j34{,}38°} \cdot 5 \cdot e^{j15°} =$$

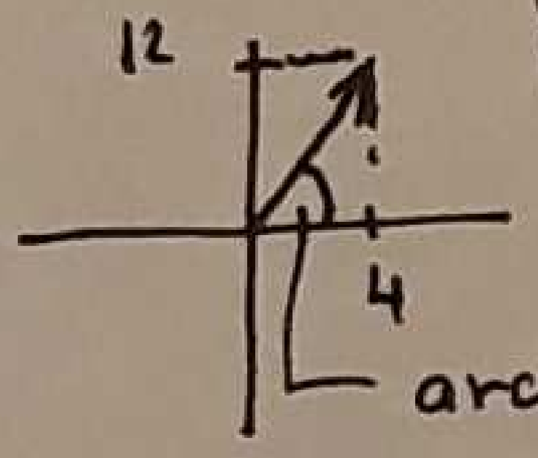
$$= \underline{5{,}75 \cdot e^{j49{,}38°}}$$



$\sqrt{4^2 + 14^2} = \underline{\underline{14{,}56}}$

$180° - 74{,}05° = \underline{\underline{105{,}95°}}$

$\text{arctg}\,\frac{14}{4} = \underline{74{,}05°}$



$\sqrt{4^2 + 12^2} = \underline{\underline{12{,}65}}$

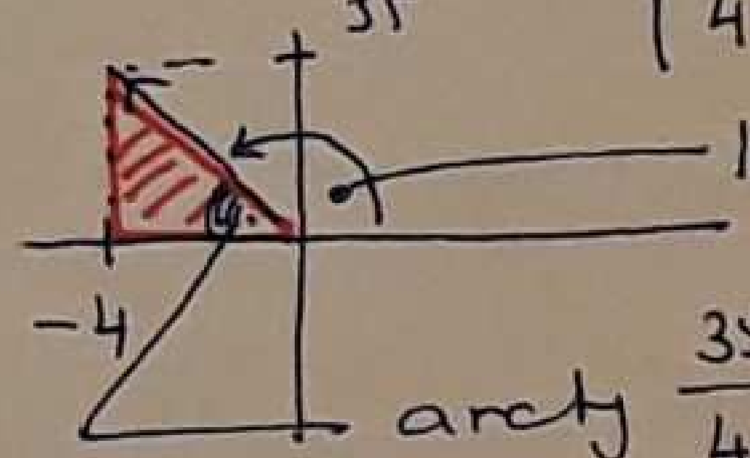
$\text{arctg}\,\frac{12}{\underset{3}{4}} = \underline{\underline{71{,}57°}}$

b.) $\hat{u} = 5e^{15°}$, $\omega = 5$

$$W(j5) = \frac{7\cdot j5 - 4}{\underbrace{(j5)^2}_{-25} + 6\cdot j\cdot 5 + 8} = \frac{-4 + j35}{-17 + j30} = \frac{35{,}23\, e^{j96{,}52°}}{34{,}48\, e^{j119{,}54°}} =$$

$$= \underline{\underline{1{,}02\, e^{-j23{,}02°}}}$$

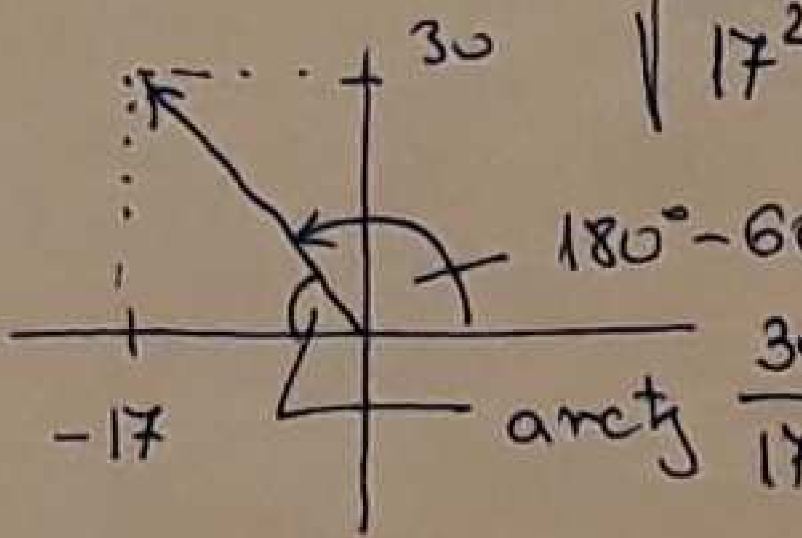
$$\hat{Y} = \bar{W}\big|_{\omega=5}\, \hat{u} = 1{,}02 \cdot e^{-j23{,}02°}\; 5 \cdot e^{j15°} = \underline{5{,}1\, e^{-j8{,}02°}}$$



$\sqrt{4^2 + 35^2} = \underline{\underline{35{,}23}}$

$180° - 83{,}48° = \underline{\underline{96{,}52°}}$

$\text{arctg}\,\frac{35}{4} = \underline{83{,}48°}$



$\sqrt{17^2 + 30^2} = \underline{\underline{34{,}48}}$

$180° - 60{,}46° = \underline{119{,}54°}$

$\text{arctg}\,\frac{30}{17} = \underline{60{,}46°}$

$(t) = 5\cos(2t + 15°)$

$W(j\omega) = \dfrac{7j\omega - 4}{(j\omega)^2 + 6j\omega + 8}$

$y(t) = 5{,}75\cos(2t + 49{,}58°)$. $y(t) = ($ y_{tr} Me^{λ}

$(t) = 5\cos(5t + \boxed{15°})$

$y(t) = 5{,}1\cos(2t - 8{,}02°)$

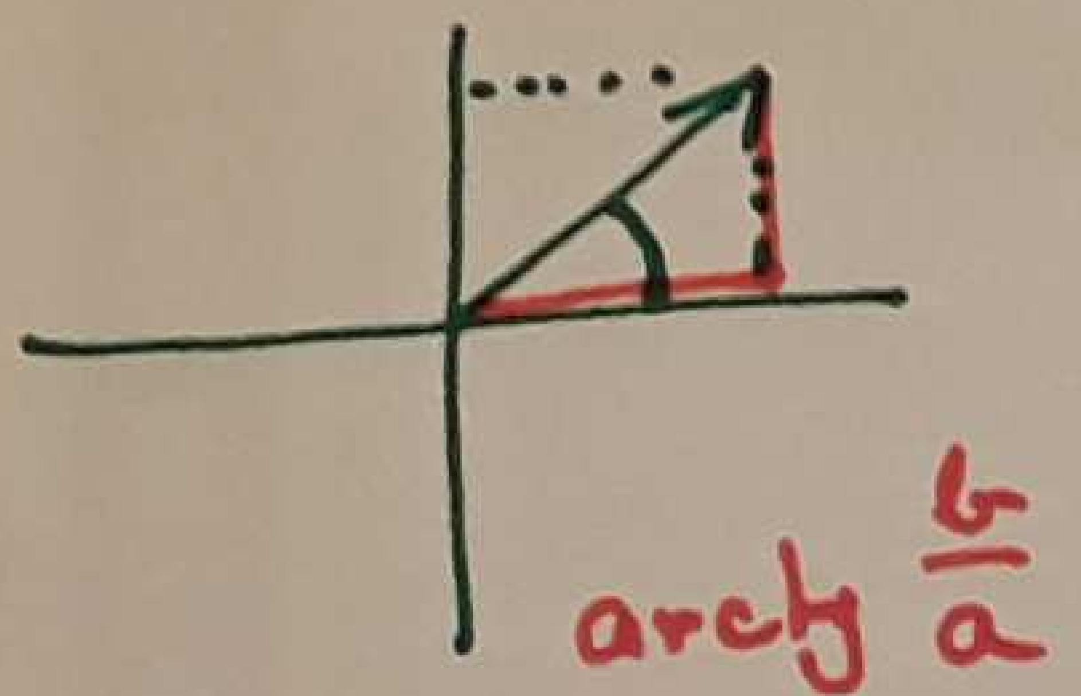
$=2$

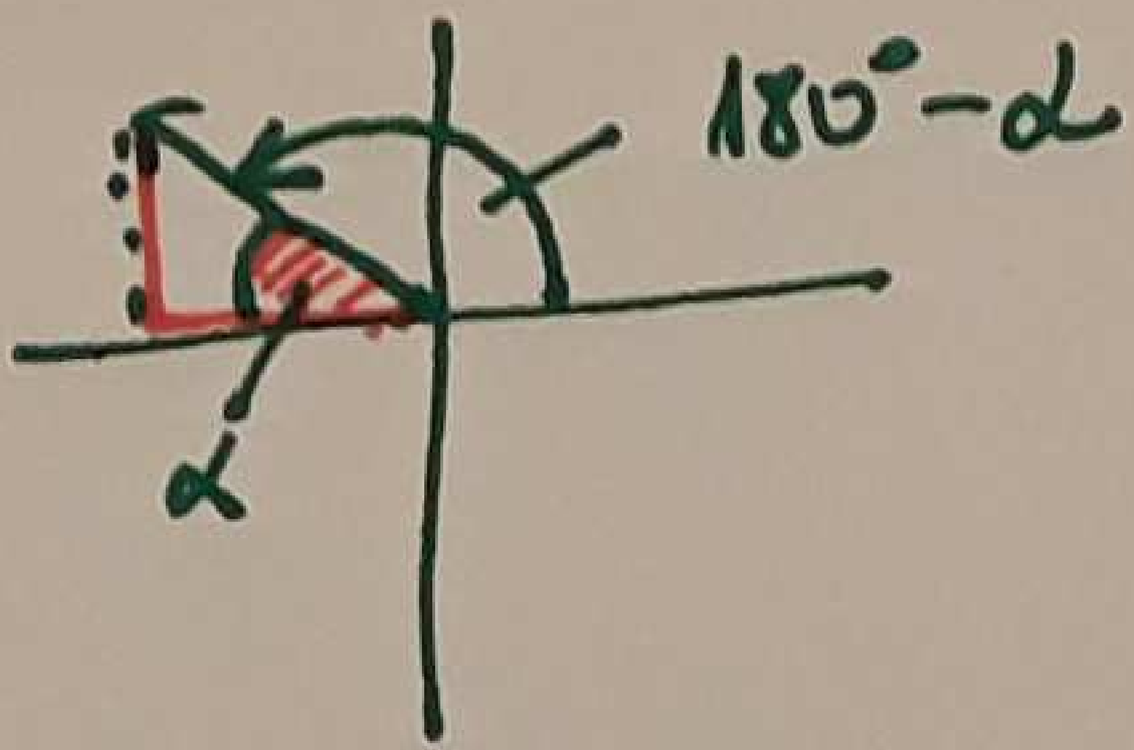
$\pm^{14}$ $\sqrt{4^2 + 14^2} = 1$

$\underbrace{\omega t}_{rad}$ $\frac{rad}{s} \cdot s = rad$

4 $+8$ $=$ $1{,}05° = 105$

$8°$



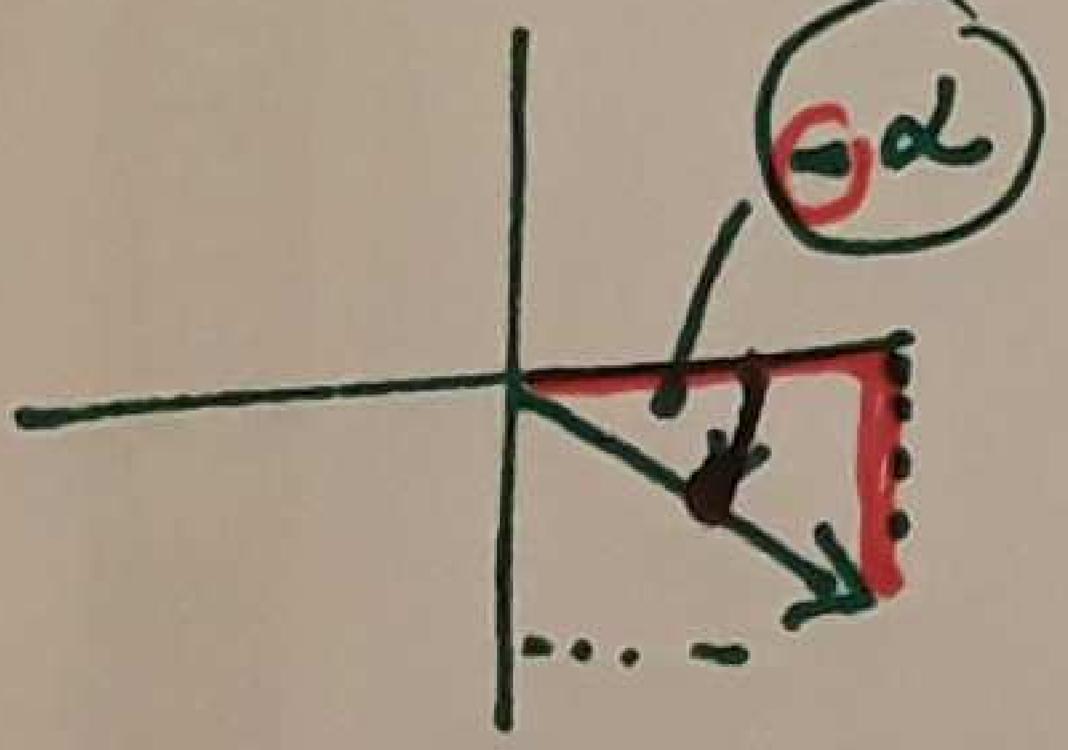


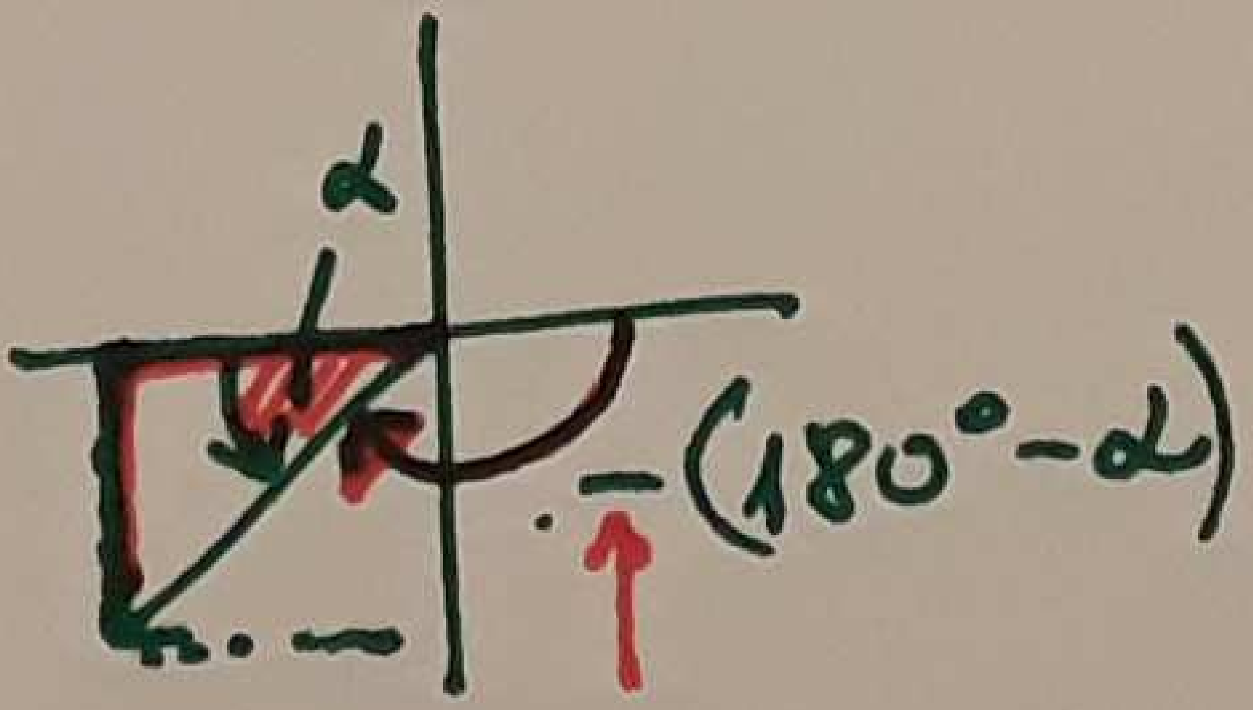
0.... 180°

$+12^2 = 1$

e^{j3}

$\frac{2}{\frac{1}{3}} = 71{,}5$





−180°.... 0°

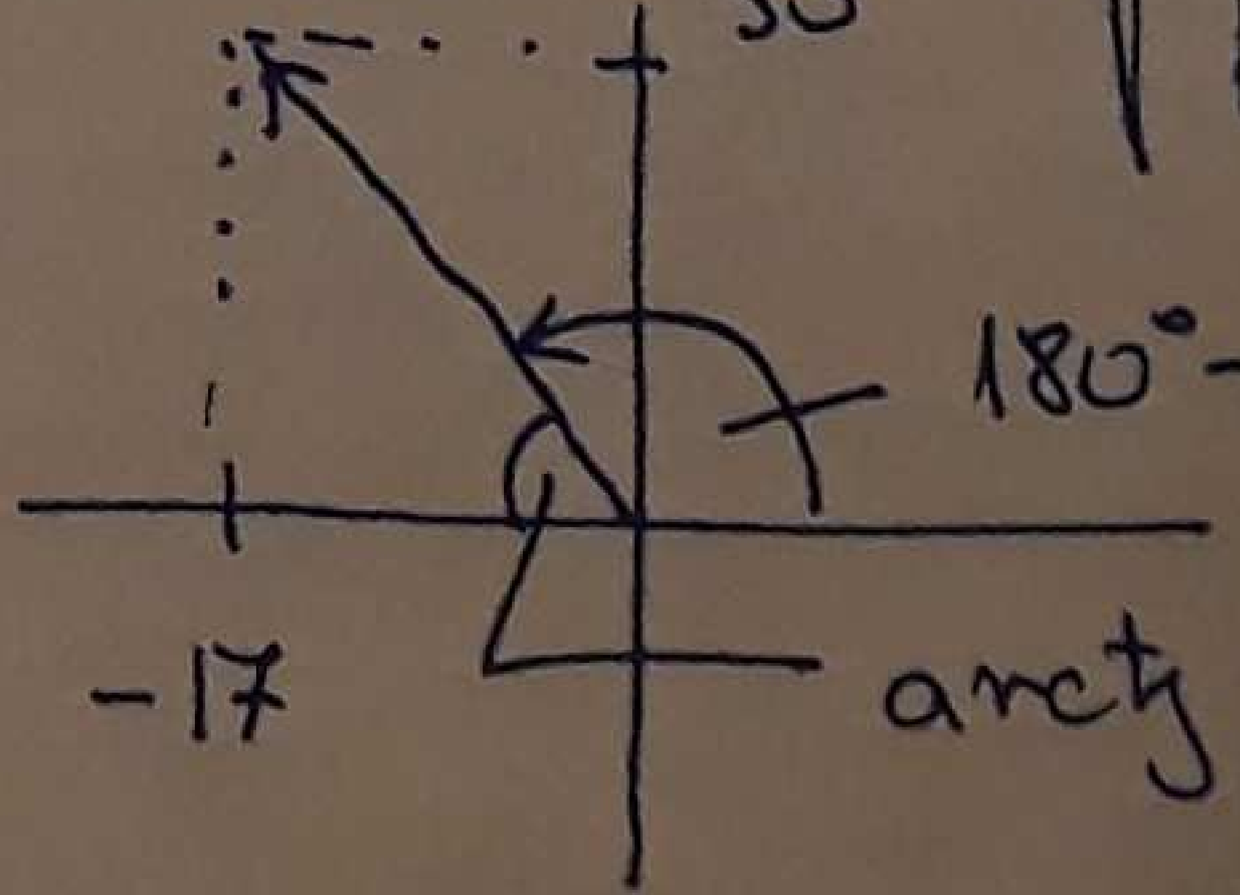
$=$

$35^2 = 3$

$° - 83{,}48°$

$= \underline{83{,}48°}$

$e^{-j\,23{,}02°}$ $5 \cdot e^{j15°} = \underline{\underline{5{,}1\, e^{-j\,8{,}02°}}}$



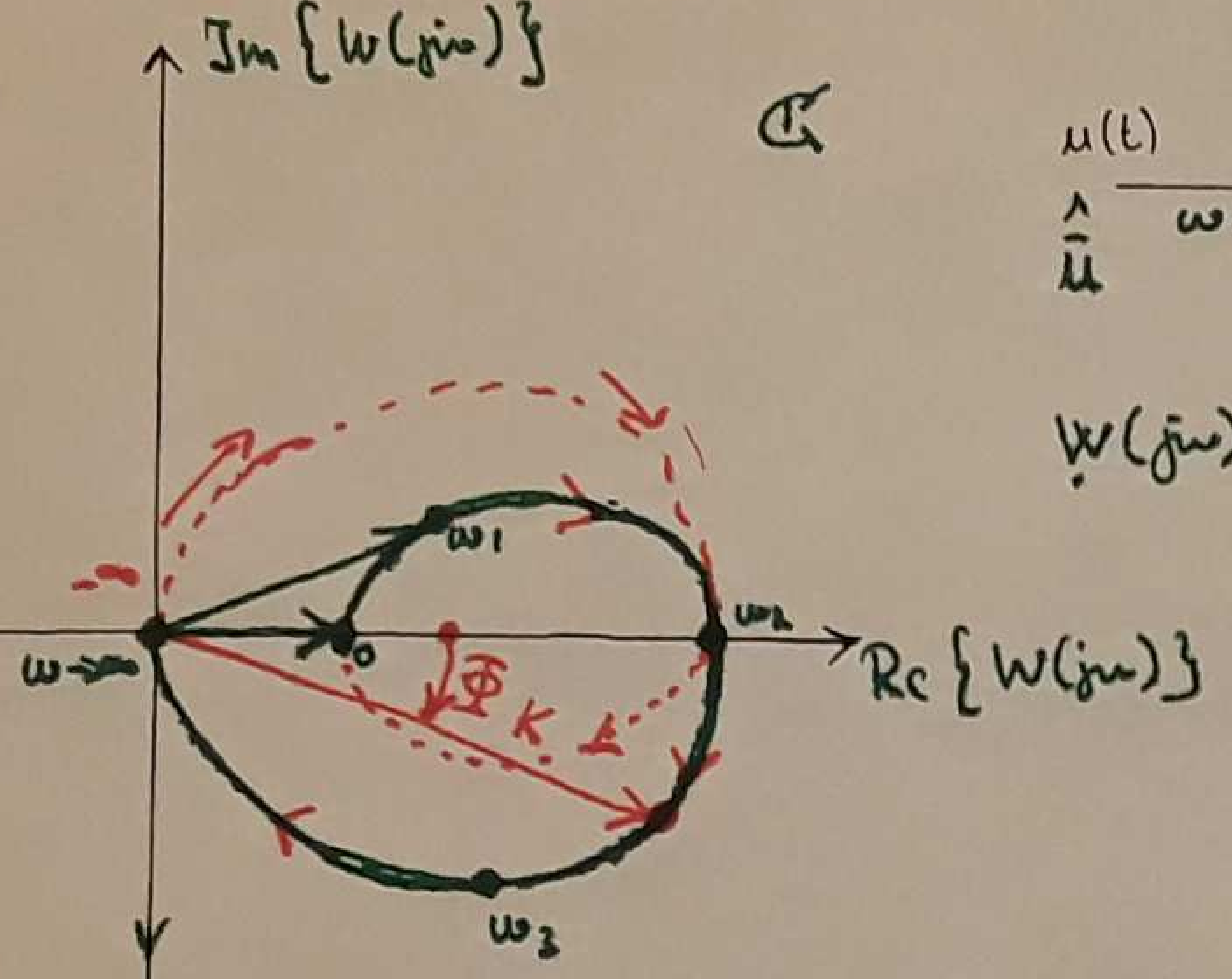
$\sqrt{17^2 + 30^2} =$

$180° - 60{,}46° = \underline{11}$

$\text{arctg}\,\frac{30}{17} = \underline{60{,}46}$

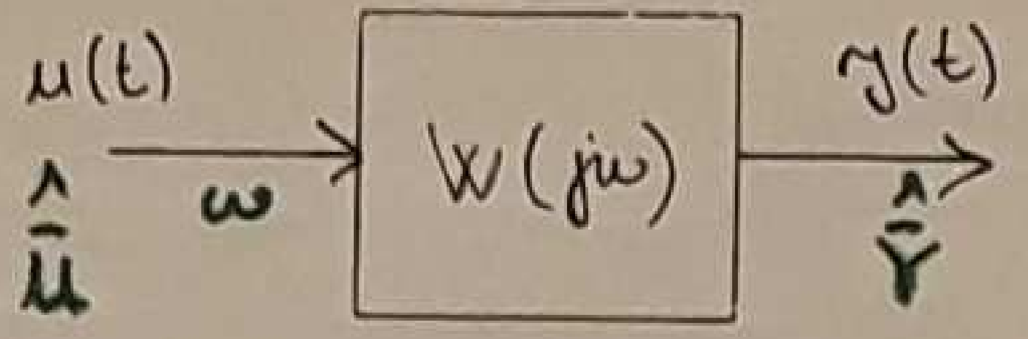
AZ ÁTVITELI KARAKTERISZTIKÁ ÁBRÁZOLÁSA

NYQUIST-DIAGRAM



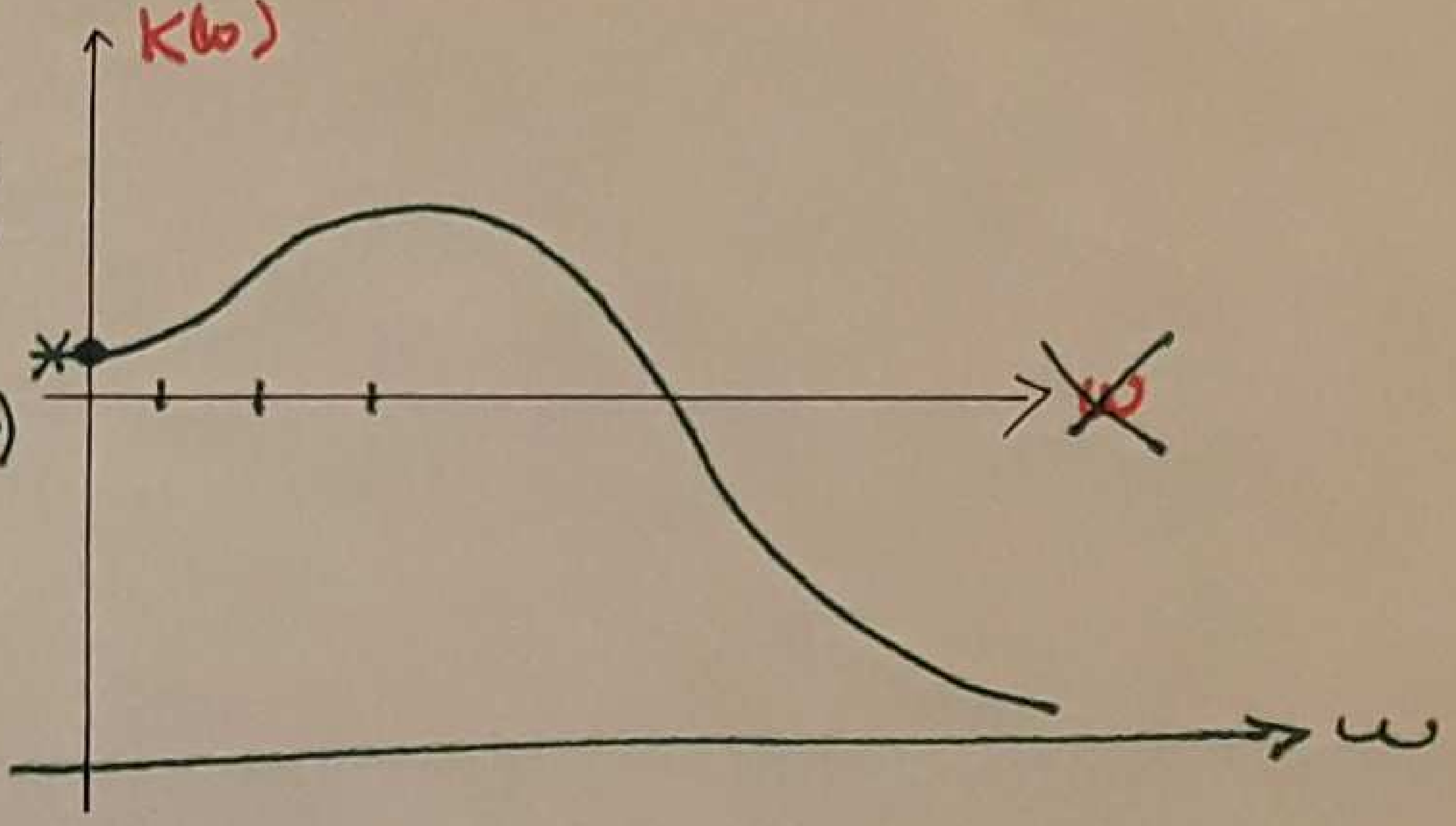
-∞..... 0..... ∞

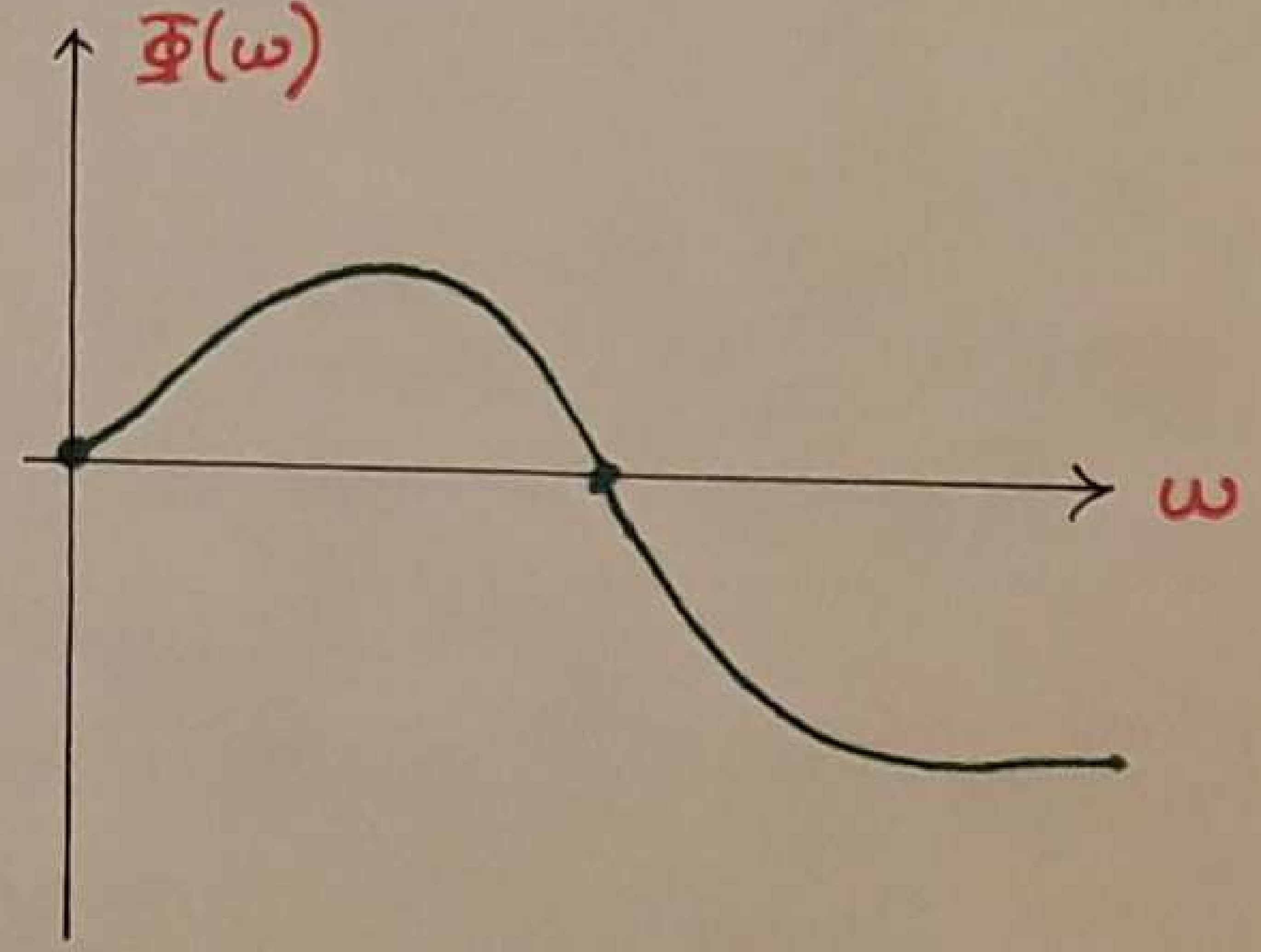
$$W(j\omega) = \frac{\hat{Y} e^{j\beta}}{\hat{U} e^{j\alpha}}$$



$$\underset{\cdot}{W}(j\omega) = \underline{K}(\omega)\, e^{j\underline{\Phi}(\omega)}$$

BODE-DIAGRAM

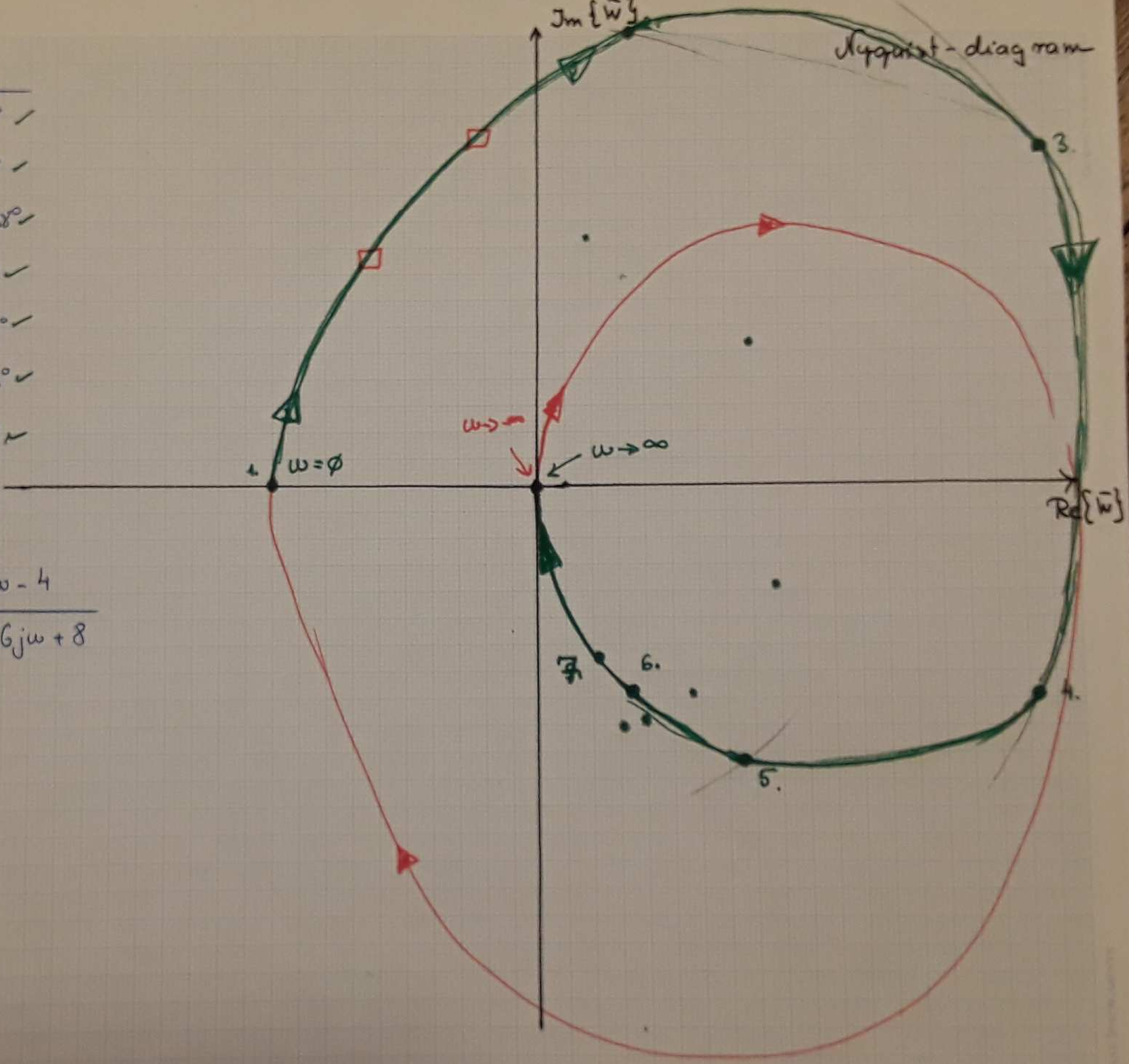


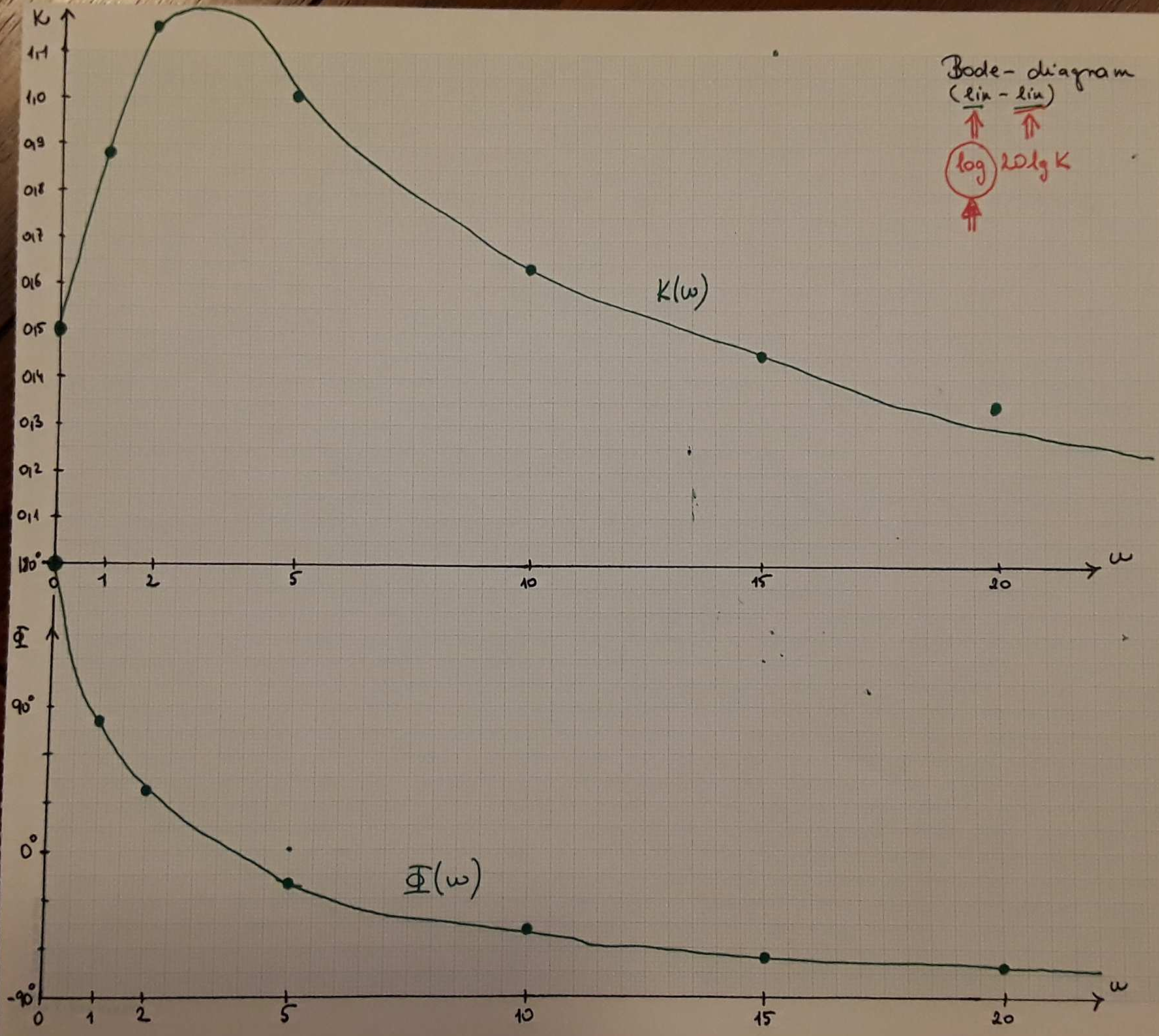


Közelítő

w	K	Φ
0	0,5	180° ✓
1	0,88	79° ✓
2	1,15	34,38° ✓
5	1,02	-23° ✓
10	0,64	-53,6° ✓
15	0,45	-65,3° ✓
20	0,34	-71,3° ✓
	↓ ∅	↓ -90°

$$W(j\omega) = \frac{7j\omega - 4}{(j\omega)^2 + 6j\omega + 8}$$

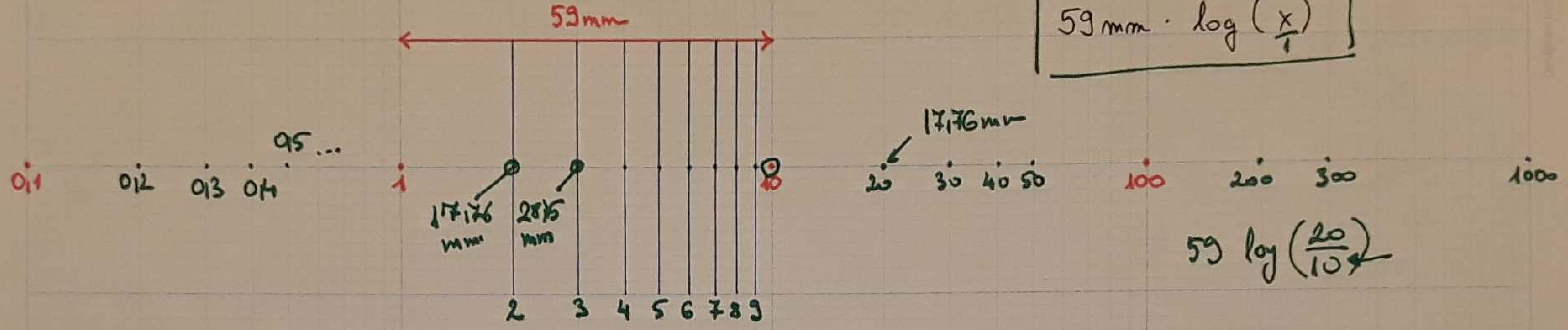




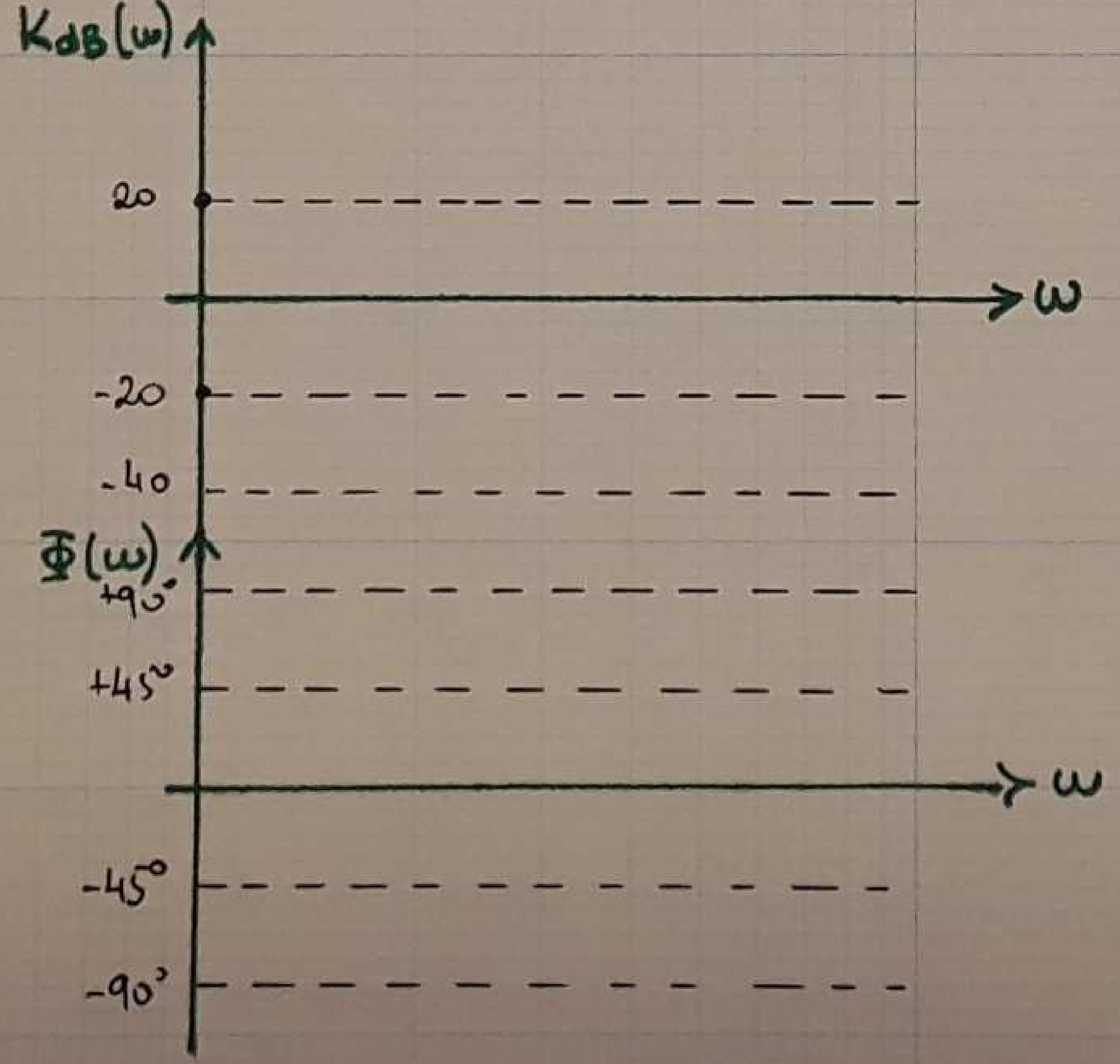
Bode- diagram
(lin - lin)
⇑ ⇑
log 20lg K
⇑

Linlog

$$\boxed{59\,\mathrm{mm} \cdot \log\left(\frac{x}{1}\right)}$$

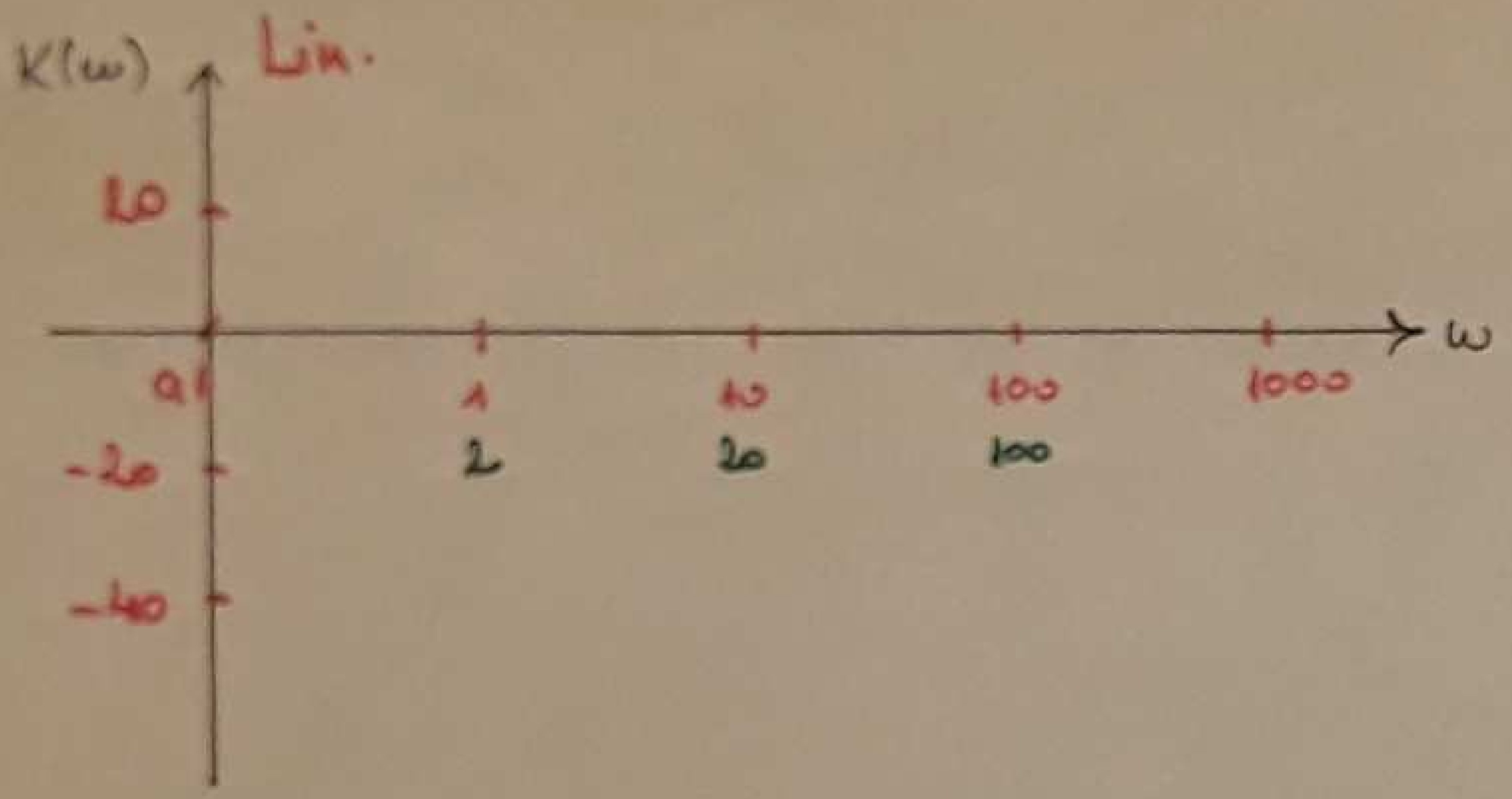


$$59 \log\left(\frac{20}{10}\right)$$



dekád

A BODE-FÉLE TÖRTVONALAS KÖZELÍTÉS



DEKÁD

Log.

$K(\omega)$

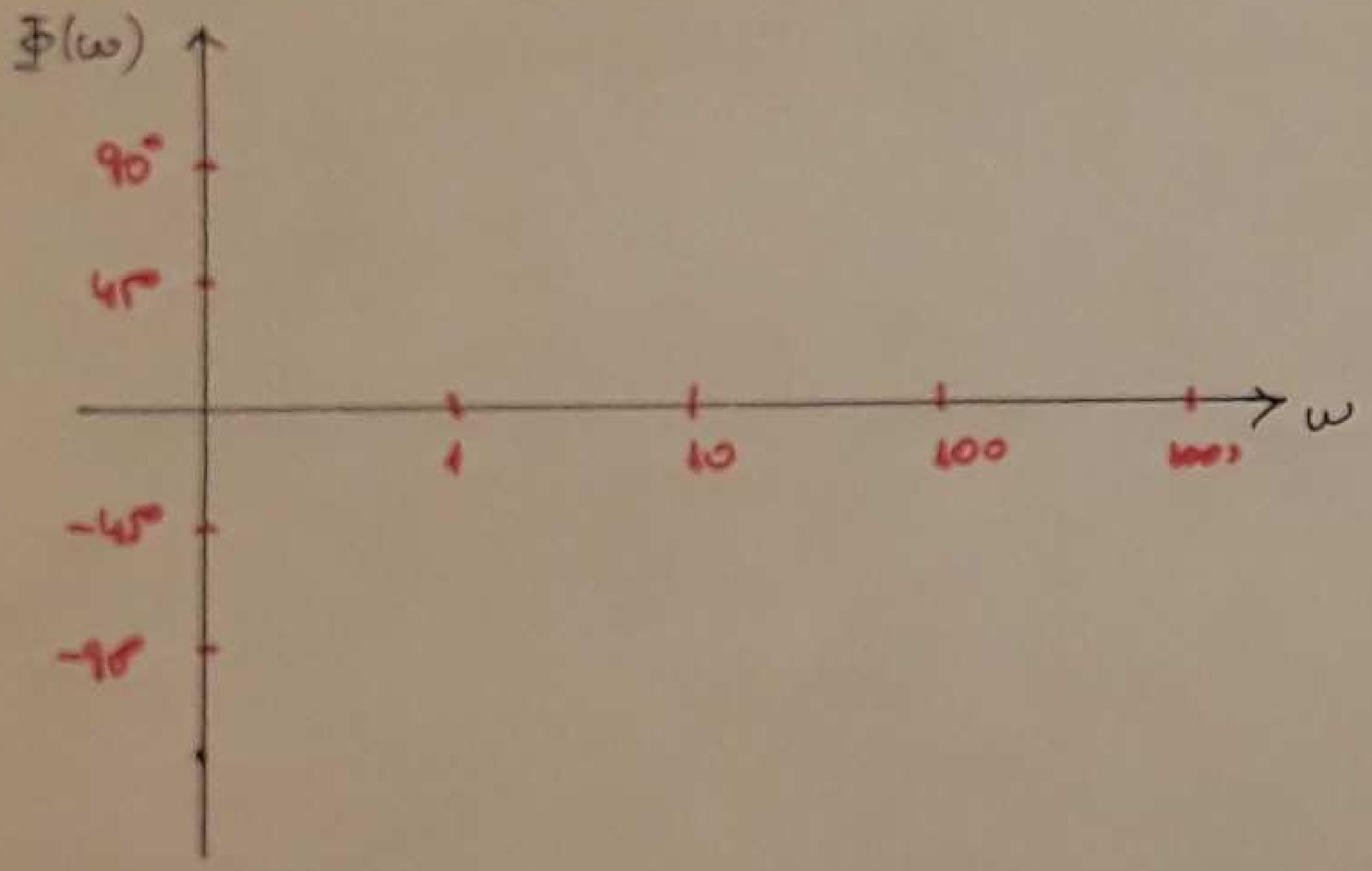
$K_{dB}(\omega) = 20 \lg K(\omega)$

decibel.

=

$$W(j\omega) = \frac{1}{1+j\omega} \qquad \frac{1}{1+\frac{j\omega}{5}} \qquad \frac{j\omega+2}{1+j\omega+(5j\omega)^2}$$

$$\frac{\text{polinom}}{\text{polinom}}$$



$$W(j\omega) = \underbrace{A}\cdot\underbrace{\left(\frac{\Omega_0}{j\omega}\right)^r}\cdot\frac{\prod_i\left(1+\frac{j\omega}{\Omega_i}\right)\cdot\prod_k\left(1+2\xi_k\frac{j\omega}{\Omega_k}+\left(\frac{j\omega}{\Omega_k}\right)^2\right)}{\prod_j\left(1+\frac{j\omega}{\Omega_j}\right)\cdot\prod_l\left(1+2\xi_l\frac{j\omega}{\Omega_l}+\left(\frac{j\omega}{\Omega_l}\right)^2\right)}$$

Ω_0 Ω_i

j

k

l

$\underbrace{\qquad\qquad}$

töréspontí körfrekvencia

$$K(\omega) = |W(j\omega)|$$

$$K(\omega) = |A|\left|\frac{\Omega_0}{j\omega}\right|^r\frac{\prod_i\left|1+\frac{j\omega}{\Omega_i}\right|\;\prod_k\left|1+2\xi_k\frac{j\omega}{\Omega_k}+\left(\frac{j\omega}{\Omega_k}\right)^2\right|}{\prod_j\left|1+\frac{j\omega}{\Omega_j}\right|\;\prod_l\left|1+2\xi_l\frac{j\omega}{\Omega_l}+\left(\frac{j\omega}{\Omega_l}\right)^2\right|}$$

$$20\lg K(\omega)$$

$$\underline{20\lg K} = \underbrace{20\lg|A|}+\underbrace{20r\lg\left|\frac{\Omega_0}{j\omega}\right|}+\underbrace{20\sum_i\lg\left|1+\frac{j\omega}{\Omega_i}\right|}-\underbrace{20\sum_j\lg\left|1+\frac{j\omega}{\Omega_j}\right|}+\underbrace{20\sum_k\lg\left|1+2\xi_k\frac{j\omega}{\Omega_k}+\left(\frac{j\omega}{\Omega_k}\right)^2\right|}-$$

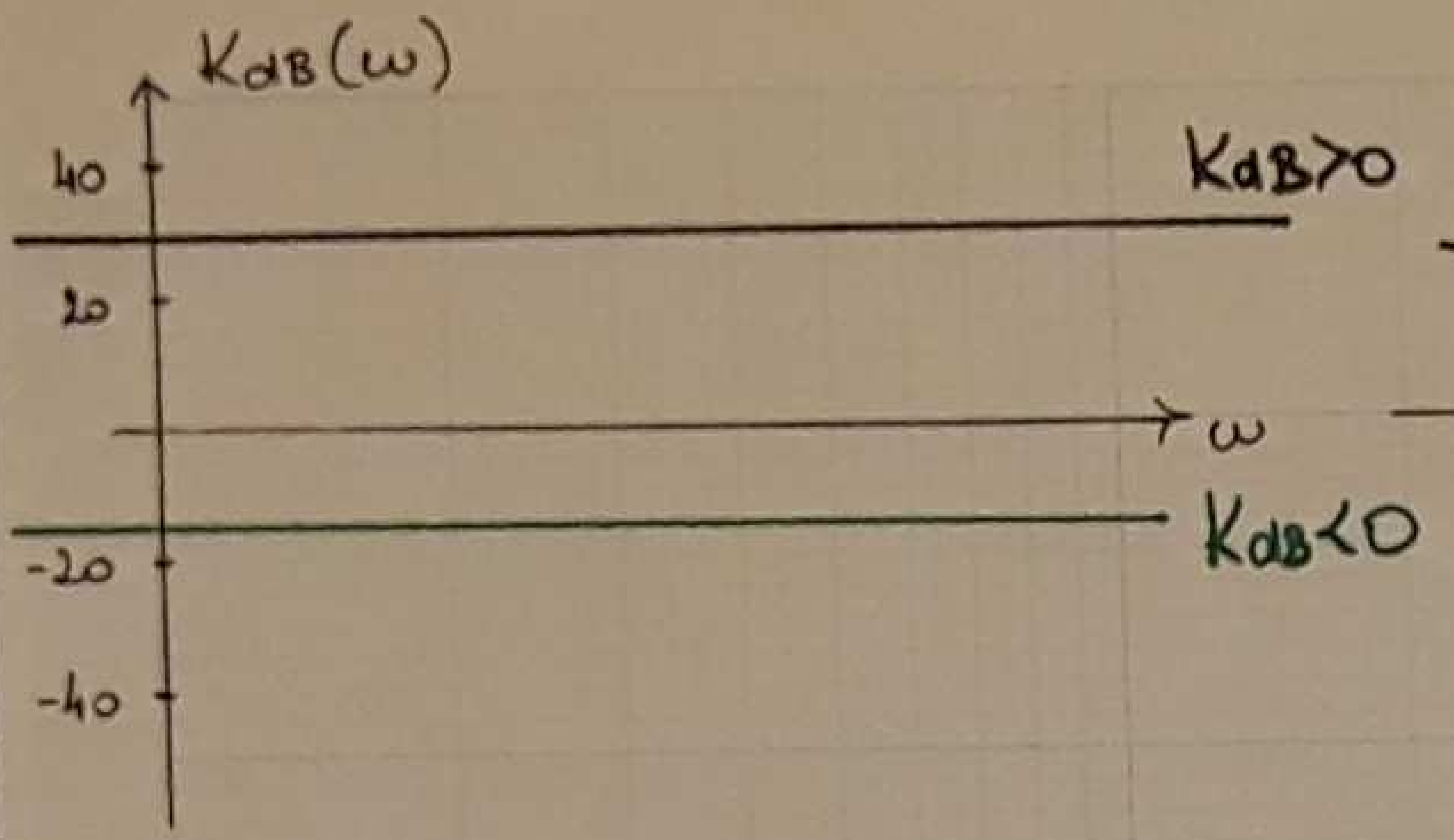
$$\underbrace{-20\sum_l\lg\left|1+2\xi_l\frac{j\omega}{\Omega_l}+\left(\frac{j\omega}{\Omega_l}\right)^2\right|}.$$

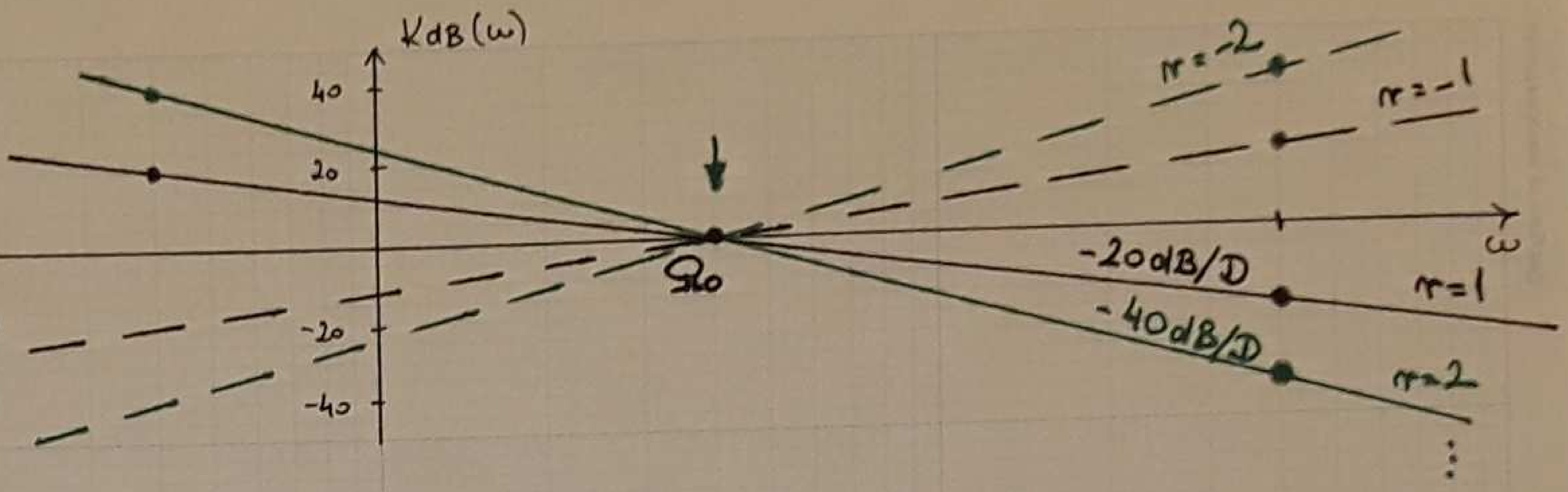
4

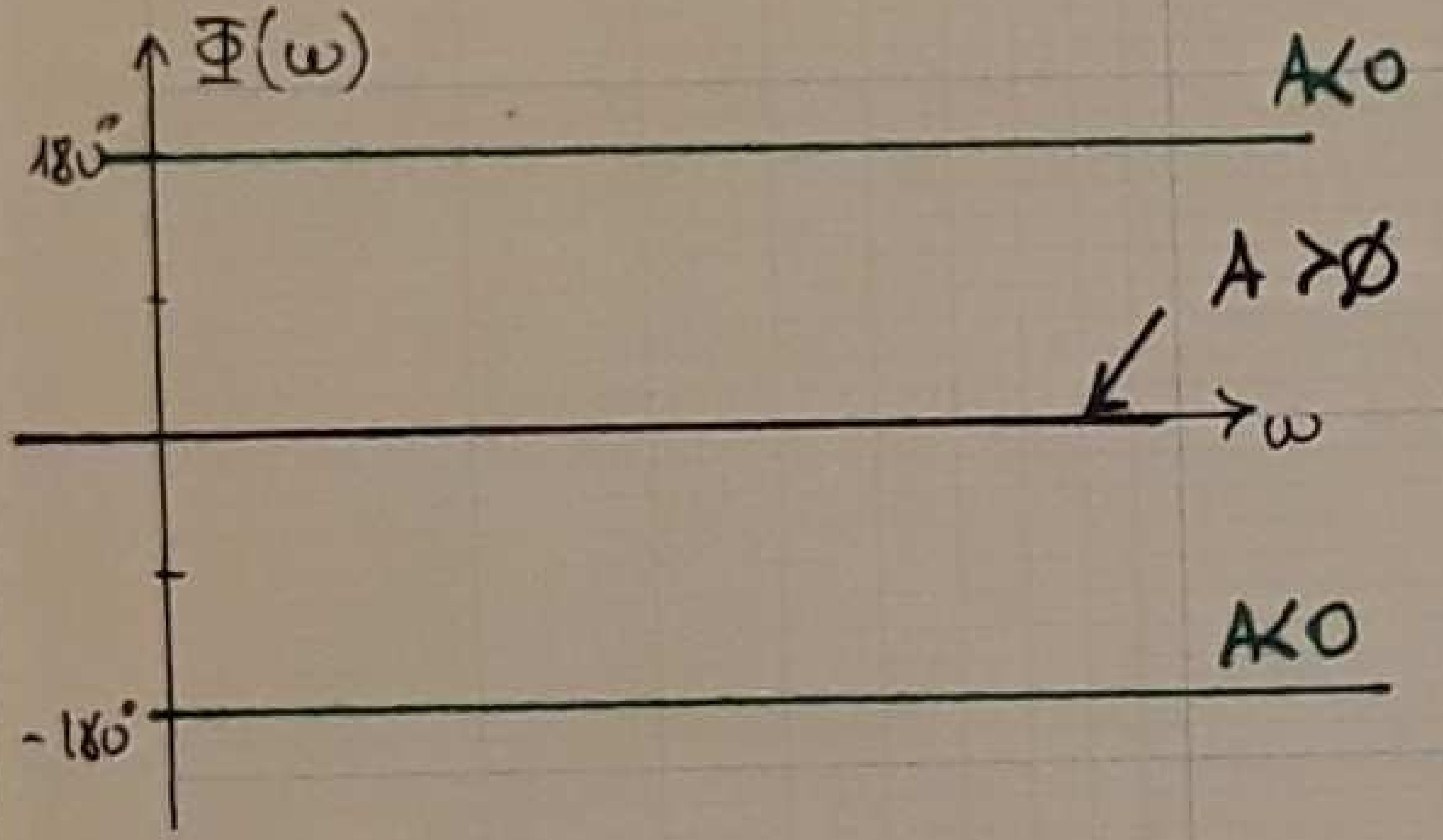
$$\Phi(\omega) = \operatorname{arc} W(j\omega)$$

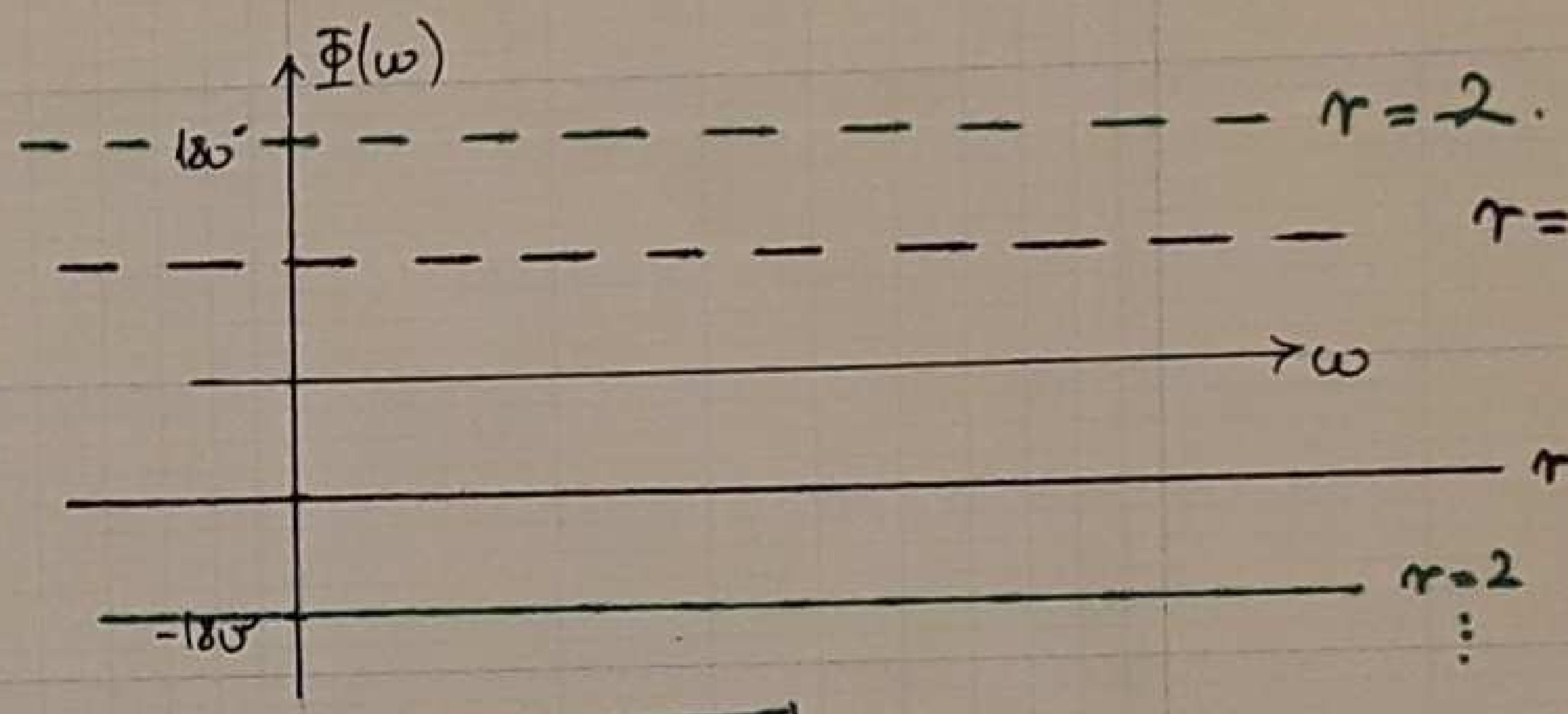
$$\Phi(\omega) = \operatorname{arc}\{A\} + r\operatorname{arc}\left\{\frac{\Omega_0}{j\omega}\right\}+\sum_i\operatorname{arc}\left\{1+\frac{j\omega}{\Omega_i}\right\}-\sum_j\operatorname{arc}\left\{1+\frac{j\omega}{\Omega_j}\right\}+\sum_k\operatorname{arc}\left\{1+2\xi_k\frac{j\omega}{\Omega_k}+\left(\frac{j\omega}{\Omega_k}\right)^2\right\}$$

$$-\sum_l\operatorname{arc}\left\{1+2\xi_l\frac{j\omega}{\Omega_l}+\left(\frac{j\omega}{\Omega_l}\right)^2\right\}.$$









$\boxed{A}$

$$K_{dB}(\omega) = 20\lg|A|$$

$|A|>1$ $K_{dB}>0$

erősítő

$|A|<1$ $K_{dB}<0$

csillapító

$$-5 = 5e^{\pm j180°}$$

$$\boxed{\left(\frac{\Omega_0}{j\omega}\right)^r}$$

$$r\,20\lg\frac{\Omega_0}{\omega}$$

$$r\,\mathrm{arc}\left\{\frac{\Omega_0}{j\omega}\right\}$$

$$\boxed{-90°r}$$

$$\boxed{\begin{array}{c} r\,20\lg\frac{\Omega_0}{10\Omega_0} \\ \hline r\,20\lg\frac{\Omega_0}{0{,}1\Omega_0} \end{array}}$$

$$\left(\frac{j\omega}{\Omega_0}\right)^r = \left(\frac{\Omega_0}{j\omega}\right)^{-r}$$

$$-r\,20\lg\frac{\Omega_0}{\omega}$$

$$+90°r$$

$$W(j\omega) = \frac{1}{1 + 2\boxed{\xi}\frac{j\omega}{\Omega} + \left(\frac{j\omega}{\Omega}\right)^2}$$

$\xi\downarrow$ $K\uparrow$

$\omega = \Omega$:) $\frac{1}{\cancel{1} + 2\xi j + \cancel{j^2}} = \frac{1}{j2\xi}$

$K(\omega) = \frac{1}{2|\xi|} \rightarrow 20 \lg \frac{1}{2|\xi|}$

$\Phi(\omega) = -90°$

$\omega = 0{,}1\Omega$:) $\frac{1}{1 + \cancel{2\xi j\,0{,}1} + \cancel{j^2\,0{,}01}} \rightarrow 1$

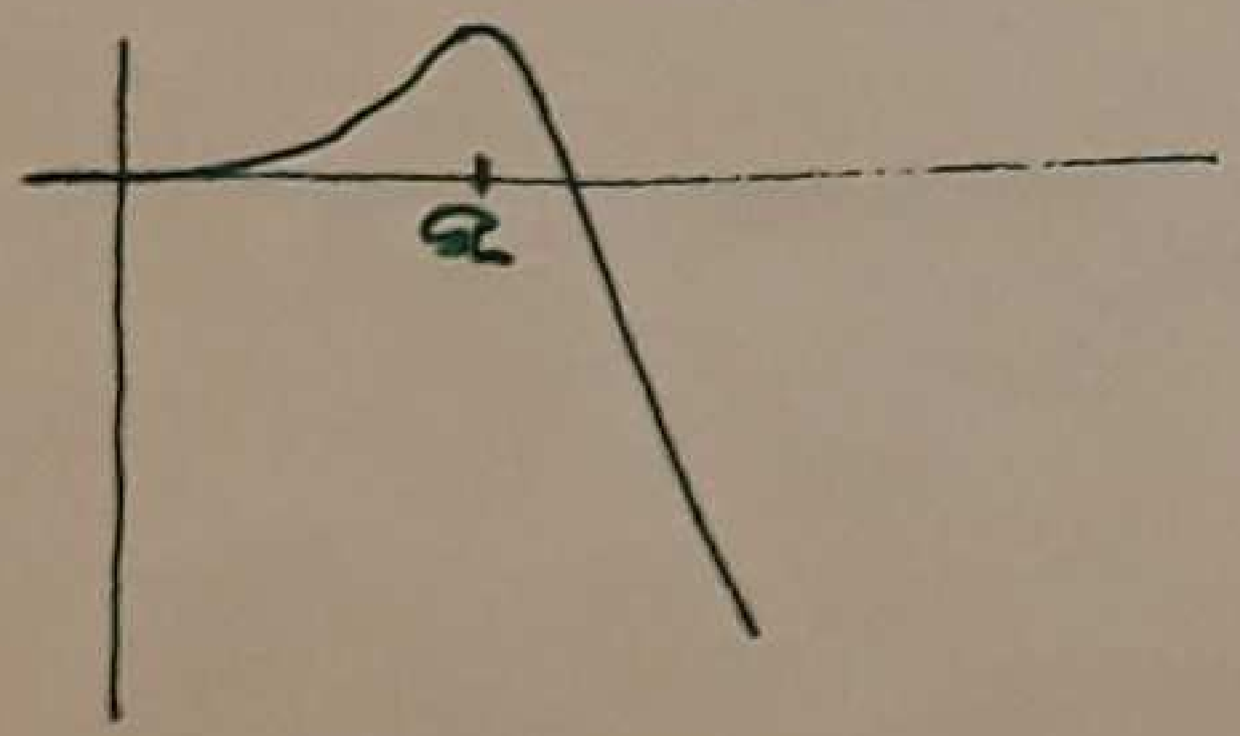
$0 dB$

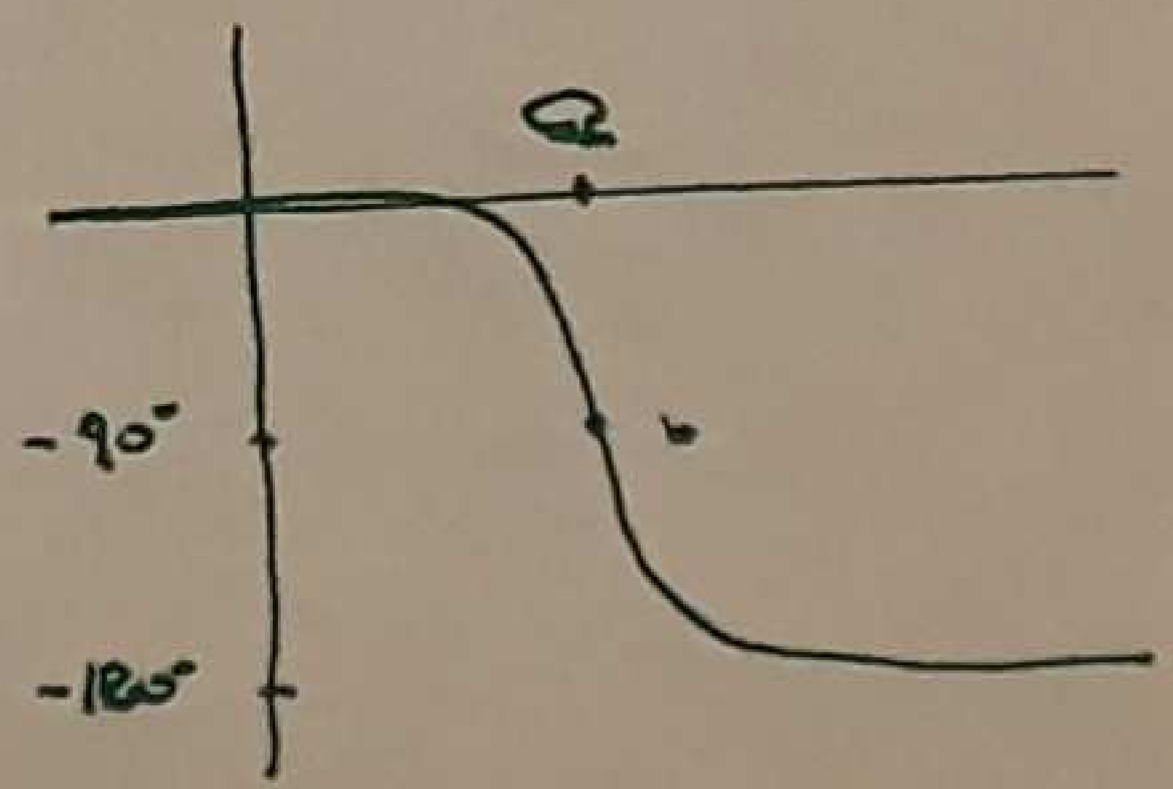
$0°$

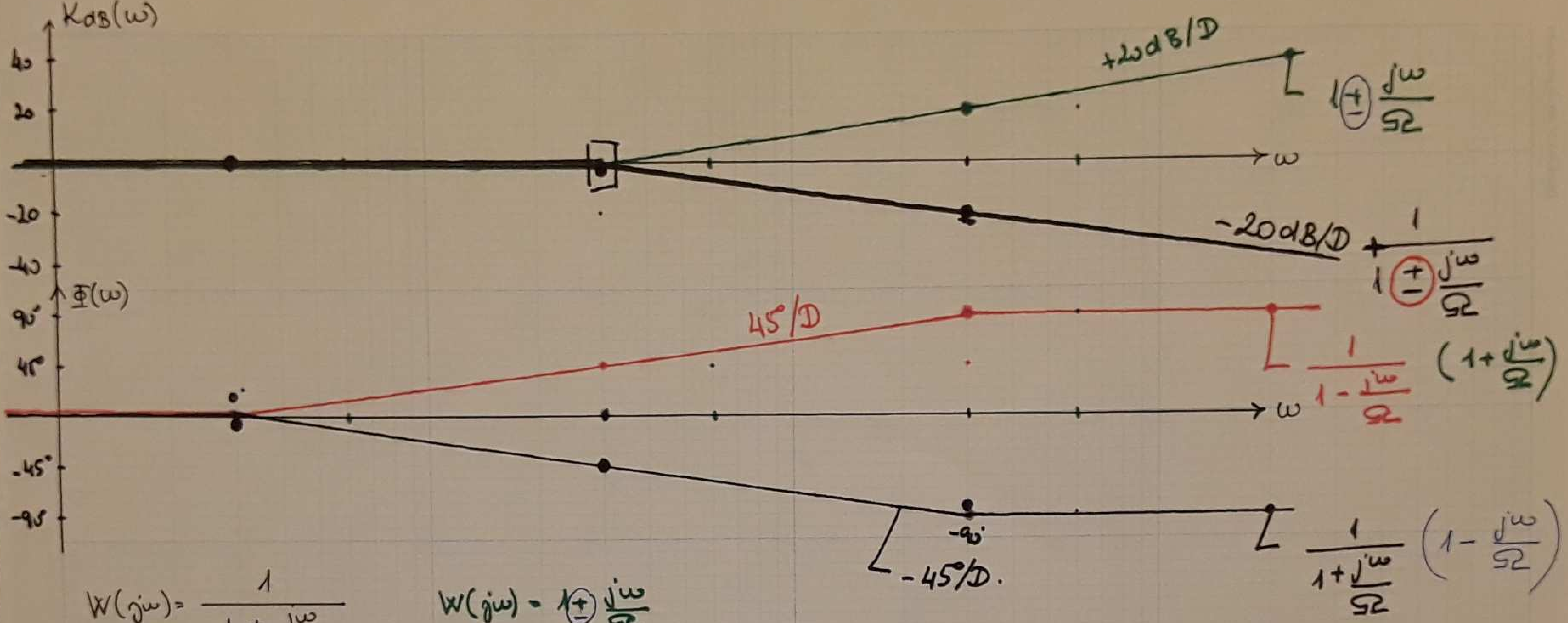
$\omega = 10\Omega$:) $\frac{1}{1 + 2\xi j\,10 - 100} = \frac{1}{-99 + \cancel{j2\xi 10}} \approx -\frac{1}{100}$

$20 \lg \frac{1}{100} = -40 dB$

$-180°$







$W(j\omega) = \dfrac{1}{1 \pm \frac{j\omega}{\Omega}}$ $\qquad$ $W(j\omega) = 1 \pm \dfrac{j\omega}{\Omega}$

$\omega = 0{,}1\Omega:)\ \dfrac{1}{1+j0{,}1}$ $\quad 20\lg 1 = \underline{\underline{0\,dB}}$ $\quad \underline{\underline{-5{,}71^\circ}}$ $\qquad$ $\omega = 0{,}1\Omega:)\ 1+j0{,}1$ $\quad 20\lg 1 = \underline{\underline{0\,dB}}$ $\quad \underline{\underline{\pm 5{,}71^\circ}}$

$\omega = \Omega:)\ \dfrac{1}{1+j}$ $\quad 20\lg \dfrac{1}{\sqrt{2}} = \underline{\underline{-3\,dB}}$ $\quad \underline{\underline{-45^\circ}}$ $\qquad$ $\omega = \Omega:)\ 1+j$ $\quad 20\lg\sqrt{2} = \underline{\underline{+3\,dB}}$ $\quad \underline{\underline{\pm 45^\circ}}$

$\omega = 10\Omega:)\ \dfrac{1}{1+j10}$ $\quad 20\lg 0{,}1 = \underline{\underline{-20\,dB}}$ $\quad \underline{\underline{-84{,}29^\circ}}$ $\qquad$ $\omega = 10\Omega:)\ 1+j10$ $\quad 20\lg 10 = \underline{\underline{+20\,dB}}$ $\quad \underline{\underline{\pm 90^\circ}}$

0,01

0,001 $\qquad -90^\circ + 5{,}71^\circ$

Ábrázoljuk a Bode-féle törtvonalas közelítést!

$$W(j\omega) = \frac{1}{(j\omega+1)(j\omega+5)}$$

$$= \frac{1}{5}\,\frac{1}{\left(1+\frac{j\omega}{1}\right)\left(1+\frac{j\omega}{5}\right)}$$

$\Omega = 1$ $\Omega = 5$

$$= 0{,}2\,\frac{1}{\left(1+\frac{j\omega}{1}\right)\left(1+\frac{j\omega}{5}\right)}$$

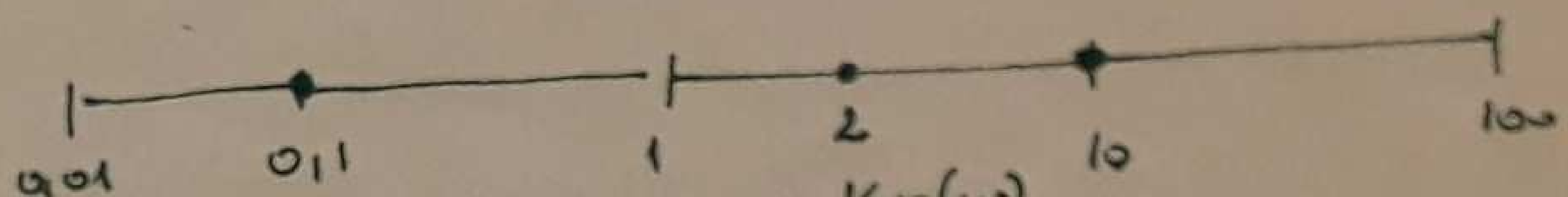
$20\lg 0{,}2 = -13{,}97\,dB \approx \boxed{-14\,dB}$

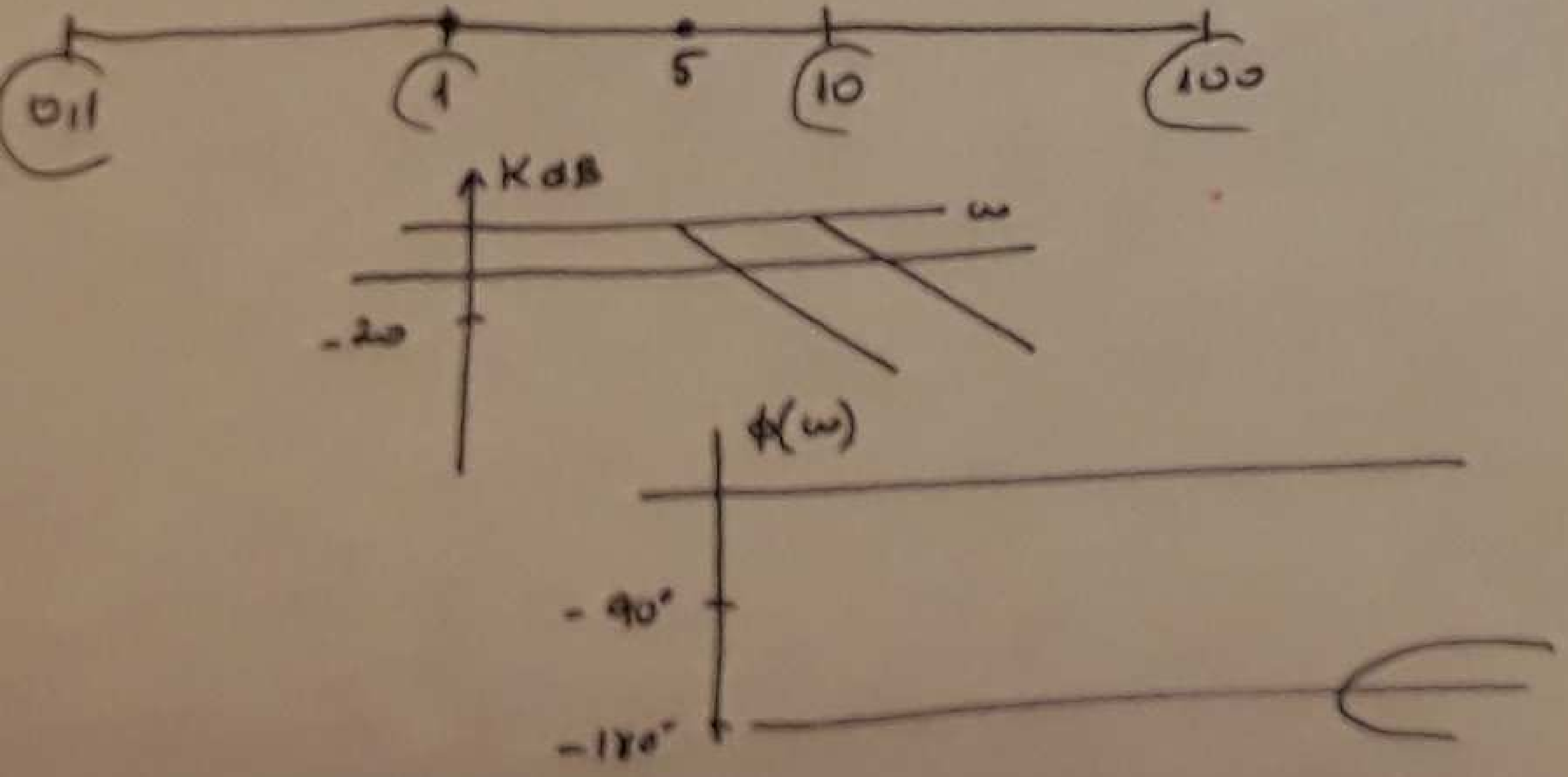
$$W(j\omega) = \frac{(j\omega+2)}{(j\omega+10)(j\omega+0{,}1)}$$

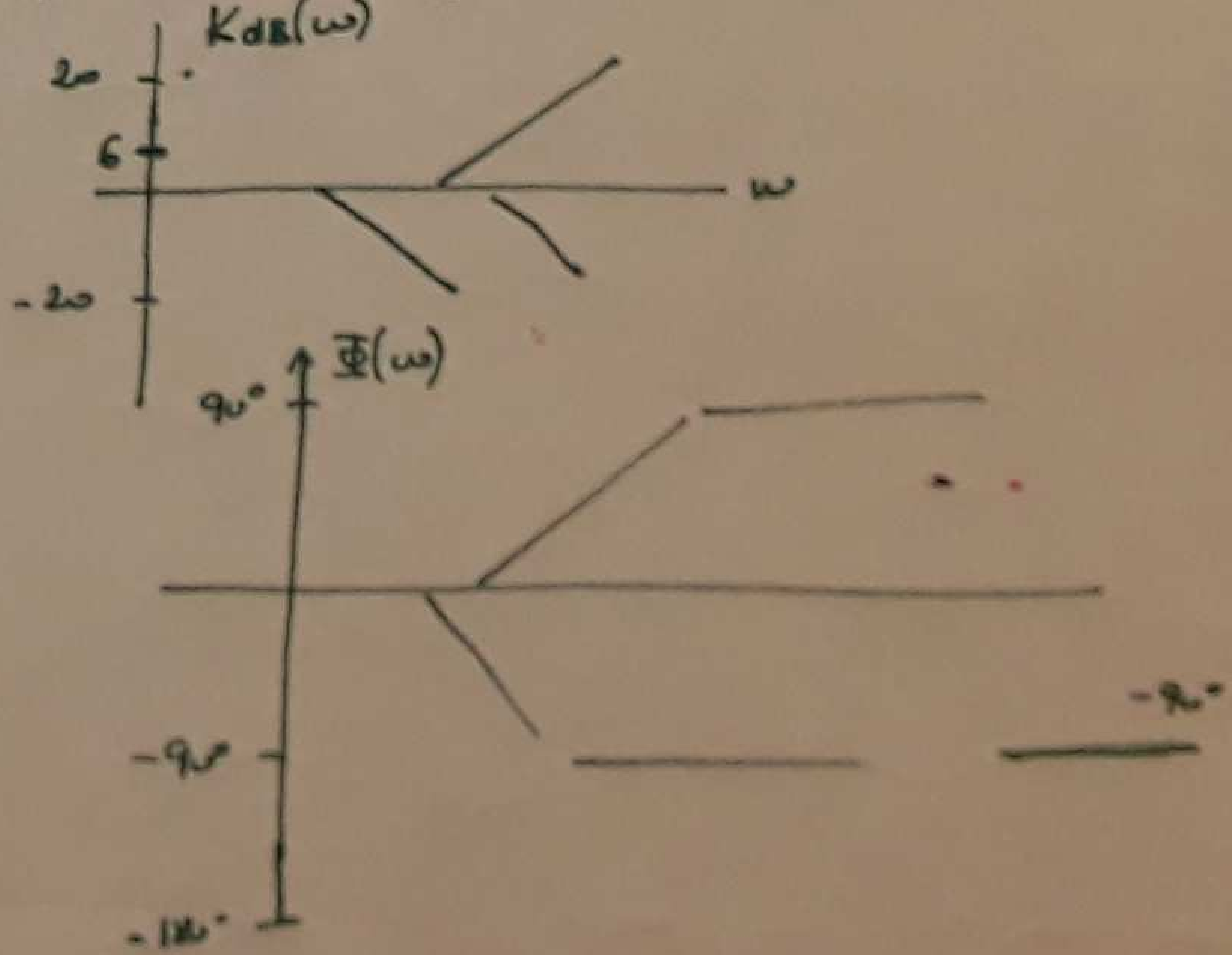
$$= \underbrace{\frac{2}{10\cdot 0{,}1}}_{2}\;\frac{\left(1+\frac{j\omega}{2}\right)}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{0{,}1}\right)}$$

$20\lg 2 = 6\,dB$

$$\frac{1}{1+\frac{j\omega}{\Omega}} \qquad 1+\frac{j\omega}{\Omega'}$$



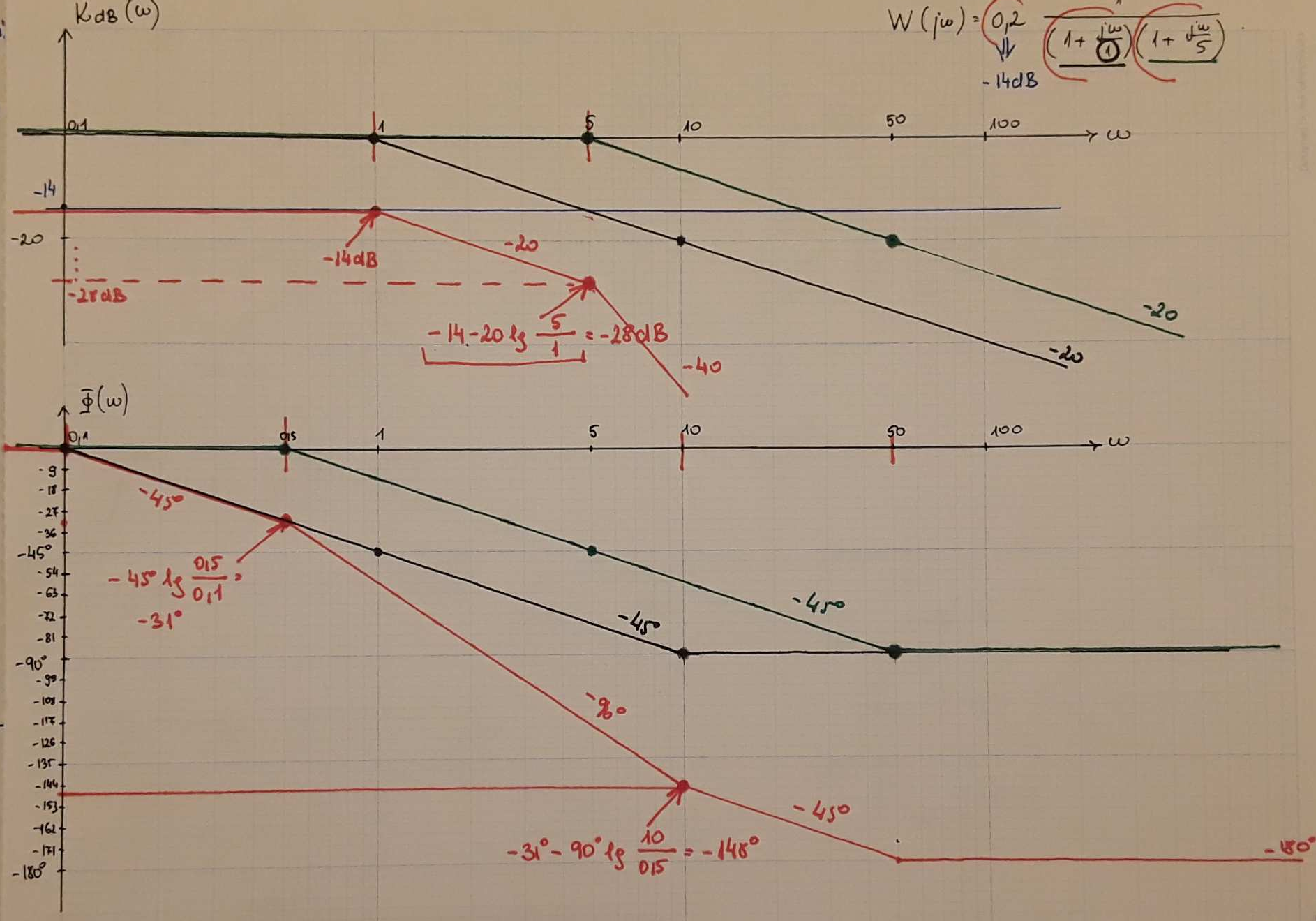




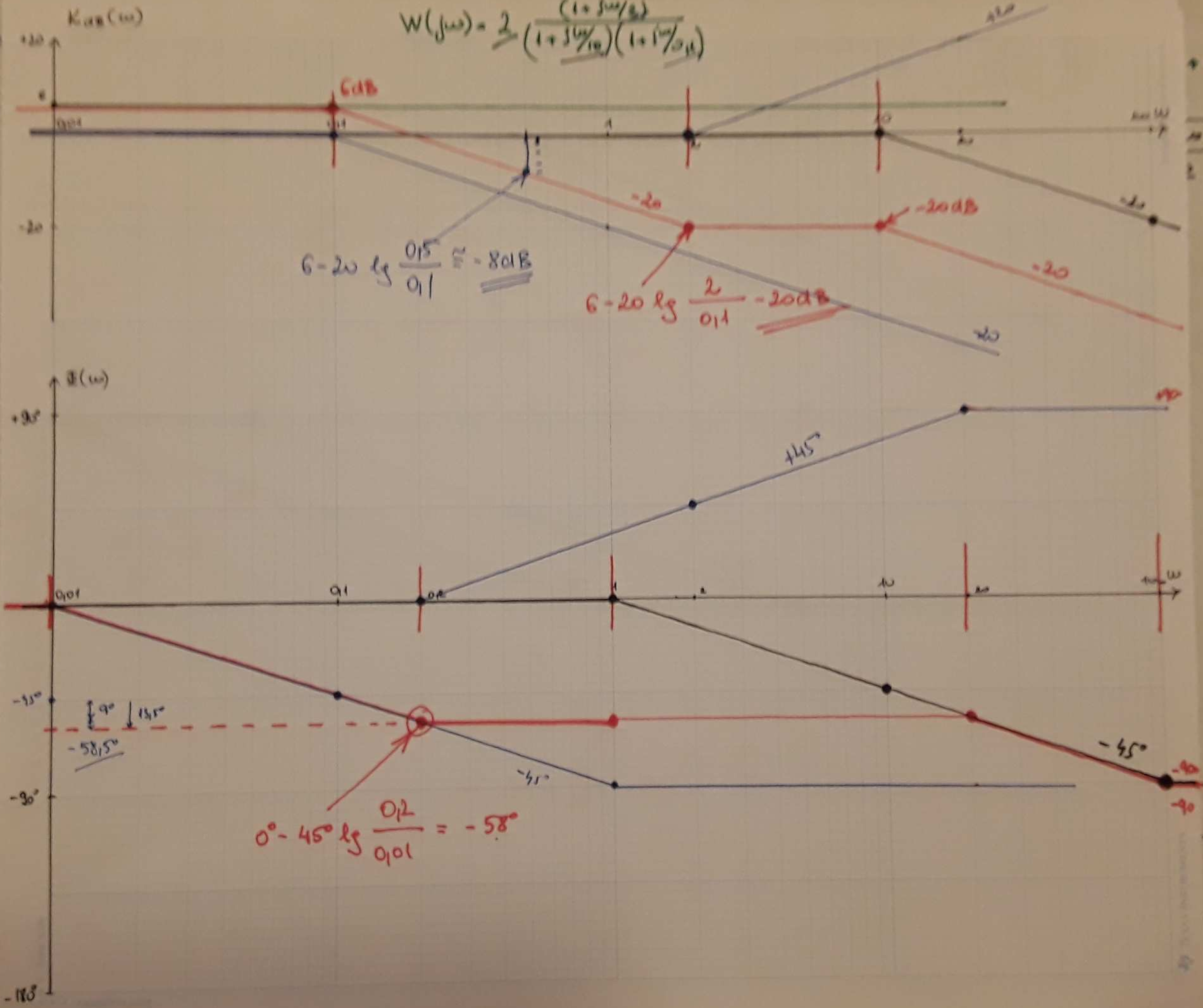
$$W(j\omega) = 0{,}2 \, \frac{1}{\left(1+\frac{j\omega}{1}\right)\left(1+\frac{j\omega}{5}\right)}$$

$\Downarrow$

$-14\,dB$



$$W(j\omega) = 2\frac{(1+j\omega/2)}{(1+j\omega/10)(1+j\omega/0{,}1)}$$



Ábrázolja a Bode-diagramot törtvonalas közelítéssel!

$$\boxed{1 \pm \frac{j\omega}{\Omega}}$$

$$W(j\omega) = \frac{7j\omega - 4}{(j\omega)^2 + 6j\omega + 8}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$j\omega \to x: \quad \overset{a}{1}x^2 + \overset{b}{6}x + \overset{c}{8} = 0$$

$$x_{1,2} = \frac{-6 \pm \sqrt{36 - 4 \cdot 1 \cdot 8}}{2 \cdot 1} = \frac{-6 \pm 2}{2} \begin{cases} -2 \\ -4 \end{cases}$$

$$x^2 + 6x + 8 = (x+2)(x+4)$$

SZORZAT

$$W(j\omega) = \frac{7j\omega - 4}{(j\omega + 2)(j\omega + 4)} = \underbrace{\frac{-4}{2 \cdot 4}} \cdot \frac{1 - \frac{7j\omega}{4}}{\left(1 + \frac{j\omega}{2}\right)\left(1 + \frac{j\omega}{4}\right)} = -0{,}5 \cdot \frac{1 - \frac{j\omega}{(4/7)}}{\left(1 + \frac{j\omega}{2}\right)\left(1 + \frac{j\omega}{4}\right)}$$

$\approx 0{,}6$

$$20 \lg |-0{,}5| = -6\,\text{dB}$$

AZ IDŐTARTOMÁNY ÉS A FREKVENCIATARTOMÁNY ÖSSZEKAPCSOLÁSA (I)

Rendszeregyenlet ⟷ Átviteli karakterisztika

$$\boxed{y^{(n)} + \sum_{i=1}^{n} a_i y^{(n-i)} = \sum_{i=0}^{n} b_i u^{(n-1)}}$$

$$y(t) \longrightarrow \hat{\bar{Y}}$$

$$y'(t) \longrightarrow j\omega \hat{\bar{Y}} \cdot$$

$$y''(t) \longrightarrow (j\omega)^2 \hat{\bar{Y}} \cdot$$

$$\vdots$$

$$\hat{\bar{Y}} \Rightarrow y(t) = \mathrm{Re}\left\{ \underbrace{\hat{\bar{Y}}} \underline{e^{j\omega t}} \right\}$$

$$y'(t) = \mathrm{Re}\left\{ \underbrace{\hat{\bar{Y}} j\omega} \underline{e^{j\omega t}} \right\}$$

$$y''(t) = \mathrm{Re}\left\{ \underbrace{\hat{\bar{Y}} (j\omega)^2} \underline{e^{j\omega t}} \right\}$$

$$(j\omega)^n \underline{\hat{\bar{Y}}} + \sum_{i=1}^{n} a_i (j\omega)^{n-i} \underline{\hat{\bar{Y}}} = \sum_{i=0}^{n} b_i (j\omega)^{n-i} \underline{\hat{\bar{U}}}$$

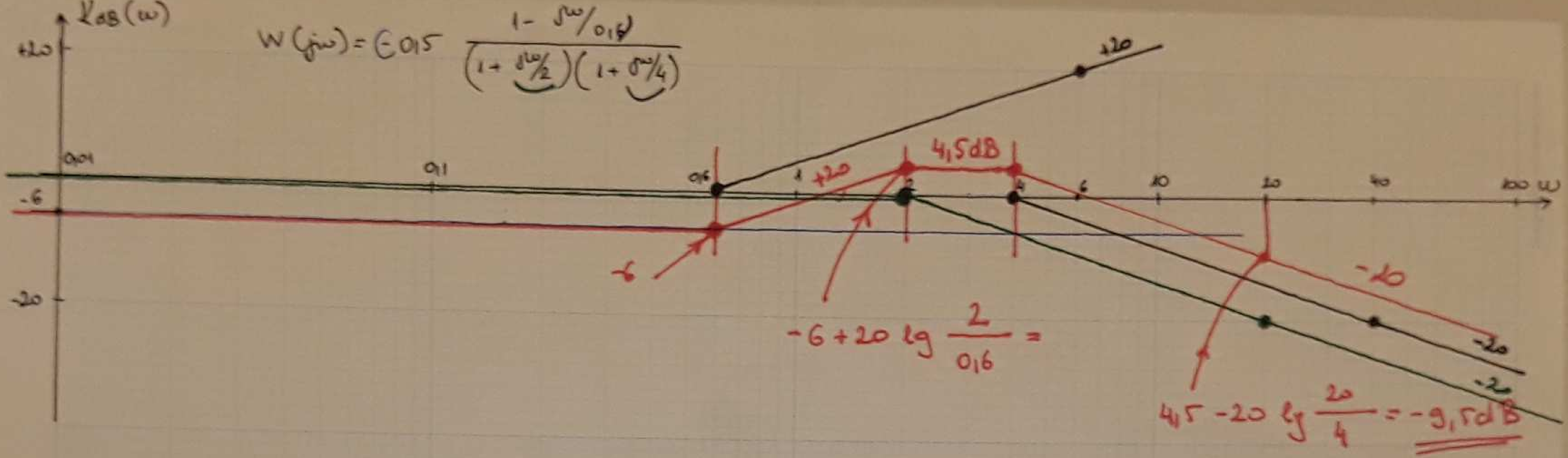
$$\hat{\bar{Y}} \left[(j\omega)^n + \sum_{i=1}^{n} a_i (j\omega)^{n-i} \right] = \hat{\bar{U}} \sum_{i=0}^{n} b_i (j\omega)^{n-i}$$

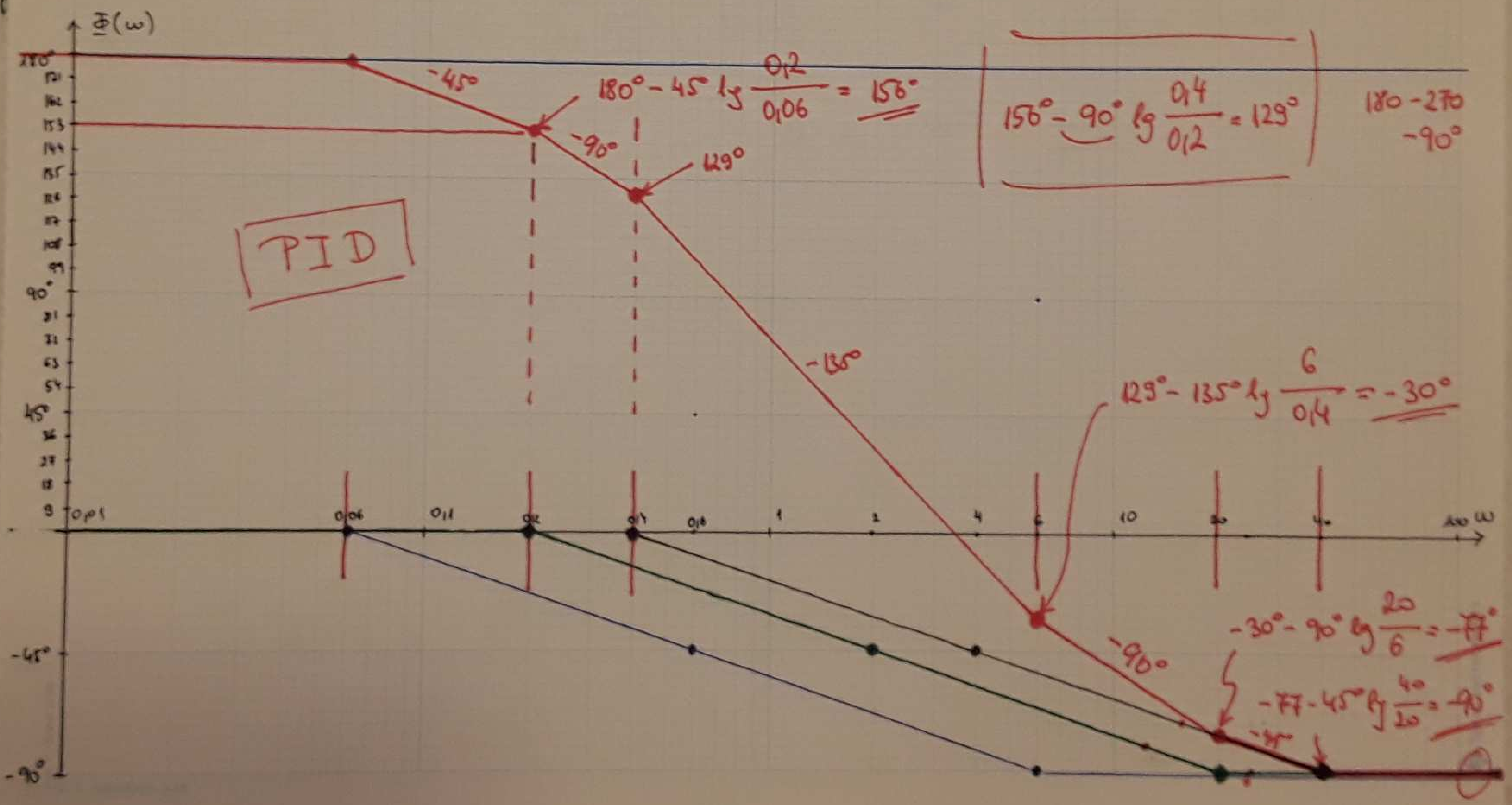
$$W(j\omega) = \frac{\hat{\bar{Y}}}{\hat{\bar{U}}} = \mathrel{\nwarrow\!\!\!\!\nearrow} \frac{\sum_{i=0}^{n} b_i (j\omega)^{n-i}}{(j\omega)^n + \sum_{i=1}^{n} a_i (j\omega)^{n-i}}$$

$$W(j\omega) = \frac{1}{1 + j\omega}$$

$$W(j\omega) = \frac{2 + j\omega}{(j\omega)^2 + 4j\omega + 5}$$

$W(j\omega) = (-)0{,}5\,\frac{1 - j\omega/0{,}6}{(1+j\omega/2)(1+j\omega/4)}$





Adjuk meg az alábbi rendszer átviteli ~~függvényét~~ ! Írjuk fel a rendszeregyenletet !

karakterisztika

$$\begin{array}{l|l}
\dot{x}_1 = -2x_1 - 4x_2 & + u \\
\dot{x}_2 = 0x_1 - 4x_2 & + 2u \\
\hline
y = 3x_1 + 2x_2 & + 0u
\end{array}$$

a.) egyenletrendszer.

$$j\omega \hat{\bar{X}}_1 = \underbrace{-2\hat{\bar{X}}_1} - 4\hat{\bar{X}}_2 + \hat{\bar{u}}$$

$$j\omega \hat{\bar{X}}_2 = -4\hat{\bar{X}}_2 + 2\hat{\bar{u}}$$

$$\hat{\bar{Y}} = 3\hat{\bar{X}}_1 + 2\hat{\bar{X}}_2$$

$$\longrightarrow \underline{(j\omega+2)}\hat{\bar{X}}_1 = \frac{-8}{j\omega+4}\hat{\bar{u}} + 1\hat{\bar{u}} = \frac{-8+j\omega+4}{j\omega+4}\hat{\bar{u}}$$

$$\longrightarrow (j\omega+4)\hat{\bar{X}}_2 = 2\hat{\bar{u}} \longrightarrow \boxed{\hat{\bar{X}}_2 = \frac{2}{j\omega+4}\hat{\bar{u}}}$$

$$\boxed{\hat{\bar{X}}_1 = \frac{j\omega-4}{(j\omega+4)(j\omega+2)}\hat{\bar{u}}}$$

$$\hat{\bar{X}} = \bar{W}_x \hat{\bar{u}}$$

$$\hat{\bar{Y}} = \frac{j3\omega - 12}{(j\omega+4)(j\omega+2)}\hat{\bar{u}} + \frac{4}{j\omega+4}\hat{\bar{u}}$$

$$\hat{\bar{Y}} = \frac{j3\omega - 12 + 4(j\omega+2)}{(j\omega+4)(j\omega+2)}\hat{\bar{u}}$$

$$\bar{W} = \frac{\hat{\bar{Y}}}{\hat{\bar{u}}} = \underline{\underline{\frac{j7\omega - 4}{(j\omega+4)(j\omega+2)}}} = \underline{\underline{\frac{7j\omega - 4}{(j\omega)^2 + 6j\omega + 8}}}$$

Rendszeregyenlet:

$$\frac{\hat{\bar{Y}}}{\hat{\bar{u}}} = \frac{7j\omega - 4}{(j\omega)^2 + 6j\omega + 8}$$

$$\underbrace{(j\omega)^2}\hat{\bar{Y}} + 6j\omega\hat{\bar{Y}} + 8\hat{\bar{Y}} = 7j\omega\hat{\bar{u}} - 4\hat{\bar{u}}$$

$$\underline{y'' + 6y' + 8y = 7u' - 4u}$$

AZ IDŐTARTOMÁNY ÉS A FREKVENCIATARTOMÁNY ÖSSZEKAPCSOLÁSA

I.

Állapotváltozós leírás ⟷ Átviteli karakterisztika

$$\boxed{\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}\,\underline{x} + \underline{b}\,u \\ y &= \underline{c}^T\underline{x} + D\,u \end{aligned}}$$

$$x(t) \longrightarrow \hat{\bar{X}}$$
$$\dot{x}(t) \longrightarrow j\omega\,\hat{\bar{X}}$$
$$\dot{\underline{x}}(t) \longrightarrow j\omega\,\hat{\bar{\underline{X}}}$$

a.) egyenletrendszerként kezelve.

b.) egy kifejezésben.

$$\left\{\begin{aligned} j\omega\,\hat{\bar{\underline{X}}} &= \underline{\underline{A}}\,\hat{\bar{\underline{X}}} + \underline{b}\,\hat{\bar{U}} \\ \hat{\bar{Y}} &= \underline{c}^T\boxed{\hat{\bar{\underline{X}}}} + D\,\hat{\bar{U}} \end{aligned}\right. \longrightarrow$$

$$j\omega\,\hat{\bar{\underline{X}}} - \underline{\underline{A}}\,\hat{\bar{\underline{X}}} = \underline{b}\,\hat{\bar{U}}$$
$$(j\omega\,\underline{\underline{E}} - \underline{\underline{A}})\,\hat{\bar{\underline{X}}} = \underline{b}\,\hat{\bar{U}}$$
$$\hat{\bar{\underline{X}}} = (j\omega\,\underline{\underline{E}} - \underline{\underline{A}})^{-1}\,\underline{b}\,\hat{\bar{U}}$$
$$\hat{\bar{Y}} = \underline{c}^T(j\omega\,\underline{\underline{E}} - \underline{\underline{A}})^{-1}\,\underline{b}\,\underline{\hat{\bar{U}}} + D\,\underline{\hat{\bar{U}}}$$

$$W(j\omega) = \frac{\hat{\bar{Y}}}{\hat{\bar{U}}} = \underline{c}^T(j\omega\,\underline{\underline{E}} - \underline{\underline{A}})^{-1}\,\underline{b} + D$$

$$\underline{\underline{W}}(j\omega) = \underline{\underline{C}}(j\omega\,\underline{\underline{E}} - \underline{\underline{A}})^{-1}\,\underline{\underline{B}} + \underline{\underline{D}}$$

$$(j\omega\,\underline{\underline{E}} - \underline{\underline{A}})^{-1} = \frac{\operatorname{adj}(j\omega\,\underline{\underline{E}} - \underline{\underline{A}})^{-1}}{|j\omega\,\underline{\underline{E}} - \underline{\underline{A}}|}$$

$$W(j\omega) = \frac{\underline{c}^T\operatorname{adj}(j\omega\,\underline{\underline{E}} - \underline{\underline{A}})\,\underline{b} + D\,|j\omega\,\underline{\underline{E}} - \underline{\underline{A}}|}{|j\omega\,\underline{\underline{E}} - \underline{\underline{A}}| \Leftarrow}.$$

$$|\lambda\,\underline{\underline{E}} - \underline{\underline{A}}|$$

b.) $W(j\omega) = \dfrac{\underline{c}^T \text{adj}(j\omega \underline{\underline{E}} - \underline{\underline{A}})\, \underline{b} + D\, \cancel{|j\omega \underline{\underline{E}} - \underline{\underline{A}}|}}{|j\omega \underline{\underline{E}} - \underline{\underline{A}}|}$

$\underline{\underline{A}} = \begin{bmatrix} -2 & -4 \\ 0 & -4 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \underline{c}^T = [3 \;\; 2]$

$D = 0.$

$$|j\omega \underline{\underline{E}} - \underline{\underline{A}}| = \left| \begin{bmatrix} j\omega & 0 \\ 0 & j\omega \end{bmatrix} - \begin{bmatrix} -2 & -4 \\ 0 & -4 \end{bmatrix} \right| = \begin{vmatrix} j\omega+2 & 4 \\ 0 & j\omega+4 \end{vmatrix} = \underbrace{(j\omega+2)(j\omega+4)} - 4 \cdot 0$$

$$\text{adj}(j\omega \underline{\underline{E}} - \underline{\underline{A}}) \rightarrow \text{adj}\begin{pmatrix} j\omega+2 & 4 \\ 0 & j\omega+4 \end{pmatrix} \rightarrow \begin{pmatrix} j\omega+4 & 0 \\ 4 & j\omega+2 \end{pmatrix} \xrightarrow[\substack{+-+ \\ -+- \\ +-+}]{\substack{+- \\ -+}} \begin{pmatrix} j\omega+4 & 0 \\ -4 & j\omega+2 \end{pmatrix} \xrightarrow{T} \underline{\underline{\begin{pmatrix} j\omega+4 & -4 \\ 0 & j\omega+2 \end{pmatrix}}}$$

$$\underbrace{[\underline{3} \;\; \underline{2}] \begin{bmatrix} j\omega+4 & -4 \\ 0 & j\omega+2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$$

$$\begin{bmatrix} j\omega+4 + -8 \\ 0 + j2\omega+4 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} j\omega - 4 \\ j2\omega + 4 \end{bmatrix}}$$

$$3(j\omega - 4) + 2(j2\omega + 4) = 3j\omega - 12 + 4j\omega + 8 = \underline{\underline{7j\omega - 4}}$$

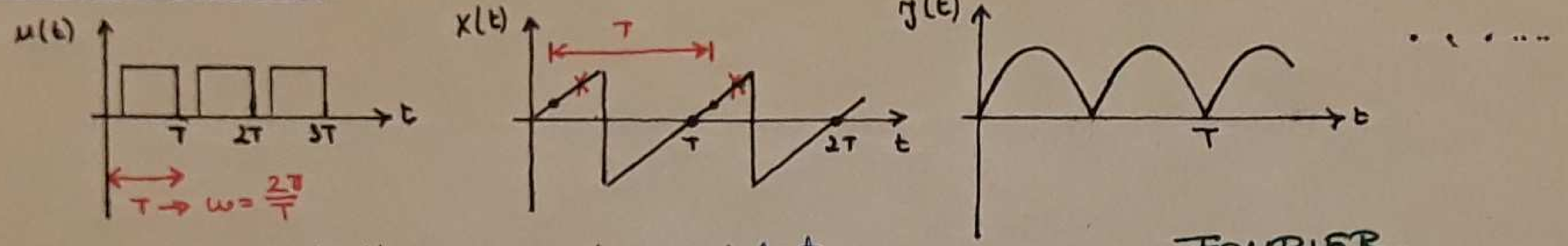
$$\underline{\underline{W(j\omega) = \frac{7j\omega - 4}{(j\omega+2)(j\omega+4)}}}$$

FOLYTONOS IDEJŰ RENDSZEREK ANALÍZISE A FREKVENCIATARTOMÁNYBAN

II.

PERIODIKUS JELRE ADOTT VÁLASZ.

Periodikus jel



. . . .

$\boxed{u(t) = u(t+T)}$

$u(t+T) = u(t)$

$\omega = \frac{2\pi}{T}$ Alapharmonikus.

A válaszjel számítása - alapgondolat

FOURIER.

$$u(t) \xrightarrow{} \boxed{\begin{matrix}\text{Lineáris}\\ \text{Invariáns}\\ \text{Kauzális}\end{matrix}} \xrightarrow{}$$

$W(j\omega)$

$$u(t) \cong U_0 + \sum_{k=1}^{n} \boxed{\hat{U}_k \cos(k\omega t + \alpha_k)}$$

$$y(t) = Y_0 + \sum_{k=1}^{n} \hat{Y}_k \cos(k\omega t + \beta_k)$$

$U_0 \longrightarrow W \cdot U_0 \quad Y_0$

$\hat{U}_1 \cos(\omega t + \alpha_1) \longrightarrow W \hat{U}_1 e^{j\alpha_1} \quad \hat{Y}_1 \cos(\omega t + \beta_1)$

$\hat{U}_2 \cos(2\omega t + \alpha_2) \longrightarrow W \hat{U}_2 e^{j\alpha_2} \quad \hat{Y}_2 \cos(2\omega t + \beta_2)$

$\hat{U}_3 \cos(3\omega t + \alpha_3) \longrightarrow \quad \vdots \quad \hat{Y}_3 \cos(3\omega t + \beta_3)$

$\vdots \qquad\qquad \vdots$

$\hat{U}_n \cos(n\omega t + \alpha_n) \longrightarrow \quad \hat{Y}_n \cos(n\omega t + \beta_n)$

$$\boxed{\hat{Y}_k = W(jk\omega)\, \hat{U}_k}$$

$k = 0, 1, \dots, n$

SZUPERPOZÍCIÓ ELV.

A FOURIER - SOR - összefoglalás.

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

$$\rightarrow u(t) \cong U_0 + \sum_{k=1}^{n} \hat{U}_k \cos(\underbrace{k\omega t}_{\alpha} + \underbrace{\alpha_k}_{\beta})$$

$$u(t) \cong U_0 + \sum_{k=1}^{n} \Big[\underset{\hat{U}_k^A}{\hat{U}_k \cos k\omega t \cdot \cos\alpha_k} - \underset{\hat{U}_k^B}{\hat{U}_k \sin k\omega t \sin\alpha_k} \Big]$$

$$\boxed{\begin{aligned} \hat{U}_k^A &= \hat{U}_k \cos\alpha_k \\ \hat{U}_k^B &= -\hat{U}_k \sin\alpha_k \end{aligned}} \ (*)$$

$$\rightarrow u(t) \cong U_0 + \sum_{k=1}^{n} \Big[\hat{U}_k^A \cos k\omega t + \hat{U}_k^B \sin k\omega t \Big]$$

$$\boxed{\begin{aligned} U_0 &= \frac{1}{T}\int_0^T u(t)\,dt \\ \hat{U}_k^A &= \frac{2}{T}\int_0^T u(t)\cos k\omega t\,dt \qquad \hat{U}_k^B = \frac{2}{T}\int_0^T u(t)\sin k\omega t\,dt \end{aligned}}$$

$$\boxed{\begin{aligned} \hat{U}_k &= \sqrt{(\hat{U}_k^A)^2 + (\hat{U}_k^B)^2} \\ \alpha_k &= -\operatorname{arc\,tg}\frac{\hat{U}_k^B}{\hat{U}_k^A} \end{aligned}}$$

$$* \quad (\hat{U}_k^A)^2 + (\hat{U}_k^B)^2 = \hat{U}_k^2\cos^2\alpha_k + \hat{U}_k^2\sin^2\alpha_k$$

$$\hat{U}_k^2 \underbrace{(\cos^2\alpha_k + \sin^2\alpha_k)}_{1}$$

$$\frac{\hat{U}_k^B}{\hat{U}_k^A} = -\operatorname{tg}\alpha_k$$

$$\mu(t) \cong \mu_0 + \sum_{k=1}^{n} \Big[\hat{\mu}_k^A \underbrace{\cos k\omega t}_{\frac{e^{jk\omega t} + e^{-jk\omega t}}{2}} + \hat{\mu}_k^B \underbrace{\sin k\omega t}_{\frac{e^{jk\omega t} - e^{-jk\omega t}}{2j}} \Big]$$

$$\mu(t) \cong \mu_0 + \sum_{k=1}^{n} \Big[\hat{\mu}_k^A \frac{e^{jk\omega t} + e^{-jk\omega t}}{2} + \hat{\mu}_k^B \frac{e^{jk\omega t} - e^{-jk\omega t}}{2j} \Big] =$$

$$\frac{1}{j} = -j$$

$$= \mu_0 + \sum_{k=1}^{n} \Big[\frac{\hat{\mu}_k^A}{2} \underline{e^{jk\omega t}} + \frac{\hat{\mu}_k^A}{2} \underline{e^{-jk\omega t}} - \frac{j\hat{\mu}_k^B}{2} \underline{e^{jk\omega t}} + \frac{j\hat{\mu}_k^B}{2} \underline{e^{-jk\omega t}} \Big] =$$

$$= \mu_0 + \sum_{k=1}^{n} \Big[\underbrace{\frac{\hat{\mu}_k^A - j\hat{\mu}_k^B}{2}}_{\hat{\mu}_k^C} e^{jk\omega t} + \underbrace{\frac{\hat{\mu}_k^A + j\hat{\mu}_k^B}{2}}_{\left(\hat{\mu}_k^C\right)^* = \hat{\mu}_{-k}^C \Longleftarrow} e^{-jk\omega t} \Big] =$$

$$\boxed{\mu(t) \cong \sum_{k=-n}^{n} \hat{\mu}_k^C e^{jk\omega t} \; !}$$

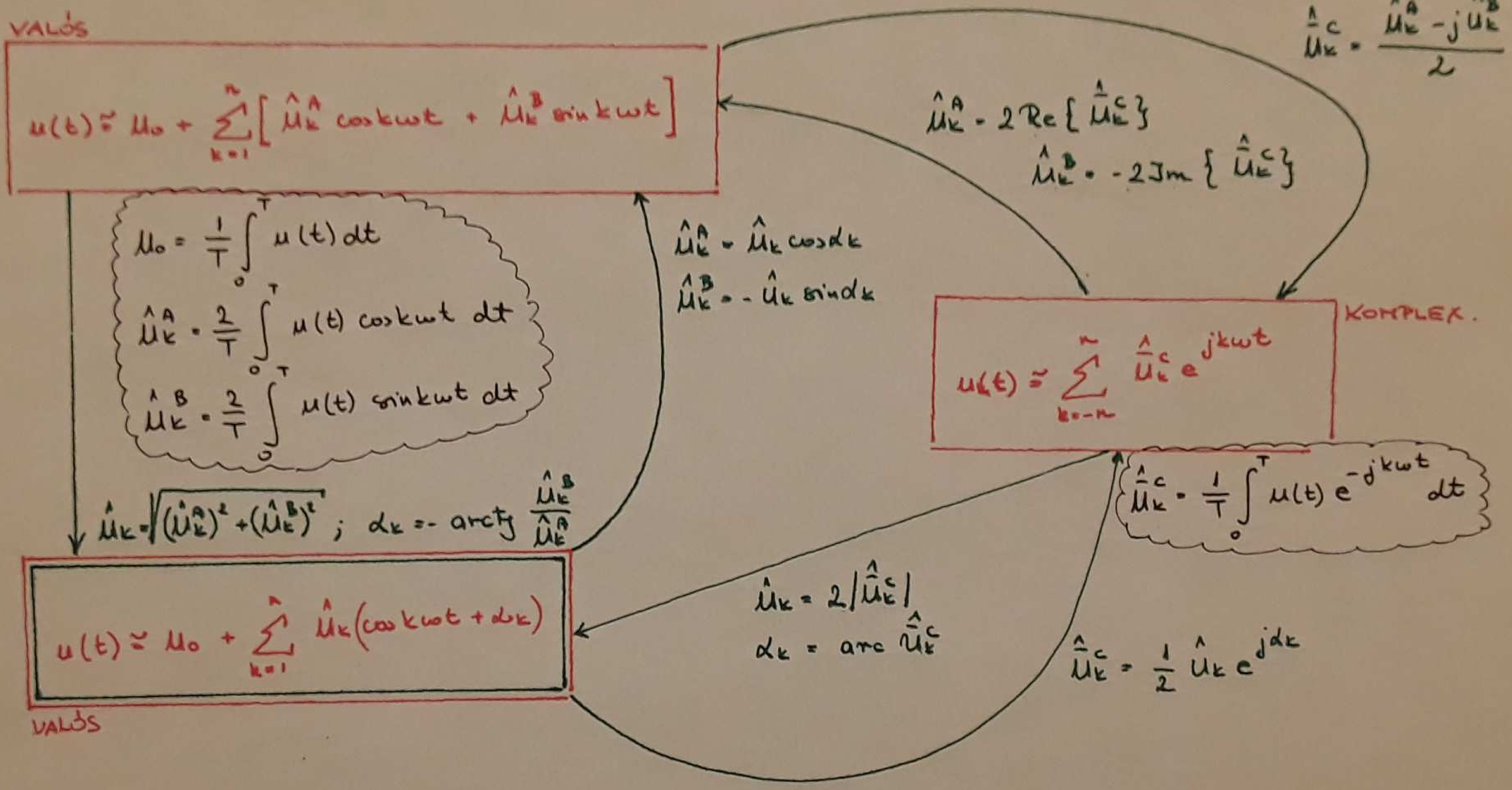
$$\hat{\mu}_k^C = \frac{\hat{\mu}_k^A - j\hat{\mu}_k^B}{2} = \frac{1}{T}\int_0^T \left[\mu(t) \underline{\cos k\omega t} - j\, \mu(t) \underline{\sin k\omega t}\right] dt =$$

$$\boxed{\hat{\mu}_k^C = \frac{1}{T}\int_0^T \mu(t)\, e^{-jk\omega t}\, dt \,.}$$

$$\left(\hat{\mu}_k^C\right)^* = \left(\frac{1}{T}\int_0^T \mu(t)\, e^{+jk\omega t}\, dt\right)^*$$

$$\boxed{\begin{aligned} \hat{\mu}_k^A &= 2\,\mathrm{Re}\{\hat{\mu}_k^C\} \\ \hat{\mu}_k^B &= -2\,\mathrm{Im}\{\hat{\mu}_k^C\} \end{aligned}} \Longrightarrow \hat{\mu}_k \;\; \alpha_k$$

FOURIER - SOR



VALÓS

$$u(t) \simeq U_0 + \sum_{k=1}^{n} \left[\hat{U}_k^A \cos k\omega t + \hat{U}_k^B \sin k\omega t \right]$$

$$U_0 = \frac{1}{T}\int_0^T u(t)\,dt$$

$$\hat{U}_k^A = \frac{2}{T}\int_0^T u(t)\cos k\omega t\,dt$$

$$\hat{U}_k^B = \frac{2}{T}\int_0^T u(t)\sin k\omega t\,dt$$

$$\hat{U}_k = \sqrt{(\hat{U}_k^A)^2 + (\hat{U}_k^B)^2}\ ; \quad \alpha_k = -\operatorname{arctg}\frac{\hat{U}_k^B}{\hat{U}_k^A}$$

$$\hat{U}_k^A = \hat{U}_k \cos\alpha_k$$

$$\hat{U}_k^B = -\hat{U}_k \sin\alpha_k$$

$$\hat{\bar{U}}_k^C = \frac{\hat{U}_k^A - j\hat{U}_k^B}{2}$$

$$\hat{U}_k^A = 2\,\mathrm{Re}\{\hat{\bar{U}}_k^C\}$$

$$\hat{U}_k^B = -2\,\mathrm{Im}\{\hat{\bar{U}}_k^C\}$$

KOMPLEX.

$$u(t) \simeq \sum_{k=-n}^{n} \hat{\bar{U}}_k^C e^{jk\omega t}$$

$$\hat{\bar{U}}_k^C = \frac{1}{T}\int_0^T u(t)\,e^{-jk\omega t}\,dt$$

$$\hat{U}_k = 2|\hat{\bar{U}}_k^C|$$

$$\alpha_k = \operatorname{arc} \hat{\bar{U}}_k^C$$

$$\hat{\bar{U}}_k^C = \frac{1}{2}\hat{U}_k e^{j\alpha_k}$$

$$u(t) \simeq U_0 + \sum_{k=1}^{n} \hat{U}_k (\cos k\omega t + \alpha_k)$$

VALÓS

A FOURIER- SOR - igazolás

$$u(t) \approx u_n(t) = U_0 + \sum_{k=1}^{n}\left[\hat{U}_k^A \cos k\omega t + \hat{U}_k^B \sin k\omega t\right]$$

$$H_n(U_0, \hat{U}_k^A, \hat{U}_k^B) = \frac{1}{T}\int_0^T \left[u_n(t) - u(t)\right]^2 dt$$

kvadratikus (négyzetes) középhiba

$$= \frac{1}{T}\int_0^T \left[U_0 + \sum_{k=1}^{n}\left[\hat{U}_k^A \cos k\omega t + \hat{U}_k^B \sin k\omega t\right] - u(t)\right]^{\textcircled{2}} dt$$

$\boxed{\underline{\min H_n}}$

$$\frac{\partial H_n}{\partial U_0} = \frac{2}{T}\int_0^T \left[U_0 + \sum_{k=1}^{n}\left[\hat{U}_k^A \cos k\omega t + \hat{U}_k^B \sin k\omega t\right] - u(t)\right] dt = 0$$

$$\frac{\partial H_n}{\partial \hat{U}_p^A} = \frac{2}{T}\int_0^T \left[U_0 + \sum_{k=1}^{n}\left[\hat{U}_k^A \cos k\omega t + \hat{U}_k^B \sin k\omega t\right] - u(t)\right]^1 \cos p\omega t \, dt = 0$$

$$\frac{\partial H_n}{\partial \hat{U}_p^B} = \frac{2}{T}\int_0^T \left[U_0 + \sum_{k=1}^{n}\left[\hat{U}_k^A \cos k\omega t + \hat{U}_k^B \sin k\omega t\right] - u(t)\right] \sin p\omega t \, dt = 0$$

1.

p = 1... n.

p = 1... n.

$\underline{\boxed{2n+1}}$

$$\int_0^T U_0 \, dt + \underbrace{\sum_{k=1}^{n}\int_0^T \left[\hat{U}_k^A \cos k\omega t + \hat{U}_k^B \sin k\omega t\right] dt}_{0} - \int_0^T u(t)\, dt = 0$$

$$U_0 \underbrace{\int_0^T dt}_{T} = \int_0^T u(t)\, dt \implies \boxed{U_0 = \frac{1}{T}\int_0^T u(t)\, dt}$$

$$\frac{\partial H_n}{\partial \hat{U}_p^A} = \frac{2}{T}\int_0^T \left[u_0 + \sum_{k=1}^{n} \left[\hat{U}_k^A \cos k\omega t + \hat{U}_k^B \sin k\omega t \right] - u(t) \right] \cos p\omega t \, dt = 0$$

$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$$

$$\sin\alpha \cos\beta = \frac{1}{2}\sin(\alpha-\beta) + \frac{1}{2}\sin(\alpha+\beta)$$

$$\underbrace{\cancel{\int_0^T u_0 \cos p\omega t \, dt}}_{0} + \sum_{k=1}^{n}\int_0^T \hat{U}_k^A \underbrace{\cos k\omega t \cos p\omega t}_{\frac{1}{2}\cos(k-p)\omega t + \frac{1}{2}\cos(k+p)\omega t} dt + \sum_{k=1}^{n}\int_0^T \hat{U}_k^B \underbrace{\sin k\omega t \cos p\omega t}_{\frac{1}{2}\sin(k-p)\omega t + \frac{1}{2}\sin(k+p)\omega t} dt = \int_0^T u(t) \cos p\omega t \, dt.$$

$$p \neq k \Rightarrow \int_0^T \ldots dt = 0$$

$$\boxed{p = k} \Rightarrow \frac{1}{2} + \frac{1}{2}\cos 2\omega t$$

$$\int_0^T \hat{U}_k^A \left(\frac{1}{2} + \underbrace{\frac{1}{2}\cos 2\omega t}_{0} \right) dt$$

$$\int_0^T \hat{U}_k^A \frac{1}{2} \, dt = \frac{1}{2}\hat{U}_k^A \int_0^T dt$$

$$= \underline{\underline{\frac{1}{2}\hat{U}_k^A T}}$$

$$p \neq k \Rightarrow \int_0^T \ldots dt = 0$$

$$p = k \Rightarrow 0$$

$$\downarrow$$

$$\longrightarrow \frac{1}{2}\hat{U}_k^A T = \int_0^T u(t) \cos k\omega t \, dt$$

$$\boxed{\hat{U}_k^A = \frac{2}{T}\int_0^T u(t) \cos k\omega t \, dt}$$

$$\frac{\partial H_m}{\partial \hat{u}_p^B} = \frac{2}{T}\int_0^T \left[u_0 + \sum_{k=1}^{n} \left[\hat{u}_k^A \cos k\omega t + \hat{u}_k^B \sin k\omega t \right] - u(t) \right] \sin p\omega t \, dt = 0$$

$$\sin\alpha \cdot \sin\beta = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)$$

$$\underbrace{\cancel{\int_0^T u_0 \sin p\omega t \, dt}}_{0} + \underbrace{\sum_{k=1}^{n} \int_0^T \hat{u}_k^A \underbrace{\cos k\omega t \sin p\omega t}\, dt}_{0} + \sum_{k=1}^{n} \int_0^T \hat{u}_k^B \underbrace{\sin k\omega t \sin p\omega t}\, dt = \int_0^T u(t) \sin p\omega t \, dt.$$

előző lap!

$$\int_0^T \cos k\omega t \sin p\omega t \, dt = 0$$

ortogonalitás

$$^1\cancel{\sin}$$

$$\frac{1}{2}\cos(k-p)\omega t - \frac{1}{2}\cos(k+p)\omega t.$$

$$p \neq k \rightarrow 0$$

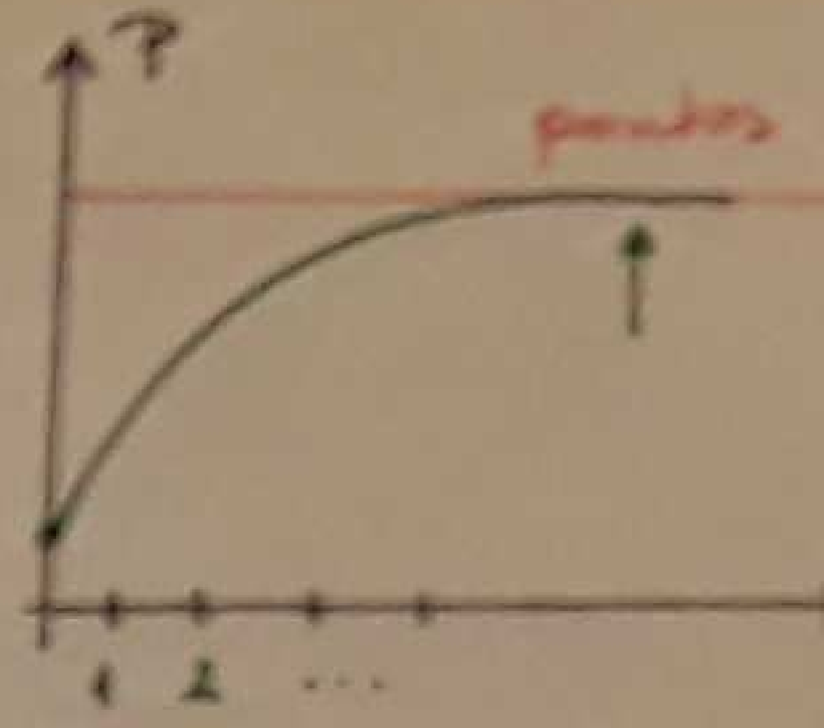
$$\boxed{p = k} \rightarrow \frac{1}{2} \underbrace{- \frac{1}{2}\cos 2\omega t}_{0}$$

$$\int_0^T \hat{u}_k^B \frac{1}{2} \, dt = \int_0^T u(t) \sin k\omega t \, dt$$

$$\frac{1}{2}\hat{u}_k^B T = \int_0^T u(t) \sin k\omega t \, dt$$

$$\boxed{\hat{u}_k^B = \frac{2}{T}\int_0^T u(t) \sin k\omega t \, dt}$$

PERIODIKUS JEL TELJESÍTMÉNYE (négyzetes középérték)



$$\boxed{P=\underbrace{\frac{1}{T}\int_0^T \boxed{u^2(t)}\,dt}_{\text{négyzetes középérték}} \cong U_0^2+\frac{1}{2}\sum_{k=1}^{n}\hat{U}_k^2}$$

$$U=\sqrt{\frac{1}{T}\int_0^T u^2(t)\,dt} \cong \sqrt{U_0^2+\frac{1}{2}\sum_{k=1}^{n}\hat{U}_k^2}$$

$$(x+y+z+u+\dots)^2 = x^2+y^2+z^2+u^2+\dots + 2xy+2xz+2xu+\dots + 2yz+2yu+\dots + 2zu+\dots$$

$$\frac{1}{T}\int_0^T\left[U_0+\sum_{k=1}^{n}\hat{U}_k\cos(k\omega t+\alpha_k)\right]^2 dt = \frac{1}{T}\int_0^T\Big[\underbrace{U_0}_{x}+\underbrace{\hat{U}_1\cos(1\omega t+\alpha_1)}_{y}+\underbrace{\hat{U}_2\cos(2\omega t+\alpha_2)}_{z}+\underbrace{\hat{U}_3\cos(3\omega t+\alpha_3)}_{u}\dots\Big]^{2} dt =$$

$$=\frac{1}{T}\int_0^T\Big[U_0^2+\hat{U}_1^2\underbrace{\cos^2(1\omega t+\alpha_1)}_{\frac{1+\cos(2[\omega t+\alpha_1])}{2}}+\hat{U}_2^2\cos^2(2\omega t+\alpha_2)+\dots+\underline{2U_0\hat{U}_1\cancel{\cos(1\omega t+\alpha_1)}}+2U_0\hat{U}_2\cancel{\cos(2\omega t+\alpha_2)}+\dots$$

$$+2\hat{U}_1\cos(\boxed{1\omega}\cancel{t+\alpha_1)\hat{U}_2}\cos(\boxed{2\omega}t+\alpha_2)+$$

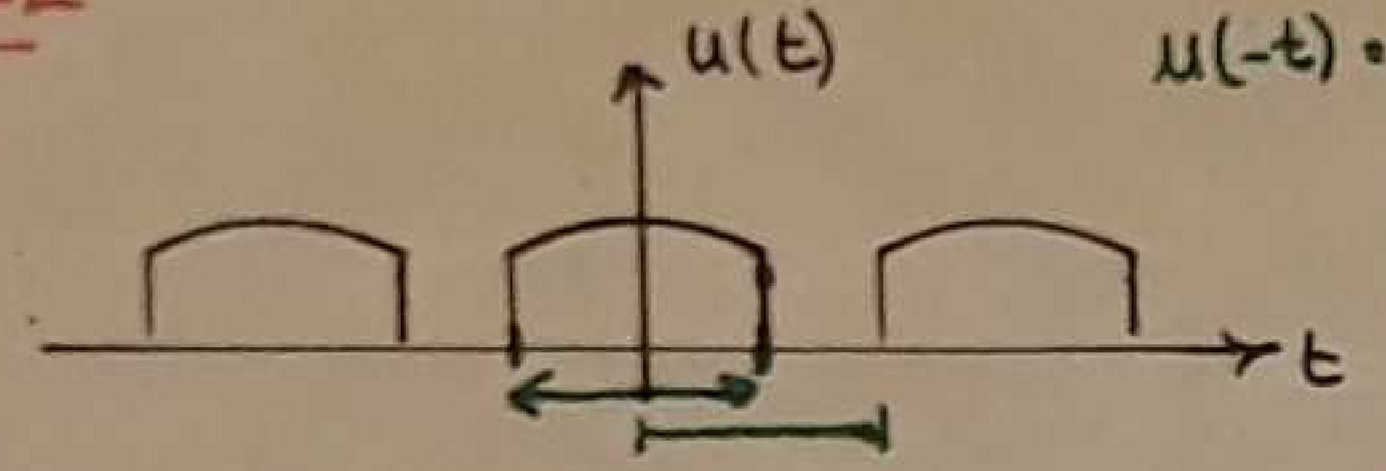
$$+2\hat{U}_1\cos(1\omega t\cancel{+\alpha_1)\hat{U}_3\cos}(3\omega t+\alpha_3)+\dots\Big]dt$$

$$=\frac{1}{T}\int_0^T\underbrace{\left[U_0^2+\hat{U}_1^2\frac{1}{2}+\hat{U}_2^2\frac{1}{2}+\dots\right]}dt = \frac{1}{T}\left[U_0^2+\frac{1}{2}\hat{U}_1^2+\frac{1}{2}\hat{U}_2^2+\dots\right]\underbrace{\int_0^T dt}_{T}$$

$$= U_0^2+\frac{1}{2}\hat{U}_1^2+\frac{1}{2}\hat{U}_2^2+\dots$$

PÁROS JEL

$u(-t) = u(t)$

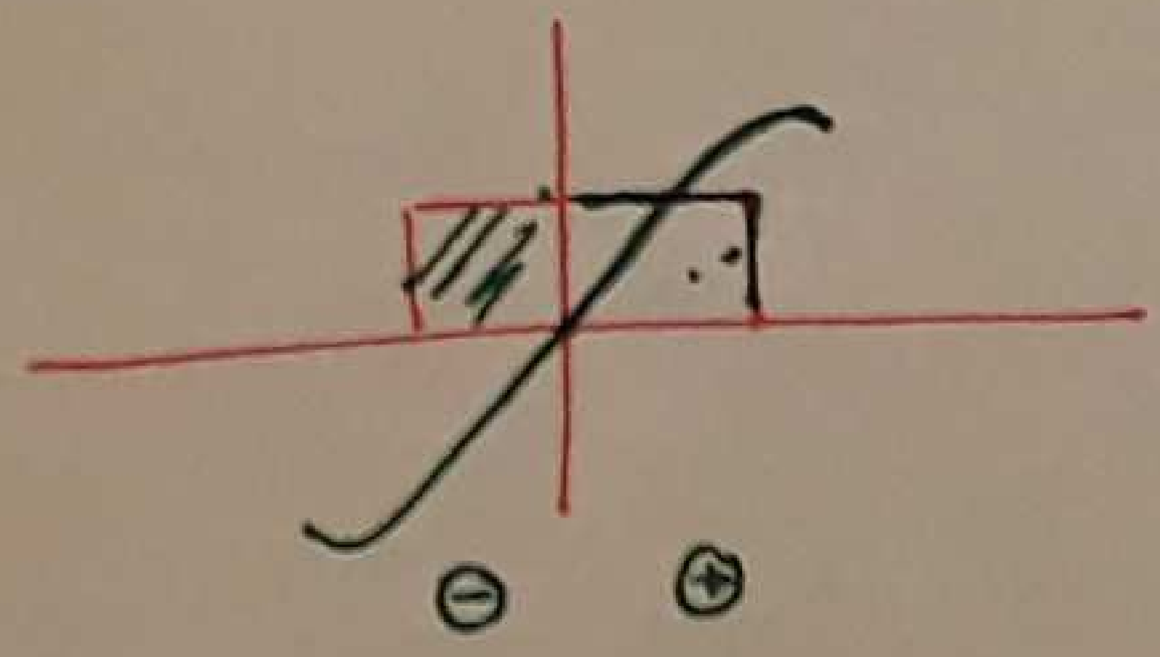


$$u(t) \cong U_0 + \sum_{k=1}^{n} \left[\hat{U}_k^A \cos k\omega t + \cancel{\hat{U}_k^B \sin k\omega t} \right]$$

$$\hat{U}_k^B \equiv 0$$

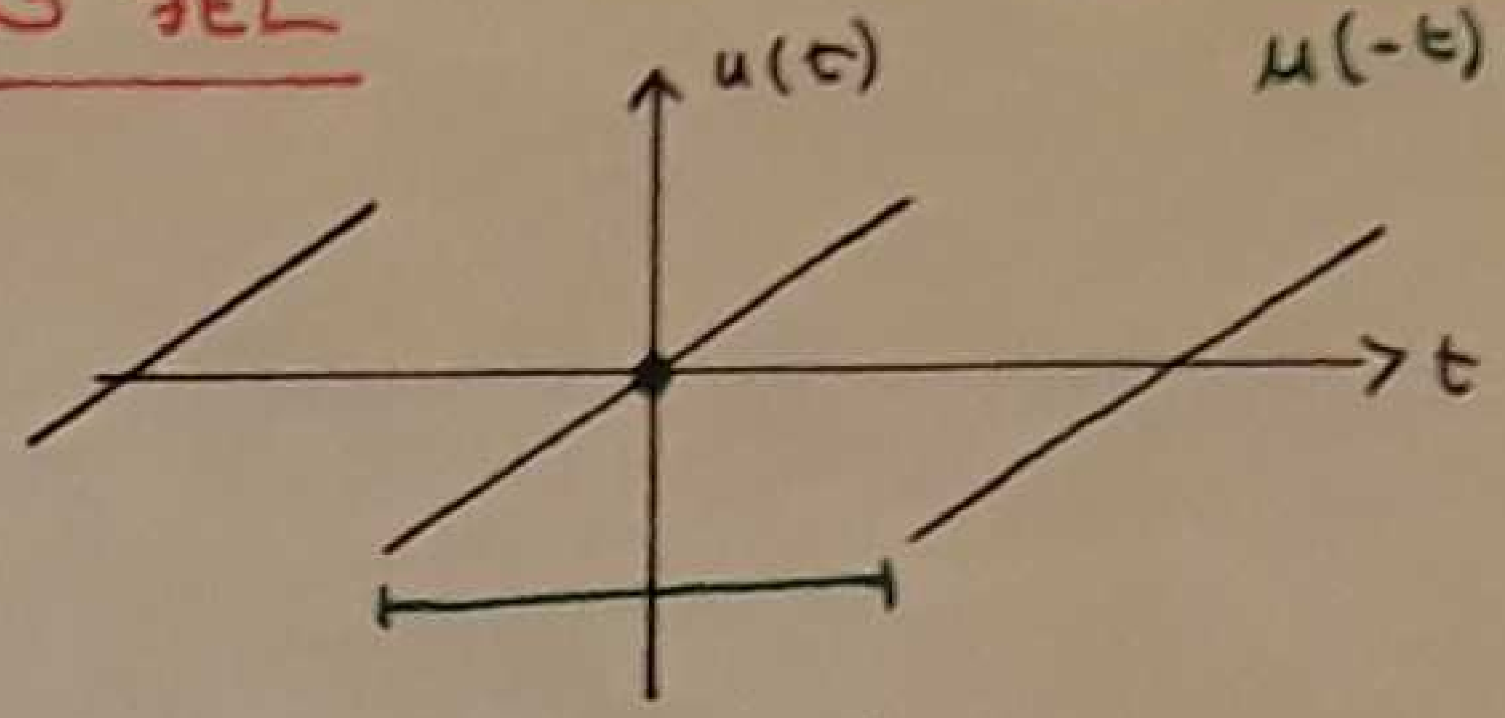
$$\hat{U}_k^B = \frac{2}{T} \int_0^T u(t) \sin k\omega t \, dt =$$

$$= \frac{2}{T} \int_{-T/2}^{T/2} u(t) \sin k\omega t \, dt = \phi$$



PÁRATLAN JEL

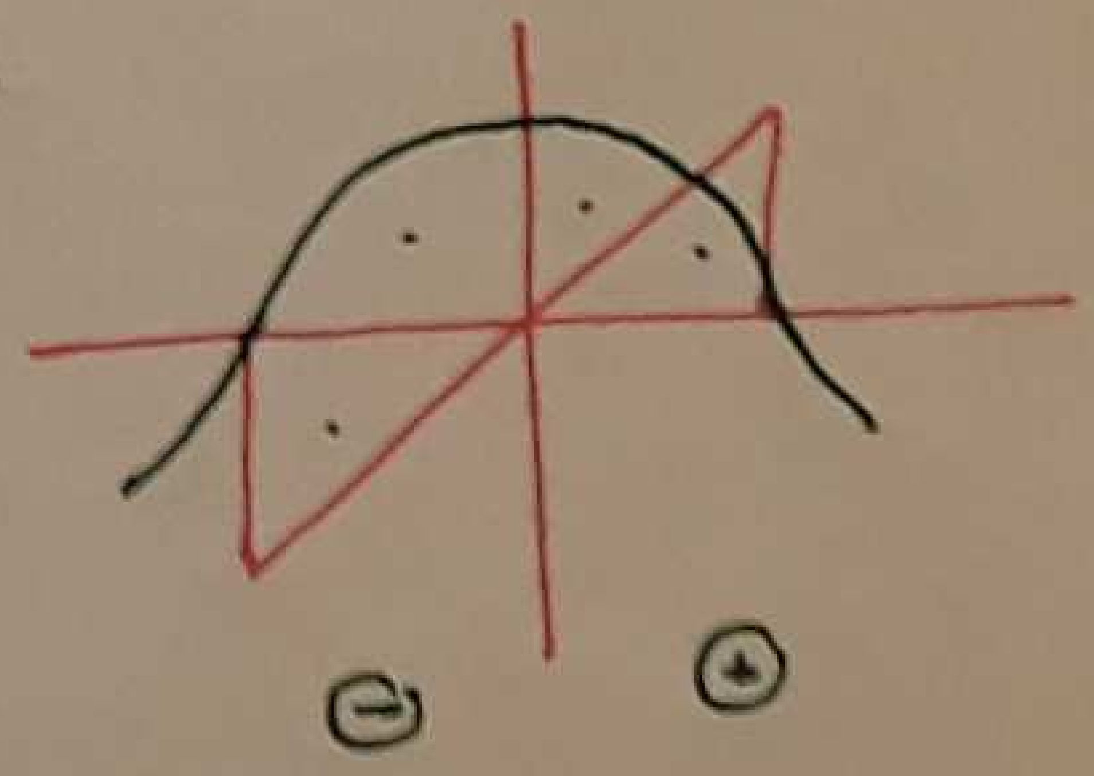
$u(-t) = -u(t)$



$$u(t) \cong U_0 + \sum_{k=1}^{n} \left[\cancel{\hat{U}_k^A \cos k\omega t} + \hat{U}_k^B \sin k\omega t \right]$$

$$\hat{U}_k^A \equiv 0$$

$$\hat{U}_k^A = \frac{2}{T} \int_{-T/2}^{T/2} u(t) \cos k\omega t \, dt = \phi.$$



FOLYTONOS IDEJŰ RENDSZEREK ANALÍZISE A FREKVENCIATARTOMÁNYBAN

II.

A SPEKTRÁLIS LEÍRÁS

A jel spektruma

Fourier-sor:

$$\boxed{u(t)=\sum_{k=-\infty}^{\infty}\hat{U}_k^c\, e^{\boxed{jk\omega}t} \quad \text{ahol} \quad \hat{U}_k^c=\frac{1}{T}\int_{-T/2}^{T/2}u(\underset{\tau}{\cancel{t}})\, e^{-jk\omega\underset{\tau}{\cancel{t}}}\,\underset{d\tau}{\cancel{dt}}}$$

periodikus jelre igaz!

$$u(t)=\sum_{k=-\infty}^{\infty}\frac{1}{T}\int_{-T/2}^{T/2}u(\tau)\,e^{-jk\omega\tau}\,d\tau\;\underline{e^{jk\omega t}}=$$

T↑

$$\omega=\frac{2\pi}{T}$$

$$\boxed{\frac{1}{T}=\frac{\omega}{2\pi}}\Rightarrow\frac{d\omega}{2\pi}$$

$$=\sum_{k=-\infty}^{\infty}\boxed{\frac{1}{T}}\int_{\boxed{-T/2}_{-\infty}}^{\boxed{T/2}^{\infty}}u(\tau)\,e^{jk\omega(t-\tau)}\,d\tau$$

$$\boxed{U(j\omega)=\mathcal{F}\{u(t)\}=\int_{-\infty}^{\infty}u(t)\,e^{-j\omega t}\,dt \quad \text{improprius.}}$$

$$u(t)=\sum_{k=-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}u(\tau)\,e^{j\underbrace{kd\omega}(t-\tau)}\,d\tau$$

$$\boxed{u(t)=\mathcal{F}^{-1}\{U(j\omega)\}=\frac{1}{2\pi}\int_{-\infty}^{\infty}U(j\omega)\,e^{j\omega t}\,d\omega}$$

$$\underline{u(t)}=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}u(\tau)\underbrace{e^{j\omega(t-\tau)}}_{e^{j\omega t}\cdot e^{-j\omega\tau}}d\tau=\frac{1}{2\pi}\int_{-\infty}^{\infty}\underbrace{\left[\int_{-\infty}^{\infty}u(\tau)\,e^{-j\omega\tau}\,d\tau\right]}_{U(j\omega)}\cdot e^{j\omega t}\,d\omega$$

SPEKTRUM.

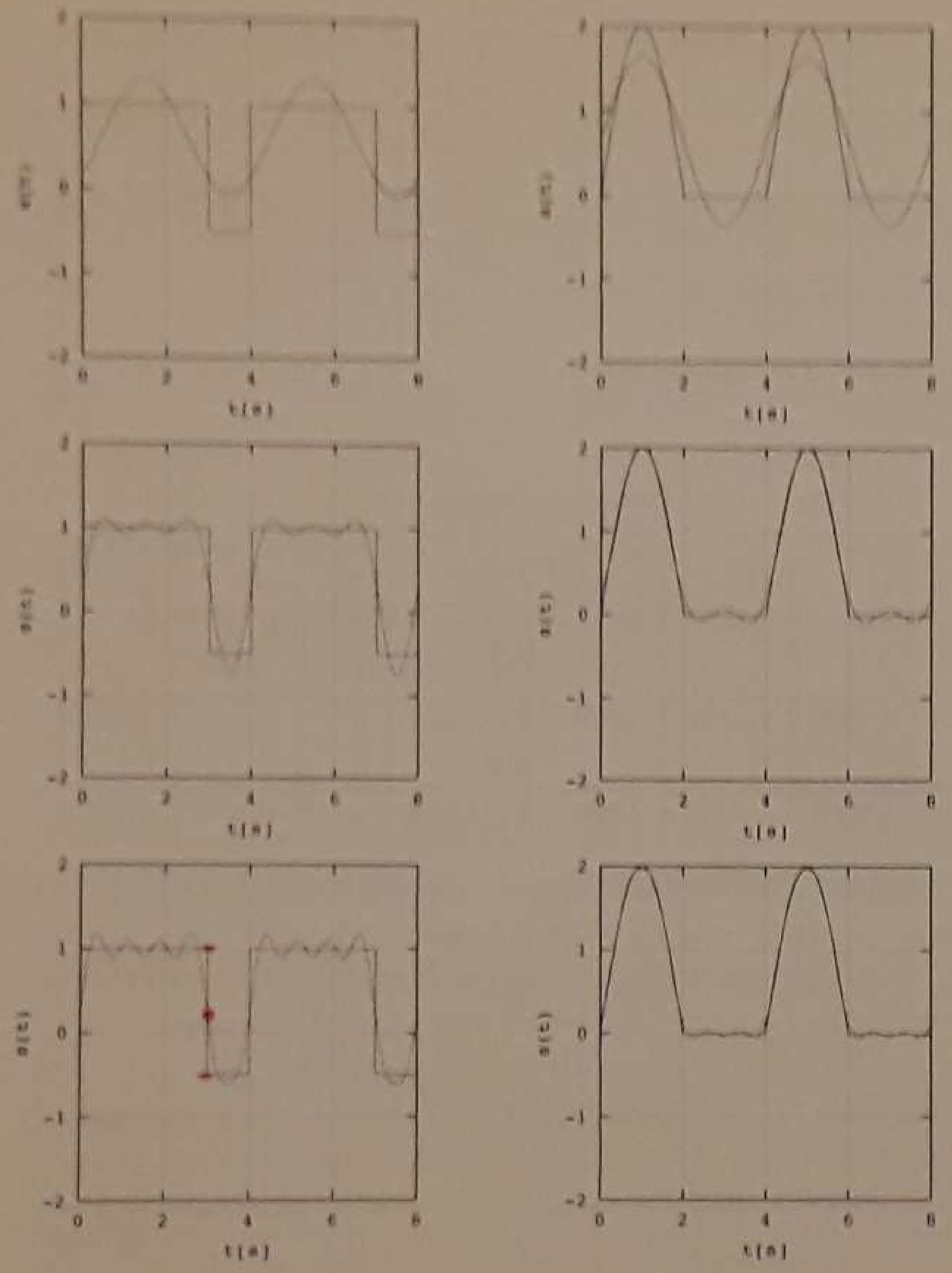
$$\int_{-\infty}^{\infty}|u(t)|\,dt<\infty \;!$$

$$u(t)=\lim_{\Delta\omega\to 0}\underline{\frac{1}{2\pi}}\sum_{k=-\infty}^{\infty}\underline{U(jk\Delta\omega)}\,e^{jk\Delta\omega t}\,\underline{\Delta\omega}$$

$$\boxed{\frac{\Delta\omega}{2\pi}U(jk\Delta\omega)}$$

amplitúdósűrűség

ILLUSZTRATÍV PÉLDA:



6.10. ábra. *A példákban szereplő függvények és a Fourier-összeggel történt közelítésük összehasonlítása $n = 1, 3, 5$ esetekre*

jobb oldali határétékek számtani közepéhez konvergál, ha $n \to \infty$:

$$s_n(t) \to \frac{s(t-\frac{3}{4}T-0)+s(t-\frac{3}{4}T+0)}{2} = 0,25,$$

Gibbs

$$u(t) \approx \underset{k=0}{U_0} + \sum_{k=1}^{\infty} \underset{k=1,2,\dots}{\hat{U}_k} \cos(\underline{k\omega t} + \alpha_k)$$

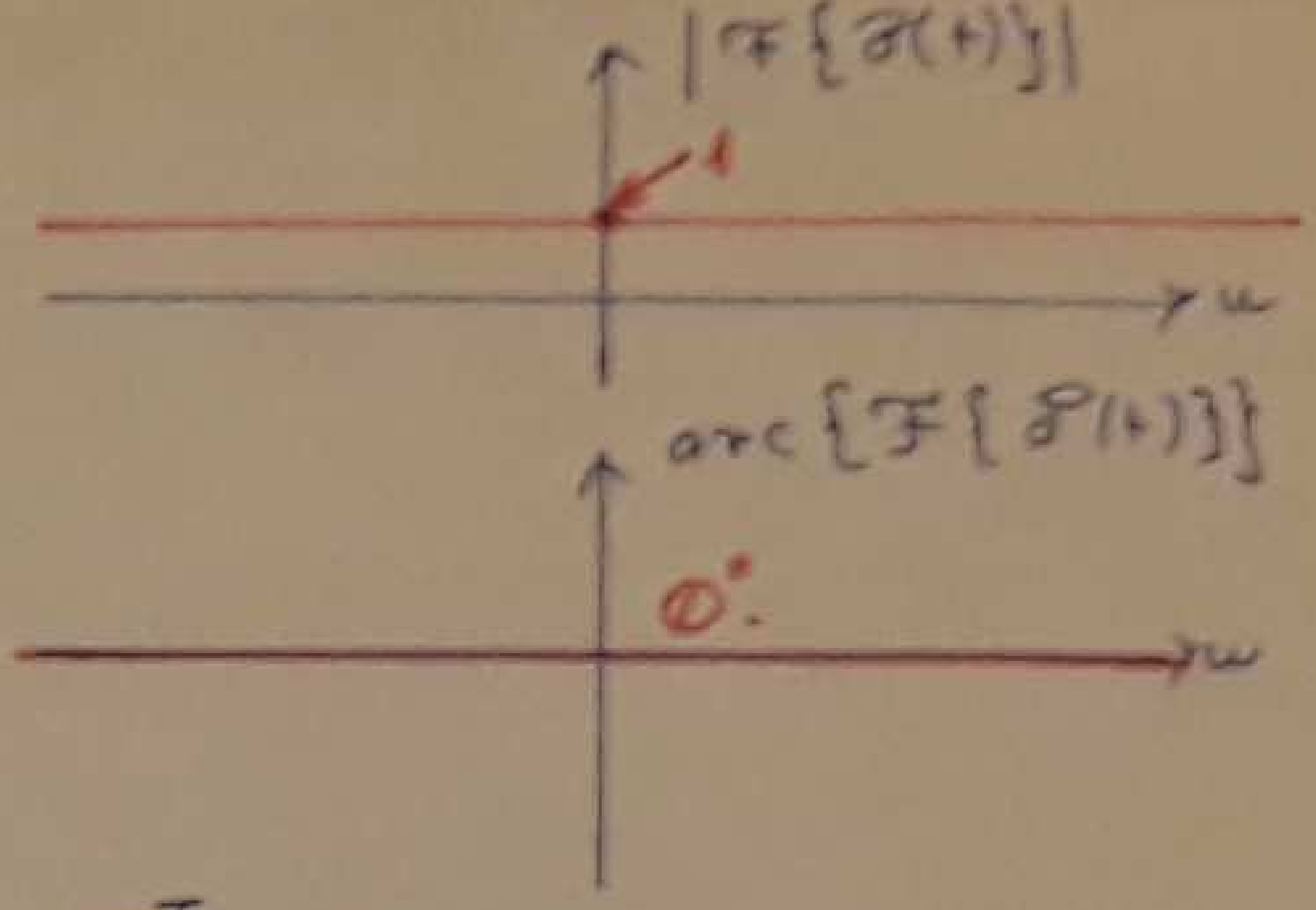
VONALAS SPEKTRUM $\boxed{\text{PER}}$

(folytonos spektrum – ált.)

Néhány jel spektruma

①
$$\mathcal{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \underline{\delta(t)}\, e^{-j\omega \underset{\substack{\uparrow \\ \emptyset}}{t}}\, dt = \int_{-\infty}^{\infty} \delta(t)\, dt = \underline{\underline{1}}$$

$$e^{-j\omega\emptyset} = 1$$

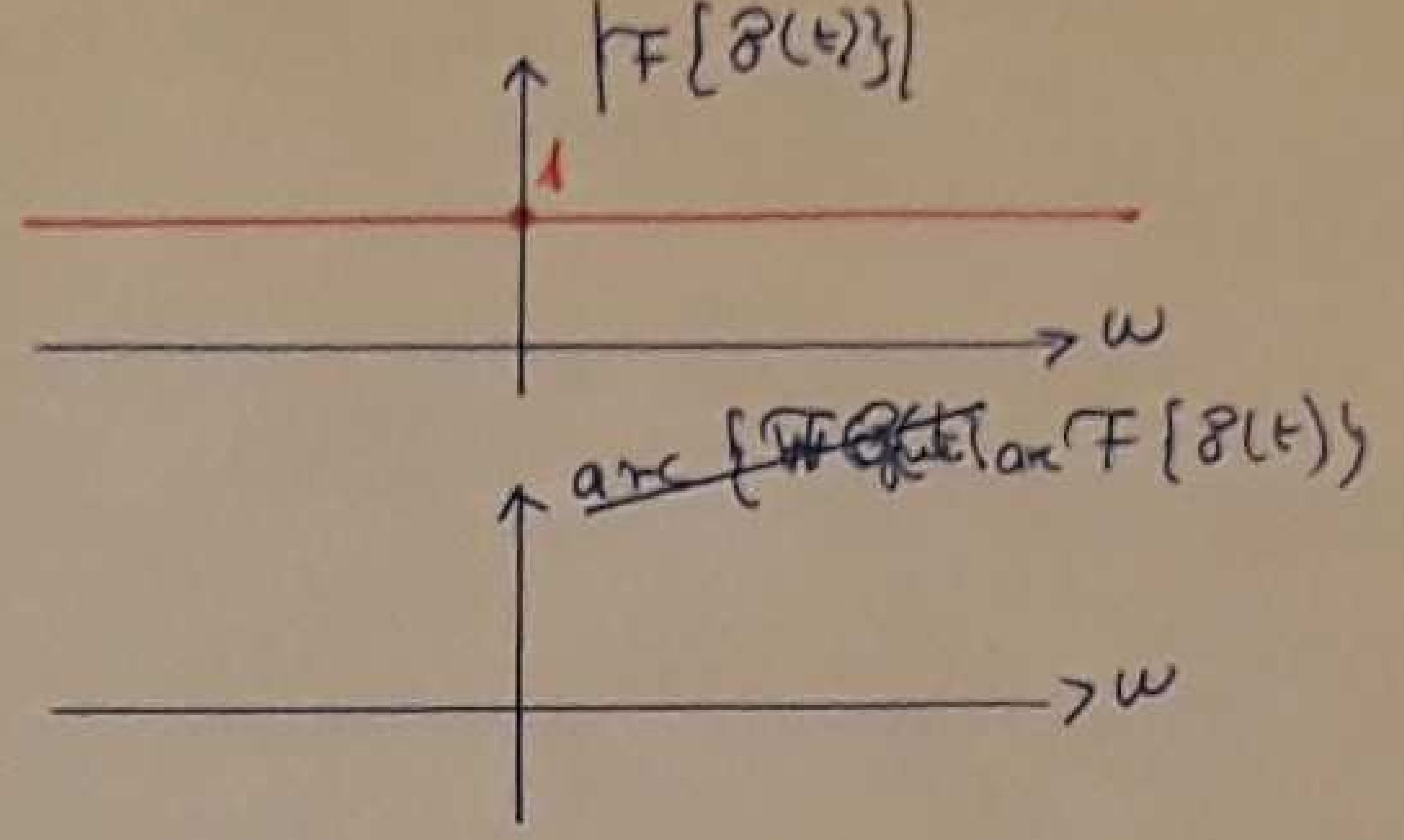


$$\mathcal{F}\{\delta(t-\tau)\} = \int_{-\infty}^{\infty} \underline{\delta(t-\tau)}\, \underbrace{e^{-j\omega \underset{\substack{\uparrow \\ \tau}}{t}}}_{e^{-j\omega\tau}}\, dt = e^{-j\omega\tau} \underbrace{\int_{-\infty}^{\infty} \delta(t-\tau)\, dt}_{1} = \underline{\underline{e^{-j\omega\tau}}}$$

$$\nearrow$$
$$1 \cdot e^{-j\omega\tau}$$

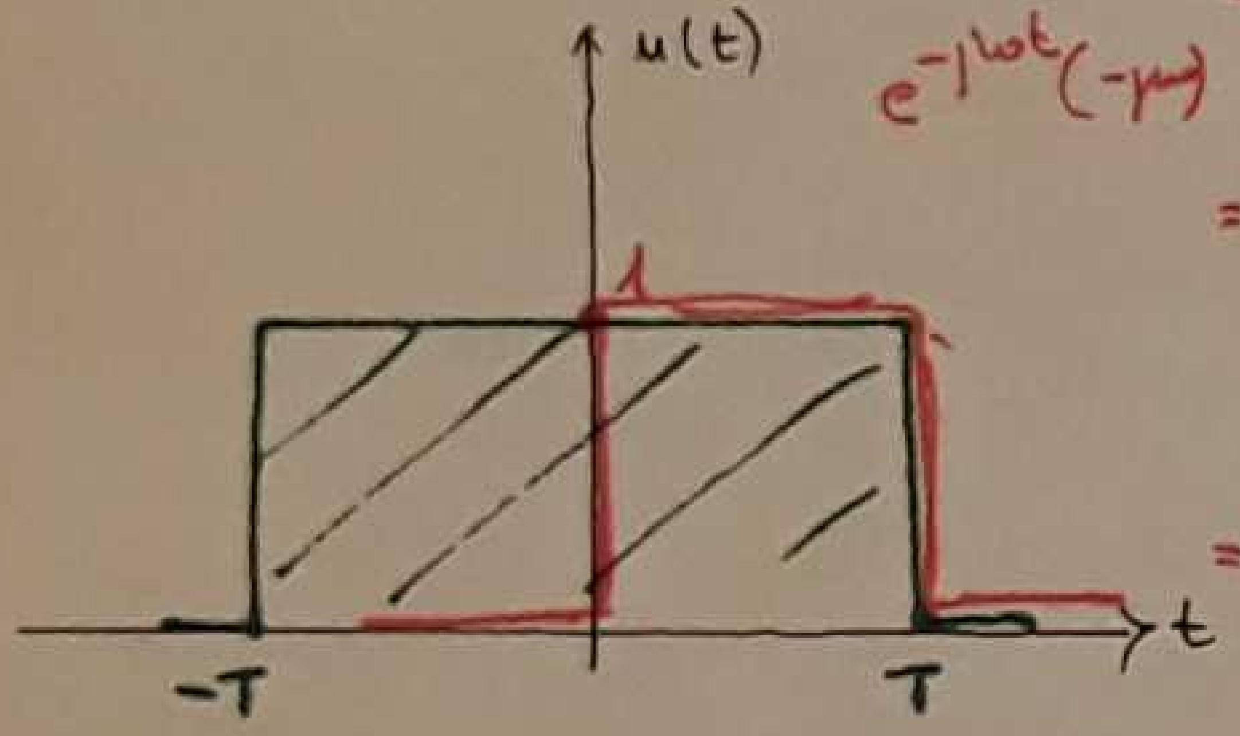
Néhány jel spektruma.

① $$\mathcal{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \underbrace{\delta(t)}\, \underbrace{e^{-j\omega t}}_{e^{-j\omega \emptyset}=1} dt = \int_{-\infty}^{\infty} \delta(t)\, dt = \underline{\underline{1}}$$



② $$\mathcal{F}\{1(t+T) - 1(t-T)\} = \int_{-\infty}^{\infty} u(t)\, e^{-j\omega t} dt = \int_{-T}^{+T} 1\, e^{-j\omega t} dt =$$

$e^{-j\omega t}(-j\omega)$

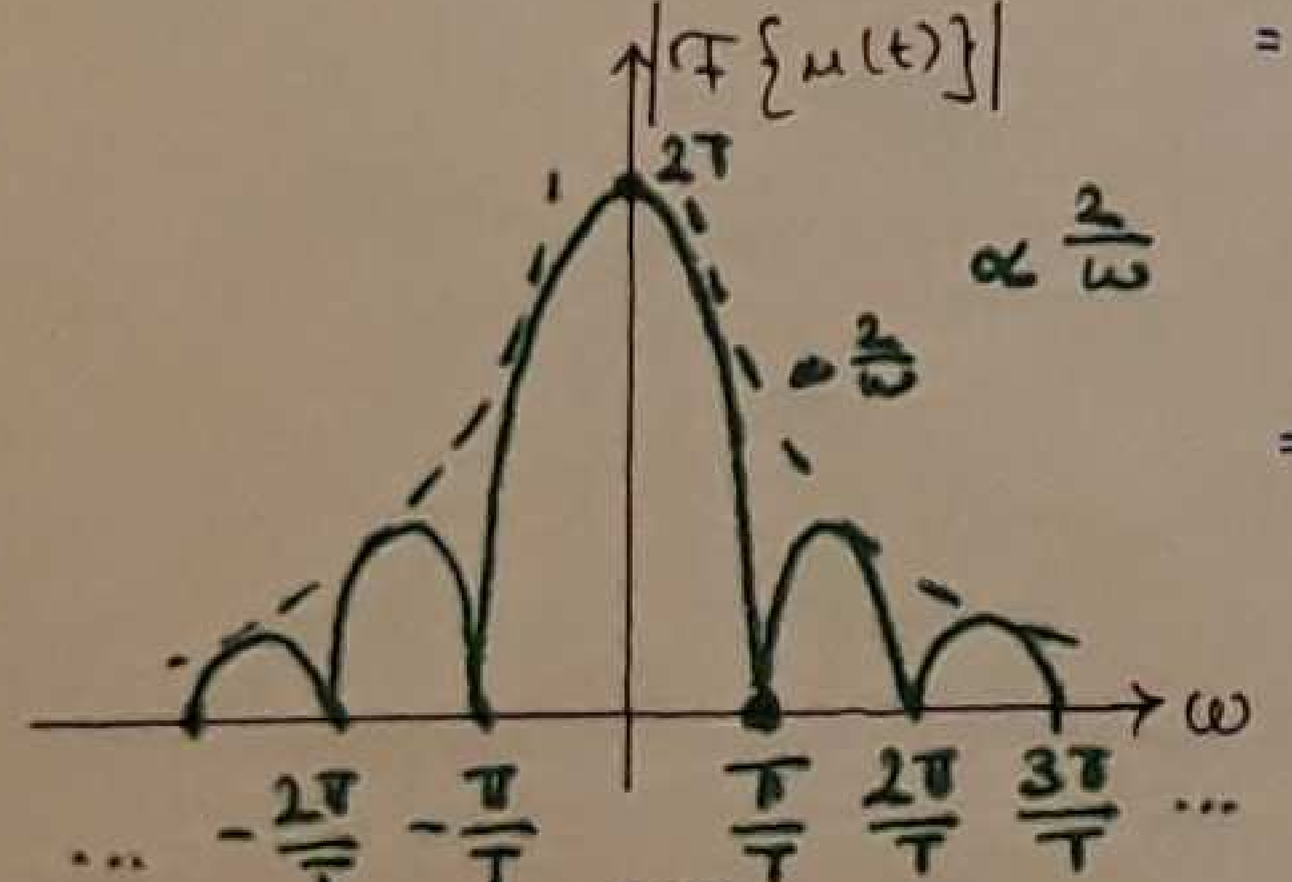


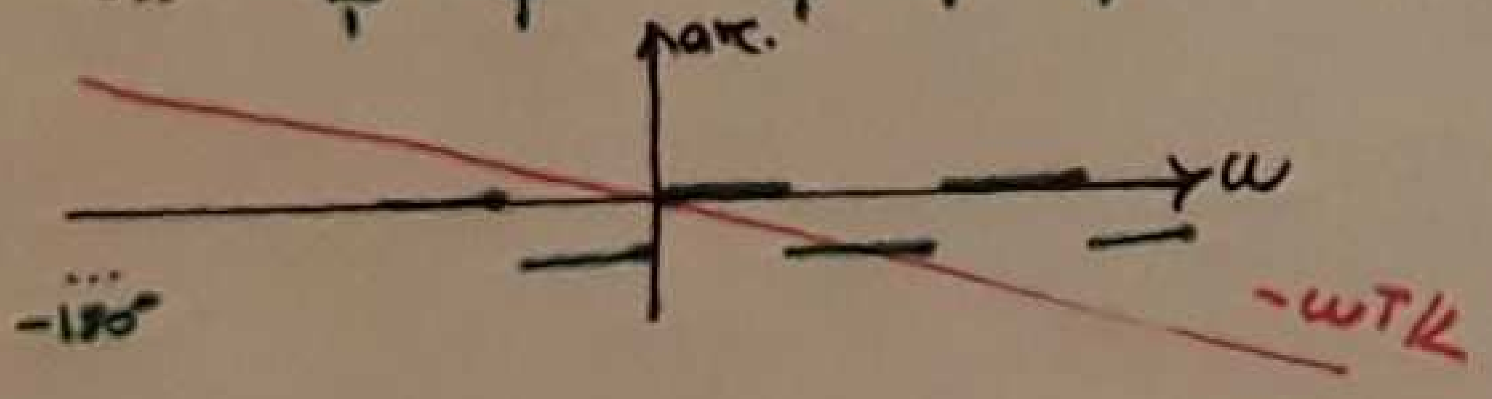
$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T}^{+T} = \frac{e^{-j\omega T} - e^{j\omega T}}{-j\omega} =$$

$$= \frac{e^{+j\omega T} - e^{-j\omega T}}{j\omega} \cdot \frac{2}{2} =$$

$$= \frac{2}{\omega}\, \underbrace{\frac{e^{j\omega T} - e^{-j\omega T}}{2j}}_{\sin \omega T} = \frac{2}{\omega} \sin \omega T$$

$$= \underline{\underline{2T \frac{\sin \omega T}{\omega T}}}$$ VALÓS !

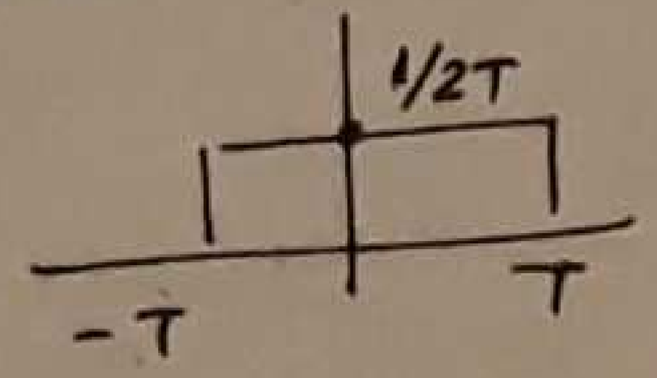




$$\frac{\sin \omega T}{\omega T} = \mathcal{F}\Big\{\underbrace{\tfrac{1}{2T}\left[1(t+T) - 1(t-T)\right]}\Big\}$$

$T \to \emptyset$

$\boxed{1}$



$$\mathcal{F}\{1(t) - 1(t-T)\} = \int_0^T e^{-j\omega t} dt =$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^T = \frac{e^{-j\omega T} - 1}{-j\omega} =$$

$$= e^{-j\omega \frac{T}{2}} \frac{e^{-j\omega T/2} - e^{j\omega T/2}}{-j\omega} =$$

$$= e^{-j\omega T/2} \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{j\omega} \frac{2}{2} =$$

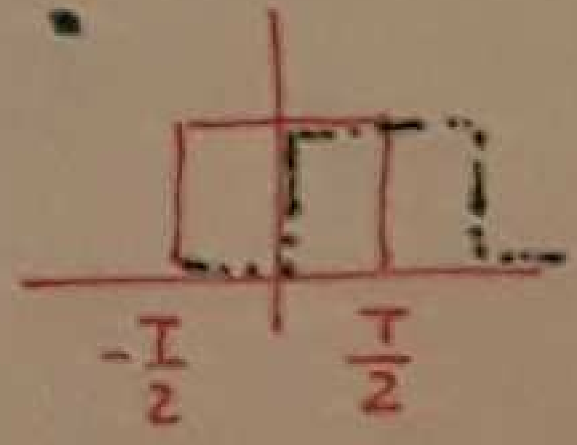
$$= e^{-j\omega T/2} \frac{2}{\omega} \underbrace{\frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j}}_{\sin \omega \frac{T}{2}} =$$

$$= e^{-j\omega T/2} \frac{2}{\omega} \sin \omega T/2 =$$

$$= \boxed{e^{-j\omega T/2}}\, T \frac{\sin \omega T/2}{\omega T/2}$$

$-\omega T/2$

eltolási tag!



Néhány jel spektruma

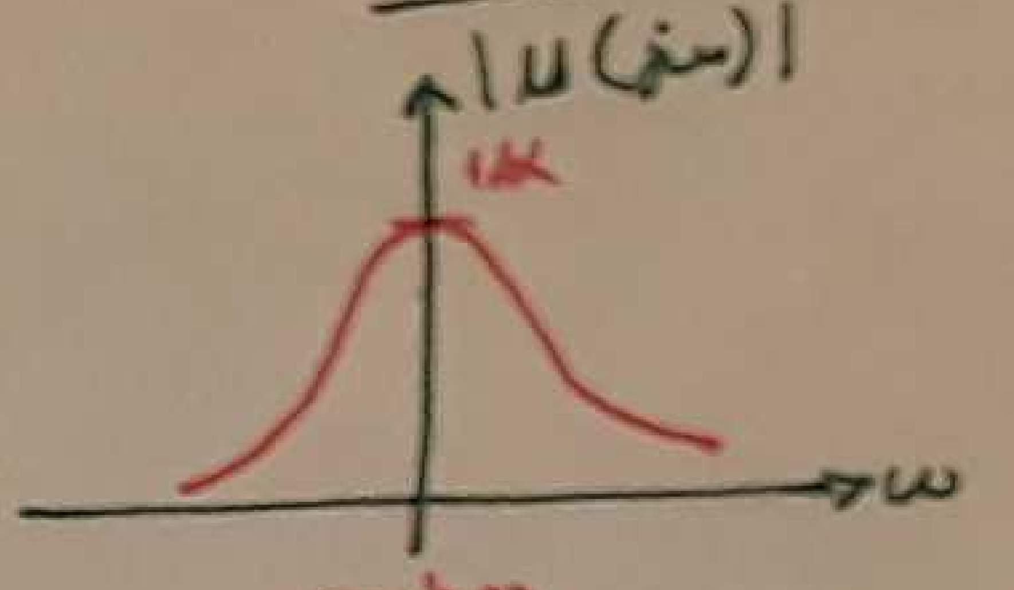
③ $\mathcal{F}\{1(t)e^{-\alpha t}\} = \int_{\cancel{-\infty}\,0}^{\infty} \boxed{1(t)} e^{-\alpha t} e^{-j\omega t}\,dt = \int_0^{\infty} e^{-(\alpha+j\omega)t}\,dt =$

$(\alpha > 0)$

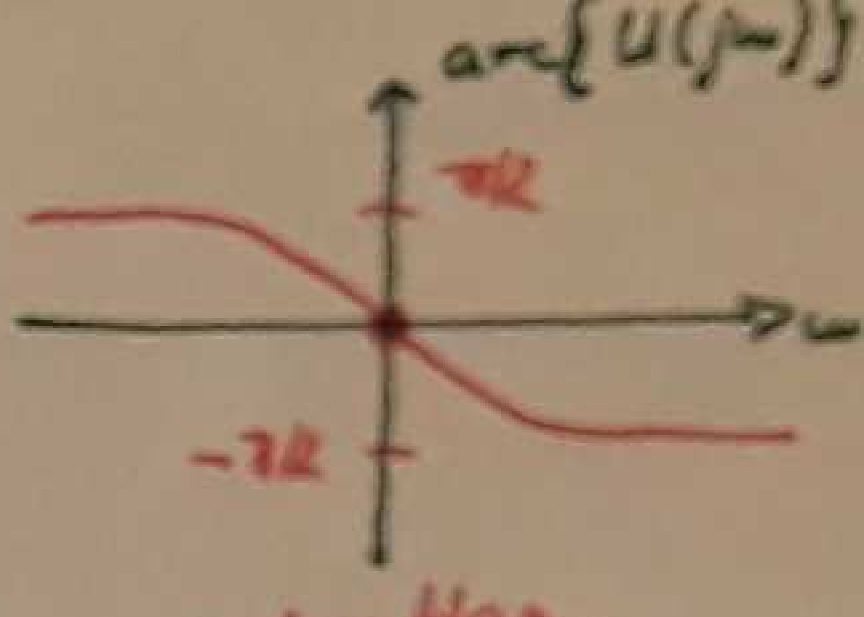
$$= \left[\frac{e^{-(\alpha+j\omega)t}}{-(\alpha+j\omega)}\right]_0^{\infty} = \frac{0-1}{-(\alpha+j\omega)} = \underline{\underline{\frac{1}{\alpha+j\omega}}}$$

$$\left|\frac{1}{\alpha+j\omega}\right| = \underline{\frac{1}{\sqrt{\alpha^2+\omega^2}}}$$

$$\underline{0^\circ - \operatorname{arctg}\frac{\omega}{\alpha}}$$

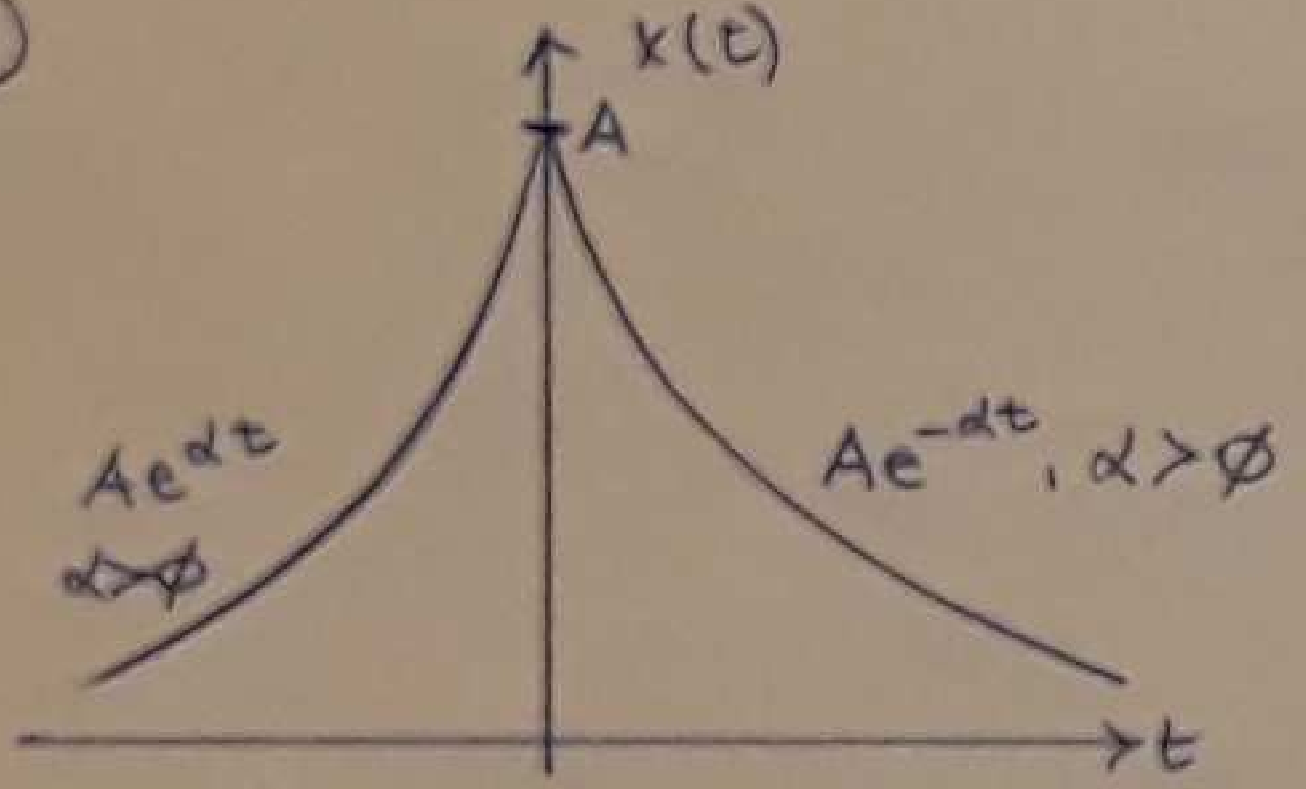


páros



páratlan.

④



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,dt =$$

$$= \int_{-\infty}^{0} \underline{Ae^{\alpha t}}\, e^{-j\omega t}\,dt + \int_0^{\infty} \underline{Ae^{-\alpha t}}\, e^{-j\omega t}\,dt =$$

$$= A\int_{-\infty}^{0} e^{(\alpha-j\omega)t}\,dt + A\int_0^{\infty} e^{-(\alpha+j\omega)t}\,dt =$$

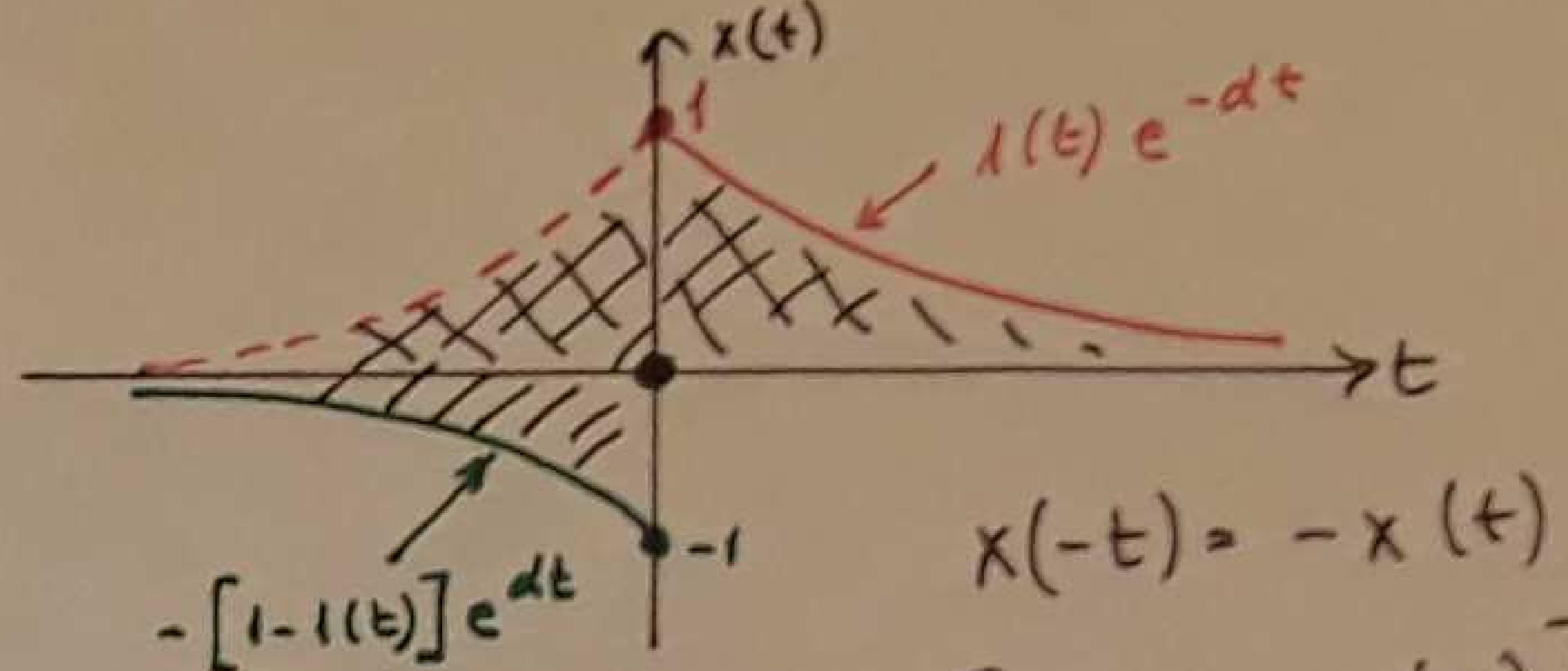
$$= A\left[\frac{e^{(\alpha-j\omega)t}}{(\alpha-j\omega)}\right]_{-\infty}^{0} + A\left[\frac{e^{-(\alpha+j\omega)t}}{-(\alpha+j\omega)}\right]_0^{\infty} = A\frac{1-0}{(\alpha-j\omega)} + A\frac{0-1}{-(\alpha+j\omega)} =$$

$(a+b)(a-b) = a^2-b^2$

$$= A\frac{1}{\alpha-j\omega} + A\frac{1}{\alpha+j\omega} = A\frac{\alpha+j\omega+\alpha-j\omega}{\alpha^2+\omega^2} = \underline{\underline{A\frac{2\alpha}{\alpha^2+\omega^2}}}$$

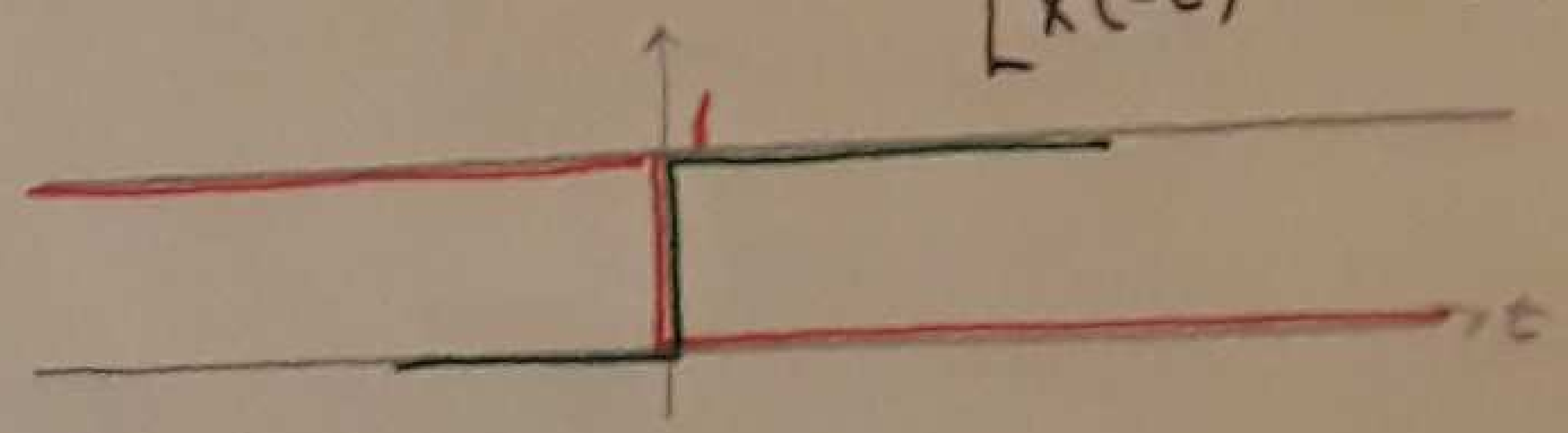
Nějaký jel spektruma

⑤ $x(t) = \underline{-[1-1(t)]e^{\alpha t}} + \underline{1(t)\, e^{-\alpha t}} \quad , \quad \alpha > 0$



$x(-t) = -x(t)$

$[x(-t) = x(t)]$



$$X(j\omega) = -\int_{-\infty}^{0} e^{\alpha t} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-\alpha t} e^{-j\omega t}\, dt =$$

$$= -\int_{-\infty}^{0} e^{(\alpha - j\omega)t}\, dt + \int_{0}^{\infty} e^{-(\alpha + j\omega)t}\, dt =$$

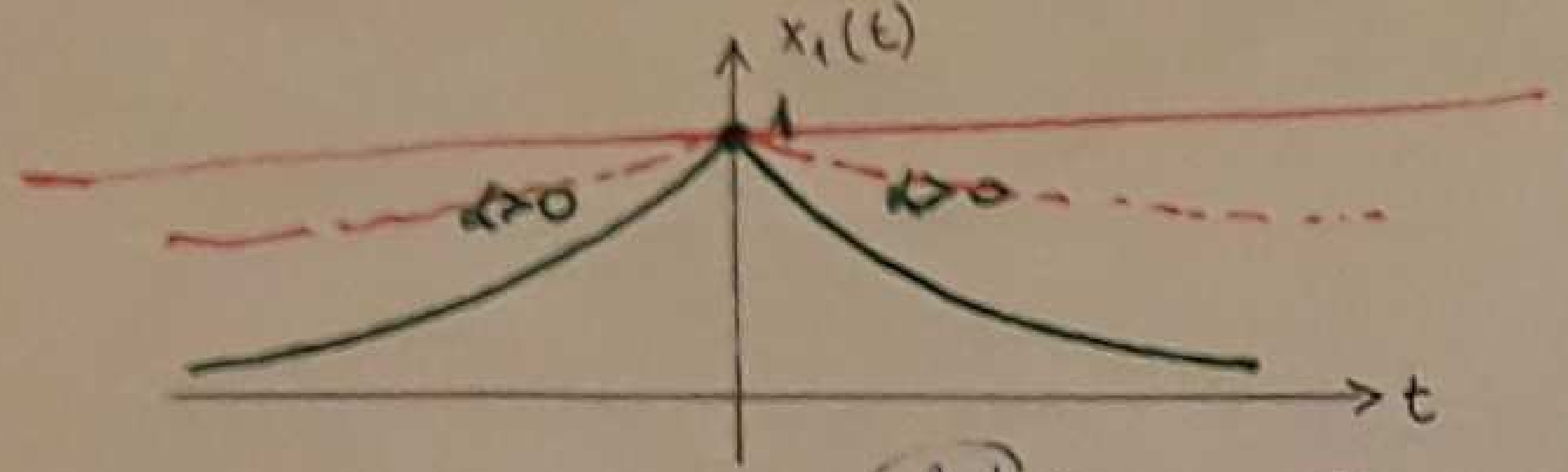
$$= -\left[\frac{e^{(\alpha - j\omega)t}}{\alpha - j\omega} \right]_{-\infty}^{0} + \left[\frac{e^{-(\alpha + j\omega)t}}{-(\alpha + j\omega)} \right]_{0}^{\infty} =$$

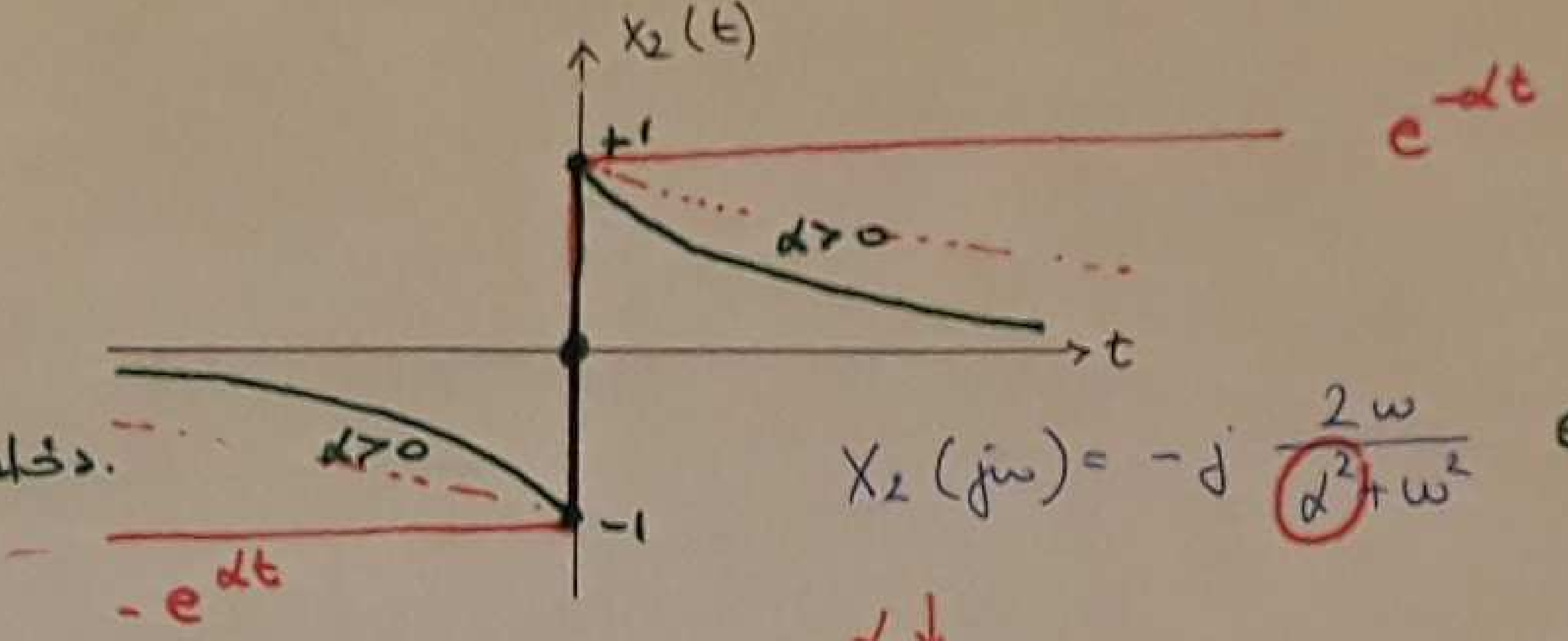
$$(\alpha - j\omega)(\alpha + j\omega) = \alpha^2 - (j\omega)^2 =$$
$$= \alpha^2 + \omega^2$$

$$= -\frac{1-0}{\alpha - j\omega} + \frac{0-1}{-(\alpha + j\omega)} = \frac{-1}{\alpha - j\omega} + \frac{1}{\alpha + j\omega} = \frac{-\alpha - j\omega + \alpha - j\omega}{\alpha^2 + \omega^2} =$$

$$= \underline{\underline{\frac{-j2\omega}{\alpha^2 + \omega^2}}} = -j \underbrace{\frac{2\omega}{\alpha^2 + \omega^2}}$$

Az előző két videó végeredménye:





$X_1(j\omega) = \frac{2\alpha}{\alpha^2+\omega^2} \in \mathbb{R}$. valós.

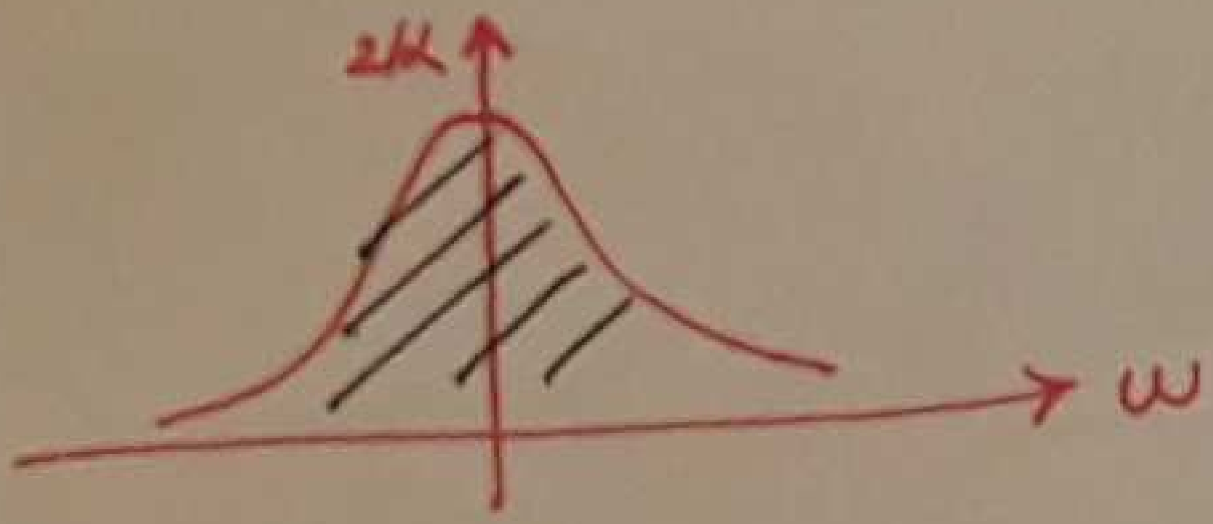
$X_2(j\omega) = -j\,\frac{2\omega}{\alpha^2+\omega^2} \in \mathbb{C}$ képzetes

$\alpha\downarrow$

$\alpha = 0$

$\alpha\downarrow$

$\boxed{\alpha = 0}$



$\frac{2\alpha}{\alpha^2} = \frac{2}{\alpha}$

$$\int_{-\infty}^{\infty} \frac{\cancel{2}\alpha}{\alpha^2+\omega^2}\,d\omega = 2\left[\operatorname{arctg}\frac{\omega}{\alpha}\right]_{-\infty}^{\infty} =$$

$$= 2\cdot\left[\frac{\pi}{2}+\frac{\pi}{2}\right] = 2\pi$$

KONSTANS!

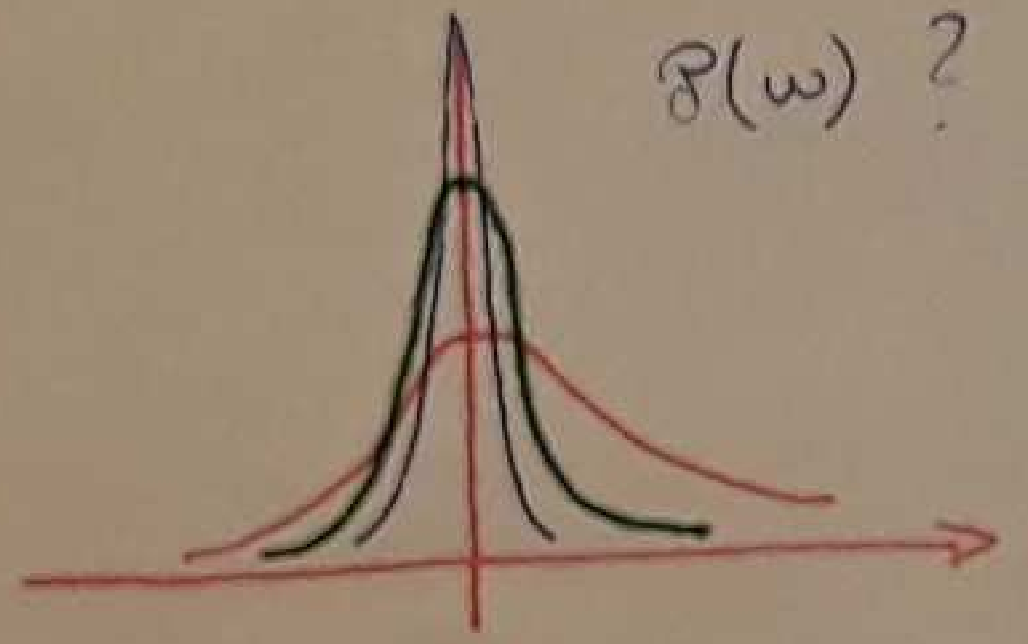
$$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x$$

$$\int \frac{\frac{1}{\alpha}}{1+\left(\frac{\omega}{\alpha}\right)^2}\,d\omega = \operatorname{arctg}\frac{\omega}{\alpha}$$

$\Downarrow \frac{d}{d\omega}$

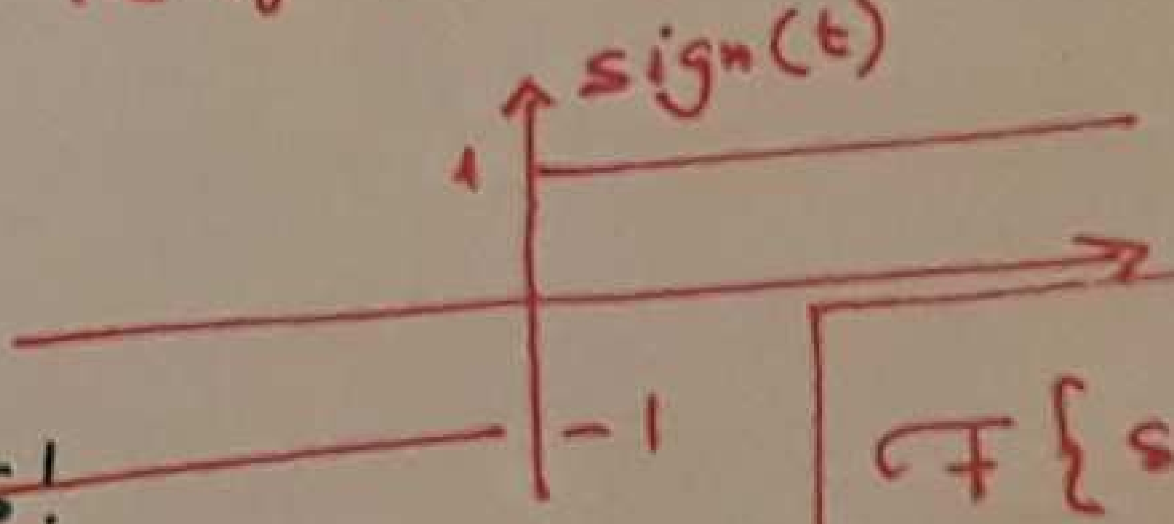
$$\frac{1/\alpha}{1+\left(\frac{\omega}{\alpha}\right)^2} = \frac{\alpha}{\alpha^2+\omega^2}$$

$\delta(\omega)$?



$\boxed{\mathcal{F}\{1\} = 2\pi\,\delta(\omega)}$

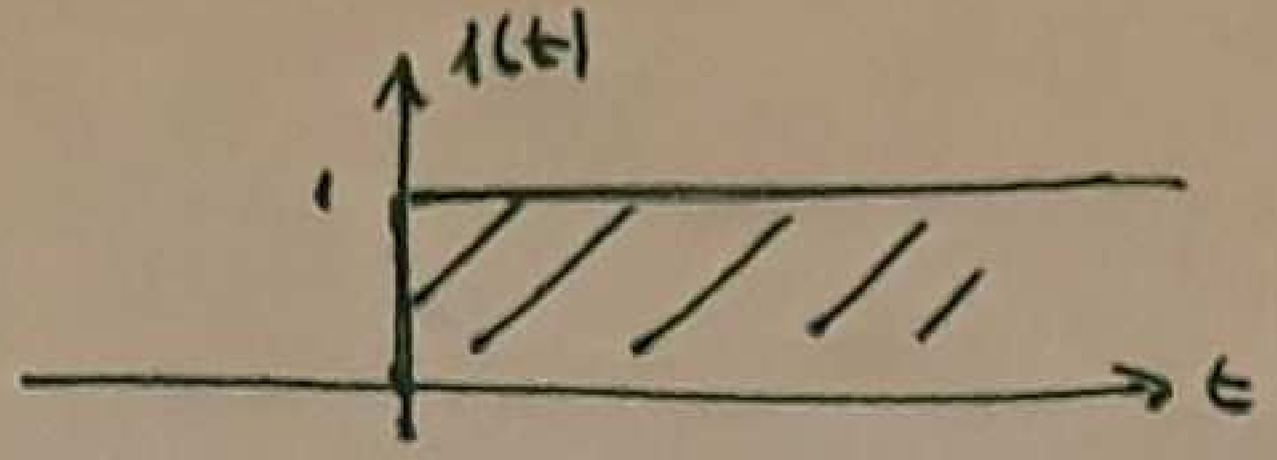
$X_2(j\omega) = -j\,\frac{2\omega}{\omega^2} = -j\,\frac{2}{\omega} = \frac{2}{j\omega}$



$\boxed{\mathcal{F}\{\operatorname{sign}(t)\} = \frac{2}{j\omega}}$

Az 1(t) jel spektruma

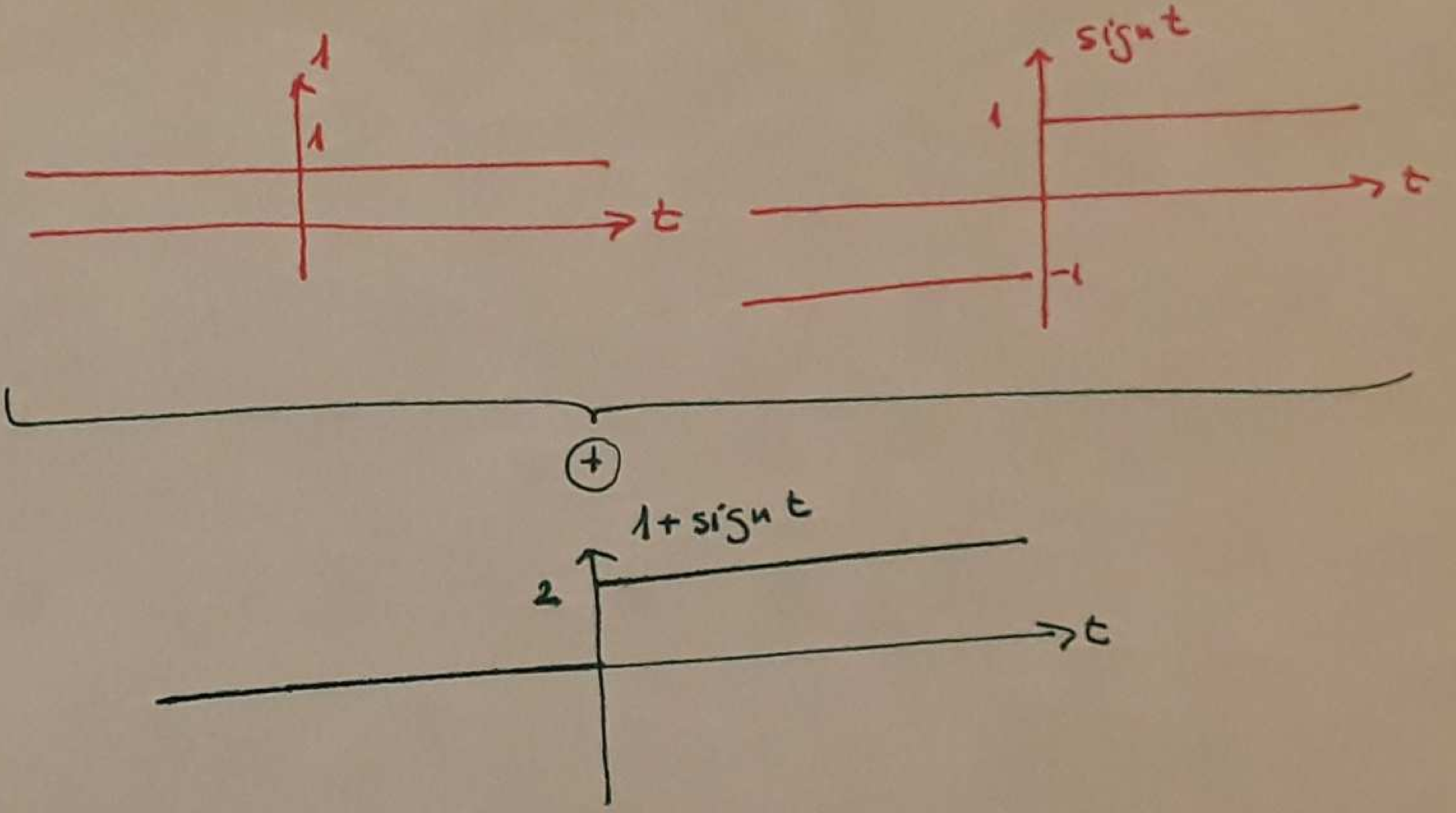
$$\mathcal{F}\{1(t)\} = \int_{-\infty}^{\infty} \underbrace{1(t)} e^{-j\omega t} dt$$



?

$$\mathcal{F}\{\text{sign}\, t\} = \frac{2}{j\omega}$$

$$\mathcal{F}\{1\} = 2\pi\delta(\omega)$$



$$\frac{1}{2} + \frac{1}{2}\,\text{sign}\, t = 1(t)$$

$$\boxed{\mathcal{F}\{1(t)\} = \pi\delta(\omega) + \frac{1}{j\omega}}$$

Parseval tétele

$$\boxed{E_u = \int_{-\infty}^{\infty} |u(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |U(j\omega)|^2\, d\omega}$$

$$\int_{-\infty}^{\infty} u(t)\, \underline{u(t)}\, dt = \int_{-\infty}^{\infty} u(t)\, \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} U(j\omega)\, e^{j\omega t}\, d\omega}\, dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \underline{u(t)} \int_{-\infty}^{\infty} \underline{U(j\omega)}\, e^{j\omega t}\, d\underline{\omega}\, d\underline{t} =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} U(j\omega) \left[\int_{-\infty}^{\infty} u(t)\, e^{-j\omega t}\, dt\right]^{*} d\omega =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} U(j\omega)\, U^{*}(j\omega)\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |U(j\omega)|^2\, d\omega .$$

$$u(t)|^2\,dt = \frac{1}{2\pi}\int^{\infty} |U(j\omega)|^2\,d\omega$$

$$(t)\,\underline{u(t)}\,dt =$$

A Fourier-transzformáció tételei:

1. Parseval tétele
2. Linearitás
3. Eltolási tétel
4. Konvolúció spektruma
5. Derivált jel, Integrálás
6. Modulációs tétel
7. Szimmetria tulajdonság
8. Skálázás megváltoztatása

$$\int_{-\infty}^{\infty} |U(j\omega)|^2\,d$$

LINEARITÁS

$$U_1(j\omega) = \int_{-\infty}^{\infty} u_1(t)\, e^{-j\omega t}\, dt$$

$$U_2(j\omega) = \int_{-\infty}^{\infty} u_2(t)\, e^{-j\omega k}\, dt$$

$$\int_{-\infty}^{\infty} [c_1 u_1(t) \pm c_2 u_2(t)]\, e^{-j\omega t}\, dt = c_1 \underbrace{\int_{-\infty}^{\infty} u_1(t)\, e^{-j\omega t}\, dt}_{U_1(j\omega)} \pm c_2 \underbrace{\int_{-\infty}^{\infty} u_2(t)\, e^{-j\omega t}\, dt}_{U_2(j\omega)}$$

$$\mathcal{F}\{c_1 u_1(t) + c_2 u_2(t)\} = c_1 \mathcal{F}\{u_1(t)\} + c_2 \mathcal{F}\{u_2(t)\}.$$

$$\mathcal{F}^{-1}\{k_1 U_1(j\omega) + k_2 U_2(j\omega)\} = k_1 \mathcal{F}^{-1}\{U_1(j\omega)\} + k_2 \mathcal{F}^{-1}\{U_2(j\omega)\}.$$

ELTOLÁSI TÉTEL

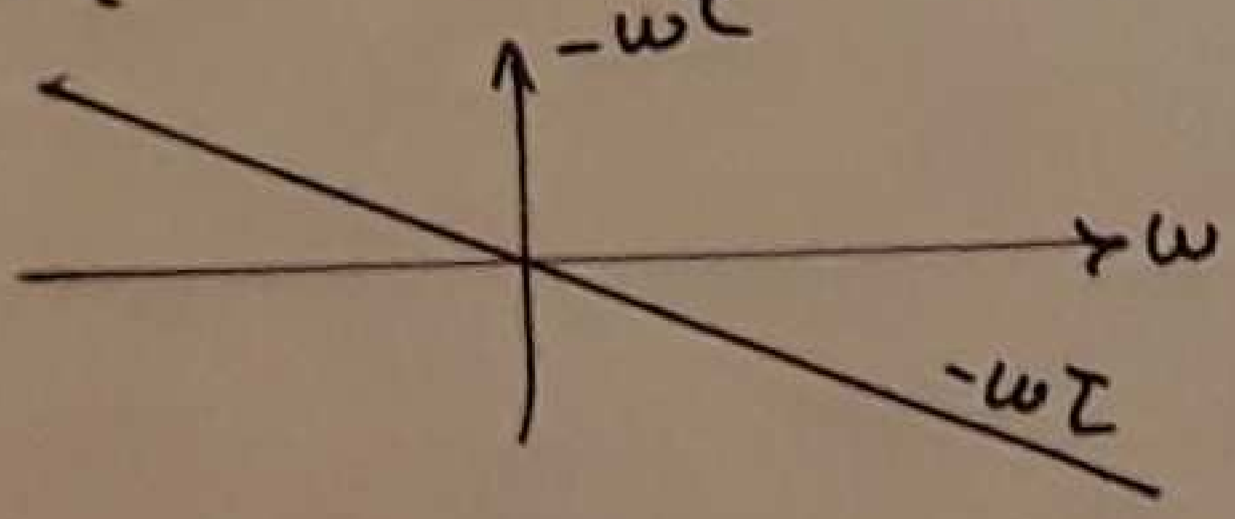
~~S(j~~

$$U(j\omega) = \mathcal{F}\{u(t)\}$$

$$\mathcal{F}\{u(t-\tau)\} = U(j\omega)\, e^{-j\omega\tau}$$

$$|e^{-j\omega\tau}| = 1$$

$$\text{arc}\{e^{-j\omega\tau}\} = -\omega\tau$$



$$u(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U(j\omega)\, e^{j\omega t}\, d\omega.$$

$$u(t-\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U(j\omega)\, e^{j\omega(t-\tau)}\, d\omega =$$

$$u(t-\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \boxed{U(j\omega)\, e^{-j\omega\tau}}\, e^{j\omega t}\, d\omega$$

KONVOLÚCIÓ SPEKTRUMA

$$\left.\begin{matrix} w(t) \\ u(t) \end{matrix}\right\} \; y(t) = \int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau \quad \Longrightarrow \quad Y(j\omega) = U(j\omega)\, W(j\omega).$$

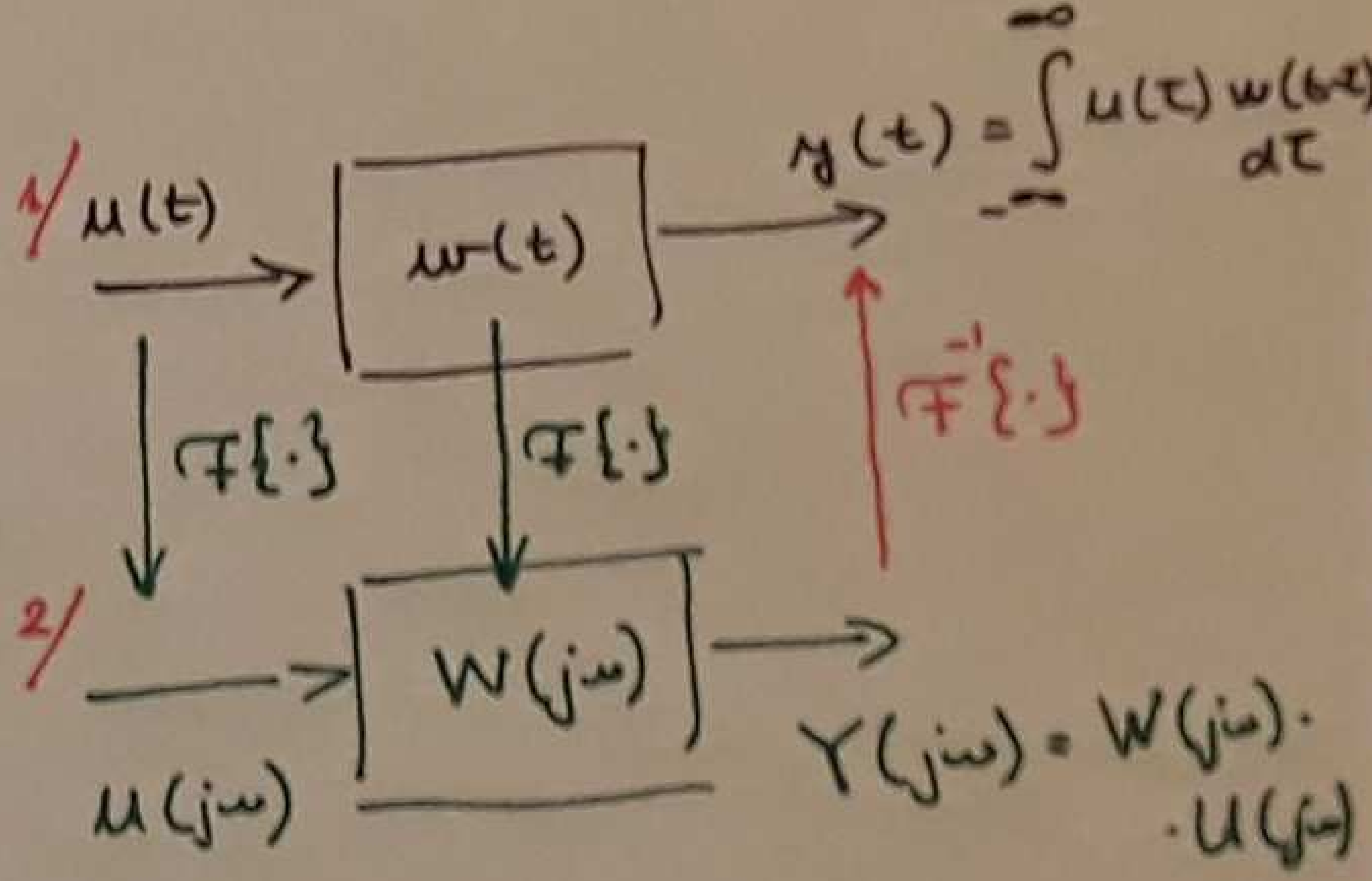
$$y(t) = \mathcal{F}^{-1}\{Y(j\omega)\} = \mathcal{F}^{-1}[U(j\omega)\, W(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underline{U(j\omega)}\, W(j\omega)\, e^{j\omega \overset{\uparrow}{t}}\, d\omega =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(\tau)\, e^{-j\omega\tau}\, d\tau \;\; W(j\omega)\, e^{j\omega t}\, d\omega =$$

$$= \int_{-\infty}^{\infty} u(\tau) \left[\frac{1}{2\pi}\int_{-\infty}^{\infty} W(j\omega)\, e^{j\omega(t-\tau)}\, d\omega \right] dt$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{W(j\omega)\, e^{-j\omega\tau}}_{w(t)|_{t \to t-\tau}}\, e^{j\omega t}\, d\omega = w(t-\tau)$$

$$= \int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau.$$

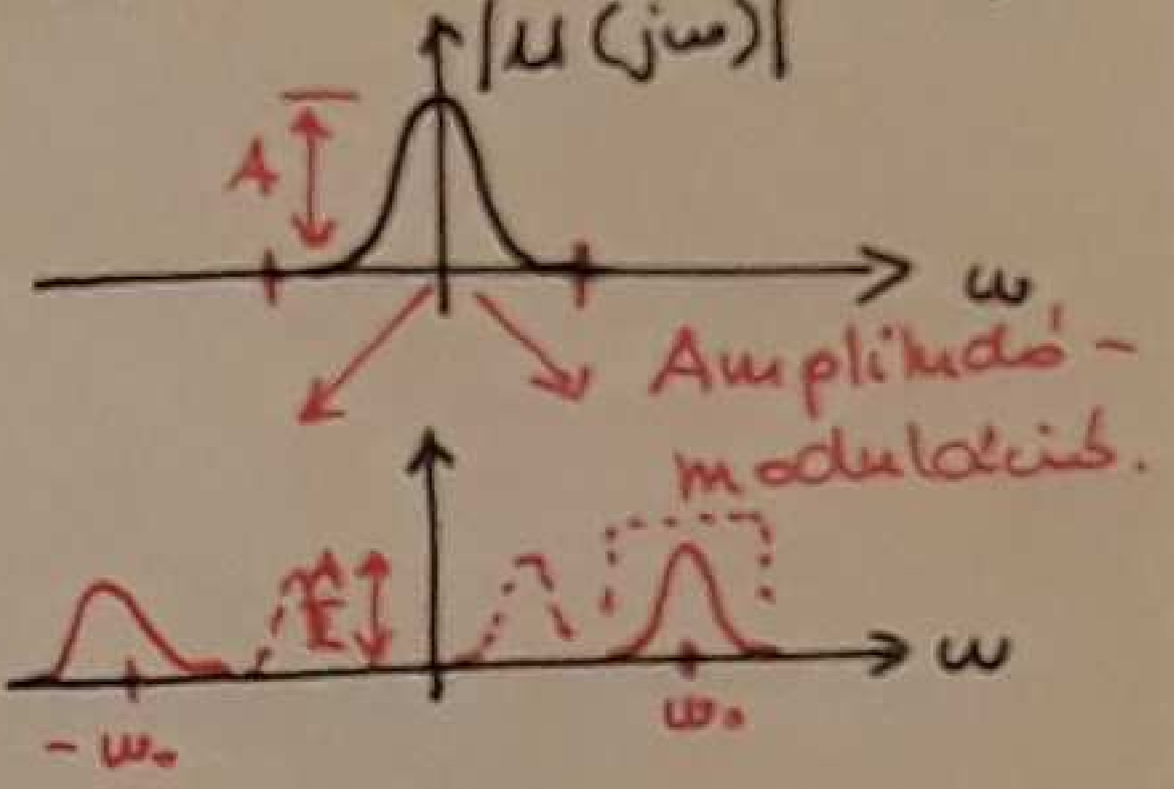


$$W(j\omega) = \mathcal{F}\{w(t)\} = \int_{-\infty}^{\infty} w(t)\, e^{-j\omega t}\, dt.$$

$$\updownarrow \quad w(t) = \mathcal{F}^{-1}\{W(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} W(j\omega)\, e^{j\omega t}\, d\omega.$$

MODULÁCIÓS TÉTEL

$$\int_{-\infty}^{\infty}\left[u(t)\,e^{j\omega_0 t}\right]e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} u(t)\,e^{-j(\omega-\omega_0)t}\,dt = U(j\omega)\Big|_{\omega\to\omega-\omega_0} = U(j[\omega-\omega_0])$$



$$\cos\omega_0 t = \frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}$$

$$\int_{-\infty}^{\infty}\left[u(t)\cos\omega_0 t\right]e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} u(t)\,\frac{e^{+j\omega_0 t}+e^{-j\omega_0 t}}{2}\,e^{-j\omega t}\,dt =$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} u(t)\,e^{-j(\omega-\omega_0)t}\,dt + \frac{1}{2}\int_{-\infty}^{\infty} u(t)\,e^{-j(\omega+\omega_0)t}\,dt = \frac{1}{2}\Big[U(j[\omega-\omega_0]) + U(j[\omega+\omega_0])\Big].$$

$$\int_{-\infty}^{\infty}\left[u(t)\sin\omega_0 t\right]e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} u(t)\,\frac{e^{j\omega_0 t}-e^{-j\omega_0 t}}{2j}\,e^{-j\omega t}\,dt = \frac{1}{2j}\Big[U(j[\omega-\omega_0]) - U(j[\omega+\omega_0])\Big].$$

DERIVÁLT JEL SPEKTRUMA, JEL INTEGRÁLJÁNAK SPEKTRUMA

$$u(t) \rightarrow U(j\omega)$$

$$\dot{u}(t) \rightarrow j\omega U(j\omega) \left< \begin{array}{l} |\mathcal{F}\{\dot{u}(t)\}| = \omega |U(j\omega)| \\ +90^\circ \end{array} \right.$$

$$\ddot{u}(t) \rightarrow (j\omega)^2 U(j\omega)$$

$$\vdots$$

$$u^{(n)}(t) \rightarrow (j\omega)^n U(j\omega)$$

$$u(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} U(j\omega)\, e^{j\omega t}\, d\omega \quad \Big/ \frac{d}{dt} \quad \Rightarrow \quad \dot{u}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \boxed{U(j\omega)\, j\omega}\, e^{j\omega t}\, d\omega$$

$$u(t) \rightarrow U(j\omega) \qquad \mathcal{F}\left\{\int_{-\infty}^{t} u(\tau)\, d\tau\right\} = \frac{1}{j\omega} U(j\omega) + \pi U(j0)\, \delta(\omega)$$

$$\int_{-\infty}^{t} u(\tau)\, d\tau = \int_{-\infty}^{t} u(\tau)\, 1(t-\tau)\, d\tau \Rightarrow \underbrace{\mathcal{F}\{u(t)\}}_{U(j\omega)} \cdot \underbrace{\mathcal{F}\{1(t)\}}_{\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]}$$

$$U(j\omega) \cdot \left[\frac{1}{j\omega} + \pi\delta(\omega)\right] = \frac{1}{j\omega} U(j\omega) + U(j\underset{\uparrow}{\omega})\, \pi\, \delta(\omega)$$

$$= \frac{1}{j\omega} U(j\omega) + \underline{\underline{\pi}}\, U(j0)\, \delta(\omega)$$

SZIMMETRIATULAJDONSÁG

$G(\omega) \in \mathbb{R}$

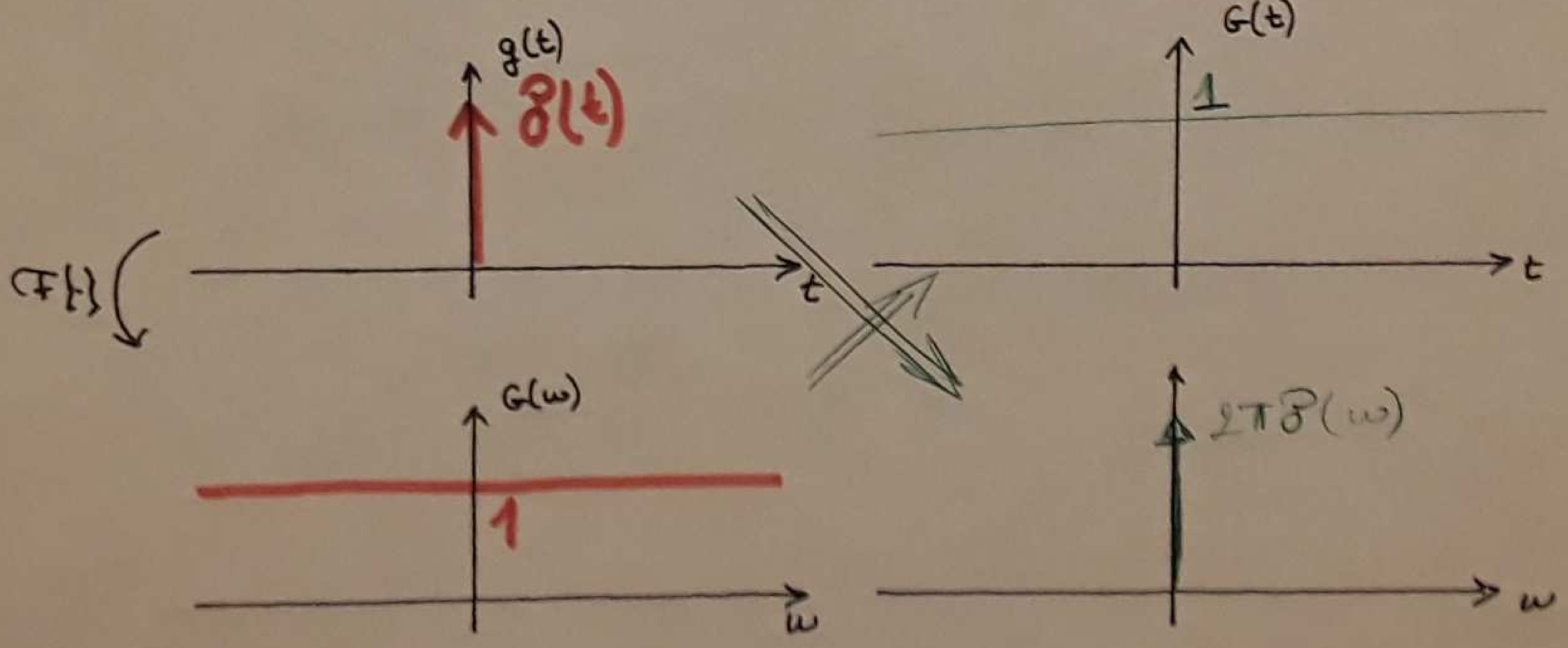
$$G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\underset{\substack{\downarrow \\ t}}{\omega})\, e^{j\omega t}\, d\omega$$

$$\boxed{\omega \rightarrow t; \quad t \rightarrow -\omega}$$

$$g(-\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(t)\, e^{-j\omega t}\, dt$$

$$\boxed{2\pi g(-\omega) = \int_{-\infty}^{\infty} G(t)\, e^{-j\omega t}\, dt}$$



SKÁLÁZÁS MEGVÁLTOZTATÁSA

$u(t) \longrightarrow U(j\omega)$

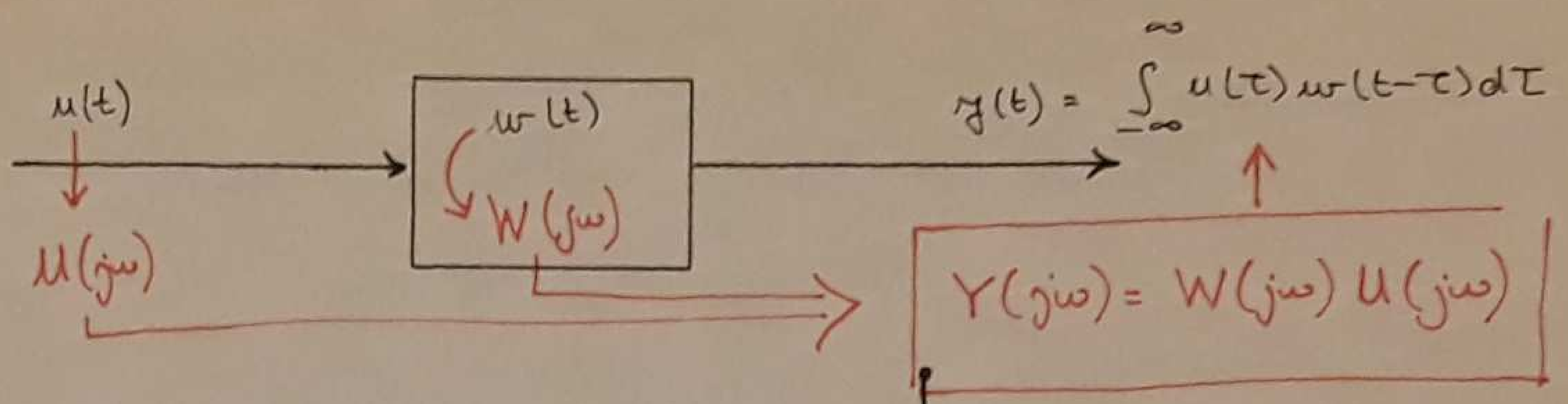
$u(\alpha t) \longrightarrow \frac{1}{\alpha} U(j\frac{\omega}{\alpha})$

$$u(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} U(j\boxed{\omega})\, e^{j\omega t}\, d\omega$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{\alpha} U\left(j\boxed{\frac{\omega}{\alpha}}\right) e^{j\omega t}\, d\omega$$

$$u(\alpha t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} U\left(j\frac{\omega}{\alpha}\right) e^{j\frac{\omega}{\alpha}\boxed{\alpha t}}\, \frac{d\omega}{\alpha}$$

A VÁLASZ SPEKTRUMA ÉS IDŐFÜGGVÉNYE



$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau$$

$$\boxed{Y(j\omega) = W(j\omega)\, U(j\omega)}$$

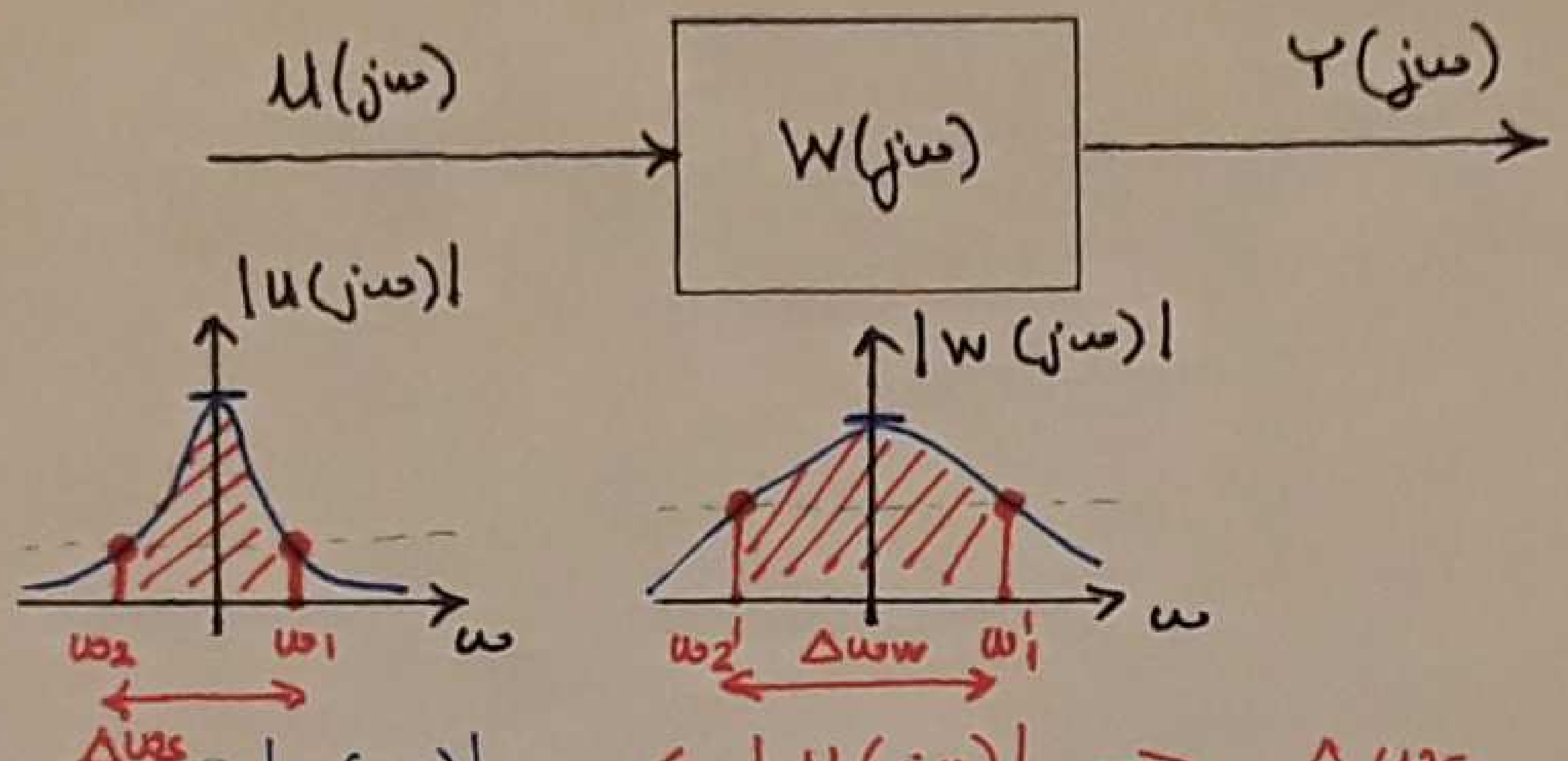
$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(j\omega)\, e^{j\omega t}\, d\omega.$$

↳ szűrőkarakterisztikák

↳ sávszélesség

⋮

SÁVSZÉLESSÉG

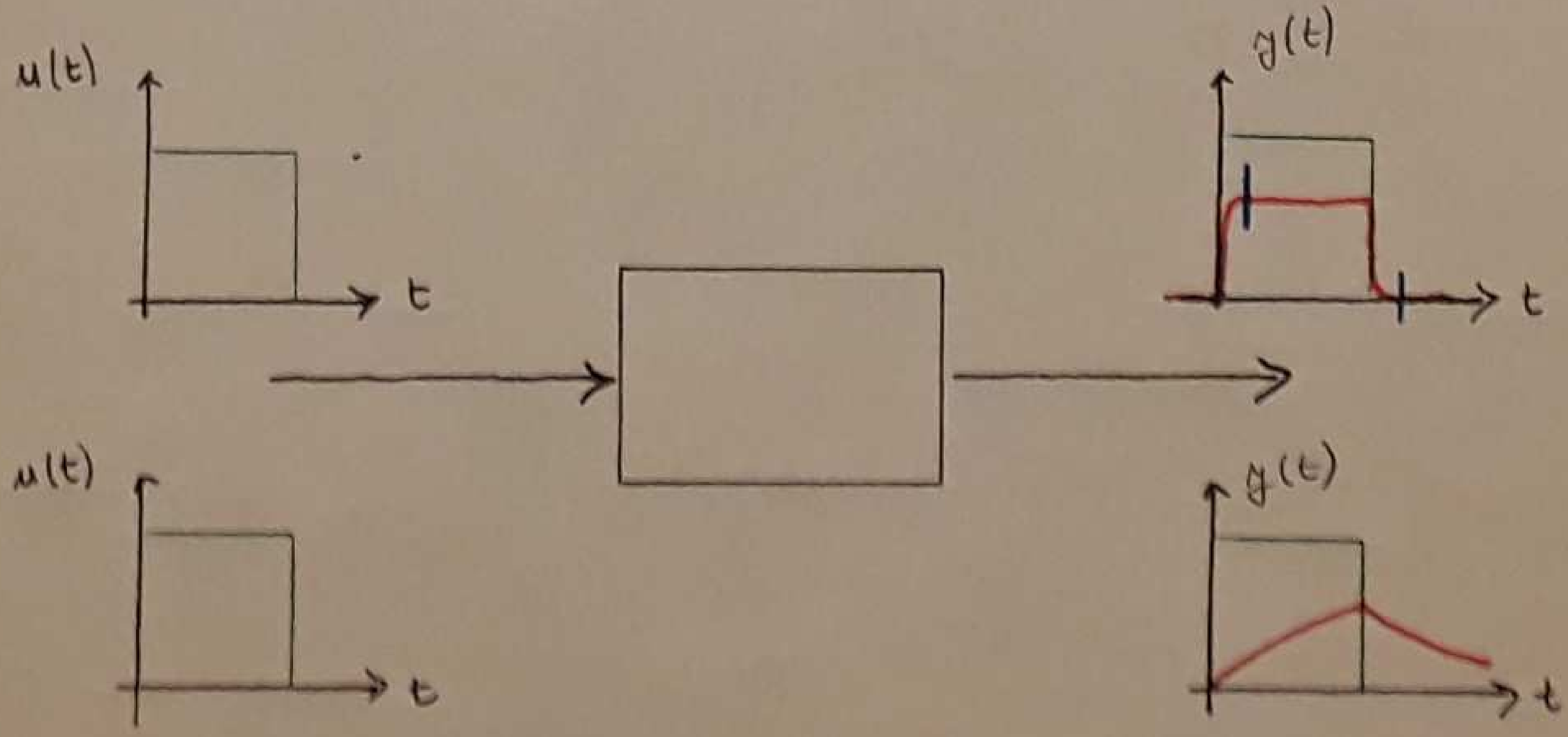


$$\varepsilon\,|U(j\omega)|_{max} \leq |U(j\omega)| \Rightarrow \Delta\omega_s$$

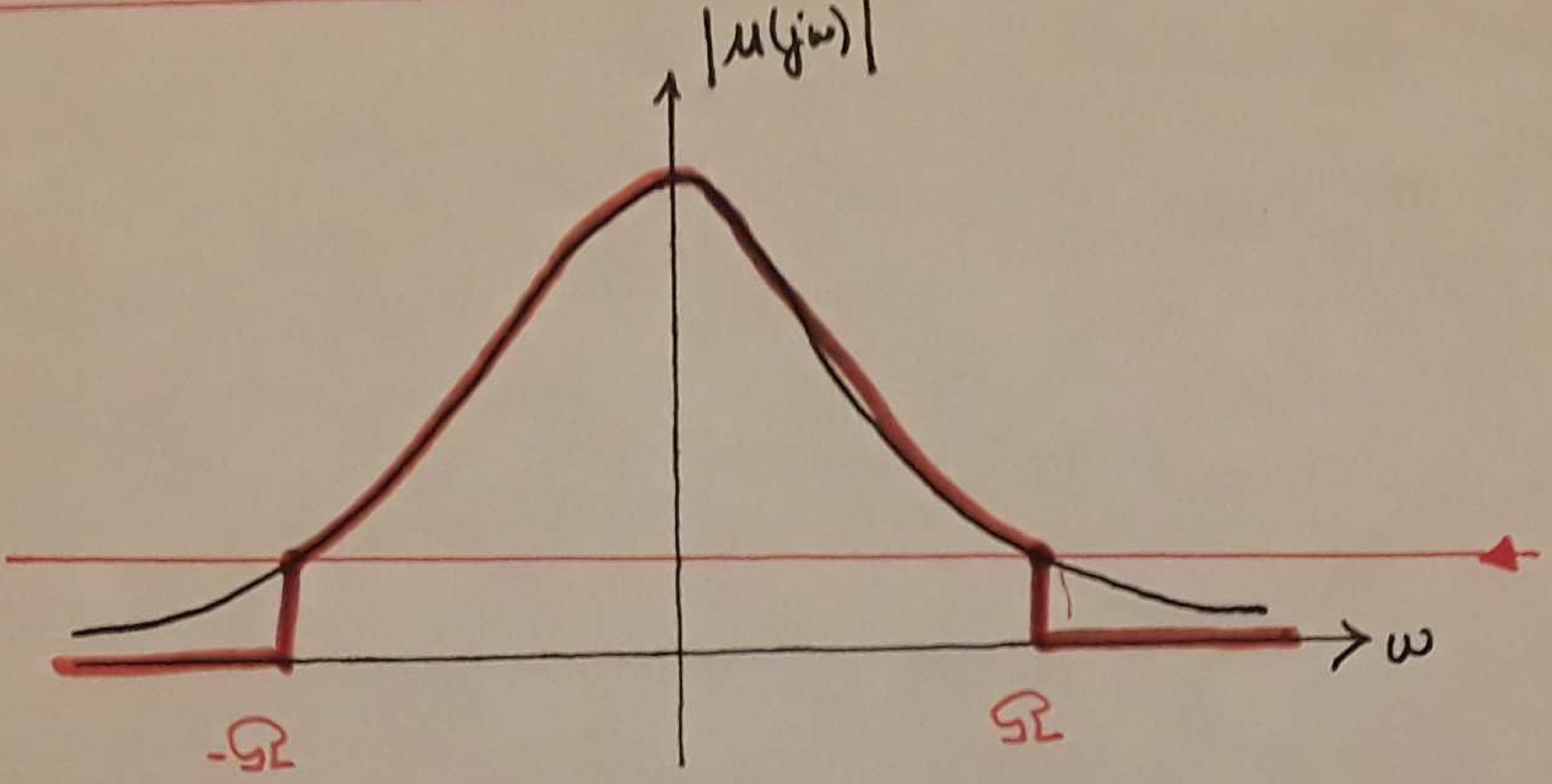
$$\eta\,|W(j\omega)|_{max} \leq |W(j\omega)| \Rightarrow \Delta\omega_w$$

Alakhű jelátvitel

$$\boxed{\Delta\omega_w \geq \Delta\omega_s}$$

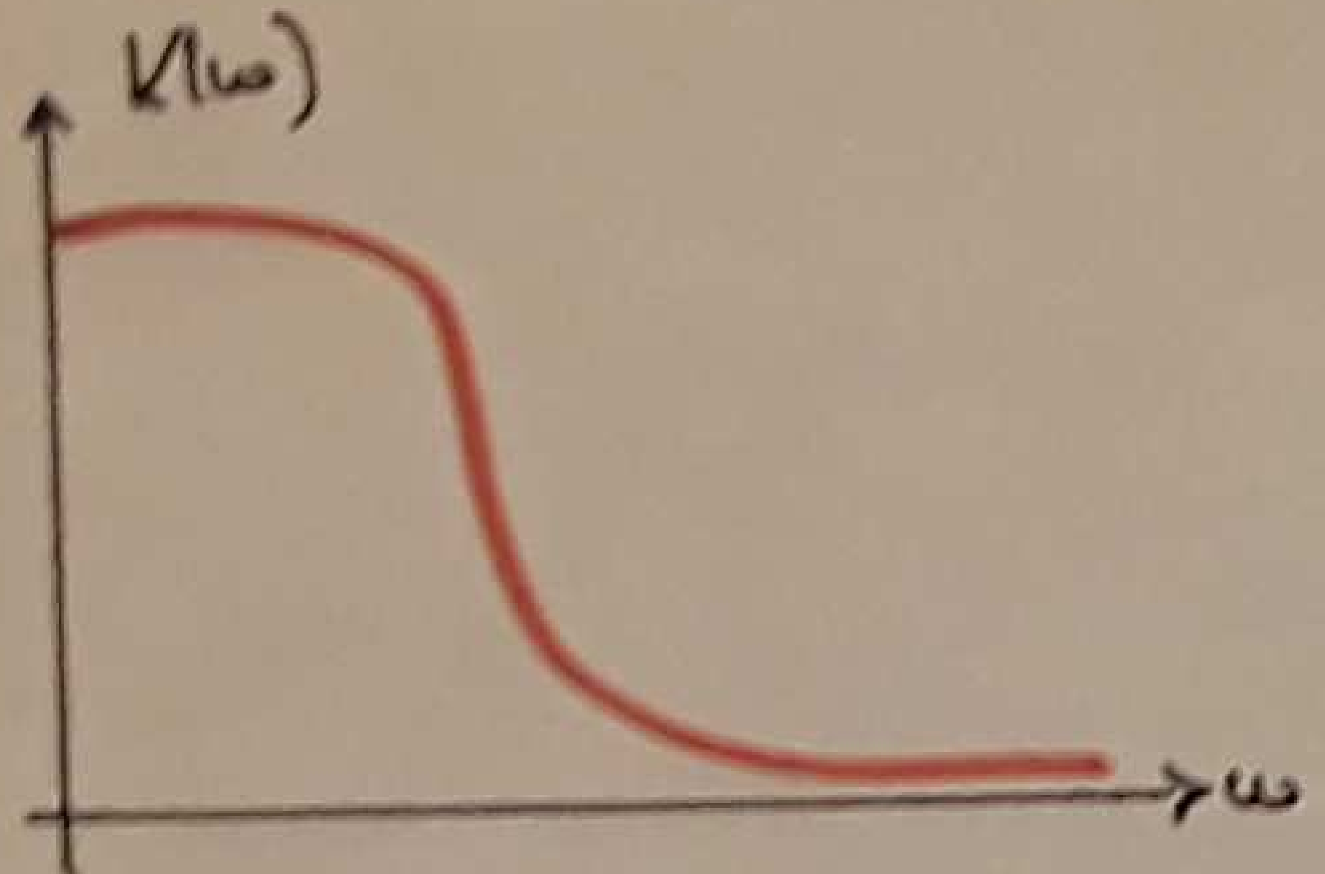


SÁVKORLÁTOZOTT JEL

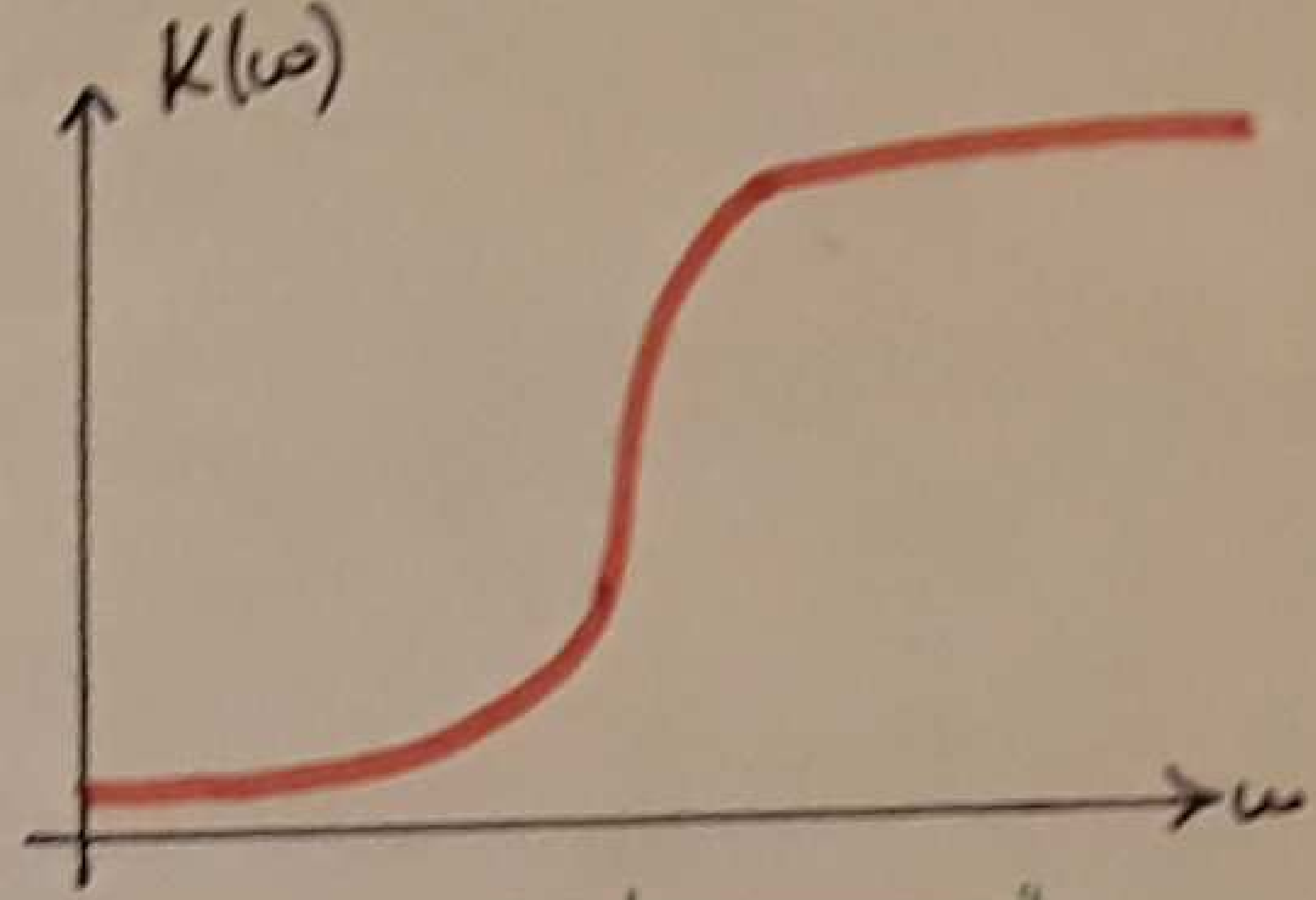


$$|U(j\omega)| = 0 \qquad |\omega| \geq \Omega$$

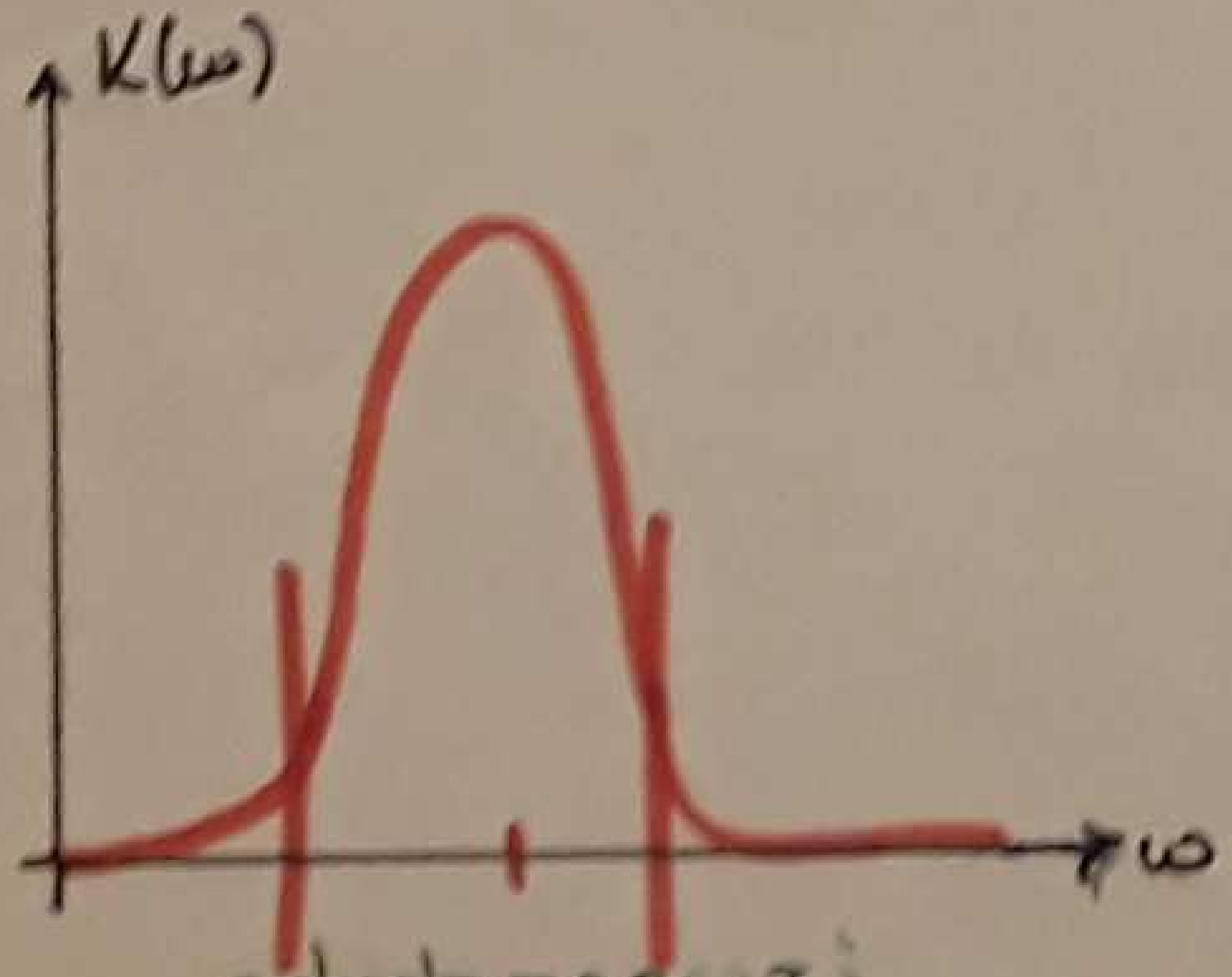
SZŰRŐK



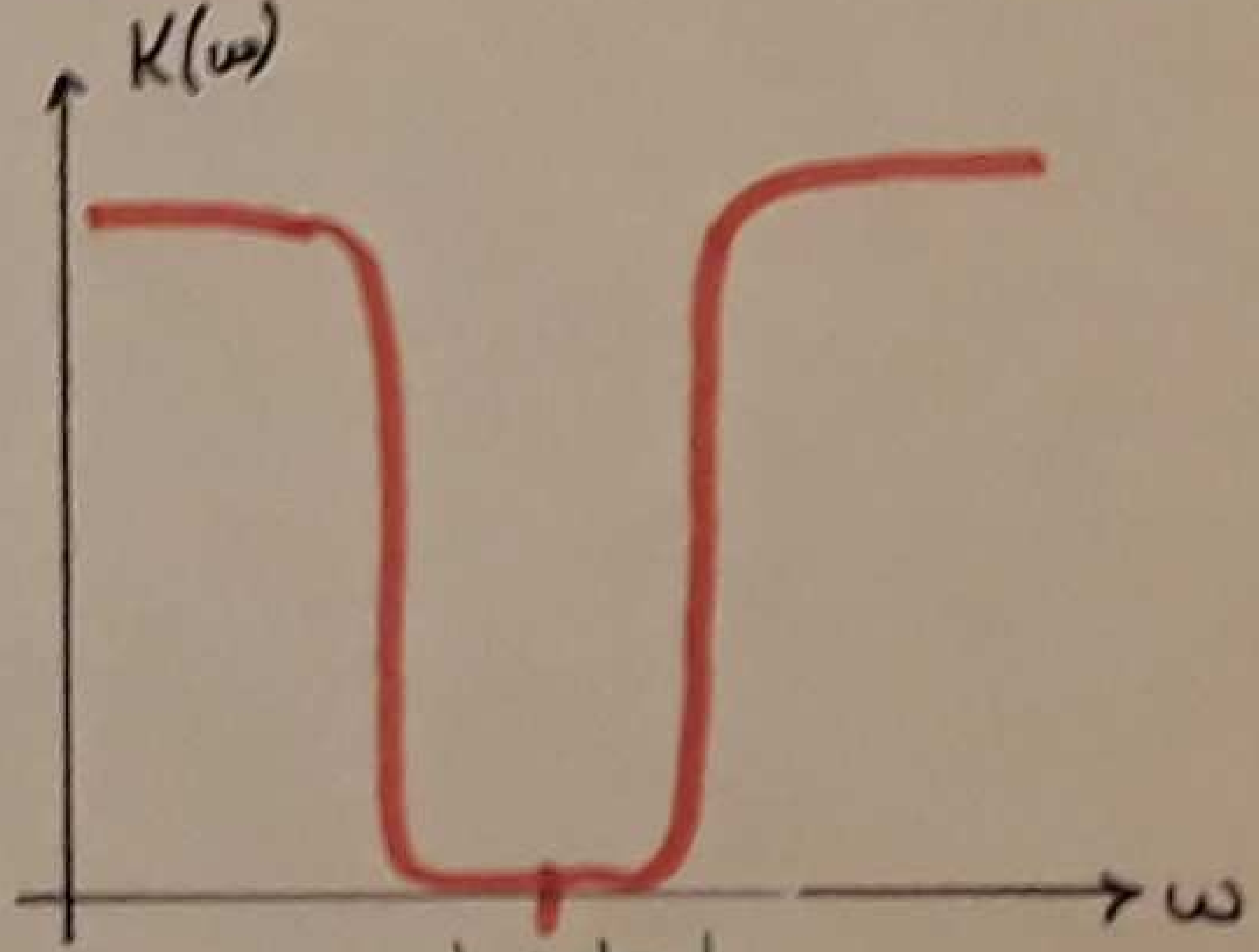
ALULÁTERESZTŐ



FELÜLÁTERESZTŐ



SÁVÁTERESZTŐ



SÁVZÁRÓ

szinuszos jel

↓

Fourier-sor

↓

Fourier-transzformáció

Laplace-transzformáció

FOLYTONOS IDEJŰ RENDSZEREK ANALÍZISE A KOMPLEX FREKVENCIATARTOMÁNYBAN.

Laplace-transzformáció:

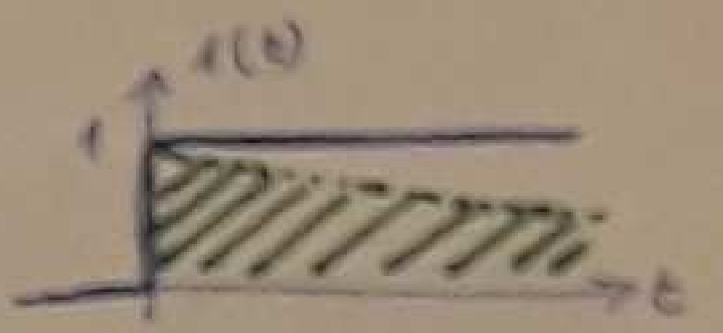
bekapcsolás!

- válasz időfüggvényének zárt alakú kifejezése a $0 \le t < \infty$ intervallumban
- egyszerű formális szabályok, matematikai módszer ↔ fizikai tartalma nincs.

$\sigma > 0$

Fourier-transzformáció:

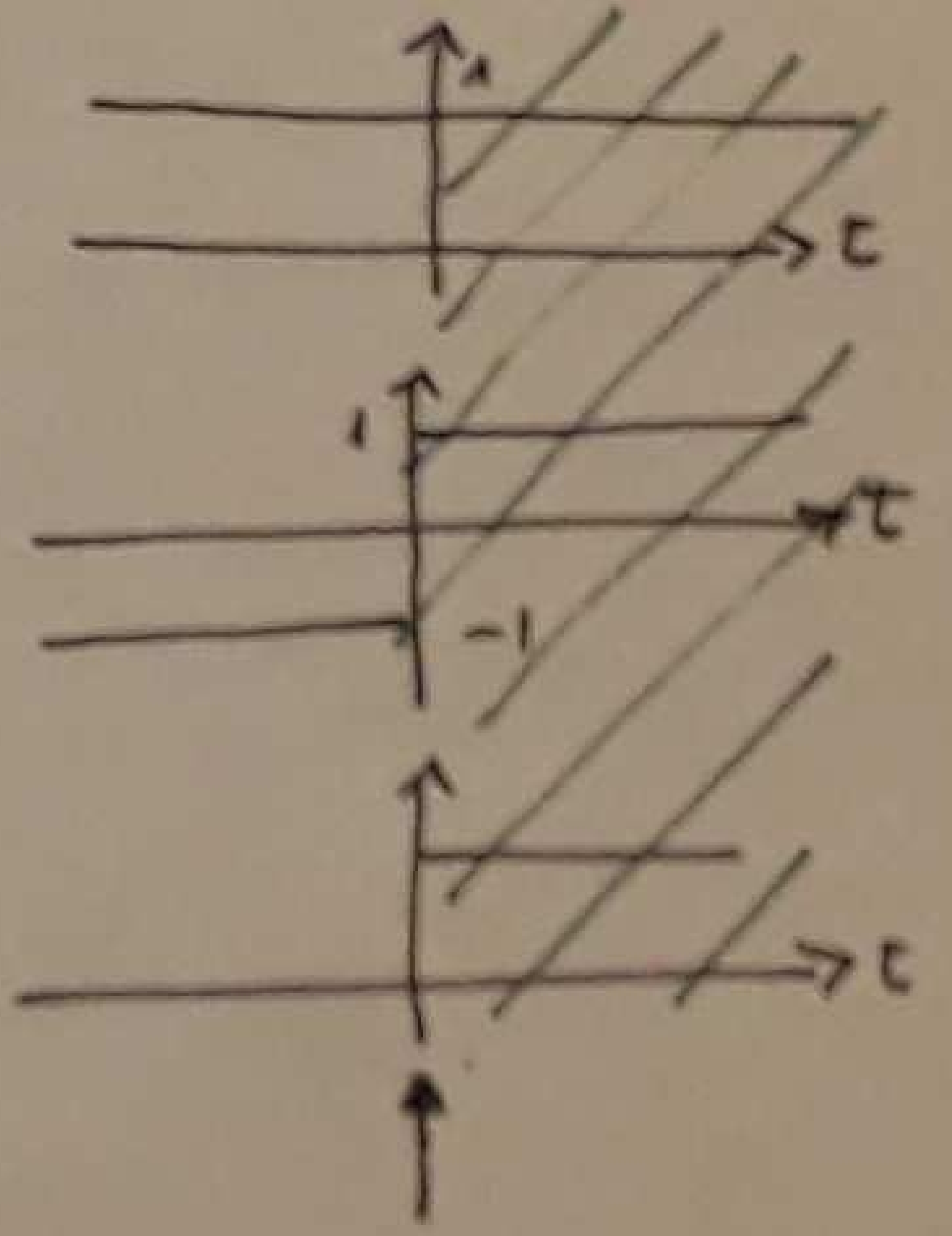
$$\underbrace{U(j\omega)}_{} = \mathcal{F}\{u(t)\} = \int_{-\infty}^{\infty} \underbrace{u(t)}_{} e^{-j\omega t}\, dt \quad , \quad \text{ahol} \int_{-\infty}^{\infty} |u(t)|\, dt < \infty .$$



$\cdot e^{-\sigma t}$

$$\int_{-\infty}^{\infty} \underbrace{1(t)\, u(t)\, e^{-\sigma t}}_{} e^{-j\omega t}\, dt$$

$$\int_{0}^{\infty} u(t) \underbrace{e^{-\sigma t} e^{-j\omega t}}_{e^{-(\sigma+j\omega)t} = e^{-st}} dt \qquad \overset{(p)}{s = \sigma + j\omega}$$



$$\boxed{U(s) = \mathcal{L}\{u(t)\} = \int_{-0}^{\infty} u(t)\, e^{-st}\, dt}$$

Inverz transzformáció:

(!) FFT

$$\begin{tikzcd} u(t) \arrow[r, bend left, "\mathcal{L}"] & U(s) \arrow[l, bend left, "\mathcal{L}^{-1}"] \end{tikzcd}$$

$$u(t) = \mathcal{F}^{-1}\{U(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} U(j\omega)\, e^{j\omega t}\, d\omega.$$

$$1(t)\,u(t)\,e^{-\sigma t} = \frac{1}{2\pi}\int_{-\infty}^{\infty} U(\sigma + j\omega)\, e^{j\omega t}\, d\omega \qquad / e^{\sigma t}$$

$$1(t)\,u(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} U(\underbrace{\sigma + j\omega}_{s})\, \underbrace{e^{\sigma t}\, e^{j\omega t}}_{e^{(\sigma + j\omega)t} = e^{st}}\, \boxed{d\omega}$$

$s = \sigma + j\omega$

$s = \sigma + j\omega$

$ds = j\,d\omega \rightarrow \boxed{d\omega = \frac{ds}{j}}$

$$1(t)\,u(t) = \frac{1}{2\pi}\int_{\sigma - j\infty}^{\sigma + j\infty} U(s)\, e^{st}\, \frac{ds}{j}$$

$$\boxed{1(t)\,u(t) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} U(s)\, e^{st}\, ds = \mathcal{L}^{-1}\{U(s)\}}$$

Fourier-Mellin

Inverziós integrál.

↳ egyszerű szabályok.

Néhány jel Laplace-transzformáltja:

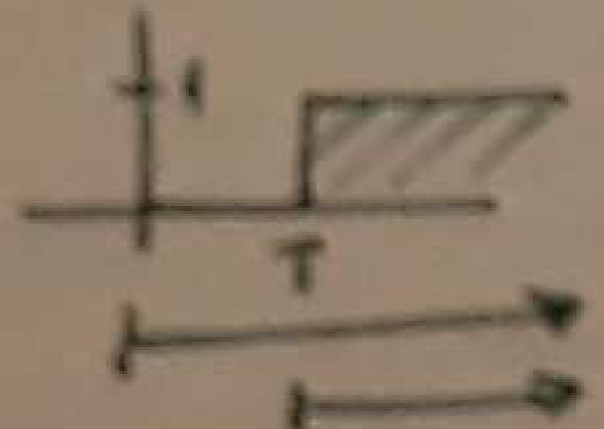
①
$$\mathcal{L}\{\delta(t)\} = \int_{-0}^{\infty} \underbrace{\delta(t)\, e^{-st}}_{t=0} dt = \int_{-0}^{\infty} \delta(t) \underbrace{e^{-s\cdot 0}}_{1} dt = \int_{-0}^{\infty} \delta(t)\, dt = \underline{1}$$

$$\mathcal{L}\{\delta(t-\tau)\} = \int_{-0}^{\infty} \underbrace{\delta(t-\tau)\, e^{-st}}_{t=\tau \qquad \tau} dt = \int_{-0}^{\infty} \delta(t-\tau) \underbrace{e^{-s\tau}} dt = e^{-s\tau} \int_{-0}^{\infty} \delta(t-\tau)\, dt = \underline{e^{-s\tau}}$$

$$\left(\frac{e^{-st}}{-s}\right)' = \frac{e^{-st}\,\cancel{(-s)}}{\cancel{-s}}$$

②
$$\mathcal{L}\{1(t)\} = \int_{-0}^{\infty} \underbrace{1(t)}_{1}\, e^{-st} dt = \int_{0}^{\infty} e^{-st} dt = \left[\frac{e^{-st}}{-s}\right]_0^{\infty} = \frac{0-1}{-s} = \underline{\frac{1}{s}}$$

$$\mathcal{L}\{1(t) - 1(t-T)\} = \int_0^{\infty} [1(t) - 1(t-T)]\, e^{-st} dt = \underbrace{\int_0^{\infty} 1(t)\, e^{-st} dt}_{\frac{1}{s}} - \int^{\infty} 1(t-T)\, e^{-st} dt = \underline{\frac{1}{s} - \frac{1}{s} e^{-sT}}$$



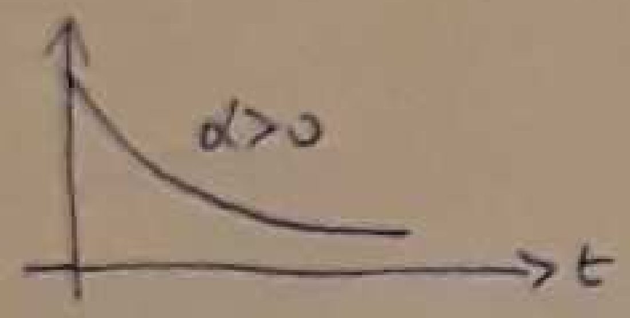
$$\int_T^{\infty} e^{-st} dt = \left[\frac{e^{-st}}{-s}\right]_T^{\infty} = \frac{0 - e^{-sT}}{-s} =$$

$$= \underline{\frac{1}{s} e^{-sT}}$$

Néhány jel Laplace-transzformáltja:

③ $\mathcal{L}\{1(t)\,e^{-\alpha t}\} = \int_0^\infty e^{-\alpha t} e^{-st}\,dt = \int_0^\infty e^{-(\alpha+s)t}\,dt = \left[\frac{e^{-(s+\alpha)t}}{-(s+\alpha)}\right]_0^\infty =$

$(\alpha > 0)$



$$= \frac{0-1}{-(s+\alpha)} = \underline{\underline{\frac{1}{s+\alpha}}}$$

$$(s-j\omega)(s+j\omega) = s^2-(j\omega)^2 =$$
$$= s^2 - j^2\omega^2 =$$
$$= s^2 \underset{\uparrow}{+} \omega^2 \quad !$$

$$\mathcal{L}\{1(t)\,e^{j\omega t}\} = \frac{1}{s-j\omega}$$

$$\mathcal{L}\{1(t)\,e^{-j\omega t}\} = \frac{1}{s+j\omega}$$

$$\mathcal{L}\{1(t)\cos\omega t\} = \mathcal{L}\left\{1(t)\,\frac{e^{j\omega t}+e^{-j\omega t}}{2}\right\} = \frac{1}{2}\,\frac{1}{s-j\omega} + \frac{1}{2}\,\frac{1}{s+j\omega} = \frac{s+\cancel{j\omega}+s-\cancel{j\omega}}{s^2+\omega^2}\cdot\frac{1}{2} = \underline{\underline{\frac{s}{s^2+\omega^2}}}$$

$$\mathcal{L}\{1(t)\sin\omega t\} = \mathcal{L}\left\{1(t)\,\frac{e^{j\omega t}-e^{-j\omega t}}{2j}\right\} = \frac{1}{2j}\,\frac{1}{s-j\omega} - \frac{1}{2j}\,\frac{1}{s+j\omega} = \frac{\cancel{s}+j\omega-\cancel{s}+j\omega}{s^2+\omega^2}\,\frac{1}{2j} = \underline{\underline{\frac{\omega}{s^2+\omega^2}}}$$

Néhány jel Laplace-transzformáltja

④ $\mathcal{L}\{1(t)t\} = \int_0^{\infty} \underbrace{t}_{v} \underbrace{e^{-st}}_{u'} dt =$

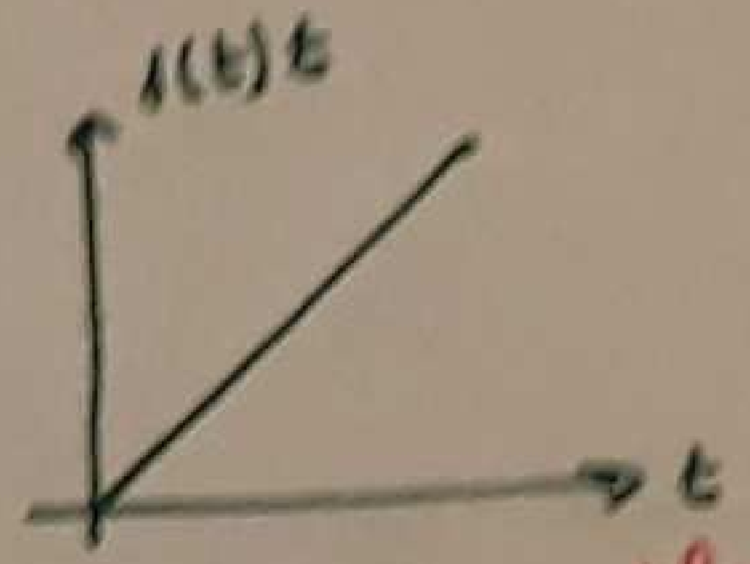
$\int u'v = uv - \int uv'$

$u' = e^{-st}$ $\qquad u = \frac{e^{-st}}{-s}$

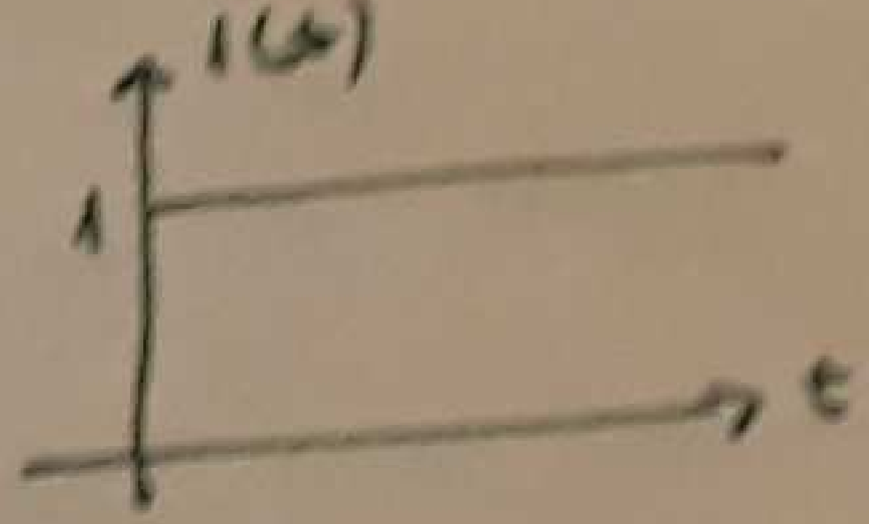
$v = t$ $\qquad v' = 1$

$$= \left[\frac{e^{-st}}{-s} t \right]_0^{\infty} - \int_0^{\infty} \frac{e^{-st}}{-s} \cdot 1 \, dt =$$

$$= \emptyset + \frac{1}{s} \underbrace{\int_0^{\infty} e^{-st} \, dt}_{\frac{1}{s}} = \frac{1}{s^2}$$



vizsgálójel.



$\mathcal{L}\{c_1 u_1(t) + c_2 u_2(t) + \dots\} = c_1 \mathcal{L}\{u_1(t)\} + c_2 \mathcal{L}\{u_2(t)\} +$

$\mathcal{L}^{-1}\{b_1 U_1(s) + b_2 U_2(s) + \dots\} = b_1 \mathcal{L}^{-1}\{U_1(s)\} + b_2 \mathcal{L}^{-1}\{U_2(s)\}$

A Laplace-transzformáció tételei

1. Linearitás
2. Eltolási tétel
3. Konvolúció transzformáltja
4. Derivált jel, jel integrálja
5. Csillapítási tétel
6. Kezdeti-érték, végérték

$: =$

$= T + \tau \quad dt$

$+\tau) \quad dT =$

$e^{-s\tau}$

$T) e^{-sT} dT$

$=$

$U(s)$

LINEARITÁS

$$\left.\begin{aligned} U(s) &= \int_{-0}^{\infty} u(t)\, e^{-st}\, dt \\ u(t) &= \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} U(s)\, e^{st}\, ds \end{aligned}\right\}$$

$$\int_0^{\infty} \left[c_1 u_1(t) + c_2 u_2(t)\right] e^{-st}\, dt = \int_0^{\infty} c_1 u_1(t)\, e^{-st}\, dt + \int_0^{\infty} c_2 u_2(t)\, e^{-st}\, dt$$

$$= c_1 \underbrace{\int_0^{\infty} u_1(t)\, e^{-st}\, dt}_{U_1(s)} + c_2 \underbrace{\int_0^{\infty} u_2(t)\, e^{-st}\, dt}_{U_2(s)}$$

szuperpozíció.

$$\mathcal{L}\{c_1 u_1(t) + c_2 u_2(t) + \dots\} = c_1 \mathcal{L}\{u_1(t)\} + c_2 \mathcal{L}\{u_2(t)\} + \dots$$

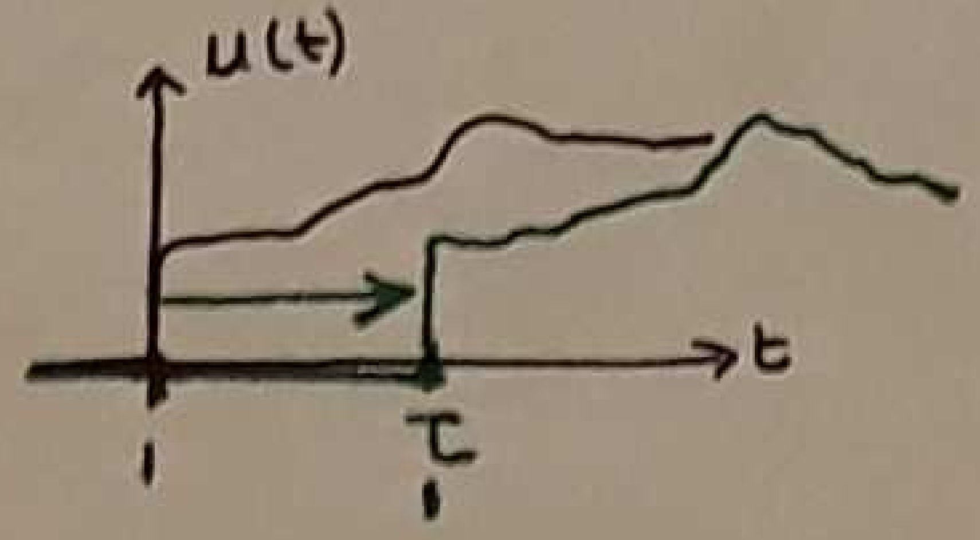
$$\mathcal{L}^{-1}\{b_1 U_1(s) + b_2 U_2(s) + \dots\} = b_1 \mathcal{L}^{-1}\{U_1(s)\} + b_2 \mathcal{L}^{-1}\{U_2(s)\} + \dots$$

ELTOLÁSI TÉTEL

$$\mathcal{L}\{u(t)\} = U(s)$$

$$\mathcal{L}\{u(t-\underset{\uparrow}{\tau})\} = \underline{U(s)}\,\underline{e^{-s\tau}}$$

$$\mathcal{L}\{1(t-\tau)\, u(t-\tau)\} = \int_{\tau}^{\infty} u(\underline{t-\tau})\, e^{-st}\, dt =$$



$$T = t-\tau \longrightarrow t = T+\tau \qquad dt = dT$$

$$= \int_0^{\infty} u(T)\, \underbrace{e^{-s(T+\tau)}}_{e^{-sT}\, e^{-s\tau}}\, dT =$$

$$= \int_0^{\infty} u(T)\, e^{-sT}\, e^{-s\tau}\, dT = e^{-s\tau} \underbrace{\int_0^{\infty} u(T)\, e^{-sT}\, dT}_{U(s)} = \underline{\underline{\boxed{e^{-s\tau}}\, U(s)}}$$

A KONVOLÚCIÓ LAPLACE-TRANSZFORMÁLTJA

$$W(s)=\int_{-0}^{\infty} w(t)e^{-st}\,dt$$

$$U(s)=\int_{-0}^{\infty} u(t)\,e^{-st}\,dt.$$

$$\left.\begin{matrix}w(t)\\u(t)\end{matrix}\right\}\ y(t)=\int_0^t u(\tau)\,w(t-\tau)\,d\tau \quad\xRightarrow{\mathcal{L}}\quad Y(s)=\underline{W(s)}\,U(s).$$

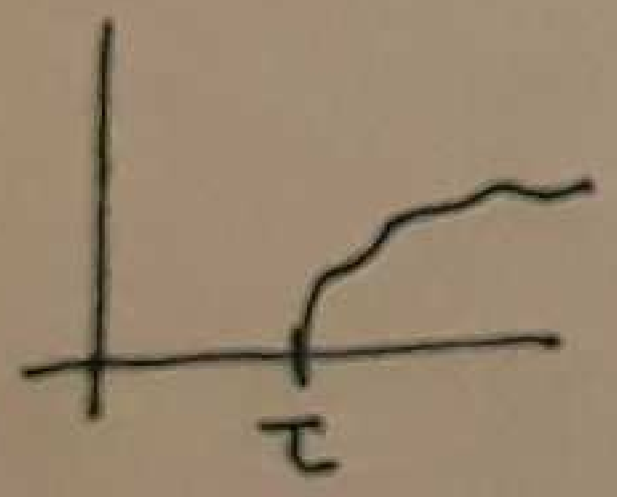
↗ átviteli függvény.

$$Y(s)=\mathcal{L}\{y(t)\}=\int_{-0}^{\infty}\underline{y(t)}\,e^{-st}\,dt=\int_{-0}^{\infty}\int_{-0}^{\cancel{t}\,\infty} u(\tau)\,\underbrace{w(t-\tau)}_{1(t-\tau)\,w(t-\tau)}\,d\tau\ e^{-st}\,dt=$$

$$=\int_{-0}^{\infty}\int_{-0}^{\infty} u(\tau)\,1(t-\tau)\,w(t-\tau)\overset{d\tau}{\wedge}e^{-st}\,dt=$$

$$\uparrow \qquad \uparrow$$

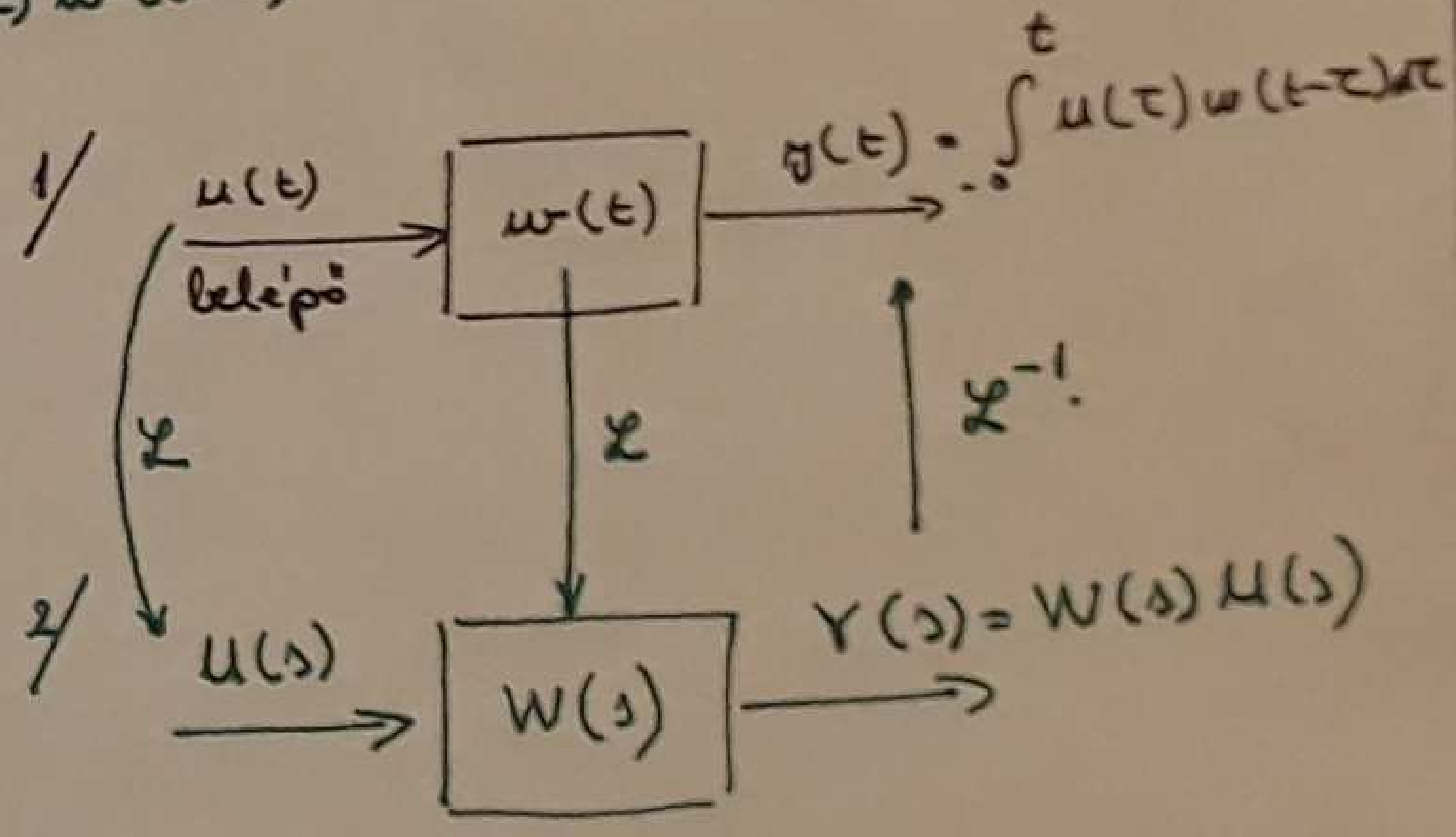
$$=\int_{-0}^{\infty} u(\tau)\left(\int_{-0}^{\infty} 1(t-\tau)\,w(t-\tau)\,e^{-st}\,dt\right)d\tau=$$



$$=\int_{-0}^{\infty} u(\tau)\boxed{\int_{\tau}^{\infty} w(t-\tau)\,e^{-st}\,dt}\,d\tau=$$

$$=\int_{-0}^{\infty} u(\tau)\boxed{W(s)\,e^{-s\tau}}\,d\tau=$$

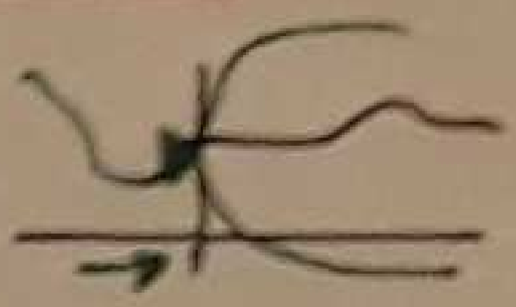
$$=W(s)\int_{-0}^{\infty} u(\tau)\,e^{-s\tau}\,d\tau=\underline{\underline{W(s)U(s).}}$$



DERIVÁLT JEL LAPLACE-TRANSZFORMÁLTJA; INTEGRÁLT JEL LAPLACE-TRANSZFORMÁLTJA

$\mathcal{L}\{u(t)\} = U(s) \longrightarrow \mathcal{L}\{\dot{u}(t)\} = sU(s) - u(-0)$

$u(t)$ ⟨ nem belépő $u(-0) \neq 0$; belépő, $\mathcal{L}\{\dot{u}(t)\} = sU(s)$



$$\int_{-0}^{\infty} \underbrace{\dot{u}(t)}_{u'} \underbrace{e^{-st}}_{v} dt = \left[u(t)e^{-st}\right]_{-0}^{\infty} - \int_{-0}^{\infty} u(t)(-s)e^{-st} dt =$$

$\int u'v = uv - \int uv'$

$u' = \dot{u}$ $\quad u = u$

$v = e^{-st}$ $\quad v' = -se^{-st}$

$$= -u(-0) + s\int_{-0}^{\infty} u(t)e^{-st} dt = \underline{\underline{sU(s) - u(-0)}}.$$

$\mathcal{L}\{\ddot{u}\} = s[sU(s) - u(-0)] - \dot{u}(-0)$

$= s^2U(s) - su(-0) - \dot{u}(-0)$

$\mathcal{L}\{u'''\} = s\mathcal{L}\{\ddot{u}\} - \ddot{u}(-0) =$

$= s^3U(s) - s^2u(-0) - s\dot{u}(-0) - \ddot{u}(-0)$

* * *

$\mathcal{L}\{u(t)\} = U(s) \longrightarrow \mathcal{L}\left\{\int_{-0}^{t} u(\tau)d\tau\right\} = \frac{1}{s}U(s)$

$$\int_{-0}^{\infty} \underbrace{\int_{-0}^{t} u(\tau)d\tau}\ \underbrace{e^{-st}} dt = \left[\frac{e^{-st}}{-s} \cdot \int_{-0}^{t} u(\tau)d\tau\right]_{-0}^{\infty} - \int_{-0}^{\infty} u(t)\frac{e^{-st}}{-s} dt = \frac{1}{s}\int_{-0}^{\infty} u(t)e^{-st} dt = \underline{\underline{\frac{1}{s}U(s)}}$$

$u' = e^{-st}$ $\quad u = \frac{e^{-st}}{-s}$

$v = \int_{-0}^{t} u(\tau)d\tau$ $\quad v' = u(t)$

$\cancel{v = w'}$

$$\boxed{\begin{array}{l} w = v' \\ W = sV. \end{array}}$$

$v(t) = \int_{-0}^{t} w(\tau)d\tau$

$V(s) = \frac{1}{s}W(s)$

CSILLAPÍTÁSI TÉTEL

$$\mathcal{L}\{u(t)\} = U(s)$$

$$\mathcal{L}\{u(t)\,e^{-\alpha t}\} = U(s)\Big|_{s \to s+\alpha} = U(s+\alpha)$$

$\alpha > 0$

$$\int_{-0}^{\infty} u(t)\,\underbrace{e^{-\alpha t}\,e^{-st}}_{e^{-(\alpha+s)t}}\,dt = \int_{-0}^{\infty} u(t)\,\underbrace{e^{-(s+\alpha)t}}_{e^{-st}}\,dt = \underline{\underline{U(s+\alpha)}}.$$

KEZDETÉRTÉK- TÉTEL

$$u(+0) = \lim_{s \to \infty} s\,U(s)$$

$$U(s) = \mathcal{L}\{u(t)\}$$

VÉGÉRTÉKTÉTEL

$$u(\infty) = \lim_{s \to 0} s\,U(s)$$

$$\lim_{s \to 0} \int_{-0}^{\infty} \dot{u}(t)\,e^{-st}\,dt = \int_{-0}^{\infty} \dot{u}(t)\,dt = \underline{u(\infty) - u(-0)}.$$

$$\lim_{s \to 0} \int_{-0}^{\infty} \dot{u}(t)\,e^{-st}\,dt = \underbrace{\lim_{s \to 0}\left[s\,U(s) - u(-0)\right]}$$

$$u(\infty) - \cancel{u(-0)} = \lim_{s \to 0}\left[s\,U(s) - \cancel{u(-0)}\right.$$

$$u(\infty) = \lim_{s \to 0} s\,U(s).$$

Példa (a kezdetiérték-tételhez) és a végértéktétel igazolása így egyszerűbb)

$$\begin{tikzcd}
\mathcal{L}\{1(t)t\} & = \frac{1}{s^2} \\
\mathcal{L}\{1(t)\} \arrow[u, "s"] & = \frac{1}{s} \arrow[u, "\frac{1}{s}"']
\end{tikzcd}$$

$$\begin{tikzcd}
\int t\,dt = \frac{t^2}{2} \arrow[d] & \Rightarrow \mathcal{L}\{1(t)\frac{t^2}{2}\} = \frac{1}{s^3} \arrow[d, bend left, "\cdot\frac{1}{s}"] \\
\int \frac{t^2}{2}\,dt = \frac{t^3}{6} \arrow[d] & \Rightarrow \mathcal{L}\left\{1(t)\frac{t^3}{6}\right\} = \frac{1}{s^4} \arrow[d, bend left, "\frac{1}{s}"] \\
\int \frac{t^3}{6}\,dt = \frac{t^4}{24} & \Rightarrow \mathcal{L}\left\{1(t)\frac{t^4}{24}\right\} = \frac{1}{s^5}
\end{tikzcd}$$

$\vdots$

$$\boxed{\mathcal{L}\left\{1(t)\frac{t^n}{n!}\right\} = \frac{1}{s^{n+1}}}$$

Az $u(t)$ jel Taylor-sora:

$$u(t)\Big|_{t=\emptyset} = 1(t)\left[u(0) + \frac{u'(0)}{1!}t + \frac{u''(0)}{2!}t^2 + \frac{u'''(0)}{3!}t^3 + \dots\right]$$

Laplace-transzformáltja:

$$U(s) = \frac{u(0)}{s} + \frac{u'(0)}{s^2} + \frac{u''(0)}{s^3} + \frac{u'''(0)}{s^4} + \dots$$

$$\Downarrow$$

~~$\cdot s$~~

$$sU = u(0) + \underbrace{\frac{u'(0)}{s} + \frac{u''(0)}{s^2} + \frac{u'''(0)}{s^3}}_{}$$

$$\boxed{\lim_{s\to\infty} sU = u(0)}$$

$$f(t)\Big|_{t=a} = \sum_{n=0}^{\infty} \frac{f^{(n)}(t)|_a}{n!}(t-a)^n$$

Taylor-sor

$a = \emptyset$

$$f(0) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}t^n =$$

$$= \sum_{n=0}^{\infty} f^{(n)}(0)\frac{t^n}{n!}$$

LAPLACE-TRANSZFORMÁCIÓ TÉTELEI

Összefoglalás

Laplace-transzformációs „táblázat"

$u(t)$	$U(s)$
$\delta(t)$	1 ✓
$1(t)$	$\frac{1}{s}$ ✓
$1(t)t$	$\frac{1}{s^2}$ ✓
$1(t)e^{-\alpha t}$ $\alpha>0$	$\frac{1}{s+\alpha}$ ✓
$1(t)te^{-\alpha t}$	$\frac{1}{(s+\alpha)^2}$ ✓ ←
$1(t)\cos\omega t$	$\frac{s}{s^2+\omega^2}$ ✓
$1(t)\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$ ✓

- Linearitás: $\mathcal{L}\{c_1u_1(t)+c_2u_2(t)+\dots\} = c_1\mathcal{L}\{u_1(t)\} + c_2\mathcal{L}\{u_2(t)\}+\dots$
- Eltolási tétel: $U(s) = \mathcal{L}\{u(t)\}$ $u(t)$ belépő
  $\mathcal{L}\{u(t-\tau)\} = U(s)e^{-s\tau}$
- Konvolúció: $Y(s) = W(s)U(s)$
  $W(s) = \int_{-0}^{\infty} w(t)e^{-st}\,dt = \mathcal{L}\{w(t)\}$
- Derivált jel: $\mathcal{L}\{\dot{u}(t)\} = sU(s) - u(-0)$
- Integrált jel: $\mathcal{L}\left[\int_{-0}^{t} u(\tau)\,d\tau\right\} = \frac{1}{s}U(s)$
- Csillapítási tétel: $\mathcal{L}\{u(t)e^{-\alpha t}\} = U(s+\alpha)$
- Kezdeti érték-tétel: $u(+0) = \lim_{s\to\infty} sU(s)$
- Végérték tétel: $u(\infty) = \lim_{s\to 0} sU(s)$

Határozzuk meg az alábbi függvények Laplace-transzformáltját!

$x_a(t) = 1(t) - 1(t-T)$

$x_b(t) = [1(t) - 1(t-T)]\, A \frac{t}{T}$

$x_c(t) = [1(t) - 1(t-T)]\, A \left(1 - \frac{t}{T}\right)$

$x_d(t) = [1(t) - 1(t-T)]\, M e^{-\alpha t} \quad (\alpha > 0)$

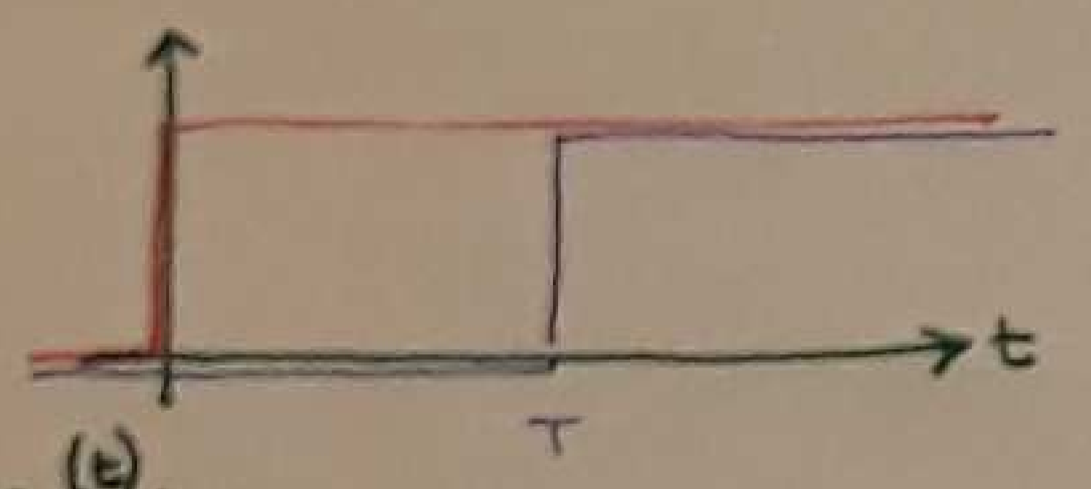
$x_e(t) = 1(t)\, t\, e^{-\alpha t}$

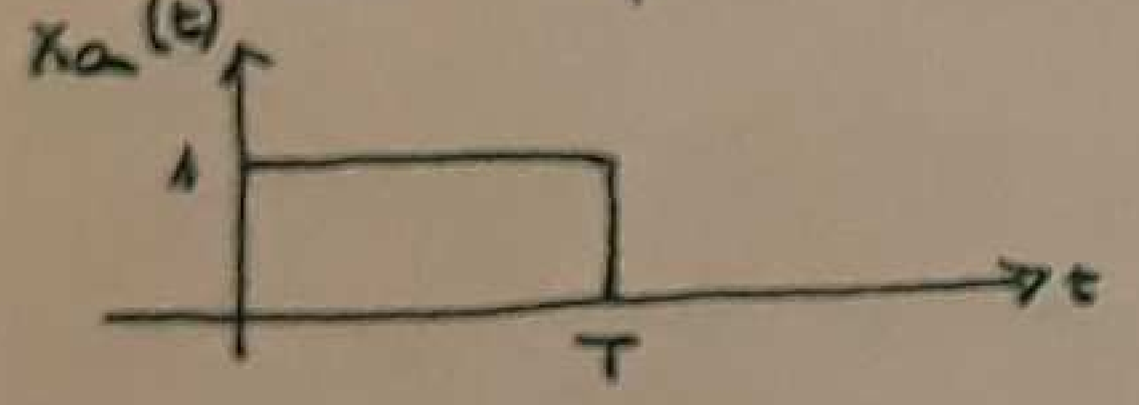
$x_f(t) = 1(t)\, e^{-\alpha t} \cos \omega t$

$x_g(t) = 1(t)\, e^{-\alpha t} \sin \omega t$

$$X(s) = \int_{-0}^{\infty} K x(t)\, e^{-st}\, dt .$$

$K \cdot X(s)$

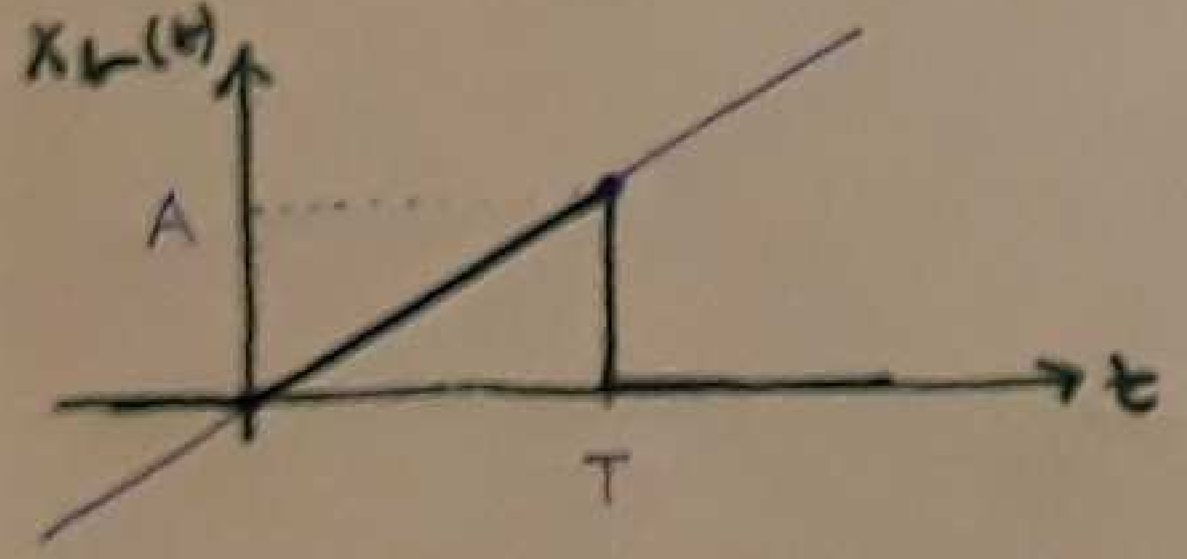




$x_a(t) = 1(t) - 1(t-T)$

$\llcorner \quad \uparrow \quad \llcorner$

$$X_a(s) = \frac{1}{s} - \frac{1}{s} e^{-sT} = \frac{1 - e^{-sT}}{s} .$$



$$x_b(t) = [1(t) - 1(t-T)]\, A \frac{t}{T}$$

$$= 1(t) \boxed{A \frac{t}{T}} - 1(\underline{t-T})\, A \frac{\underline{t - T + T}}{T} =$$

$$= 1(t)\, A \frac{t}{T} - 1(\underline{t-T})\, A \frac{\underline{t-T}}{T} - 1(\underline{t-T})\, A \frac{\cancel{T}}{\cancel{T}}$$

$$= \frac{A}{T} 1(t)\, t - \frac{A}{T} 1(t-T)(t-T) - A \cdot 1(t-T)$$

$$\underline{\underline{X_b(s) = \frac{A}{T} \frac{1}{s^2}}} - \frac{A}{T} \frac{1}{s^2} e^{-sT} - A \cdot \frac{1}{s} e^{-sT}$$

$x_c(t) = [1(t) - 1(t-T)] A \left(1 - \frac{t}{T}\right) = A \cdot 1(t) - A \cdot 1(t) \frac{t}{T} - A \cdot 1(t-T) + \underbrace{A \cdot 1(t-T) \frac{t - T + T}{T}}$

$= A \cdot 1(t) - \frac{A}{T} \cdot 1(t) t - \cancel{A \cdot 1(t-T)} + A \cdot 1(t-T) \frac{t-T}{T} + \cancel{A \cdot 1(t-T) \frac{T}{T}}$

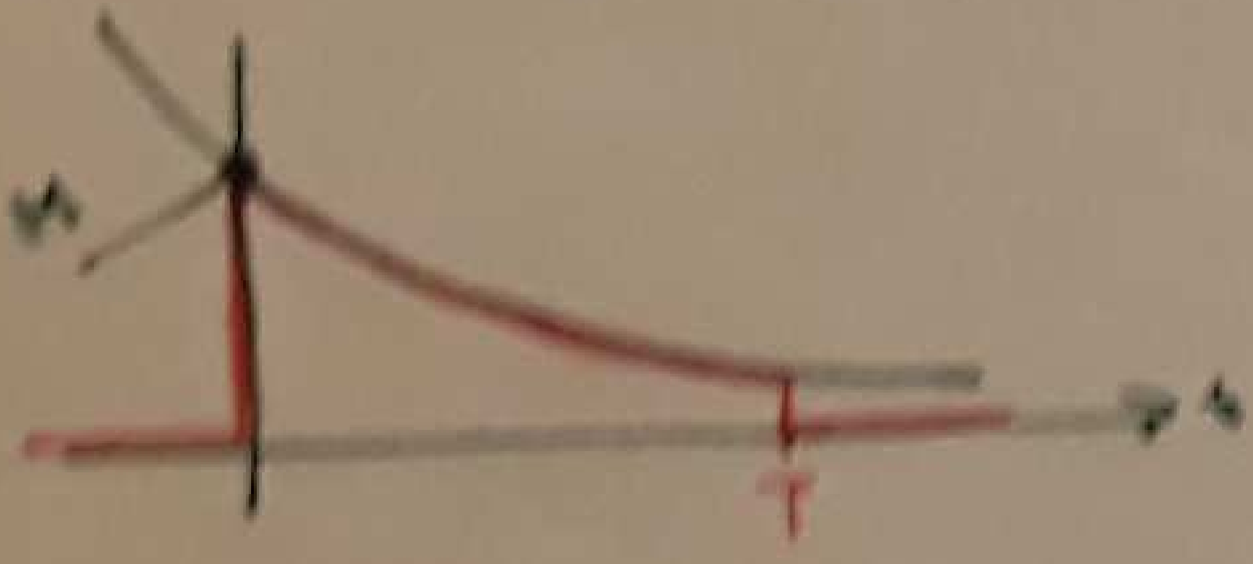
$= A \cdot 1(t) - \frac{A}{T} \cdot 1(t) t + \frac{A}{T} \cdot 1(t-T)(t-T)$

$X_c(s) = A \cdot \frac{1}{s} - \frac{A}{T} \cdot \frac{1}{s^2} + \frac{A}{T} \frac{1}{s^2} \cdot e^{-sT}.$

$x_d(t) = [1(t) - 1(t-T)] M e^{-\alpha t}, \quad (\alpha > 0)$

$e^{-\alpha(t-T)} \cdot e^{-\alpha T}$

$= 1(t) M e^{-\alpha t} - 1(t-T) M e^{-\alpha(t - T + T)} =$



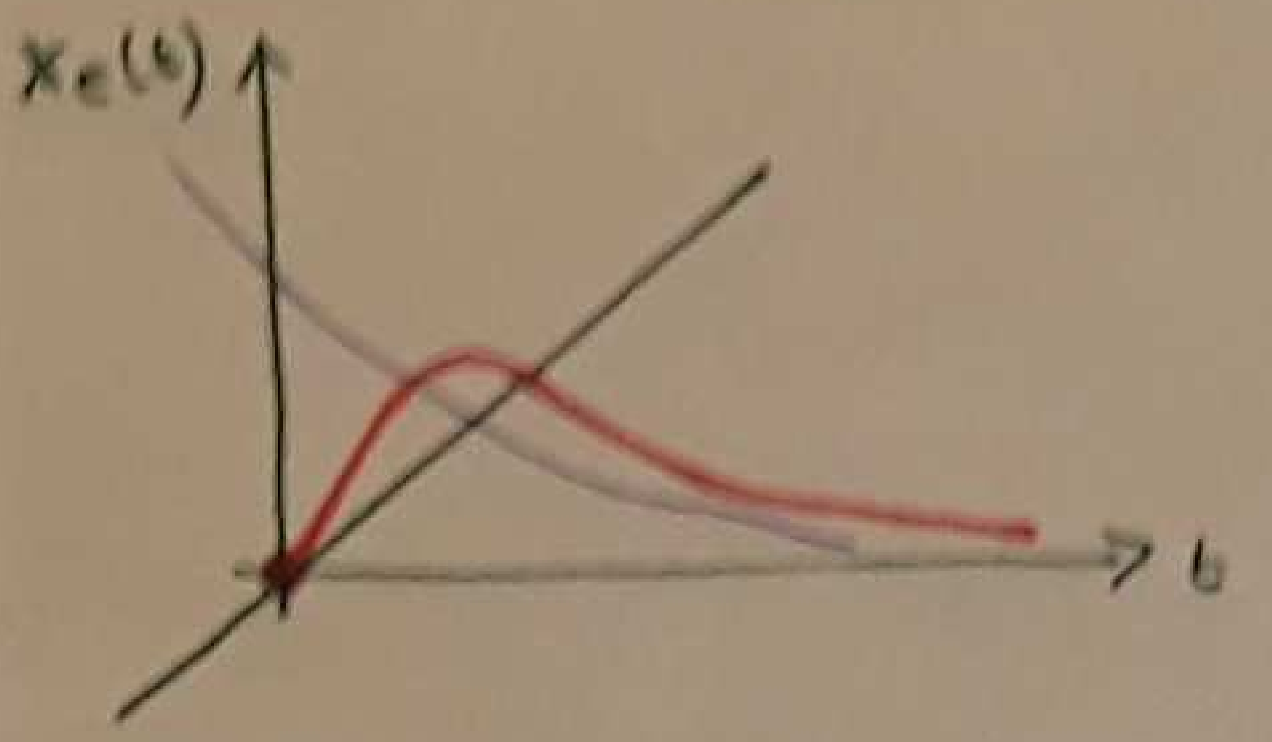
$= M \cdot 1(t) e^{-\alpha t} - 1(t-T) M e^{-\alpha(t-T)} \cdot \underbrace{e^{-\alpha T}}$

$= \underbrace{M \cdot 1(t) e^{-\alpha t}} - \underbrace{(M e^{-\alpha T}) \, 1(t-T) e^{-\alpha(t-T)}}$

$1(t) e^{-\alpha t}$

$X_d(s) = \frac{M}{s+\alpha} - \frac{M e^{-\alpha T}}{s+\alpha} e^{-sT}$

$x_e(t) = 1(t)\,\underline{t}\,\underline{e}^{-\alpha t}$



$$\underline{\boxed{1(t)\,t}}\,\underline{\boxed{e^{-\alpha t}}}$$

$\frac{1}{s^2}$ $\quad s \to s+\alpha$

$$\underline{\underline{X_e(s) = \frac{1}{(s+\alpha)^2}}}$$

$$\int_0^\infty t\,e^{-\alpha t}\,e^{-st}\,dt.$$

parciális integrálás:

$$\int u'v = uv - \int uv'$$

$$\int_0^\infty t\,e^{-(s+\alpha)t}\,dt =$$

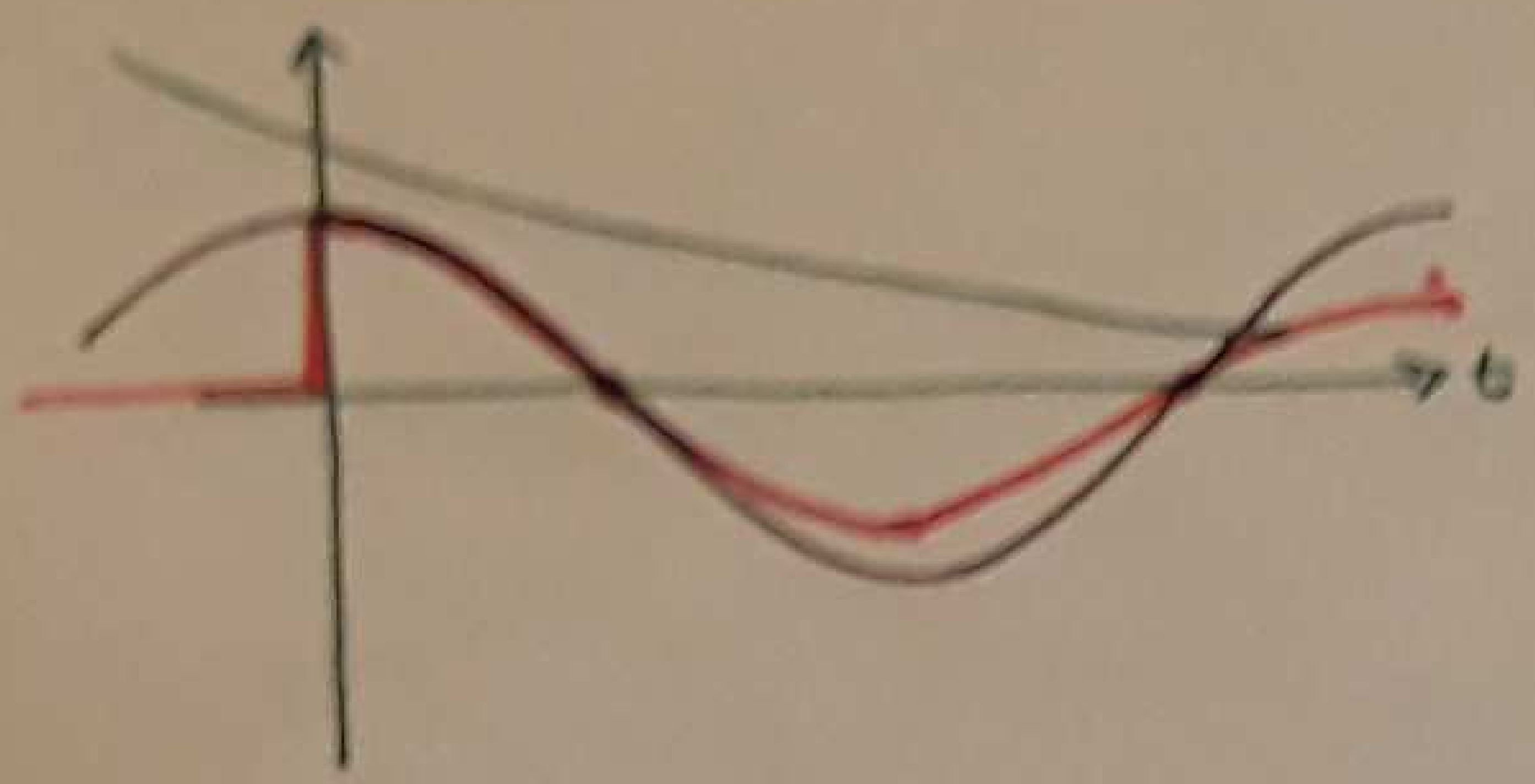
$u' = e^{-(s+\alpha)t}$ $\quad u = \frac{e^{-(s+\alpha)t}}{-(s+\alpha)}$

$v = t$ $\quad v' = 1$

$$= \underbrace{\left[t\,\frac{e^{-(s+\alpha)\textcircled{t}}}{-(s+\alpha)}\right]_0^\infty}_{0} - \int_0^\infty \frac{e^{-(s+\alpha)t}}{-(s+\alpha)}\,dt = \frac{1}{s+\alpha}\int_0^\infty e^{-(s+\alpha)t}\,dt = \frac{1}{s+\alpha}\left[\frac{e^{-(s+\alpha)t}}{-(s+\alpha)}\right]_0^\infty =$$

$$= \frac{1}{s+\alpha}\,\frac{0-1}{-(s+\alpha)} = \underline{\underline{\frac{1}{(s+\alpha)^2}}}$$

$x_f(t) = 1(t)\boxed{e^{-\alpha t}}\boxed{\cos \omega t}$



$$\frac{s}{s^2+\omega^2} \qquad s \rightarrow s+\alpha$$

$$X_f(s) = \underline{\frac{s+\alpha}{(s+\alpha)^2+\omega^2}}$$

$x_g(t) = 1(t)\, e^{-\alpha t} \sin \omega t$

$$X_g(s) = \left.\frac{\omega}{s^2+\omega^2}\right|_{s \rightarrow s+\alpha}$$

$$= \underline{\frac{\omega}{(s+\alpha)^2+\omega^2}}$$

LAPLACE-TRANSZFORMÁCIÓ ÉS A FOURIER-TRANSZFORMÁCIÓ KAPCSOLATA:

$$U(j\omega) = \mathcal{F}\{u(t)\} = \int_{-\infty}^{\infty} u(t)\, e^{-j\omega t}\, dt \qquad \int_{-\infty}^{\infty} |u(t)|\, dt < \infty$$

$$U(s) = \mathcal{L}\{u(t)\} = \int_{-0}^{\infty} u(t)\, e^{-st}\, dt$$

Jelek

$u(t)$ belépő

abszolút integrálható

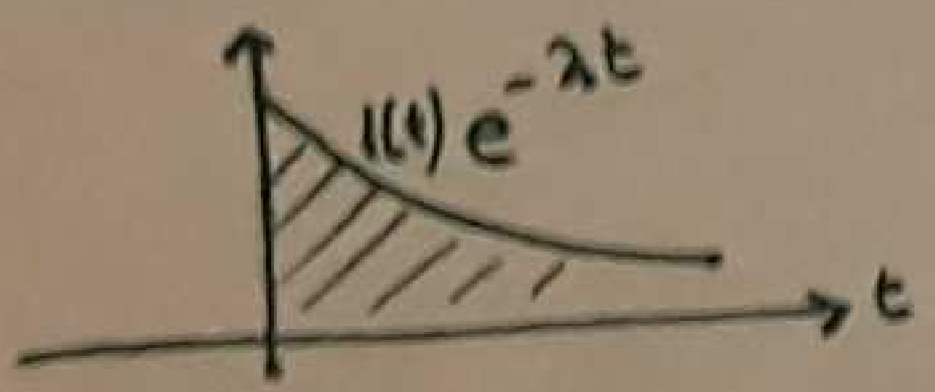
$$\boxed{U(j\omega) = U(s)\big|_{s=j\omega}}$$

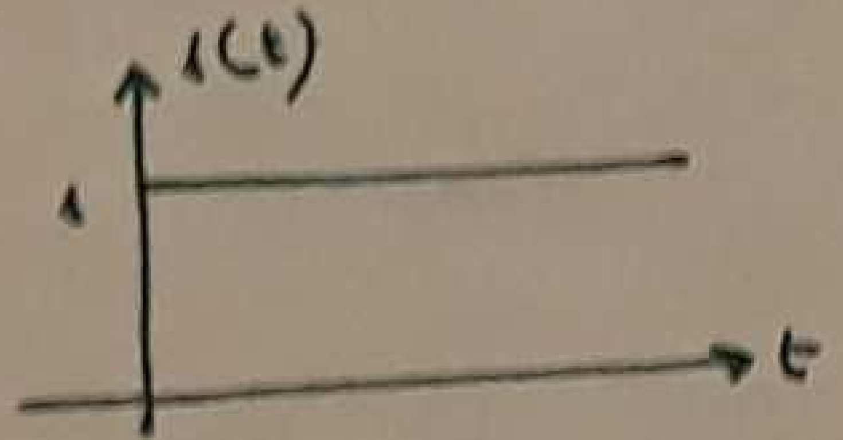
Rendszer.

$w(t)$ belépő = kauzális

abs. integrálható = G-V-stabil.

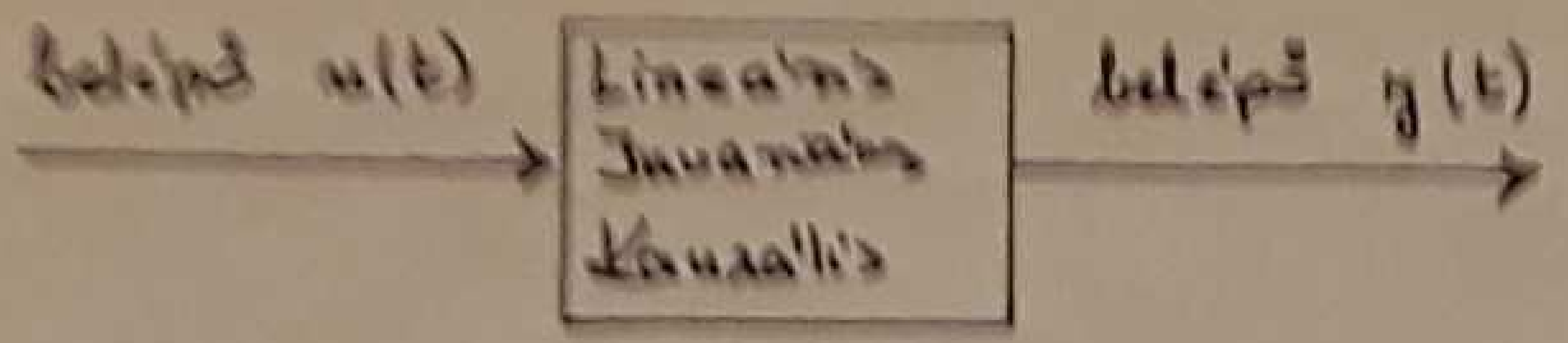
$$\boxed{W(j\omega) = W(s)\big|_{s=j\omega}}$$





$$\frac{1}{s} \overset{?}{\leftrightarrow} \frac{1}{j\omega} + \pi\delta(\omega)$$

ÁTVITELI FÜGGVÉNY

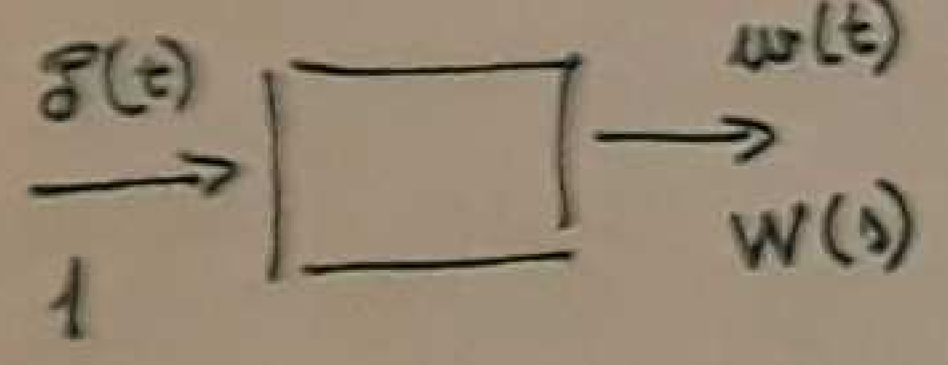


$$\begin{tikzcd} u(t) \arrow[d, "\mathcal{L}"] \\ U(s) \end{tikzcd} \qquad \begin{tikzcd} w(t) \arrow[d, "\mathcal{L}"] \\ W(s) \end{tikzcd} \qquad \begin{tikzcd} y(t) = \int_0^t u(\tau)\, w(t-\tau)\, d\tau \arrow[d, "\mathcal{L}"] \\ Y(s) = W(s)\cdot U(s) \arrow[r, "?", "\mathcal{L}^{-1}"'] & y(t) \end{tikzcd}$$

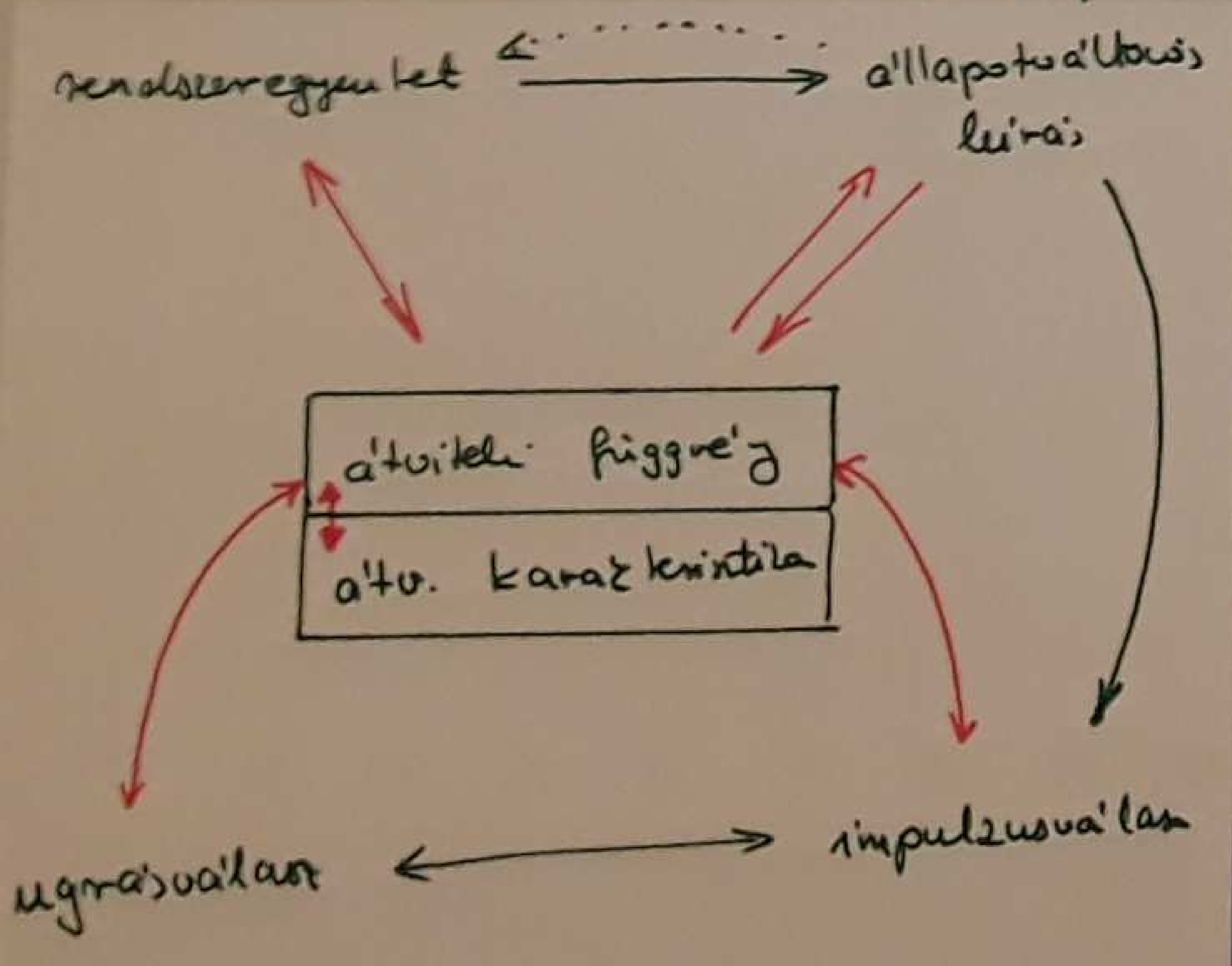
$$\underbrace{Y(s) = W(s)\cdot U(s)}_{\boxed{W(s) = \frac{Y(s)}{U(s)}}}$$

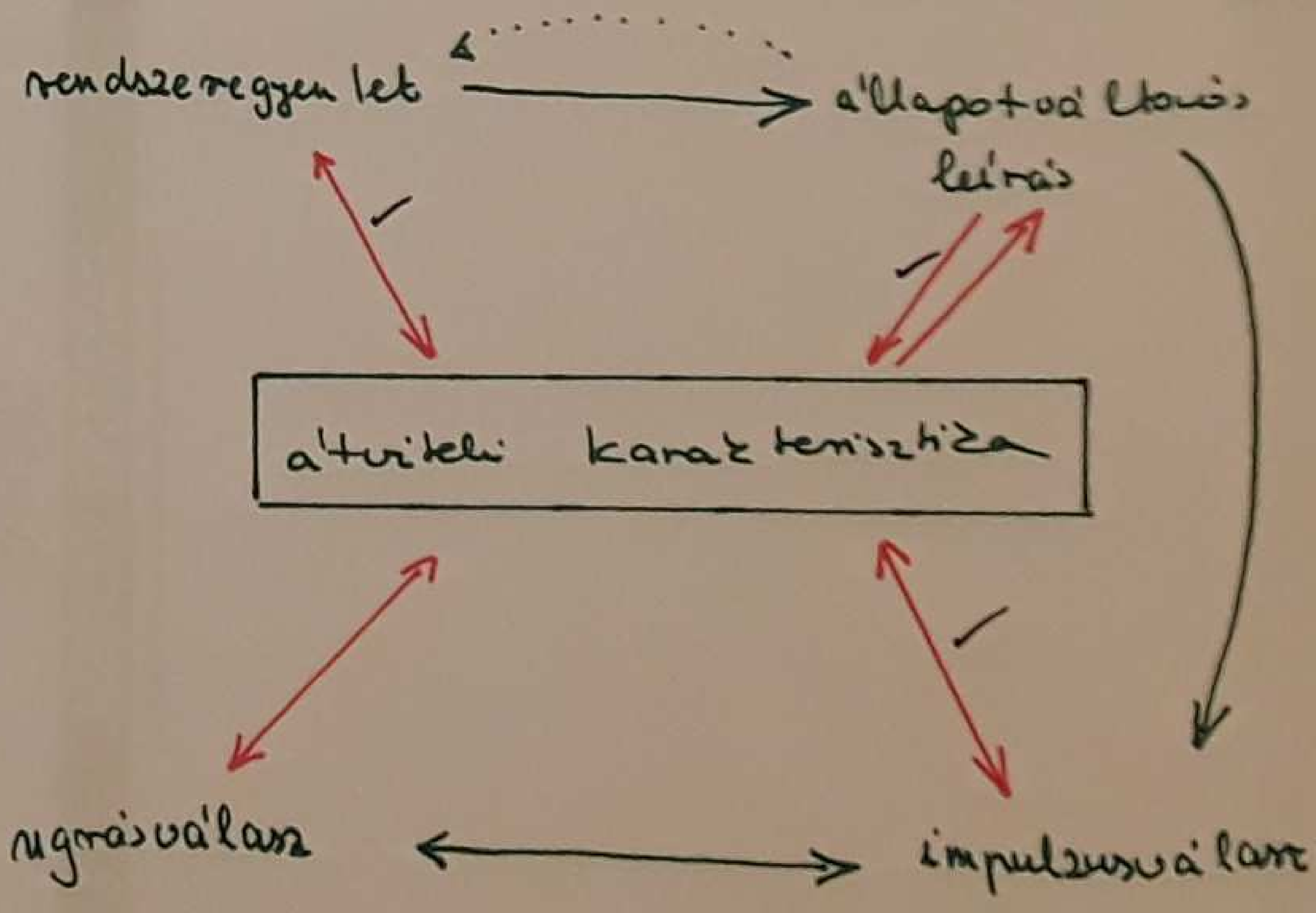
$U=1 \rightarrow W = Y$

$u(t) = \delta(t) \longrightarrow \mathcal{L}\{\delta(t)\} = 1$



$$\left.\right\} \begin{array}{|c|} \hline W(s) = \mathcal{L}\{w(t)\} \\ \hline w(t) = \mathcal{L}^{-1}\{W(s)\} \\ \hline \end{array}$$





AZ IDŐTARTOMÁNY ÉS A KOMPLEX FREKVENCIATARTOMÁNY ÖSSZEKAPCSOLÁSA

Ⓘ

Rendszeregyenlet ⟷ Átviteli függvény

$$\boxed{y^{(n)} + \sum_{i=1}^{n} a_i y^{(n-i)} = \sum_{i=0}^{n} b_i u^{(n-i)}}$$

$$\Big\Downarrow \mathcal{L}$$

$$s^n \underline{Y(s)} + \sum_{i=1}^{n} a_i s^{n-i} \underline{Y(s)} = \sum_{i=0}^{n} b_i s^{n-i} \underline{U(s)}$$

$$Y(s)\left[s^n + \sum_{i=1}^{n} a_i s^{n-i}\right] = U(s) \sum_{i=0}^{n} b_i s^{n-i}$$

$$W(s) = \frac{Y(s)}{U(s)} \rightleftarrows \frac{\sum_{i=0}^{n} b_i s^{n-i}}{s^n + \sum_{i=1}^{n} a_i s^{n-i}}$$

$\mathcal{L}\{u(t)\} = U(s)$

$\mathcal{L}\{\dot{u}(t)\} = sU(s) - u(-0)$

$\mathcal{L}\{\ddot{u}(t)\} = s[sU(s) - u(-0)] - \dot{u}(-0)$

$\vdots$

belépő $u(t)$: $u(-0) = 0$

$\dot{u}(-0) = 0 \ldots$

$\dfrac{\text{polinom}}{\text{polinom}}$

- számláló → $= 0$ → zérusok.
- nevező → $= 0$ → pólusok → STAB.

példa: $W(s) = \dfrac{s+1}{s^2+2s+2} \rightleftarrows \dfrac{Y}{U}$

$$Us + U = Ys^2 + 2Ys + 2Y$$

$$u' + u = y'' + 2y' + 2y$$

$$s^2Y + 2Ys + 2Y = sU + U$$

$$Y(s^2+2s+2) = U(s+1)$$

$$\frac{Y}{U} = \frac{s+1}{s^2+2s+2}$$

AZ IDŐTARTOMÁNY ÉS A KOMPLEX FREKVENCIATARTOMÁNY ÖSSZEKAPCSOLÁSA

Állapotváltozós leírás ⟷ Átviteli függvény

$$\boxed{\begin{aligned}\dot{\underline{x}} &= \underline{A}\,\underline{x} + \underline{b}\,u\\ y &= \underline{c}^T\underline{x} + Du\end{aligned}}$$

$$\mathcal{L}\{\dot{x}(t)\} = sX(s) \underbrace{- x(-0)}_{=0}$$

$$\begin{cases} s\underline{X} = \underline{A}\,\underline{X} + \underline{b}\,U \\ Y = \underline{c}^T\underline{X} + DU \end{cases}$$

a.) egyenletrendszer.

b.) összefüggés Y/U

$$W(s) = \frac{\underline{c}^T \operatorname{adj}(s\underline{E} - \underline{A})\,\underline{b} + D\,|s\underline{E} - \underline{A}|}{|s\underline{E} - \underline{A}|}$$

polinom

polinom

mátrix/skalár
vigyázz !!!

AZ IDŐTARTOMÁNY ÉS A KOMPLEX FREKVENCIATARTOMÁNY ÖSSZEKAPCSOLÁSA

III.

$$W(s) = \mathcal{L}\{w(t)\} = \int_{-0}^{\infty} w(t)\, e^{-st}\, dt$$

$$w(t) = \mathcal{L}^{-1}\{W(s)\} \longrightarrow$$

$$w(t) = v'(t)$$

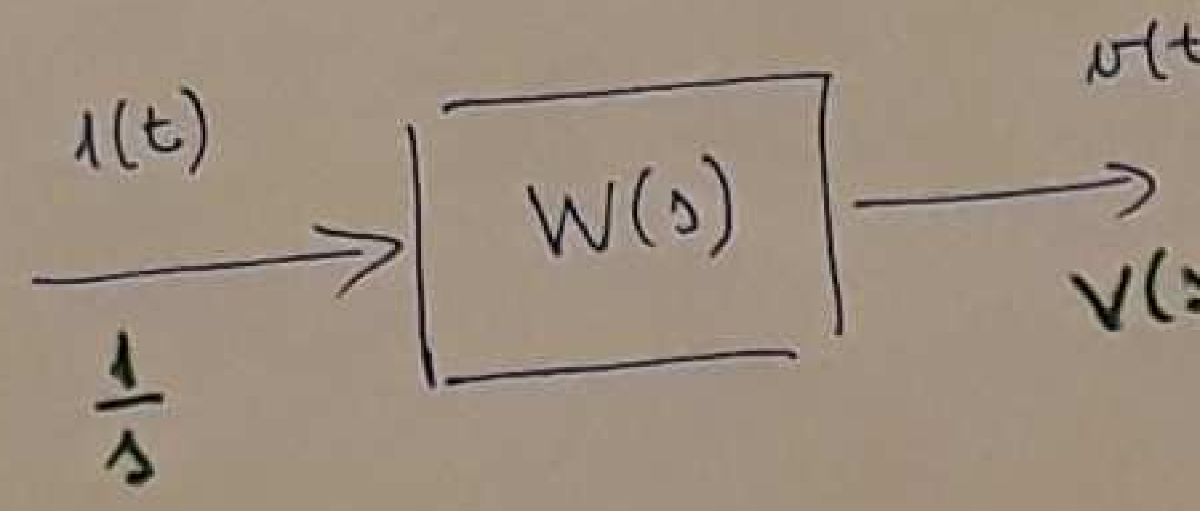
$$W(s) = \cancel{L}\, sV(s) - \underbrace{v(-0)}_{0}$$

$$\boxed{W(s) = sV(s)}$$

$$v(t) = \int_{-0}^{t} w(\tau)\, d\tau$$

$$V(s) = \frac{1}{s} W(s)$$

$$v(t) = \mathcal{L}^{-1}\{V(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} W(s)\right\} \longrightarrow$$

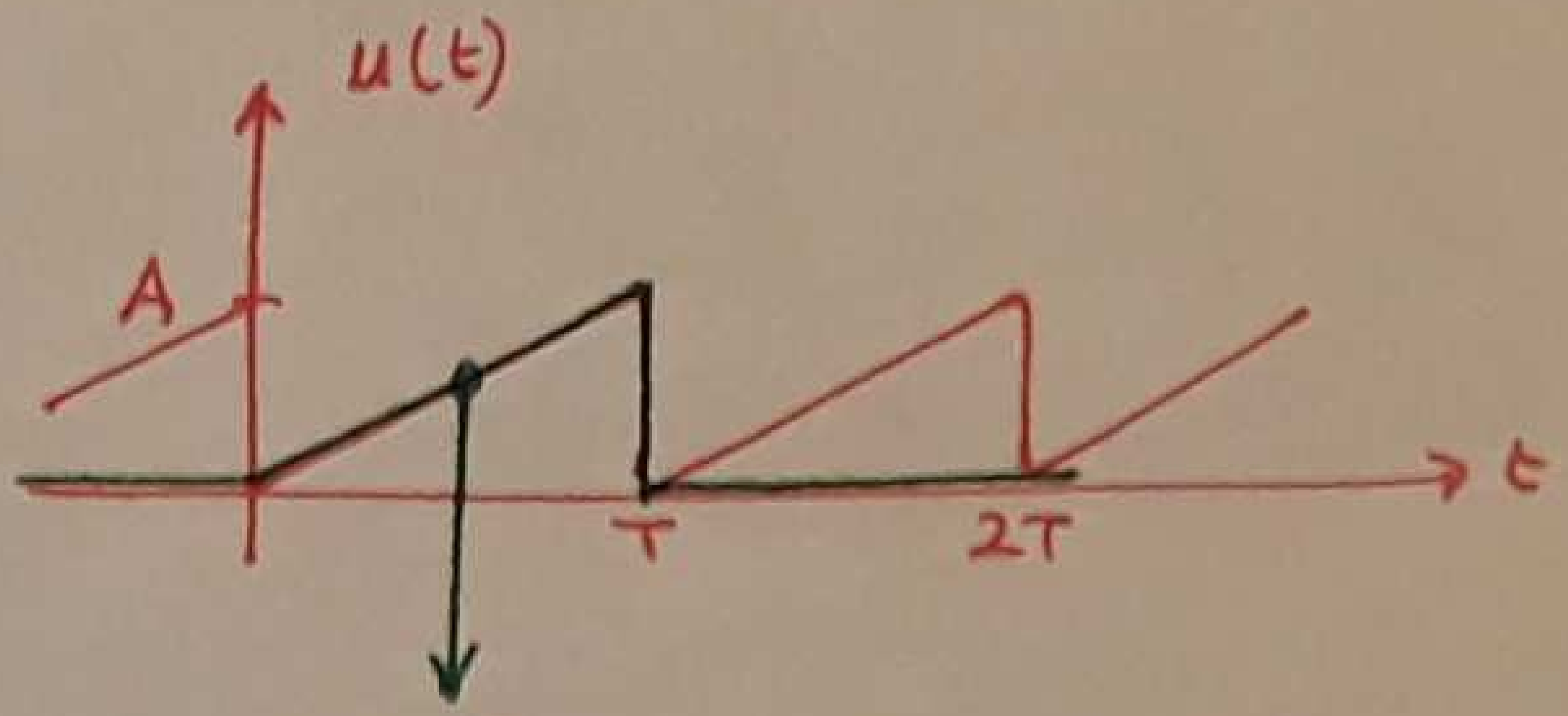


$$V(s) = \frac{1}{s} W(s).$$

Kapcsolat a Fourier-sorral

$$\hat{U}_k^c = \frac{1}{T}\int_0^T u(t)\, e^{-\boxed{jk\omega}t}\, dt$$

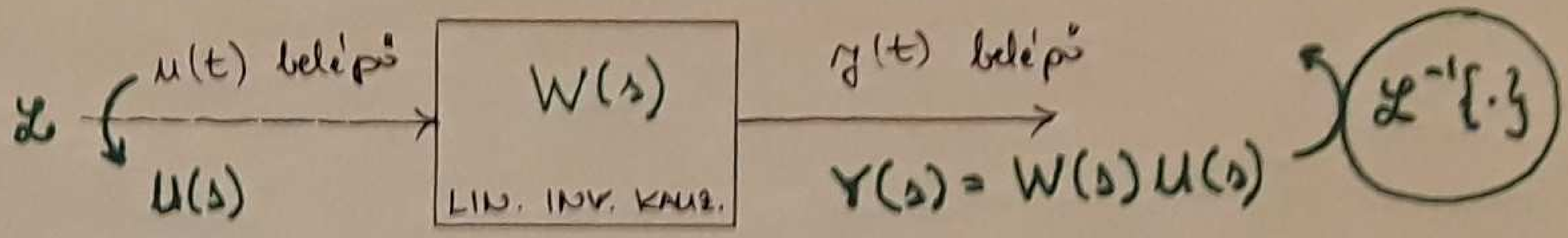
$$U(s) = \int_0^\infty u(t)\, e^{-st}\, dt$$



$$U(s) = \int_0^T u(t)\, e^{-\boxed{s}t}\, dt$$

$$\boxed{\hat{U}_k^c = \frac{1}{T}\, U(s)\Big|_{s\to jk\omega}.}$$

A VÁLASZJEL SZÁMÍTÁSA



$$W(s) = \mathcal{L}\{w(t)\}$$

Határozzuk meg a rendszer $w(t)$ impulzusválaszát, $v(t)$ ugrásválaszát és az $u(t) = 5\cdot 1(t)\,e^{-2t}$ jelre adott $y(t)$ válaszát!

$$W(s) = \frac{5s+1}{\boxed{s^2+4s+3}}$$

parciális (részlet) törtekre bontás.

$(s-p_1)(s-p_2)\cdots$

$$w(t) = \mathcal{L}^{-1}\{W(s)\}$$

$$s^2+4s+3 = 0 \qquad p_{1,2} = \frac{-4 \pm \sqrt{16-4\cdot 1\cdot 3}}{2} = \frac{-4\pm 2}{2} \begin{cases} p_1 = -1 \\ p_2 = -3 \end{cases} \Bigg\} \quad s^2+4s+3 = (s+1)(s+3)$$

- $$W(s) = \frac{5s+1}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3} = \frac{A(s+3)+B(s+1)}{(s+1)(s+3)} \longrightarrow$$

számláló: $\boxed{As + 3A + Bs + B = 5s+1}$

$$\begin{cases} A+B = 5 \\ 3A+B = 1 \end{cases}$$

1. −2.)

$-2A = 4$

$A = -2$

1) $B = 5 - A \Rightarrow B = 7$

$$= \frac{-2}{s+1} + \frac{7}{s+3}$$

$$A = \frac{5(-1)+1}{-1+3} = \frac{-5+1}{2} = \frac{-4}{2} = -2$$

$$B = \frac{5(-3)+1}{-3+1} = \frac{-15+1}{-2} = \frac{-14}{-2} = 7$$

$$W(s) = \frac{-2}{s+1} + \frac{7}{s+3}$$

$$w(t) = 1(t)\left[-2e^{-1t} + 7e^{-3t}\right]$$

d/dt

∫

- $$v(t) = \mathcal{L}^{-1}\{V(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s}W(s)\right\}$$

$$V(s) = \frac{1}{s}W(s) = \frac{5s+1}{s(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$A = \frac{5\cdot 0+1}{(0+1)(0+3)} = \frac{1}{3}$$

$$B = \frac{5(-1)+1}{(-1)(-1+3)} = \frac{-4}{-2} = 2$$

$$C = \frac{5(-3)+1}{(-3)(-3+1)} = \frac{-14}{6} = -\frac{7}{3}$$

$$V(s) = \frac{1/3}{s} + \frac{2}{s+1} + \frac{-7/3}{s+3}$$

$$v(t) = 1(t)\left[\frac{1}{3} + 2e^{-1t} - \frac{7}{3}e^{-3t}\right]$$

$$u(t) = 5 \cdot 1(t) \cdot e^{-2t}$$

$$\underline{U(s) = \frac{5}{s+2}}$$

$$Y(s) = W(s)\,U(s) = \frac{5s+1}{(s+1)(s+3)} \cdot \frac{5}{s+2}$$

$$= \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s+2}$$

$$A = \frac{5(-1)+1}{(-1+3)(-1+2)}\,5 = 5 \cdot \frac{-4}{2 \cdot 1} = \underline{\underline{-10}}$$

$$B = \frac{5(-3)+1}{(-3+1)(-3+2)} \cdot 5 = 5\,\frac{-14}{(-2)(-1)} = \underline{\underline{-35}}$$

$$C = \frac{5(-2)+1}{(-2+1)(-2+3)} \cdot 5 = 5\,\frac{-9}{(-1)(1)} = \underline{\underline{45}}$$

$$Y(s) = \frac{-10}{s+1} + \frac{-35}{s+3} + \frac{45}{s+2}$$

$$\underline{y(t) = 1(t)} \cdot \left[-10 \cdot e^{-1t} - 35 \cdot e^{-3t} + 45 e^{-2t}\right] \longleftarrow$$

$$w(t) = 1(t)\left[-2 \cdot e^{-1t} + 7 e^{-3t}\right]$$

$$u(t) = 5 \cdot 1(t)\, e^{-2t}$$

$$\Downarrow$$

$$y(t) = \int_0^t 5 \cdot e^{-2\tau} \cdot \left[-2 e^{-1(t-\tau)} + 7 e^{-3(t-\tau)}\right] d\tau$$

Határozzuk meg a rendszer átviteli függvényét és a válaszjelet!

$$w(t) = 2\delta(t) + 1(t)\left[e^{-2t} + 3e^{-5t}\right] \xrightarrow{\mathcal{L}} W(s) = 2 + \frac{1}{s+2} + \frac{3}{s+5} = \frac{2(s+2)(s+5)+(s+5)+3(s+2)}{(s+2)(s+5)}$$

$$2(s^2+7s+10)+s+5+3s+6$$

$$\underline{2s^2} + \underset{3}{\underline{14s}} + \underset{1}{20} + \underset{3}{s} + \underset{1}{5} + \underset{3}{3s} + \underset{1}{6}$$

$$= \frac{2s^2+18s+31}{(s+2)(s+5)}$$

$$u(t) = 5 \cdot 1(t)\, e^{-2t} \longrightarrow U(s) = \frac{5}{s+2}$$

$$Y(s) = W(s) \cdot U(s) = \frac{2s^2+18s+31}{(s+2)(s+5)} \cdot \frac{5}{(s+2)}$$

$$= 5 \cdot \frac{2s^2+18s+31}{(s+2)^{②}(s+5)} = \frac{\boxed{A}^{!}}{s+2} + \frac{\boxed{B}}{(s+2)^2} + \frac{\boxed{C}}{s+5}$$

$$B = 5\frac{2(-2)^2+18\cdot(-2)+31}{-2+5} = 5\,\frac{8+(-36)+31}{3} = \underline{\underline{5}}$$

$$C = 5 \cdot \frac{2(-5)^2+18(-5)+31}{(-5+2)^2} = 5\,\frac{50-90+31}{9} = \underline{\underline{-5}}$$

$$Y(s) = \frac{A}{s+2} + \frac{5}{(s+2)^2} - \frac{5}{s+5} = \frac{A(s+2)(s+5)+5(s+5)-5(s+2)^2}{(s+2)^2(s+5)}$$

számláló: $A(s^2+7s+10) + 5s + 25 - 5(s^2+4s+4)$

$$\underline{\underline{As^2}} + \underline{7As} + \underline{10A} + \underline{5s} + \underline{25} \underline{\underline{-5s^2}} \underline{-20s} \underline{-20}$$

$$\begin{cases} A - 5 = 10 \longrightarrow \underline{\underline{A = 15}} \\ 7A + 5 - 20 = 90 \rightarrow \underline{\underline{A = 15}} \\ 10A + 25 - 20 = 155 \rightarrow \underline{\underline{A = 15}} \end{cases} \checkmark$$

$$Y(s) = \frac{15}{s+2} + \frac{5}{(s+2)^2} - \frac{5}{s+5}$$

$$\underline{\underline{y(t)}} = 1(t) \cdot \left[15e^{-2t} + 5te^{-2t} - 5e^{-5t}\right]$$

Határozzuk meg az alábbi rendszer súlyfüggvényét!

$$W(s) = \frac{3s^2+17s+10}{s^2+4s+3}$$

$$(3s^2+17s+10) : (s^2+4s+3) = \overset{\downarrow}{3} + \overset{\downarrow}{\frac{5s+1}{s^2+4s+3}} = 3 + \frac{-2}{s+1} + \frac{7}{s+3}$$

$$\frac{-(3s^2+12s+9)}{5s+1}$$

$$w(t) = \boxed{3\delta(t)} + 1(t)\left[-2\cdot e^{-1t} + 7e^{-3t}\right]$$

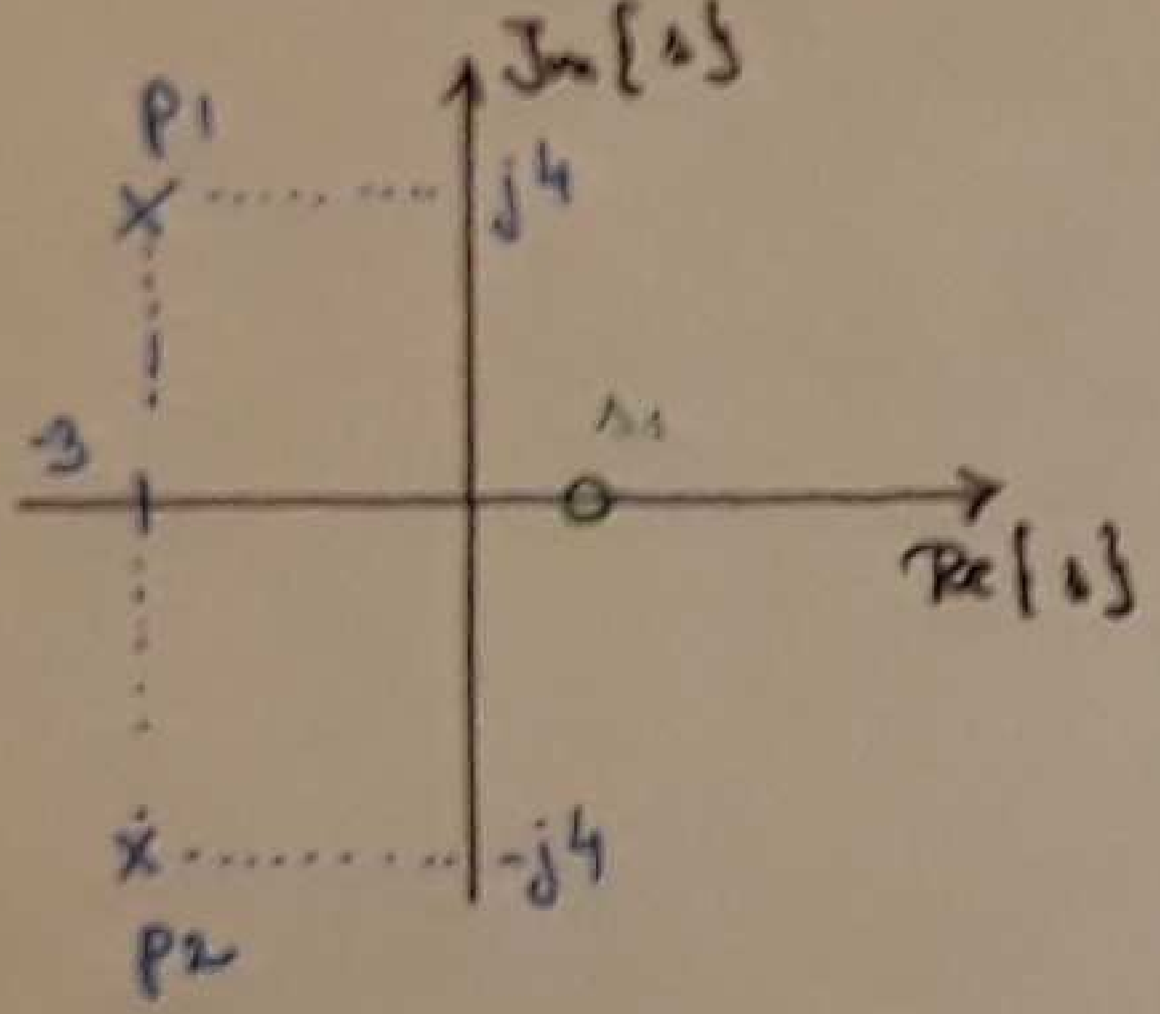
Vázoljuk fel az alábbi rendszer pólus-zérus elrendezését!

$$W(s) = \frac{s-1}{s^2+6s+25} \quad \begin{matrix} \circ \\ \times \end{matrix}$$

pólus: $s^2+6s+25 = (s-p_1)(s-p_2) = 0$

$$p_{1,2} = \frac{-6 \pm \sqrt{36-4\cdot 25}}{2} = \frac{-6 \pm j8}{2} = -3 \pm j4$$

zérus: $s-1 = 0 \qquad s_1 = 1$



Pólus-zérus kiejtés - illusztráció.

$$\begin{pmatrix} \underline{\underline{A}} & \underline{b} \\ \underline{c}^T & D \end{pmatrix} \rightarrow \varphi(\lambda) = \boxed{|\lambda \underline{\underline{E}} - \underline{\underline{A}}|} = 0 \rightarrow \lambda_1\ \lambda_2 \dots \lambda_N$$

$\mathrm{Re}\{\lambda_i\} < 0 \quad i = 1 \dots N$ aszimptotikusan stabil.

$$\downarrow$$

$$W(s) = \frac{\underline{c}^T \operatorname{adj}(s\underline{\underline{E}} - \underline{\underline{A}})\,\underline{b} + D\,|s\underline{\underline{E}} - \underline{\underline{A}}|}{\boxed{|s\underline{\underline{E}} - \underline{\underline{A}}|}} \longrightarrow \cancel{p_1}\ p_2 \dots p_N \qquad \frac{\cancel{(s-s_1)}(s-s_2)\dots}{\cancel{(s-p_1)}(s-p_2)\dots}$$

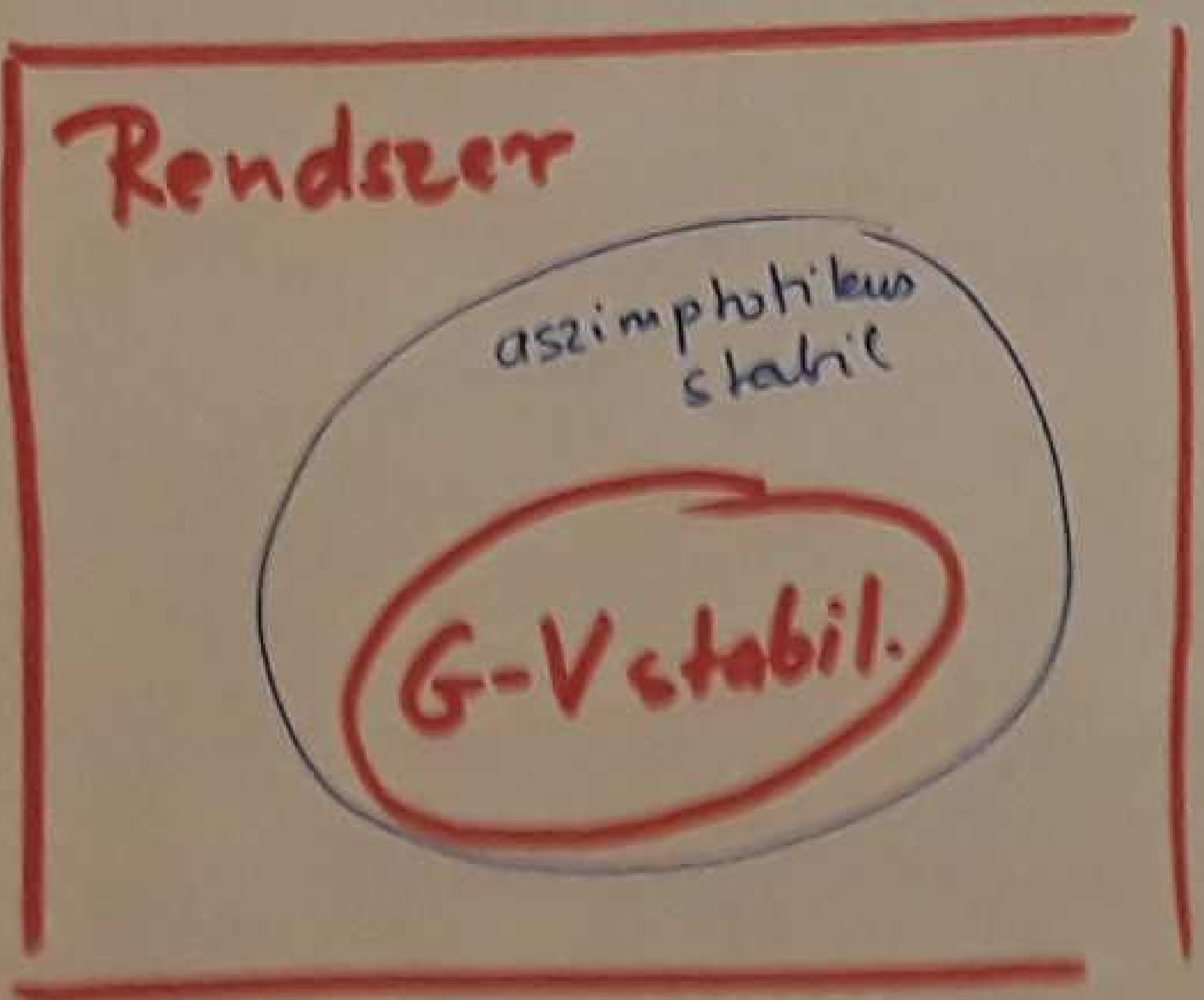
$$\Downarrow$$

$w(t)$

$s_1 = p_1$ pl.

$\mathrm{Re}\{p_i\} < 0 \quad i = 1 \dots N$ gerjesztés-válasz stabil

$\downarrow$ ✓ $\uparrow$?



pl. $\dfrac{\cancel{(s-s_1)}\dots}{\cancel{(s-p_1)}\dots}$

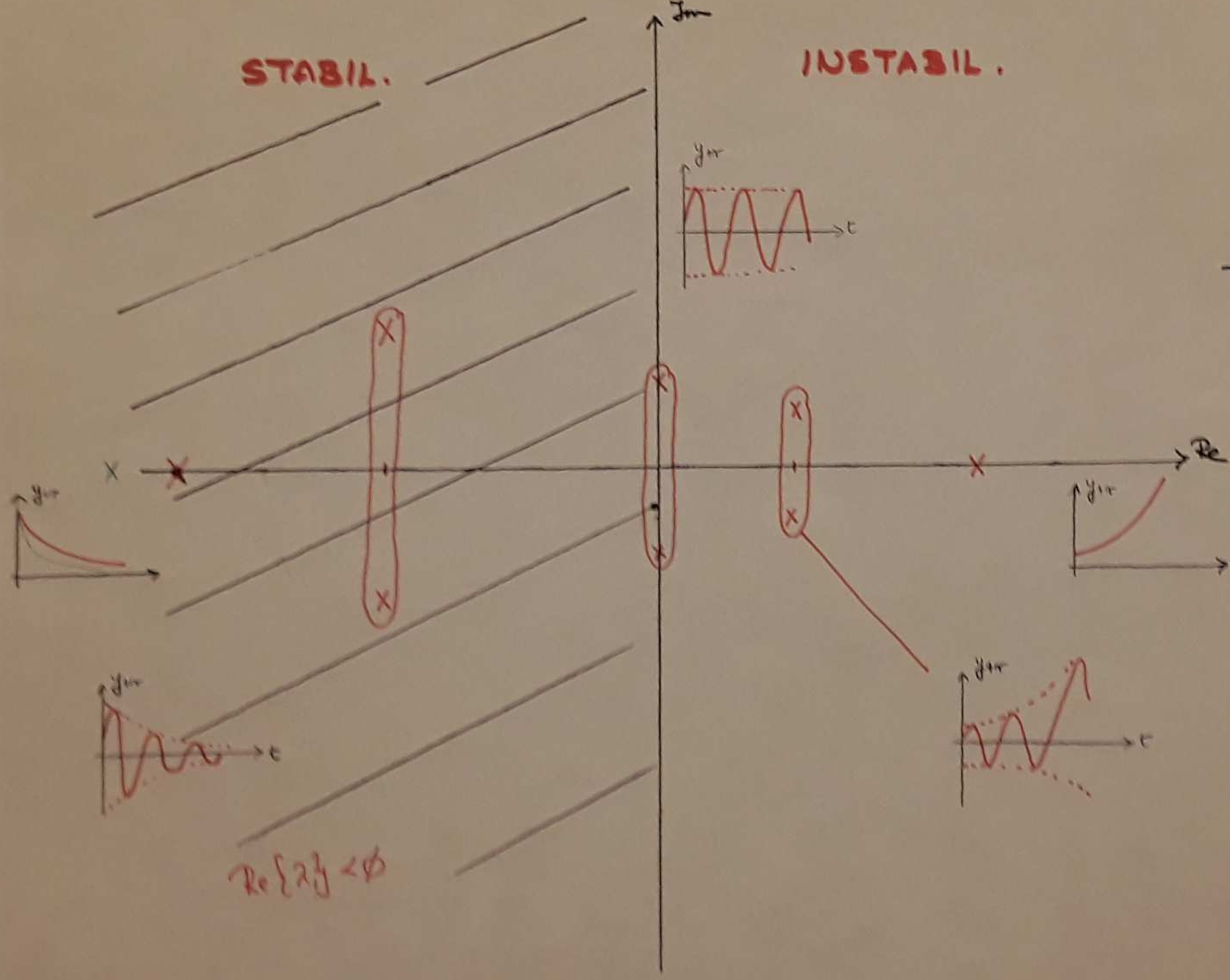
$\mathrm{Re}\{p_1\} > 0$

$\lambda_1\ \lambda_2 \dots \lambda_N$

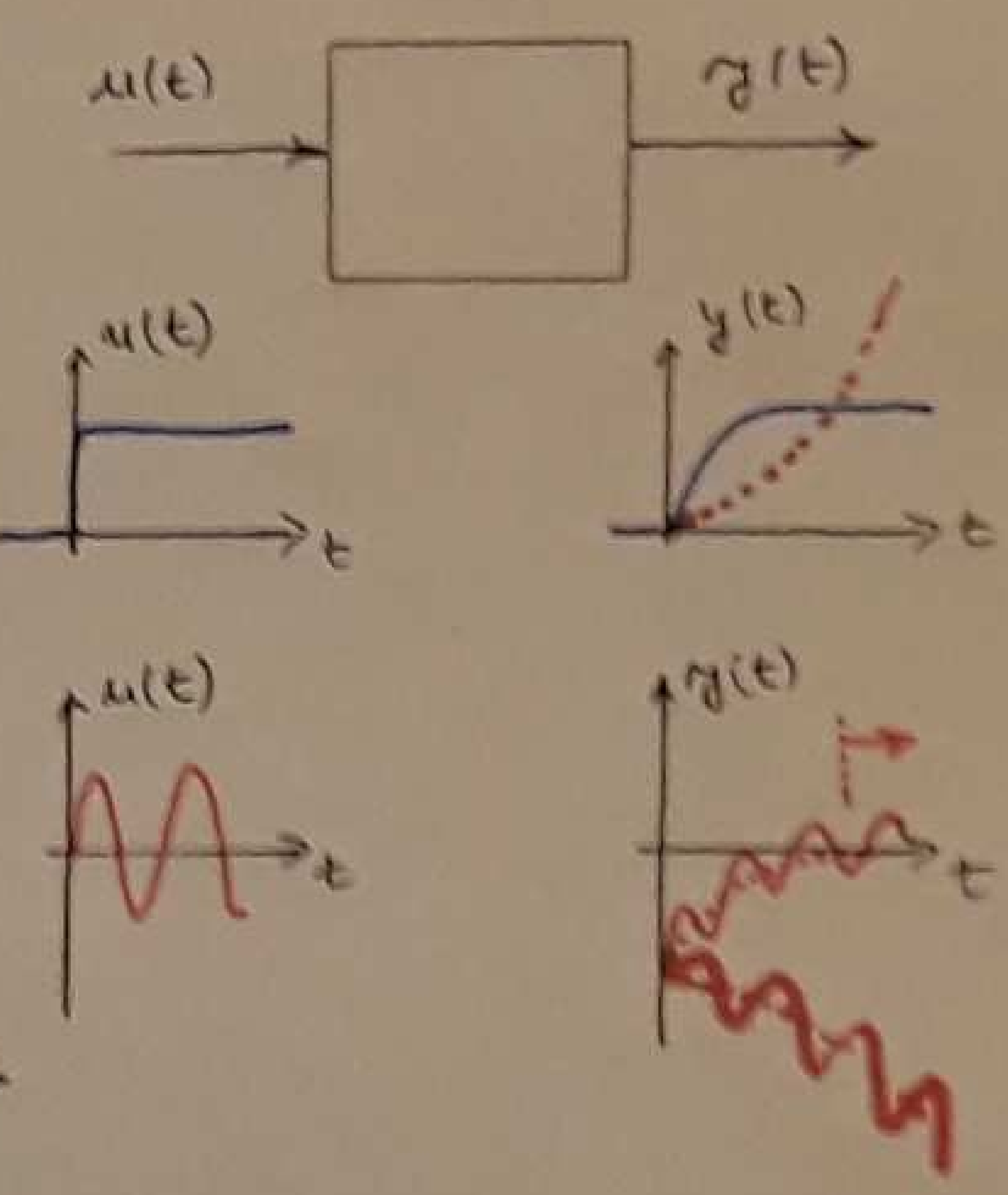
$p_1\ p_2 \dots p_n$

$n \le N$

TRANZIENS JELLEGE ÉS A SAJÁTÉRTÉKEK (PÓLUSOK)



$y(t) = y_{tr}(t) + y_{st}(t)$



$Me^{\lambda t}$	$+ \; y_{st}$
TRANZIENS	STACIONÁRIUS ÁLLANDÓSULT
HOMOGÉN ÁLTALÁNOS	INHOMOGÉN PARTIKULÁRIS
SZABAD VÁLASZ	GERJESZTETT VÁLASZ

AZ $e^{\underline{A}t}$ MÁTRIX FÜGGVÉNY MEGHATÁROZÁSA

$$w(t) = 1(t)\,\underline{c}^T e^{\underline{A}t}\,\underline{b} + D\delta(t)$$

$$\downarrow \mathcal{L}$$

$$W(s) = \underline{c}^T (s\underline{E} - \underline{A})^{-1}\,\underline{b} + D$$

$$\boxed{\mathcal{L}\{e^{\underline{A}t}\} = (s\underline{E} - \underline{A})^{-1}} \qquad \mathcal{L}\{e^{\alpha t}\} = \frac{1}{s+\alpha}$$

$$\hookrightarrow \frac{\mathrm{adj}\,(s\underline{E} - \underline{A})}{|s\underline{E} - \underline{A}|}$$

Példa

$$\underline{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \qquad s\underline{E} - \underline{A} = \begin{pmatrix} s & -1 \\ 2 & s+3 \end{pmatrix}$$

$$|s\underline{E} - \underline{A}| = s(s+3) + 2 = s^2 + 3s + 2 = \underline{\underline{(s+1)(s+2)}}$$

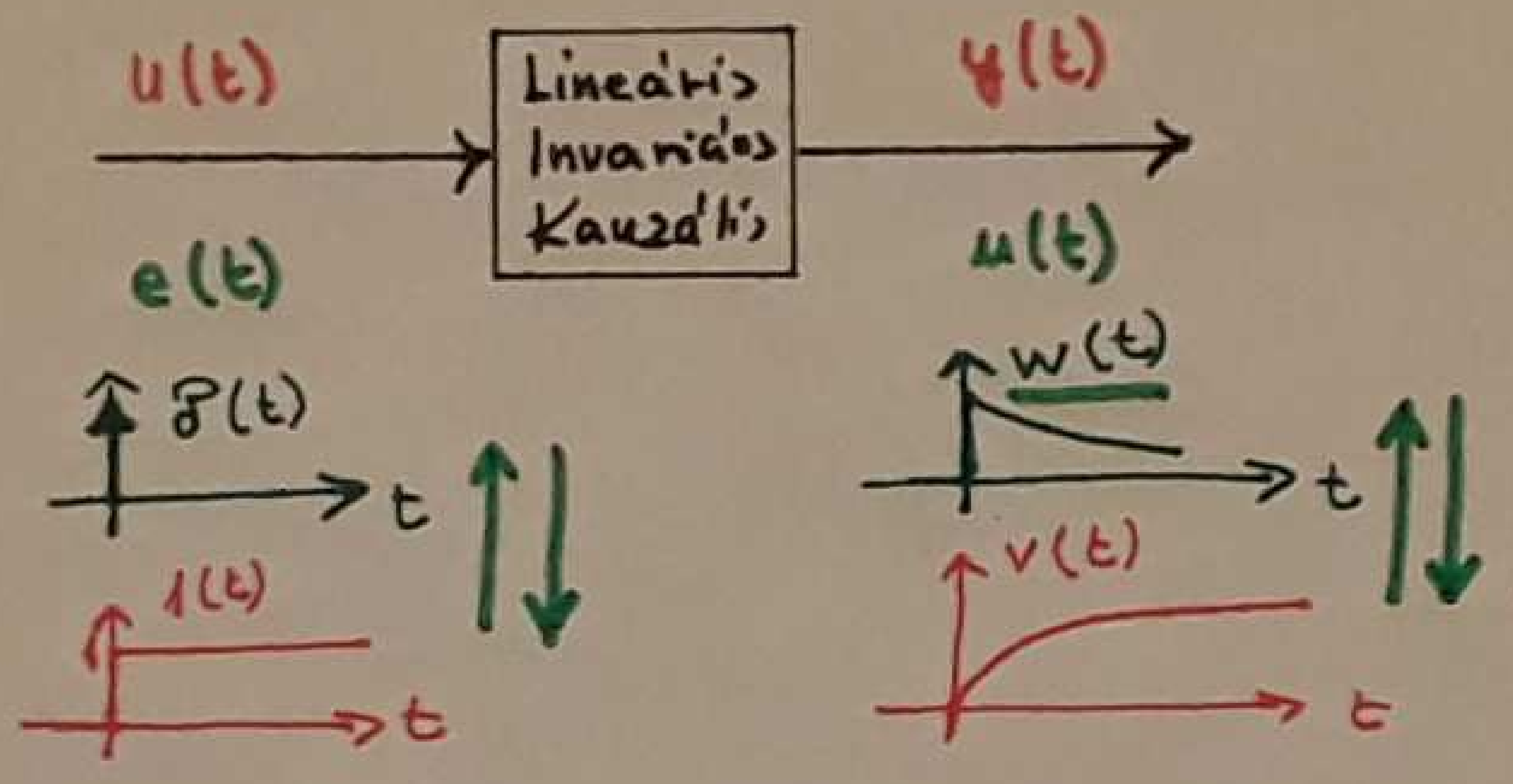
$$\left(\begin{array}{c|c} s+3 & 2 \\ \hline -1 & s \end{array}\right) \xrightarrow{\substack{+\,- \\ -\,+}} \begin{pmatrix} s+3 & -2 \\ 1 & s \end{pmatrix} \xrightarrow{T} \underline{\underline{\begin{pmatrix} s+3 & 1 \\ -2 & s \end{pmatrix}}}$$

$$\begin{bmatrix} \dfrac{s+3}{(s+1)(s+2)} & \dfrac{1}{(s+1)(s+2)} \\[2ex] \dfrac{2}{(s+1)(s+2)} & \dfrac{s}{(s+1)(s+2)} \end{bmatrix} = \left[\begin{array}{c|c} \dfrac{2}{s+1} + \dfrac{-1}{s+2} & \dfrac{1}{s+1} + \dfrac{-1}{s+2} \\[2ex] \hline \dfrac{-2}{s+1} + \dfrac{2}{s+2} & \dfrac{-1}{s+1} + \dfrac{2}{s+2} \end{array}\right] \Rightarrow e^{\underline{A}t} = \left[\begin{array}{c|c} 1(t)\left[2e^{-1t} - 1e^{-2t}\right] & 1(t)\left[1e^{-1t} - 1e^{-2t}\right] \\ \hline 1(t)\left[-2e^{-1t} + 2e^{-2t}\right] & 1(t)\left[-1e^{-1t} + 2e^{-2t}\right] \end{array}\right]$$

Lagrange. Hermite.

LINEÁRIS RENDSZEREK LEÍRÁSI MÓDSZEREI

- ÖSSZEFOGLALÁS -



impulzusválasz
ugrásválasz

rendszeregyenlet

$\boxed{\text{állapotváltozós leírás}}$

átviteli karakterisztika

$\boxed{\text{átviteli függvény}}$

T.J.D.

$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, w(t-\tau)\, d\tau$$

$$y^{(n)} + \sum_{i=1}^{n} a_i y^{(n-i)} = \sum_{i=0}^{n} b_i u^{(n-i)}$$

$\downarrow\uparrow$

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}\,u$$

$$y = \underline{c}^T\underline{x} + D\,u$$

$$u(t) = \hat{U}\,\underline{\cos}(\underset{\uparrow}{\omega} t + \alpha) \longrightarrow y(t) = \hat{Y}\,\underline{\cos}(\omega t + \beta)$$

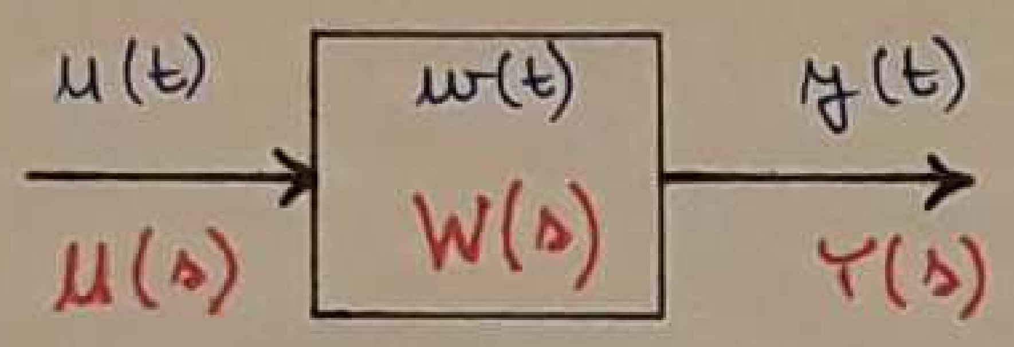
$$W(j\omega) = \frac{\hat{\underline{Y}}}{\hat{\underline{U}}} = \frac{\mathcal{F}\{y(t)\}}{\mathcal{F}\{u(t)\}}$$

$\updownarrow$

$$W(s) = \frac{\mathcal{L}\{y(t)\}}{\mathcal{L}\{u(t)\}}$$

AZ ÁTVITELI FÜGGVÉNY FORMÁI

– ÖSSZEFOGLALÓ –



$$W(s) = \frac{Y(s)}{U(s)}$$

$$W(s) = \frac{Y(s)}{U(s)} = \frac{\boxed{b_0} s^n + b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_{n-1} s + b_n \;{}^{=0}}{1 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n = 0}$$

polinom / polinom

racionális tört-függvény

számláló fokszáma ≤ nevező fokszáma !

$$= k \frac{(s-s_1)(s-s_2)\dots \quad (s-s_n)}{(s-p_1)(s-p_2)(s-p_3)\dots \quad (s-p_n)}$$

$(= b_0)$

zérus / pólus

gyöktényezős alak (faktorizált)

$$s - s_1 = -s_1\left(\frac{s}{-s_1} + 1\right) = -s_1\left(1 + s\tau_1\right)$$

$$\tau_1 = -\frac{1}{s_1}$$

$s - s_1 \big|_{s_1=0} = s$ $\quad = 0$

VALÓS

KONJUGÁLT KOMPLEX PÁR

$$= \frac{K}{s^i} \cdot \underbrace{\frac{(1+s\tau_1)(1+s\tau_2)\dots}{(1+sT_1)(1+sT_2)\dots} \cdot \frac{(1+2\mu_1\tau_1' s + s^2\tau_1'^2)(1+2\mu_2\tau_2' s + s^2\tau_2'^2)\dots}{(1+2\xi_1 T_1' s + s^2 T_1'^2)(1+2\xi_2 T_2' s + s^2 T_2'^2)\dots}}_{W_1(s)\big|_{s=0} = 1}$$

időállandók (valós!)

$$= \boxed{A\left(\frac{\Omega_0}{s}\right)^i} \frac{\left(1+\frac{s}{\Omega_1}\right)\left(1+\frac{s}{\Omega_2}\right)\dots}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)\dots} \quad \frac{\left(1 + 2\mu_1\frac{s}{\Omega_1'} + \left(\frac{s}{\Omega_1}\right)^2\right)\dots}{\left(1 + 2\xi_1\frac{s}{\omega_1} + \left(\frac{s}{\omega_1}\right)^2\right)\dots}$$

Bode

$(s-s_1)(s-s_2)$ $\quad s_1 = \underline{a+jb}$ $\quad s_2 = \underline{a-jb}$ LENGŐ

$$(s-[a+jb])(s-[a-jb])$$

$$s^2 - s[a-jb] - [a+jb]s + \underbrace{[a+jb][a-jb]}_{a^2+b^2}$$

$$s^2 - \underline{sa} + \cancel{sjb} - \underline{sa} - \cancel{sjb} + a^2+b^2$$

$$s^2 - 2as + \underline{(a^2+b^2)}$$

$$(a^2+b^2)\left[1 - \frac{2a}{a^2+b^2}s + \underbrace{\boxed{\frac{1}{a^2+b^2}}s^2}_{\tau^2 s^2}\right]$$

SAJÁTFREKUENCIA

$$\boxed{\Omega^2 = a^2+b^2}$$

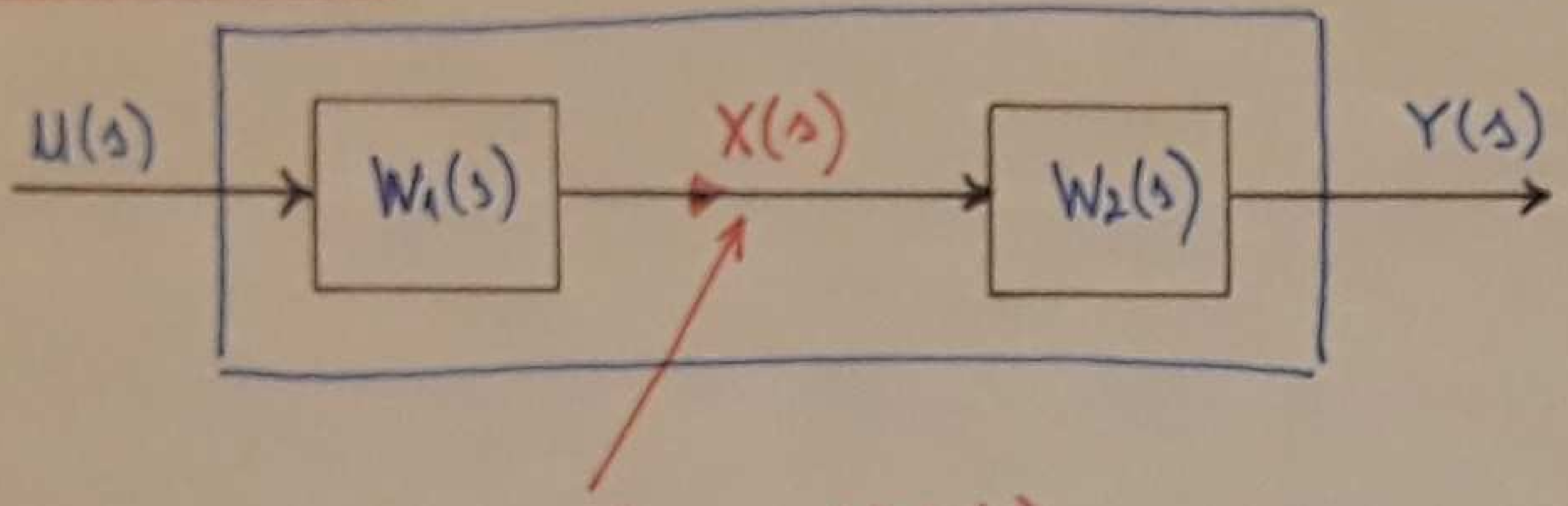
CSILLAPITÁSI TÉNYEZŐ $\rightarrow$ $\boxed{\xi = -\frac{a}{\Omega}}$ $\quad -2\xi\tau s$ $\quad \tau^2 = \frac{1}{\Omega^2}$ $\quad \boxed{\tau = \frac{1}{\Omega}}$

$$-2\frac{a}{\Omega^2} = -2\xi\tau$$

$$\boxed{\Omega^2(1+2\xi\tau s+\tau^2 s^2)}$$

ALAPKAPCSOLÁSOK

SOROS KAPCSOLÁS

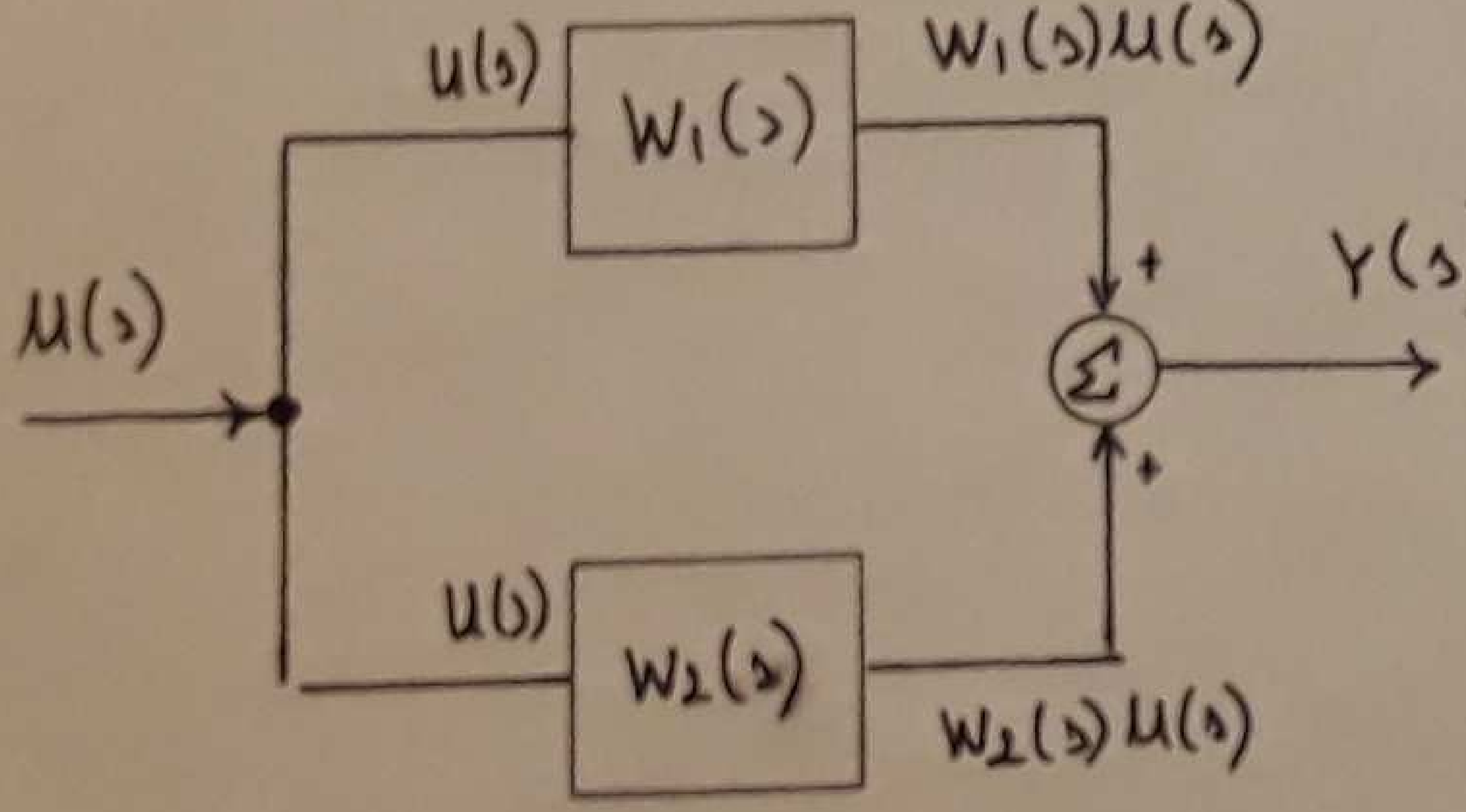


$Y(s) = W_2(s)\cdot X(s) = \boxed{W_2(s)W_1(s)}U(s)$

$\boxed{W_e(s) = W_2(s)W_1(s)}$

$X(s) = W_1(s)U(s)$

PÁRHUZAMOS KAPCSOLÁS

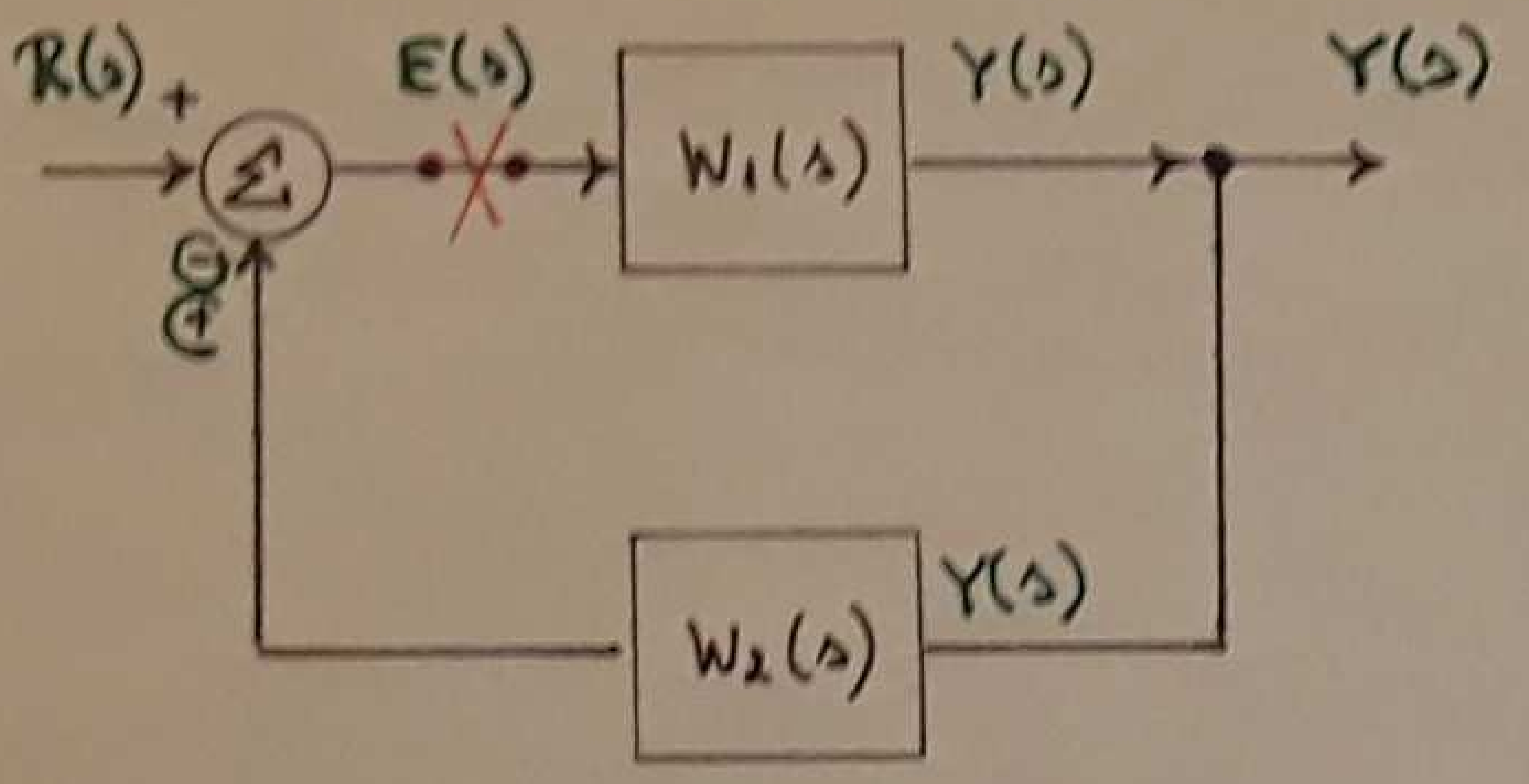


$Y(s) = W_1(s)U(s) + W_2(s)U(s)$

$Y(s) = (W_1(s) + W_2(s))U(s)$

$\boxed{W_e(s) = W_1(s) + W_2(s)}$

VISSZACSATOLÁSOS KAPCSOLÁS



$$Y(s) = W_1(s)\,E(s)$$

$$E(s) = R(s) \mp W_2(s)\,Y(s)$$

$$\boxed{W_0 = W_1 W_2}$$

↑ OPEN LOOP

felnyitott kör átviteli függvénye (hurokátvitel)

$$Y(s) = W_1(s)\left[R(s) \mp W_2(s)\,Y(s)\right]$$

$$Y(s) = W_1(s)\,R(s) \mp W_1(s)\,W_2(s)\,Y(s)$$

$$\left[1 \pm W_1(s)W_2(s)\right] Y(s) = W_1(s)\,R(s)$$

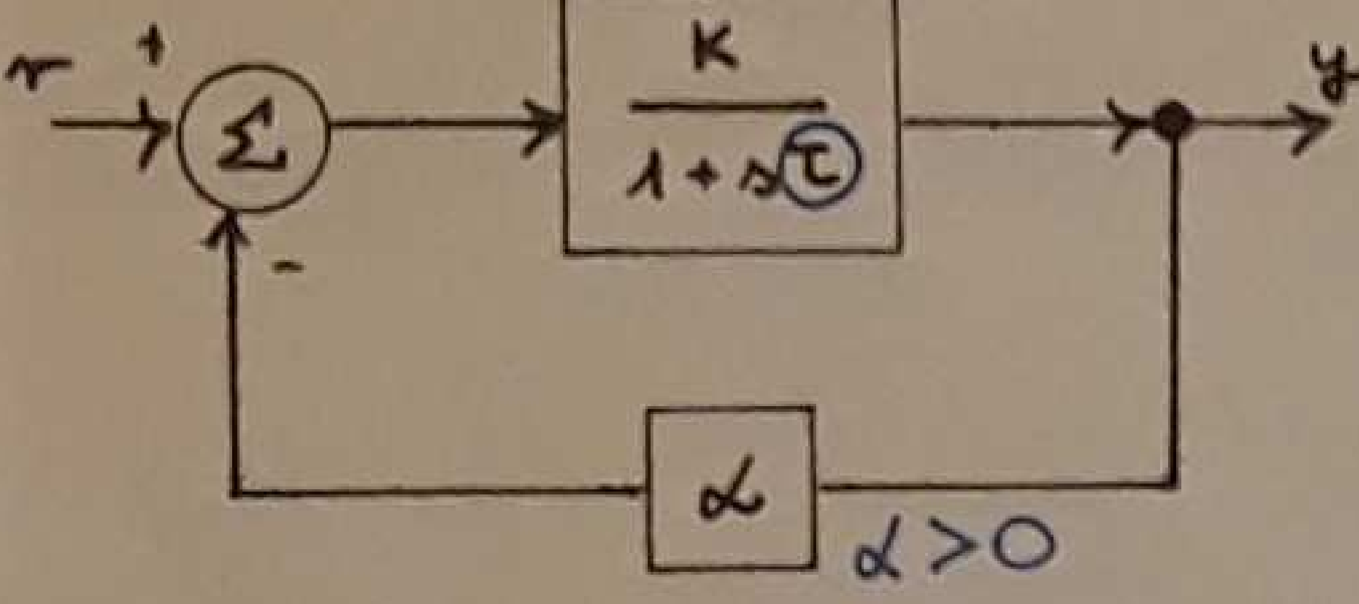
$$W_e(s) = \frac{Y(s)}{R(s)} = \boxed{\frac{W_1(s)}{1 \pm W_1(s)W_2(s)}}$$

$$\boxed{W_e(s) = \frac{W_1(s)}{\boxed{1 + W_1(s)W_2(s)}}}$$

$1 + W_0 = 0 \Rightarrow$ STABILITA

A VISSZACSATOLÁS NÉHÁNY LÉNYEGES TULAJDONSÁGA

JAVÍT A TRANZIENS VISELKEDÉSEN

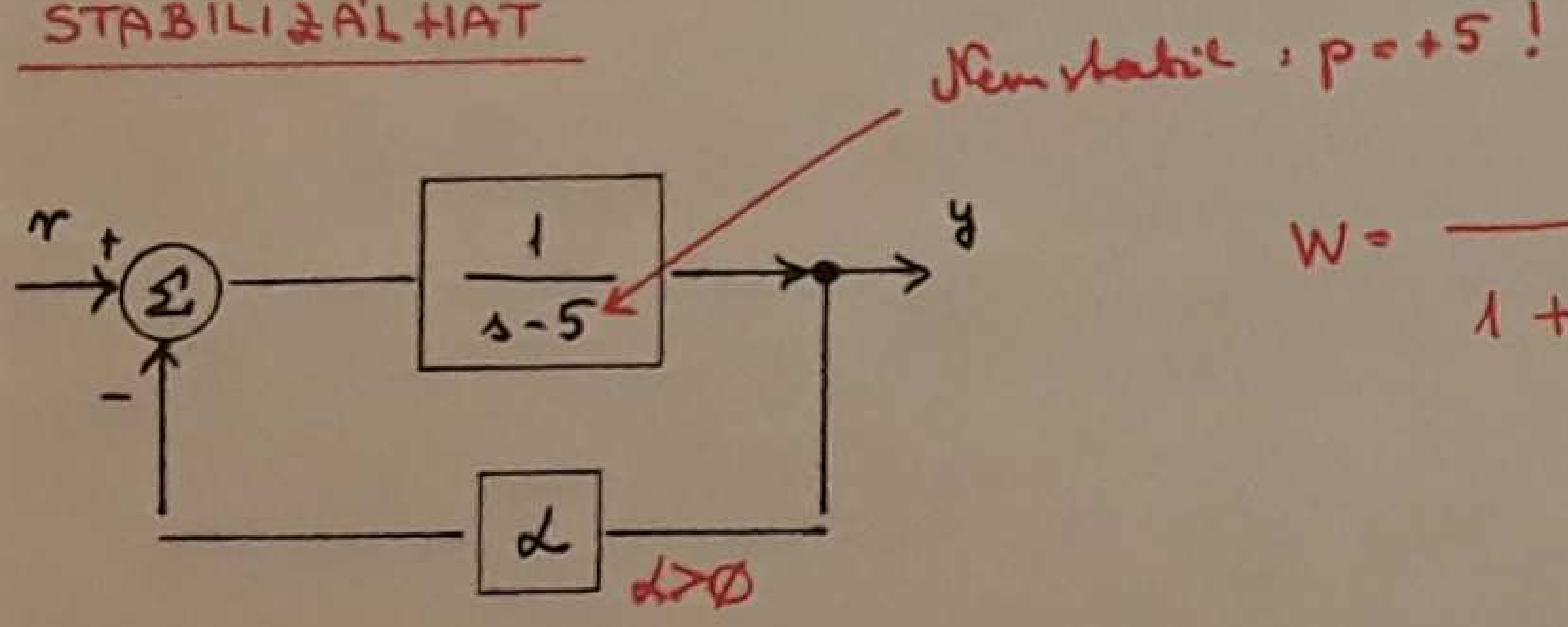


$\tau \rightarrow \tau' < \tau$

$$W = \frac{Y}{R} = \frac{\frac{K}{\boxed{1+s\tau}}}{1+\alpha\cdot\frac{K}{\boxed{1+s\tau}}} = \frac{K}{1+s\tau+\alpha K} = \frac{K}{1+\alpha K}\,\frac{1}{1+\frac{s\tau}{1+\alpha K}}$$

$$= \frac{K}{1+\alpha K}\,\frac{1}{1+s\tau'} \qquad \tau' = \frac{\tau}{1+\alpha K} \; >1 \qquad \tau' < \tau \; ✓$$

STABILIZÁLHAT



Nem stabil : p = +5 !

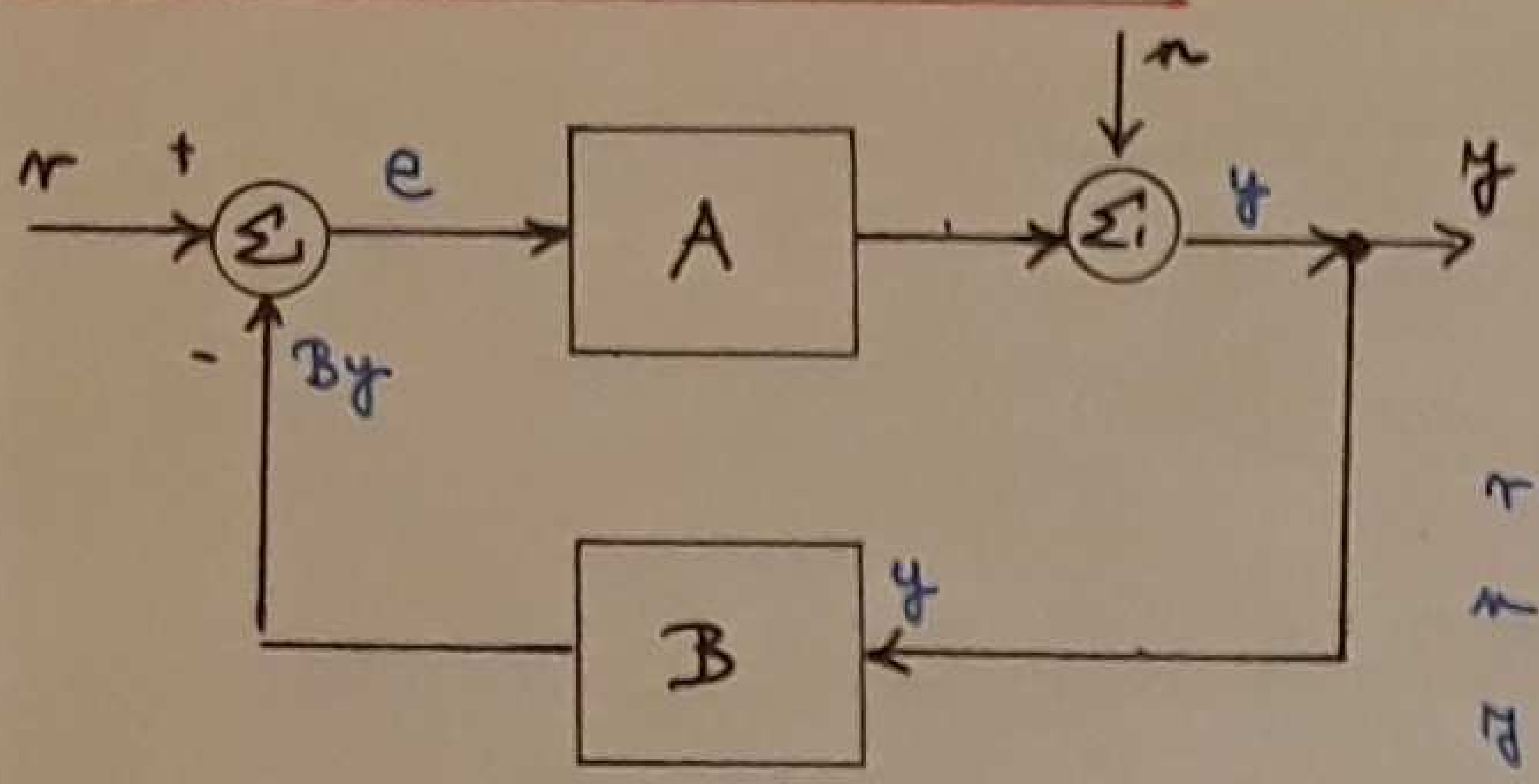
$$W = \frac{\frac{1}{s-5}}{1+\alpha\frac{1}{s-5}} = \frac{1}{s-5+\alpha} = \frac{1}{s + \boxed{(\alpha-5)}}$$

$p > 0$

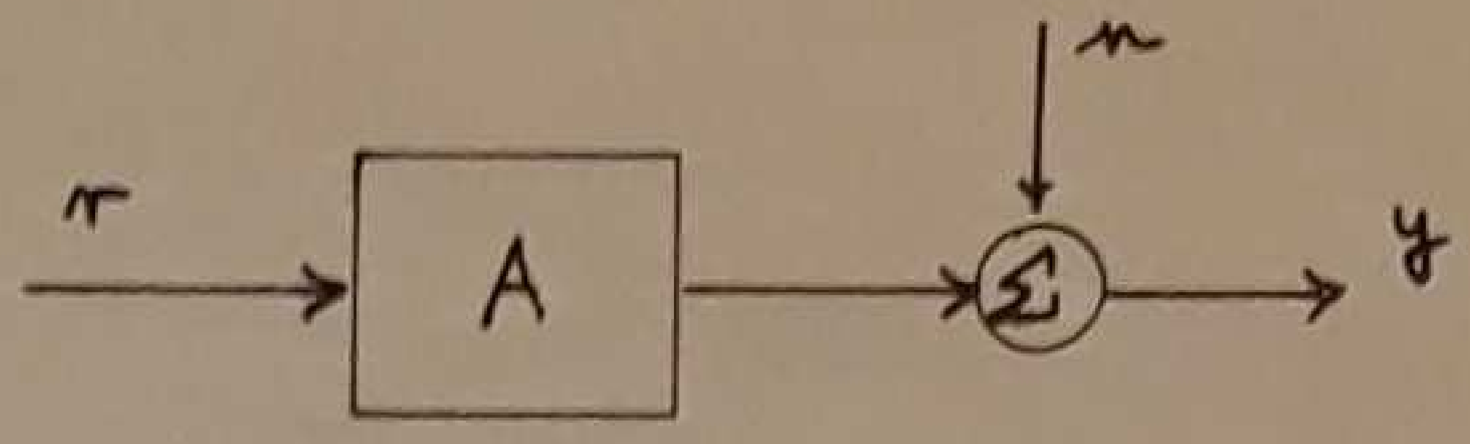
$\alpha - 5 > 0$

$\boxed{\alpha > 5}$

ZAVAR HATÁSÁT CSÖKKENTI



$r = r(t)$
$n = n(t)$
$y = y(t)$



$y = Ar + n$

$y(t) = Ar(t) + \boxed{n(t)}$

!

$e = r - By$

$y = Ae + n = A(r - By) + n$

$y = Ar \underline{- ABy} + n$

$(1 + AB)\,y = Ar + n$

$$\boxed{y = \frac{A}{\boxed{1+AB}}\,r + \boxed{\frac{1}{\boxed{1+AB}}\,n}}$$

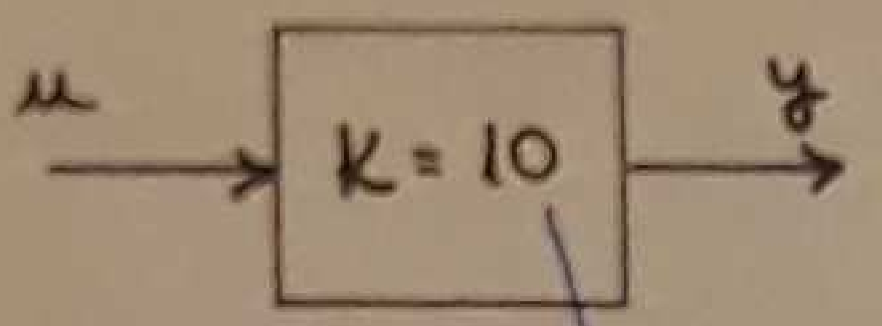
$A \cdot B \uparrow$

nagy hurokerősítés !

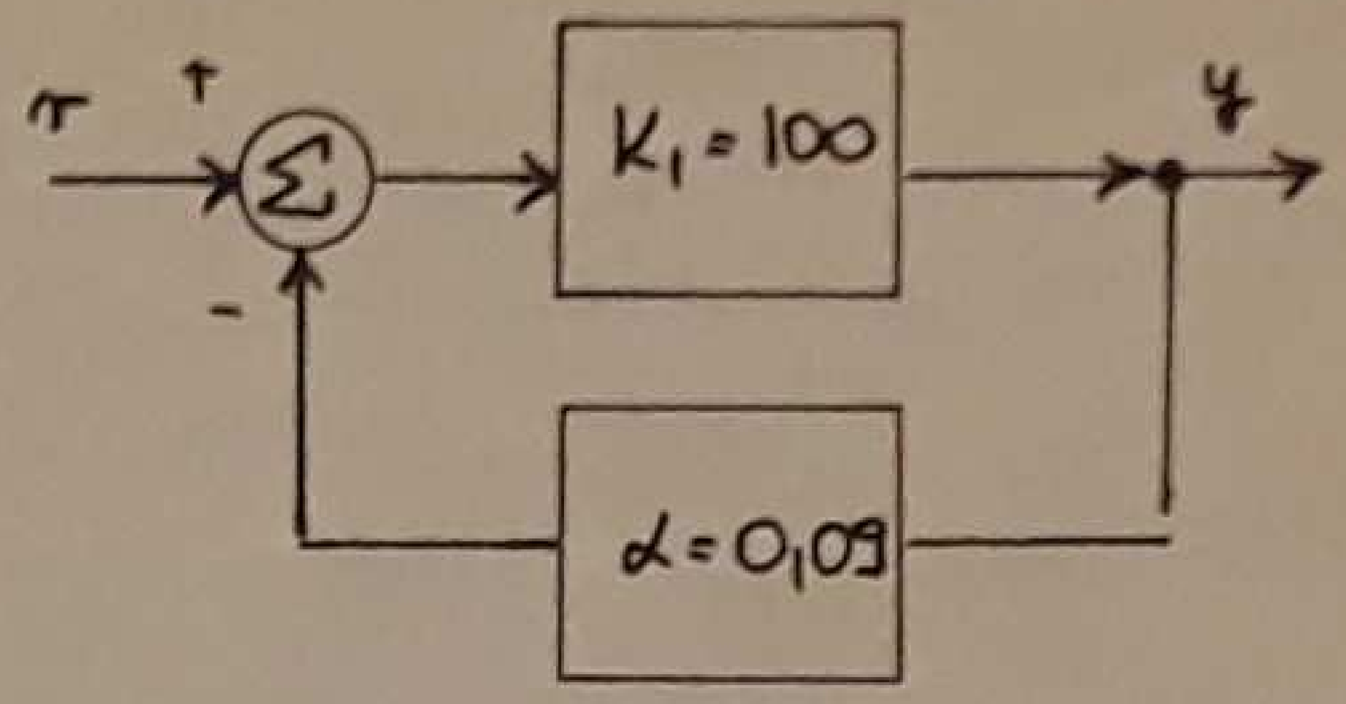
?

$$\boxed{y = Ar + \boxed{n}}$$

ROBUSZTUSSÁ TESZI A ZÁRT KÖRT



$\equiv$



$$\frac{K_1}{1+\alpha K_1} = \frac{100}{\underbrace{1+0{,}09\cdot 100}_{9}} = \underline{\underline{10}}$$

$u = r = 10$

(a) $\underline{\underline{y = 100}}$

(b) +5%

$K = 10{,}5$

$y = 10{,}5 \cdot 10 = \boxed{\underline{\underline{105}}}$

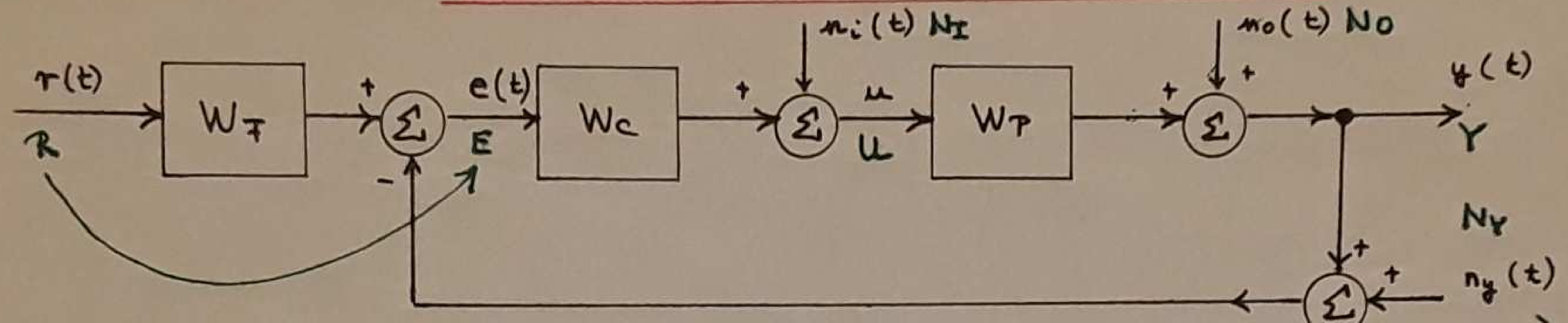
$\underline{\underline{y = 100}}$

$K_1 = 105$

$$\frac{K_1}{1+\alpha K_1} = \frac{105}{1+0{,}09\cdot 105} = \underline{\underline{10{,}048}}$$

$y = 10{,}048 \cdot 10 = \boxed{\underline{\underline{100{,}48}}}$

AZ ÁLTALÁNOS SZABÁLYOZÁSI KÖR



$$E = W_F R - (N_Y + N_O + W_P U) = W_F R - (N_Y + N_O + W_P N_I + W_P W_C E)$$

SZUPERPOZÍCIÓ.

$$U = N_I + W_C E$$

$$E = W_F R - N_Y - N_O - W_P N_I \underline{- W_P W_C E}$$

$$(1 + W_C W_P) E = W_F R - N_Y - N_O - W_P N_I \longrightarrow \boxed{E = \frac{W_F}{\boxed{1+W_C W_P}} R - \frac{W_P}{1+W_C W_P} N_I - \frac{1}{1+W_C W_P} N_O - \frac{1}{1+W_C W_P} N_Y}$$

$$U = N_I + W_C E = \underbrace{N_I} + \frac{W_C W_F}{1+W_C W_P} R - \underbrace{\frac{W_C W_P}{1+W_C W_P} N_I} - \frac{W_C}{1+W_C W_P} N_O - \frac{W_C}{1+W_C W_P} N_Y$$

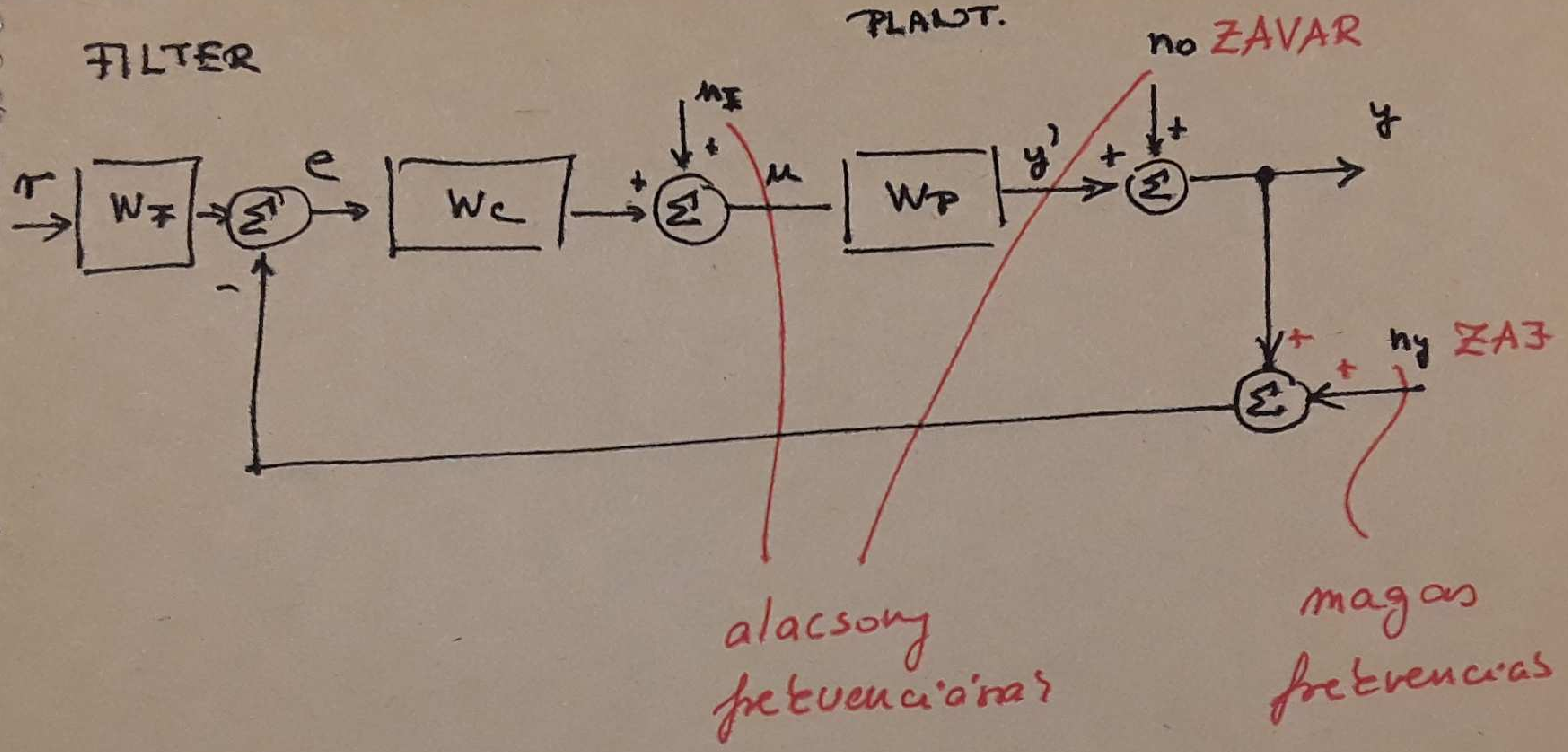
$$\frac{1}{1+W_C W_P} N_I$$

$$\boxed{U = \frac{W_C W_F}{\boxed{1+W_C W_P}} R + \frac{1}{1+W_C W_P} N_I - \frac{W_C}{1+W_C W_P} N_O - \frac{W_C}{1+W_C W_P} N_Y}$$

$$Y = N_O + W_P U = \underbrace{N_O} + \frac{W_C W_P W_F}{1+W_C W_P} R + \frac{W_P}{1+W_C W_P} N_I - \underbrace{\frac{W_C W_P}{1+W_C W_P} N_O} - \frac{W_C W_P}{1+W_C W_P} N_Y$$

$$1 + \underbrace{W_C W_P}_{W_O} \quad \text{open loop}$$

$$\boxed{Y = \frac{W_C W_P W_F}{\boxed{1+W_C W_P}} R + \frac{W_P}{1+W_C W_P} N_I + \frac{1}{1+W_C W_P} N_O - \frac{\boxed{W_C W_P}}{1+W_C W_P} N_Y}$$


FILTER
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+
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ny ZAJ
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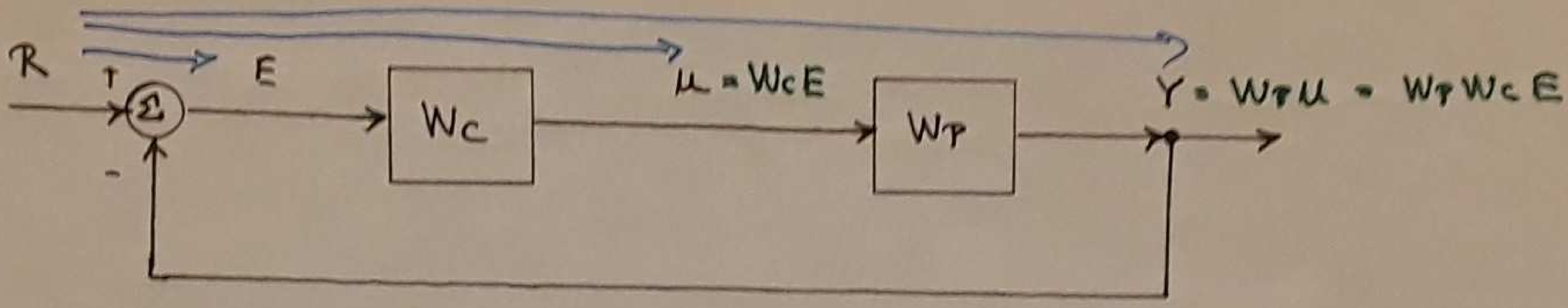
Σ
alacsony frekvenciára
magas frekvenciás

AZ EGYSZERŰ SZABÁLYOZÁSI KÖR



$$E = R - Y = R - W_p W_c E$$

$$(1 + W_c W_p) E = R \longrightarrow \boxed{E = \frac{1}{1+W_c W_p} R}$$

$$\boxed{W_0 = W_c W_p}$$

$$u = W_c E = \frac{W_c}{1+W_c W_p} R \longrightarrow \boxed{u = \frac{W_c}{1+W_c W_p} R}$$

$$Y = W_c W_p E \longrightarrow \boxed{Y = \frac{W_c W_p}{1+W_c W_p} R}$$

A FELNYITOTT KÖR ÁTVITELI FÜGGVÉNYE

$W_o = W_c W_P$

$$W_o(s) = \underbrace{\boxed{\frac{\overset{\text{körerősítés}}{K}}{s^{\overset{\text{típusszám}}{i}}}}}_{\text{STACIONÁRIUS ÁLLAPOT}} \cdot \underbrace{\underbrace{\frac{(1+s\tau_1)(1+s\tau_2)\ \ldots\ldots}{(1+sT_1)(1+sT_2)\ \ldots\ldots}}_{\text{egytárolós tag}} \underbrace{\frac{[1+2\mu_1\tau_1's+(\tau_1's)^2]\cdot\ldots\ldots}{[1+2\xi_1T_1's+(T_1's)^2]\cdot\ldots\ldots}}_{\text{kéttárolós lengő tag}} \underbrace{e^{-sT_h}}_{\text{holtidő}}}_{\text{TRANZIENS ÖSSZETEVŐ}}$$

$$W_{o1}(s)\Big|_{s=\underset{\uparrow}{0}} = 1$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,Y(s)$$

W_o $\quad 1 + W_o$

$\xi > 1$ (a) aperiodikus ~~nem~~ nem lengő

$\xi = 1$ (b) aperiodikus határeset

$0 < \xi < 1$ (c) periodikus lengő

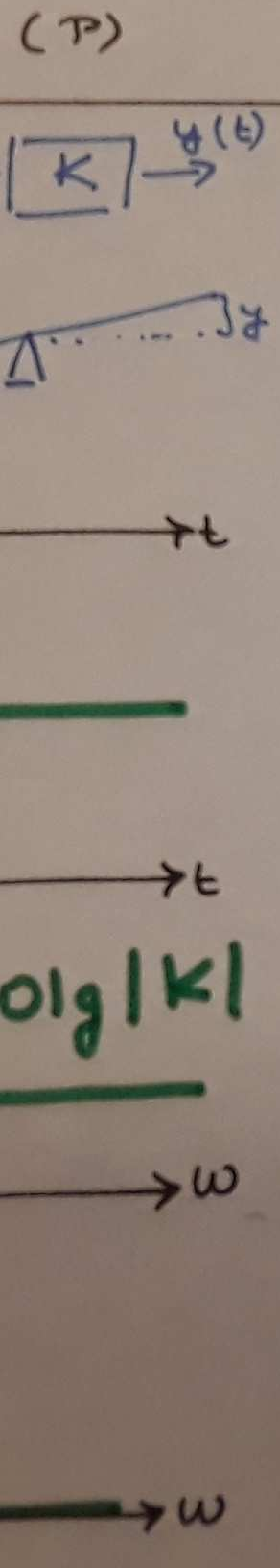
$$\frac{1}{1+2\xi\tau s+\tau^2 s^2}$$

(c) $0 < \xi \le 1$. Lengő

(b) $\xi = 1$ Aperiodikus határeset

(a) $\xi > 1$ Nincs lengés (Aperiodikus)

ALAPTAGOK



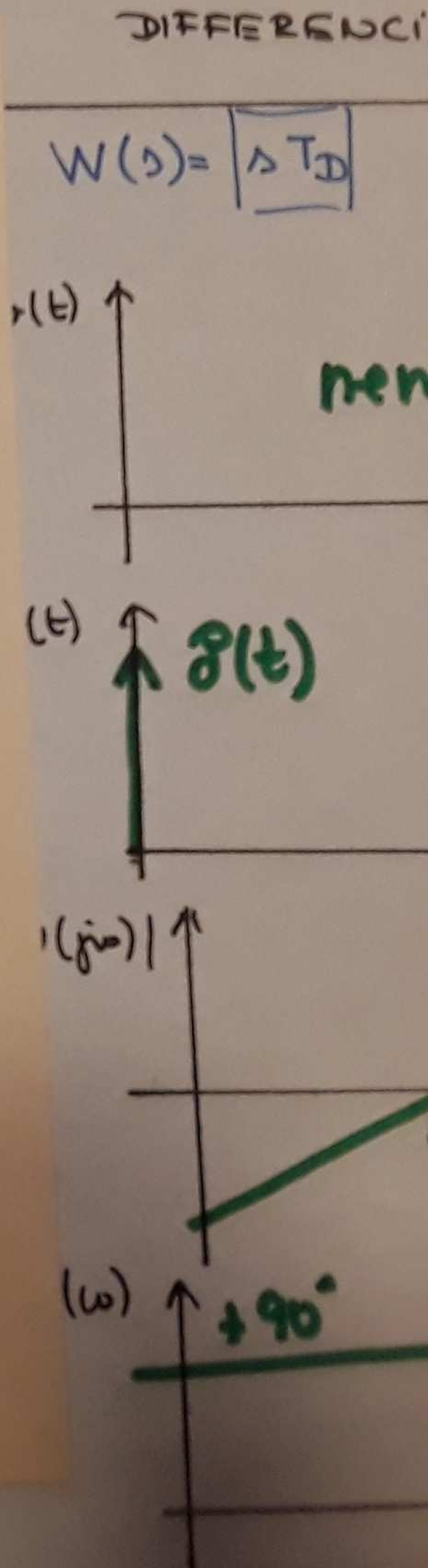
Szabályozás technikai alaptagok

ARÁNYOS (PROPORCIONÁLIS)
INTEGRÁLÓ
DIFFERENCIÁLÓ

IDEÁLIS

EGYTÁROLÓS
KÉTTÁROLÓS LENGŐ
HOLTIDŐ

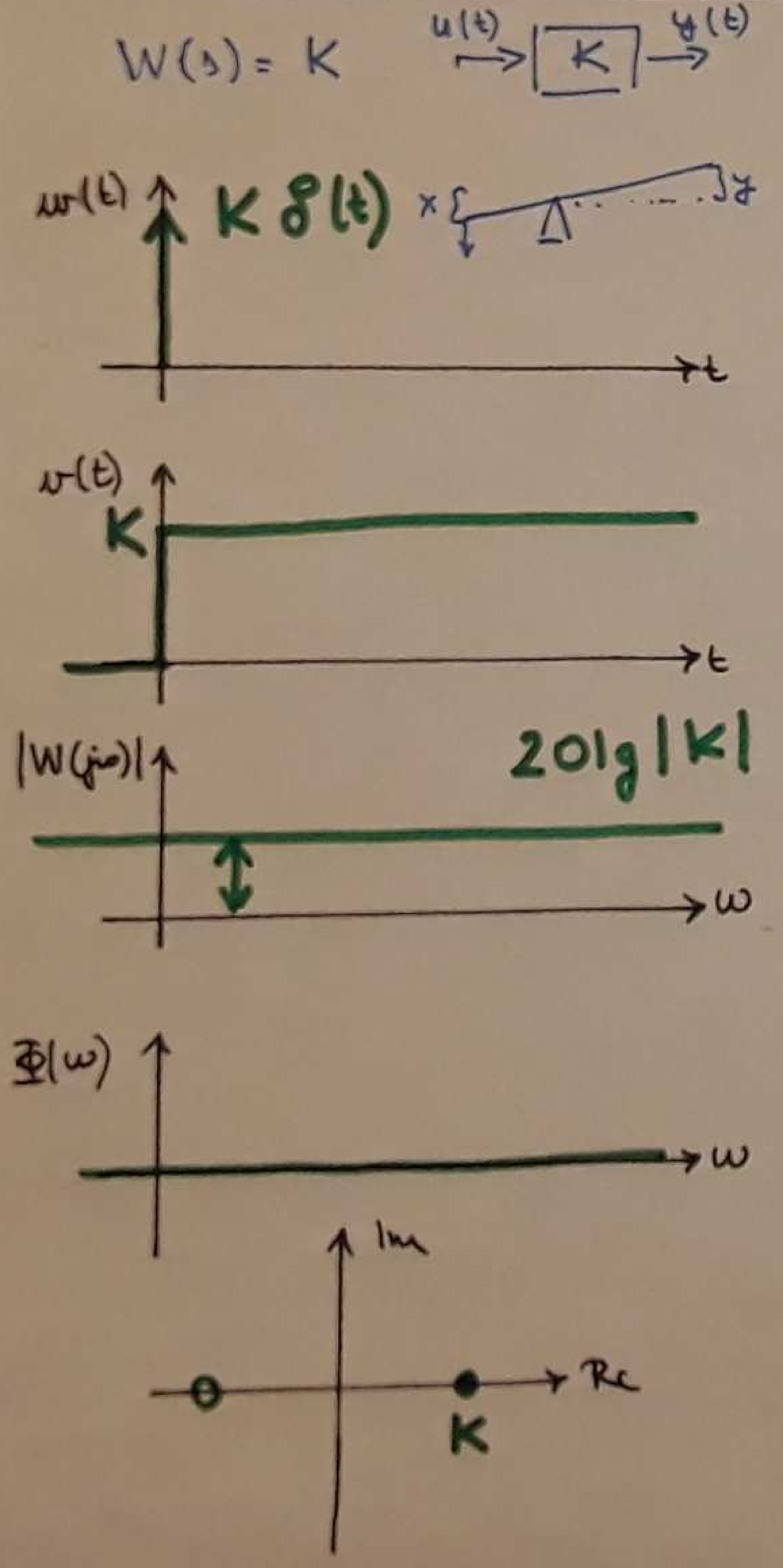
ÁTVITELI FÜGGVÉNY
BODE-DIAGRAM
NYQUIST-DIAGRAM
IMPULZUSVÁLASZ
UGRÁSVÁLASZ



ALAPTAGOK

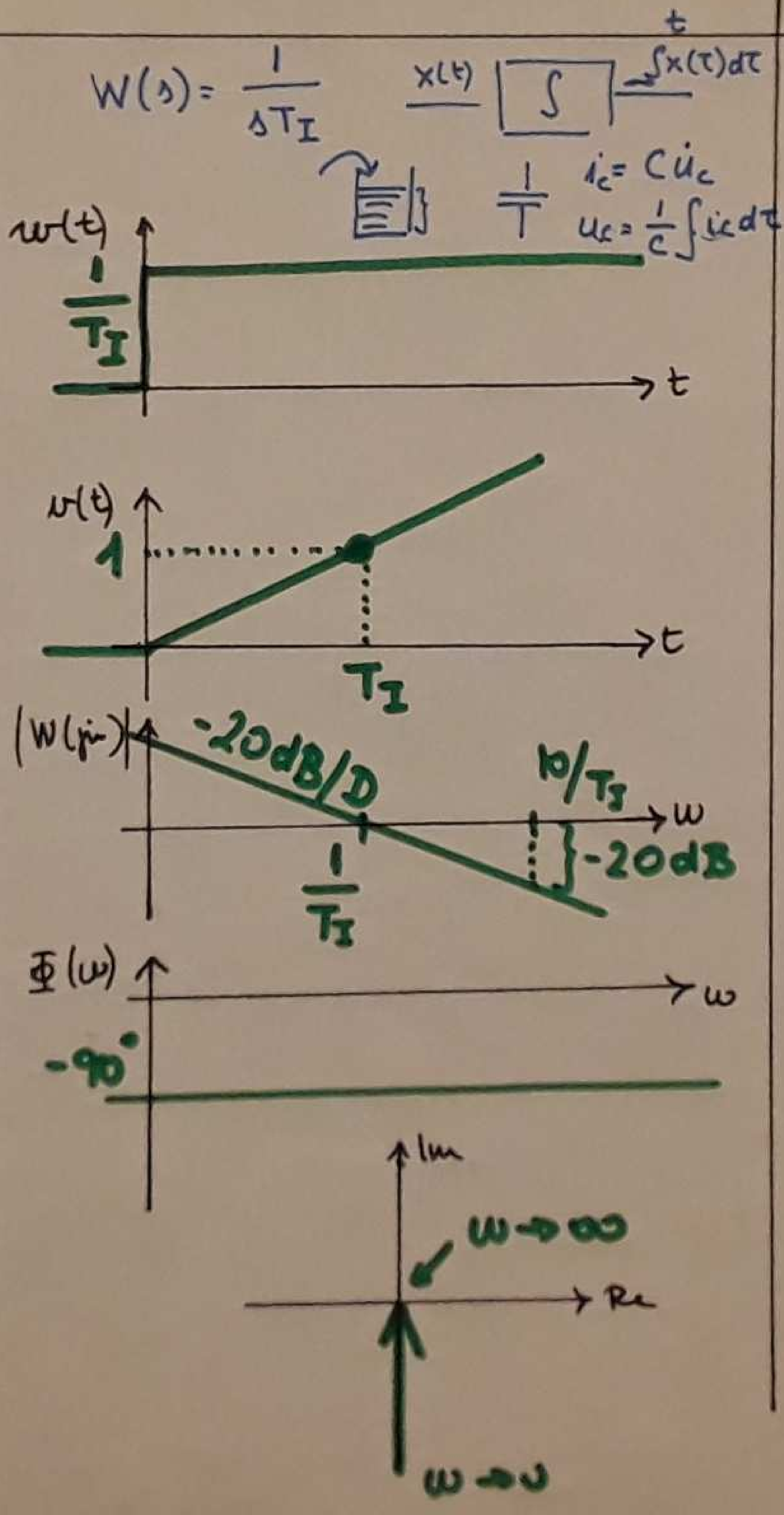
ARÁNYOS (P)

$W(s) = K$



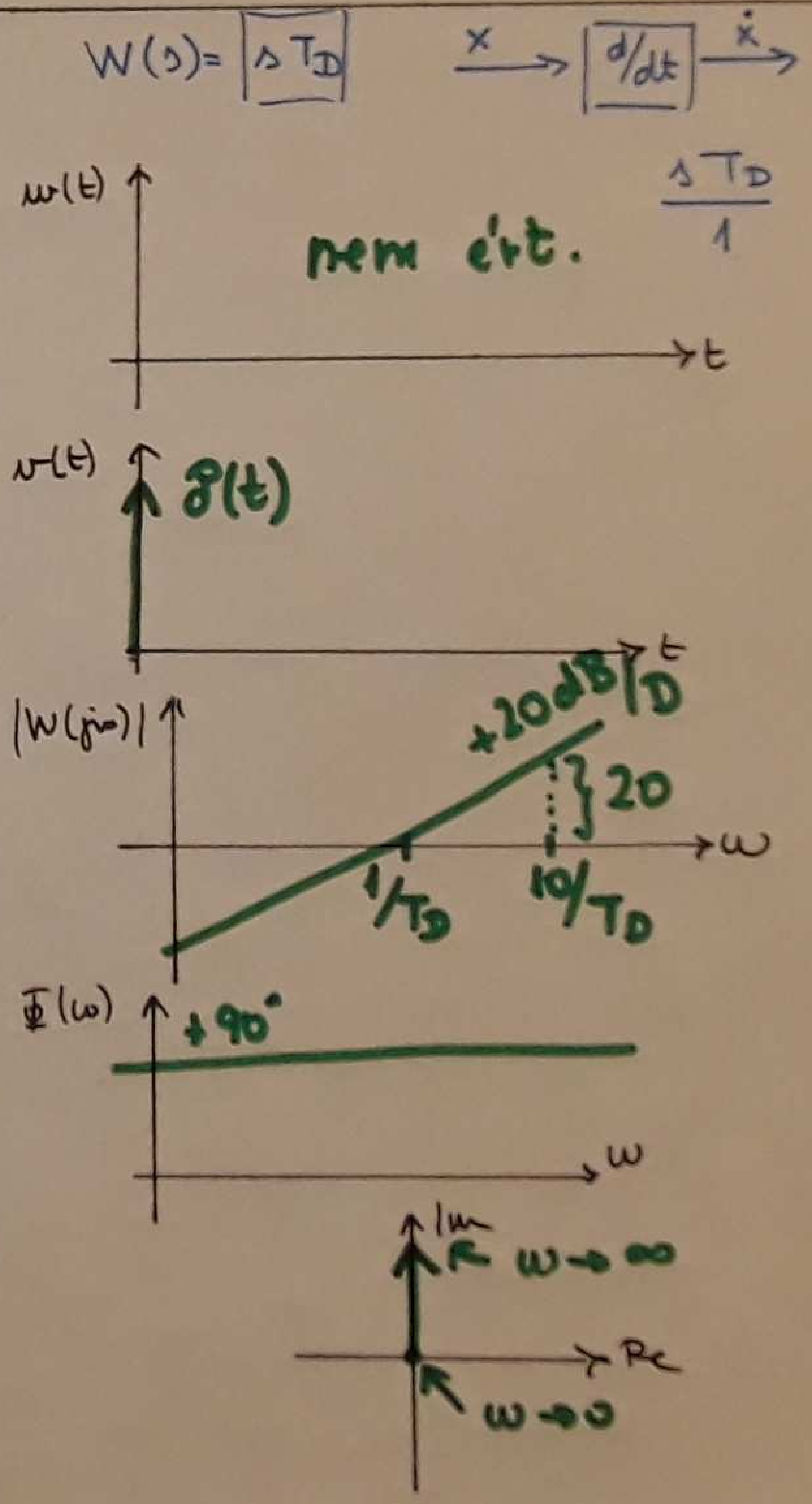
INTEGRÁLÓ (I)

$W(s) = \frac{1}{sT_I}$



DIFFERENCIÁLÓ (D)

$W(s) = \boxed{sT_D}$



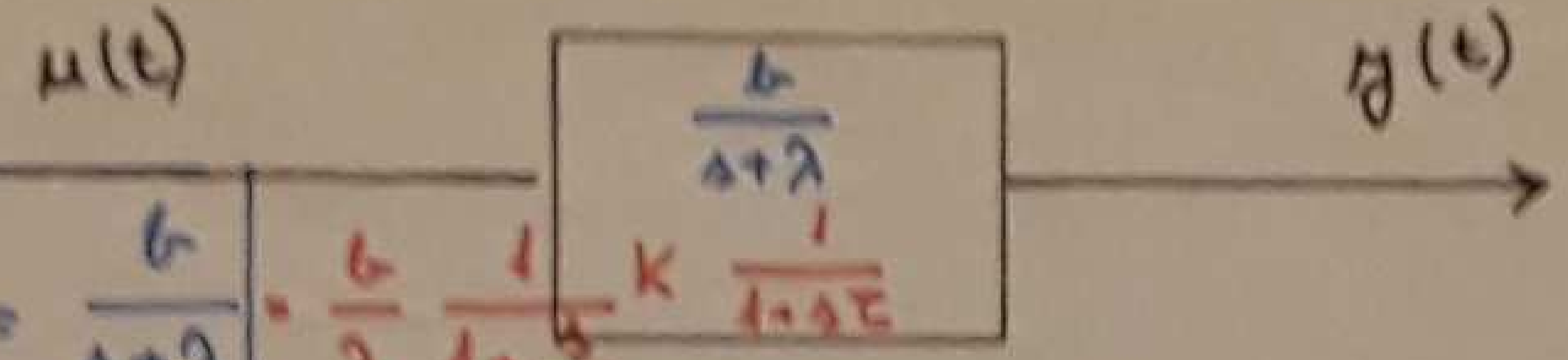
ALAPTAGOK

EGYTÁROLÓS TAG

$\dot{y} + \lambda y = b u \quad / \mathcal{L}$

$sY + \lambda Y = bU$

$(s+\lambda) Y = bU \longrightarrow \boxed{W = \frac{Y}{U} = \frac{b}{s+\lambda}} = \frac{b}{\lambda} \frac{1}{1+\frac{s}{\lambda}}$

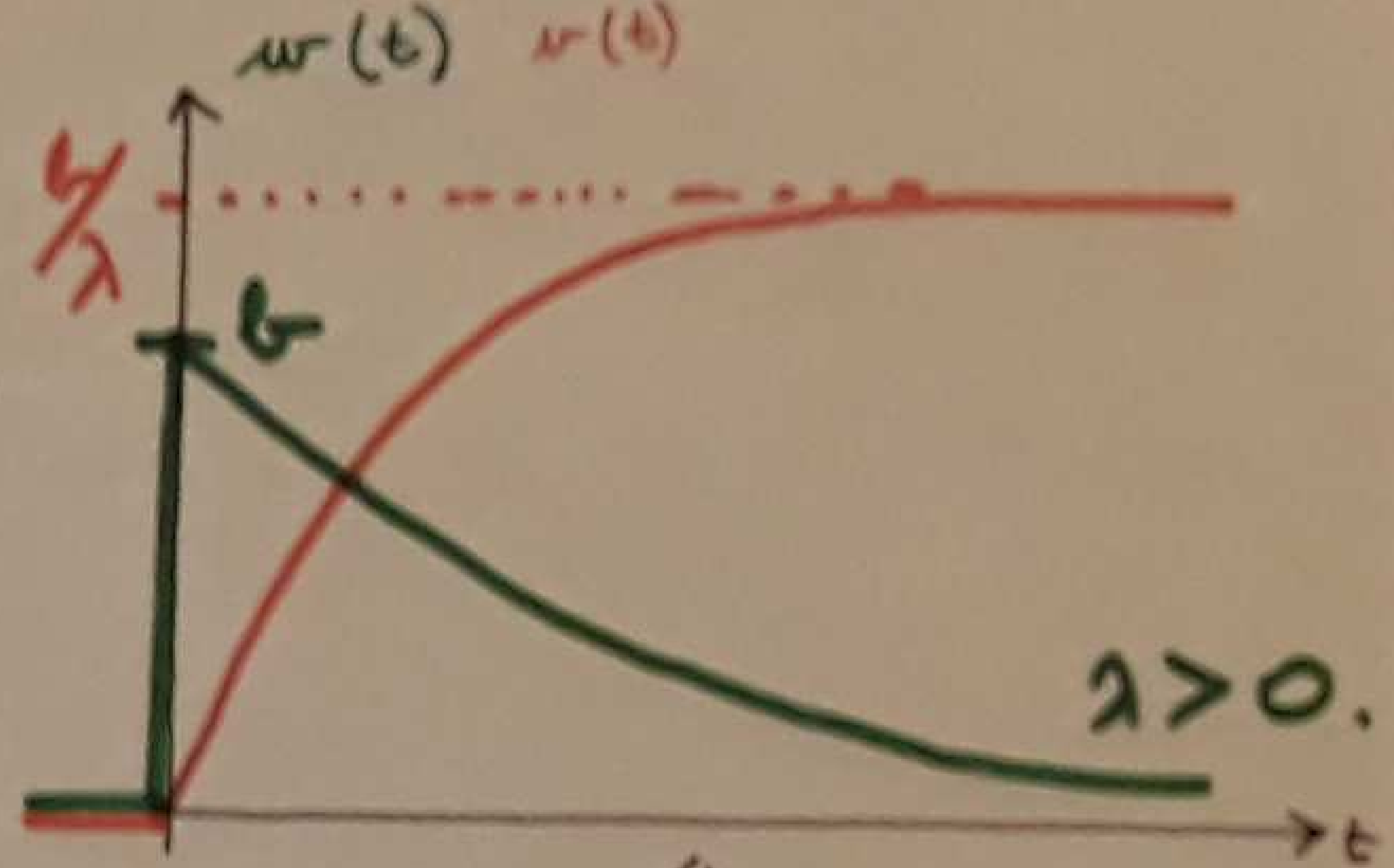


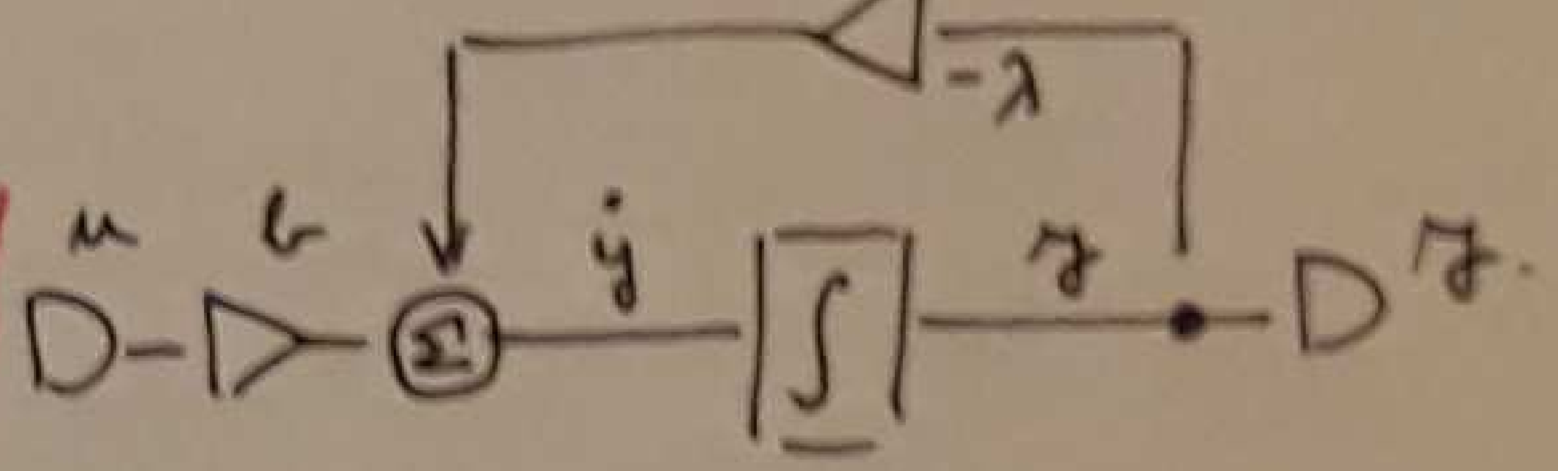
$W = \frac{b}{s+\lambda} \longrightarrow w(t) = 1(t)\, b\, e^{-\lambda t}.$

$\dot{y} = -\lambda y + b u$

$V = \frac{b}{s(s+\lambda)} \longrightarrow V = \frac{b/\lambda}{s} + \frac{-b/\lambda}{s+\lambda}$

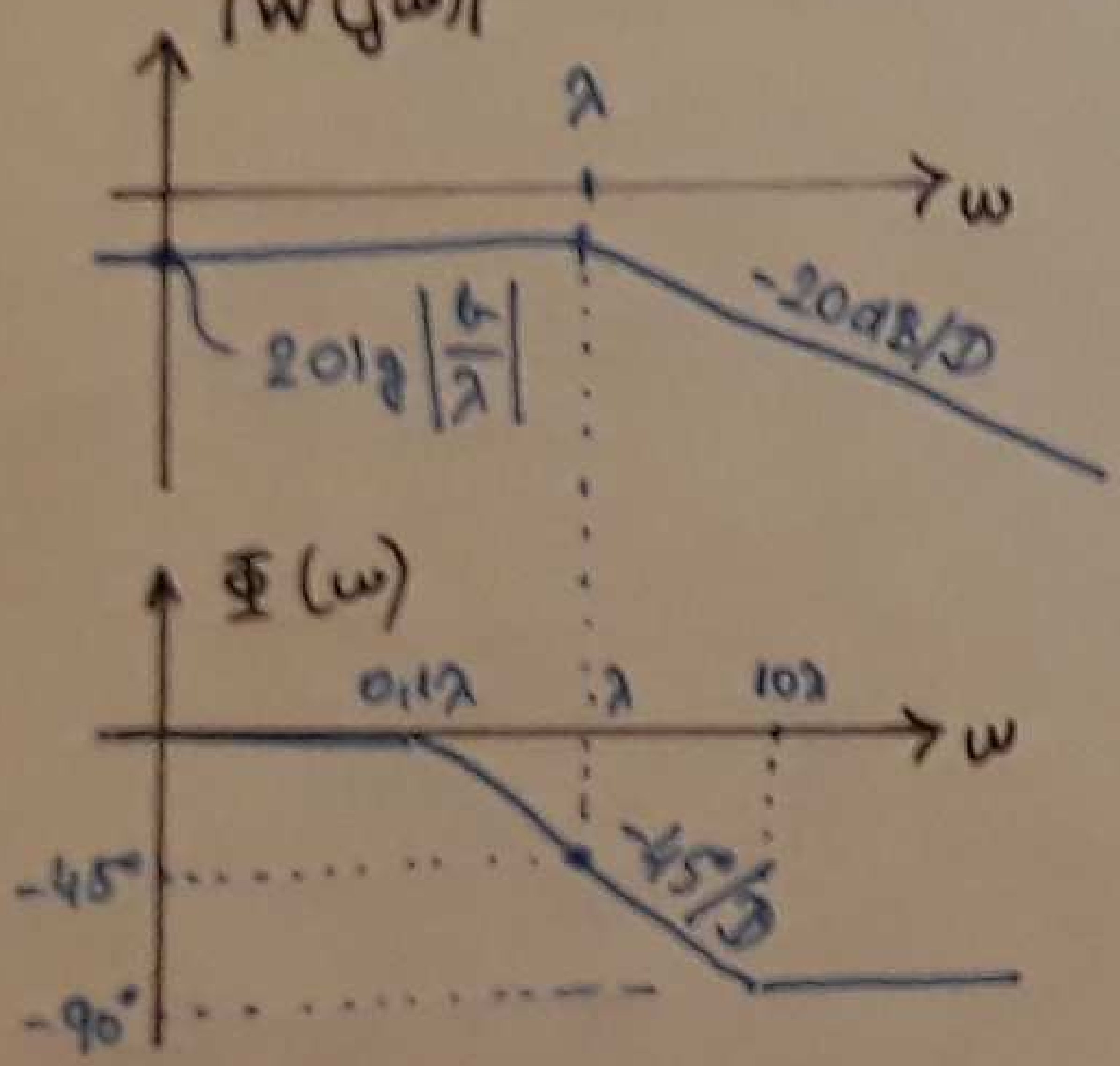
$v(t) = 1(t) \frac{b}{\lambda} \left[1 - e^{-\lambda t} \right]$

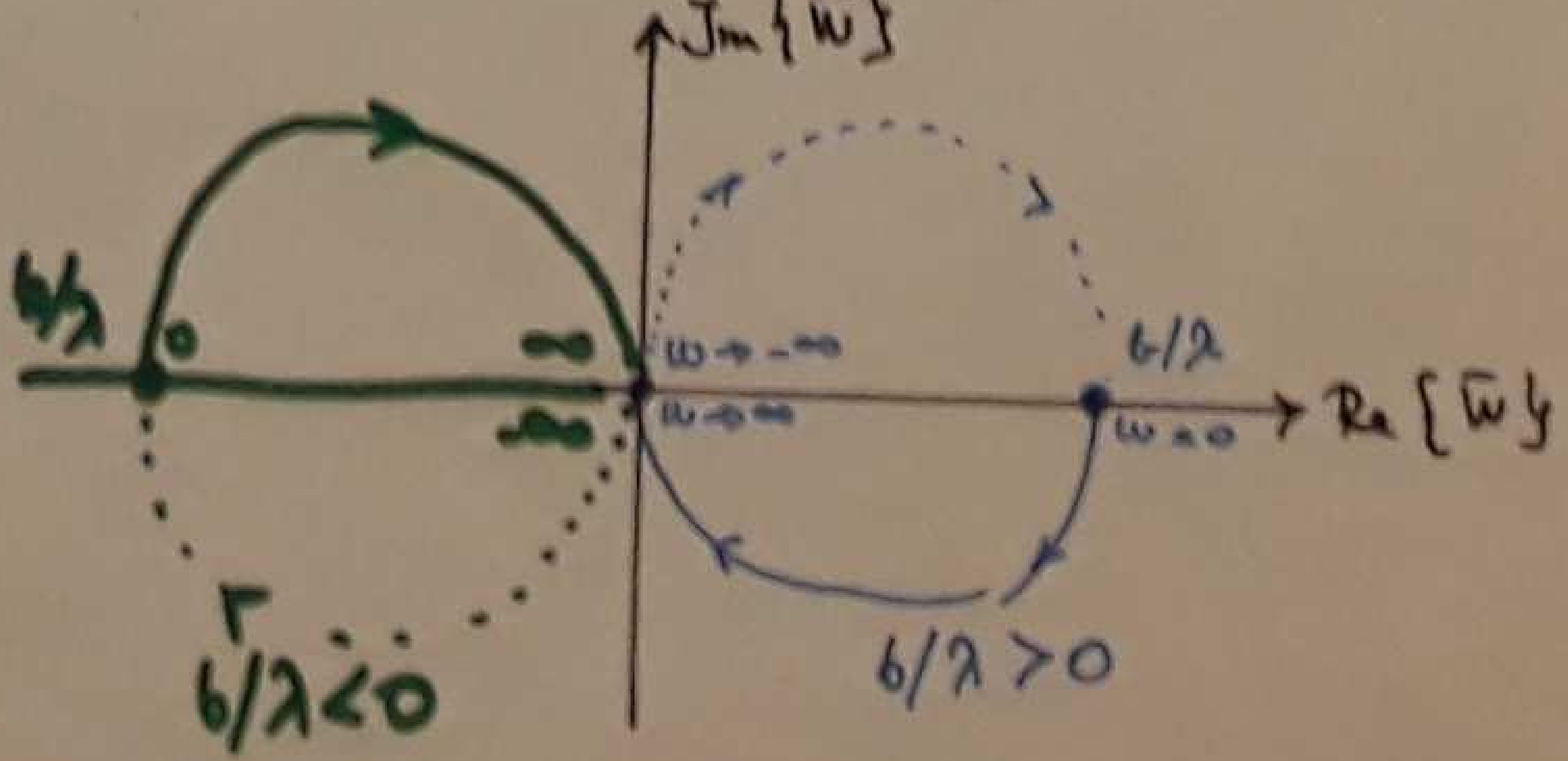




$W(j\omega) = \frac{b}{j\omega + \lambda} = \left|\frac{b}{\lambda}\right| \boxed{\frac{1}{1+\frac{j\omega}{\lambda}}}$

$|\overline{W}| = \left|\frac{b}{\lambda}\right| \frac{1}{\sqrt{1+\left(\frac{\omega}{\lambda}\right)^2}}$





ALAPTAGOK

KÉTTÁROLÓS TAG $\xi > 1.$

$$W(s) = \frac{1}{\underbrace{1}_{c} + \underbrace{2\xi\tau}_{b} s + \underbrace{\tau^2}_{a} s^2}$$

$\xi > 1$

$$p_{1,2} = \frac{-2\xi\tau \pm \sqrt{4\xi^2\tau^2 - 4\tau^2 \cdot 1}}{2\tau^2} = \frac{-2\xi\tau \pm 2\tau\sqrt{\xi^2-1}}{2\tau^2} = \frac{-\xi \pm \sqrt{\xi^2-1}}{\tau} \begin{cases} p_1 = -\frac{1}{T_1} \\ p_2 = -\frac{1}{T_2} \end{cases}$$

$$W(s) = \frac{\frac{1}{T_1T_2}}{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)} = \frac{\frac{1}{T_1-T_2}}{s+\frac{1}{T_1}} + \frac{-\frac{1}{T_1-T_2}}{s+\frac{1}{T_2}} \Rightarrow w(t) = 1(t)\left[e^{-t/T_1} - e^{-t/T_2}\right]\frac{1}{T_1-T_2}$$

$$\frac{\frac{1}{T_1T_2}}{-\frac{1}{T_1}+\frac{1}{T_2}} = \frac{\frac{1}{\cancel{T_1T_2}}}{\frac{-T_2+T_1}{\cancel{T_1T_2}}} = \frac{1}{T_1-T_2}$$

$$\frac{\frac{1}{T_1T_2}}{-\frac{1}{T_2}+\frac{1}{T_1}} = \frac{\frac{1}{\cancel{T_1T_2}}}{\frac{-T_1+T_2}{\cancel{T_1T_2}}} = \frac{1}{T_2-T_1} = \frac{-1}{T_1-T_2}$$

$$V(s) = \frac{\frac{1}{T_1T_2}}{s\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)} = \frac{1}{s} + \frac{\frac{-T_1}{T_1-T_2}}{s+\frac{1}{T_1}} + \frac{\frac{T_2}{T_1-T_2}}{s+\frac{1}{T_2}} \Rightarrow v(t) = 1(t)\left[1 - \frac{T_1}{T_1-T_2}e^{-t/T_1} + \frac{T_2}{T_1-T_2}e^{-t/T_2}\right.$$

$$\frac{\frac{1}{T_1T_2}}{\frac{1}{T_1}\cdot\frac{1}{T_2}}$$

$$\frac{\frac{1}{T_1T_2}}{-\frac{1}{T_1}\left(-\frac{1}{T_1}+\frac{1}{T_2}\right)} = \frac{\frac{1}{\cancel{T_1T_2}}}{-\frac{1}{T_1}\,\frac{-T_2+T_1}{\cancel{T_1T_2}}} = \frac{-T_1}{-T_2+T_1}$$

$$1 - \frac{T_1}{T_1-T_2} + \frac{T_2}{T_1-T_2}$$

$$1 - \frac{T_1-T_2}{T_1-T_2} = 0$$

$$\frac{\frac{1}{T_1T_2}}{-\frac{1}{T_2}\left(-\frac{1}{T_2}+\frac{1}{T_1}\right)} = \frac{\frac{1}{\cancel{T_1T_2}}}{-\frac{1}{T_2}\,\frac{-T_1+T_2}{\cancel{T_1T_2}}} = \frac{-T_2}{T_2-T_1} = \frac{T_2}{T_1-T_2}$$

$$p_{12} = \frac{-2\tau \pm \sqrt{4\tau^2 - 4\tau^2}}{\ } = \frac{-2\tau}{\ \tau^2} = -\frac{1}{\tau} \in \mathbb{R}$$

$$\frac{1}{(s-p_1)(s-p_2)} = \frac{①}{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)} = \frac{T_1T_2}{T_1T_2}\,\frac{1}{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}$$

$$= \frac{\boxed{T_1T_2}}{(1+sT_1)(1+sT_2)} \qquad \frac{1}{1+2\xi\tau s+\tau^2 s^2}$$

$$1+s(T_1+T_2)+s^2T_1T_2 \leftrightarrow$$

$$\boxed{\xi > 1}$$

$$\boxed{\frac{\frac{1}{T_1T_2}}{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}} = \frac{1}{1+2\xi\tau s+\tau^2 s^2}$$

$$\boxed{\begin{tikzcd} {} \arrow[r] & \boxed{\frac{1}{T_1}\,\frac{1}{s+\frac{1}{T_1}}} \arrow[r] & \boxed{\frac{1}{T_2}\,\frac{1}{s+\frac{1}{T_2}}} \arrow[r] & {} \end{tikzcd}}$$

$$\left[\,1 \quad e^{-t/\tau} \quad \frac{1}{\ }\,t\,e^{-t/\tau}\,\right]$$

ha $\underline{\xi = 1}$ (aperiodikus határeset)

$$W = \frac{1}{\underbrace{1}_{c} + \underbrace{2\cancel{\xi}}_{b}\tau s + \underbrace{\tau^2}_{a} s^2}$$

$$p_{12} = \frac{-2\tau \pm \sqrt{4\tau^2 - 4\tau^2}}{2\tau^2} = \frac{-2\tau}{2\tau^2} = \underline{\underline{-\frac{1}{\tau}}} \in \mathbb{R}$$

$\xi = 1$

$$W = \frac{\frac{1}{\tau^2}}{\left(s + \frac{1}{\tau}\right)^2} \longrightarrow \boxed{w(t) = 1(t)\,\frac{1}{\tau^2}\,\underline{t e^{-t/\tau}}}$$

$$V(s) = \frac{\boxed{\frac{1}{\tau^2}}}{s\left(s+\frac{1}{\tau}\right)^2} = \frac{1}{s} + \frac{\cancel{B} - 1}{s + \frac{1}{\tau}} + \frac{-\frac{1}{\tau}}{\left(s+\frac{1}{\tau}\right)^{②}} = \frac{\boxed{\left(s+\frac{1}{\tau}\right)^2 + Bs\left(s+\frac{1}{\tau}\right) - \frac{1}{\tau}s}}{s\left(s+\frac{1}{\tau}\right)^2}$$

$\left(s+\frac{1}{\tau}\right)\left(s+\frac{1}{\tau}\right)$

$$\frac{\frac{1}{\tau^2}}{\frac{1}{\tau^2}}$$

$$\frac{\frac{1}{\tau^2}}{-\frac{1}{\tau}} = \frac{-\tau}{\tau^2} = -\frac{1}{\tau}$$

$$\cancel{s}\quad \boxed{s^2} + \underline{2s\frac{1}{\tau}} + \frac{1}{\tau^2} + \boxed{Bs^2} + \underline{Bs\frac{1}{\tau}} \; \underline{-\frac{1}{\tau}s} = \frac{1}{\tau^2} + 0s^2 + 0s$$

$$\cancel{s}\quad 1 + B = 0 \qquad\qquad \frac{2}{\cancel{\tau}} + \frac{B}{\cancel{\tau}} - \frac{1}{\cancel{\tau}} = 0$$

$$\underline{\underline{B = -1}} \qquad\qquad 1 + B = 0$$

$$\boxed{v(t) = 1(t)\left[1 - e^{-t/\tau} - \frac{1}{\tau}t e^{-t/\tau}\right]}$$

ALAPTAGOK

KÉTTÁROLÓS LENGŐ TAG $0 < \xi < 1$

$$W(s) = \frac{1}{\underbrace{1}_{c} + \underbrace{2\xi\tau s}_{b} + \underbrace{\tau^2 s^2}_{a}}$$

$$\sqrt{\xi^2-1} = \sqrt{-1(1-\xi^2)} = j\sqrt{1-\xi^2}$$

$$p_{1,2} = \frac{-2\xi\tau \pm \sqrt{4\xi^2\tau^2 - 4\tau^2}}{2\tau^2} = \frac{-2\xi\tau \pm 2\tau\sqrt{\xi^2-1}}{2\tau^2} = \frac{-\xi \pm \sqrt{\xi^2-1}}{\tau} \Rightarrow$$

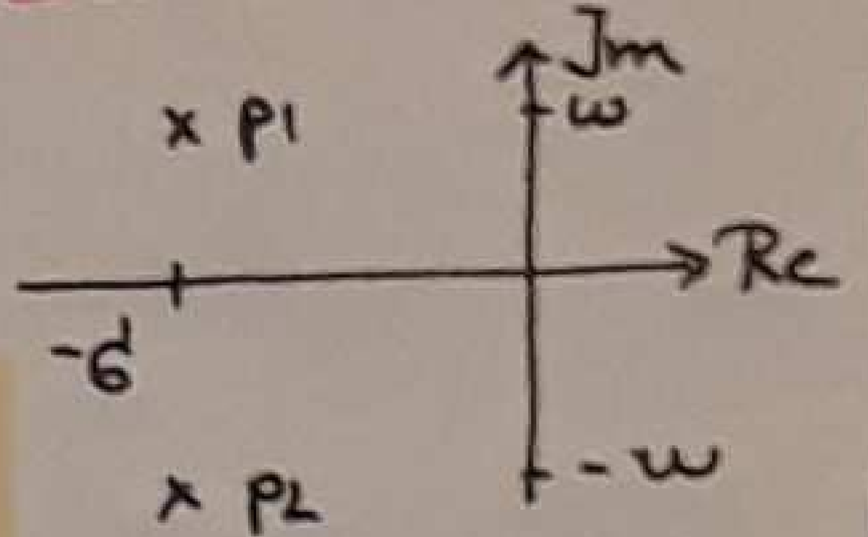
$$p_{1,2} = \frac{-\xi \pm j\sqrt{1-\xi^2}}{\tau} = -\underbrace{\boxed{\xi\Omega}}_{\sigma} \pm j\underbrace{\boxed{\Omega\sqrt{1-\xi^2}}}_{\omega}$$

$$\boxed{p_{1,2} = -\sigma \pm j\omega}$$

$$\boxed{\Omega = \frac{1}{\tau}}$$

SAJÁT FREKVENCIA

$$\sigma^2 + \omega^2 = \xi^2\Omega^2 + \Omega^2(1-\xi^2) = \Omega^2$$



$$\boxed{\begin{aligned}\sigma &= \xi\Omega \\ \omega &= \Omega\sqrt{1-\xi^2}\end{aligned}}$$

$$\boxed{\sigma^2 + \omega^2 = \Omega^2}$$

$W(s) =$

$\frac{1}{s+\alpha} \rightarrow$

$\frac{1}{s-\alpha} \rightarrow$

$w(t$

$$\frac{1\Omega^2}{(s-p_1)(s-p_2)} = \frac{1\Omega^2}{(s\underline{-[-\sigma+j\omega]})(s\underline{-[-\sigma-j\omega]})}$$

$$\underbrace{\frac{1\Omega^2}{(-[-\sigma+j\omega])(-[-\sigma-j\omega])}}_{\frac{1}{(-\sigma+j\omega)(-\sigma-j\omega)}} \underbrace{\frac{1}{\left(1+\frac{s}{-[-\sigma+j\omega]}\right)\left(1+\frac{s}{-[-\sigma-j\omega]}\right)}}_{\boxed{\frac{1}{1+2\xi\tau s+\tau^2 s^2}}}$$

$$\frac{1}{\sigma^2+\omega^2}$$

$$\boxed{\frac{1}{\Omega^2}}$$

$$\boxed{\frac{\Omega^2}{(s-p_1)(s-p_2)}} \overset{!}{=} \frac{1}{1+2\xi\tau s+\tau^2 s^2} \qquad \frac{\frac{1}{T_1T_2}}{\frac{\frac{1}{T^2}}{\Omega^2}}$$

$$\frac{\Omega^2}{\sigma+j\omega)-(-\sigma-j\omega)} = \frac{\Omega^2}{-\sigma+j\omega+\sigma+j\omega} = \underline{\underline{\frac{\Omega^2}{2j\omega}}}$$

$$\frac{\Omega^2}{\sigma-j\omega)-(-\sigma+j\omega)} = \frac{\Omega^2}{-\sigma-j\omega+\sigma-j\omega} = \underline{\underline{\frac{\Omega^2}{-2j\omega}}}$$

$$\underline{\underline{e^{-\sigma t}\sin\omega t}}$$

$j\omega t$

ALAPTAGOK

KÉTTÁROLÓS LENGŐ TAG $0 < \xi < 1$

$$\sqrt{\xi^2-1} = \sqrt{-1(1-\xi^2)} = j\sqrt{1-\xi^2}$$

$$W(s) = \frac{1}{\underbrace{1}_{c} + \underbrace{2\xi\tau s}_{b} + \underbrace{\tau^2 s^2}_{a}}$$

$$p_{1|2} = \frac{-2\xi\tau \pm \sqrt{4\xi^2\tau^2 - 4\tau^2}}{2\tau^2} = \frac{-2\xi\tau \pm 2\tau\sqrt{\xi^2-1}}{2\tau^2} = \frac{-\xi \pm \sqrt{\xi^2-1}}{\tau} \Rightarrow$$

$$p_{1|2} = \frac{-\xi \pm j\sqrt{1-\xi^2}}{\tau} = -\underbrace{\boxed{\xi\Omega}}_{\sigma} \pm j\underbrace{\boxed{\Omega\sqrt{1-\xi^2}}}_{\omega}$$

$$\boxed{p_{1|2} = -\sigma \pm j\omega}$$

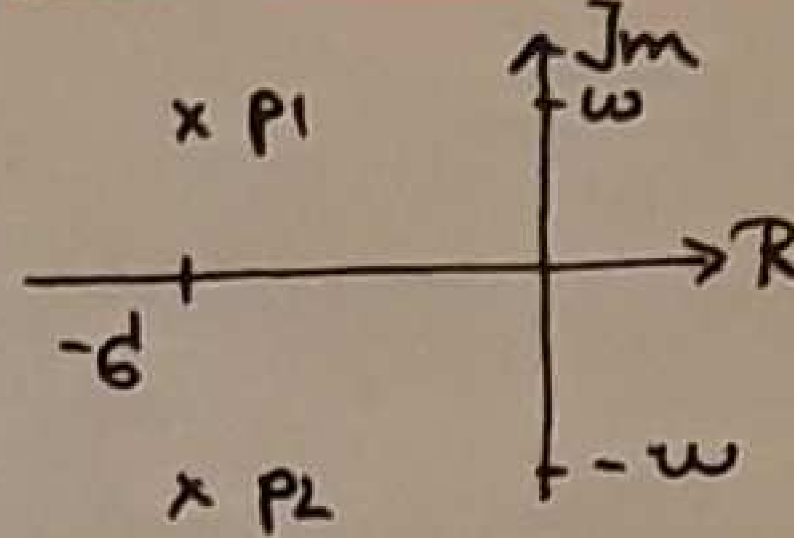
$$\boxed{\Omega = \frac{1}{\tau}}$$

SAJÁT FREKVENCI

$$\boxed{\begin{aligned}\sigma &= \xi\Omega \\ \omega &= \Omega\sqrt{1-\xi^2}\end{aligned}}$$

$$\boxed{\sigma^2 + \omega^2 = \Omega^2}$$

$$\sigma^2 + \omega^2 = \xi^2\Omega^2 + \Omega^2(1-\xi^2) = \Omega^2$$



$$W(s) = \frac{\Omega^2}{(s-p_1)(s-p_2)} = \underbrace{\frac{\Omega^2}{(s-[-\sigma+j\omega])(s-[-\sigma-j\omega])}}_{} =$$

$$= \frac{\frac{\Omega^2}{2j\omega}}{s-[-\sigma+j\omega]}\,{}^{A} + \frac{\frac{\Omega^2}{-2j\omega}}{s-[-\sigma-j\omega]}\,{}^{B=A^x}$$

$$A = \frac{\Omega^2}{(-\sigma+j\omega)-(-\sigma-j\omega)} = \frac{\Omega^2}{-\sigma+j\omega+\sigma+j\omega} = \frac{\Omega^2}{2j\omega}$$

$$B = \frac{\Omega^2}{(-\sigma-j\omega)-(-\sigma+j\omega)} = \frac{\Omega^2}{-\sigma-j\omega+\sigma-j\omega} = \frac{\Omega^2}{-2j\omega}$$

$$\frac{1}{s+\alpha} \to e^{-\alpha t}$$

$$\frac{1}{s-\alpha} \to e^{\alpha t}$$

$$w(t) = 1(t)\left[\frac{\Omega^2}{2j\omega}\underbrace{e^{(-\sigma+j\omega)t}}_{e^{-\sigma t}\cdot e^{j\omega t}} - \frac{\Omega^2}{2j\omega}\underbrace{e^{(-\sigma-j\omega)t}}_{e^{-\sigma t}\cdot e^{-j\omega t}}\right]$$

$$w(t) = 1(t)\frac{\Omega^2}{\omega}\underbrace{e^{-\sigma t}}_{}\underbrace{\sin\omega t}_{}$$

$\neq$

$$= 1(t)\frac{\Omega^2}{\omega}e^{-\sigma t}\left[\frac{e^{j\omega t}}{2j} - \frac{e^{-j\omega t}}{2j}\right] = 1(t)\frac{\Omega^2}{\omega}e^{-\sigma t}\underbrace{\frac{e^{j\omega t} - e^{-j\omega t}}{2j}}_{\sin\omega t}$$

$$V(s) = \frac{\Omega^2}{s\,(s-[-\sigma+j\omega])(s-[-\sigma-j\omega])} = \frac{1}{s}^{A} + \frac{\frac{-\sigma-j\omega}{2j\omega}}{s-[-\sigma+j\omega]}^{B} + \frac{\frac{-\sigma+j\omega}{-2j\omega}}{s-[-\sigma-j\omega]}^{C}$$

$$A = \frac{\Omega^2}{(+[-\sigma+j\omega])(+[-\sigma-j\omega])} = \frac{\Omega^2}{\sigma^2+\omega^2} = \underline{\underline{1}}$$

$$B = \frac{\Omega^2}{(-\sigma+j\omega)(-\sigma+j\omega-[-\sigma-j\omega])} = \frac{\Omega^2}{(-\sigma+j\omega)(-\sigma+j\omega+\sigma+j\omega)} = \frac{\Omega^2}{2j\omega\underbrace{(-\sigma+j\omega)\cdot\frac{-\sigma-j\omega}{-\sigma-j\omega}}} =$$

$$= \frac{\cancel{\Omega^2}(-\sigma-j\omega)}{2j\omega\,\cancel{(\sigma^2+\omega^2)}} = \frac{-\sigma-j\omega}{2j\omega} \qquad \underline{C = B^*} = \frac{-\sigma+j\omega}{-2j\omega}$$

$$v(t) = 1(t)\left[1 + \frac{-\sigma-j\omega}{2j\omega}e^{(-\sigma+j\omega)t} + \frac{-\sigma+j\omega}{-2j\omega}e^{(-\sigma-j\omega)t}\right] = \qquad \cos\omega t = \frac{e^{j\omega t}+e^{-j\omega t}}{2}$$

$$= 1(t)\left[1 - \underbrace{\frac{\sigma}{2j\omega}}e^{(-\sigma+j\omega)t} - \underbrace{\frac{1}{2}}e^{-(\sigma+j\omega)t} + \underbrace{\frac{\sigma}{2j\omega}}e^{(-\sigma-j\omega)t} - \underbrace{\frac{1}{2}}e^{-(\sigma-j\omega)t}\right] = \qquad \sin\omega t = \frac{e^{j\omega t}-e^{-j\omega t}}{2j}$$

$$= 1(t)\left[1 - \frac{\sigma}{\omega}e^{-\sigma t}\underbrace{\frac{e^{j\omega t}-e^{-j\omega t}}{2j}}_{\sin\omega t} - e^{-\sigma t}\underbrace{\frac{e^{j\omega t}+e^{-j\omega t}}{2}}_{\cos\omega t}\right]$$

$$\underline{\underline{v(t) = 1(t)\left[1 - e^{-\sigma t}\left(\frac{\sigma}{\omega}\sin\omega t + \cos\omega t\right)\right]}}$$

A KÉTTÁROLÓS LENGŐ TAG

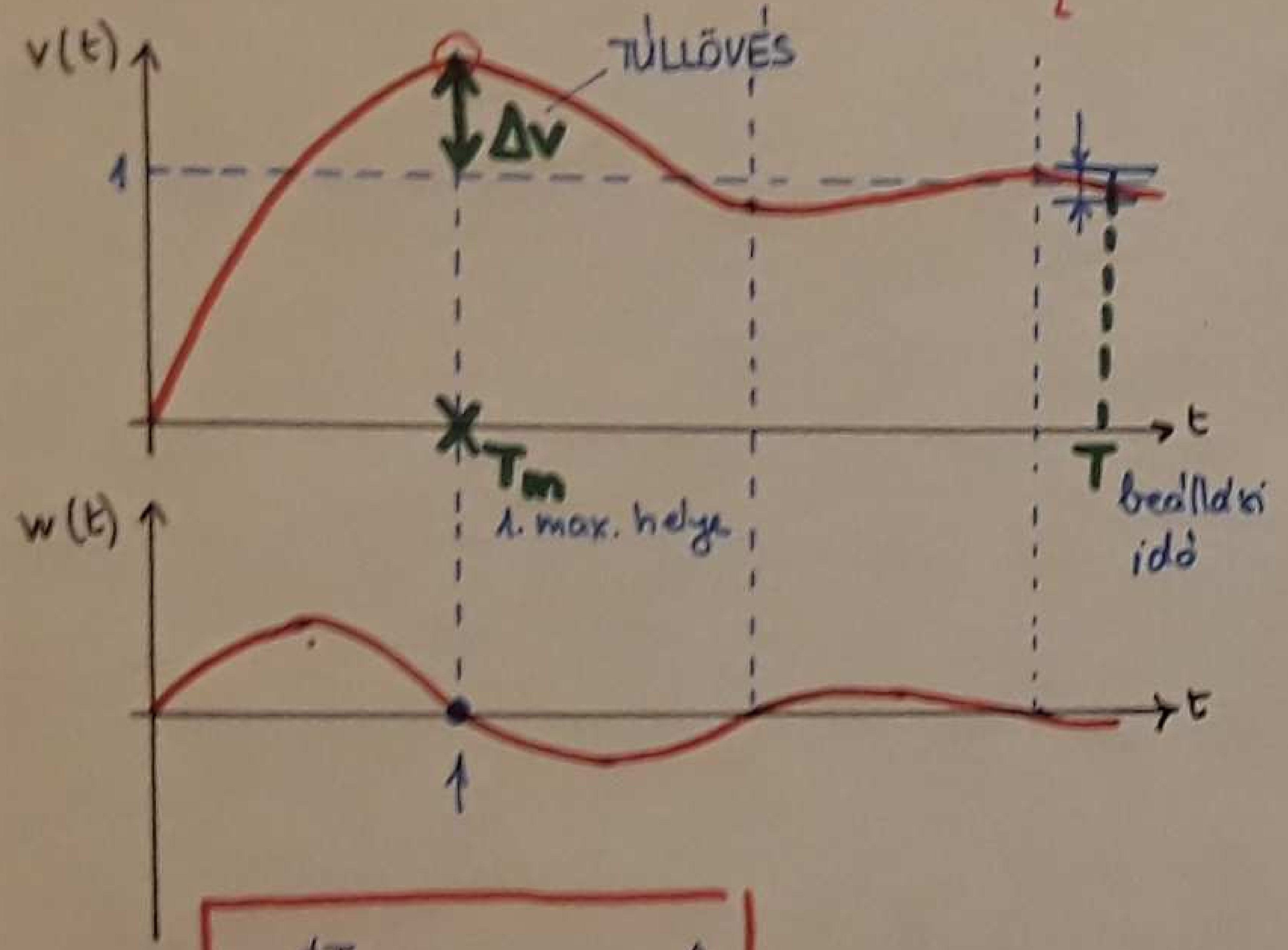
- impulzusválasza : $w(t) = 1(t)\frac{\Omega^2}{\omega}e^{-\sigma t}\sin\omega t$
- ugrásválasza : $v(t) = 1(t)\left[1 - e^{-\sigma t}\left(\frac{\sigma}{\omega}\sin\omega t + \cos\omega t\right)\right]$

$\Omega = \frac{1}{\tau}$

$\sigma = \xi\Omega$

$\omega = \Omega\sqrt{1-\xi^2}$

$\sigma^2 + \omega^2 = \Omega^2$



$$\boxed{e^{-\sigma T} = \Delta\% = \frac{\Delta}{100}} \longrightarrow T$$

5%

$e^{-\sigma T} = 0{,}05 \quad / \ln$

$-\sigma T = \ln 0{,}05 = -3$

$T = \frac{3}{\sigma} \quad (5\%)$

$w(t) = v'(t)$

$\sin\omega t = \emptyset$

$\omega t = \pi$

$\Omega\sqrt{1-\xi^2}\, t = \pi$ itt:

$\Omega\sqrt{1-\xi^2}\, T_m = \pi \longrightarrow \boxed{T_m = \frac{\pi}{\Omega\sqrt{1-\xi^2}}}$

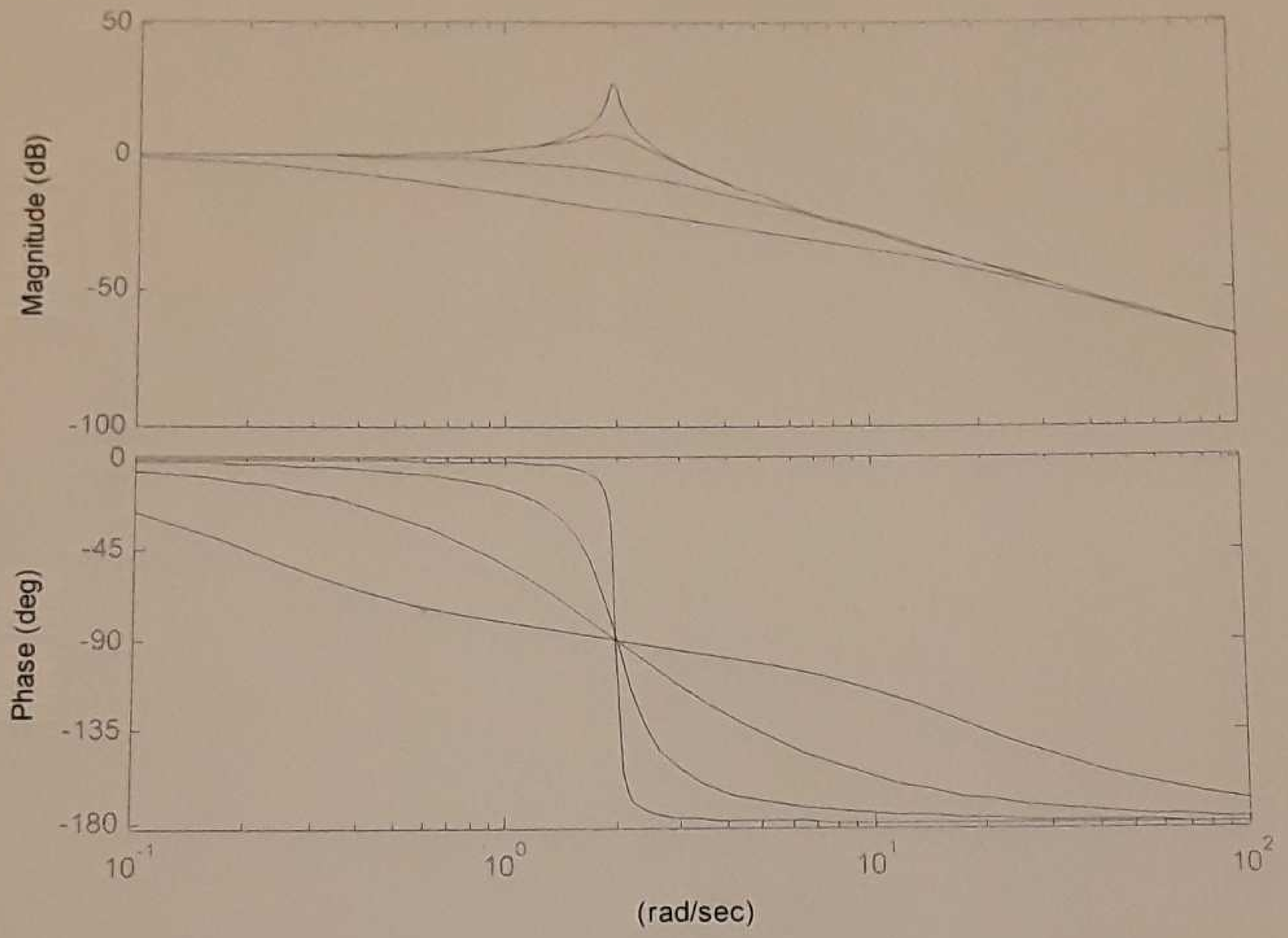
$$\Delta v = v(T_m) - 1$$

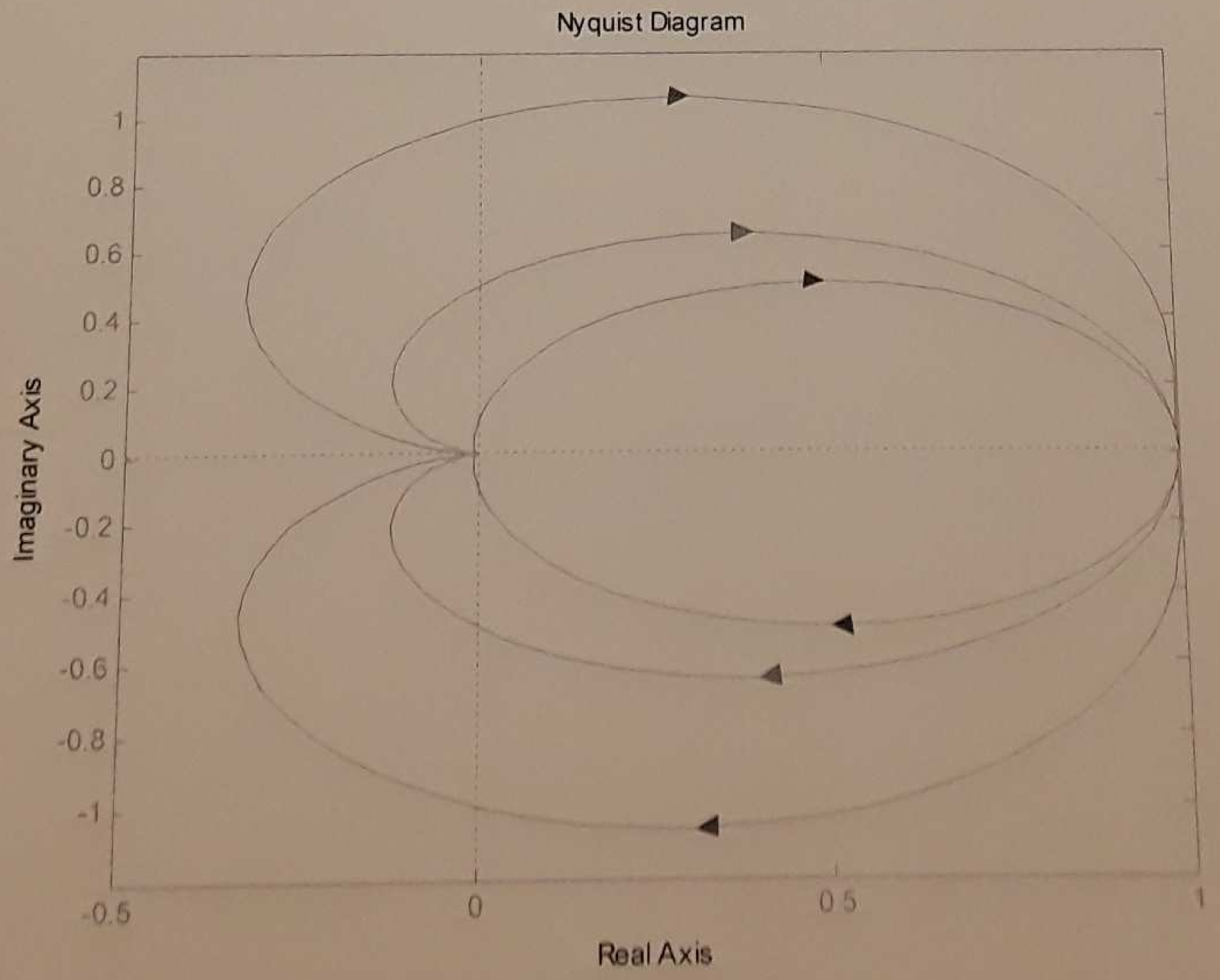
$$= 1 - e^{-\sigma T_m}\left(\frac{\sigma}{\omega}\underbrace{\sin\overbrace{\omega T_m}^{\pi}}_{\emptyset} + \underbrace{\cos\overbrace{\omega T_m}^{\pi}}_{-1}\right) - 1$$

$$= e^{-\sigma T_m}$$

$$\Delta v = e^{-\xi\Omega\frac{\pi}{\Omega\sqrt{1-\xi^2}}}$$

$$\boxed{\Delta v = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}$$


50
0
-50
-100
Magnitude (dB)
0
-45
-90
-135
-180
Phase (deg)
10^{-1}
10^{0}
10^{1}
10^{2}
(rad/sec)


Nyquist Diagram
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Imaginary Axis
-0.5
0
0.5
1
Real Axis

KÉTTÁROLÓS LENGŐ TAG REZONANCIÁJA

$0 < \xi < 1$

$$W(s) = \frac{1}{1 + 2\xi T s + T^2 s^2}$$

$$\Omega = \frac{1}{\tau}$$

$|W(j\omega| > 1$

$$W(j\omega) = \frac{1}{1 + 2\xi\tau j\omega - \tau^2\omega^2} = \frac{1}{(1-\tau^2\omega^2) + j2\xi\tau\omega}$$

$$|W(j\omega)| = \frac{1}{\sqrt{(1-\tau^2\underline{\omega}^2)^2 + 4\xi^2\tau^2\underline{\omega}^2}} \;\textcircled{>}\; 1$$

$$(1-\tau^2\omega^2)^2 + 4\xi^2\tau^2\omega^2 < 1$$

$$\cancel{1} \underline{- 2\tau^2\omega^2} + \tau^4\omega^4 + \underline{4\xi^2\tau^2\omega^2} < \cancel{1}$$

$$\underline{\tau^4\omega^4 + \tau^2\omega^2(4\xi^2 - 2) < 0}$$

Hol a minimum?

$$4\tau^4\omega^3 + 2\tau^2\omega(4\xi^2 - 2) = 0 \qquad /\tau^2\omega$$

$$4\tau^4\omega^3 + 4\tau^2\omega(2\xi^2 - 1) = 0 \qquad /:4\tau^2\omega$$

$$\tau^2\omega^2 + (2\xi^2 - 1) = 0$$

$$\omega^2 = \frac{1-2\xi^2}{\tau^2} \Rightarrow \boxed{\omega_{res} = \frac{\sqrt{1-2\xi^2}}{\tau}}$$

$$1 - 2\xi^2 > 0$$

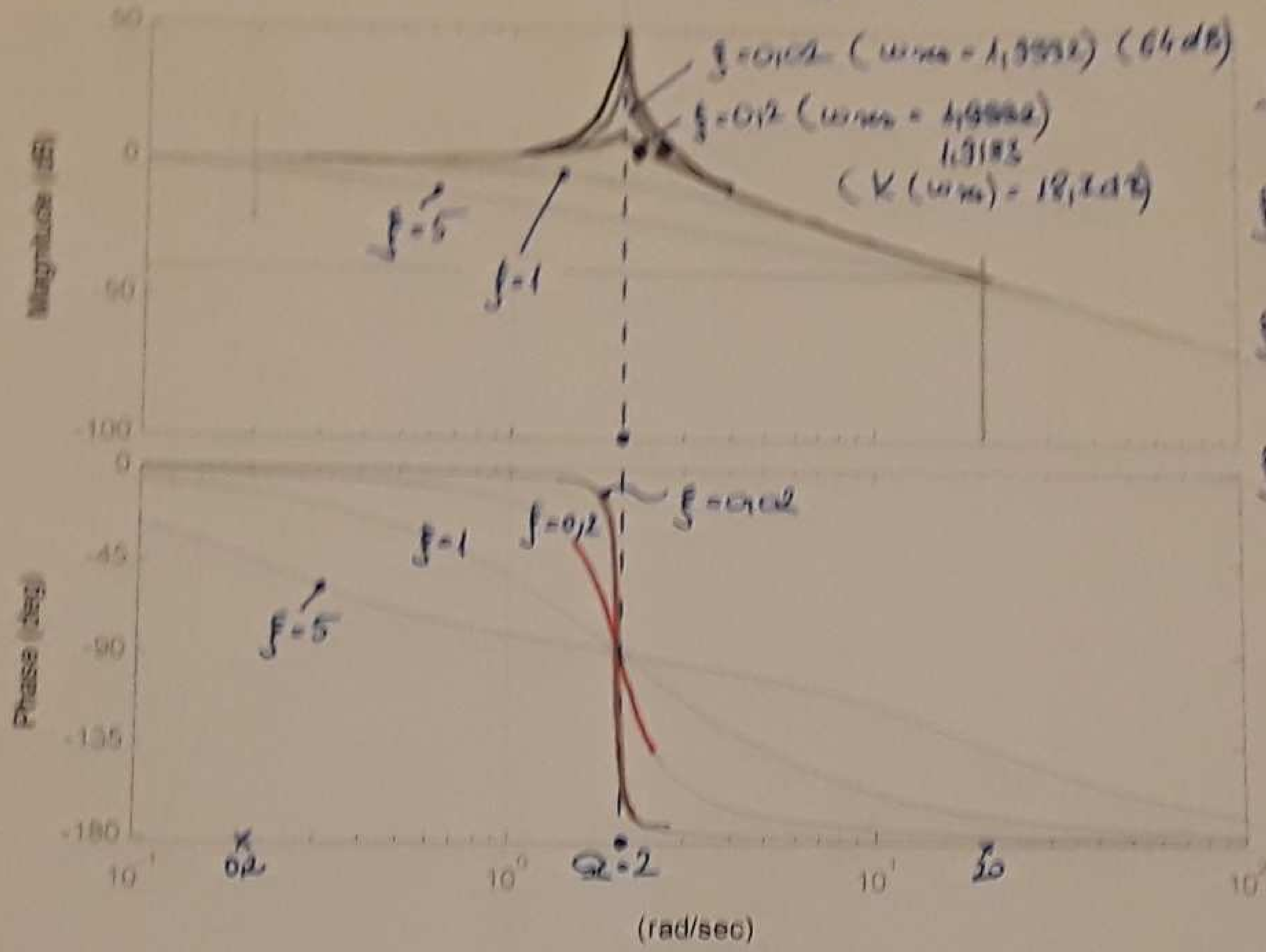
$$1 > 2\xi^2$$

$$\boxed{\xi < \frac{1}{\sqrt{2}}}$$

$$|W(j\omega_{res})| = \frac{1}{\sqrt{\underbrace{\underbrace{\left(1 - \cancel{\tau^2}\cdot\frac{1-2\xi^2}{\cancel{\tau^2}}\right)^2}_{1-1+2\xi^2}}_{4\xi^4} + 4\xi^2\cancel{\tau^2}\cdot\frac{1-2\xi^2}{\cancel{\tau^2}}}} = \frac{1}{\sqrt{4\xi^4 + 4\xi^2 - 8\xi^4}} = \frac{1}{\sqrt{4\xi^2 - 4\xi^4}} = \boxed{\frac{1}{2\xi\sqrt{1-\xi^2}}}$$

$$W(s) = \frac{1}{1 + 2\xi T s + T^2 s^2} \qquad \tau = 0{,}5,\ \Omega = \frac{1}{\tau} = 2$$

$$\omega_{rez} = \frac{\sqrt{1-2\xi^2}}{\tau} \qquad K(\omega_{rez}) = \frac{1}{2\xi\sqrt{1-\xi^2}} \quad (*)$$

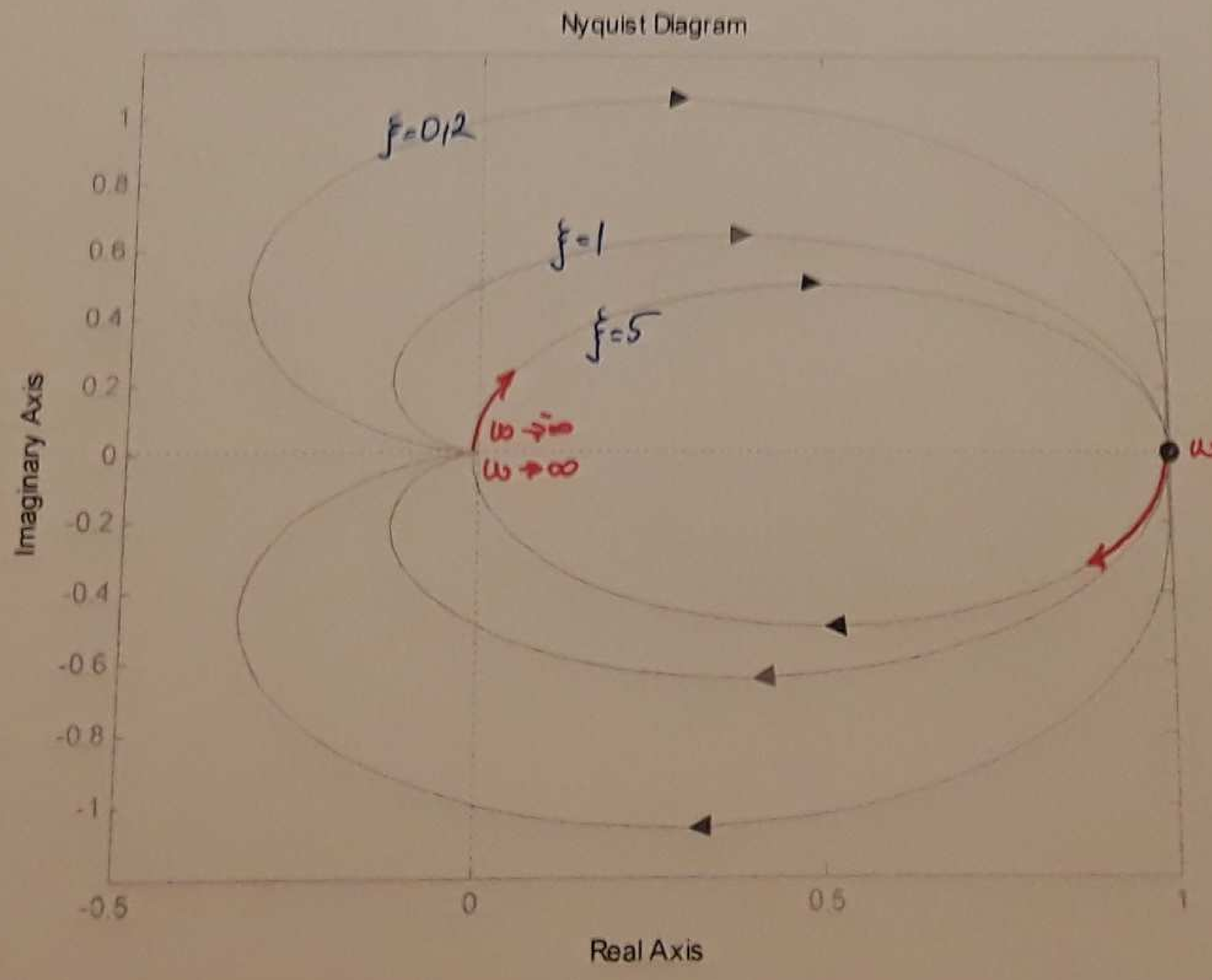


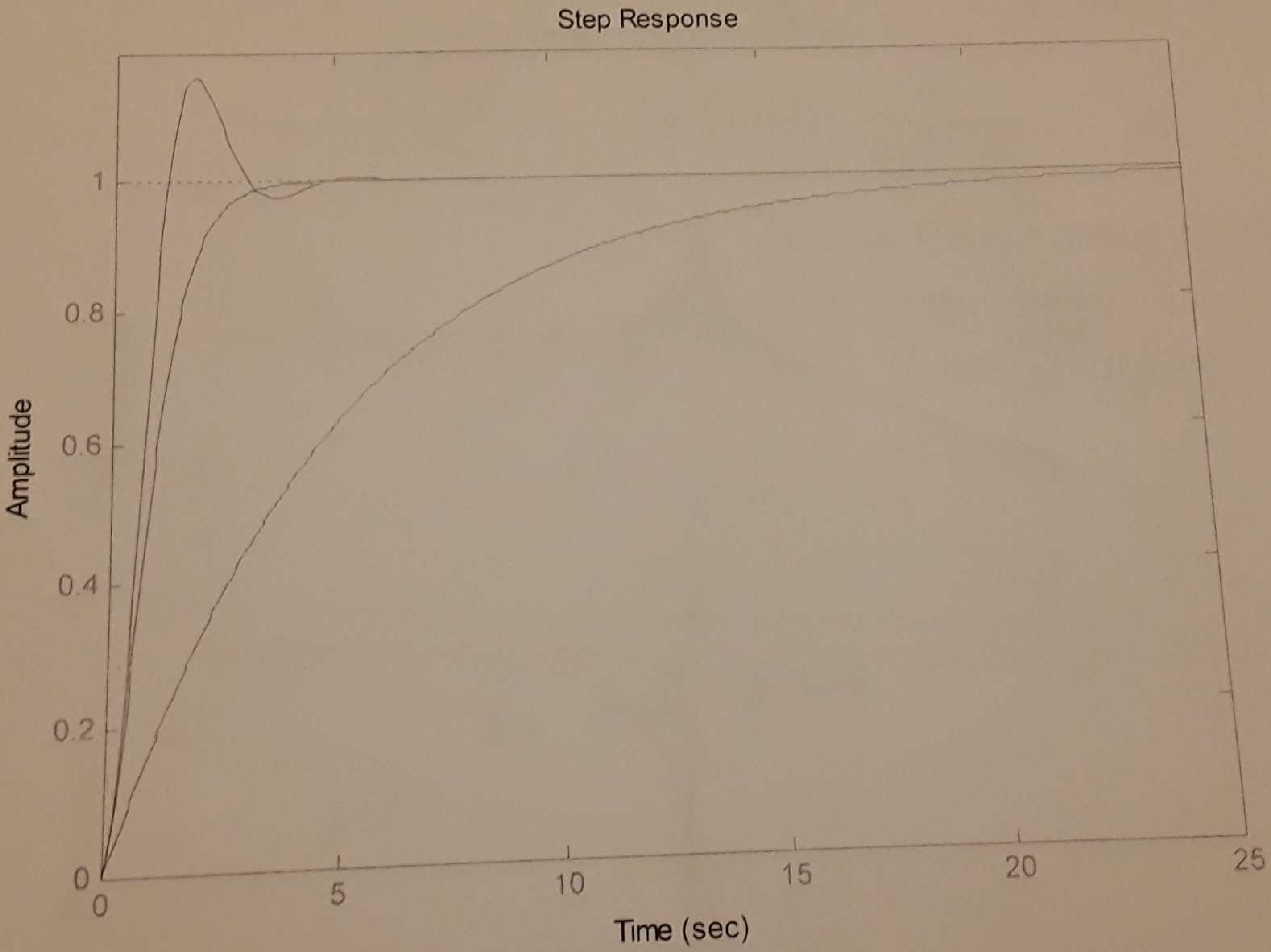
ξ = 5 → 0,2 ve 20
2 kırılma noktası

ξ = 1 → 2 katlı kök
kırılma noktası

ξ = 0,2 $\quad p_{1,2} = -\xi\Omega \pm j\Omega\sqrt{1-\xi^2}$
$= -0{,}4 \pm j\,1{,}96$

ξ = 0,02 $\quad p_{1,2} = -0{,}04 \pm j\,1{,}99$

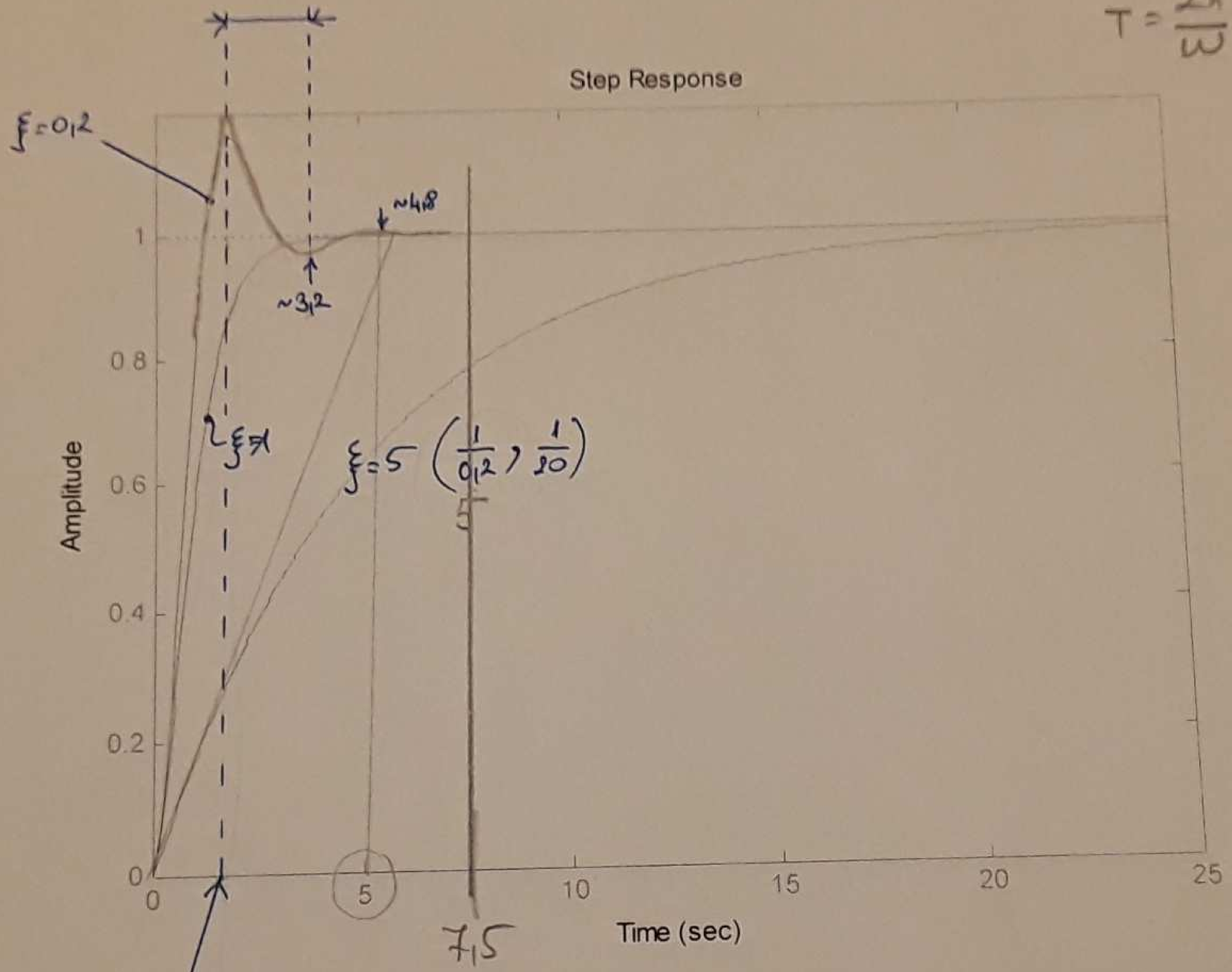



Step Response
Amplitude
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
Time (sec)

④ $\omega = \frac{2\pi}{T} = \Omega\sqrt{1-\xi^2} = 1{,}96 \rightarrow T = 3{,}2$

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$



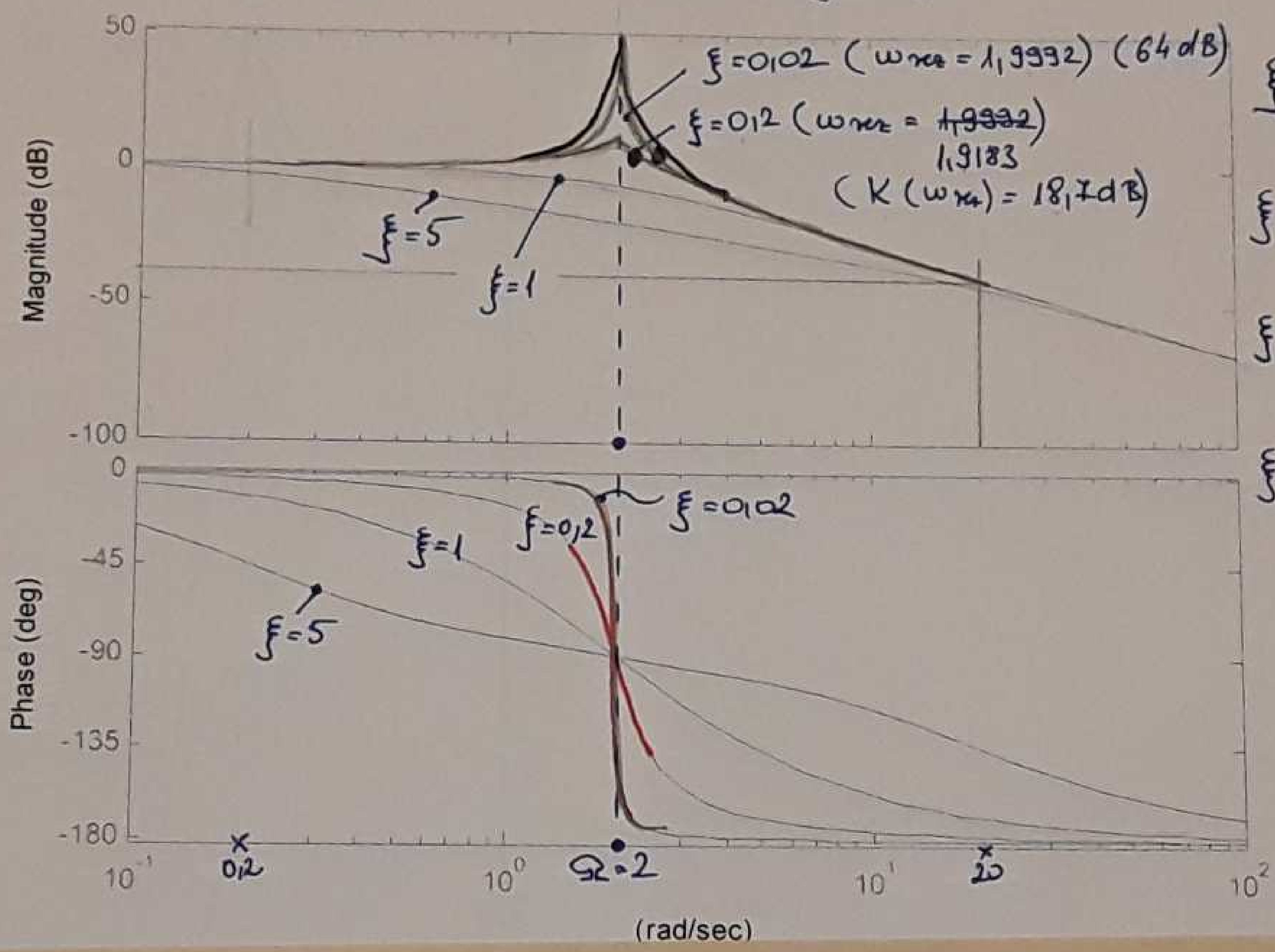
① $T_m = \frac{\pi}{\Omega\sqrt{1-\xi^2}} = 1{,}6$

② $\Delta v = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = 0{,}52$

③ $\sigma = 0{,}4 \rightarrow T = \frac{3}{\sigma} = 7{,}5$
$(\xi = 0{,}2)$

$$W(s)=\frac{1}{1+2\xi Ts+T^2s^2} \qquad \tau=0{,}5,\ \Omega=\frac{1}{\tau}=2$$

$$\omega_{rez}=\frac{\sqrt{1-2\xi^2}}{\tau} \qquad K(\omega_{rez})=\frac{1}{2\xi\sqrt{1-\xi^2}} \qquad (*)$$



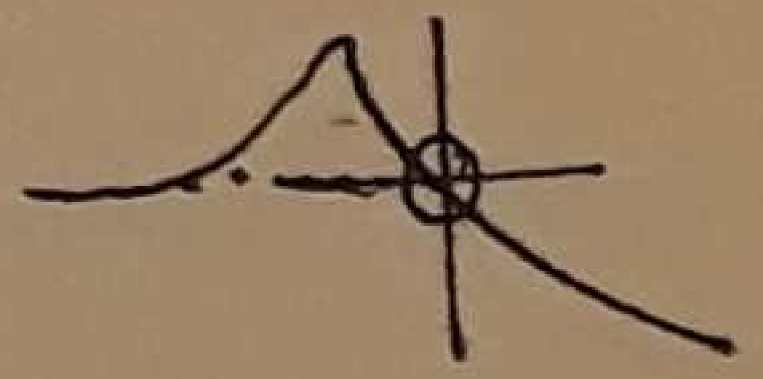
$\xi=5 \to \sim 0{,}2 \quad \sim 20$
2 töréspont

$\xi=1 \to$ 2 helyen 2db
töréspont

$\xi=0{,}2 \quad p_{12}=-\xi\Omega\pm j\Omega\sqrt{1-\xi^2}$
$=-0{,}4\pm j\,1{,}96$

$\xi=0{,}02 \quad p_{12}=-0{,}04\pm j\,1{,}99$

(*) AHOL METSZI A VÍZSZINTES TENGELYT:



$$W(j\omega)=\frac{1}{1+2\xi\tau j\omega+\tau^2(j\omega)^2}=$$

$$\left|\frac{1}{(1-\tau^2\omega^2)+j2\xi\tau\omega}\right|=1 \qquad \lg 1=0\,\text{dB}$$

$$(1-\tau^2\omega^2)^2+4\xi^2\tau^2\omega^2=1$$

$$\cancel{1}-2\underline{\tau^2\omega^2}+\underline{\tau^4\omega^4}+4\xi^2\underline{\tau^2\omega^2}=\cancel{1} \qquad /:\tau^2\omega^2$$

$$-2+\tau^2\omega^2+4\xi^2=0$$

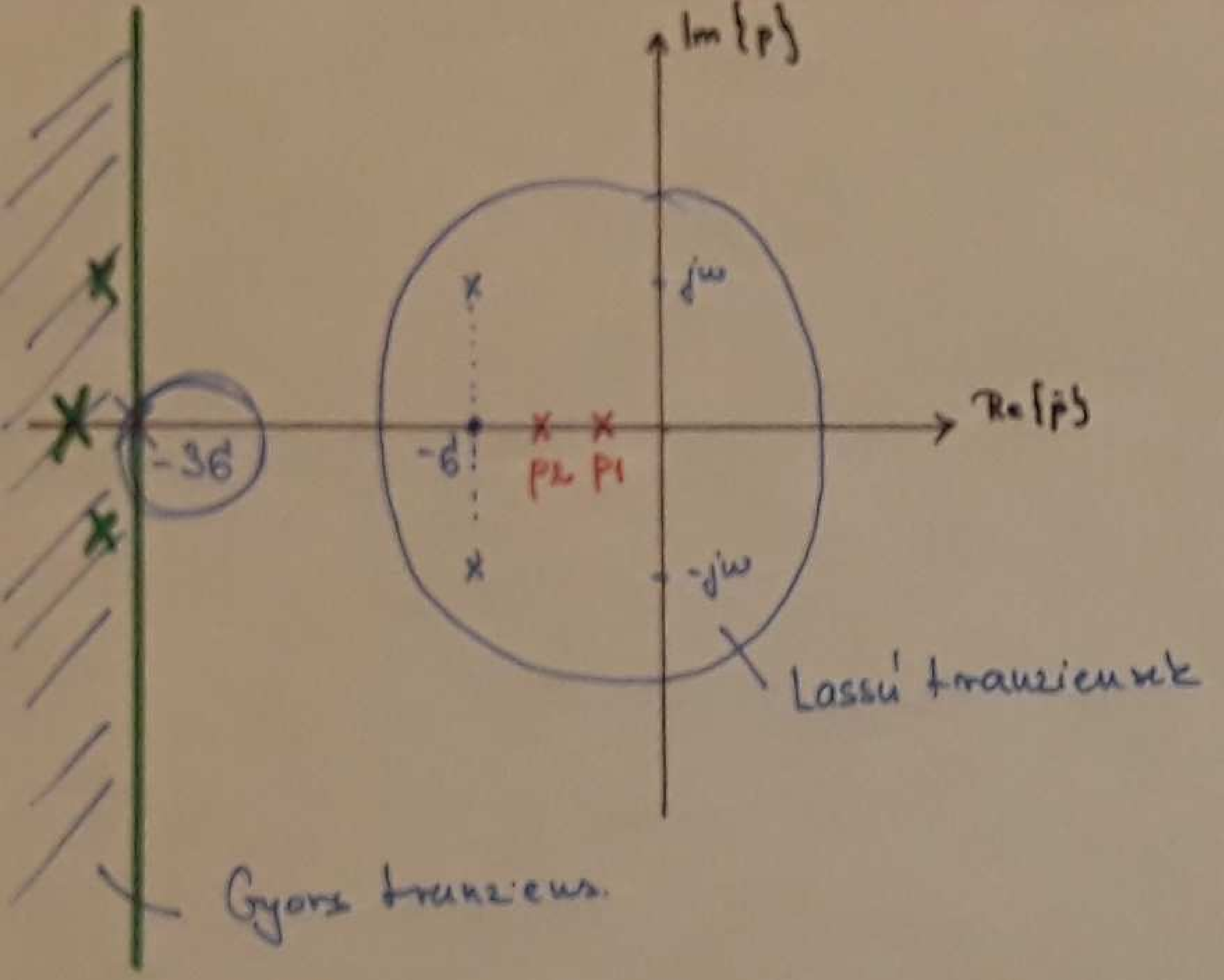
$$\tau^2\omega^2=2-4\xi^2$$

$$\omega=\sqrt{\frac{2-\cancel{4}\xi^2}{\tau^2}}=\underline{\underline{\frac{\sqrt{2(1-2\xi^2)}}{\tau}}}$$

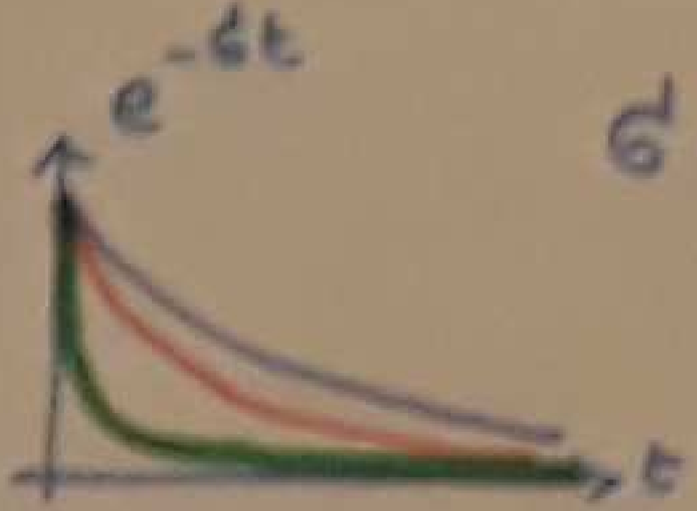
$\xi=0{,}2 \quad \omega=2{,}71$

$\xi=0{,}02 \quad \omega=2{,}82$

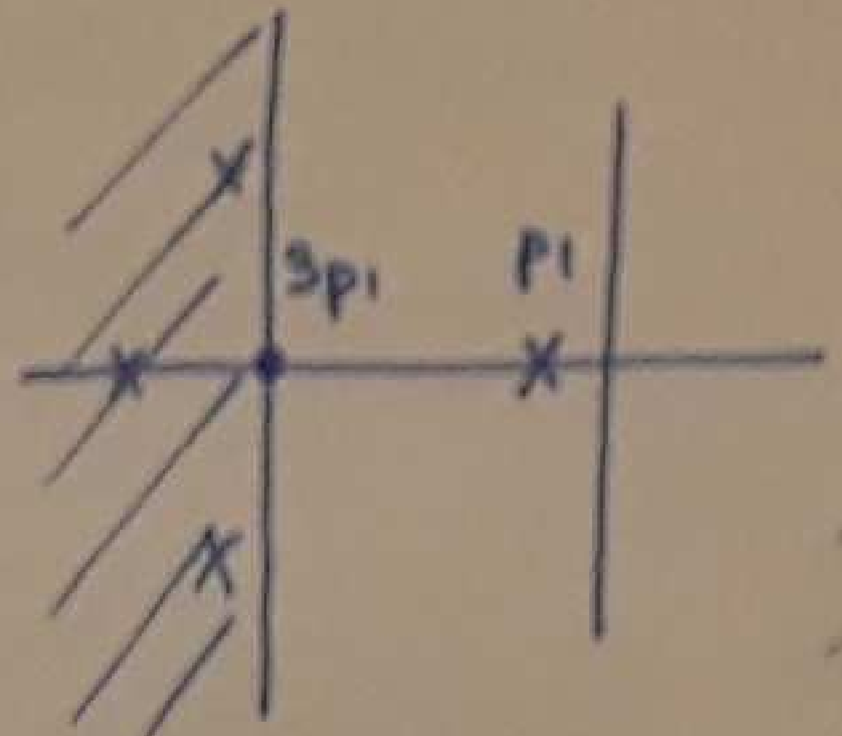
A DOMINÁNS PÓLUS(OK) / PÓLUSPÁR

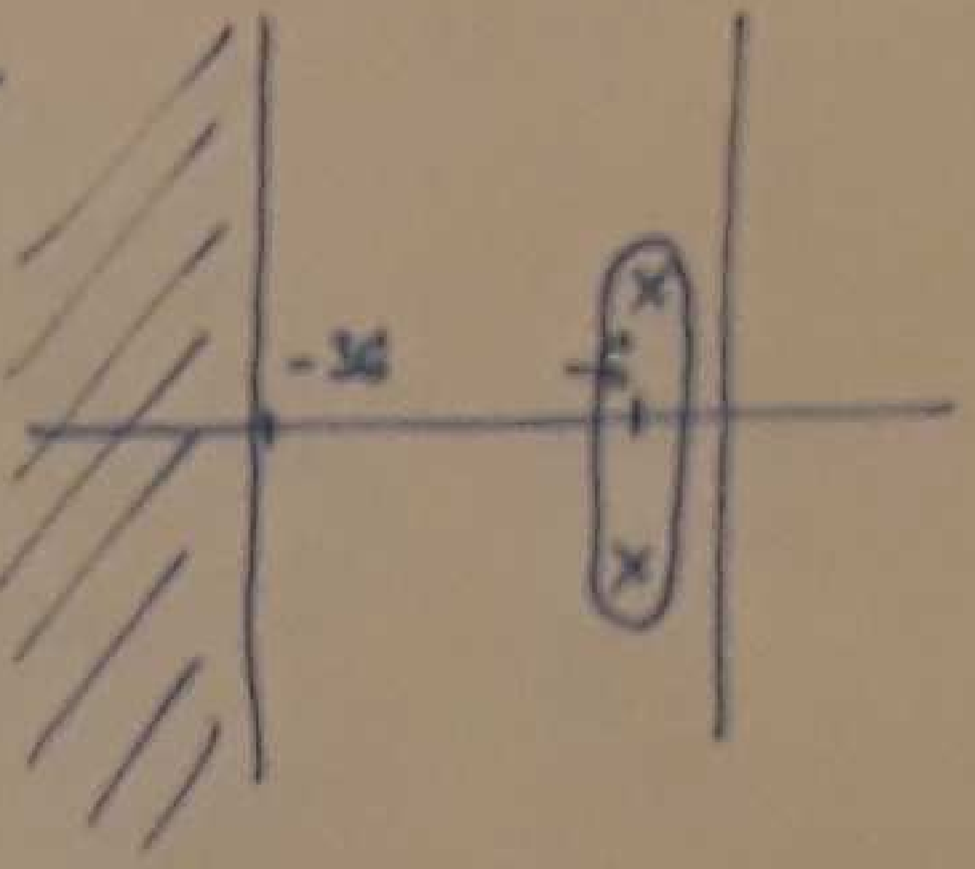


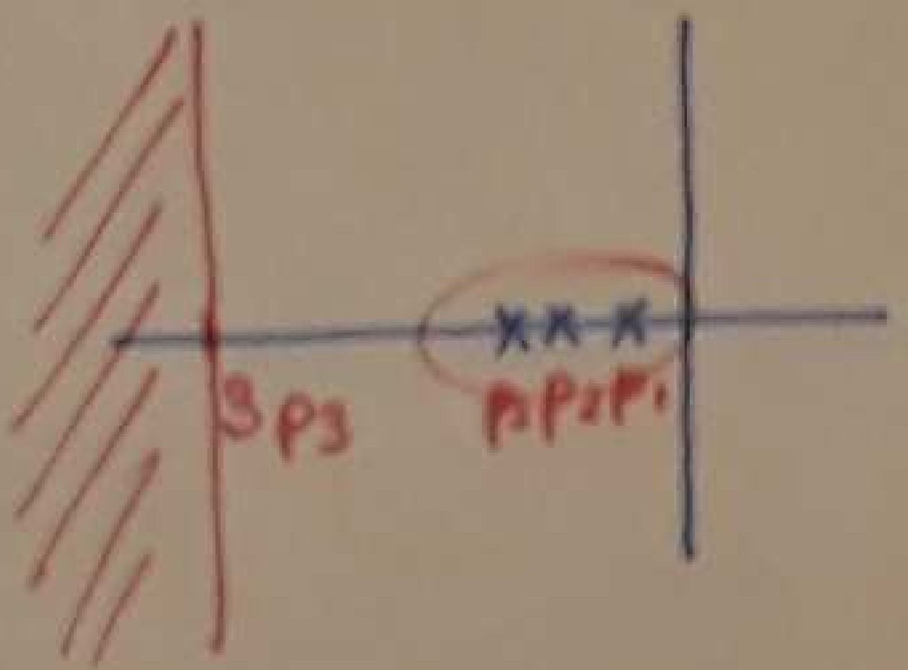
$e^{-\sigma t}$ ↑ ......



$\sigma < \sigma < \sigma$

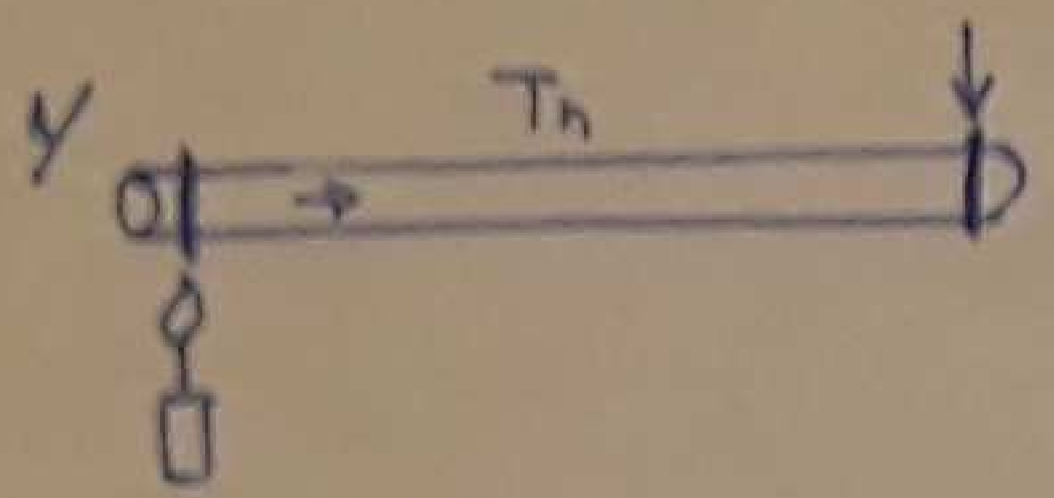




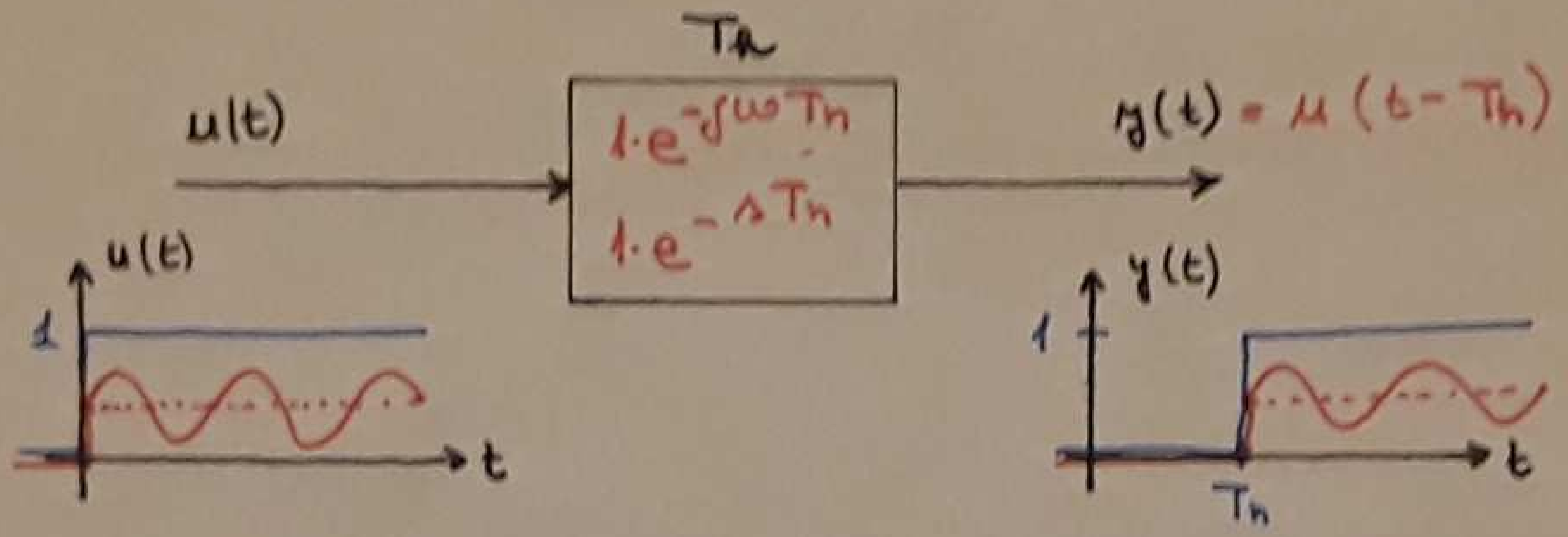


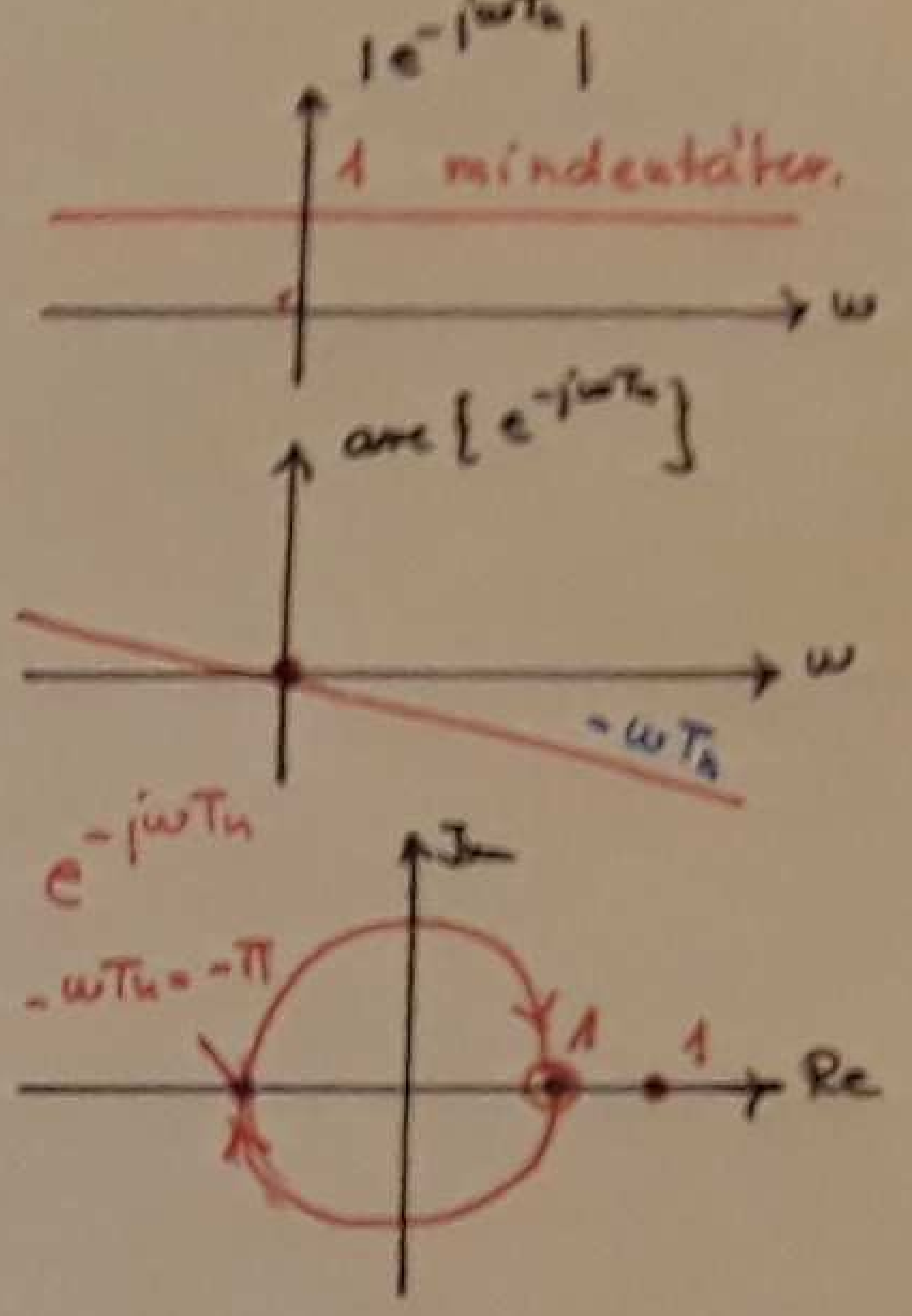
ALAPTAGOK

HOLTIDŐ



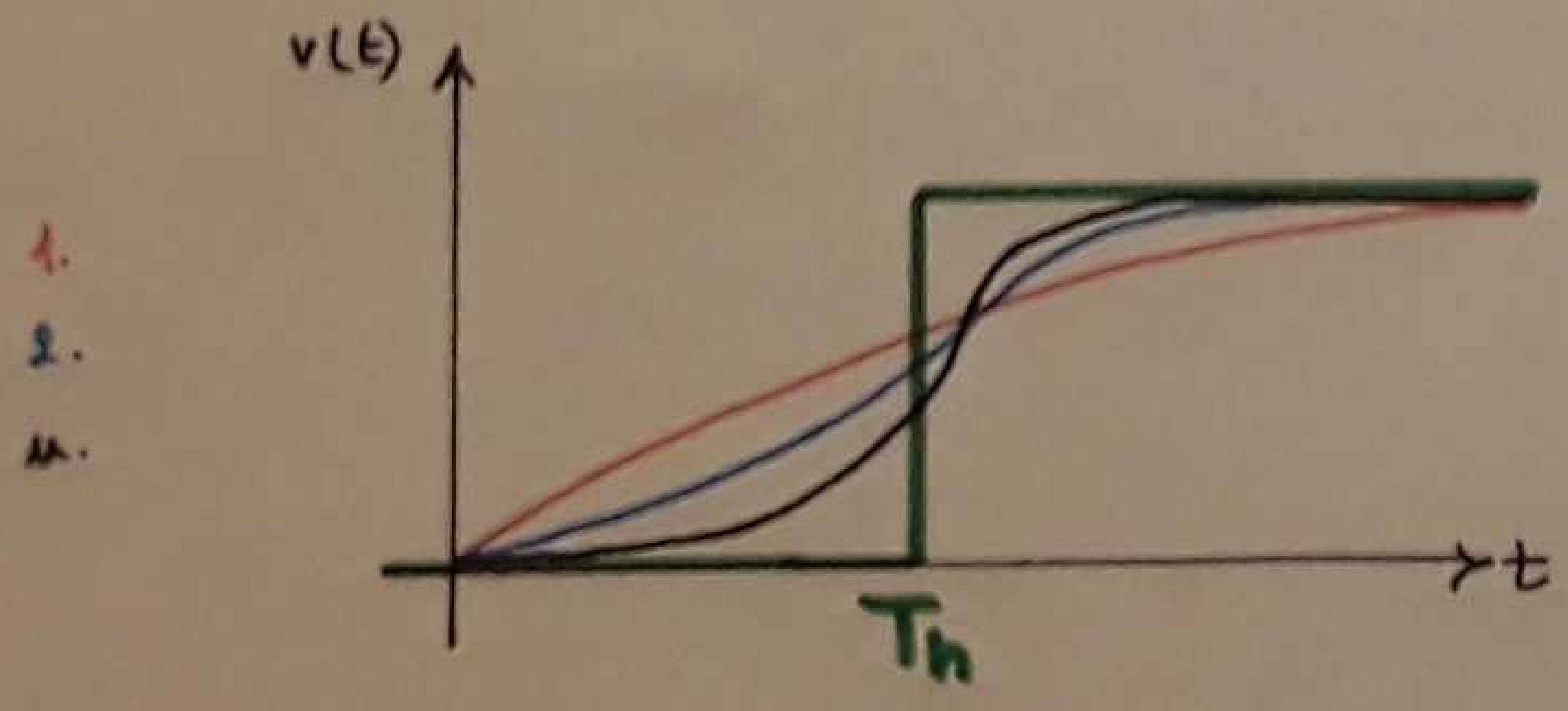
2/ közelítés





STREJC- KÖZELÍTÉS

$$e^{-\overset{sT_n}{x}} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{x}{n}\right)^n} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{T_n}{n}s\right)^n} \approx \frac{1}{\left(1+s\frac{T_n}{n}\right)^n}$$



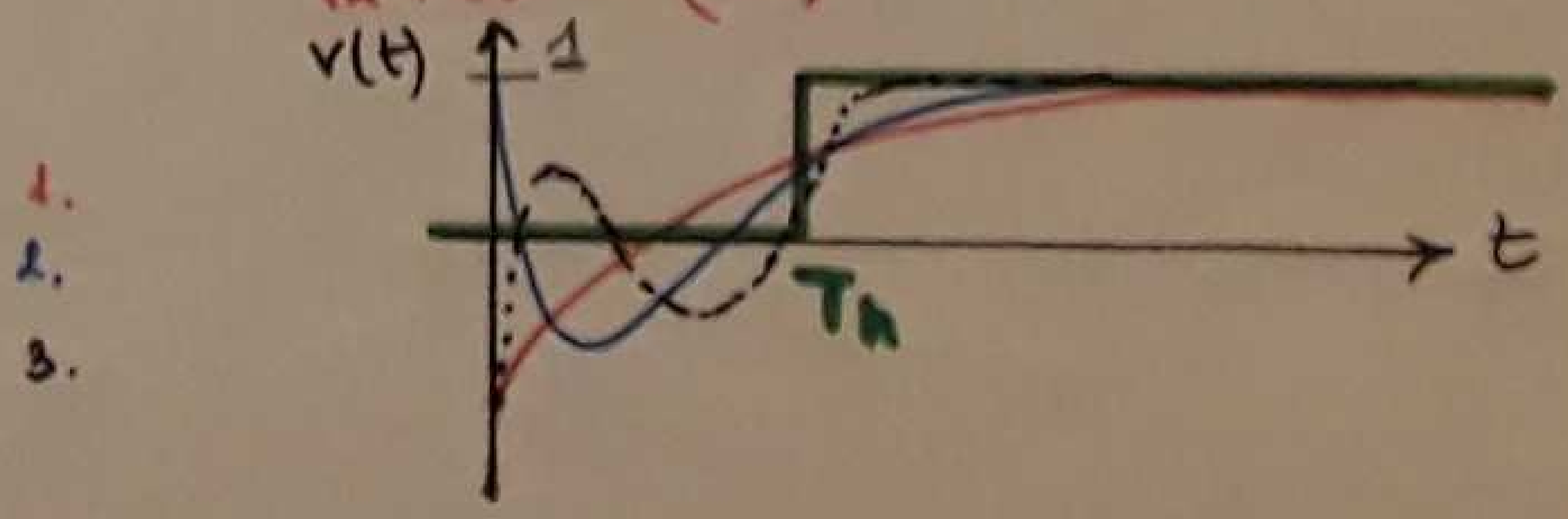
PADÉ- KÖZELÍTÉS

$$e^{-sT_n} \approx \frac{(s-s_1)(s-s_2)\dots(s-s_n)}{(s+s_1)(s+s_2)\dots(s+s_n)}$$ (Taylor-sorok)

$$e^{-sT_n} \approx \frac{2-sT_n}{2+sT_n}$$

$$e^{-sT_n} \approx \frac{120-60sT_n+12(sT_n)^2-(sT_n)^3}{120+60sT_n+12(sT_n)^2+(sT_n)^3}$$

$$e^{-sT_n} \approx \frac{12-6sT_n+(sT_n)^2}{12+6sT_n+(sT_n)^2}$$



A DINAMIKUS MINŐSÉGI JELLEMZŐK ÉS A DOMINÁNS PÓLUSPÁR KAPCSOLATA

$p_{1,2} = -\sigma \pm j\omega = -\xi\Omega \pm j\Omega\sqrt{1-\xi^2}$ $\quad (1+2\xi\tau s+(\tau s)^2$

$v(t) = 1(t)\left[1 - e^{-\sigma t}\left(\frac{\sigma}{\omega}\sin\omega t + \cos\omega t\right)\right]$

Szokásos előírások:

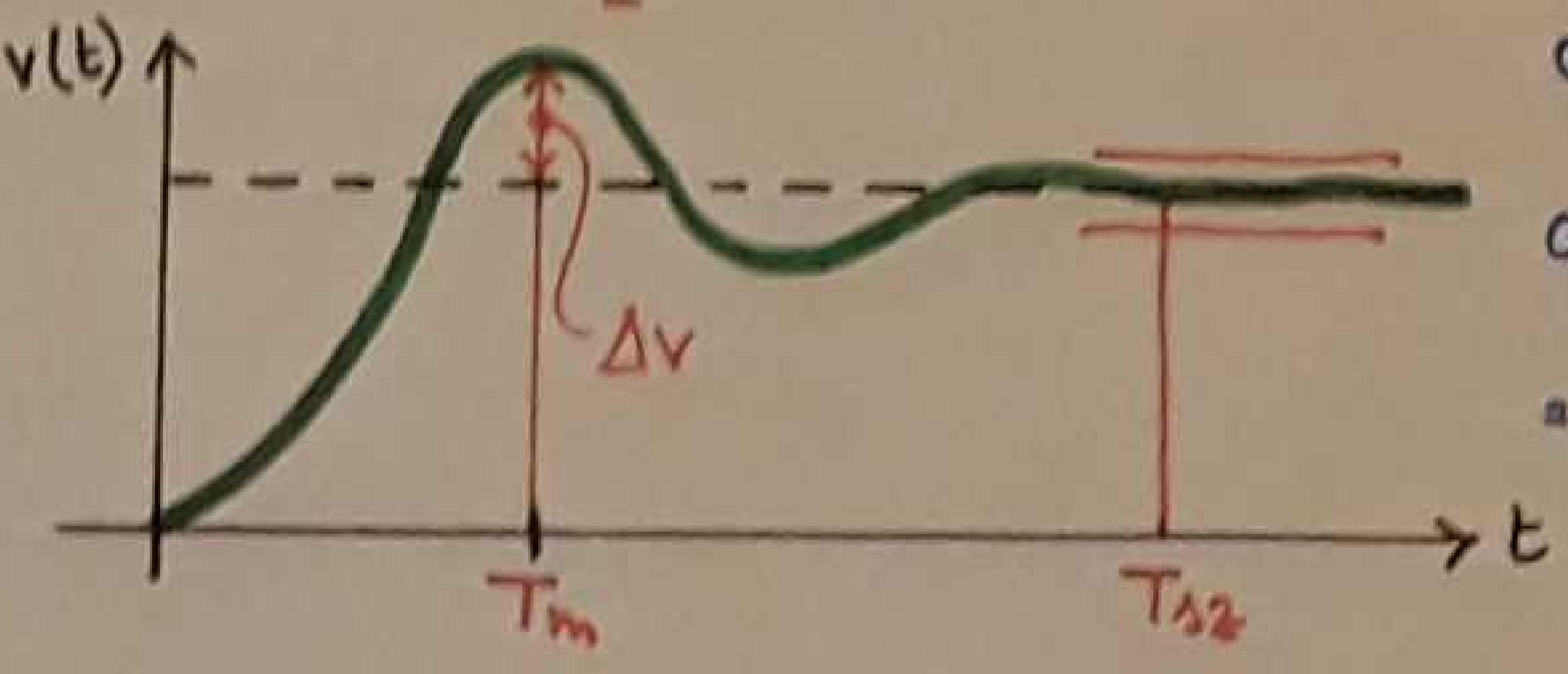
1.) maximális túllövés : $\Delta v_{max} \rightarrow \xi_{min}$

2.) maximális szabályozási idő : $T_{sz,max} \rightarrow \sigma_{min}$

3.) maximális belengési idő : $T_{m,max} \rightarrow \omega_{min}$

4.) maximális nagyfrekvenciás lengés : ω_{max}

↳ milyen domináns póluspárt válasszunk ?

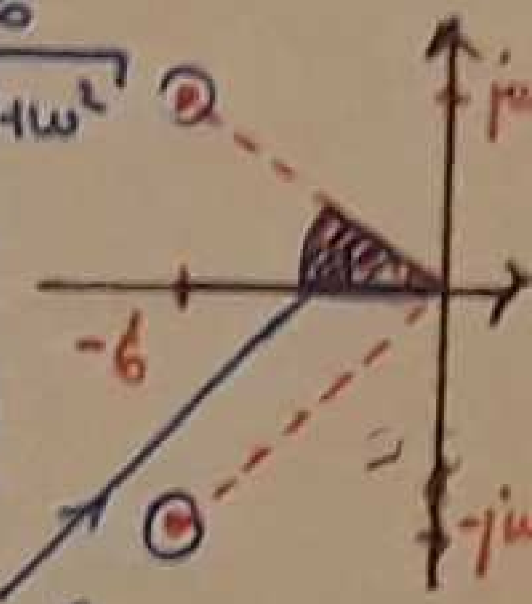


$\cos\varphi = \frac{\sigma}{\sqrt{\sigma^2+\omega^2}}$

$\cos\varphi = \frac{\sigma}{\Omega}$

$= \frac{\xi\Omega}{\Omega} = \xi$

$\cos\varphi = \xi \ldots$



$$\Delta v = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\ln\Delta v = -\frac{\xi\pi}{\sqrt{1-\xi^2}}$$

$$\ln^2\Delta v = +\frac{\xi^2\pi^2}{1-\xi^2}$$

$$\ln^2\Delta v - \xi^2\ln^2\Delta v = \xi^2\pi^2$$

$$\ln^2\Delta v = \xi^2(\pi^2+\ln^2\Delta v)$$

$$\xi^2 = \frac{\ln^2\Delta v}{\pi^2+\ln^2\Delta v} = \frac{1}{1+\frac{\pi^2}{\ln^2\Delta v}}$$

$$\xi_{min} = \frac{1}{\sqrt{1+\pi^2/\ln^2\Delta v_{max}}}$$

$$e^{-\sigma T_{sz}} = \frac{\Delta}{100} \quad [\Delta] = \%$$

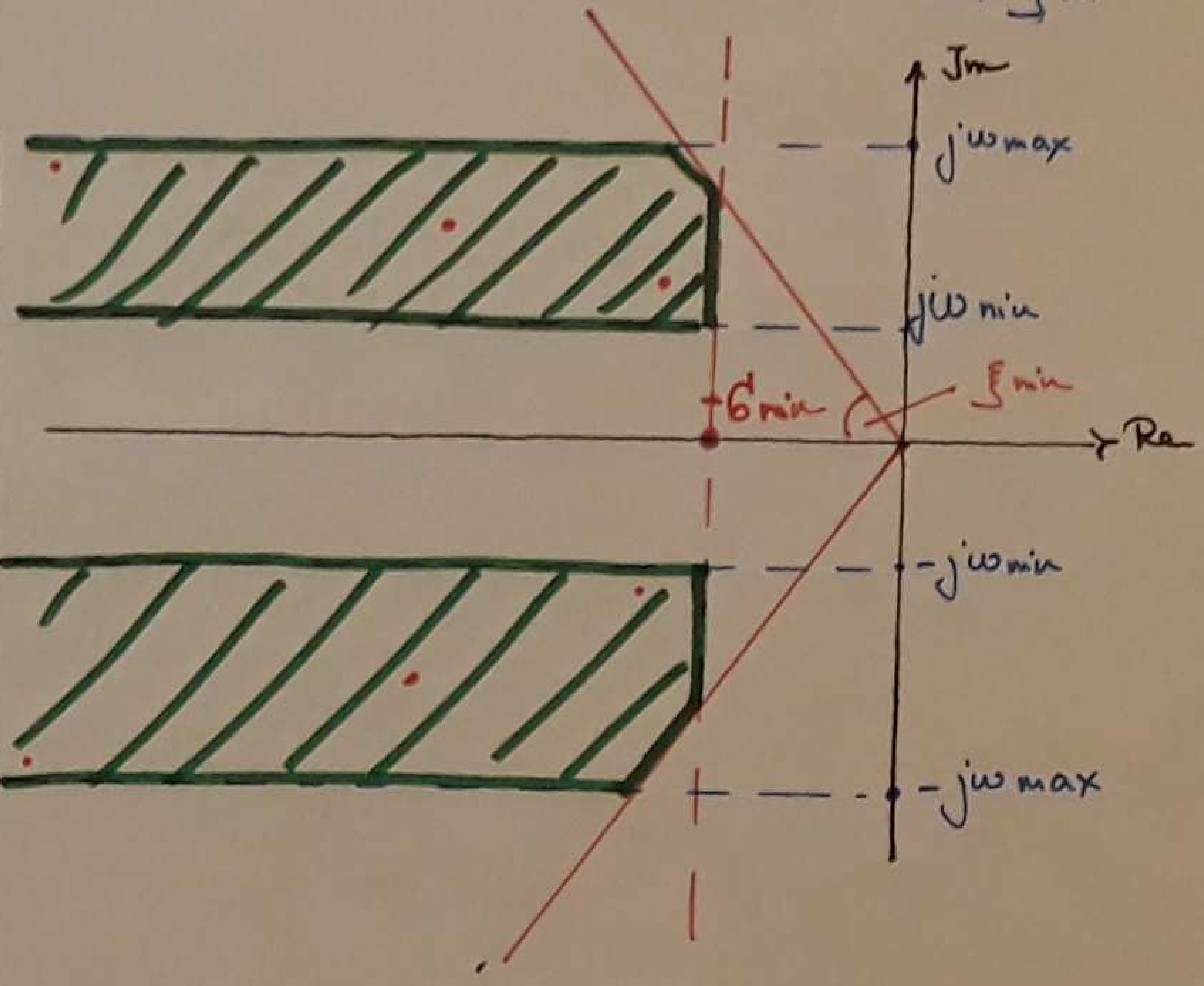
$$-\sigma T_{sz} = \ln\frac{\Delta}{100}$$

$$\sigma = -\frac{\ln\frac{\Delta}{100}}{T_{sz}}$$

$$\sigma_{min} = -\frac{\ln\frac{\Delta}{100}}{T_{sz,max}}$$

$$T_m = \frac{\pi}{\Omega\sqrt{1-\xi^2}} = \frac{\pi}{\omega}$$

$$\omega_{min} = \frac{\pi}{T_{m,max}}$$



A zárt rendszer domináns póluspárját az alábbi követelményekhez kell igazítani:

$\Delta v \leq 5\%$

$T_{s2} \leq 2\,s \quad (\Delta = 2\%)$

Javasoljunk domináns póluspárt! $p_{1|2} = -\sigma \pm j\omega = -\xi\Omega \pm j\Omega\sqrt{1-\xi^2}$

$$\xi_{min} = \frac{1}{\sqrt{1 + \pi^2/\ln^2 \Delta v_{max}}} = \frac{1}{\sqrt{1+\pi^2/\ln^2 0{,}05}} = 0{,}69 \approx \underline{\underline{0{,}7}} \qquad \boxed{\xi = 0{,}9} \Rightarrow \sigma = \xi\Omega \rightarrow \Omega = \frac{\sigma}{\xi} = \frac{5}{0{,}9} = \underline{\underline{5{,}56}}$$

$$\sigma_{min} = -\frac{\ln\frac{\Delta}{100}}{T_{s2,max}} = -\frac{\ln 0{,}02}{2} = 1{,}95 \approx \underline{\underline{2}} \qquad \boxed{\sigma = 5}$$

$$\omega = \Omega\sqrt{1-\xi^2} = 5{,}56\sqrt{1-0{,}9^2} = \underline{\underline{2{,}42}}$$

$$\boxed{p_{1|2} = -5 \pm j\,2{,}42}$$

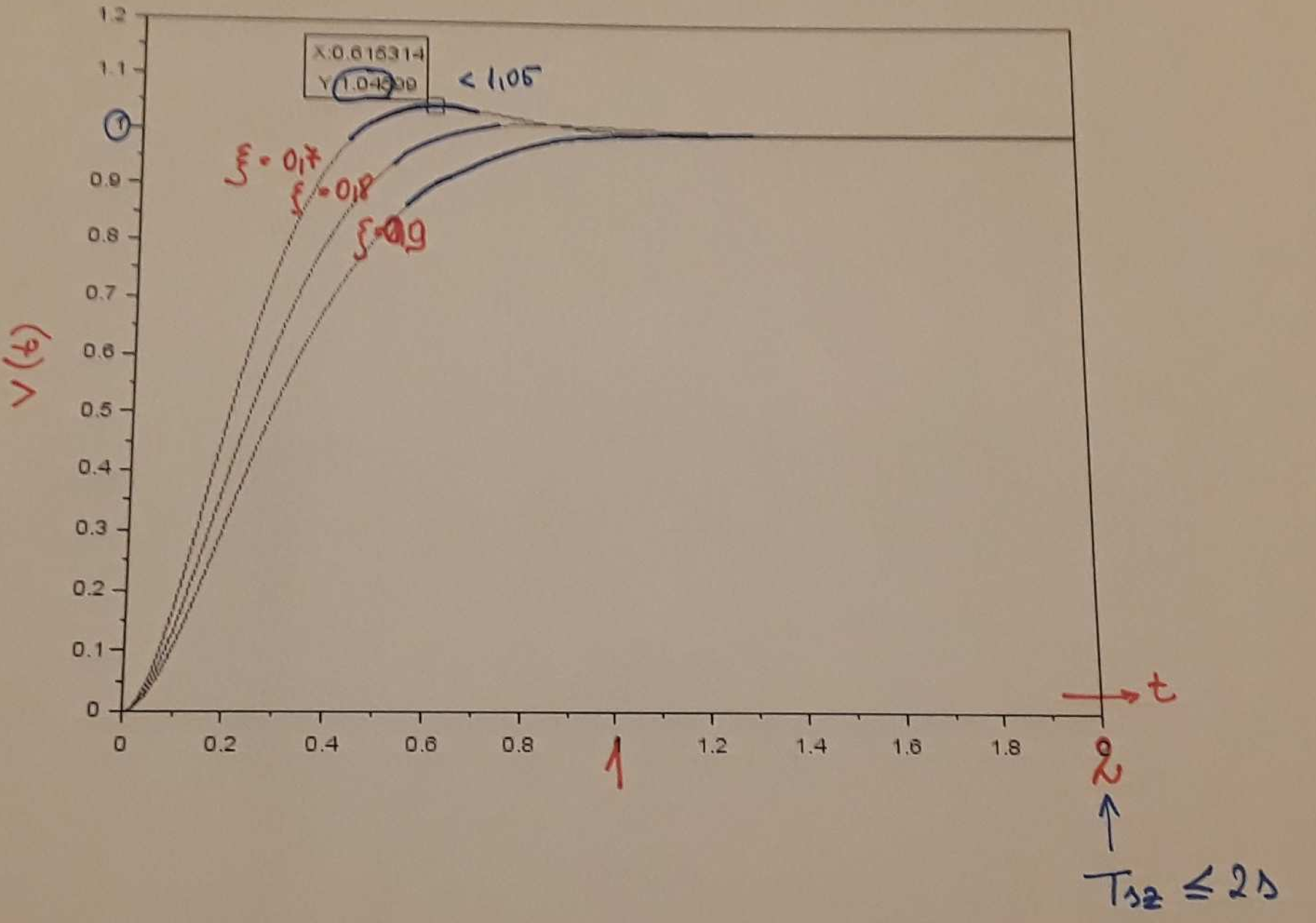
$$W = \frac{1}{(s-p_1)(s-p_2)} = \frac{1}{(s-[-5+j2{,}42])(s-[-5-j2{,}42])} = \frac{1}{s^2 - s[-5-j2{,}42] - s[-5+j2{,}42] + [25+5{,}87]} = \frac{1}{\underline{\underline{s^2 + 10s + 30{,}87}}}$$

$s^2 + 5s \underline{+j2{,}42s} + 5s \underline{-j2{,}42s} + 30{,}87$

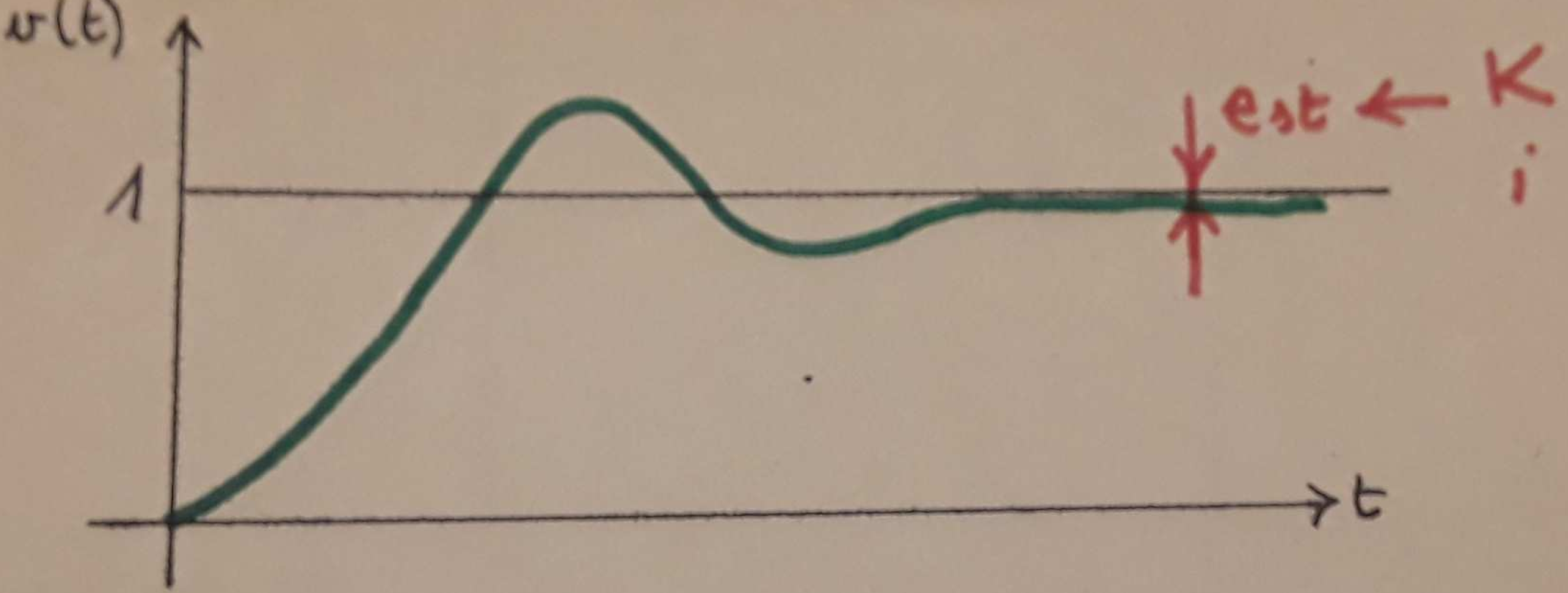
$\swarrow 30{,}87$

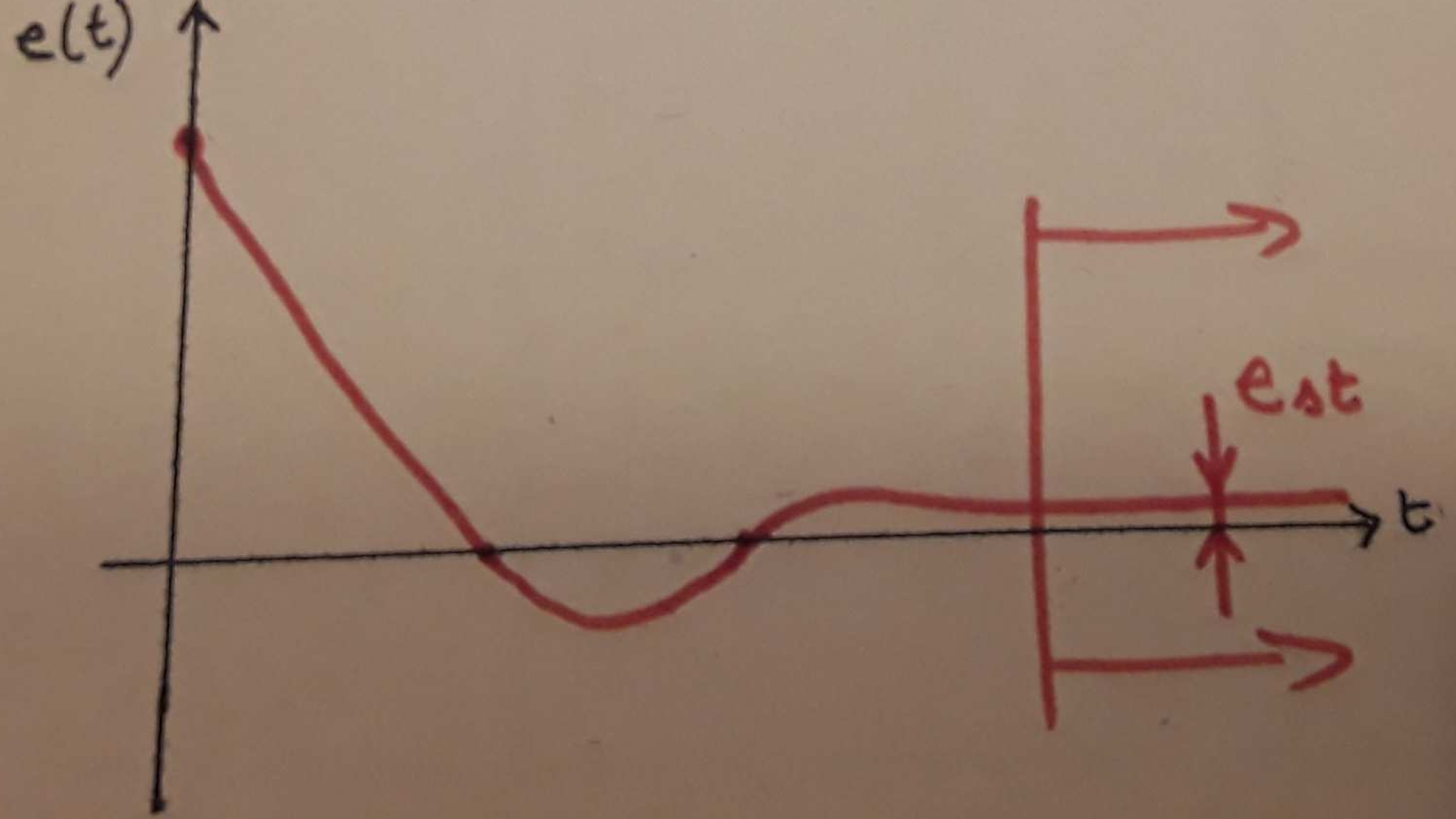
ξ	$\Omega = \sigma/\xi$	$\omega = \Omega\sqrt{1-\xi^2}$	$p_{1\vert 2}$	$W_{cl}(s)$
→ 0,7	7,14	5,10	$-5 \pm j\,5{,}10$	$51{,}02/(s^2+10s+51{,}02)$
0,8	6,25	3,75	$-5 \pm j\,3{,}75$	$39{,}06/(s^2+10s+39{,}06)$
→ 0,9	5,56	2,42	$-5 \pm j\,2{,}42$	$30{,}86/(s^2+10s+30{,}86)$
1,0	5,0	0,0	-5	$25/(s^2+10s+25)$ ↑

$$\lim_{s\to 0} \underbrace{s\frac{1}{s}W_{cl}}_{V_{cl}} = \underline{1} \Rightarrow v(t\rightarrow\infty) = 1$$

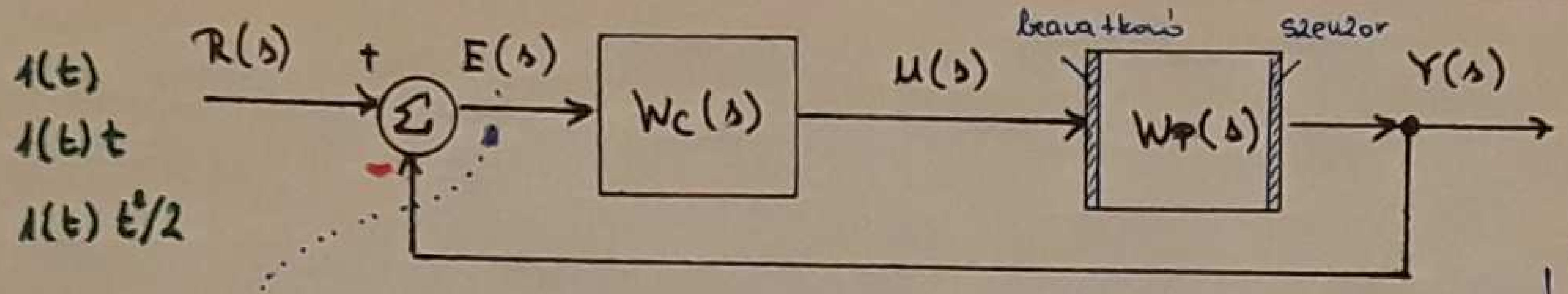

X:0.616314
Y:1.04399
< 1,05
ξ = 0,7
ξ = 0,8
ξ = 0,9
V(t)
t
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Ts2 ≤ 2s

$$\lim_{s\to 0} \boxed{s\, E(s)} \Longrightarrow E(s) = R(s) \frac{}{1+W_0(s)} \Rightarrow R(s) \frac{}{1+\frac{K}{s^i}} = R \frac{}{s^i+}$$





A STACIONÁRIUS HIBA KAPCSOLATA A KÖRERŐSÍTÉSSEL ÉS A TÍPUSSZÁMMAL.

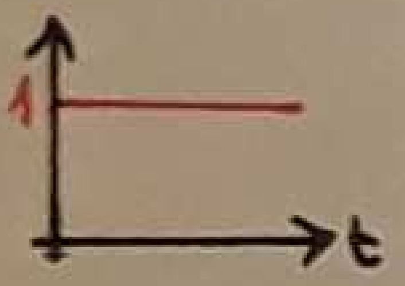
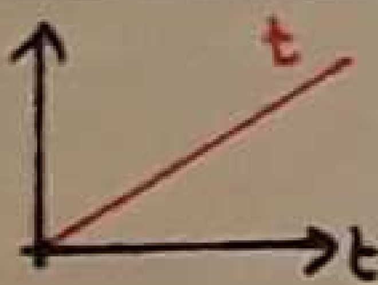
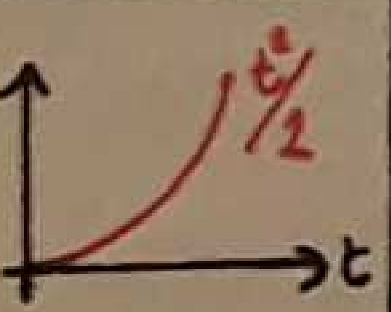


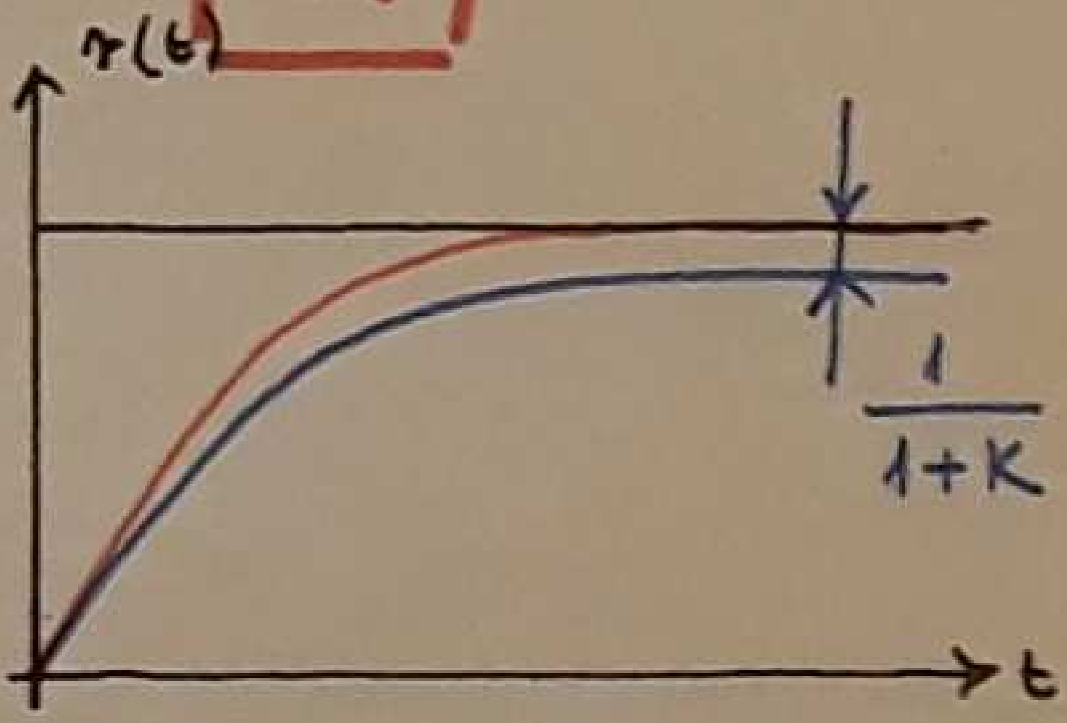
$$W_0 = W_c W_p = \frac{K}{s^i}\underbrace{W_{01}(s)}$$

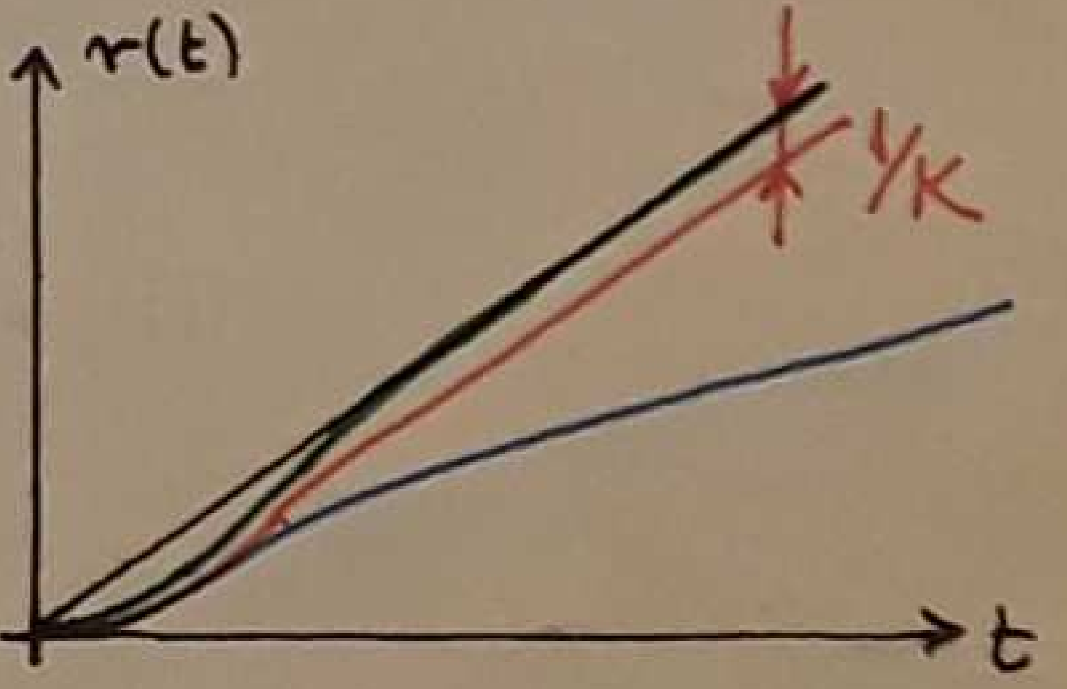
$$W_{01}(0) = 1.$$

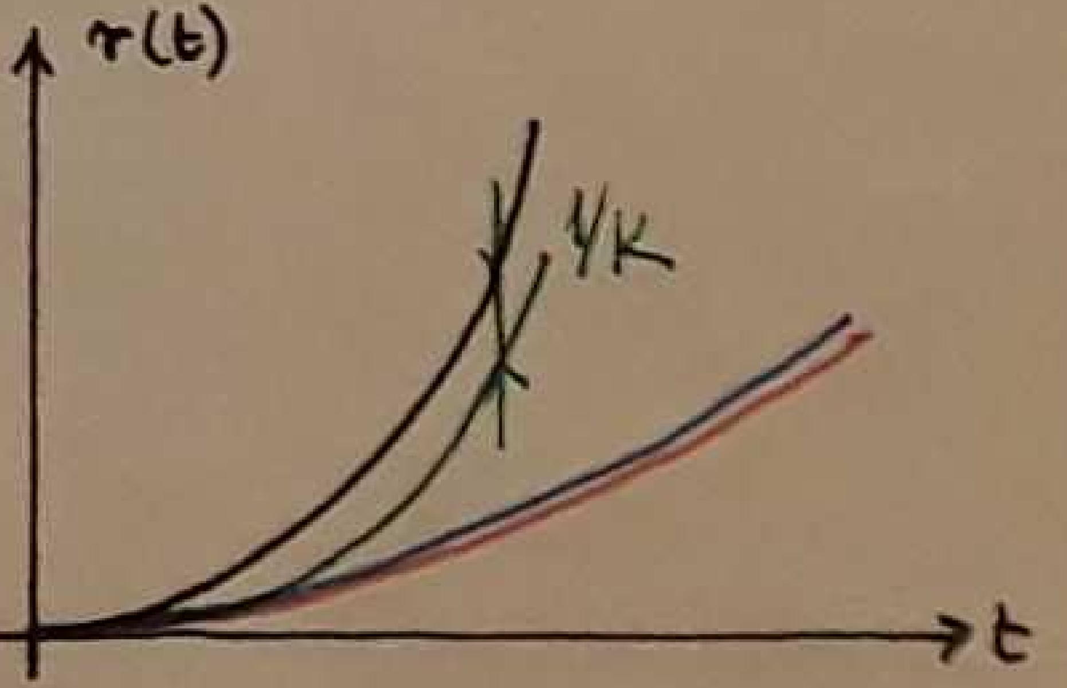
$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} \boxed{s\,E(s)} \Longrightarrow E(s) = R(s)\frac{1}{1+W_0(s)} \Rightarrow R(s)\frac{1}{1+\frac{K}{s^i}} = \boxed{R\frac{s^i}{s^i+K}s}$$

$$\boxed{\begin{matrix}K\uparrow \\ i\uparrow\end{matrix}} \longleftrightarrow \text{STAB.}$$

r(t) R(s)	i=0	i=1	i=2
 $\frac{1}{s}$	$\lim_{s\to0}\frac{1}{s}\frac{1}{1+K}s = \frac{1}{1+K}$ $\boxed{K\uparrow}$	$\lim_{s\to0}\frac{1}{s}\frac{s}{s+K}s = 0$ ✓	$\lim_{s\to0}\frac{1}{s}\frac{s^2}{s^2+K}s = 0$
 $\frac{1}{s^2}$	$\lim_{s\to0}\frac{1}{s^2}\frac{1}{1+K}s = \infty$	$\lim_{s\to0}\frac{1}{s^2}\frac{s}{s+K}s = \frac{1}{K}$ $\boxed{K\uparrow}$	$\lim_{s\to0}\frac{1}{s^2}\frac{s^2}{s^2+K}s = 0$ ✓
 $\frac{1}{s^3}$	$\lim_{s\to0}\frac{1}{s^3}\frac{1}{1+K}s = \infty$	$\lim_{s\to0}\frac{1}{s^3}\frac{s}{s+K}s = \infty$	$\lim_{s\to0}\frac{1}{s^3}\frac{s^2}{s^2+K}s = \frac{1}{K}$







STABILITÁS

Ismétlés röviden :

- gerjesztés-válasz stabil : $\int_0^\infty |w(t)|\,dt < \infty \Rightarrow \lim\limits_{t\to\infty} w(t) = 0.$

  ↳ korlátos gerjesztésre korlátos válasszal felel.

  ↑✓ ↓?

- aszimptotikus stabil : $\lim\limits_{t\to\infty} \underline{x}(t) = \underline{0}$

  ↳ egyensúlyi állapotból kimozdítva, a rendszer visszatér egyensúlyi állapotába

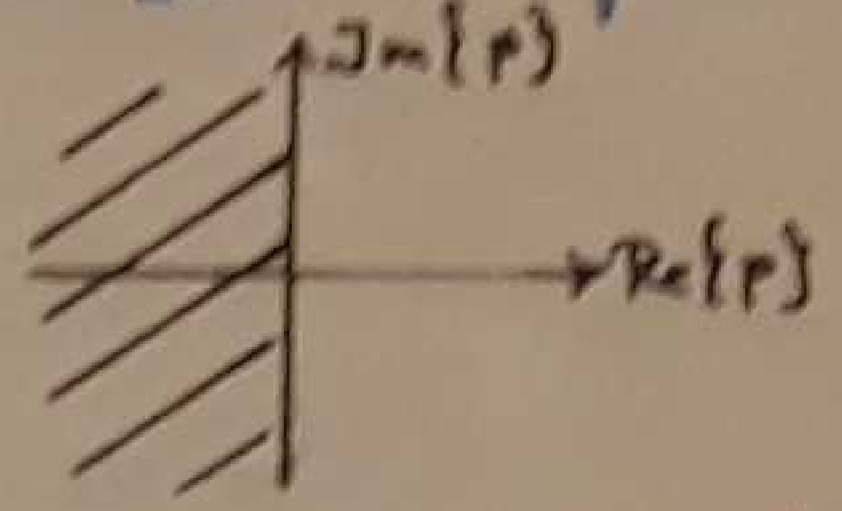
Lineáris rendszer jellemzője, nem függ a gerjesztéstől.

Zárt szabályozási rendszer belső stabilitása

$$\underline{\underline{T}} = \begin{bmatrix} \dfrac{\boxed{W_c W_p}}{1+W_c W_p} & \dfrac{\boxed{W_p}}{1+W_c W_p} \\[2ex] \dfrac{W_c}{1+W_c W_p} & \dfrac{1}{1+W_c W_p} \end{bmatrix}$$

STABILITÁSI KRITÉRIUMOK

Zárt rendszer karakterisztikus egyenlete: $1+W_0=0$. (Gyökei = zárt r. pólusai)



Megoldása:
- analitikusan, képlettel (megoldó képlet)
- numerikusan, gyökkereső eljárások
- holtidős rendszer: $1+W_0e^{-sT_h}=0$ → - közelítés racionális törtfgv.
  - frekvenciatartomány

Következtetés a megoldás nélkül: kritériumok

- Routh-séma
- Hurwitz-determináns
- Gyökhelygörbe
- Nyquist-kritérium
- Bode-kritérium

STABILITÁSVIZSGÁLAT A HURWITZ- DETERMINÁNS ALAPJÁN

$$1+W_0 \Rightarrow \quad a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \dots + a_2 s^2 + a_1 s + a_0$$

- $a_0, a_1, \dots a_n > 0$

$\Downarrow$ $n \times n$ méretű Hurwitz-determináns:

$$\begin{bmatrix} a_{n-1} & a_{n-3} & a_{n-5} & a_{n-7} & \cdots \\ a_n & a_{n-2} & a_{n-4} & a_{n-6} & \cdots \\ 0 & a_{n-1} & a_{n-3} & \vdots & \\ 0 & a_n & a_{n-2} & & \\ 0 & 0 & a_{n-1} & & \\ 0 & 0 & a_n & & \\ \vdots & \vdots & 0 & & \\ & & \vdots & & \end{bmatrix}$$

$n=2$, $a_2 s^2 + a_1 s + a_0 = 0$

$$\begin{pmatrix} a_1 & 0 \\ a_2 & a_0 \end{pmatrix}$$

$n=3$, $a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$

$$\begin{pmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{pmatrix}$$

- $a_{n-1} > 0$
- $\begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} > 0$
- $\begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix} > 0$

$\vdots$

Mi a feltétele egy másodfokú rendszer stabilitásának?

$$a_2 s^2 + a_1 s + a_0 = 0$$

$$\begin{pmatrix} a_1 & 0 \\ a_2 & a_0 \end{pmatrix}$$

- $\boxed{a_0 > 0}$ $\boxed{a_1 > 0}$ $\boxed{a_2 > 0}$
- $a_1 > 0$ ✓
- $\begin{vmatrix} a_1 & 0 \\ a_2 & a_0 \end{vmatrix} > 0$

$a_1 \cdot a_0 - 0 \cdot a_2 > 0$

$a_1 \cdot a_0 > 0$

$\Downarrow$

$a_1 > 0$ ✓ ÉS $a_0 > 0$ ✓

Mi a feltétele egy harmadrendű rendszer stabilitásának?

$$a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

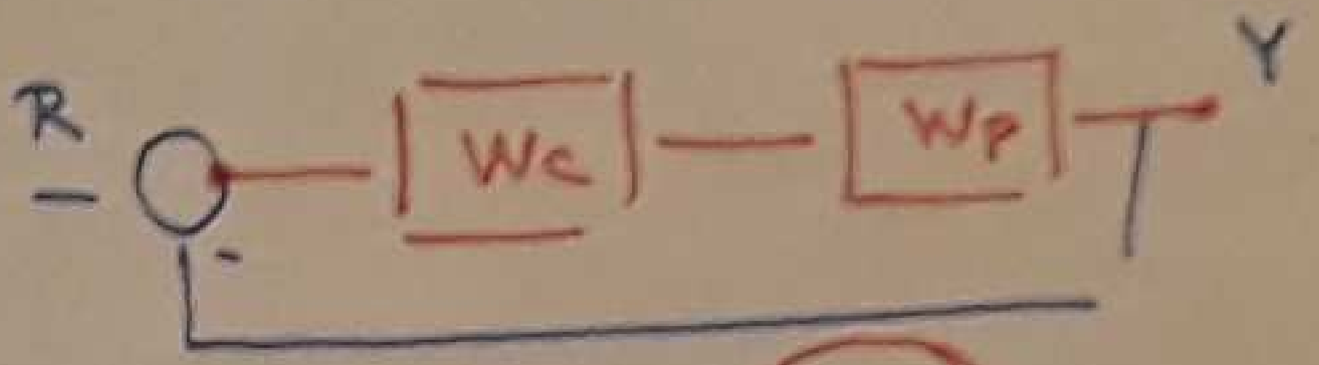
$$\begin{pmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{pmatrix}$$

- $\boxed{a_0,\ a_1,\ a_2,\ a_3 > 0}$
- $a_2 > 0$ ✓
- $\begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = \boxed{a_1 \cdot a_2 - a_0 \cdot a_3 > 0}$
- $\begin{vmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & \boxed{a_0} \end{vmatrix} = a_2 \underset{\begin{vmatrix} a_1 & 0 \\ a_2 & a_0 \end{vmatrix}}{(a_1 \cdot a_0)} + a_0 \underset{\begin{vmatrix} a_3 & 0 \\ 0 & a_0 \end{vmatrix}}{(0 - a_3 a_0)} + 0 \underset{\begin{vmatrix} a_3 & a_1 \\ 0 & a_2 \end{vmatrix}}{(..)}$

$$= a_2 a_1 \underline{a_0} - a_0 a_3 \underline{a_0}$$

$$= \underbrace{a_0}_{>0\ \checkmark} \underbrace{(a_1 a_2 - a_0 a_3)}_{>0} > 0$$

Egy felnyitott kör $W_0(s)$ átviteli függvénye ismert. K mely értékei mellett stabil a zárt kör?



$$W_0(s) = \frac{K}{s(1+s)(1+5s)}$$

$$W = \frac{W_0}{1+W_0}$$

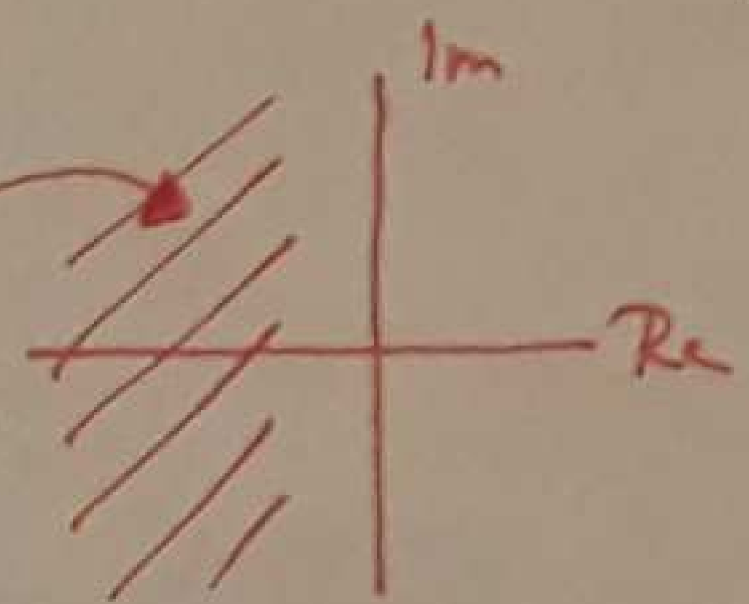
$$W_0 = \boxed{\frac{B(s)}{A(s)}}$$

$$\frac{W_0}{1+W_0} = \frac{B/A}{1+B/A}\cdot\frac{A}{A} = \frac{B}{\boxed{A+B}}$$

$$W = \frac{\frac{K}{s(1+s)(1+5s)}}{1+\frac{K}{s(1+s)(1+5s)}} = \frac{K}{\boxed{s(1+s)(1+5s)}+\boxed{K}} \leftarrow = 0$$

$$s(1+6s+5s^2)+K$$

$$\boxed{s+6s^2+5s^{\textcircled{3}}+\underset{\substack{\uparrow\\ ?}}{K}=0}$$



$$\underbrace{5s^3}_{a_3}+\underbrace{6s^2}_{a_2}+\underbrace{s}_{a_1}+\underbrace{K}_{a_0}$$

$$\begin{pmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{pmatrix} = \begin{pmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{pmatrix} = \begin{pmatrix} 6 & K & 0 \\ 5 & 1 & 0 \\ 0 & 6 & \textcircled{K} \end{pmatrix}$$

$$\boxed{0 < K < 1{,}2}$$

- $\overset{\checkmark}{5}s^3+\overset{\checkmark}{6}s^2+\overset{\checkmark}{1}s+K$ $\boxed{K>0}$
- $6>0$ ✓
- $\begin{vmatrix} 6 & K \\ 5 & 1 \end{vmatrix} = 6-5K>0$
  $6>5K \Rightarrow \boxed{K<1{,}2}$
- $K(6-5K)>0$
  $\swarrow$ $K>0$ ✓ $\searrow$ $6-5K>0$ ✓

Egy felnyitott kör $W_0(s)$ átviteli függvénye ismert. Adjuk meg K és T paraméterek azon tartományát, ahol a zárt rendszer stabil!

$$W_0(s) = K \frac{1+sT}{s(1+s)(1+5s)} = \frac{B}{A} \qquad W = \frac{W_0}{1+W_0}$$

$$\underbrace{s(1+s)(1+5s)}_{A} + \underbrace{K(1+sT)}_{B}$$

$$s + 6s^2 + 5s^3 + K + sKT = 0$$

$$\underbrace{5s^3}_{a_3} + \underbrace{6s^2}_{a_2} + \underbrace{(1+KT)}_{a_1}s + \underbrace{K}_{a_0} = 0$$

$$\boxed{K>0}$$

$$\begin{pmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & \boxed{a_2} \end{pmatrix}$$

- $a_2 > 0$ ✓
- $\begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} = a_2 a_1 - a_0 a_3 > 0$

$$6(1+KT) - K5 > 0$$

$$6 + \underbrace{K6T - 5K}_{} > 0$$

$$6 + K(6T-5) > 0$$

$$K(6T-5) > -6$$

$$K < \frac{-6}{6T-5}$$

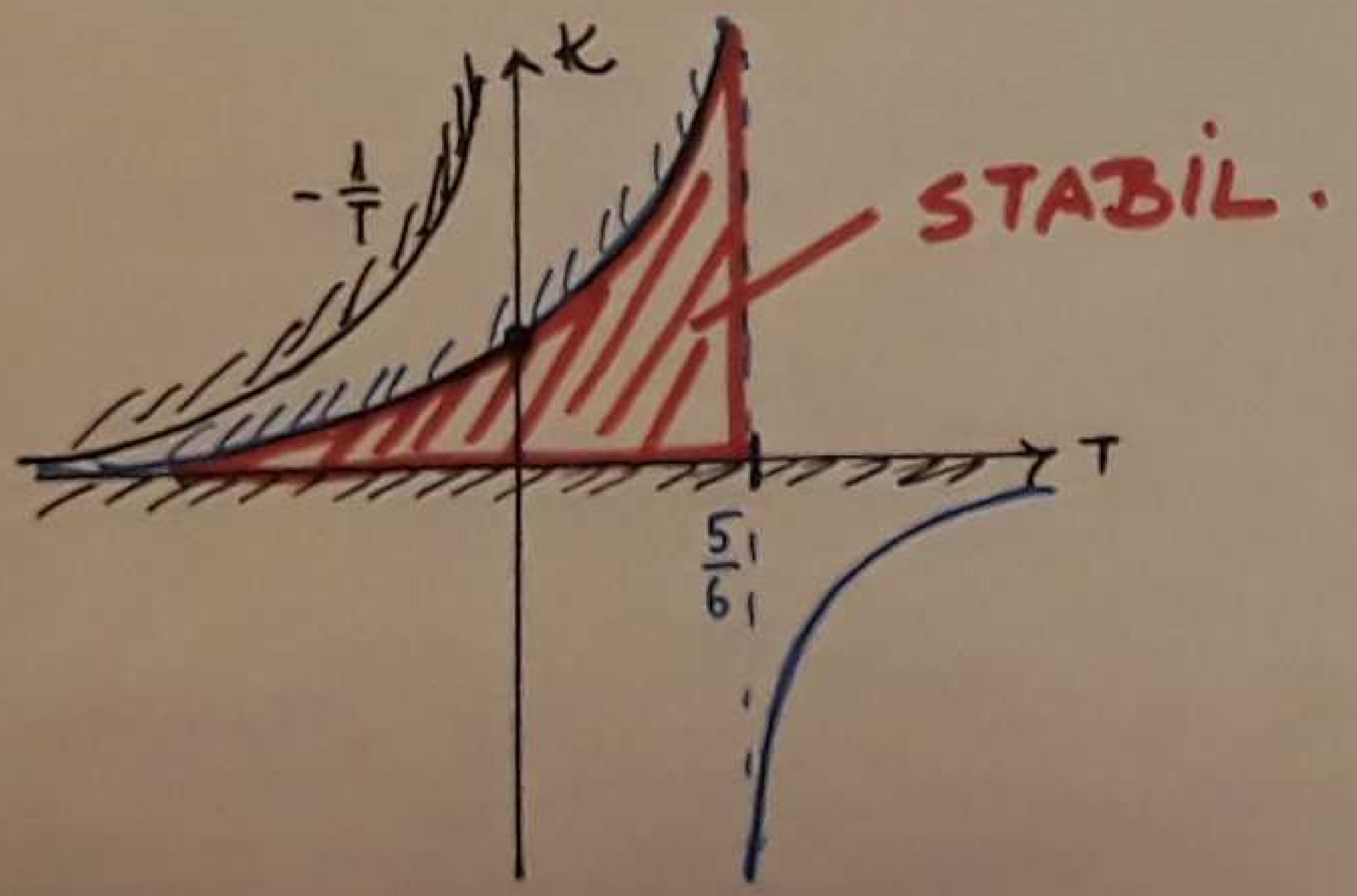
$$\boxed{K < -\frac{1}{T-\frac{5}{6}}}$$

- $a_2 > 0$ és $\begin{vmatrix} a_2 & a_0 \\ a_3 & a_1 \end{vmatrix} > 0$ ✓

$$1+KT>0$$

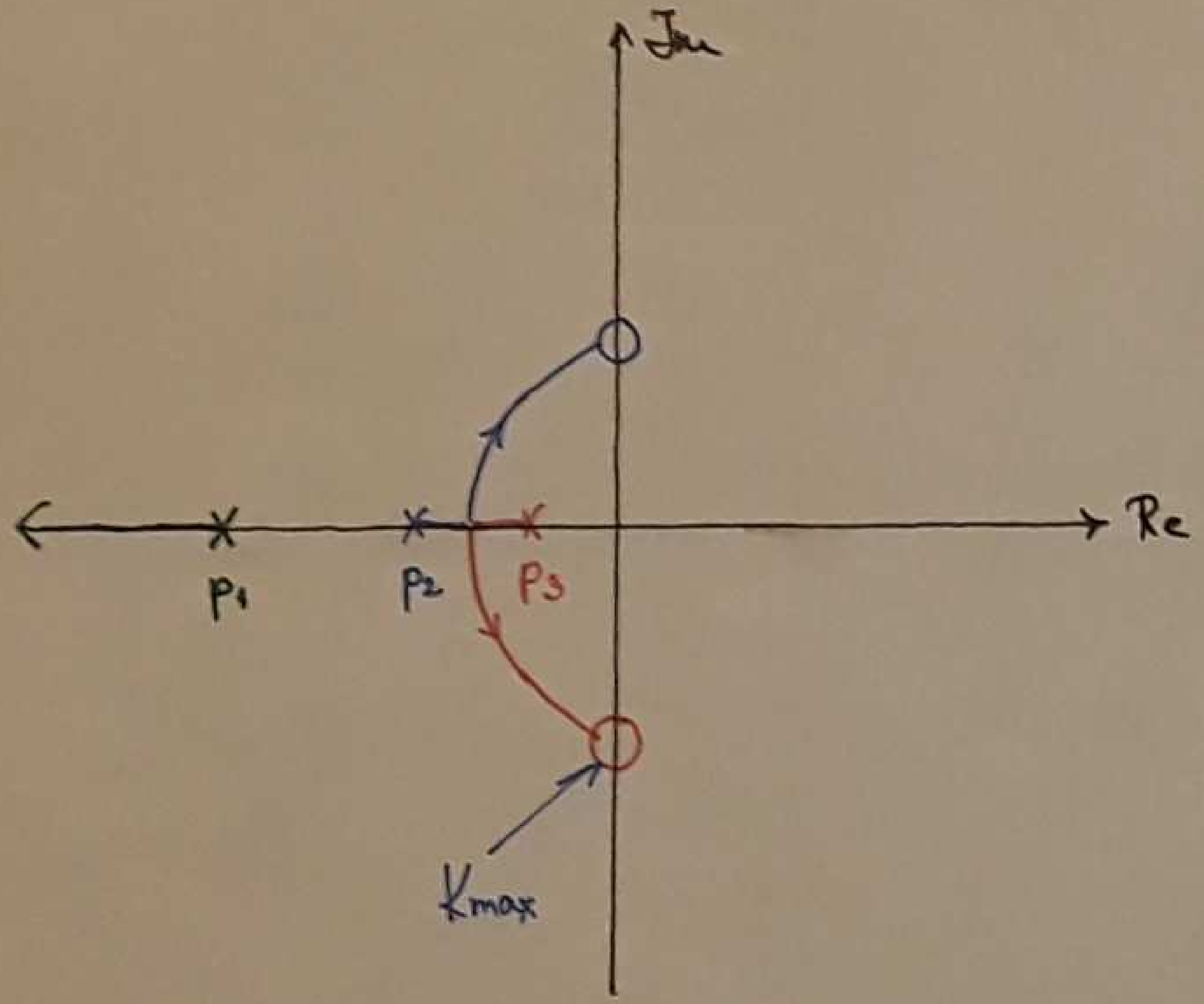
$$KT>-1$$

$$\boxed{K<-\frac{1}{T}}$$



GYÖKHELYGÖRBE (Evans)

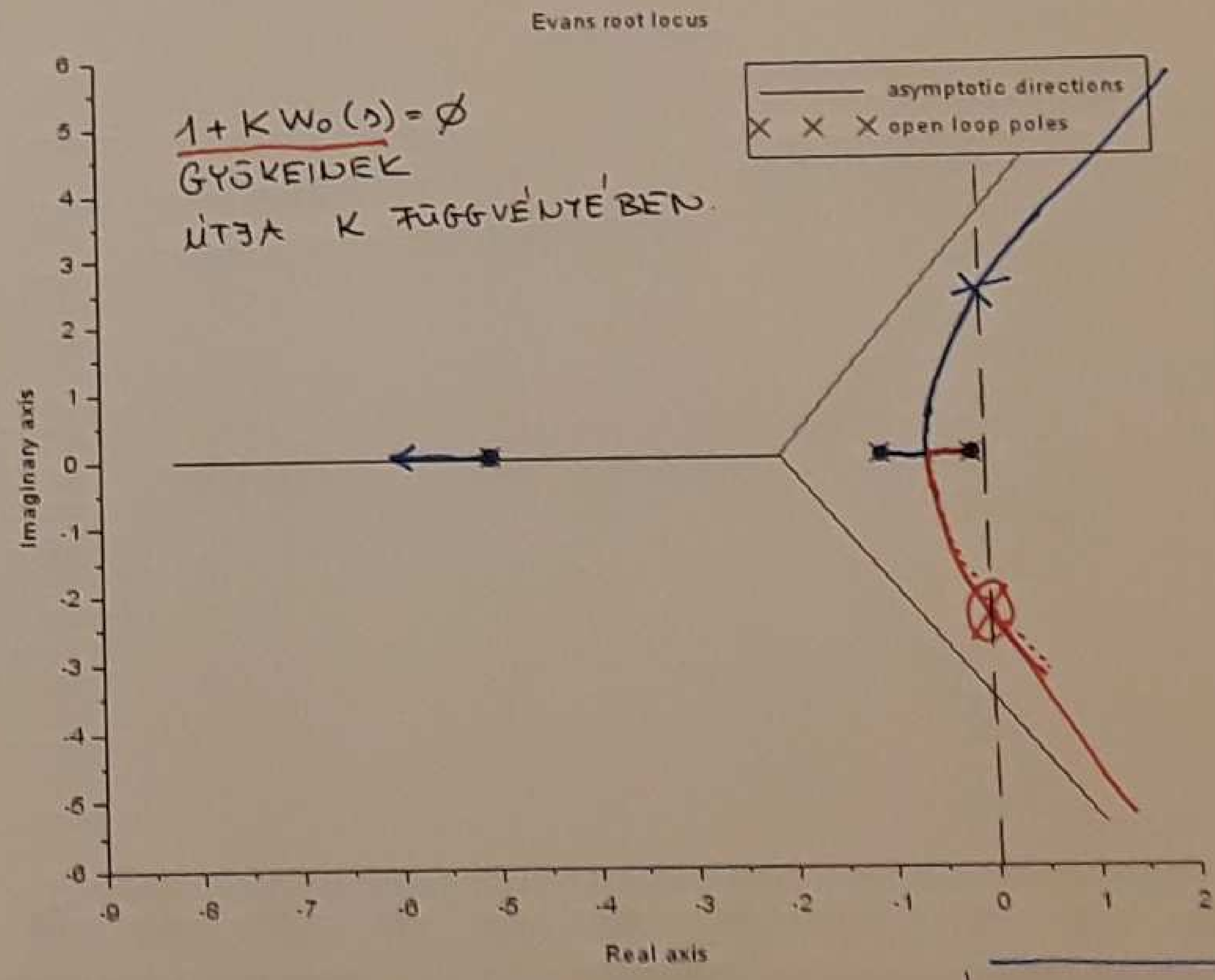
A zárt rendszer karakterisztikus egyenletének gyökeit (a zárt rendszer pólusait) ábrázolja a komplex számsíkon, miközben valamely paraméter értéke a 0 ... ∞ tartományban változik.



W_0

$1 + KW_0 = 0 \quad K = 0 \dots \infty$

$p_1 \; p_2 \dots p_n$



$1 + KW_0(s) = 0$
GYÖKEINEK
ÚTJA K FÜGGVÉNYÉBEN.

$$W_0(s) = \frac{1}{(\underbrace{1+s10}_{\frac{s}{0,1}})(\underbrace{1+s1}_{\frac{s}{1}})(\underbrace{1+s0,2}_{\frac{s}{5}})}$$

$\boxed{K\uparrow \text{ pontosabb lesz}}$

$\frac{1}{1+K} \quad \frac{1}{K}$

Egy rendszer rendszermátrix ismert:

$$\underline{\underline{A}} = \begin{bmatrix} -1 & \alpha \\ \beta & -2 \end{bmatrix}.$$

Adjuk meg α és β azon értékeit, ahol a rendszer stabil!

$$|\lambda \underline{\underline{E}} - \underline{\underline{A}}| = \left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} -1 & \alpha \\ \beta & -2 \end{pmatrix} \right| = \begin{vmatrix} \lambda+1 & -\alpha \\ -\beta & \lambda+2 \end{vmatrix} = (\lambda+1)(\lambda+2) - \alpha\beta = 0$$

$$\lambda^2 + 3\lambda + 2 - \alpha\beta = 0$$

$$\underset{a_2}{\overset{\checkmark}{1}}\lambda^2 + \underset{a_1}{\overset{\checkmark}{3}}\lambda + \underbrace{(2-\alpha\beta)}_{a_0} = 0$$

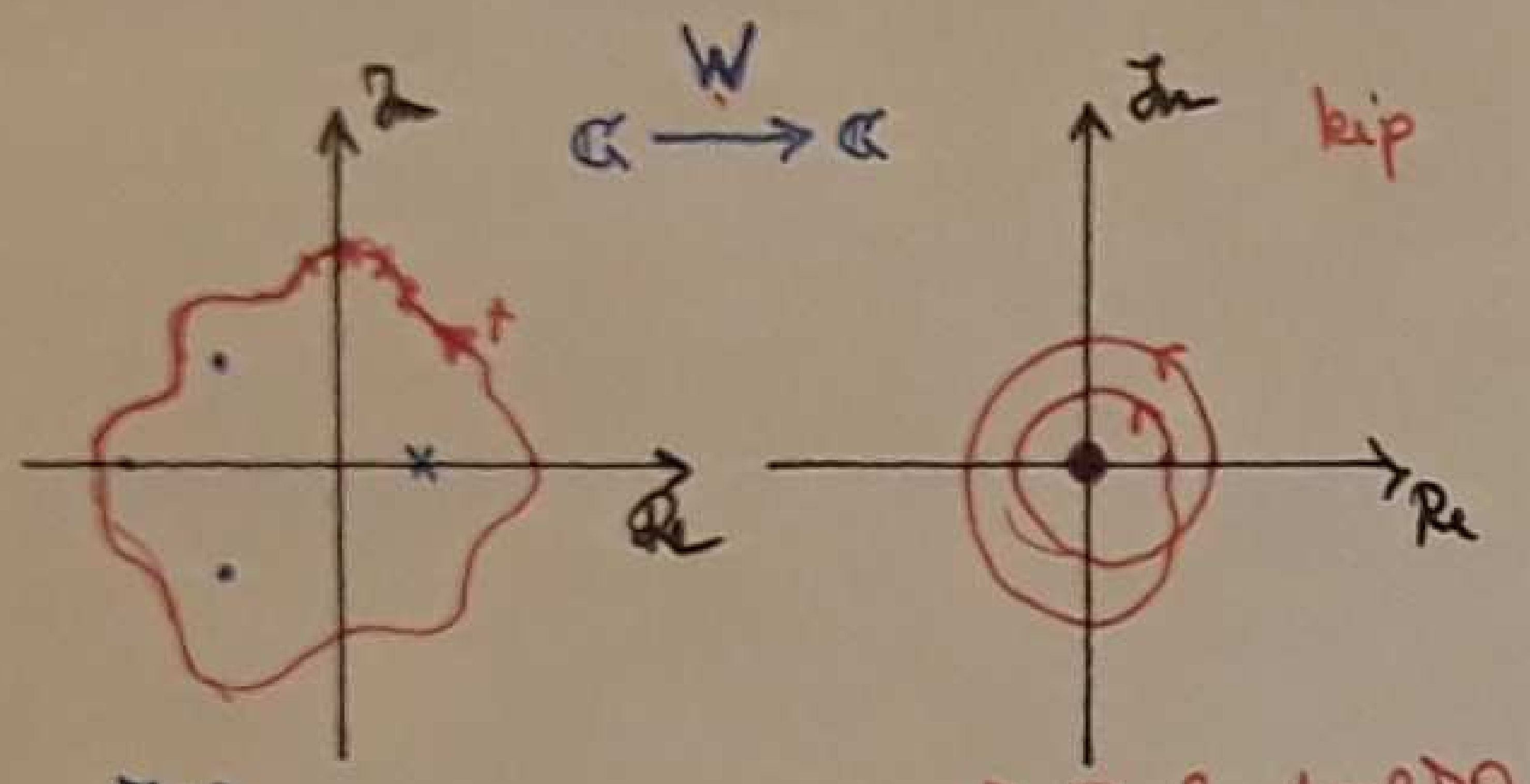
$$\begin{pmatrix} a_1 & 0 \\ a_2 & a_0 \end{pmatrix} \Rightarrow a_1 a_0 > 0$$

$$2 - \alpha\beta > 0$$

$$\boxed{2 > \alpha\beta}$$

A NYQUIST-KRITÉRIUM

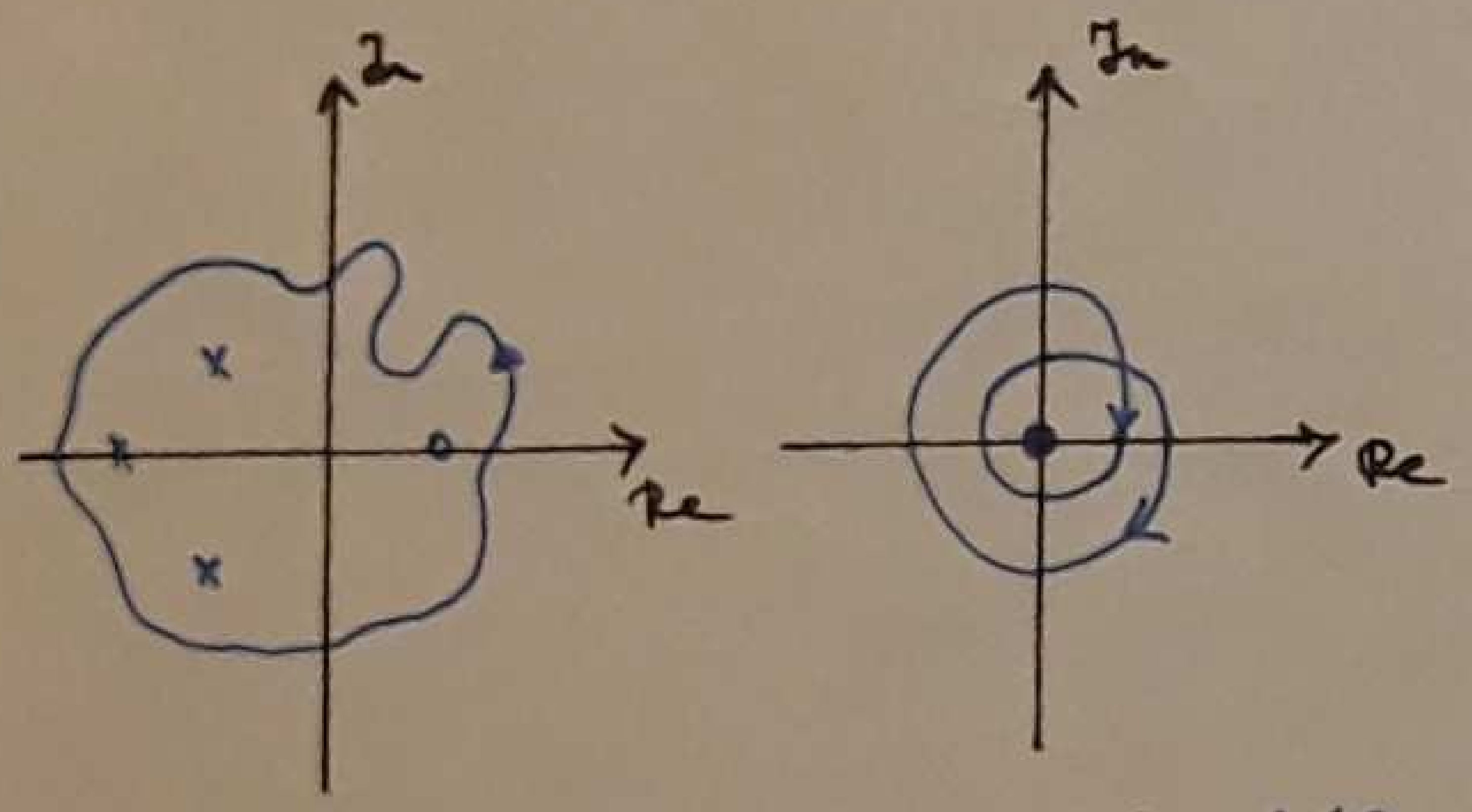
Komplex függvénytan argumentum elve



$Z=3$

$P=1$

$Z-P=3-1=2>0$

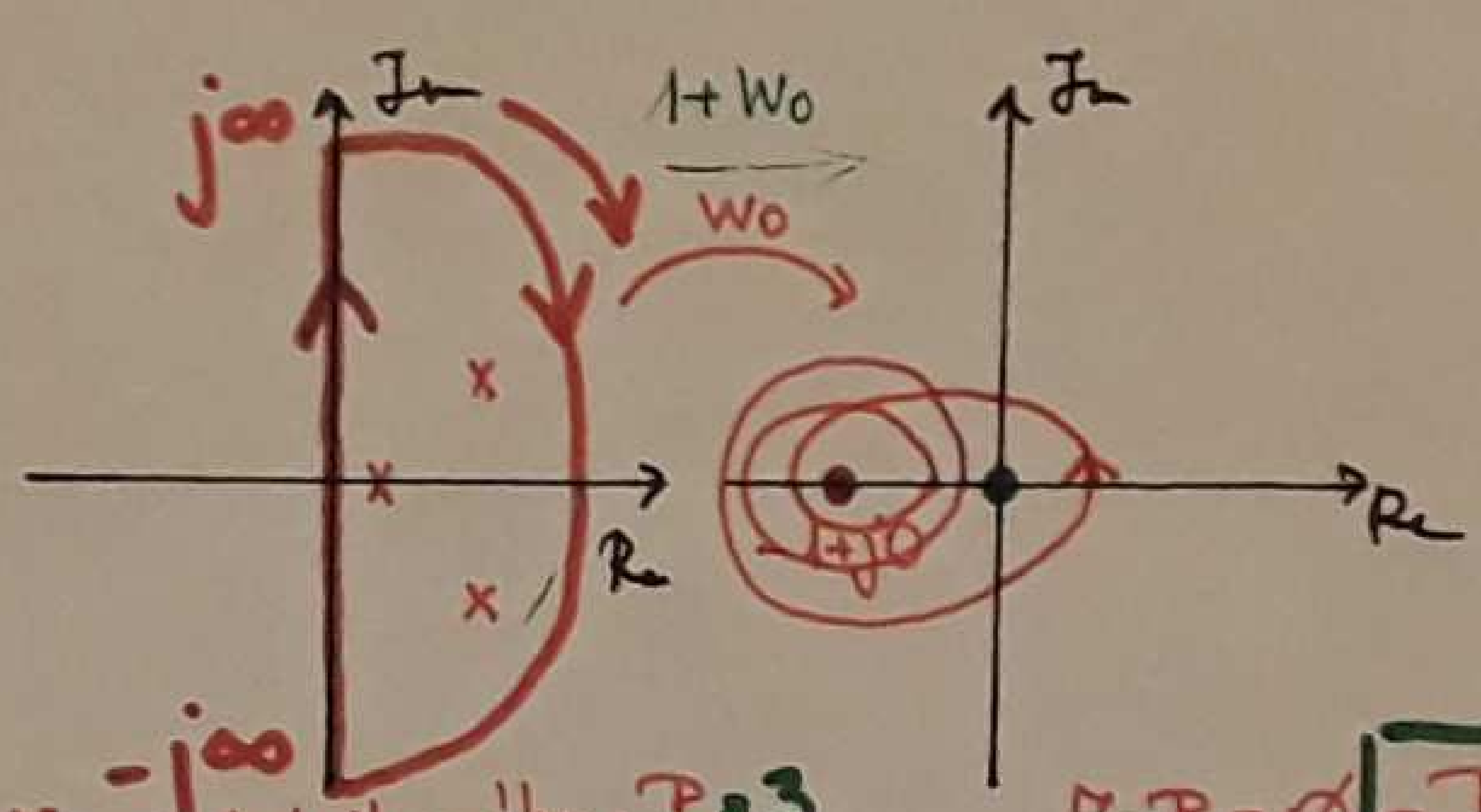


$Z=1$

$P=3$

$Z-P=1-3=-2<0$

A kritérium megfogalmazása



$$W_0 = \frac{B}{\boxed{A}}$$

$$W = \frac{W_0}{1+W_0} = \frac{\frac{B}{A}}{1+\frac{B}{A}} \cdot \frac{A}{A} = \frac{B}{\boxed{A+B}} \to P$$

$$\boxed{1+W_0} = 1+\frac{B}{A} = \frac{\boxed{A+B}}{\boxed{A}}$$

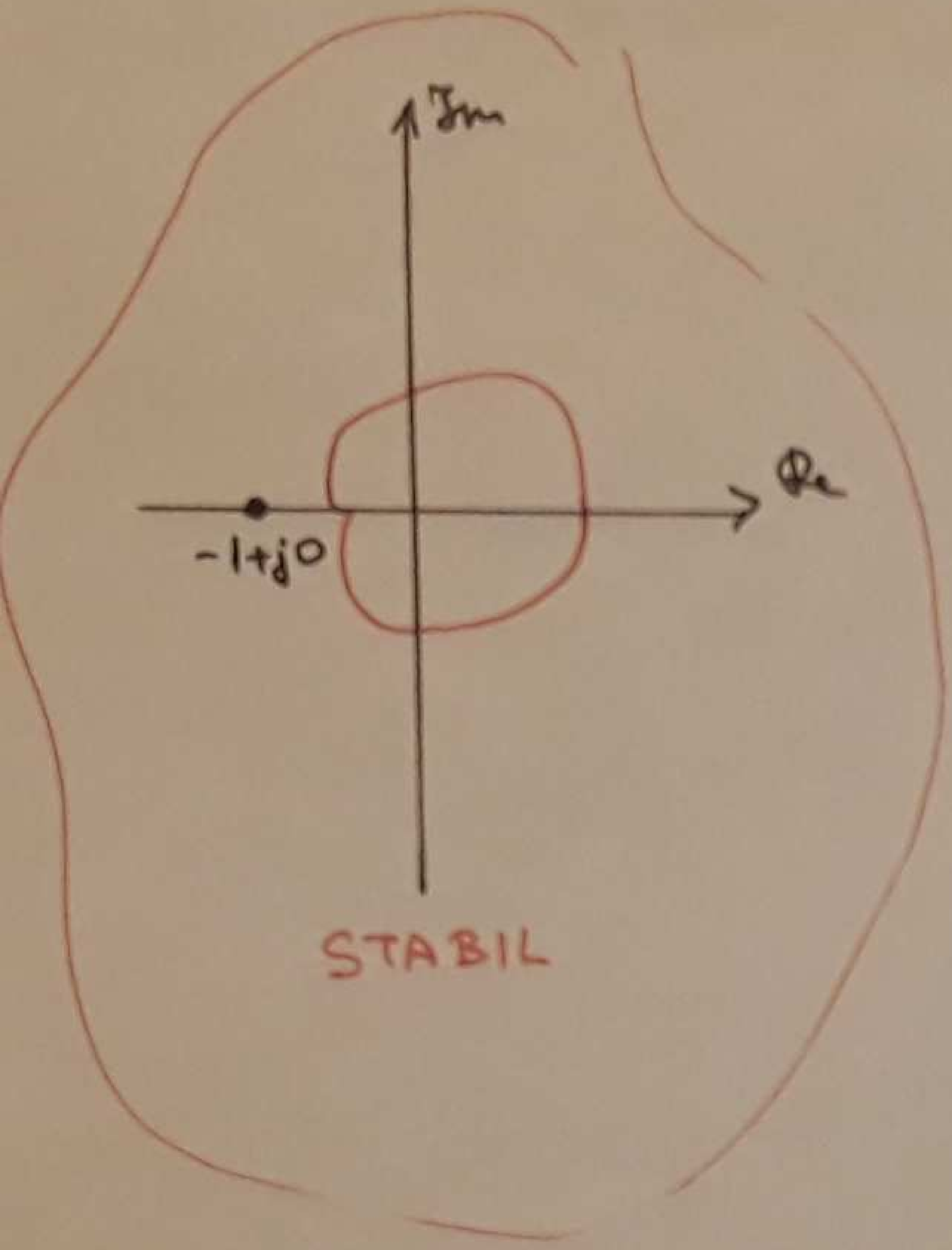
Nem stabil pólus: $P=3$

Nem stabil zérus: $\boxed{Z=0}$

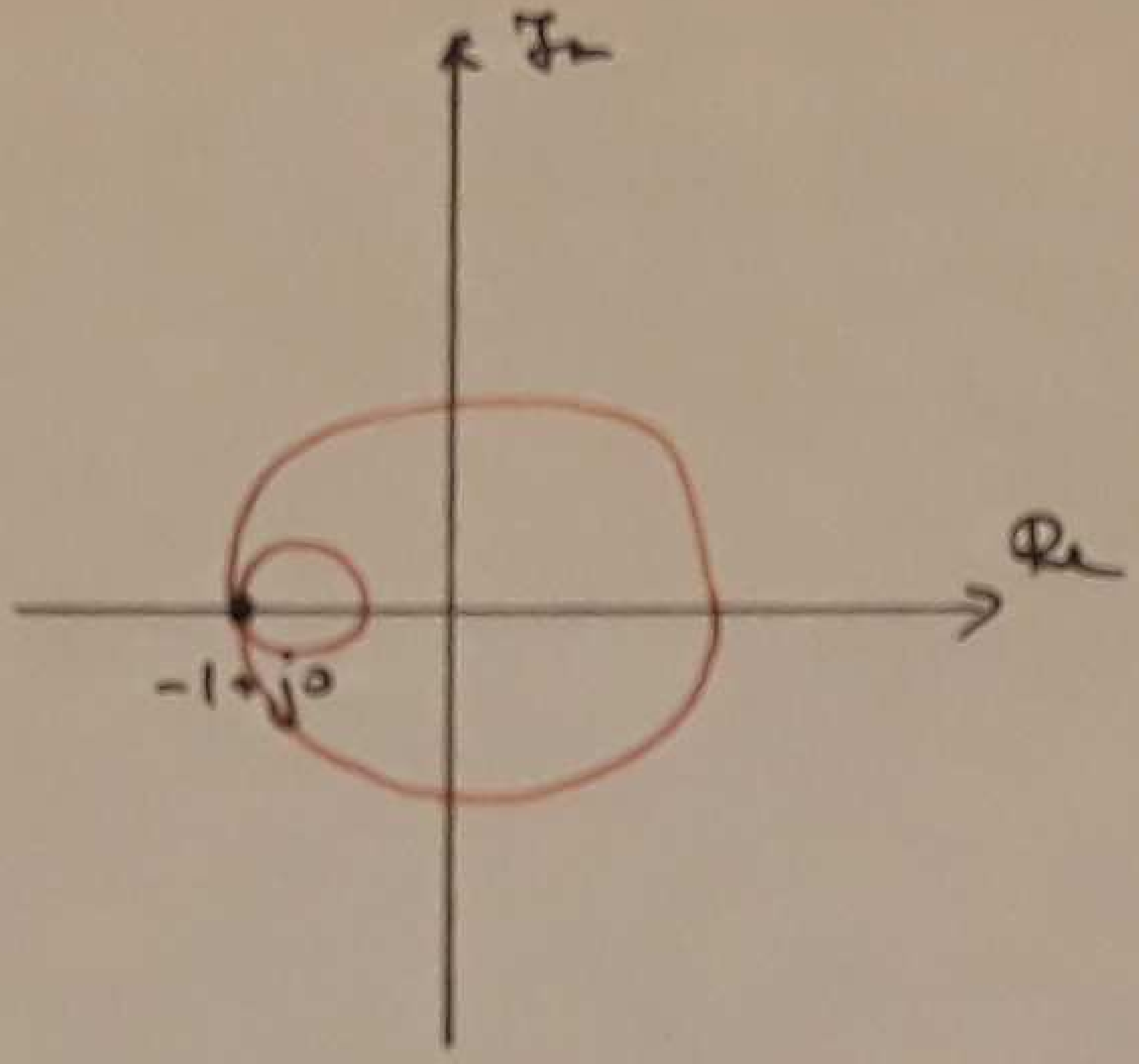
$Z-P=0 \quad \boxed{-P} = -3$

Nyquist-kritérium: ha a felnyitott kör labilis ($W_0(s)$-nek P számú pólusa van a jobb félsíkon), a zárt kör stabil, ha a felnyitott kör $W_0(j\omega)$ átviteli karakterisztika teljes Nyquist-diagramja P-szer veszi körül a $-1+j0$ pontot az óramutató járásával ellentétes irányban.

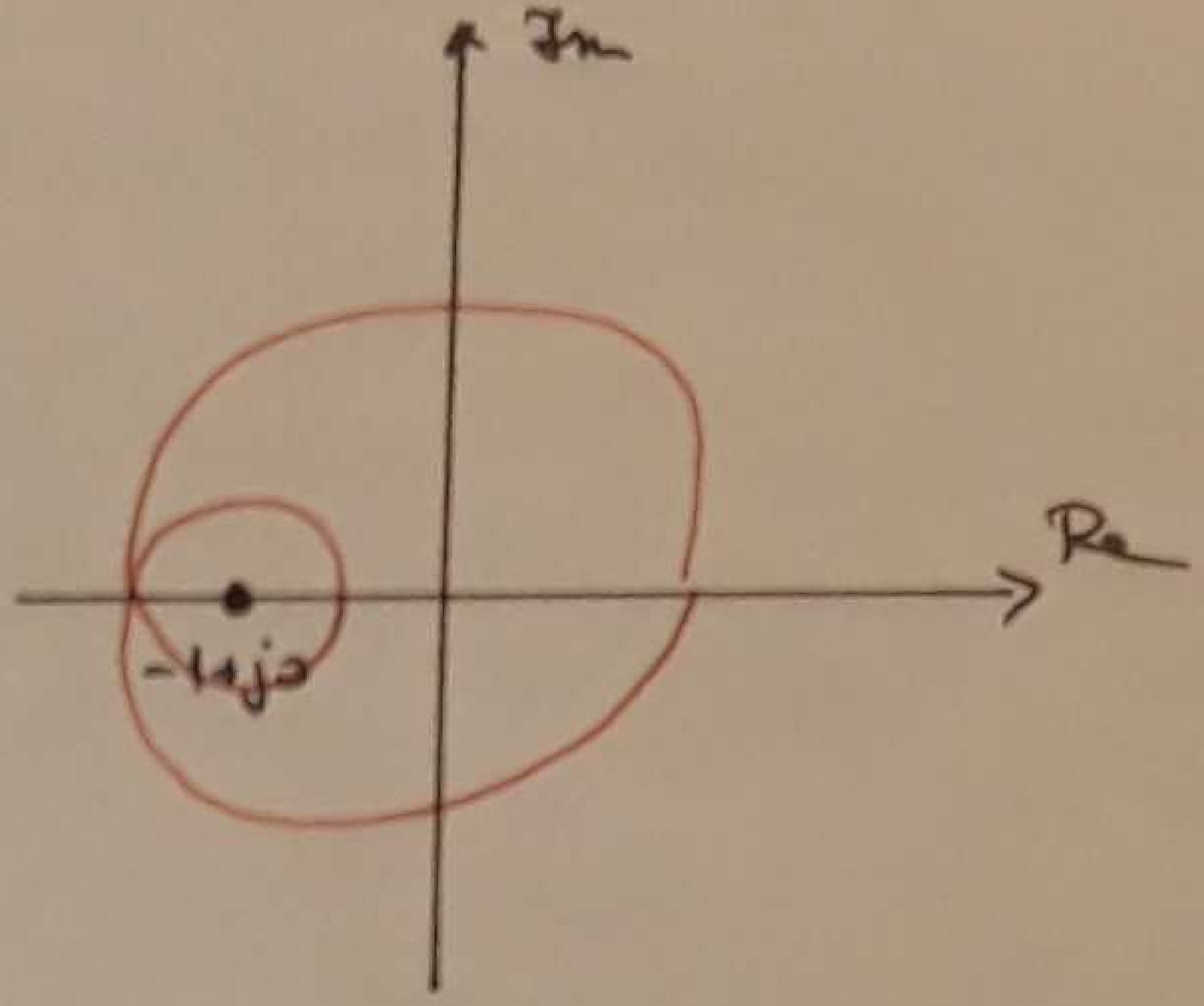
Egyszerűsített Nyquist-kritérium: ha a felnyitott kör stabil, a zárt kör akkor stabil, ha a teljes Nyquist-diagram nem veszi körül a $-1+j0$ pontot.



STABIL



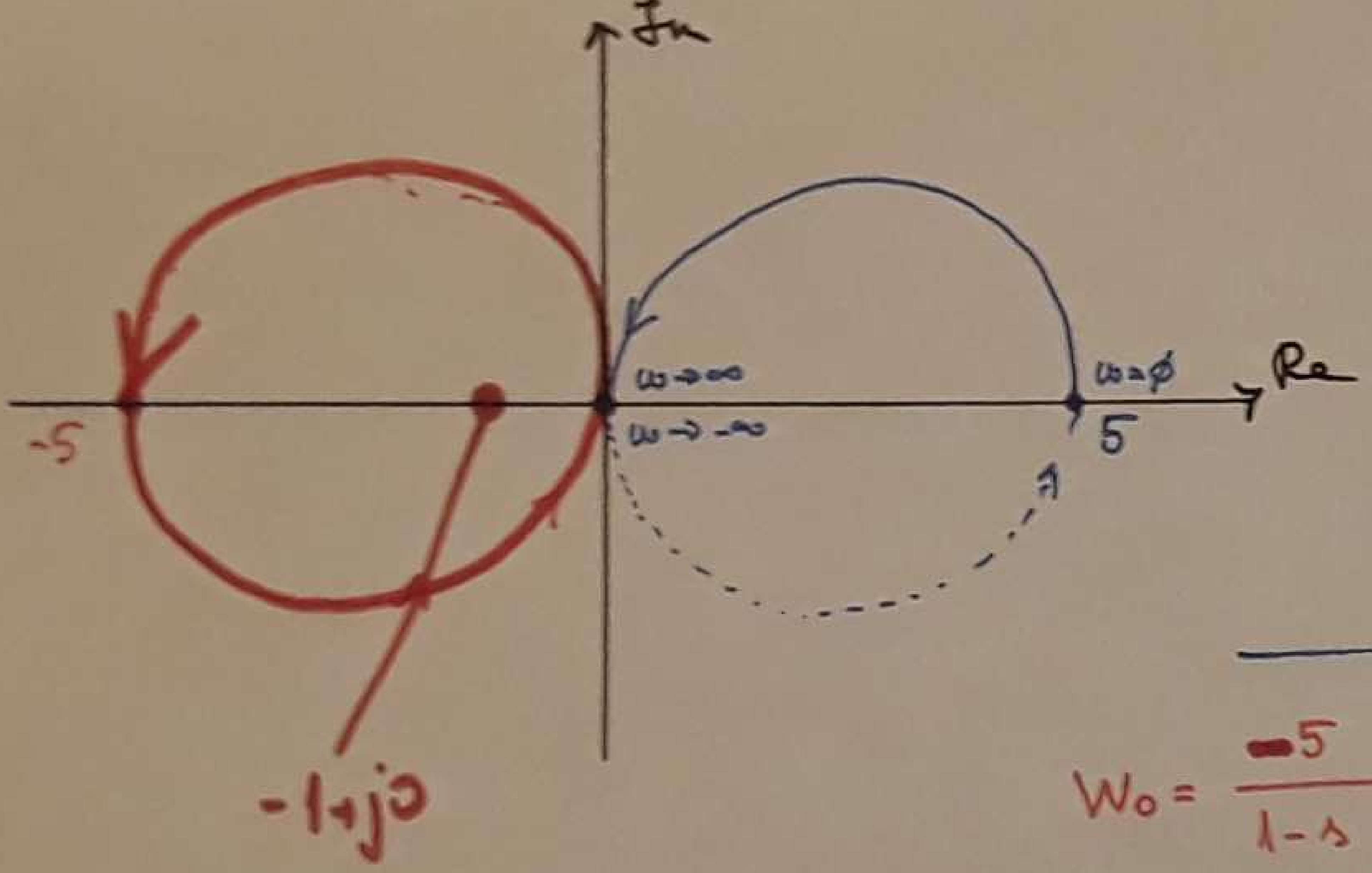
H. HELYZET



NEM STABIL

A Nyquist-kritérium segítségével állapítsuk meg, hogy a zárt kör stabil vagy sem.

$W_0 = \dfrac{5}{1-s}$ $\qquad$ $W_0 = \dfrac{-5}{1-s}$.



$1-s=0 \to p=1$ $\quad \boxed{p=1}$

$W_0 = \dfrac{5}{1-s}$ $\qquad$ $W_0(j\omega) = \dfrac{5}{1-j\omega}$ $\begin{array}{c}0^\circ\\ 0^\circ\ldots-90^\circ\\ \hline 0^\circ\ldots 90^\circ\end{array}$

$$\frac{W_0}{1+W_0} = \frac{\frac{5}{1-s}}{1+\frac{5}{1-s}} \cdot \frac{1-s}{1-s} = \frac{5}{1-s+5} = \frac{5}{\boxed{6-s}}$$

Nem stabil!

$W_0 = \dfrac{-5}{1-s}$

$$\frac{W_0}{1+W_0} = \frac{\frac{-5}{1-s}}{1+\frac{-5}{1-s}} \cdot \frac{1-s}{1-s} = \frac{-5}{1-s-5} = \frac{-5}{-4-s} = \frac{5}{\boxed{s+4}} \quad -4$$

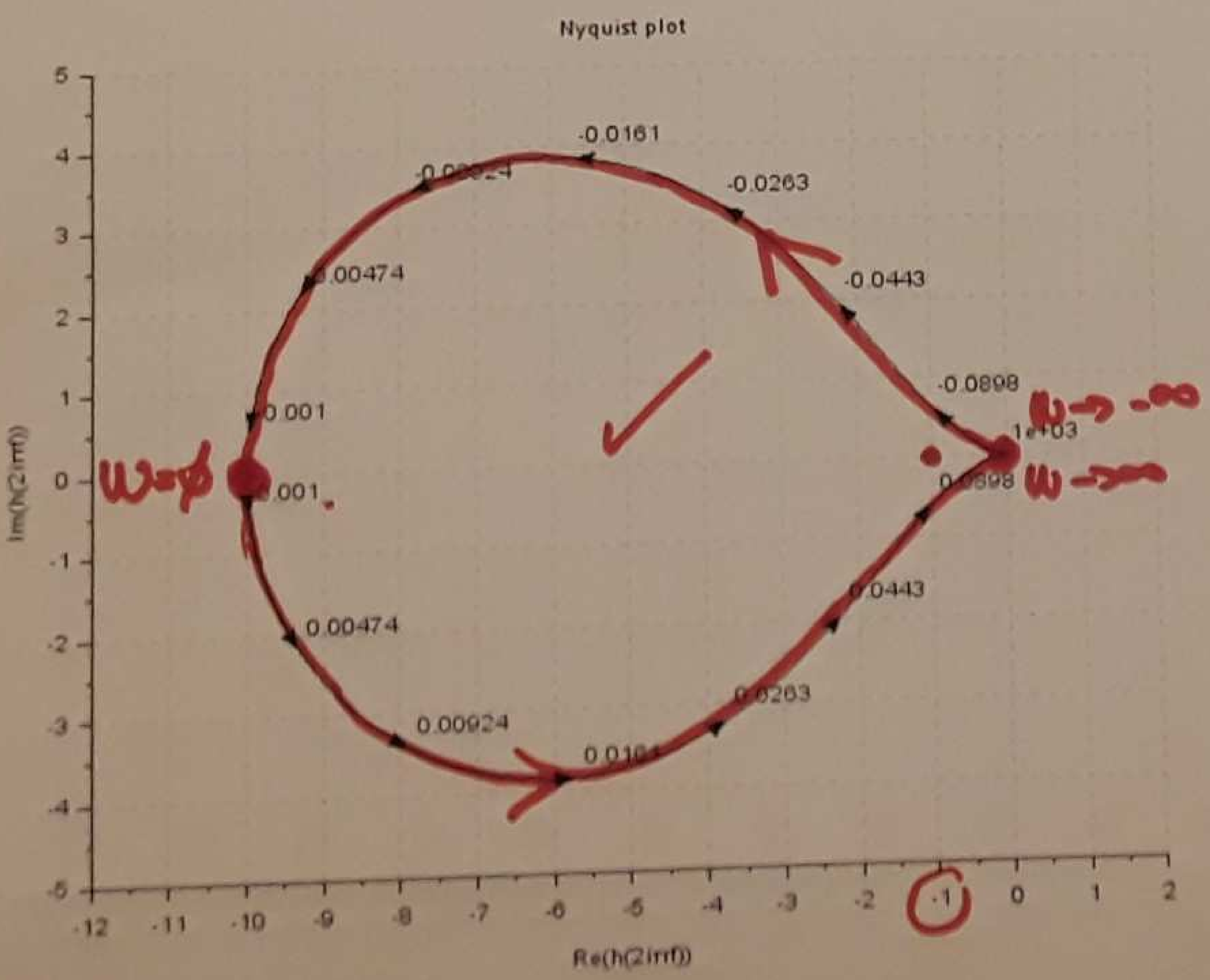
A Nyquist - kritérium segítségével döntsük el, hogy a zárt kör stabil vagy nem!

$$W_0(s) = -10 \frac{1}{(1+s2)\underline{(1-s10)}}$$

P=1

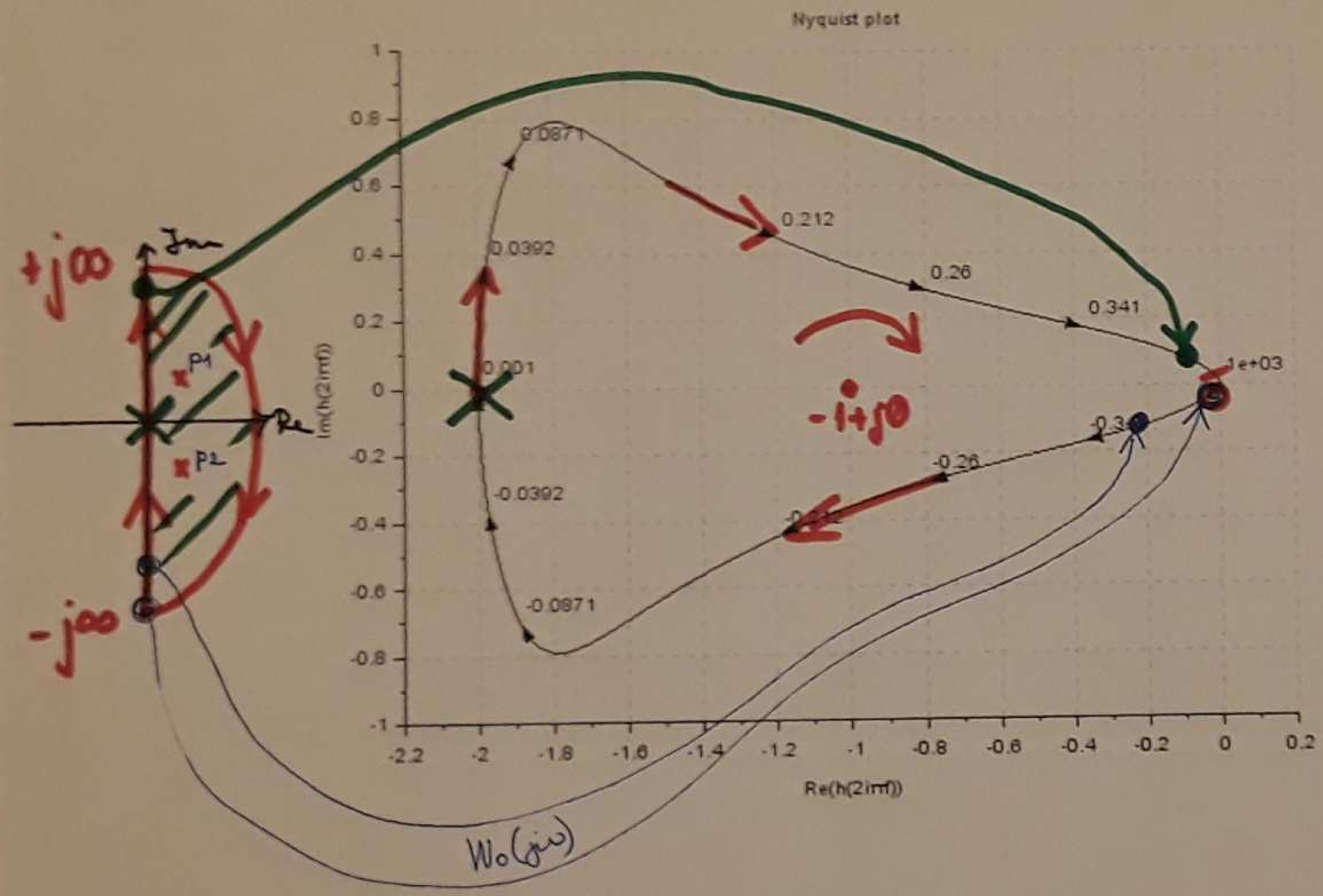
$$\frac{W_0}{1+W_0} = \frac{-10\frac{1}{(1+s2)(1-s10)}}{1-10\frac{1}{(1+s2)(1-s10)}} = \frac{-10}{(1+s2)(1-s10)-10} = \frac{-10}{1-s8-s^2 20-10} = \frac{-10}{-9-s8-s^2 20}$$

$$= \frac{10}{s^2 20 + s8 + 9}$$ **Hurwitz**



Vizsgáljuk meg az alábbi rendszert !

$$W_0(s) = 3\,\frac{s-2}{0{,}8s^4 + s^3 + 0{,}8s + 3}$$



Nyitott:

$p_{1,2} = +0{,}73 \pm j\,1{,}04$

$p_{3,4} = -1{,}35 \pm j\,0{,}65$

$P = 2$

Zárt:

$p_{1,2} = +0{,}23 \pm j\,1{,}52$

$p_3 = -2{,}37$

$p_4 = +0{,}67$

$$W_0 = \frac{B}{\boxed{A}} \qquad 1 + W_0 = 1 + \frac{B}{A} = \frac{\boxed{A+B}}{\boxed{A}}$$

$$W = \frac{W_0}{1+W_0} = \frac{\frac{B}{A}}{1+\frac{B}{A}} \cdot \frac{A}{A} = \frac{B}{\boxed{A+B}}$$

$P = 2$

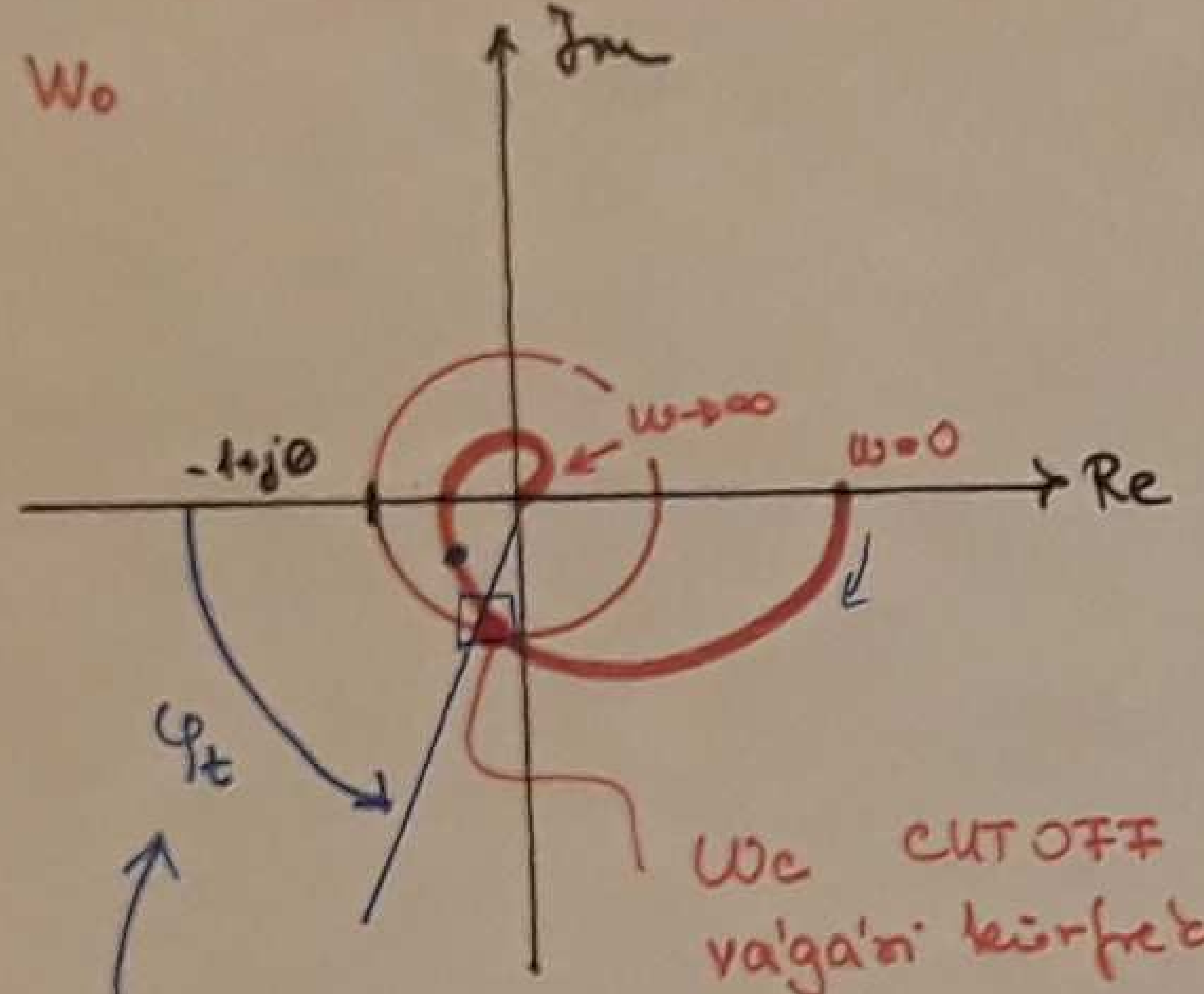
$Z - P = +1$

$Z - 2 = 1$

$\boxed{Z = 3}$

BODE- KRITÉRIUM

Az egyszerűsített Nyquist-kritérium megfogalmazása a Bode-diagram segítségével.

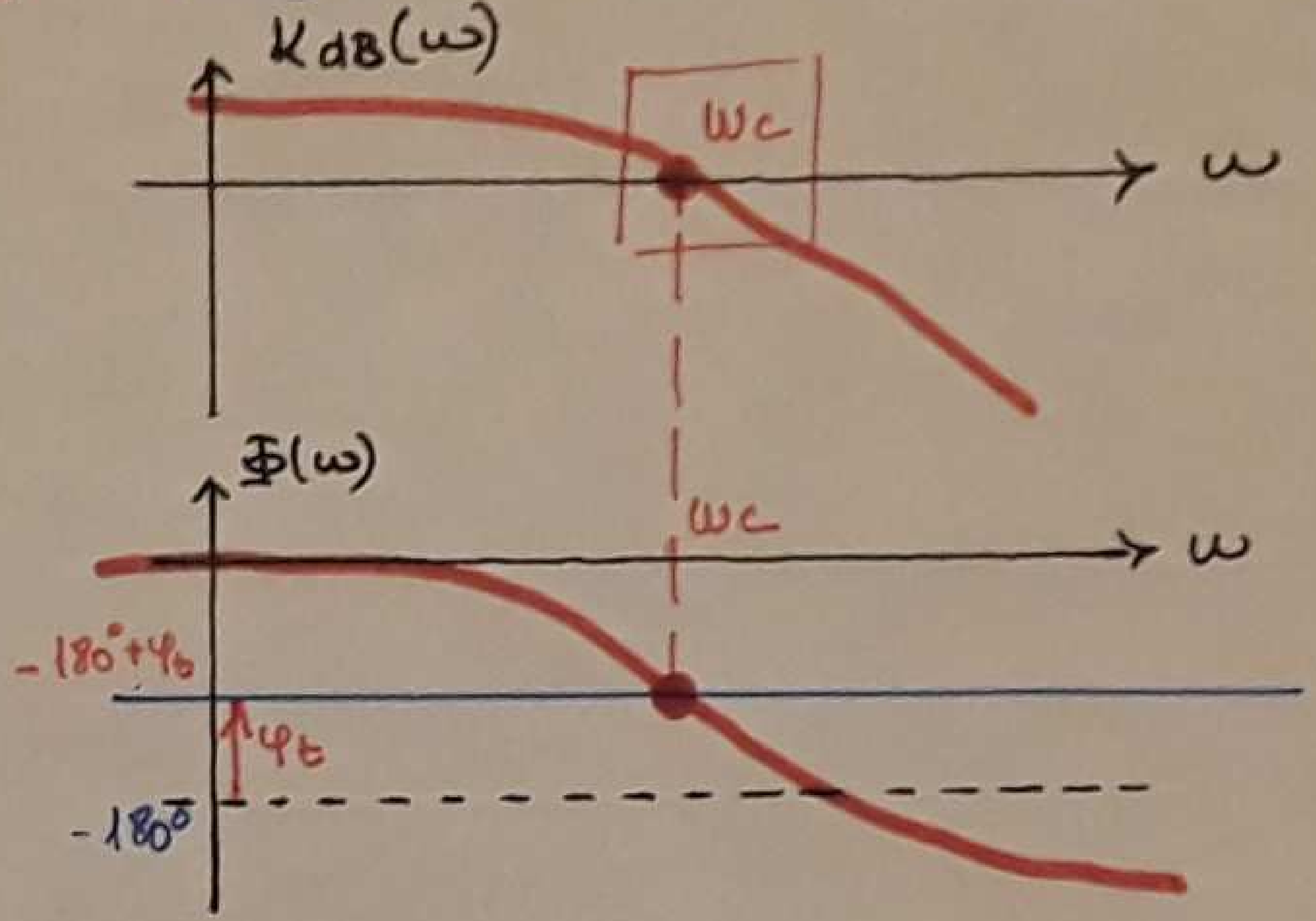


ωc CUTOFF
vágási körfrekvencia

fázistartalék

$|W_0(j\omega_c)| = 1$

$\lg 1 = 0dB$



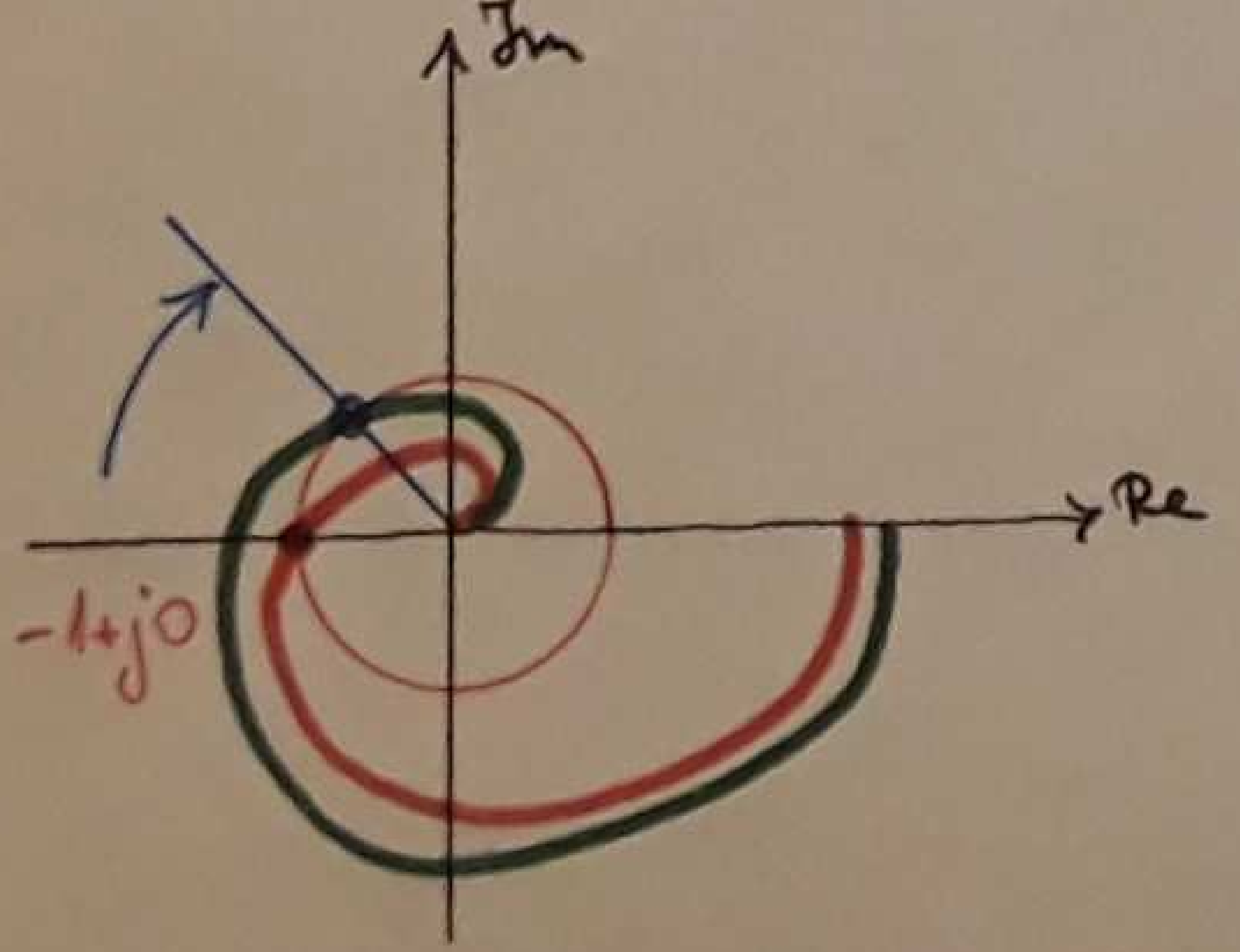
$$W_0 = \frac{1}{(1+s)(1+10s)}$$

$\boxed{\varphi_t = 60°}$

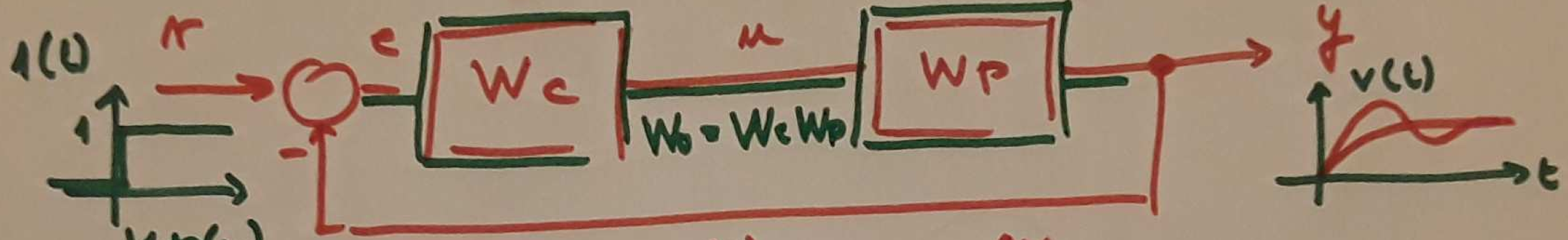
$\underbrace{\omega_c \uparrow}$ $\varphi_t \downarrow$ STABIL { LABILIS

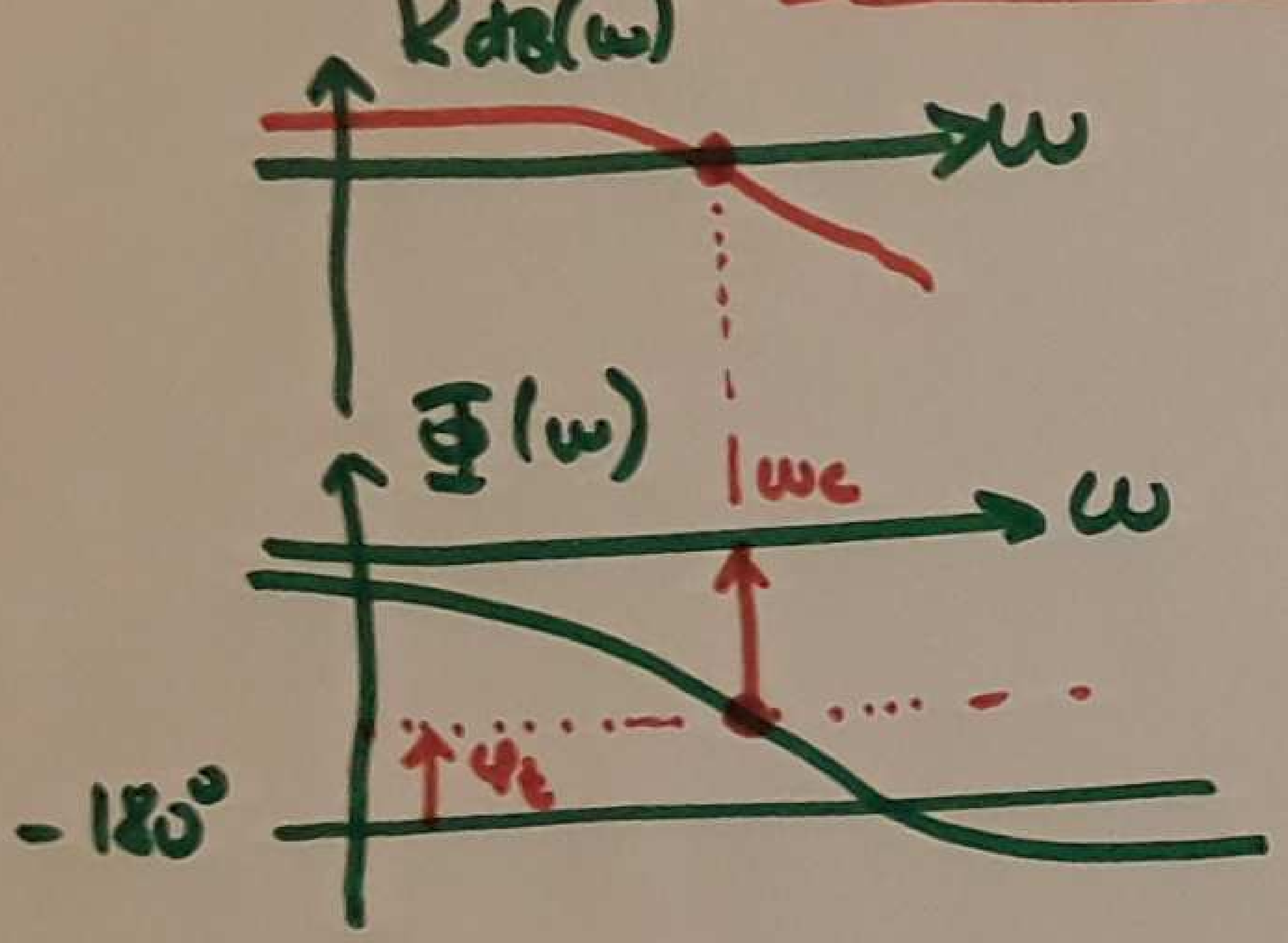
lengő $\Delta v \uparrow$

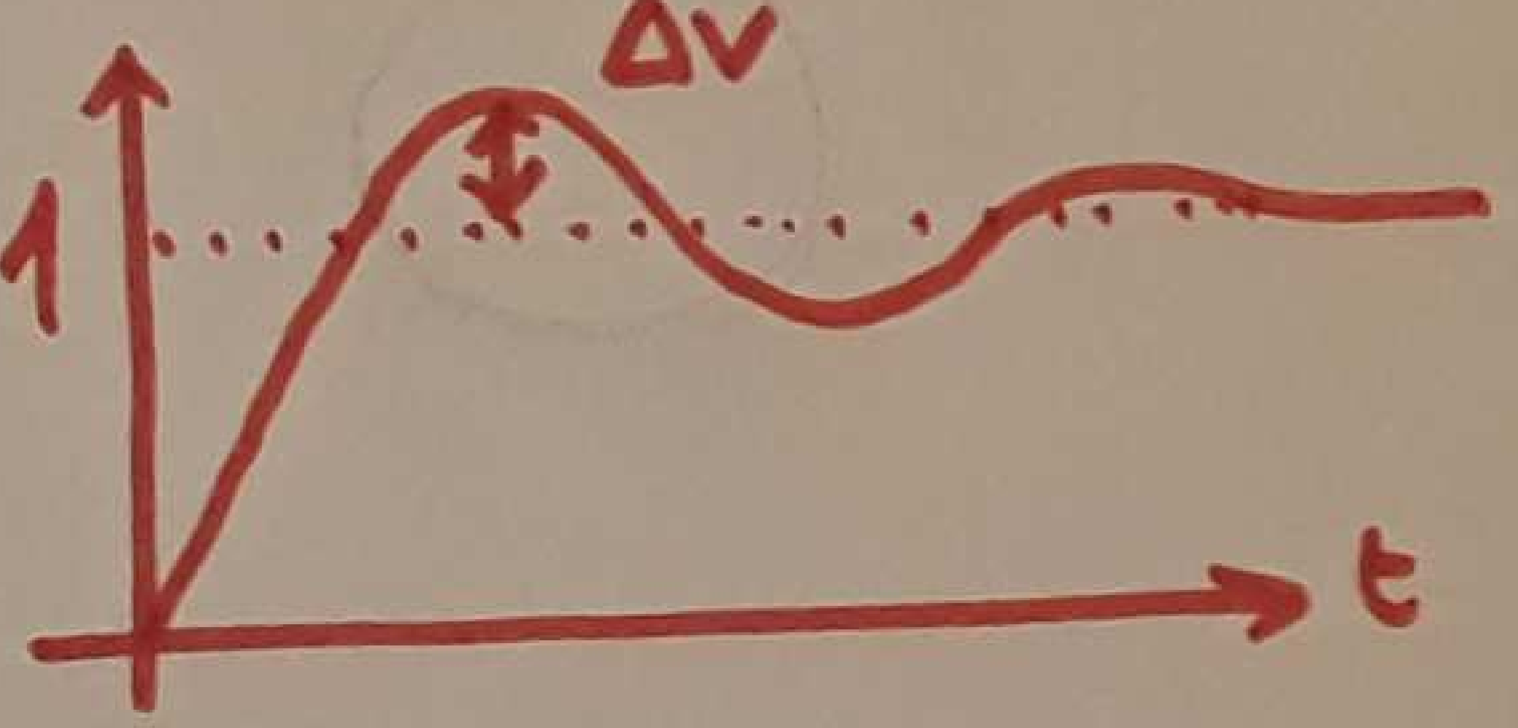
GYORSITÁS



$\boxed{\omega_c \uparrow \text{ gyorsul.}}$ $\Delta v \uparrow$


1(t)
1
r
e
-
Wc
u
Wo = WcWp
Wp
y
v(t)
t


KdB(ω)
ω
Φ(ω)
ωc
ω
-180°
φt


v(t)
Δv
1
t

A ZÁRT RENDSZER Δv TÚLLÖVÉSE ÉS A FELNYITOTT KÖR FÁZISTÖBBLETE KÖZTI KAPCSOLAT

példa : $W_0 = \frac{K}{s(1+s)} \Rightarrow W = \frac{W_0}{1+W_0} = \frac{K}{s(1+s)+K} = \frac{K}{s^2+s+K} = \frac{1}{1+\underline{\frac{1}{K}}s+\underline{\frac{1}{K}}s^2} = \frac{1}{1+\underline{2\xi}\tau s+\underline{\tau^2}s^2}.$

$$\Rightarrow \begin{cases} \tau = \frac{1}{\sqrt{K}} \\ \xi = \frac{1}{2\sqrt{K}} \end{cases}$$

1.) ZÁRT RENDSZER Δv előírás

$$\xi = \frac{1}{\sqrt{1+\pi^2/\ln^2 \underline{\Delta v}}} \Rightarrow K = \frac{1}{4\xi^2} \Rightarrow W_0(s).$$

2.) FELNYITOTT KÖR ÁTVITELI KARAKTERISZTIKÁJÁBÓL:

$$|W_0| = 1 = \frac{K}{\sqrt{\omega_c^4+\omega_c^2}} \Rightarrow \omega_c^4+\omega_c^2-K^2 = 0$$

$$\omega_c^2 = \frac{-1+\sqrt{1+4K^2}}{2}$$

$$\Phi(\omega_c) = -180^\circ + \varphi_t$$

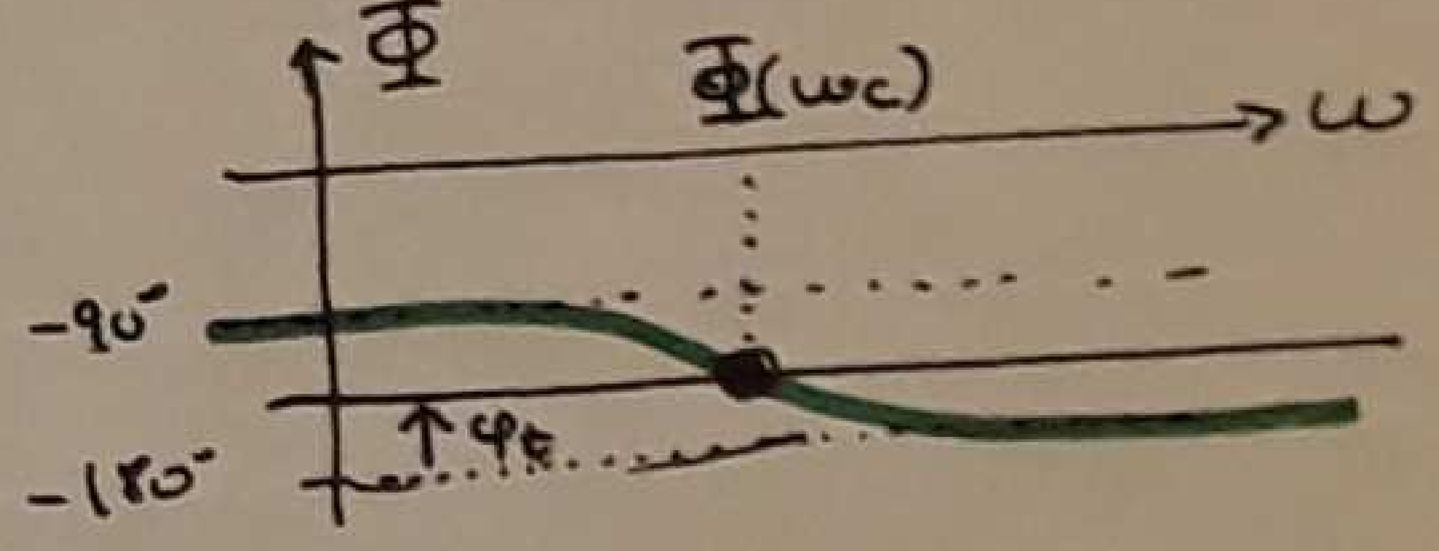
$$\varphi_t = 180^\circ + \Phi(\omega_c)$$

$$\varphi_t = 180^\circ + (-90^\circ - \operatorname{arctg} \omega_c)$$

Δv_z	ξ	K	ω_c	φ_t
1%	0,826	0,366	0,346	70,9°
5%	0,690	0,525	0,474	64,6°
10%	0,591	0,715	0,611	$\underline{58,6°}$
20%	0,456	1,202	0,896	48,1°
50%	0,215	5,386	2,216	24,3°
Δv↑	ξ↓	K↑	ω_c↑	φ_t↓

$$W_0(j\omega) = \frac{K}{j\omega(1+j\omega)} = \frac{K}{-\omega^2+j\omega}$$

$$\omega_c:)\ 0^\circ - \underline{(-90^\circ - \operatorname{arctg}\omega)}$$



K↑ Δv↑ ξ↓

ω_c↑ φ_t↓

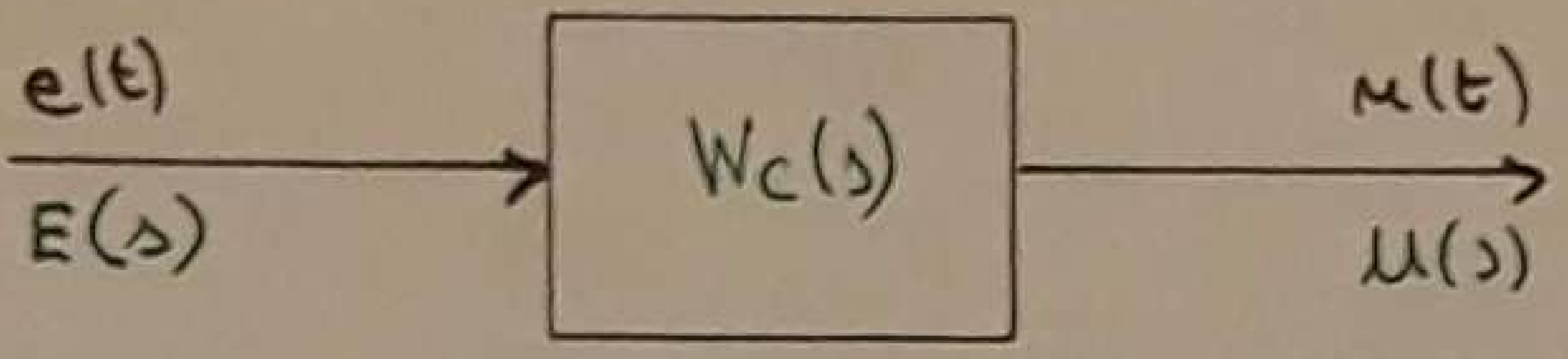
$$\boxed{\varphi_t = 60^\circ \quad \Delta v = 10\%}$$

A PID-CSALÁD ELEMEI

$v(t) = \mathcal{L}^{-1}\{V(s)\}$

$\uparrow$

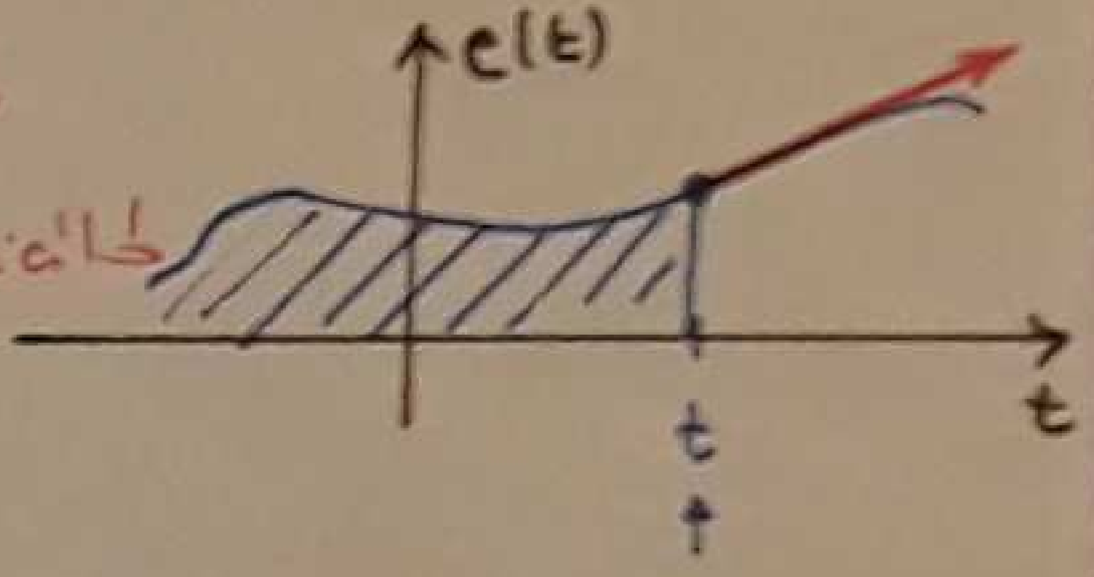
$\frac{1}{s}W(s)$

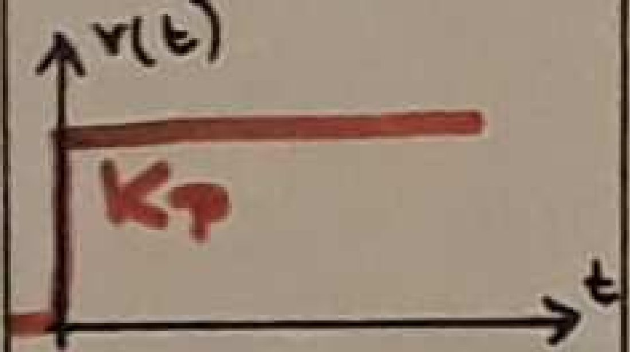
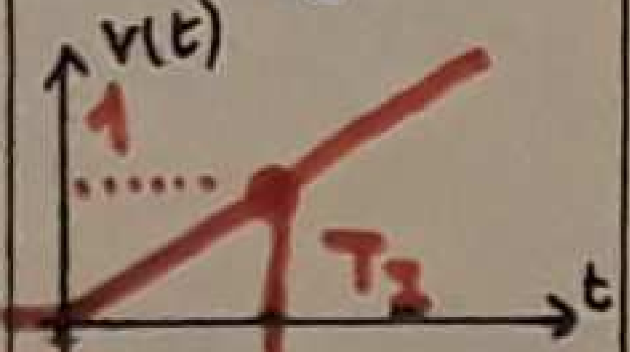
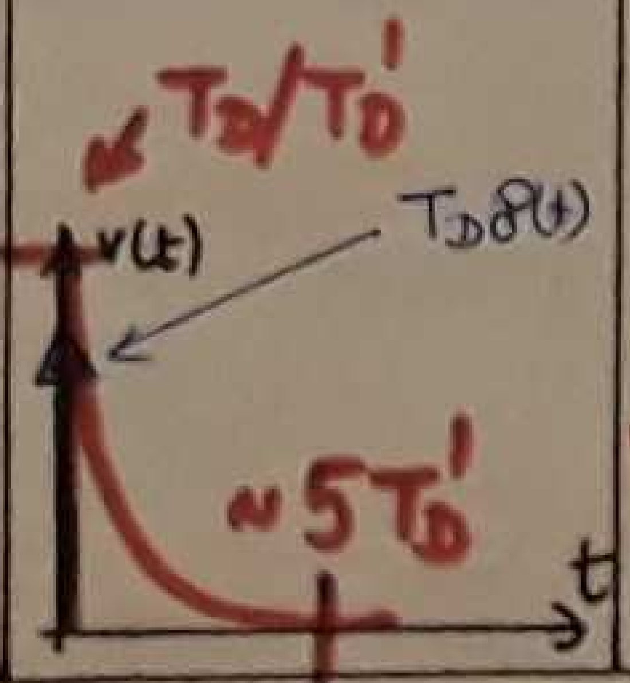


P - proporcionális (arányos)

I - integráló

D - differenciáló



	e(t) → u(t)	Wc(s)	v(t)	
P	$u(t) = K_P e(t)$	$U = K_P E$ $\frac{U}{E} = K_P$	$V = \frac{1}{s} K_P$ $v(t) = K_P 1(t)$ 	
I	$u(t) = K_I \int_{-\infty}^{t} e(\tau)d\tau$	$U = K_I E \frac{1}{s}$ $\frac{U}{E} = \frac{K_I}{s} = \frac{1}{sT_I}$	$V = \frac{1}{s^2}\frac{1}{T_I}$ $v(t) = \frac{1}{T_I} 1(t) t$ 	
D	$u(t) = K_D \frac{de(t)}{dt}$ $\frac{T_D}{T_D'} \leq 10$ ✓	$U = K_D E s$ $= E s T_D$ $\frac{U}{E} = sT_D = \frac{sT_D}{1}$ $sT_D \cong \frac{sT_D}{1+sT_D'}$ $T_D' << T_D$	$V = T_D$ $v(t) = T_D \delta(t)$ 	$V = \frac{T_D}{1+sT_D'}$ $= \frac{T_D}{T_D'} \frac{1}{s+\frac{1}{T_D'}}$ $v(t) = 1(t) \underbrace{\frac{T_D}{T_D'}} e^{-t/T_D'}$

$$W_{PID} = K\left(1 + \frac{1}{sT_I} + \underline{\underline{sT_D}}\right) =$$

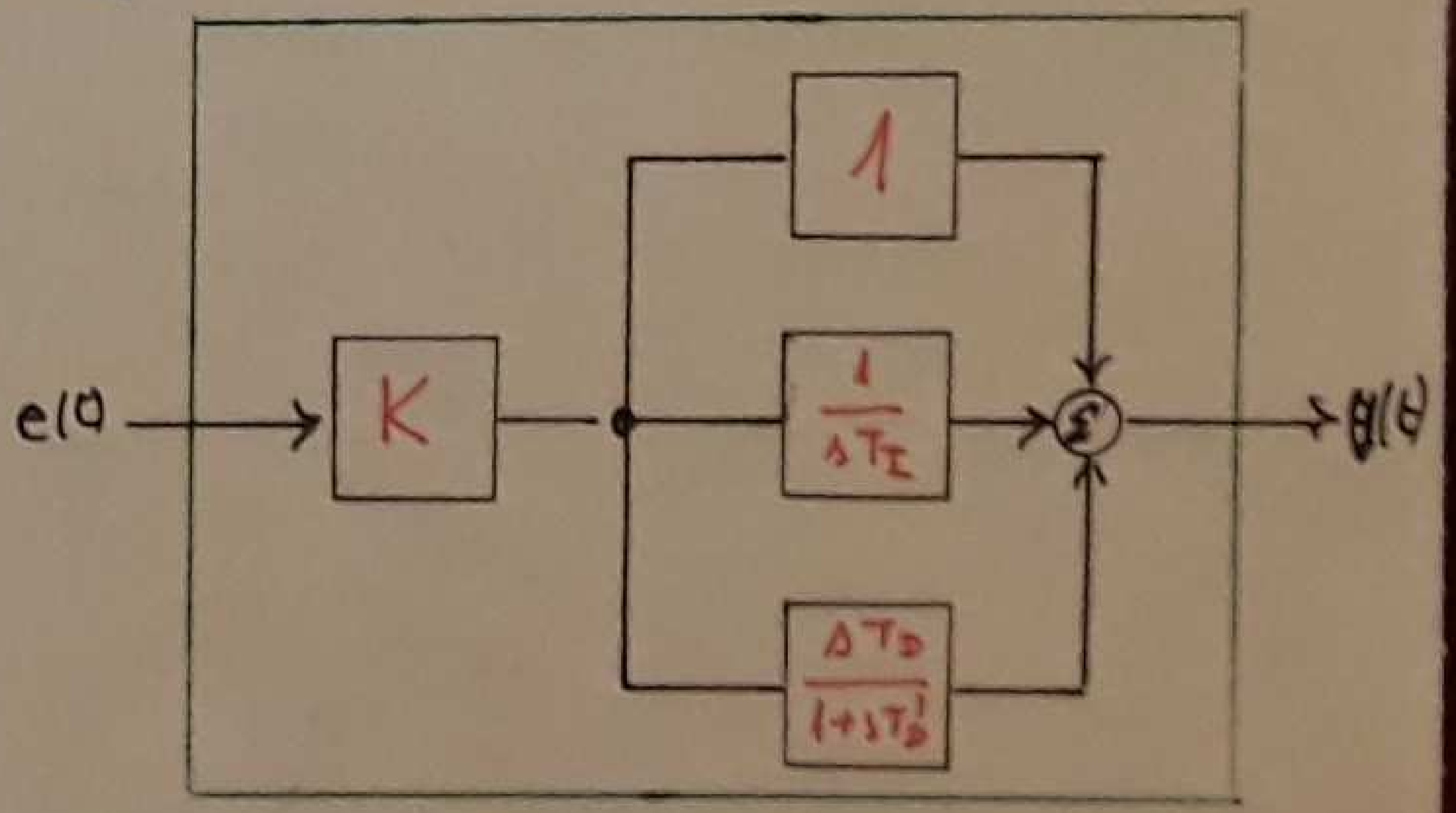
$$= K\frac{sT_I + 1 + s^2T_IT_D}{sT_I} = K\frac{1 + sT_I + s^2T_IT_D}{sT_I}$$

$$\tilde{W}_{PID} = K\left(1 + \frac{1}{sT_I} + \frac{sT_D}{1+sT_D'}\right) = K\frac{sT_I(1+sT_D') + (1+sT_D') + sT_DsT_I}{sT_I(1+sT_D')}$$

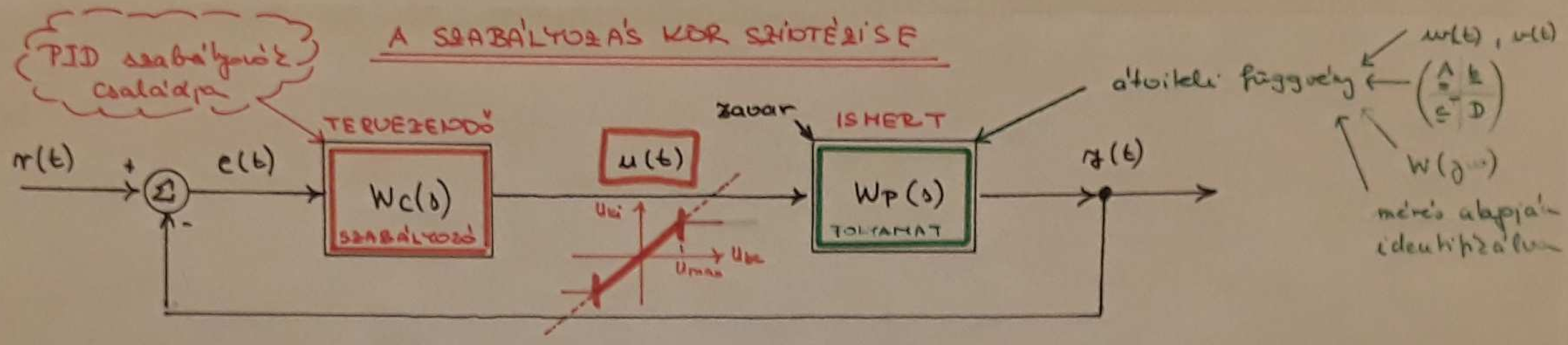
$$= K\frac{sT_I + s^2T_IT_D' + 1 + sT_D' + s^2T_DT_I}{sT_I(1+sT_D')} = K\frac{1+s(T_I+T_D') + s^2T_I(T_D+T_D')}{sT_I(1+sT_D')}$$

$$\tilde{\tilde{W}}_{PID} = K \underbrace{\frac{1+sT_I}{sT_I}}_{PI} \cdot \underbrace{\frac{1+sT_D}{1+sT_D'}}_{PD}$$

PI - PD

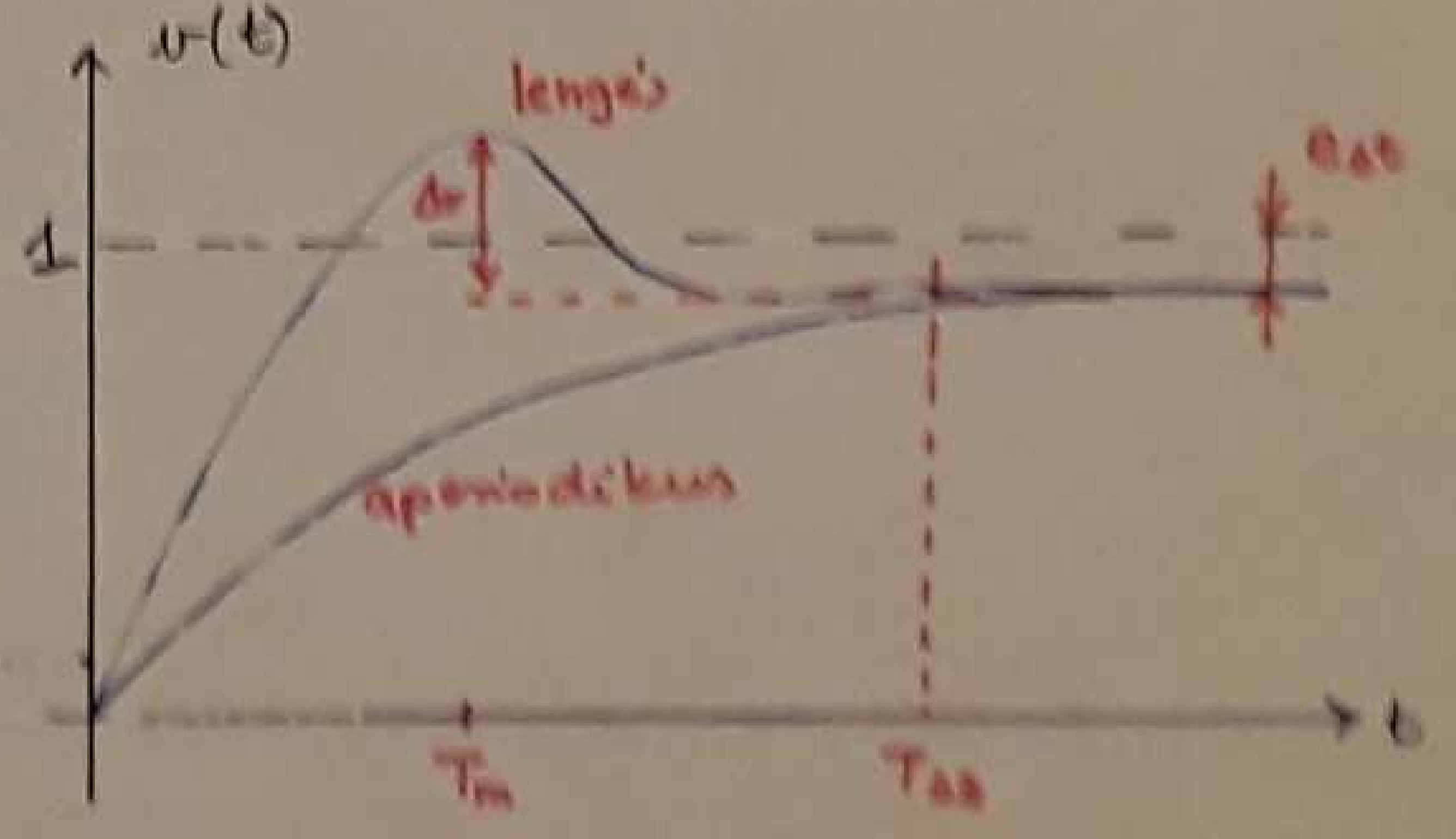


A SZABÁLYOZÁS KÖR SZINTÉZISE



Szintézis (méretezés, tervezés) feladata: a zavaró hatásoknak kitett folyamat olyan u(t) beavatkozó jelének a meghatározása, amellyel a rendszer valamilyen minőségi kritérium szerint optimálisan hozható a kívánt állapotba.

- stabilitás
- statikus pontosság alapjelkövetésre
- zavarelhárítás
- zajelnyomás
- előírt tranziensviselkedés, gyorsaság
- robusztusság
- megvalósíthatóság < analóg / digitális



Nyquist ↔ Bode

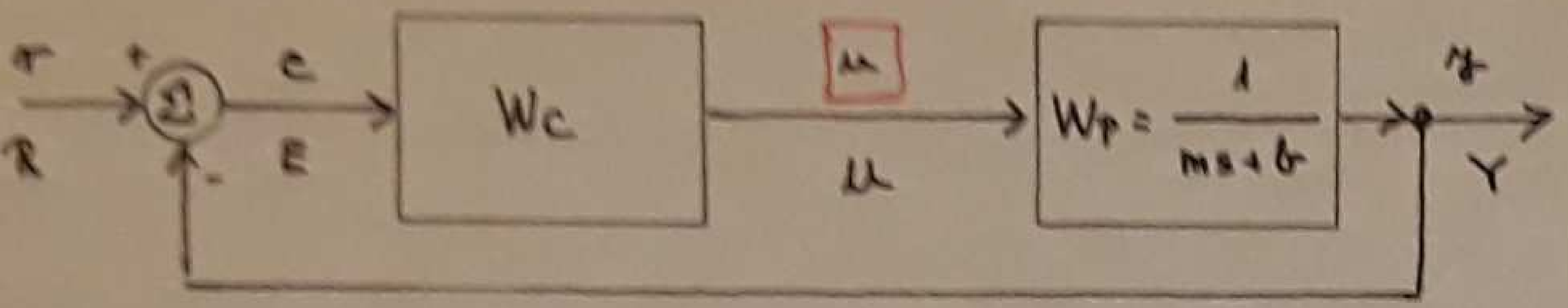
Szabályozó: struktúra + paraméterek

$1 + 2\xi\tau s + \tau^2 s^2$

PI ⇒ $K\left(1 + \frac{1}{sT_I}\right) = K\frac{1+sT_I}{sT_I}$

PD ⇒ $K\left(1 + \frac{sT_D}{1+sT_D'}\right) = K\frac{1+s\overset{T_D}{\boxed{(T_D+T_D')}}}{1+sT_D'}$

Egy egyszerű példa a működés illusztrálására.



jármű modell, $m = 1000 \text{ kg}$

$b = 50 \frac{Ns}{m}$

$r = 10 \frac{m}{s}$

$T_{rise} < 5s$

$\Delta v < 10\%$

$e_{st} < 2\%$

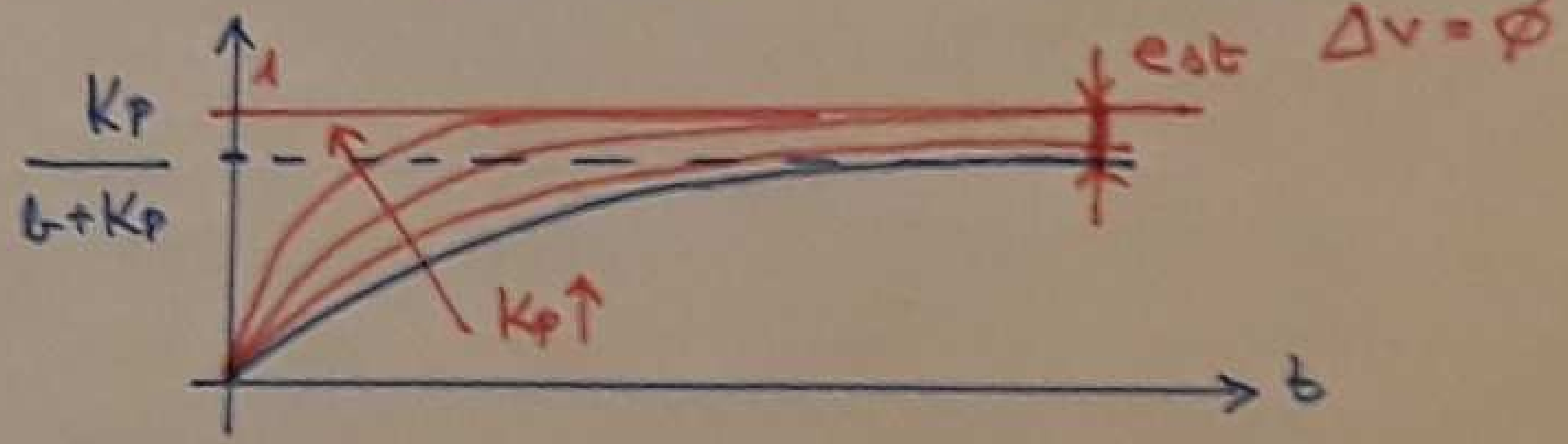
---

a.) **P-szabályozó : $W_c = K_p$**

$$W = \frac{W_o}{1+W_o} = \frac{K_p \frac{1}{ms+b}}{1 + K_p \frac{1}{ms+b}} = \frac{K_p}{ms + b + K_p} = \frac{K_p}{m} \frac{1}{s + \frac{b+K_p}{m}} \rightarrow V = \frac{K_p}{m} \frac{1}{s\left(s + \frac{b+K_p}{m}\right)}$$

$$V = \frac{K_p}{m}\left(\frac{\frac{m}{b+K_p}}{s} + \frac{-\frac{m}{b+K_p}}{s + \frac{b+K_p}{m}}\right) \Rightarrow Y = \frac{\frac{K_p}{b+K_p}}{s} - \frac{\frac{K_p}{b+K_p}}{s + \frac{b+K_p}{m}}$$

$$v(t) = \frac{K_p}{b+k_p}\left[1 - e^{-\boxed{\frac{b+K_p}{m}}t}\right] 1(t)$$



$$e_{st} = 1 - \frac{K_p}{b+K_p} = \frac{b}{b+K_p} \leq 0{,}02$$

$$b \leq 0{,}02 b + 0{,}02 K_p$$

$$0{,}98 b \leq 0{,}02 K_p$$

$$K_p \geq \frac{0{,}98 b}{0{,}02} = \underline{\underline{2450}}$$

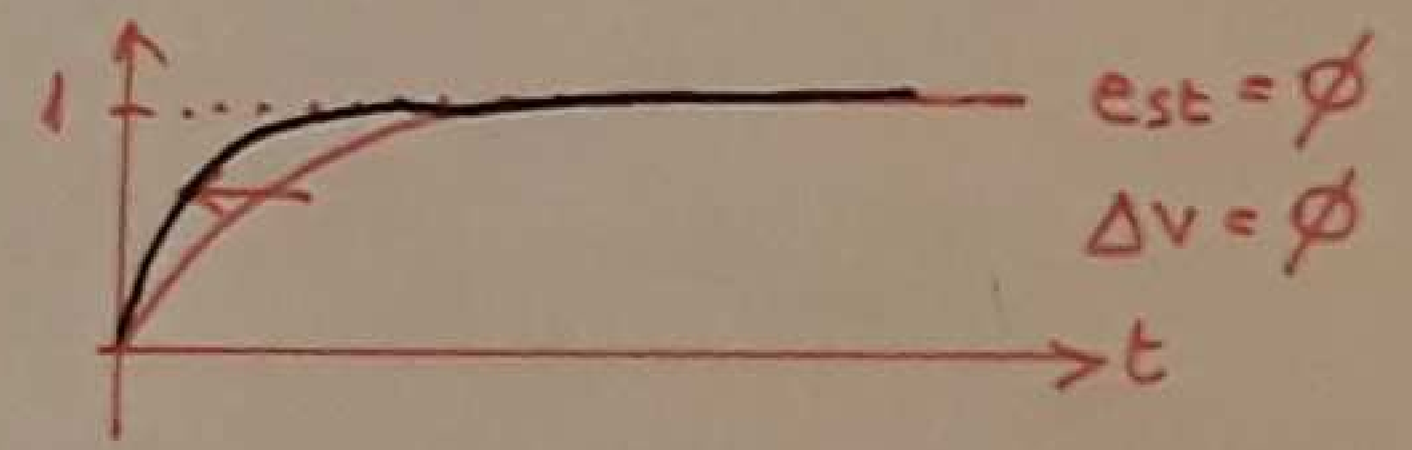
b.) PI - szabályozó : $W_C = K_{PI} \frac{1+sT_I}{sT_I}$

$$W_0 = K_{PI} \frac{1+sT_I}{sT_I} \cdot \frac{1}{ms+b} = K_{PI} \frac{\cancel{1+sT_I}}{sT_I} \cdot \frac{1/b}{\cancel{1+s\frac{m}{b}}} = K_{PI} \frac{1}{\boxed{s}T_I} \frac{1}{b} \qquad T_I = \frac{m}{b} = \underline{\underline{20}}$$

$$W = \frac{W_0}{1+W_0} = \frac{\frac{K_{PI}}{bsT_I}}{1+\frac{K_{PI}}{bsT_I}} \cdot \frac{bsT_I}{bsT_I} = \frac{K_{PI}}{bsT_I + K_{PI}} = \frac{K_{PI}}{bT_I} \frac{1}{s+\frac{K_{PI}}{bT_I}}$$

$$V = \frac{1}{s} W = \frac{K_{PI}}{bT_I} \frac{1}{s\left(s+\frac{K_{PI}}{bT_I}\right)} = \frac{K_{PI}}{bT_I} \left(\frac{\frac{bT_I}{K_{PI}}}{s} + \frac{-\frac{bT_I}{K_{PI}}}{s+\frac{K_{PI}}{bT_I}} \right) = \frac{1}{s} - \frac{1}{s+\frac{K_{PI}}{bT_I}}$$

$$v(t) = 1 \cdot 1(t) \left[1 - e^{-\frac{\boxed{K_{PI}}}{bT_I} t} \right].$$



$$t_2 - t_1 = \frac{bT_I}{K_{PI}} \ln 9 < 5$$

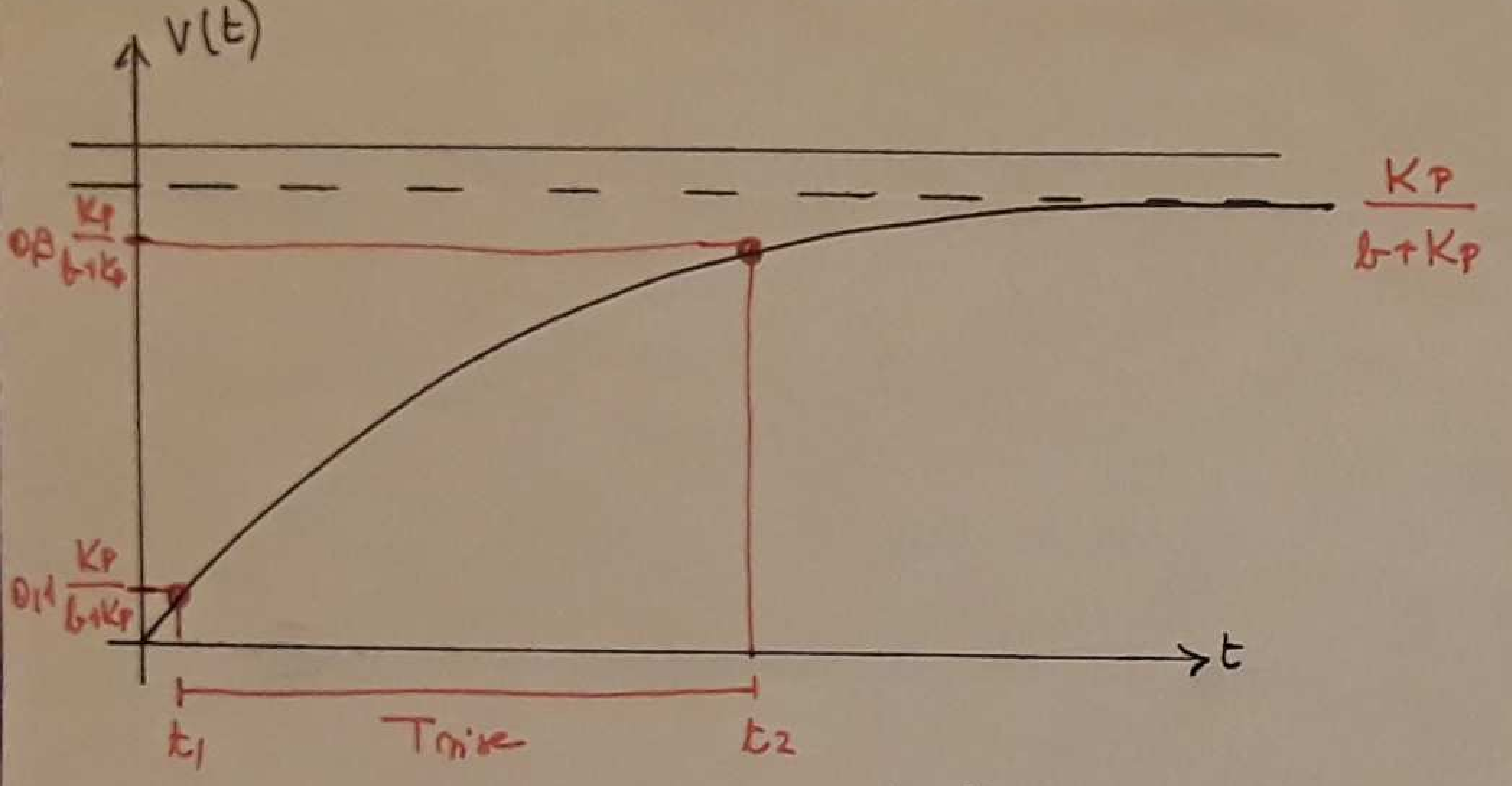
$$K_{PI} > \frac{bT_I \ln 9}{5} = 439{,}44 \approx \underline{\underline{440}} \quad (2450)$$

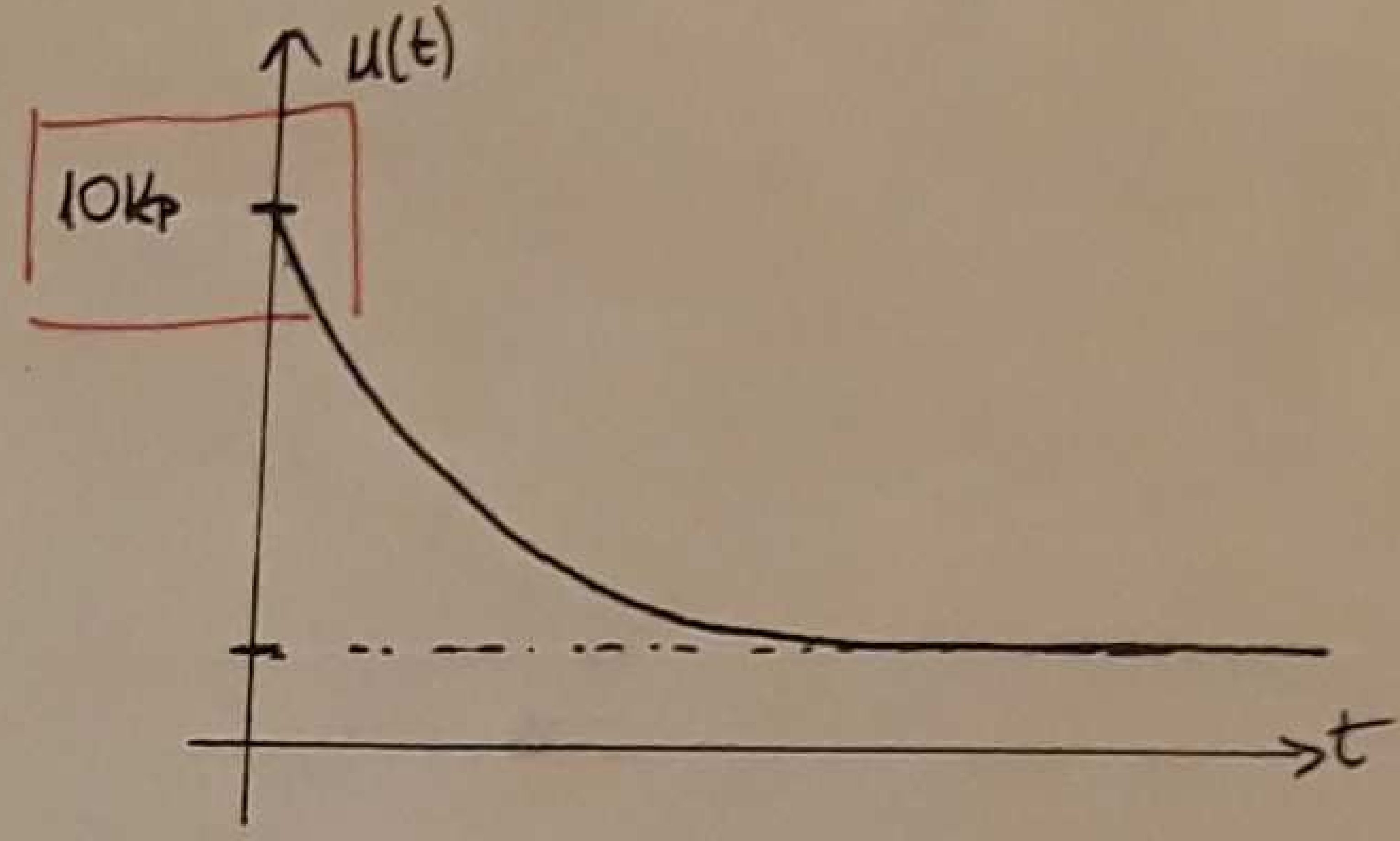
$$\frac{\boxed{\frac{1}{s}} + T_I}{T_I}$$

$$U = R \frac{W_C}{1+W_0} = \frac{10}{s} \frac{K_{PI} \frac{1+sT_I}{sT_I}}{1+\frac{K_{PI}}{b}\frac{1}{sT_I}} \Rightarrow$$

$$u(0) = \lim_{s\to\infty} s\,U(s) = \lim_{s\to\infty} \cancel{s} \frac{10}{\cancel{s}} \frac{K_{PI}\frac{1+sT_I}{sT_I}}{1+\boxed{\frac{K_{PI}}{b}\frac{1}{sT_I}}} = \underline{\underline{10 K_{PI}}}$$

$$u(\infty) = \lim_{s\to 0} s\,U(s) = \lim_{s\to\cancel{\infty}} \cancel{s} \frac{10}{\cancel{s}} \frac{K_{PI}\frac{1+sT_I}{sT_I}}{1+\frac{K_{PI}}{b}\frac{1}{sT_I}} = \lim_{s\to 0} \frac{10K_{PI}\frac{1+\boxed{sT_I}}{T_I}}{\boxed{s}+\frac{K_{PI}}{bT_I}}$$





$$0{,}1 \frac{K_p}{\cancel{b+K_p}} = \frac{K_p}{\cancel{b+K_p}}\left(1-e^{-\frac{b+K_p}{m}t_1}\right) \Rightarrow 0{,}9 = e^{-\frac{b+K_p}{m}t_1} \Rightarrow$$

$$0{,}9 \frac{K_p}{\cancel{b+K_p}} = \frac{K_p}{\cancel{b+K_p}}\left(1-e^{-\frac{b+K_p}{m}t_2}\right) \Rightarrow 0{,}1 = e^{-\frac{b+K_p}{m}t_2} \Rightarrow$$

$$\boxed{\begin{aligned} t_1 &= -\frac{m}{b+K_p}\ln 0{,}9 \\ t_2 &= -\frac{m}{b+K_p}\ln 0{,}1 \end{aligned}}$$

$$T_{rise} = t_2 - t_1 = \frac{m}{b+\boxed{K_p}}\ln 9$$

pl. $T_{rise} = \frac{1000}{50+2450}\ln 9 = 0{,}87s \leq 5s$ ✓

~~$u = R$~~ $\quad u = R\frac{W_c}{1+W_0} = R\frac{K_p}{1+K_p\frac{1}{ms+b}}$

$$u(0) = \lim_{s\to\infty} s\,U(s) = \lim_{s\to\infty} \cancel{s}\frac{10}{\cancel{s}}\frac{K_p}{1+K_p\frac{1}{ms+b}} = 10K_p$$

$$u(\infty) = \lim_{s\to 0} s\,U(s) = \lim_{s\to 0} \cancel{s}\frac{10}{\cancel{s}}\frac{K_p}{1+K_p\frac{1}{ms+b}} = \frac{10K_p}{1+\frac{K_p}{b}} = \frac{10K_pb}{K_p+b}.$$

$$\frac{\frac{10\cancel{K_{PI}}}{\cancel{T_I}}}{\frac{\cancel{K_{PI}}}{b\cancel{T_I}}} = \underline{\underline{10b}}$$

$$\frac{10K_{PI}}{T_I}\,\frac{bT_I}{K_{PI}} = 10b.$$

$$\frac{K_{PI}\,\frac{1+sT_I}{sT_I}}{1+\frac{K_{PI}}{b}\,\frac{1}{sT_I}} = \frac{K_{PI}\,(1+\cancel{sT_I})}{\cancel{sT_I}+\frac{K_{PI}}{b}} = b.$$

c.) PD - szabályozó: $W_C = K_{PD} \dfrac{1+sT_D}{1+sT_D'}$

$$T_D = \frac{m}{b} = \frac{1000}{50} = \underline{\underline{20}}$$

$$T_D' < T_D \qquad T_D' = \underline{\underline{2}} \quad (\underline{\underline{0{,}5{,}1}})$$

$$W_0 = W_C W_P = K_{PD} \frac{\cancel{1+sT_D}}{1+sT_D'} \cdot \frac{1/b}{\cancel{1+sT/b}} = K_{PD} \frac{1/b}{1+sT_D'}$$

$$W = \frac{W_0}{1+W_0} = \frac{K_{PD}\frac{1/b}{1+sT_D'}}{1+K_{PD}\frac{1/b}{1+sT_D'}} \cdot \frac{1+sT_D'}{1+sT_D'} = \frac{\frac{K_{PD}}{b}}{1+sT_D' + K_{PD}/b} = \frac{K_{PD}}{bT_D'} \frac{1}{s+\left(\frac{1+K_{PD}/b}{T_D'}\right.}$$

$$\frac{K_{PD}}{b\cancel{T_D'}} \cdot \frac{\cancel{T_D'}}{1+\frac{K_{PD}}{b}} = \underline{\underline{\frac{K_{PD}}{K_{PD}+b}}}$$

$$V = \dots = \frac{K_{PD}}{bT_D'}\left(\frac{+\frac{T_D'}{1+K_{PD}/b}}{s} + \frac{-\frac{T_D'}{1+K_{PD}/b}}{s+\frac{1+K_{PD}/b}{T_D'}} \right)$$

$$v(t) = \boxed{\frac{K_{PD}}{K_{PD}+b}} \left(1 - e^{-\frac{1+K_{PD}/b}{T_D'}t}\right) 1(t) \quad \textcircled{P.} \Longrightarrow \underline{\underline{K_{PD} \geq 2450}} \quad (2\%)$$

$$t_2 - t_1 = \frac{\textcircled{T_D'}}{K_{PD}+b} \ln 9 \qquad \frac{2}{2450+50} \ln 9 = \underline{0{,}00175 \text{ sec.}}$$

ÁRAUAN!!!

$$u(0) = \lim_{s\to\infty} \cancel{s} \cdot \cancel{\frac{10}{s}} \frac{K_{PD}\frac{1+sT_D}{1+sT_D'}}{1+\frac{K_{PD}/b}{1+sT_D'}} \to 1 \qquad \left(\frac{\frac{1}{s}+T_D}{\frac{1}{s}+T_D'}\right. = \boxed{10 K_{PD}}\ \boxed{\frac{T_D}{T_D'}} \leftarrow 10\,!$$

24.500 · 10

P => 2450 · 10

PI => 440 · 10

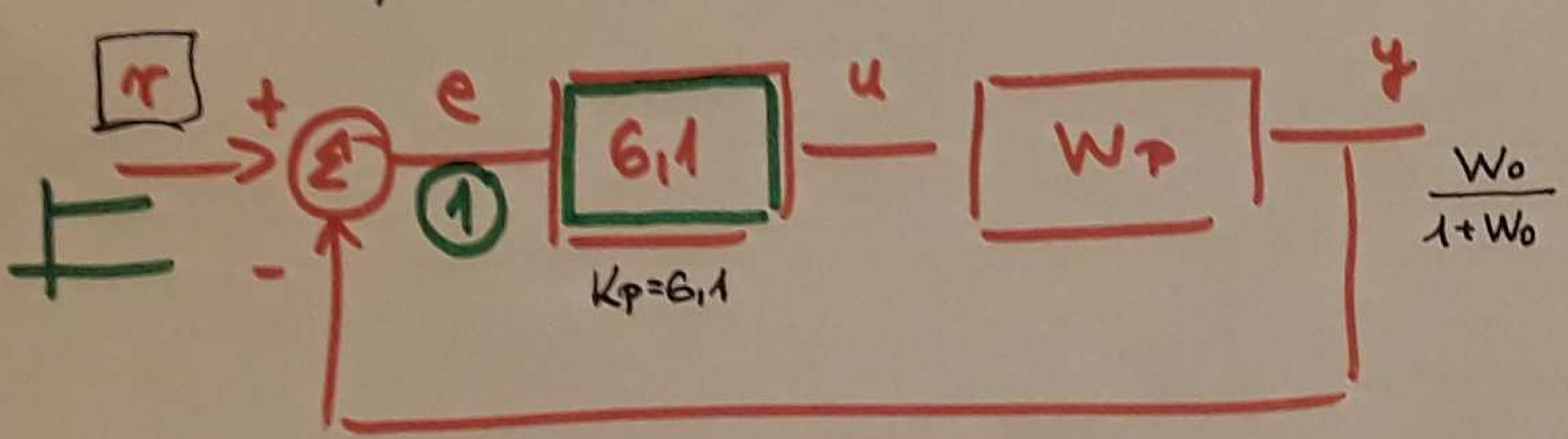
PD => 2450 · 10 · 10

A P-SZABÁLYOZÓ HANGOLÁSA

$$W_P(s) = \frac{1}{(1+s10)(1+s)(1+s0{,}2)} \qquad \varphi_t = 60° \qquad W_C(s) = K_P.$$

$$W_0 = \underset{\substack{\uparrow \\ K_P = 1}}{W_C} W_P = 1 \cdot \frac{1}{\left(1+\frac{s}{\boxed{0{,}1}}\right)\left(1+\frac{s}{\boxed{1}}\right)\left(1+\frac{s}{\boxed{5}}\right)}$$

$$W = \frac{W_0}{\boxed{1+W_0}} \longrightarrow \begin{matrix} p_1(K_P) \\ p_2(K_P) \\ p_3(K_P) \end{matrix}$$



$$-107{,}9° - 115° \lg \frac{\omega_c}{0{,}5} = -120°$$

$$135 \lg \frac{\omega_c}{0{,}5} = 12{,}1$$

$$\lg \frac{\omega_c}{0{,}5} = 0{,}089$$

$$\frac{\omega_c}{0{,}5} = 10^{0{,}089} = 1{,}23$$

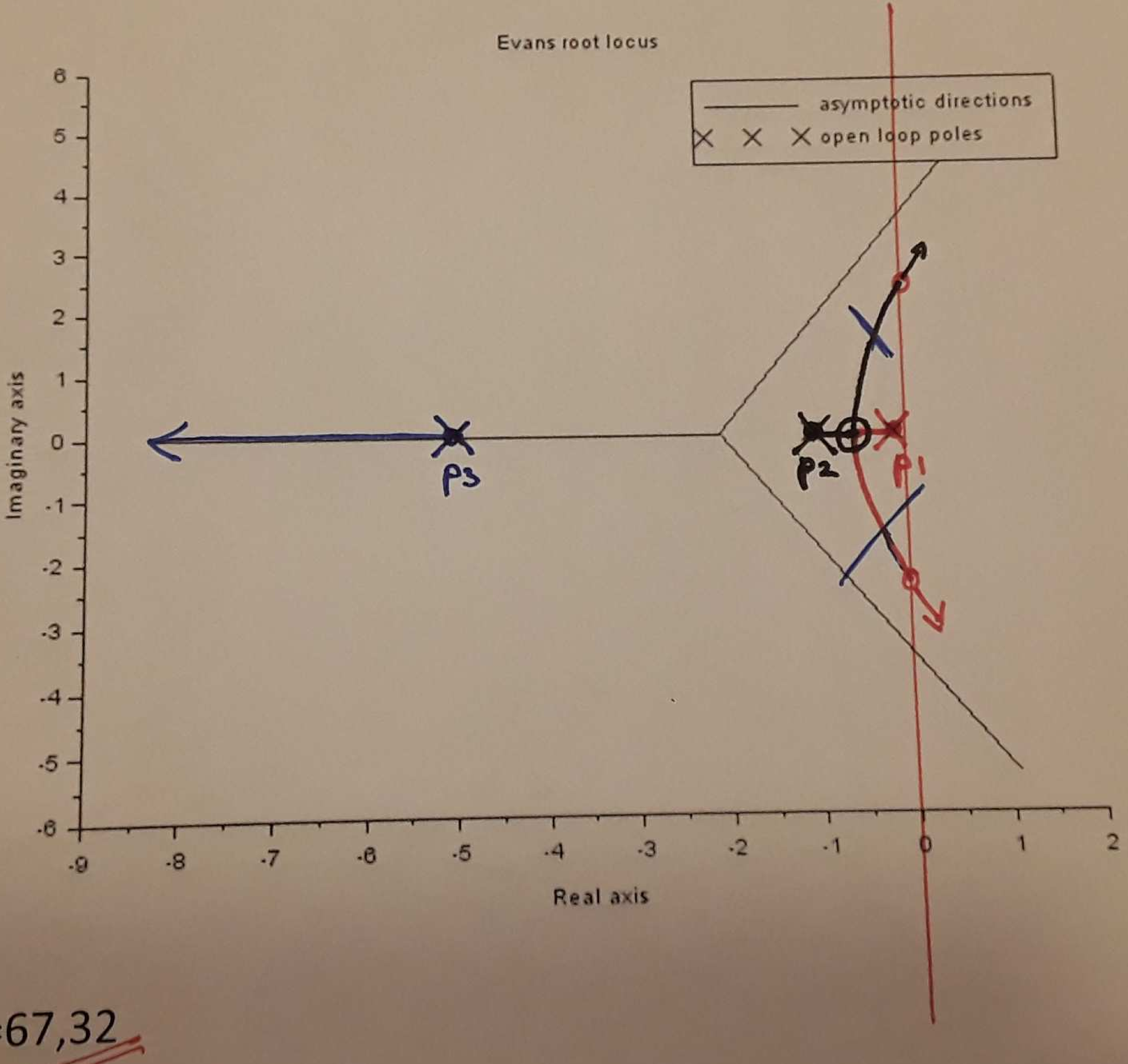
$$\omega_c = 0{,}5 \cdot 1{,}23$$

$$\boxed{\omega_c = 0{,}61}$$

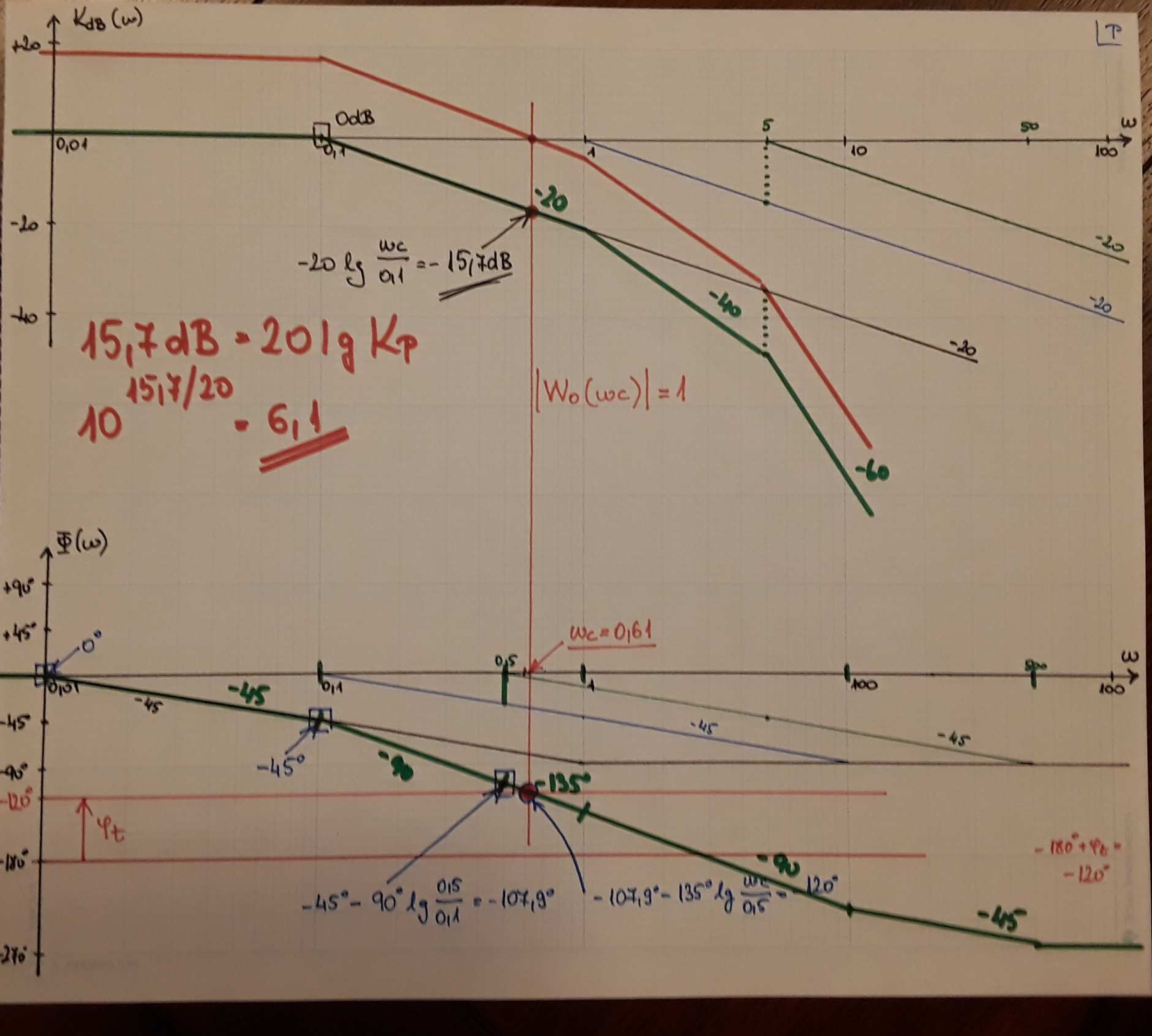
$$Y = R \frac{W_0}{1+W_0} = \frac{1}{s} \frac{W_0}{1+W_0}$$

$$U = R \frac{W_C}{1+W_0} = \frac{1}{s} \frac{W_C}{1+W_0}$$

P-szabályozó



K_{max}=67,32



$-20\lg\frac{\omega c}{0{,}1} = -15{,}7\,dB$

$15{,}7\,dB = 20\lg K_P$

$10^{15{,}7/20} = 6{,}1$

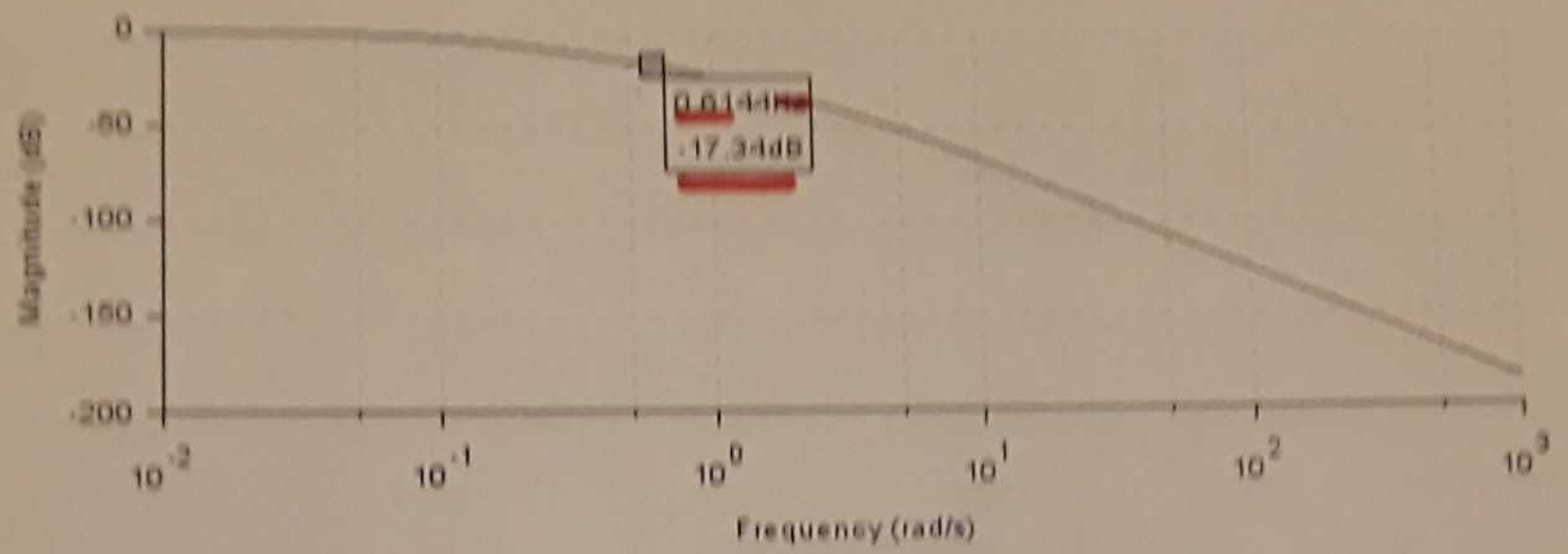
$|W_o(\omega c)| = 1$

$\omega c = 0{,}61$

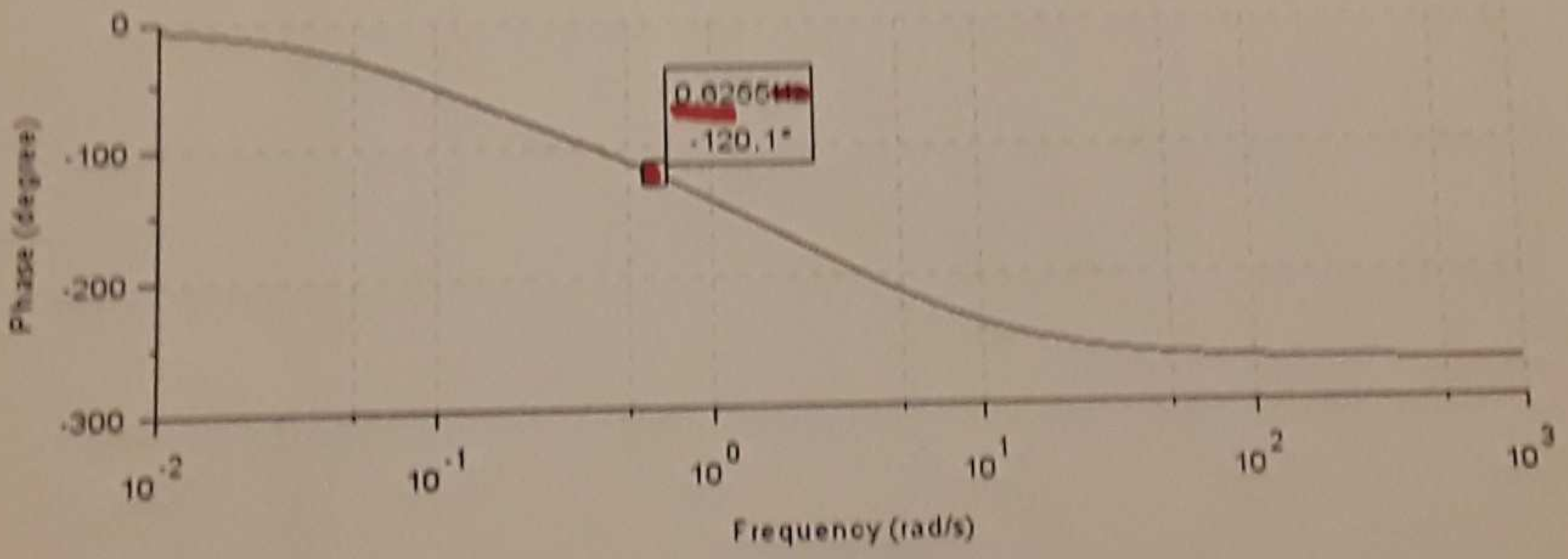
$-45° - 90°\lg\frac{0{,}5}{0{,}1} = -107{,}9°$

$-107{,}9° - 135°\lg\frac{\omega c}{0{,}5} = -120°$

$-180° + \varphi_t = -120°$



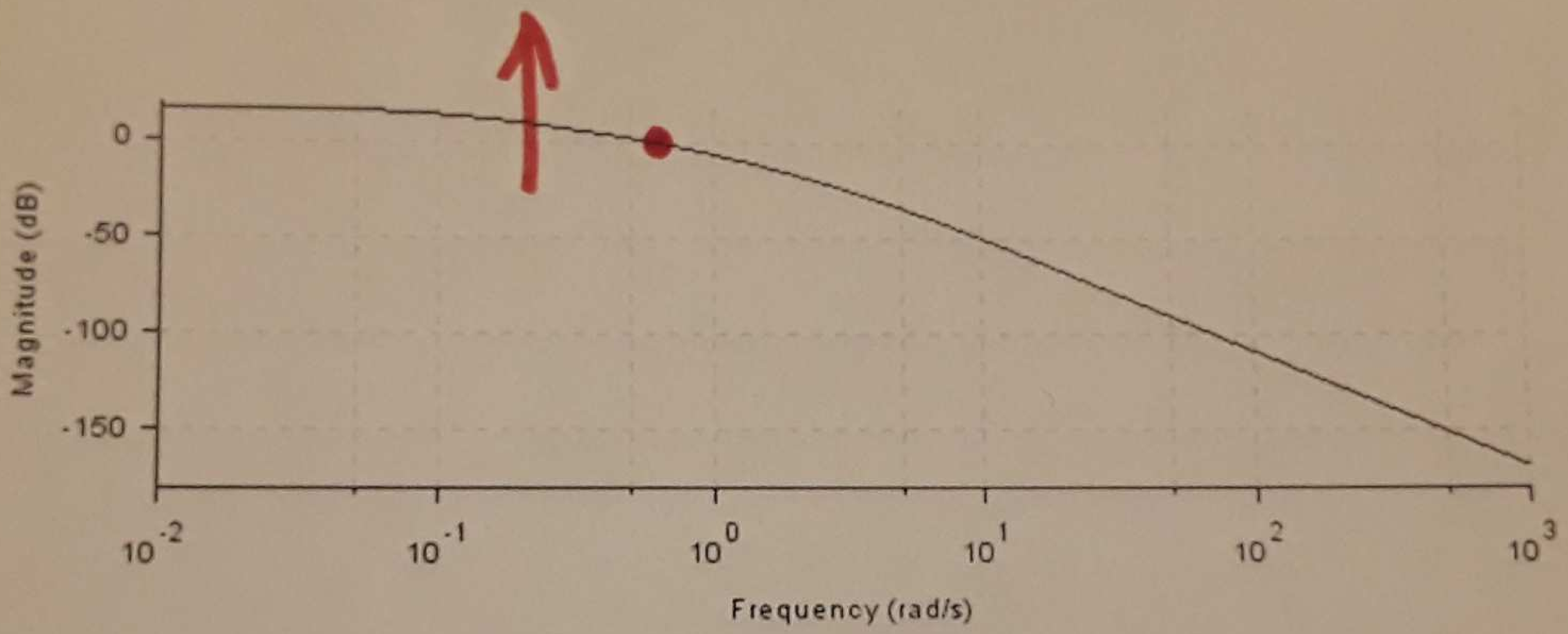
-15,7dB

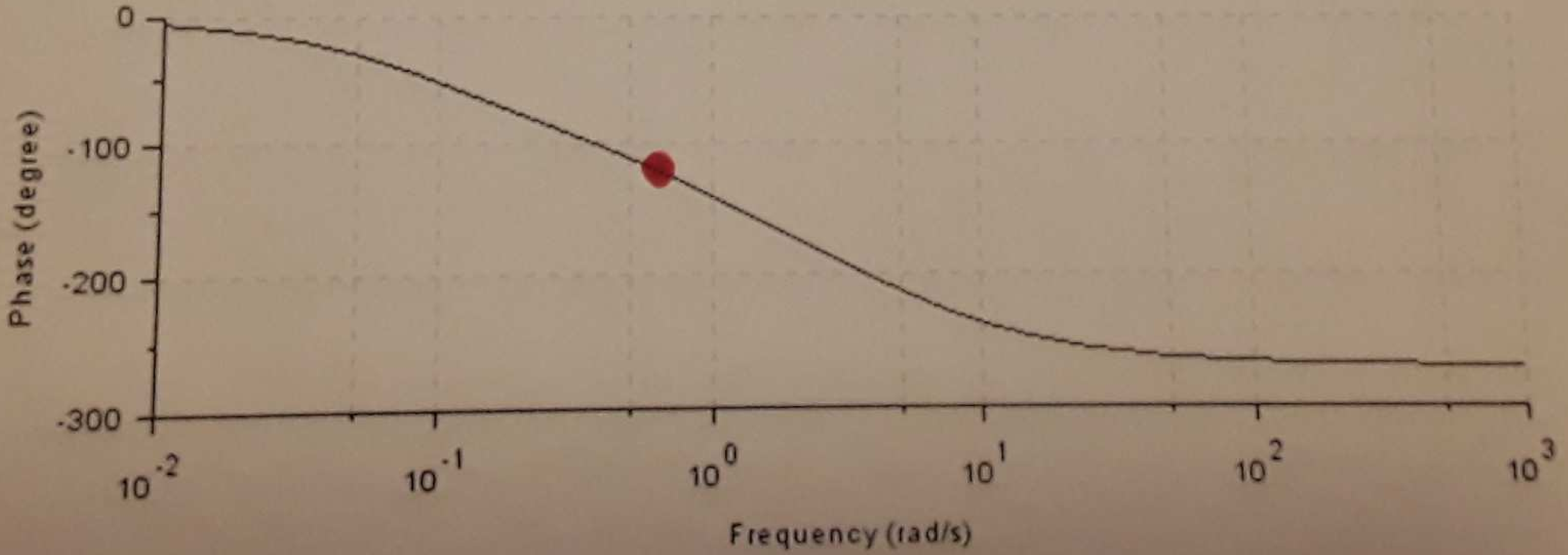


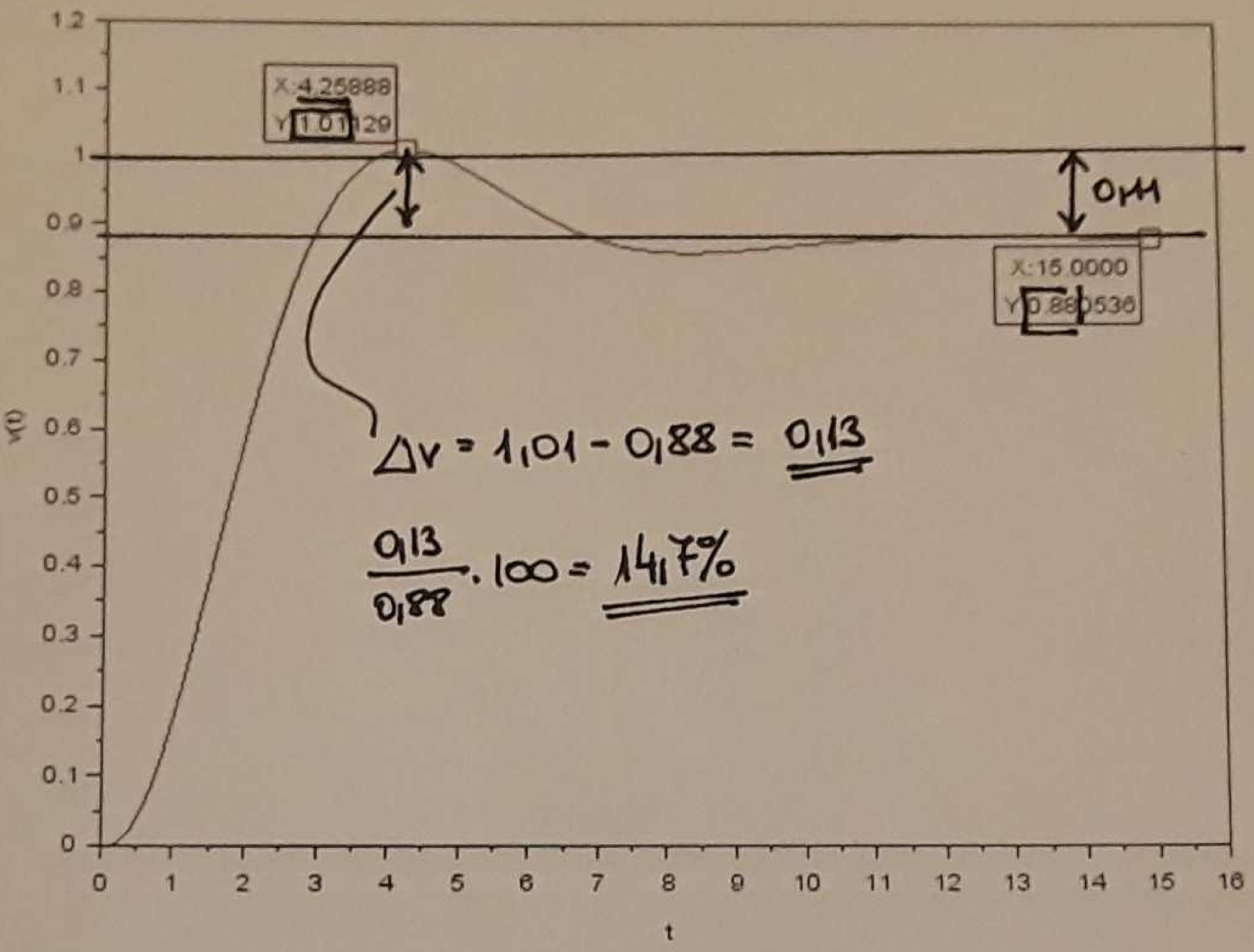
0,61

17,34dB -> 7,36

$10^{17,34/20}$


0
-50
-100
-150
Magnitude (dB)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)


0
-100
-200
-300
Phase (degree)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)



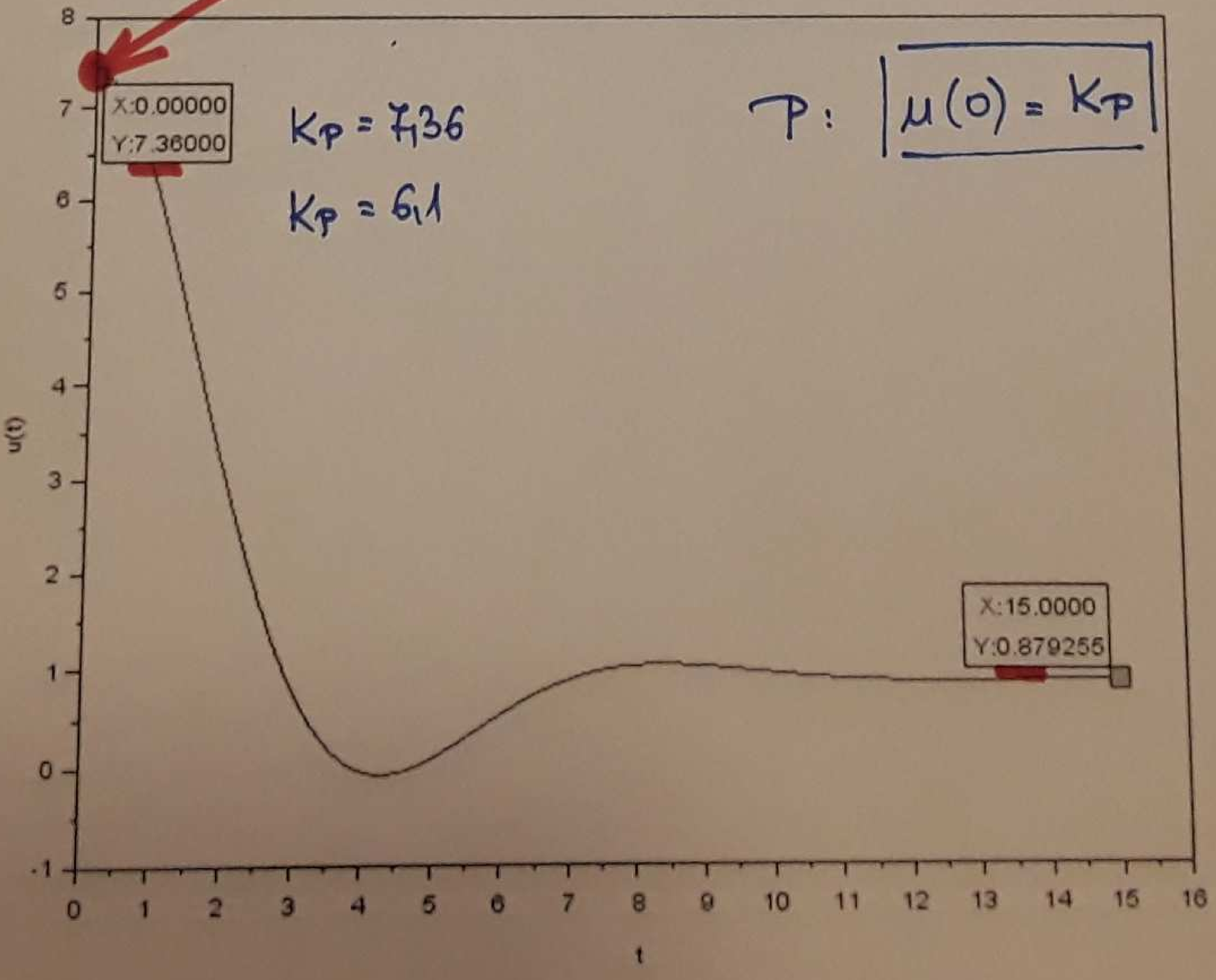
0,11

$e_{st} = \frac{1}{1+K_p} = \underline{\underline{0,14}}$

$K_p \uparrow$

$\Delta v = 1,01 - 0,88 = \underline{\underline{0,13}}$

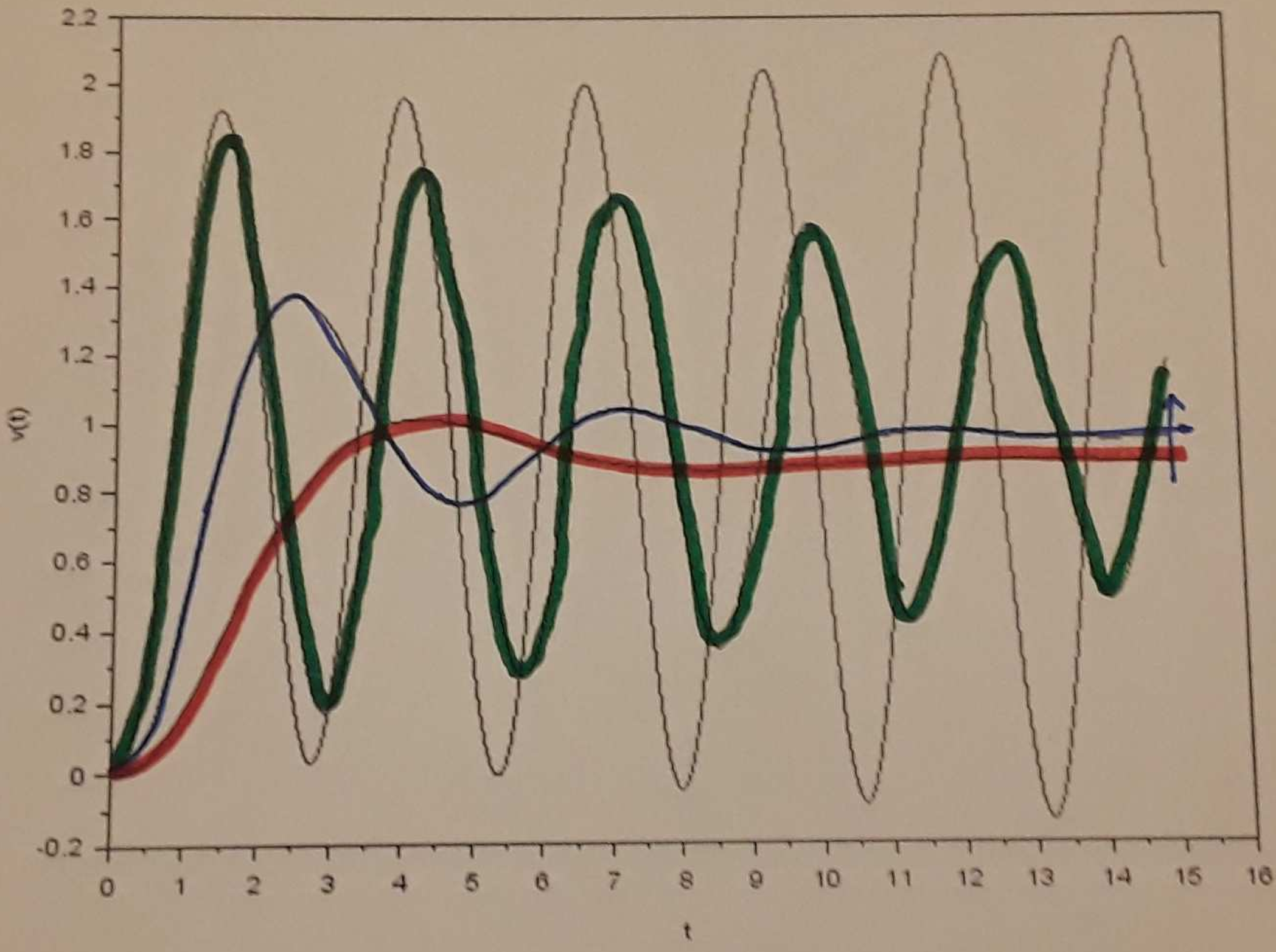
$\frac{0,13}{0,88} \cdot 100 = \underline{\underline{14,7\%}}$



$K_p = 7,36$

$K_p = 6,1$

P: $\boxed{\mu(0) = K_p}$



K_p=7,36; 20; 60; 70

P → pontosabb → PI

P → gyorsít → PD

A PI - SZABÁLYOZÓ HANGOLÁSA

$$W_P(s) = \frac{1}{(1+s10)(1+s)(1+s0{,}2)} \qquad \varphi_t = 60° \qquad W_C = K_{PI}\frac{1+sT_I}{sT_I}$$

$$\boxed{T_I = 10}$$

$$W_0 = W_C W_P = K_{PI}\frac{\cancel{1+sT_I}}{sT_I}\cdot\frac{1}{\cancel{(1+s10)}(1+s)(1+s0{,}2)} = K_{PI}\frac{1}{\underset{\uparrow\left(\frac{s}{0{,}1}\right)}{s10}(1+s)(1+s0{,}2)}$$

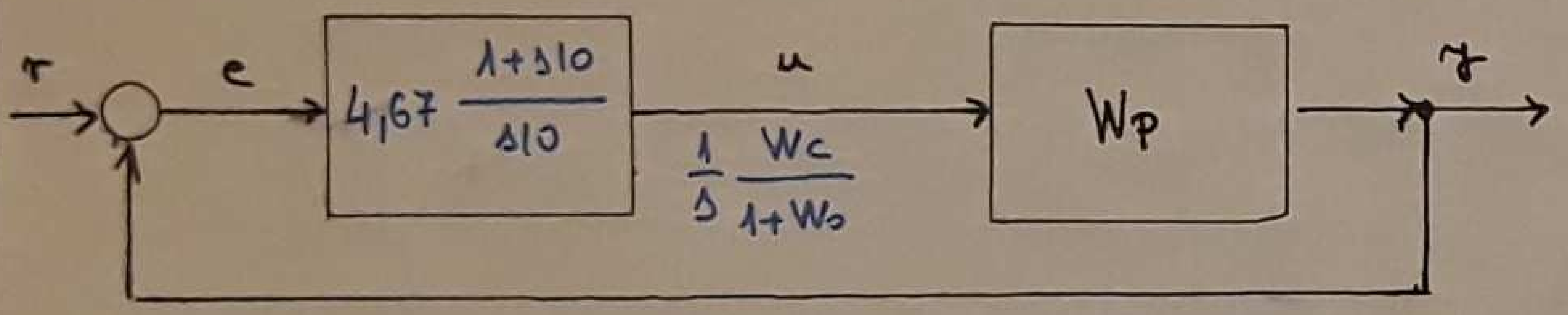
$$-90° - 45° \lg\frac{\omega_c}{0{,}1} = -120°$$

$$45\lg\frac{\omega_c}{0{,}1} = 30$$

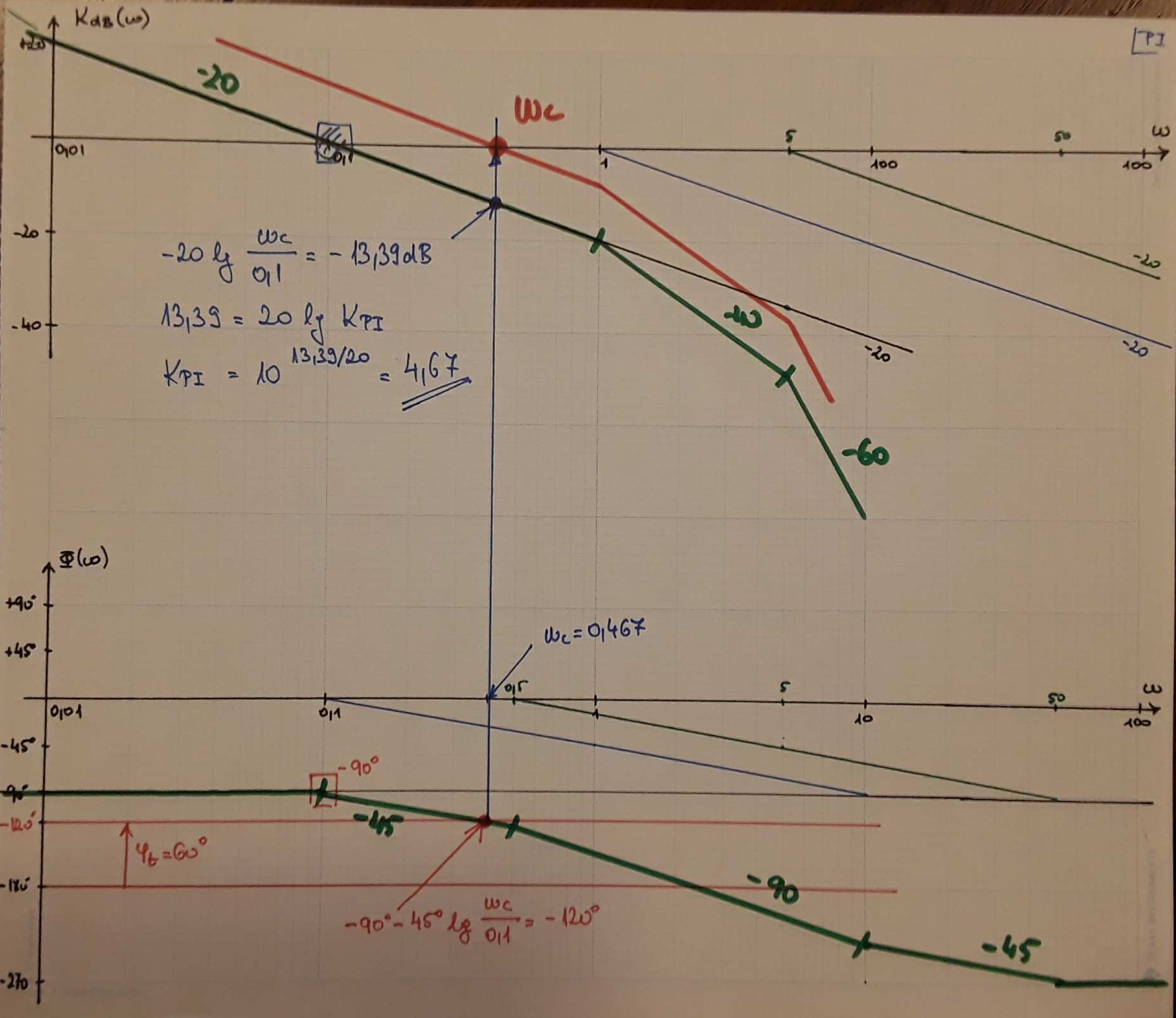
$$\lg\frac{\omega_c}{0{,}1} = 0{,}67$$

$$\frac{\omega_c}{0{,}1} = 10^{0{,}67} = 4{,}67$$

$$\boxed{\omega_c = 0{,}467}$$



$$R\cdot\frac{W_0}{1+W_0}$$



$-20 \lg \frac{\omega_c}{0,1} = -13,39 dB$

$13,39 = 20 \lg K_{PI}$

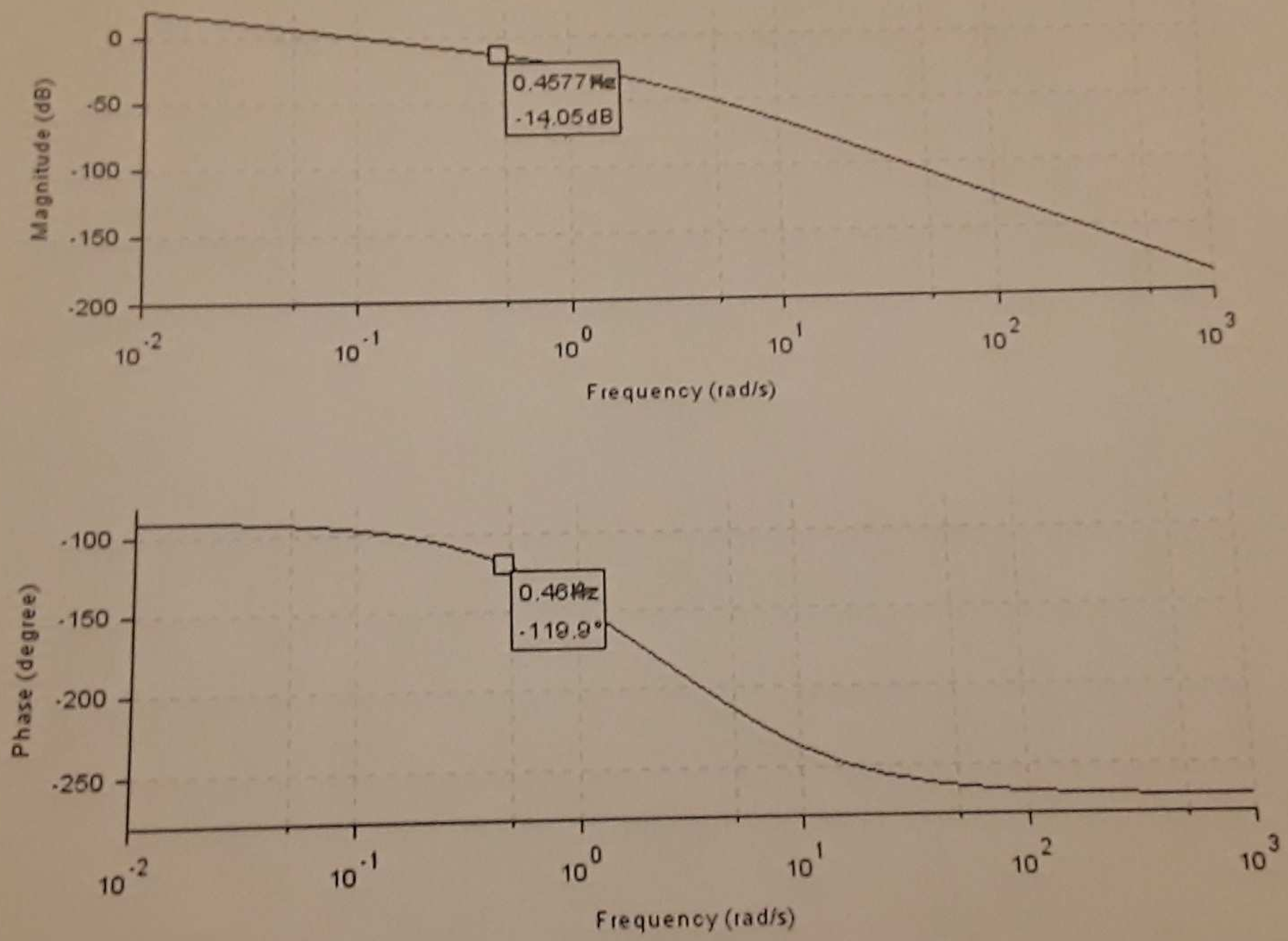
$K_{PI} = 10^{13,39/20} = 4,67$

$\omega_c = 0,467$

$-90° - 45° \lg \frac{\omega_c}{0,1} = -120°$

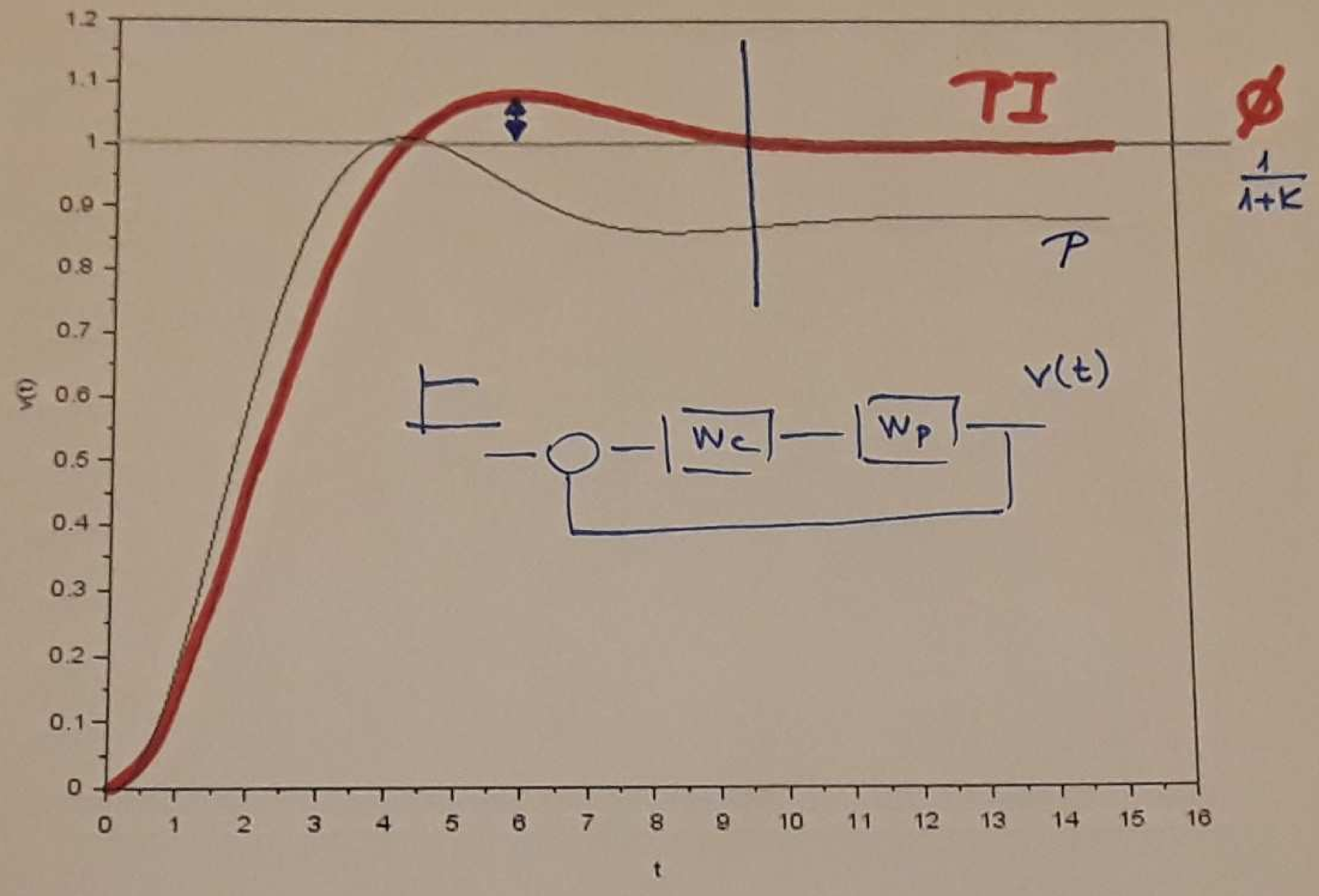
PI-szabályozó

0,46 $\frac{rad}{s}$

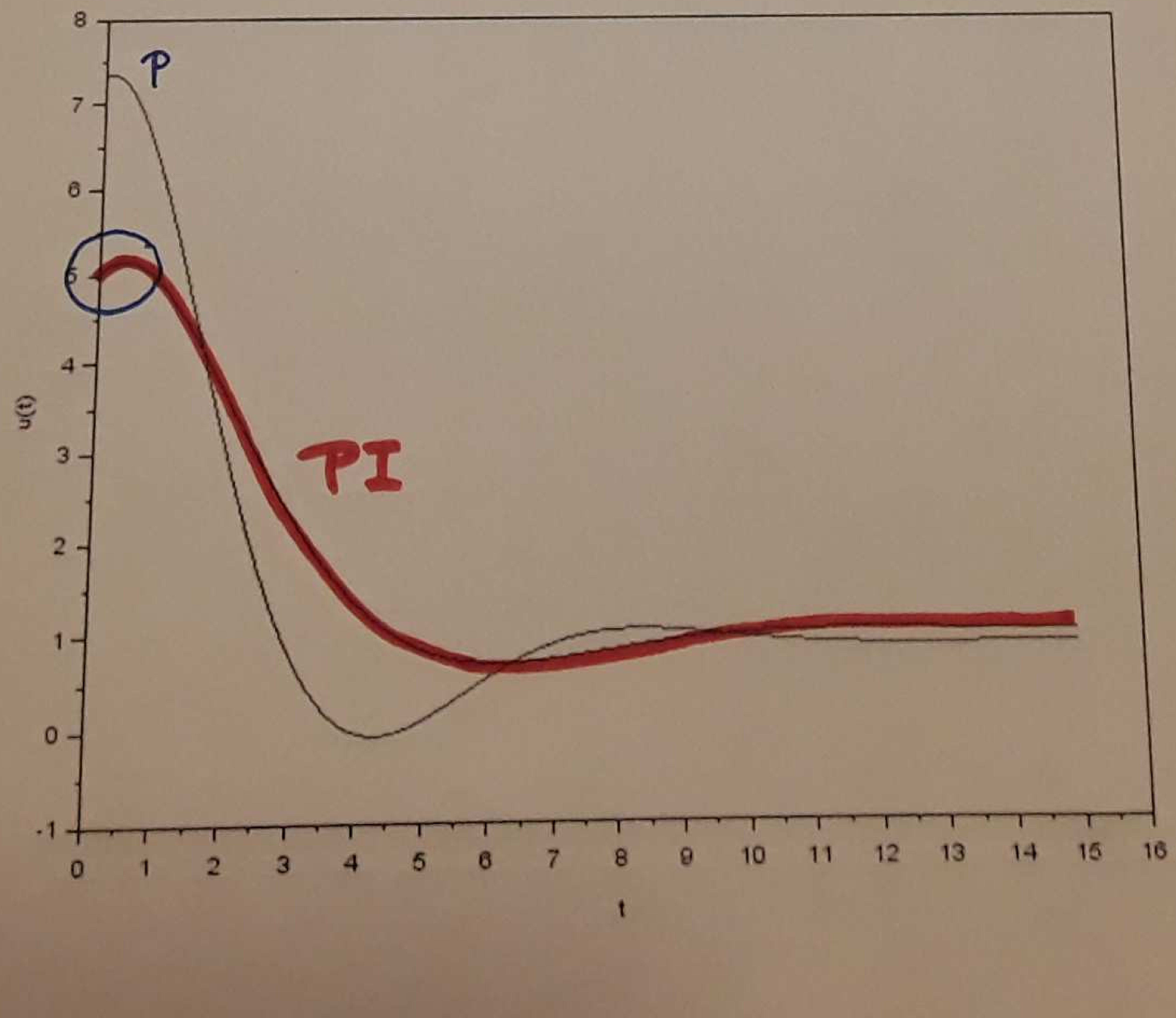


14,05dB -> 5,04

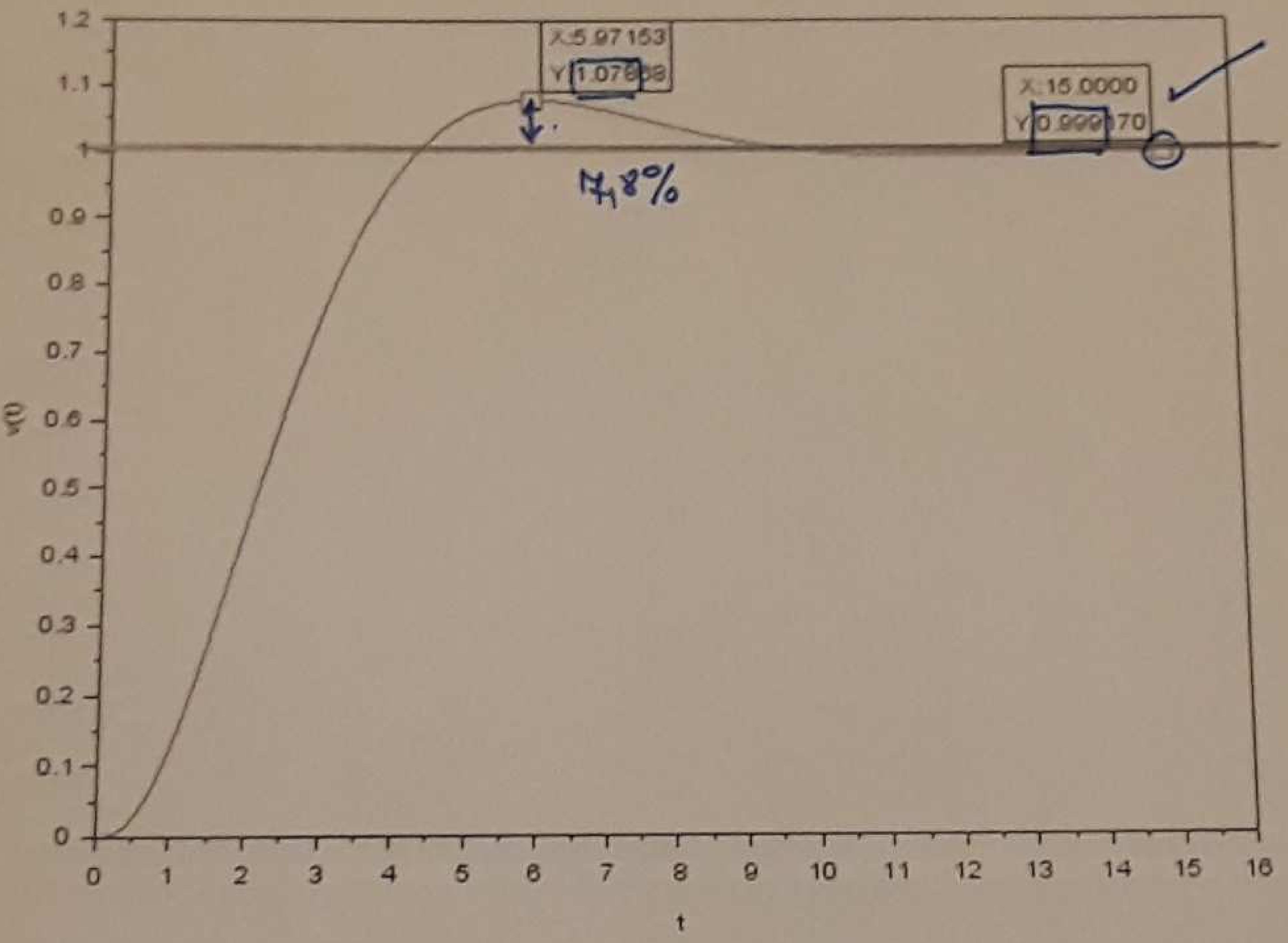
$K_{PI} = 5,04$



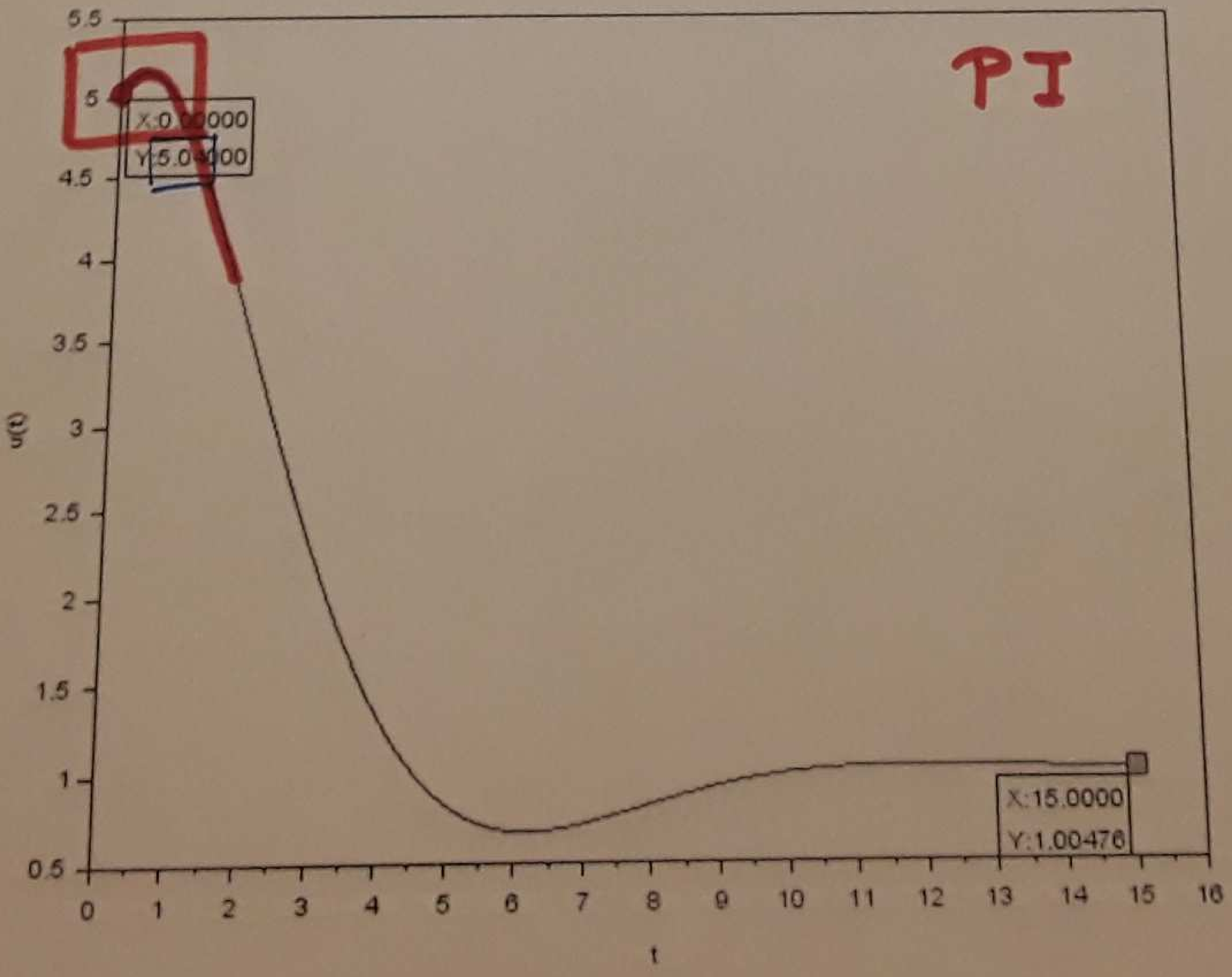
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
v(t)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
t
PI
∅
1/1+K
P
v(t)
Wc
Wp



8
7
6
5
4
3
2
1
0
-1
u(t)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
t
P
PI


X:5.97153
Y:1.07868
X:15.0000
Y:0.999970
y(t)
t
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

7,8%


X:0.00000
Y:5.00000
X:15.0000
Y:1.00476
u(t)
t
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

PI

A PD-SZABÁLYOZÓ HANGOLÁSA

$$W_P(s) = \frac{1}{(1+s10)(1+s)(1+s0{,}2)}$$

$$\varphi_t = 60°$$

$$W_C = K_{PD}\frac{1+sT_D}{1+sT_D'}$$

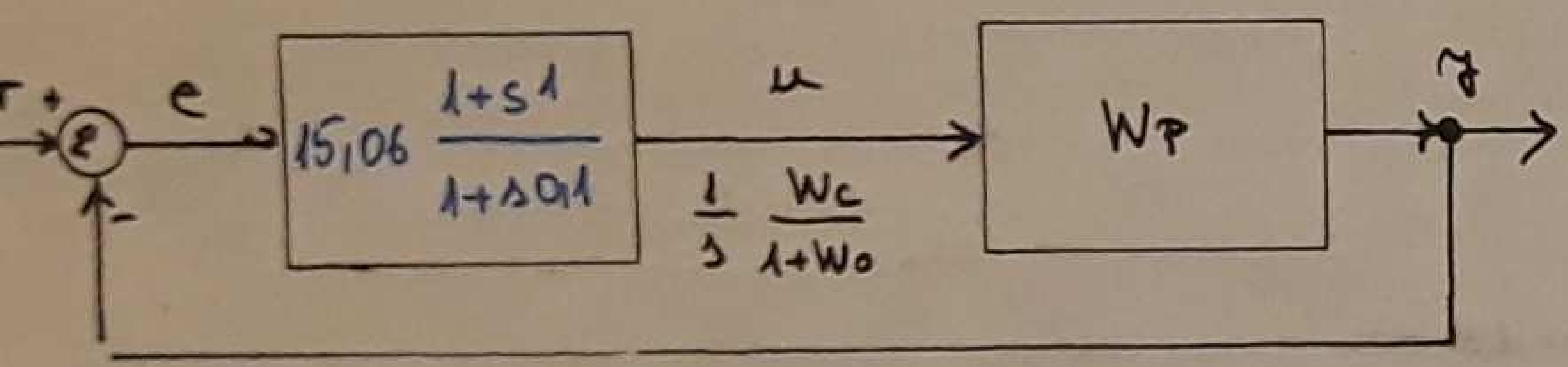
$$T_D' < T_D$$

$$T_D = 1$$

$$T_D' = 0{,}1$$

$$\frac{T_D}{T_D'}$$

$$W_o = W_C W_P = K_{PD}\frac{\cancel{1+sT_D}}{1+sT_D'}\,\frac{1}{(1+s10)\cancel{(1+s)}(1+s0{,}2)} = K_{PD}\frac{1}{(1+\underline{s10})(1+\underline{s0{,}2})(1+\underline{s0{,}1})}$$



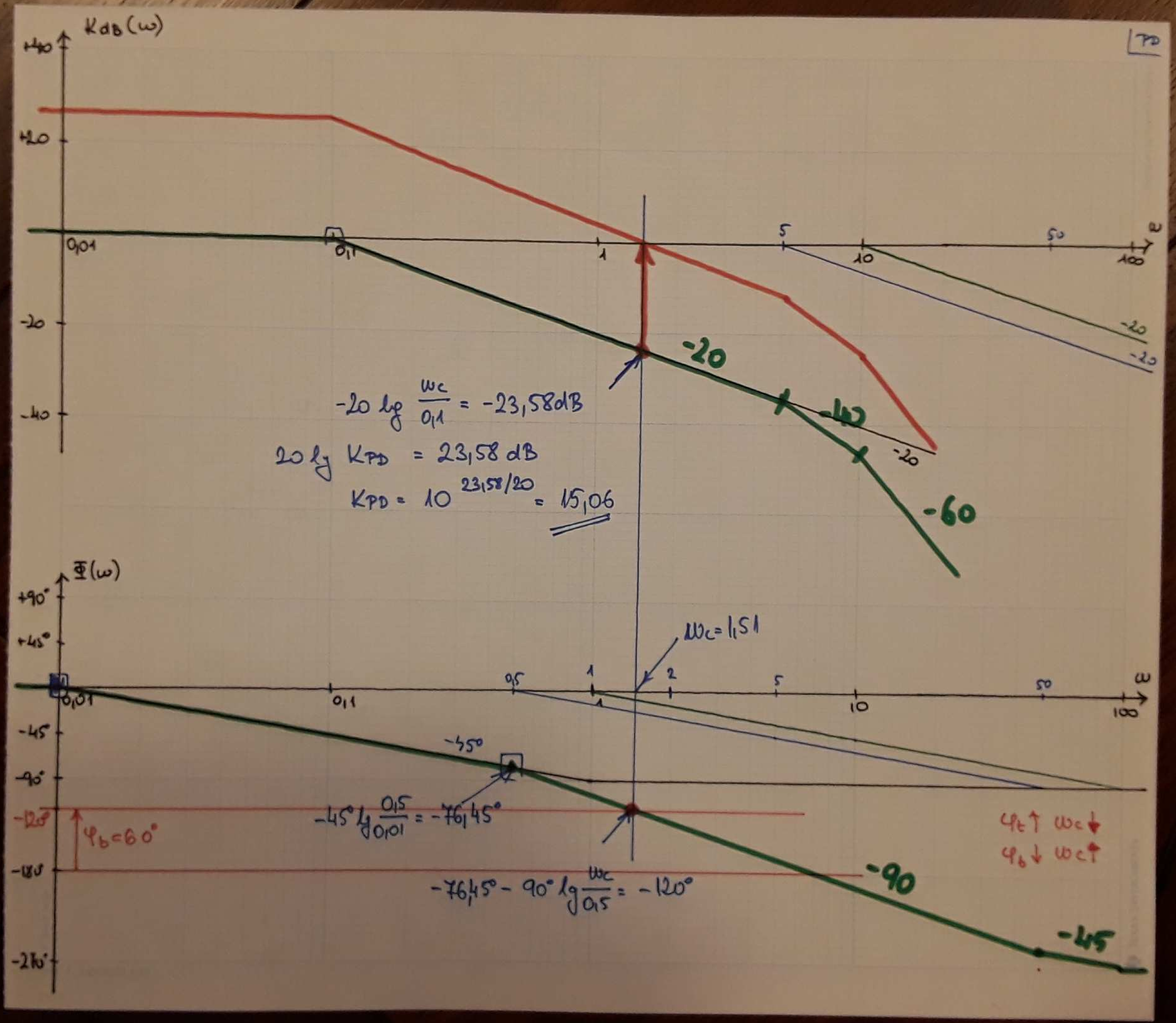
$$\frac{1}{s}\,\frac{W_o}{1+W_o}$$

$$-76{,}45° - 90°\,\lg\frac{\omega_c}{0{,}5} = -120°$$

$$\lg\frac{\omega_c}{0{,}5} = \frac{120-76{,}45}{90} = 0{,}48$$

$$\frac{\omega_c}{0{,}5} = 10^{0{,}48}$$

$$\omega_c = 0{,}5\cdot 10^{0{,}48} = \underline{\underline{1{,}51}}$$



$-20 \lg \frac{\omega_c}{0{,}1} = -23{,}58 dB$

$20 \lg K_{PD} = 23{,}58 dB$

$K_{PD} = 10^{23{,}58/20} = \underline{\underline{15{,}06}}$

$\omega_c = 1{,}51$

$-45° \lg \frac{0{,}5}{0{,}01} = -76{,}45°$

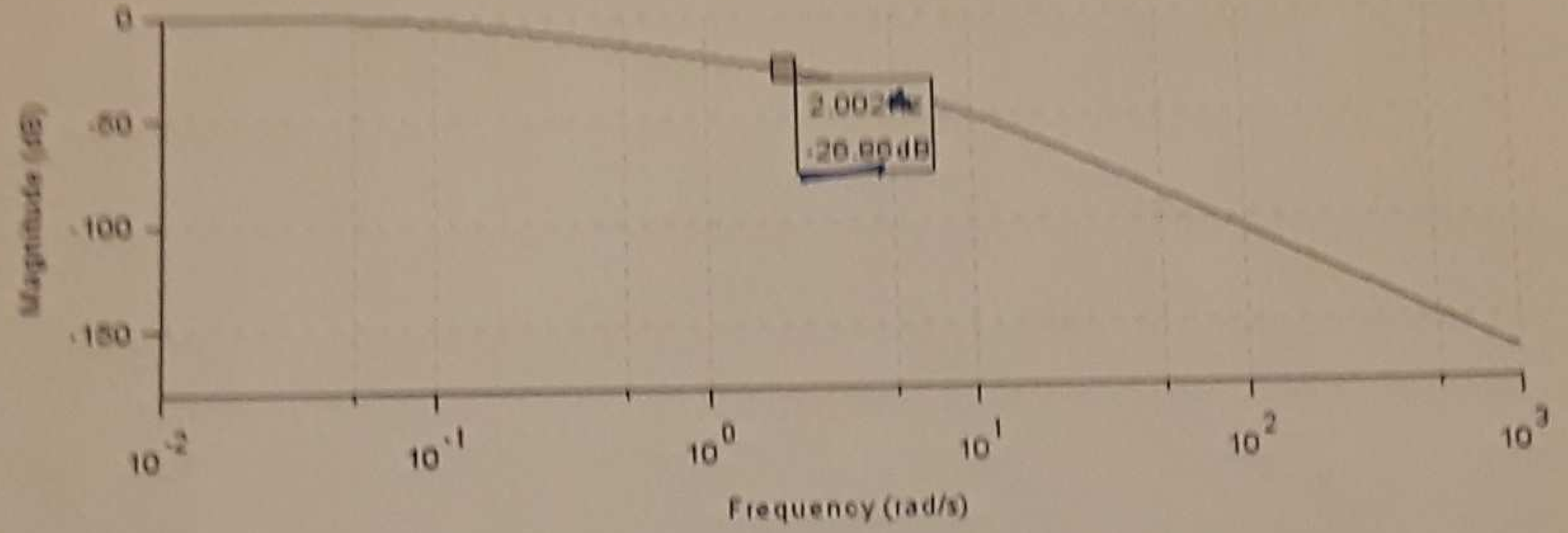
$-76{,}45° - 90° \lg \frac{\omega_c}{0{,}5} = -120°$

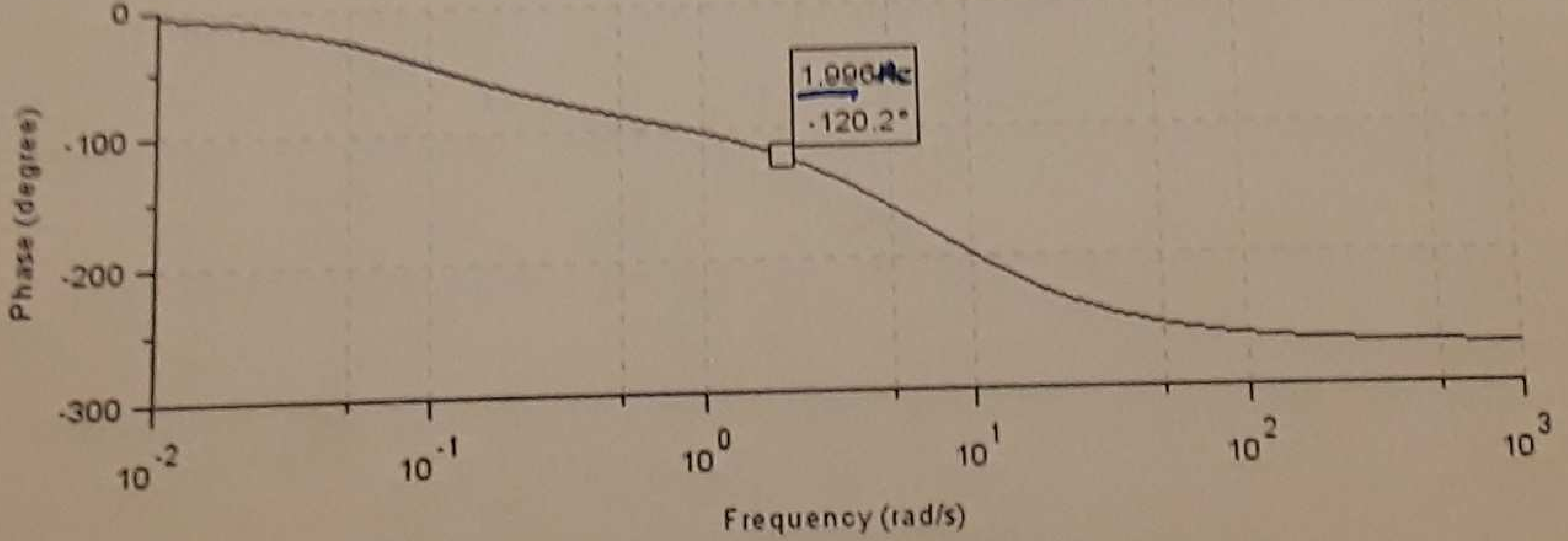
$\varphi_b = 60°$

$\varphi_t \uparrow \omega_c \downarrow$

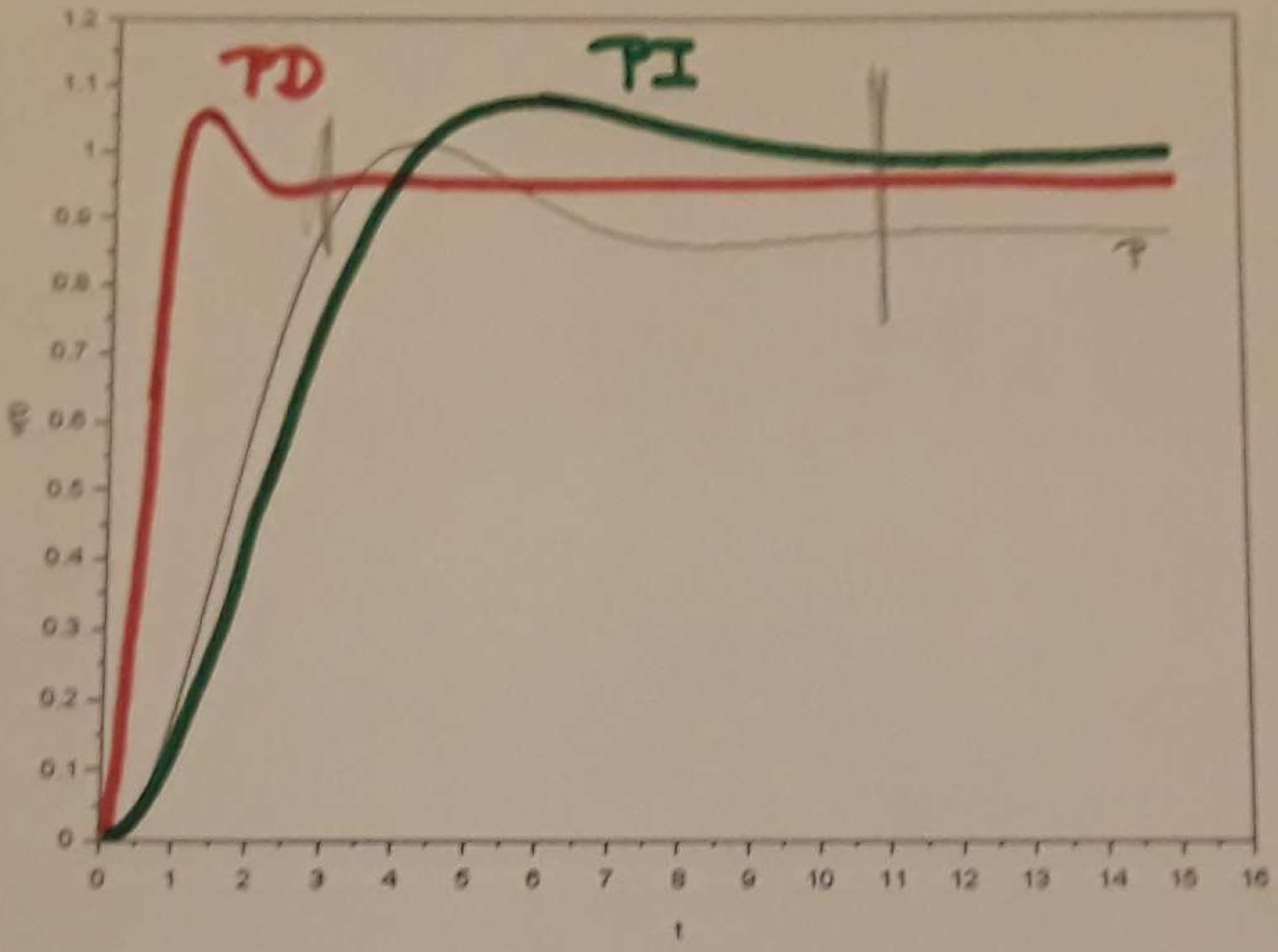
$\varphi_b \downarrow \omega_c \uparrow$

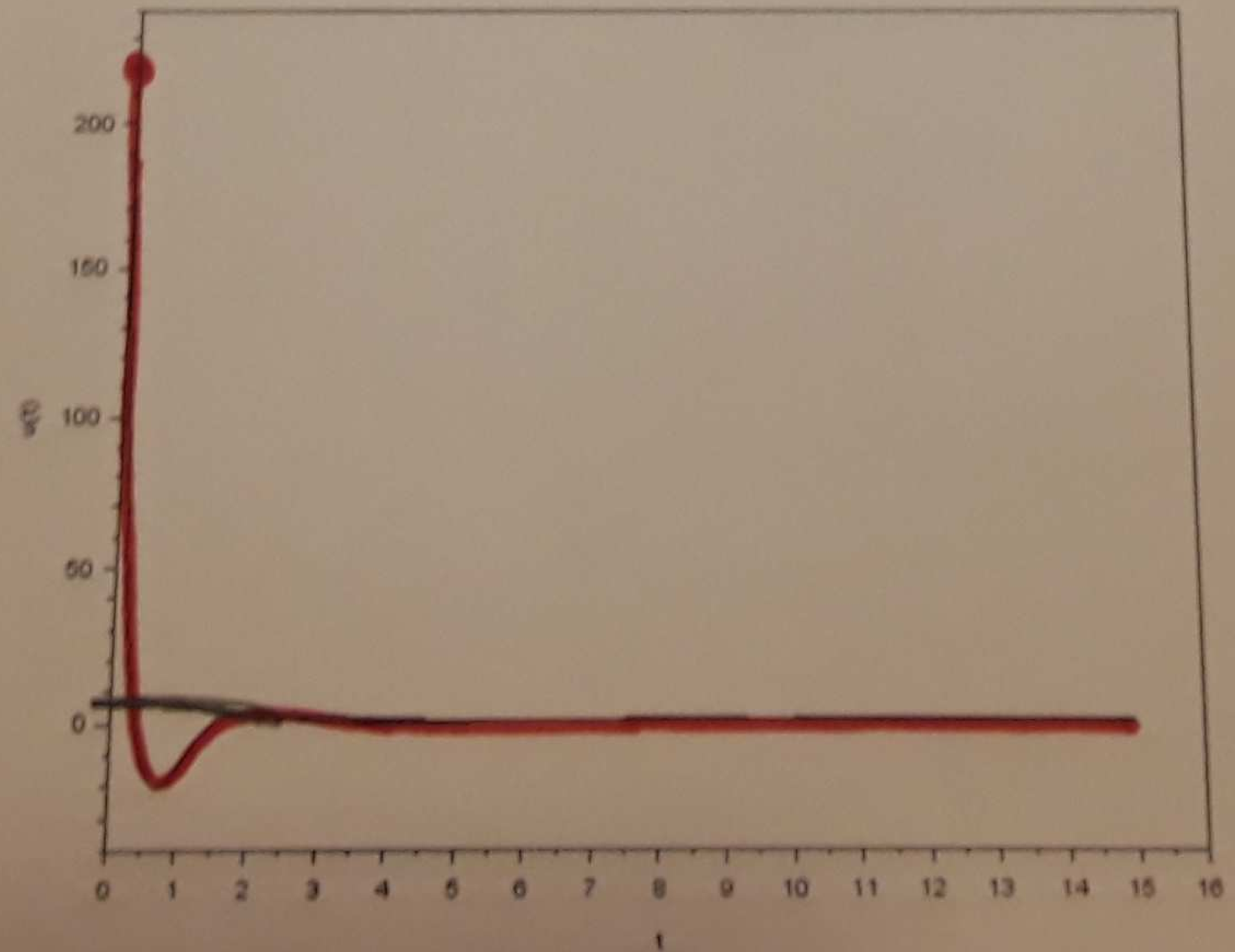
PD-szabályozó

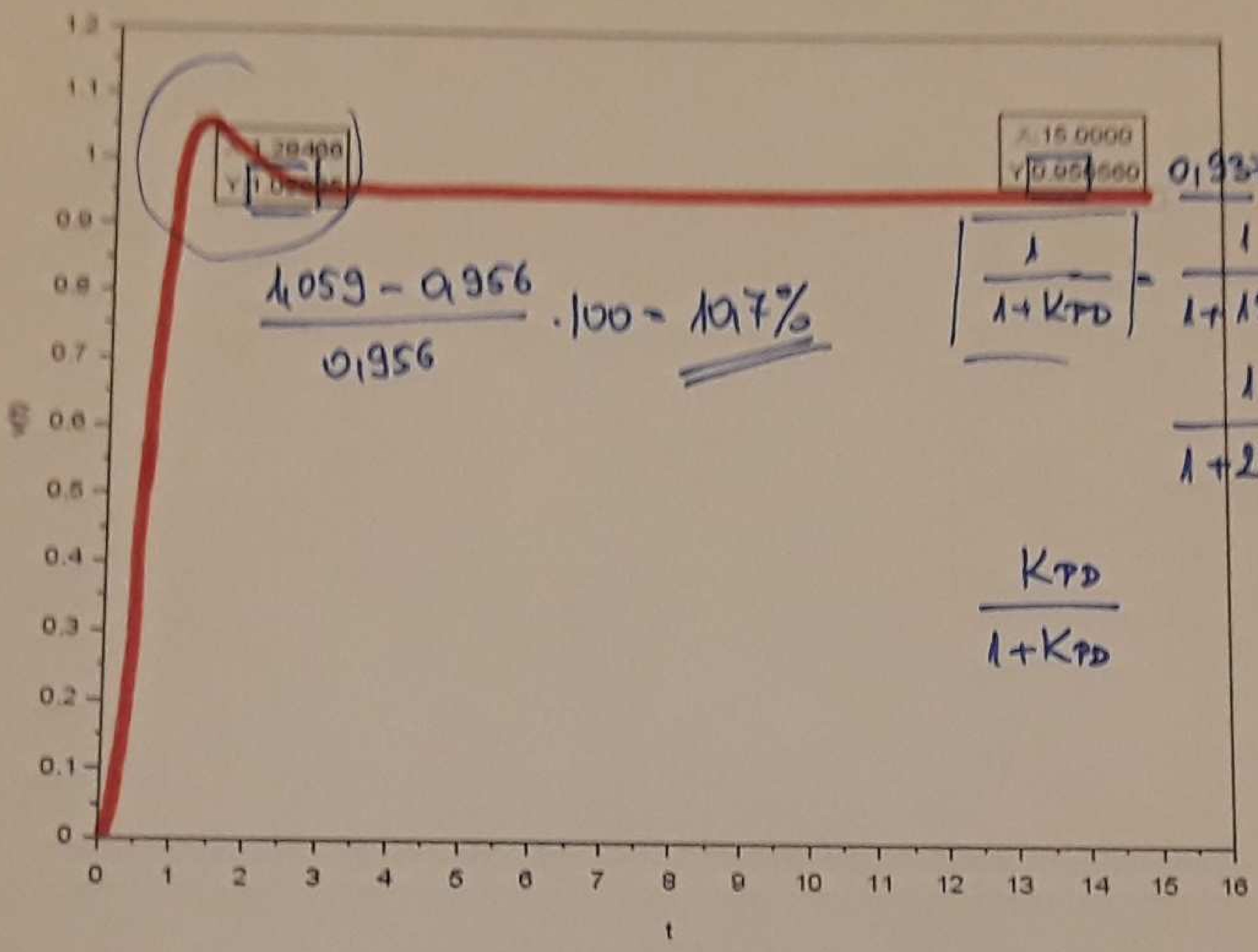




26,86dB ->(22,02 = K_{PD})


PD
PI
P
y(t)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
t


u(t)
0
50
100
150
200
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
t



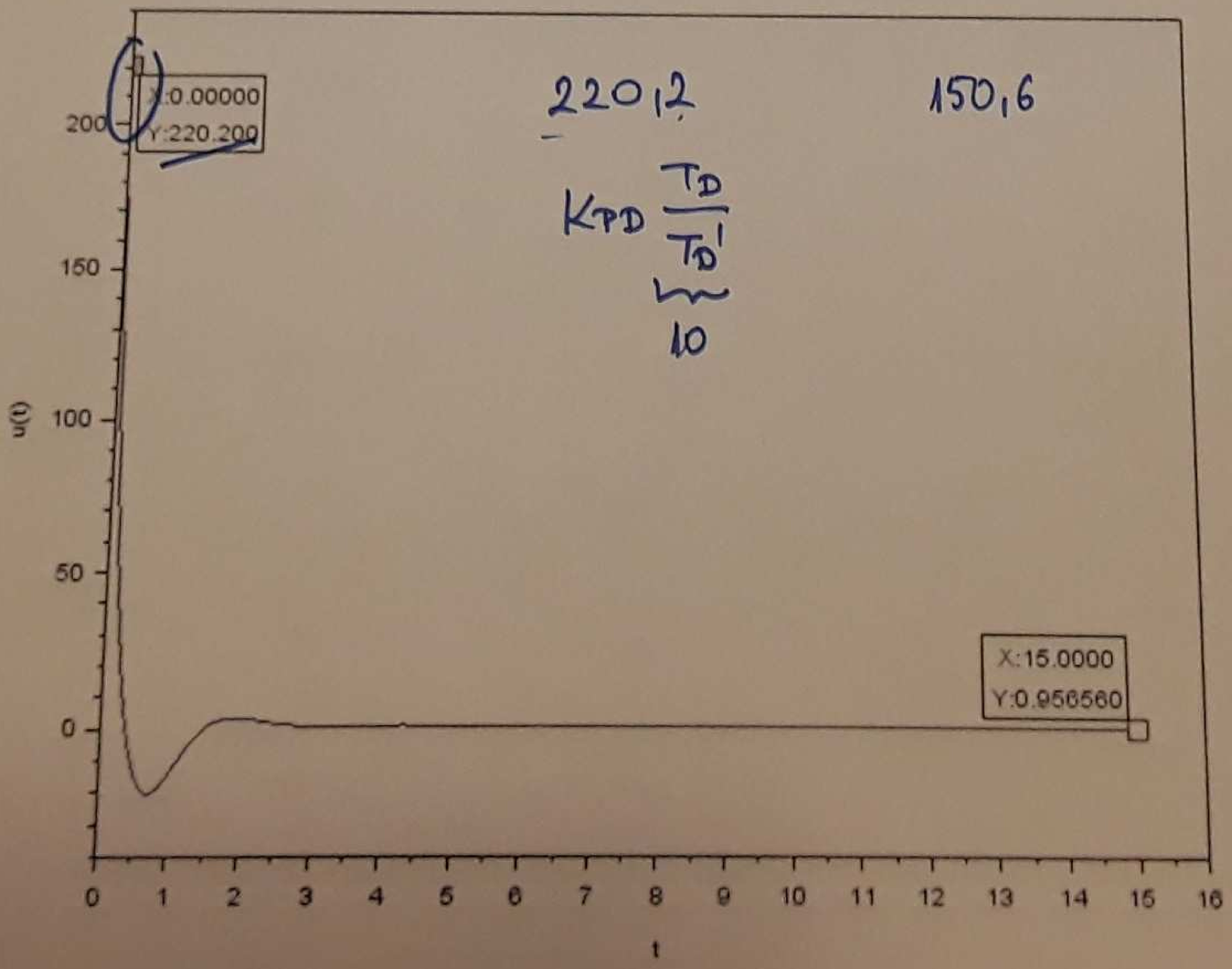
$$\frac{1{,}059 - 0{,}956}{0{,}956} \cdot 100 = \underline{10{,}7\%}$$

$0{,}937$

$$\left|\frac{1}{1+K_{PD}}\right| = \frac{1}{1+15{,}06} = 0{,}062$$

$$\frac{1}{1+22{,}02} = 0{,}043$$

$$\frac{K_{PD}}{1+K_{PD}}$$



$220{,}2$

$150{,}6$

$$K_{PD} \underbrace{\frac{T_D}{T_D'}}_{10}$$

A PID- SZABÁLYOZÓ HANGOLÁSA

$$W_P(s) = \frac{1}{(1+s10)(1+s)(1+s0{,}2)} \qquad \varphi_t = 60^\circ$$

$$W_C = K_{PID}\,\frac{1+sT_I}{sT_I}\,\frac{1+sT_D}{1+sT_D'} \qquad \text{PI-PD}$$

$$W_0 = W_C W_P = K_{PID}\,\frac{\cancel{1+sT_I}}{sT_I}\,\frac{\cancel{1+sT_D}}{1+sT_D'}\,\frac{1}{\cancel{(1+s\underline{10})}\cancel{(1+\underline{s})}(1+s\underline{0{,}2})} = K_{PID}\,\frac{1}{s10\,(1+s0{,}2)(1+s0{,}1)}$$

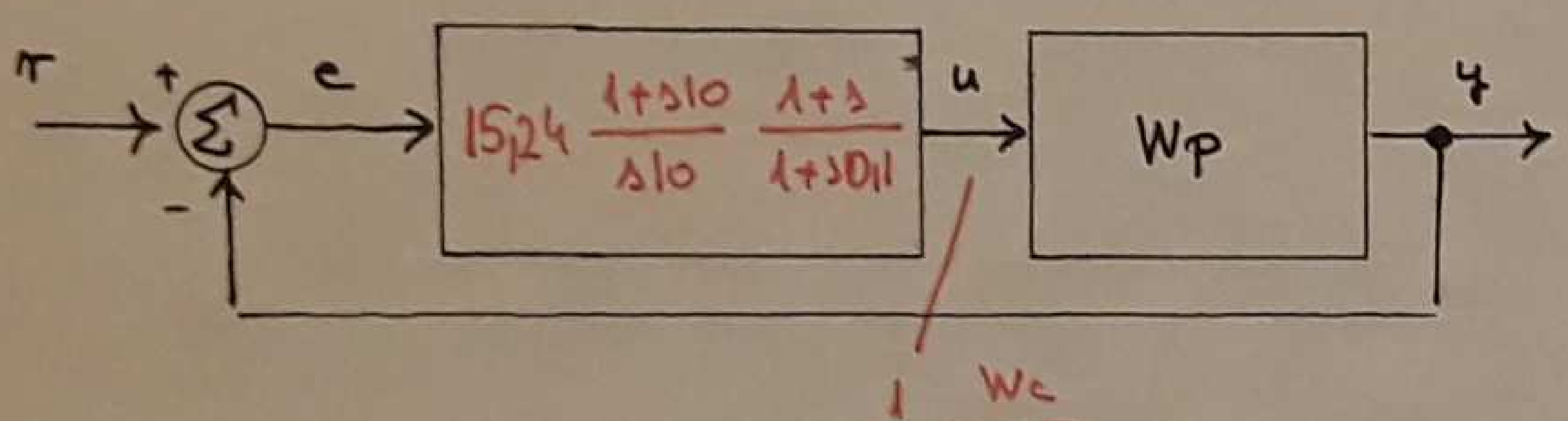
$T_I = 10$

$T_D = 1 \quad T_D' = 0{,}1$

$$-103{,}5 - 90\,\lg\omega_c = -120$$

$$\lg\omega_c = \frac{120-103{,}5}{90} = 0{,}183$$

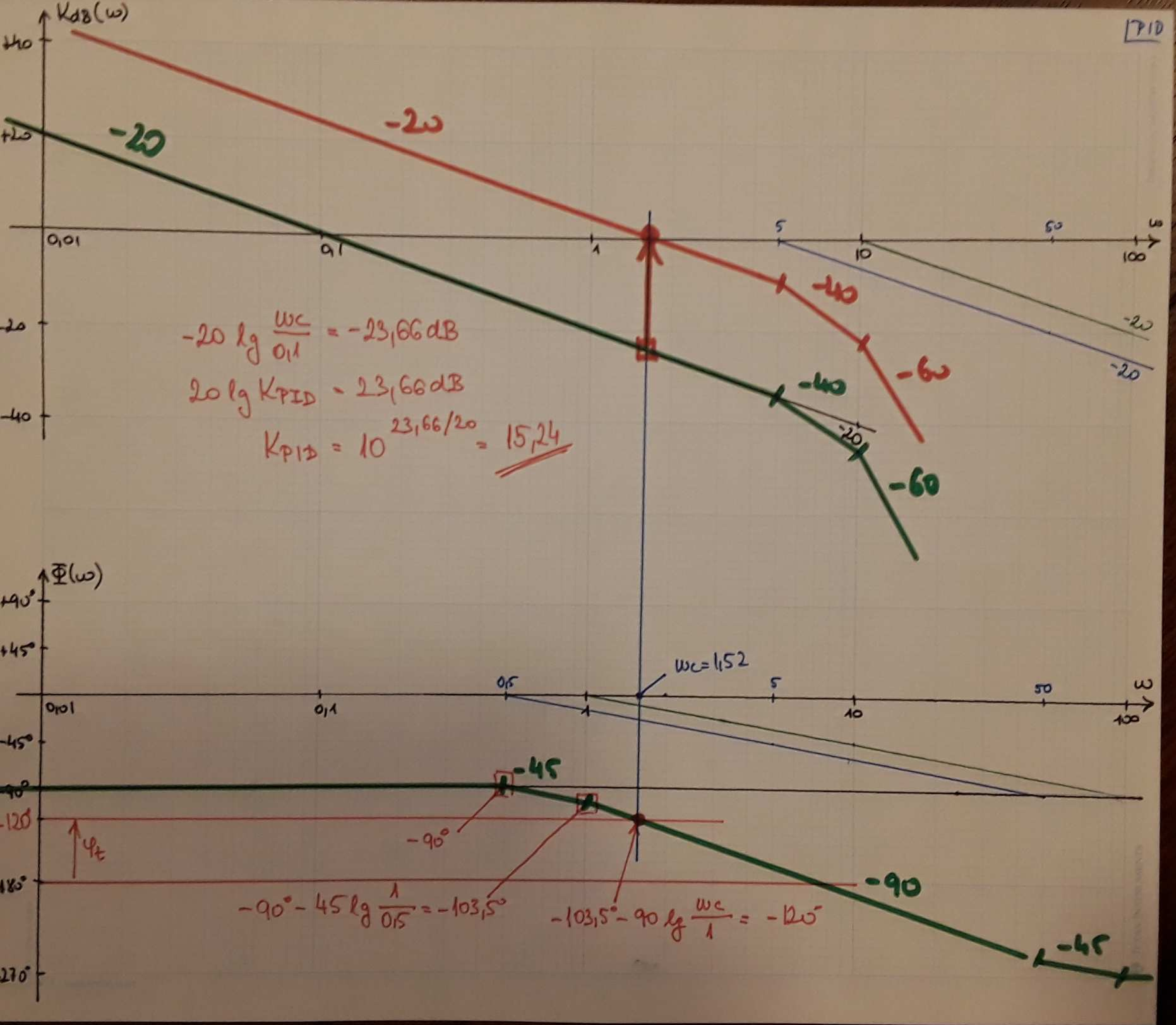
$$\omega_c = 10^{0{,}183} = \underline{\underline{1{,}52}}$$



$$15{,}24\,\frac{1+s10}{s10}\,\frac{1+s}{1+s0{,}1}$$

$$\frac{1}{s}\,\frac{W_0}{1+W_0}$$

$$\frac{1}{s}\,\frac{W_c}{1+W_0}$$



$-20\lg\frac{\omega_c}{0{,}1} = -23{,}66\,dB$

$20\lg K_{PID} = 23{,}66\,dB$

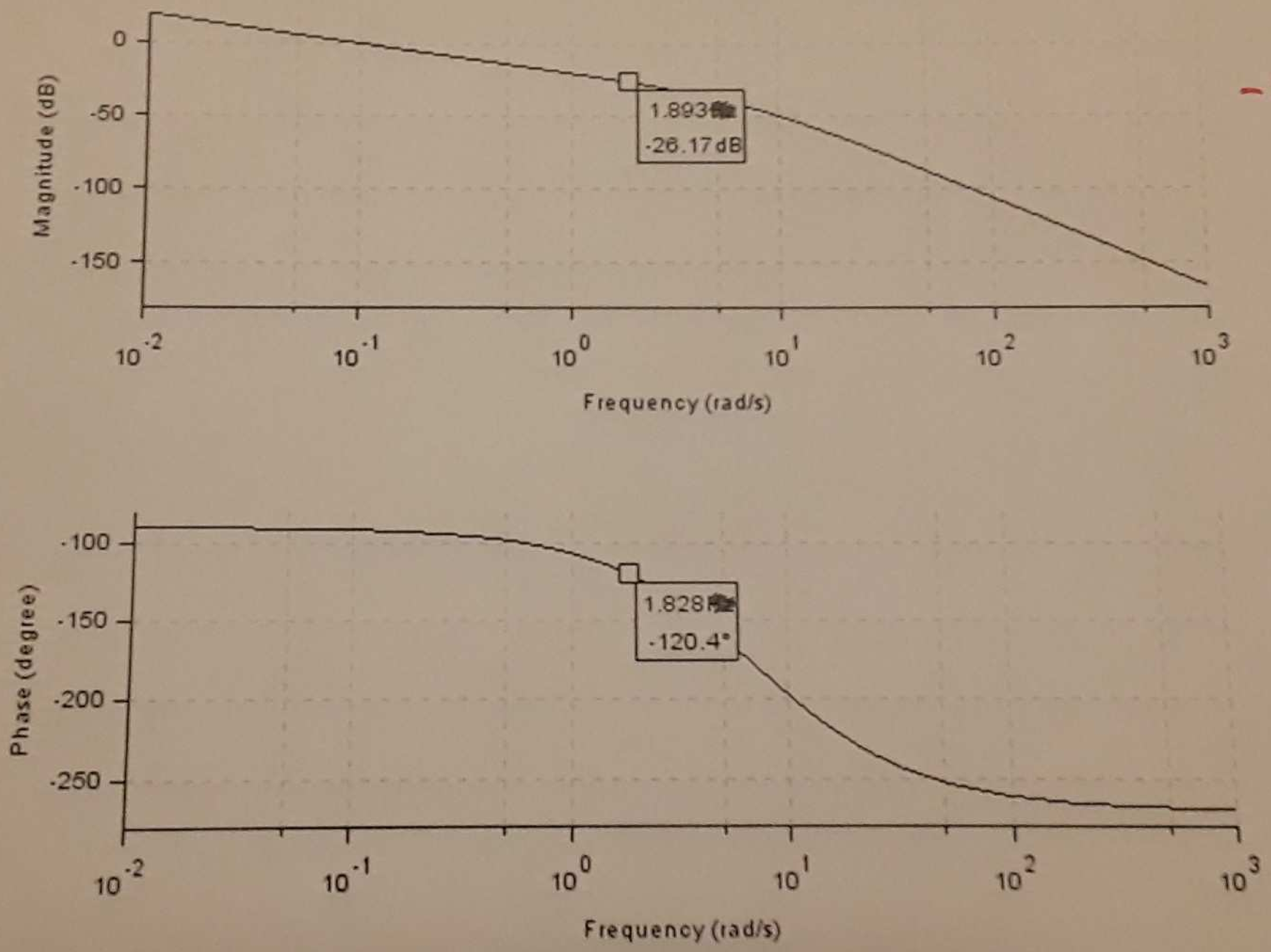
$K_{PID} = 10^{23{,}66/20} = \underline{\underline{15{,}24}}$

$\omega_c = 1{,}52$

$-90° - 45\lg\frac{1}{0{,}5} = -103{,}5°$

$-103{,}5° - 90\lg\frac{\omega_c}{1} = -120°$

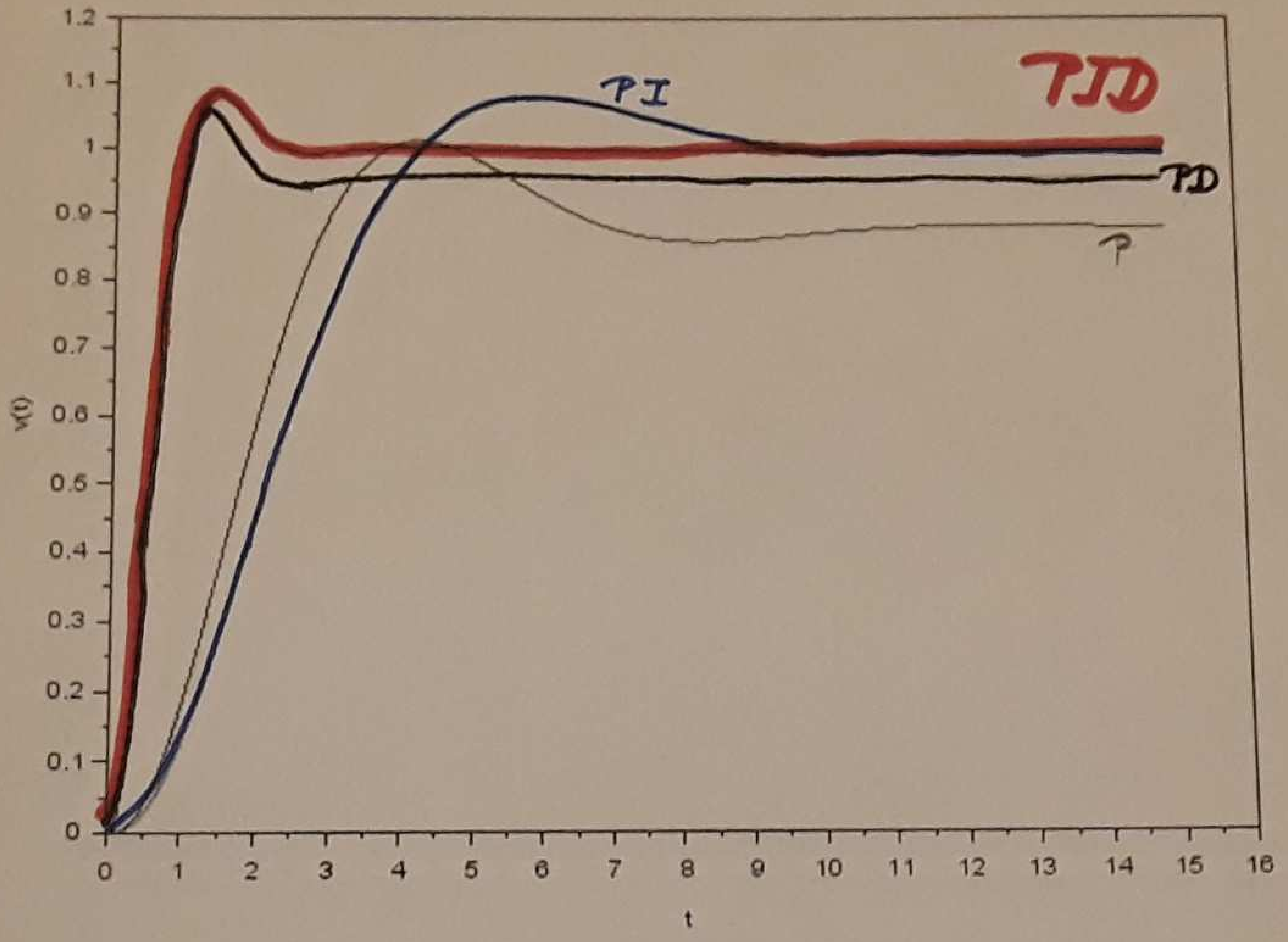
PID-szabályozó

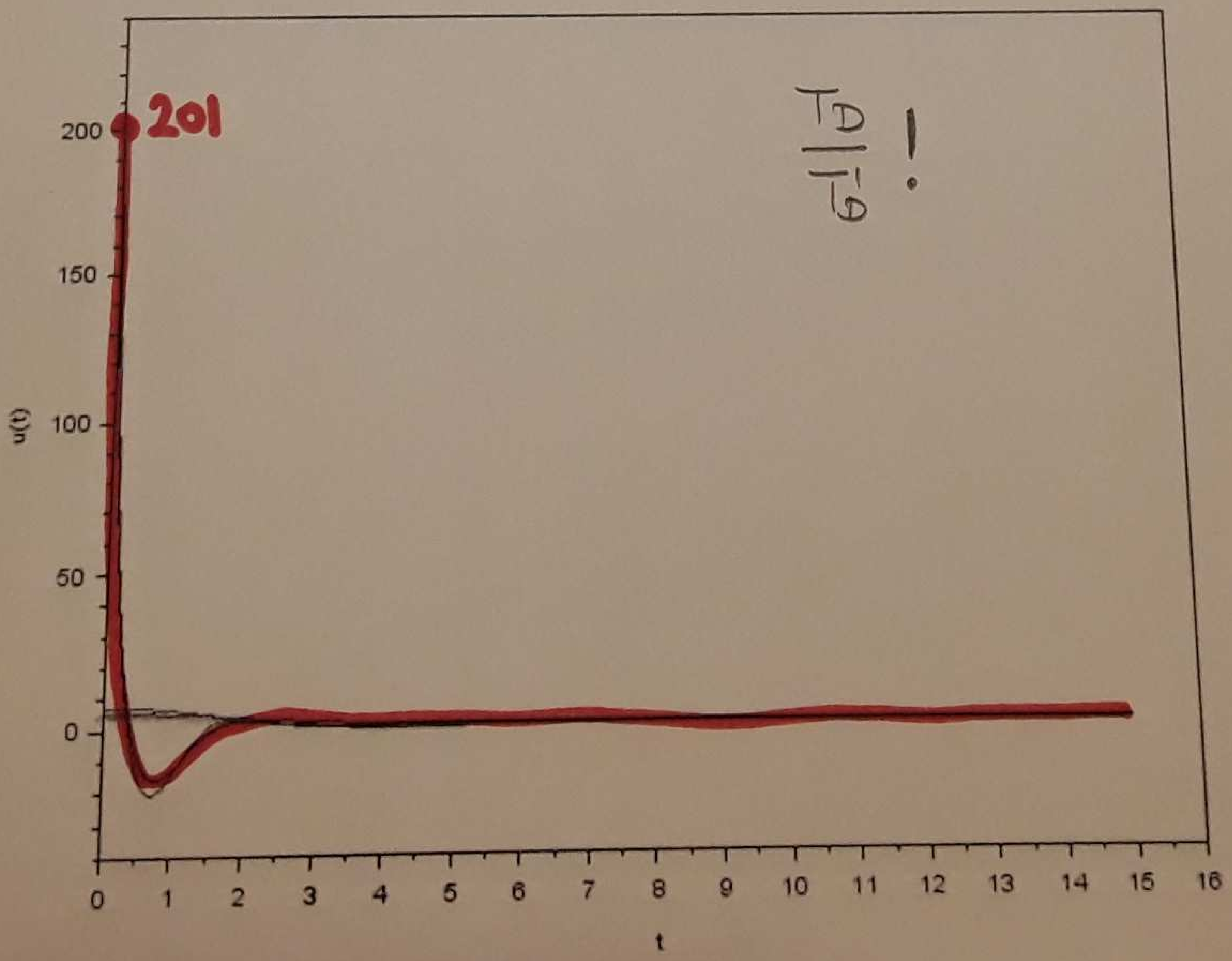


-23,66dB

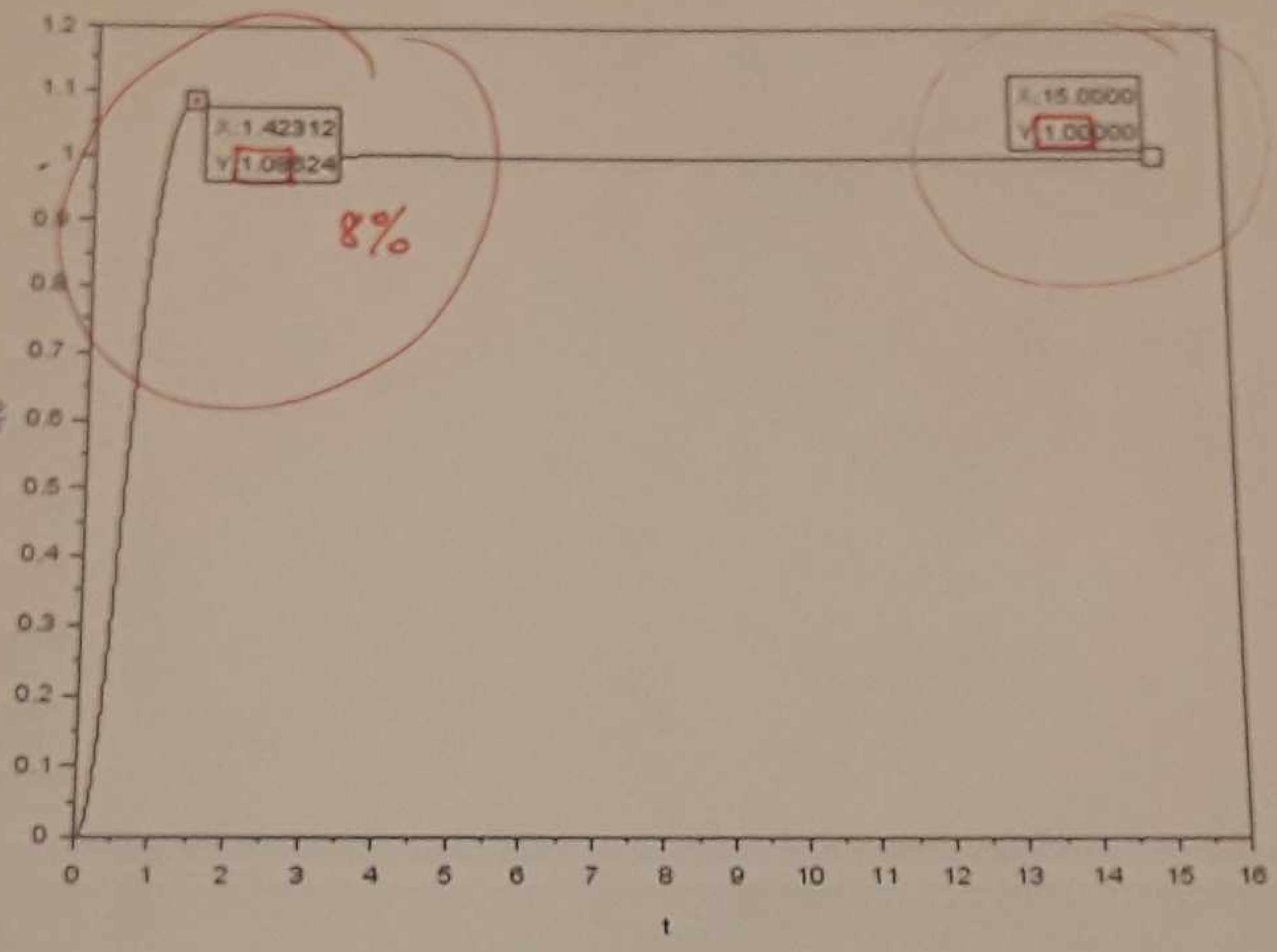
1,52

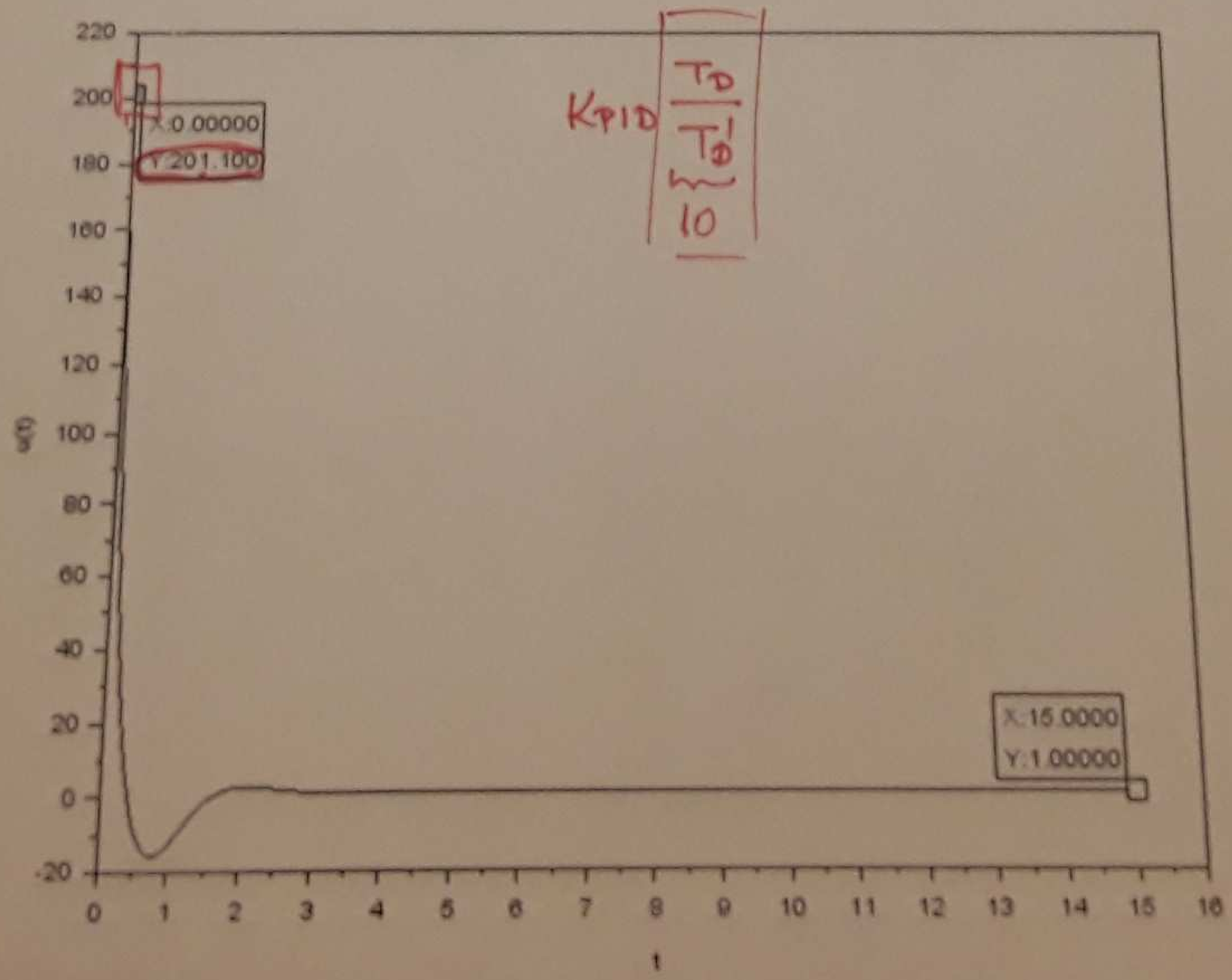
26,07dB -> 20,11


PID
PI
PD
P
y(t)
t
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16


201
u(t)
t
200
150
100
50
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

$\frac{T_D}{T_D}$!


1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y(t)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
t
X:1.42312
Y:1.08824
X:15.0000
Y:1.00000
8%

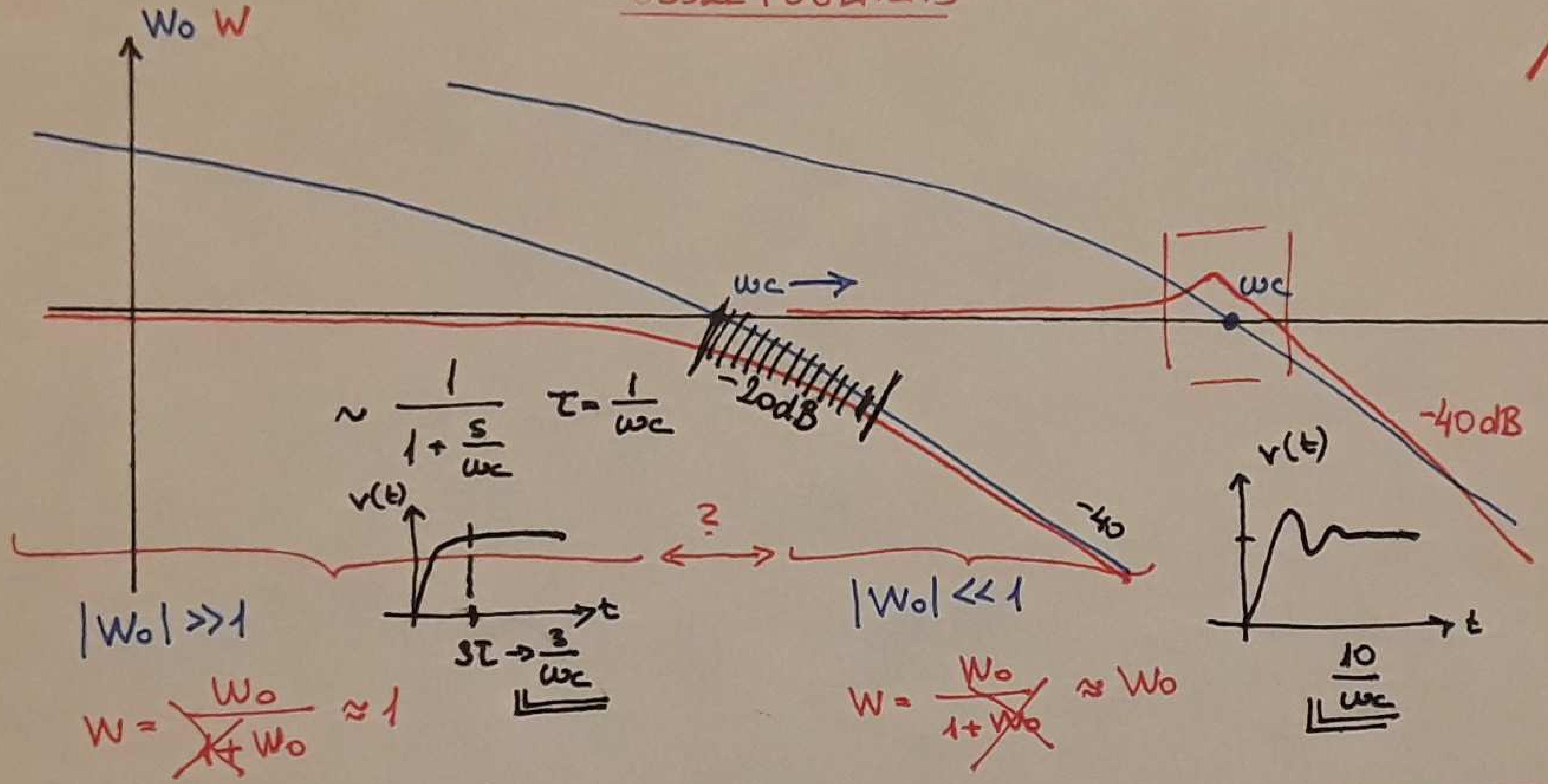

220
200
180
160
140
120
100
80
60
40
20
0
-20
u(t)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
t
X:0.00000
Y:201.100
X:15.0000
Y:1.00000
KPID
T_D
T_D'
~
10

A FELNYITOTT KÖR AMPLITÚDÓDIAGRAMJA

ÖSSZEFOGLALÁS

$W_0 = W_C W_P$

$W = \frac{W_0}{1+W_0}$

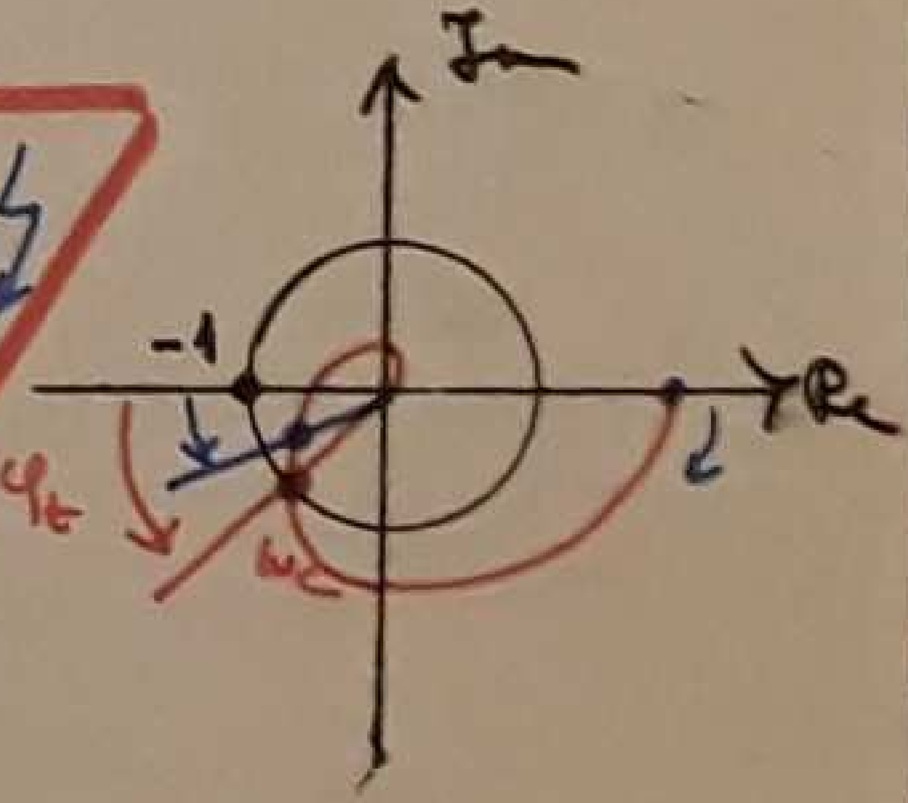


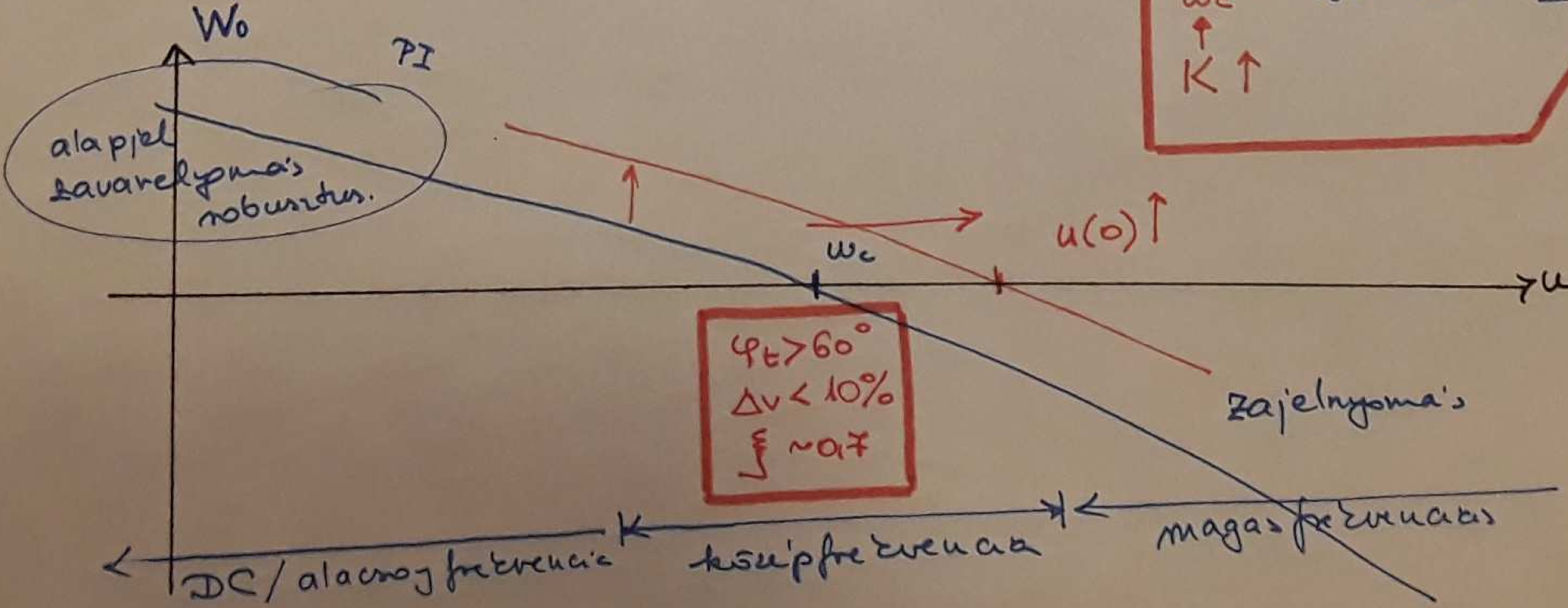
$$\boxed{\frac{3}{\omega_c} \le T_{sz} \le \frac{10}{\omega_c}}$$

$\omega_c \uparrow$ $\varphi_t \downarrow$ $\Delta v \uparrow$ $\boxed{ST}$ ↯

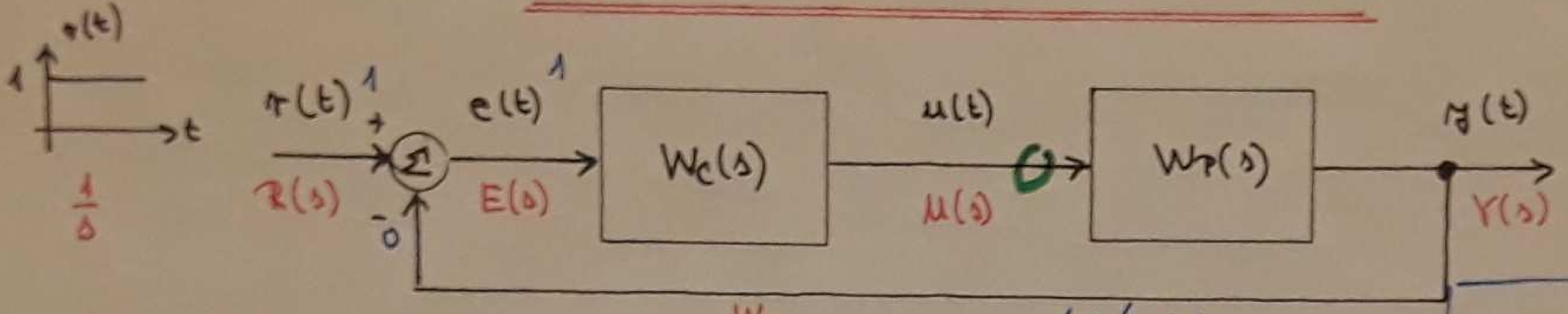
$\uparrow$

$K \uparrow$





A BEAVATKOZÓJEL MAXIMÁLIS ÉRTÉKE

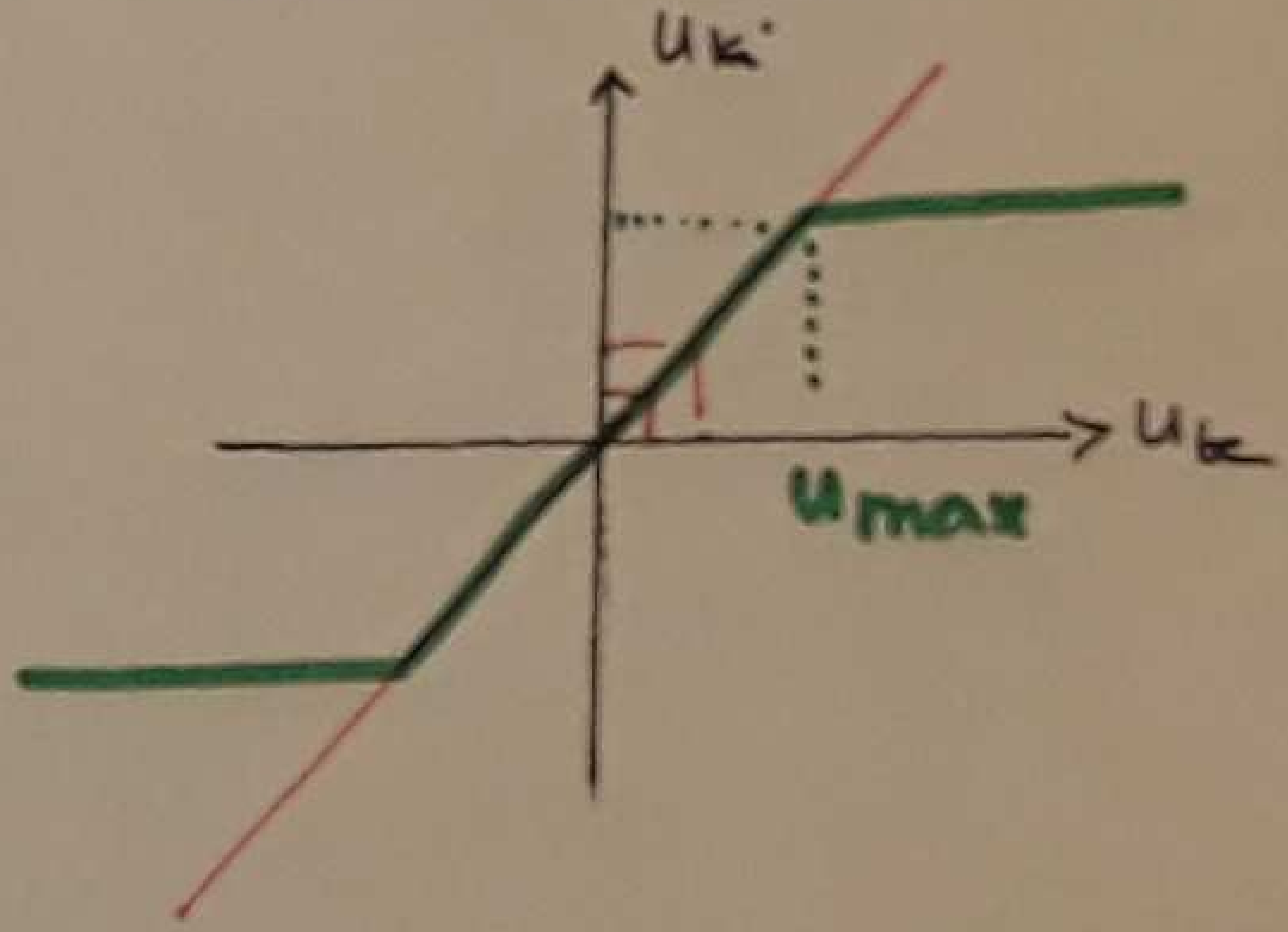


U(0)

$$u(0)=\lim_{s\to\infty} sU(s)=\lim_{s\to\infty} s\underbrace{R(s)}_{\frac{1}{s}}\frac{W_c}{1+W_0}=\lim_{s\to\infty}\cancel{s}\frac{1}{\cancel{s}}\frac{W_c}{1+\underbrace{W_0}}\Rightarrow$$

$W_0(\infty)=0$

$$\boxed{u(0)=\lim_{s\to\infty}W_c(s)}$$



P:) K_P $u(0)=K_P$

PI:) $K_{PI}\frac{1+sT_I}{sT_I}\quad\frac{1/s}{1/s}\quad K_{PI}\frac{\cancel{\frac{1}{s}}+T_I}{T_I}\rightarrow u(0)=K_{PI}$

PD:) $K_{PD}\frac{1+sT_D}{1+sT_D'}\quad\frac{1/s}{1/s}\quad K_{PD}\frac{\cancel{\frac{1}{s}}+T_D}{\cancel{\frac{1}{s}}+T_D'}\rightarrow u(0)=K_{PD}\frac{T_D}{T_D'}$

ID:) $K_{PID}\frac{1+sT_I}{sT_I}\;\frac{1+sT_D}{1+sT_D'}\rightarrow u(0)=K_{PID}\frac{T_D}{T_D'}$

ID) $K_{PID}\frac{1+s(T_D+T_D')+s^2T_I(T_D+T_D')}{sT_I(1+sT_D')}\quad\frac{1/s^2}{1/s^2}\quad K_{PID}\frac{\cancel{\frac{1}{s^2}}+\frac{1}{s}\cancel{(T_D+T_D')}+T_I(T_D+T_D')}{\cancel{\frac{1}{s}T_I}+T_IT_D'}\Rightarrow u(0)=K_{PID}\frac{T_D+T_D'}{T_D'}$

LABILIS RENDSZER SZABÁLYOZÁSA

$$\begin{pmatrix} \dfrac{\boxed{W_c W_p}}{1+W_o} & \dfrac{\boxed{W_p}}{\boxed{1+W_o}} \\[2ex] \dfrac{W_c}{1+W_o} & \dfrac{1}{1+W_o} \end{pmatrix}$$

~~[illegible]~~

$$\underbrace{\frac{(1+sT_1)}{(1+sT_2)}}_{W_c} \; \underbrace{\frac{A}{(1+sT_1)(1\underset{\uparrow}{-}sT_2)}}_{W_p}$$

/ stabilitás !?

\ nem pontos a kiejtés.

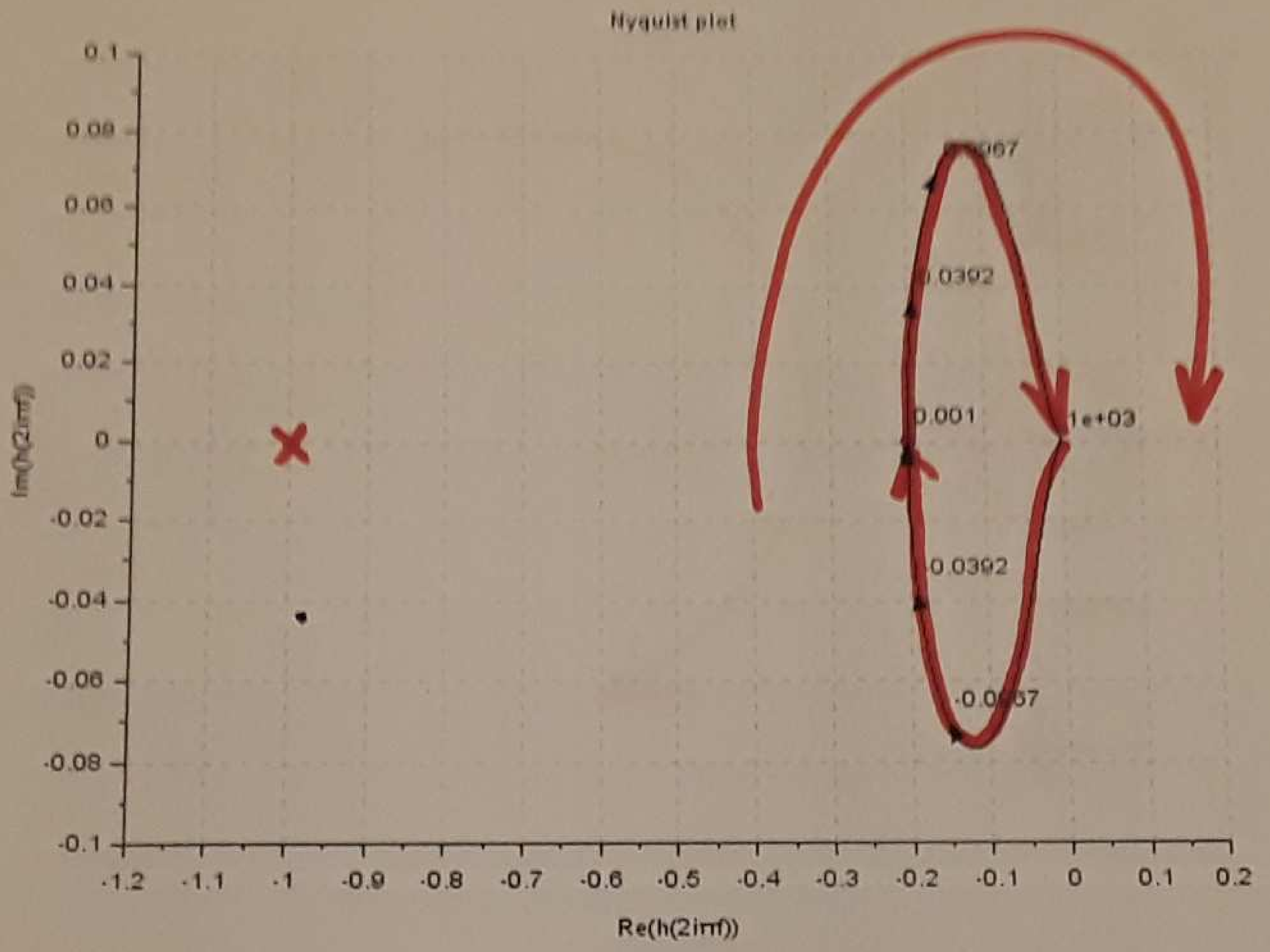
Nem stabil → ált. Nyquist-kritérium

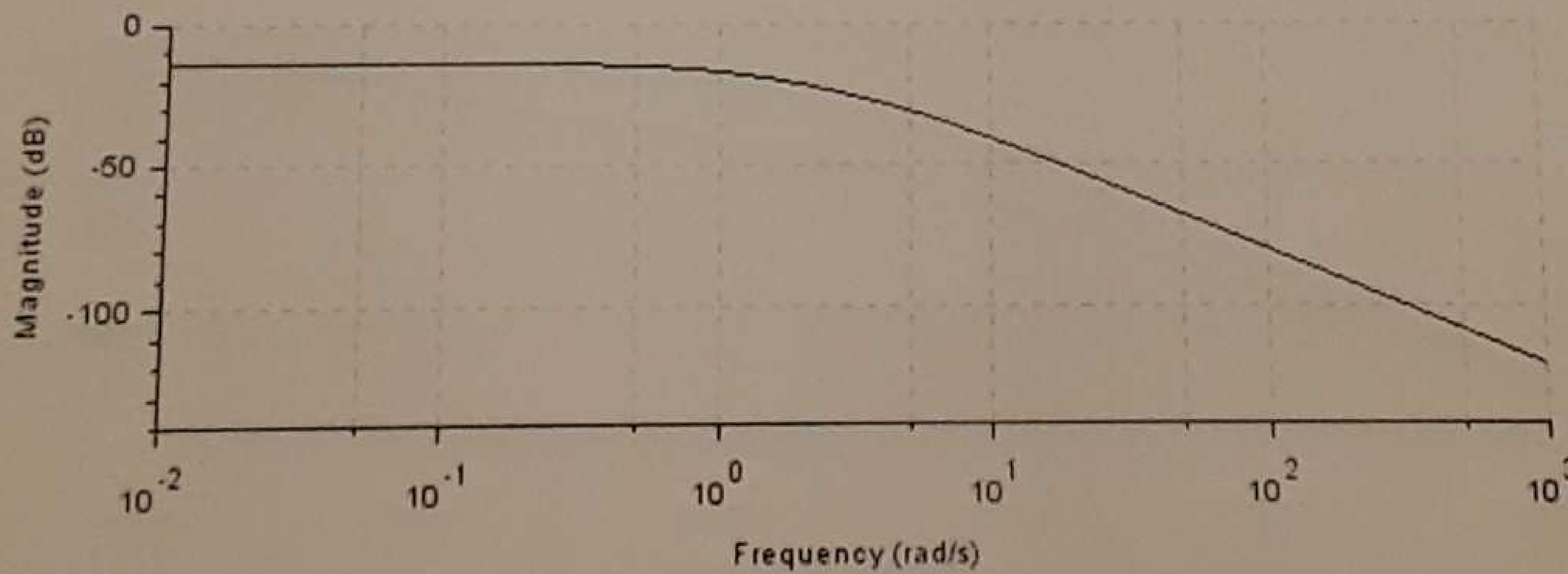
gyökhelygörbe

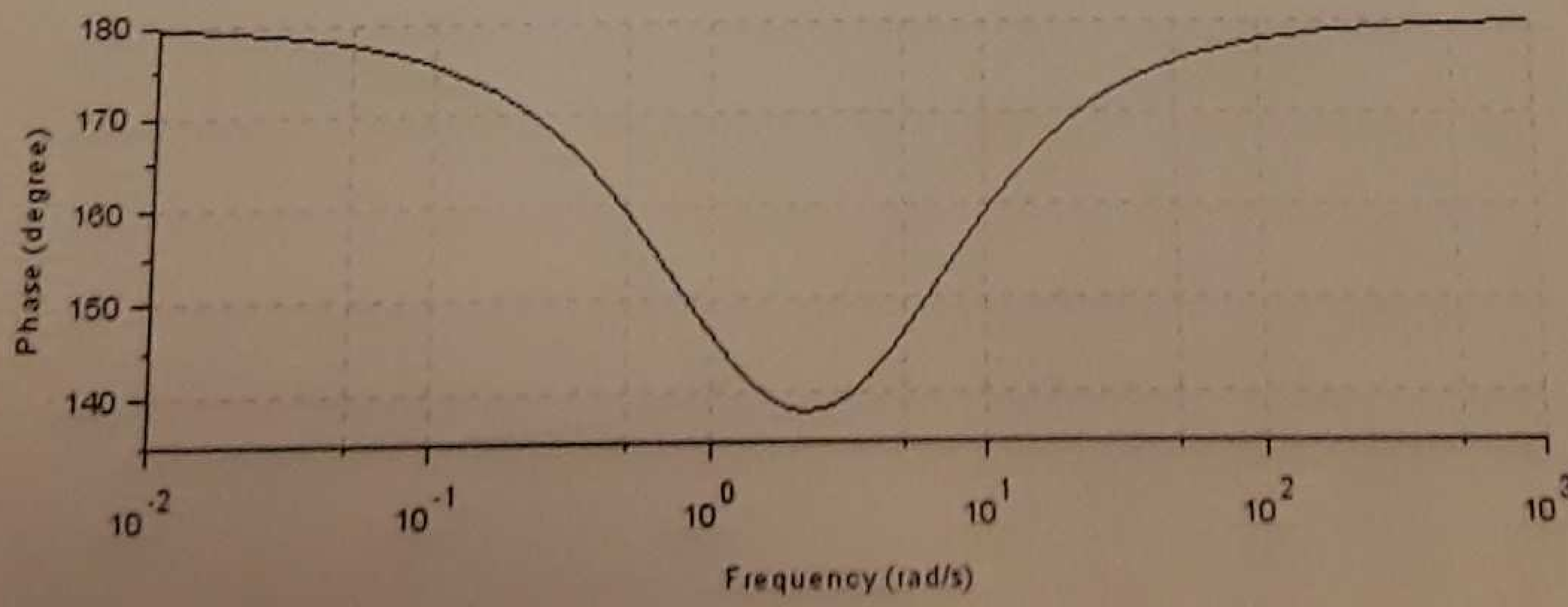
$$W_P = \frac{-0{,}2}{(1+s)\,(\underbrace{1-s0{,}2}_{T=1})}$$

$$W_0 = K_P \, \frac{-0{,}2}{(1+s)\,(1-s0{,}2)}$$

P





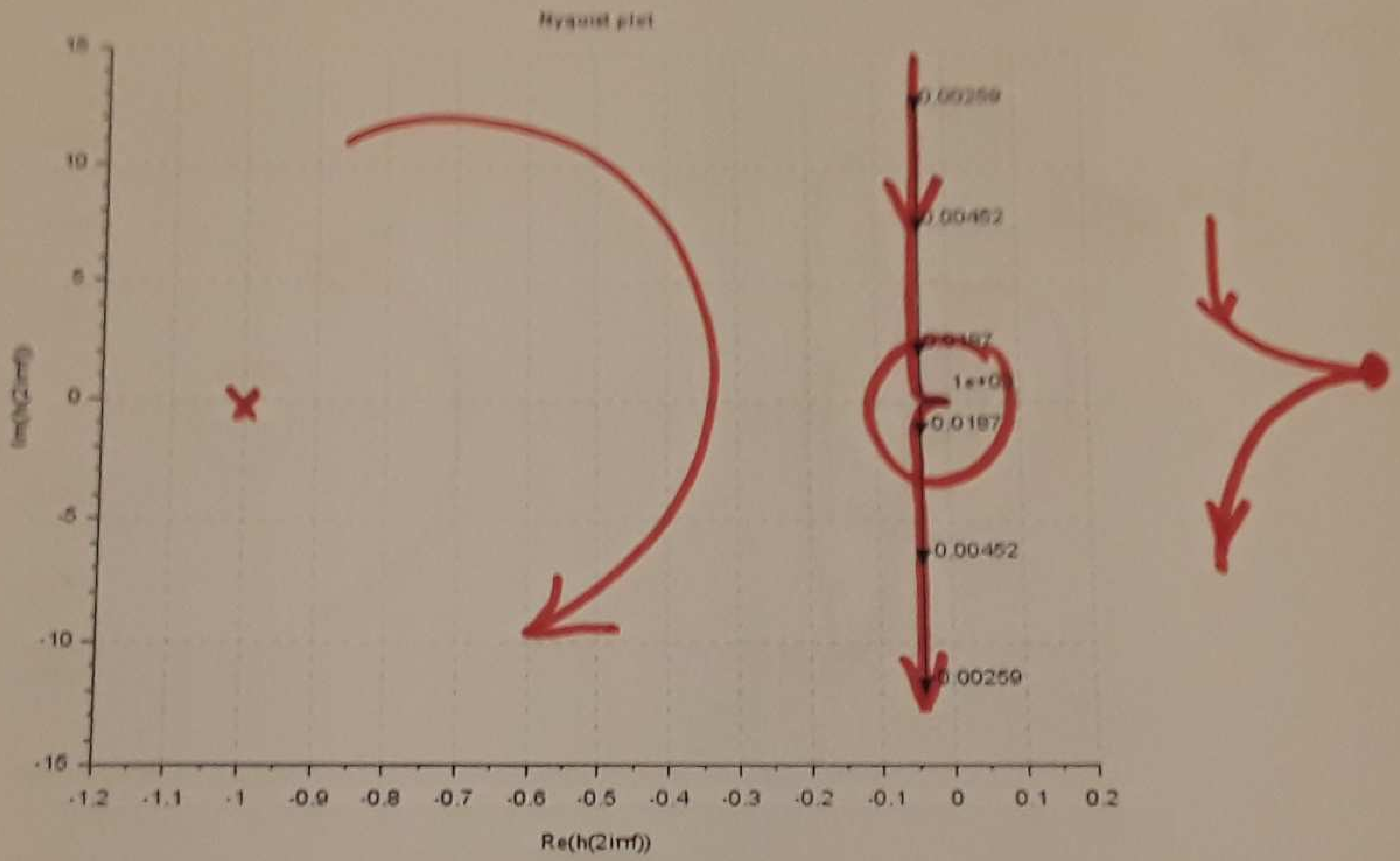


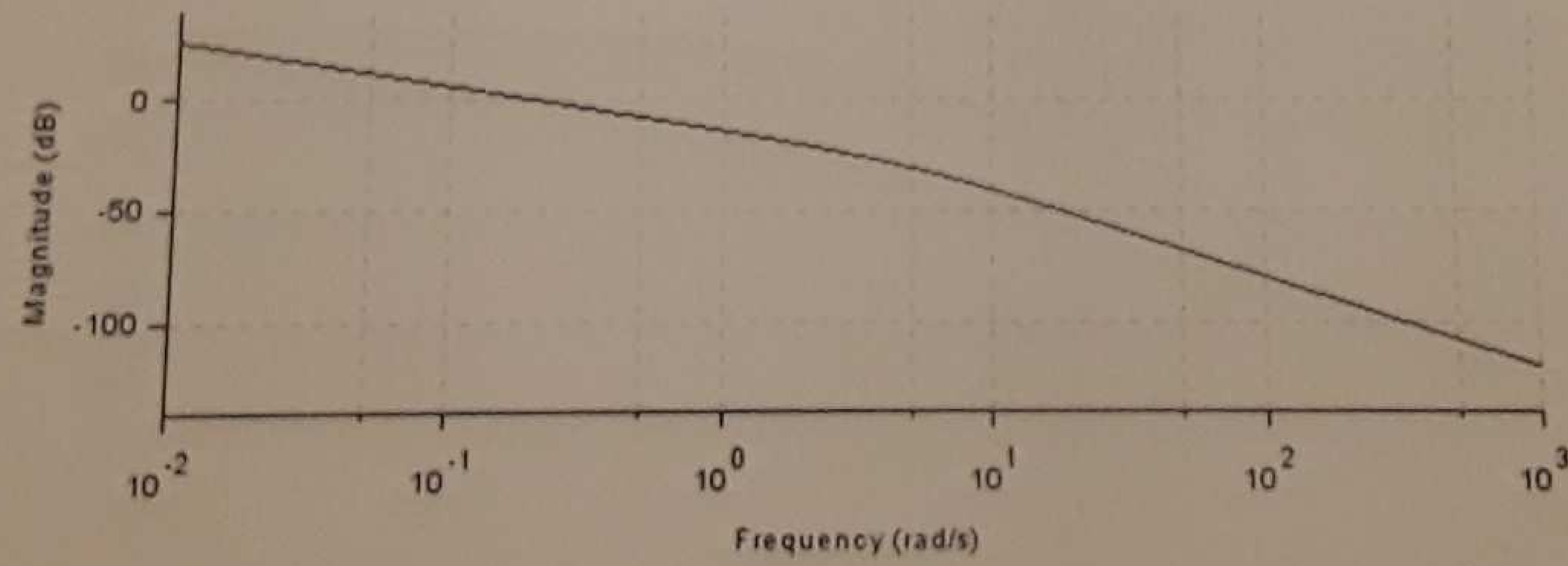
$$W_P = \frac{-0{,}2}{(1+s)\underbrace{(1-s0{,}2)}_{T=1}}$$

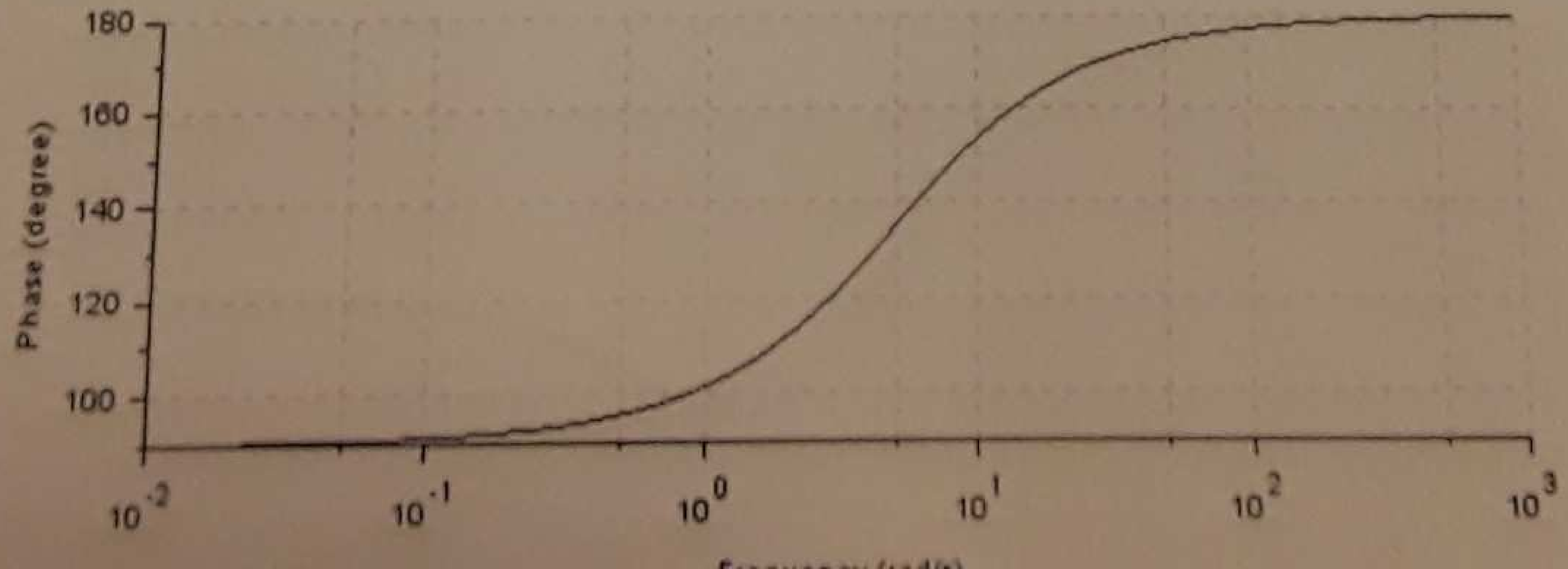
$$W_0 = \underbrace{K_{PI}\frac{\cancel{1+sT_I}}{sT_I}}_{}\frac{-0{,}2}{\cancel{(1+s)}(1-s0{,}2)}$$

$T_I = 1$

PI







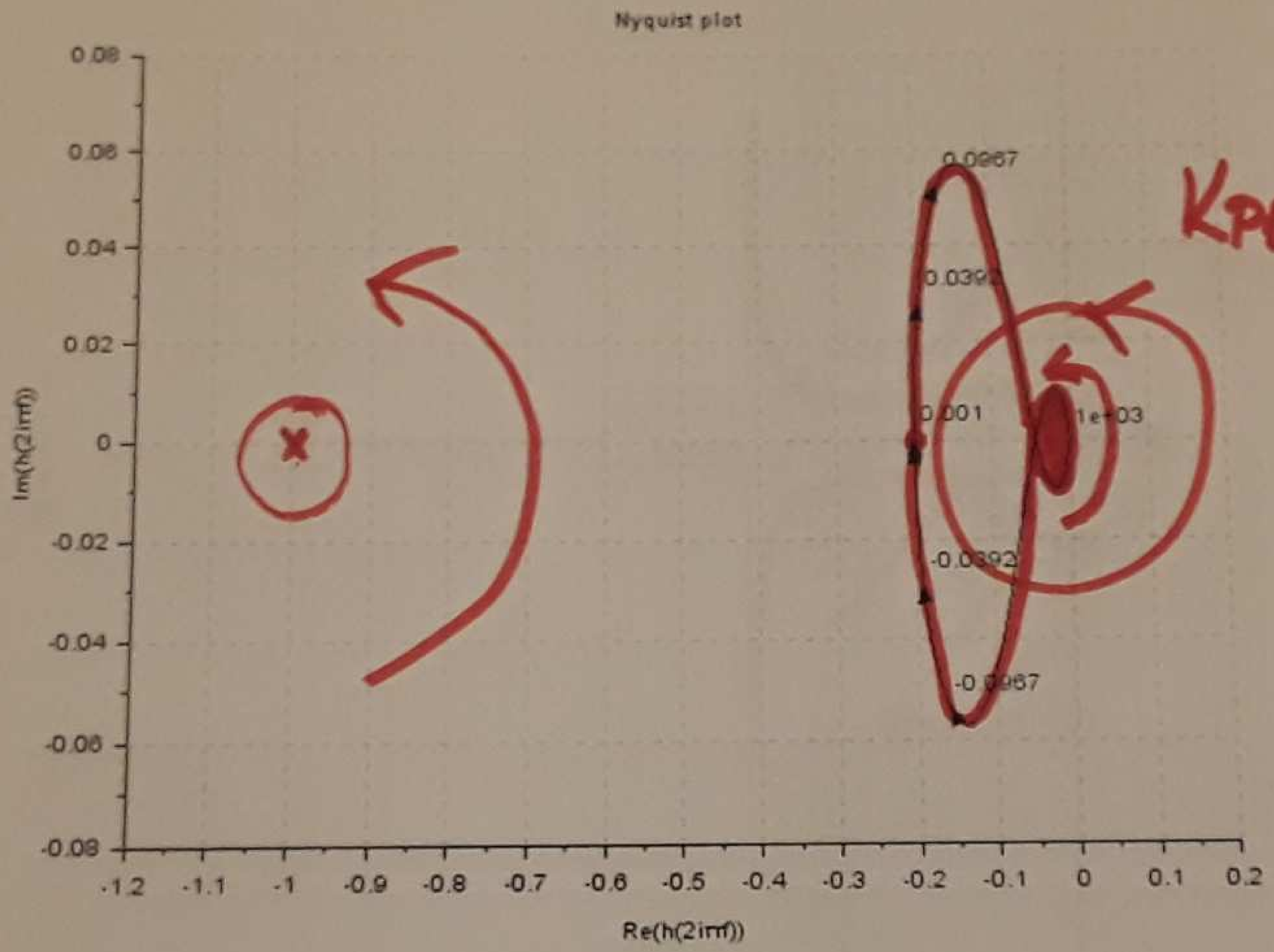
$$W_P = \frac{-0{,}2}{(1+s)\underset{P=1}{\underline{(1-s0{,}2)}}}$$

$$W_0 = \underbrace{\boxed{K_{PD}}\,\frac{1+sT_D}{1+sT_D'}}\,\frac{-0{,}2}{(1+s)(1-s\underline{0{,}2})}$$

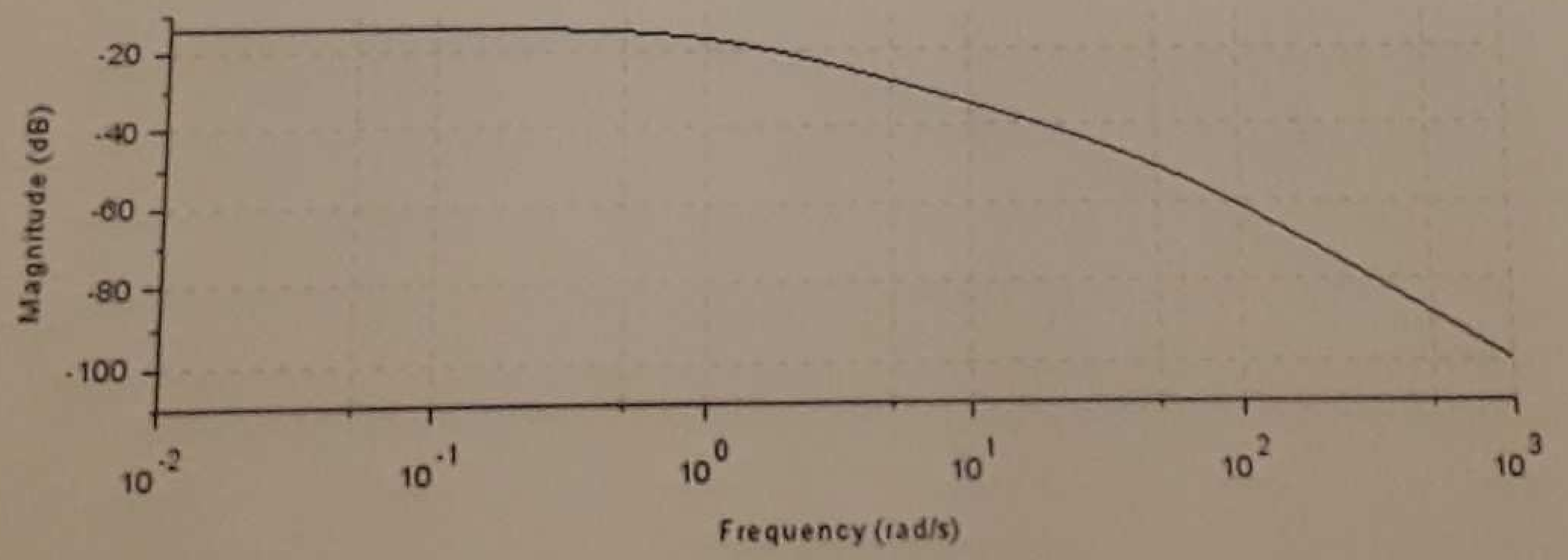
$T_D = 0{,}2$

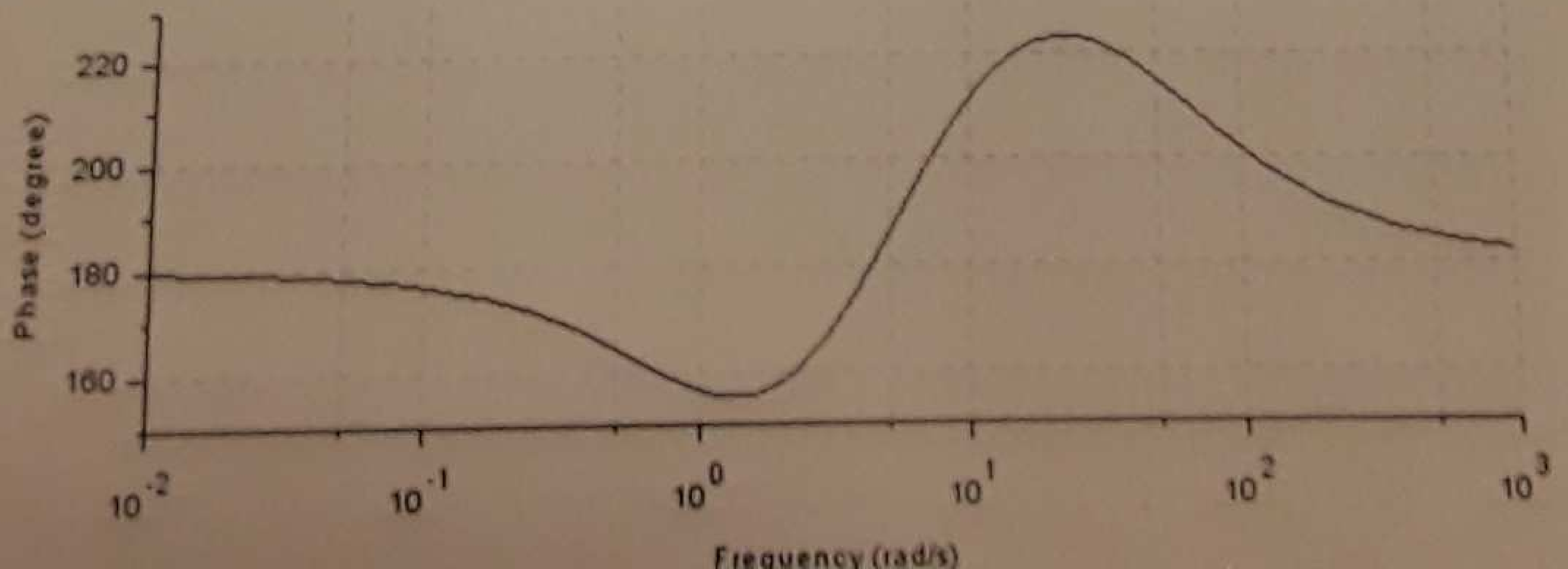
$T_D' = 0{,}02$

PD

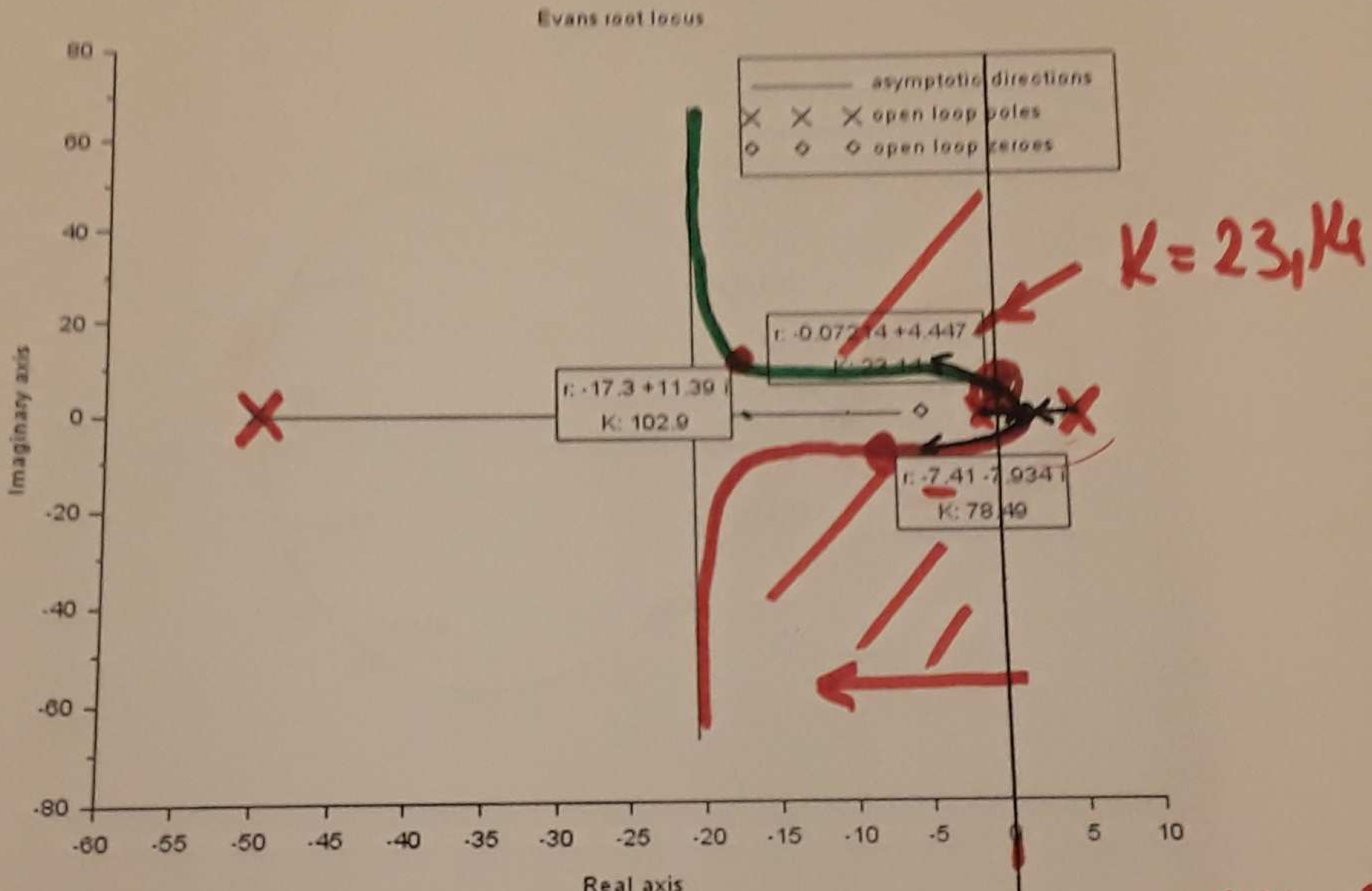


$K_{PD} = 1$



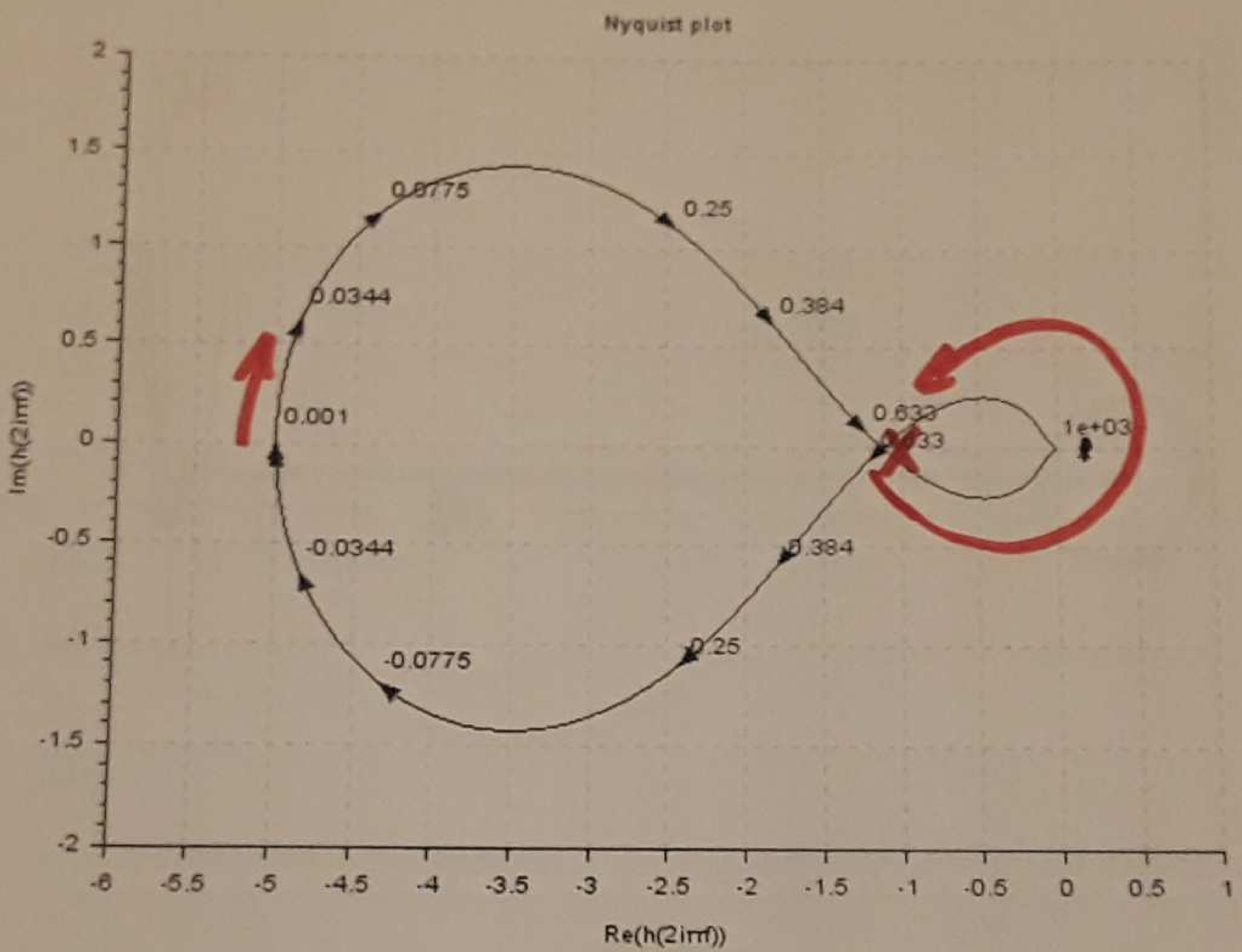


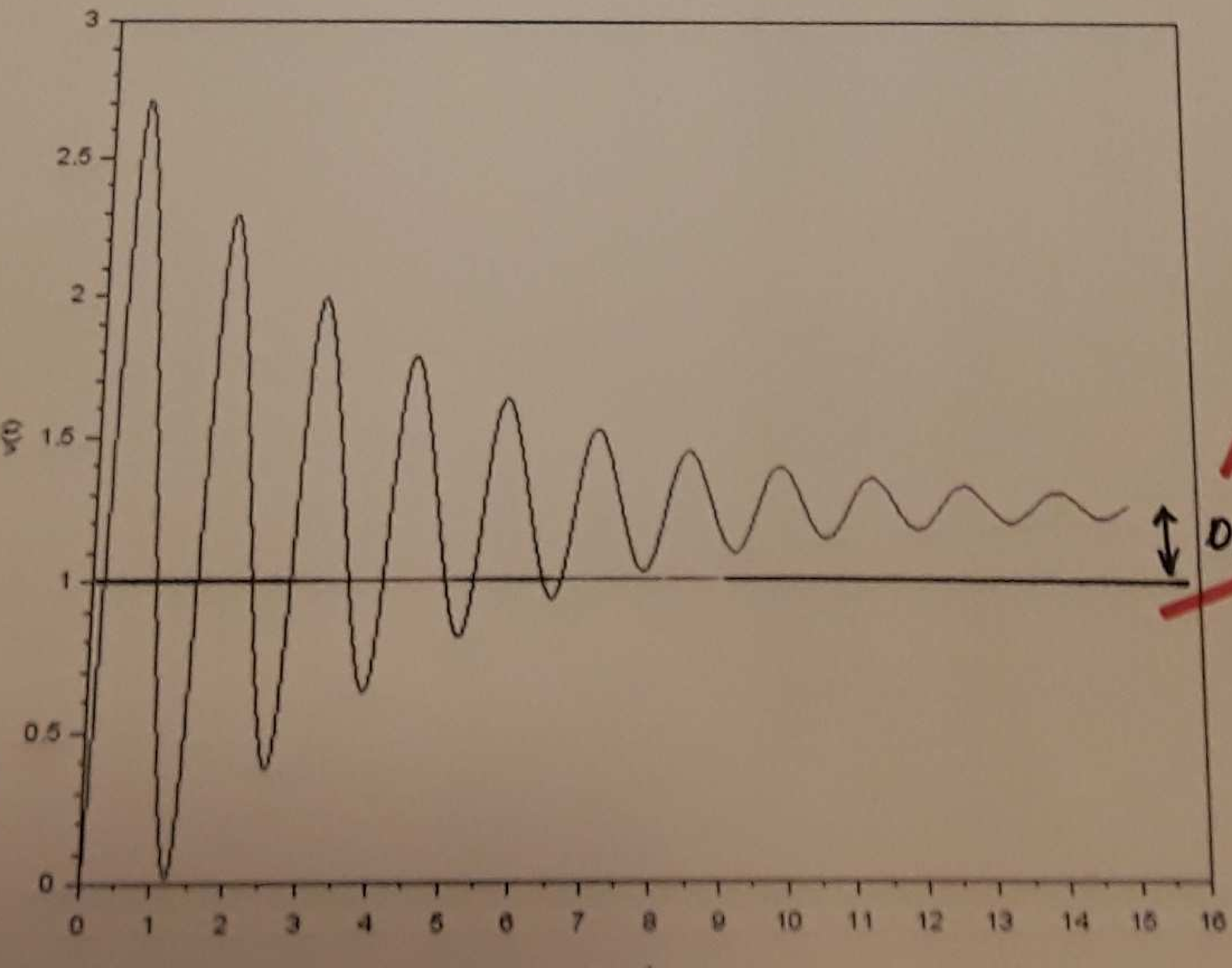
PD


Evans root locus
asymptotic directions
open loop poles
open loop zeroes
Imaginary axis
Real axis
80
60
40
20
0
-20
-40
-60
-80
-60
-55
-50
-45
-40
-35
-30
-25
-20
-15
-10
-5
0
5
10
r: -0.07214 +4.447
K: 23.14
r: -17.3 +11.39 i
K: 102.9
r: -7.41 -7.934 i
K: 78.49
K=23,14

K>22,4.

PD

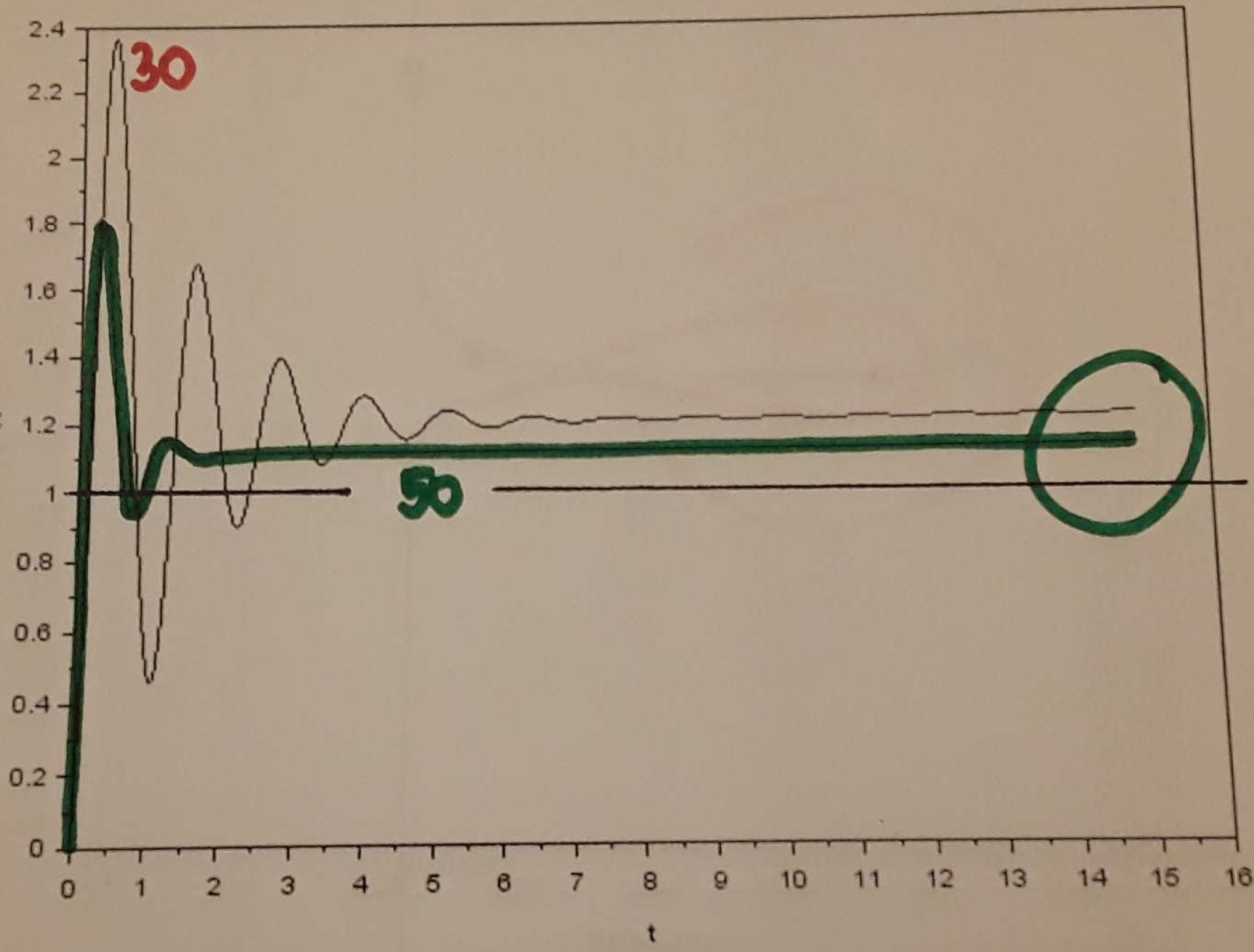

Nyquist plot
Im(h(2iπf))
Re(h(2iπf))
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-6
-5.5
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
0.0775
0.25
0.0344
0.384
0.001
0.633
0.633
1e+03
-0.0344
0.384
-0.0775
0.25


y(t)
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

1,25

0,25

PDI

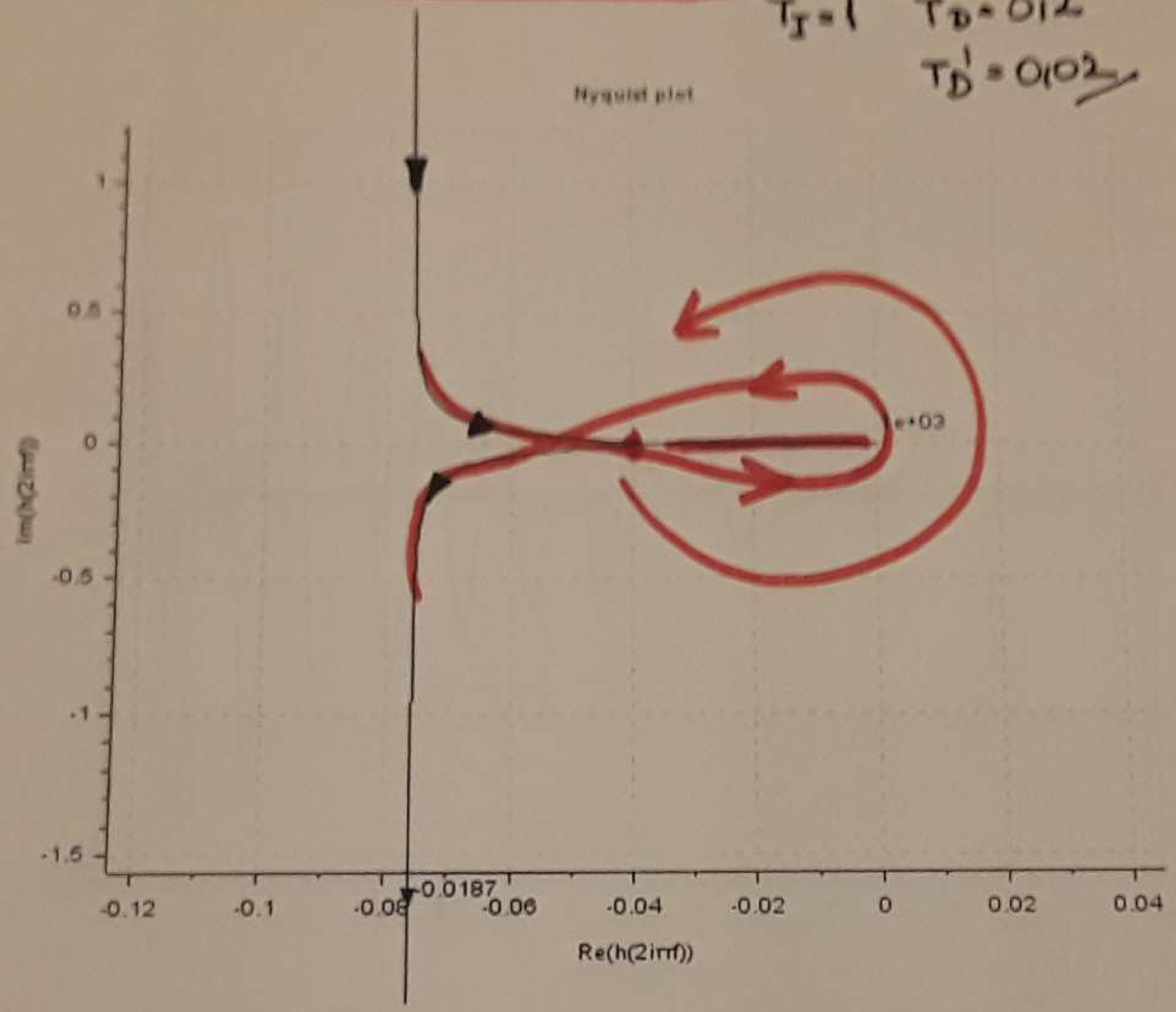

30
50
y(t)
t
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

$$W_0 = \underbrace{K_{PID} \frac{\cancel{1+sT_I}}{sT_I} \frac{1+sT_D}{1+sT_D'}} \frac{-0,2}{\cancel{(1+s)}(1-s0,2)}$$

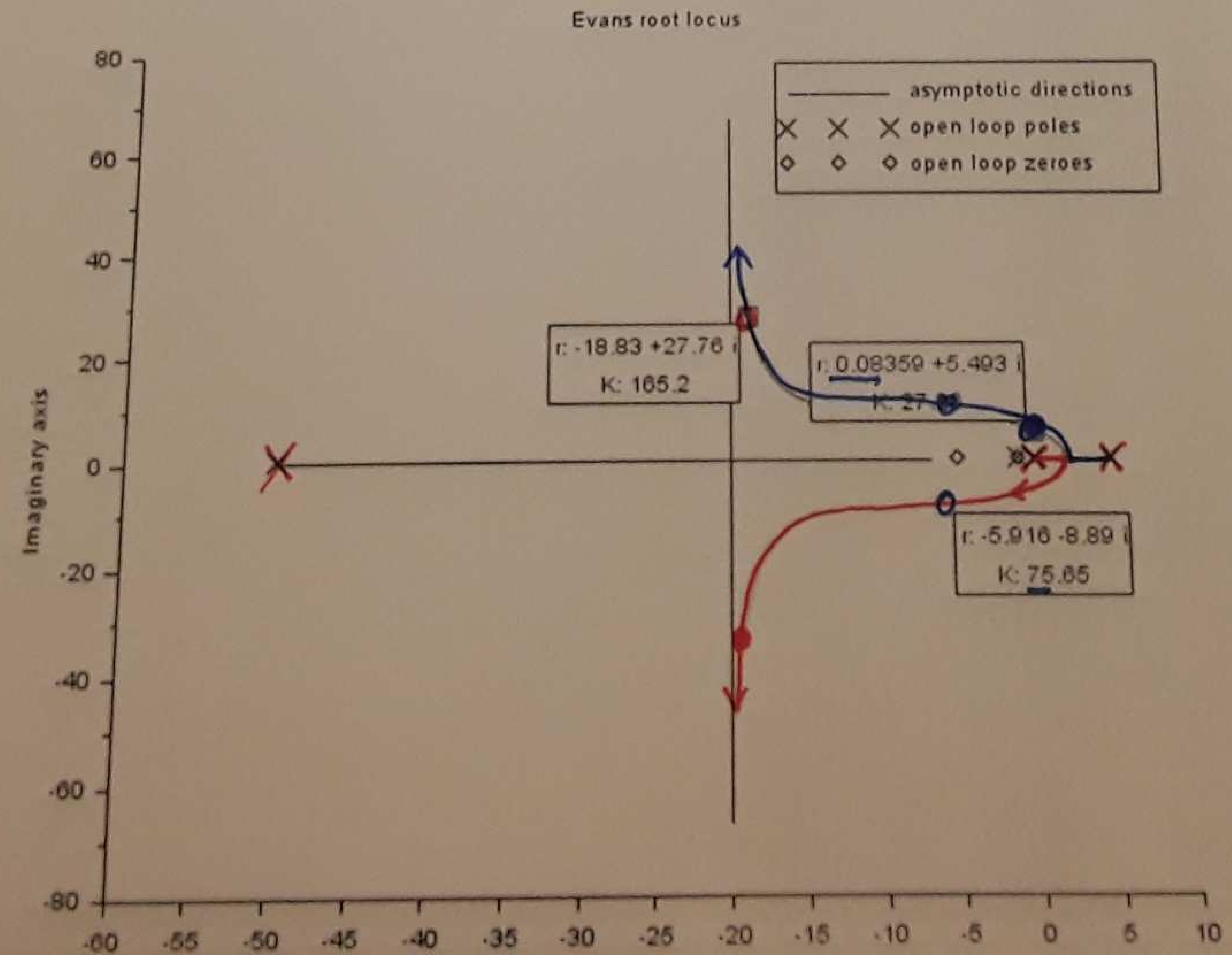
PID

$T_I = 1$ $T_D = 0,2$

$T_D' = 0,02$



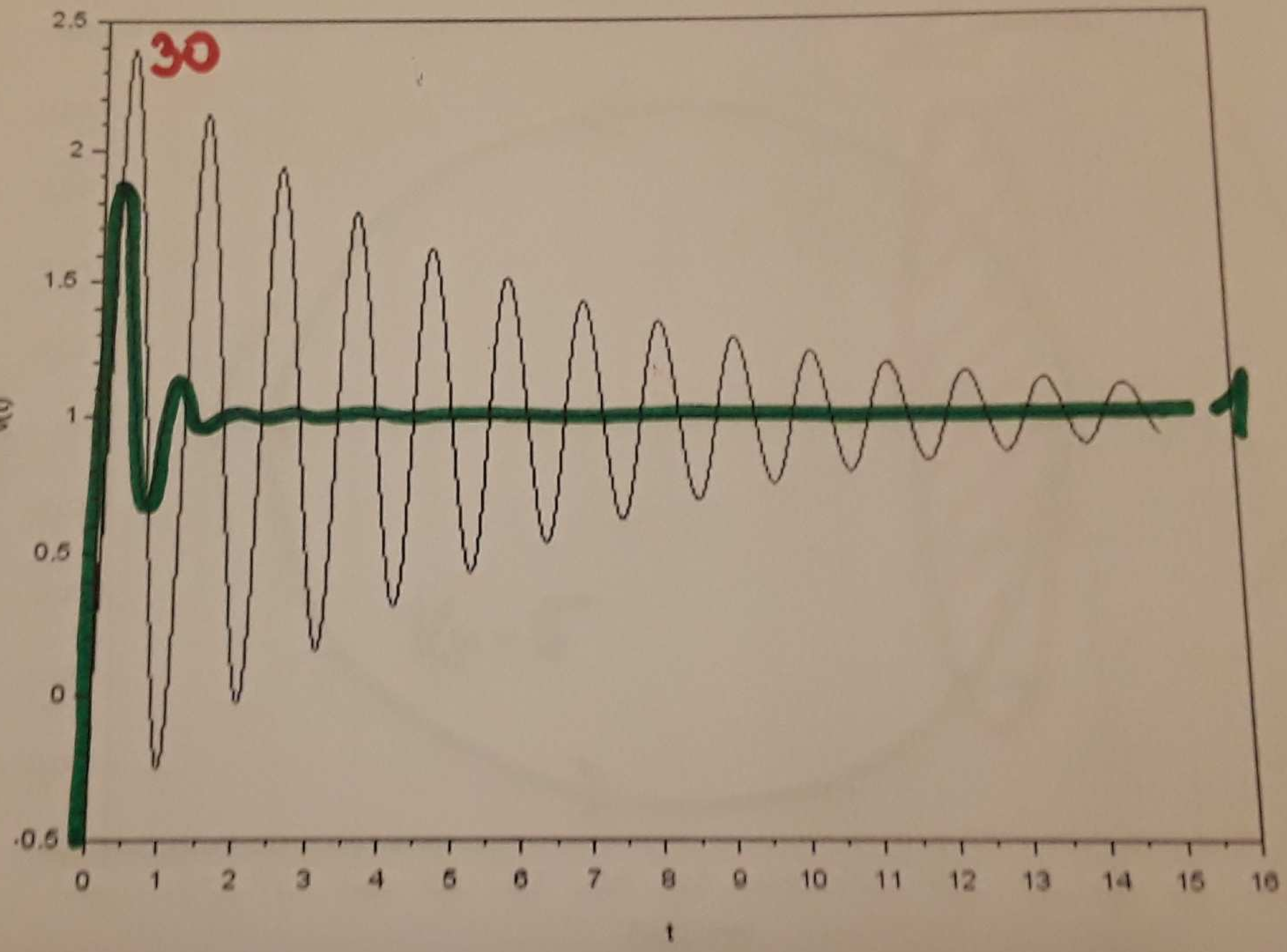
Nyquist plot



Evans root locus

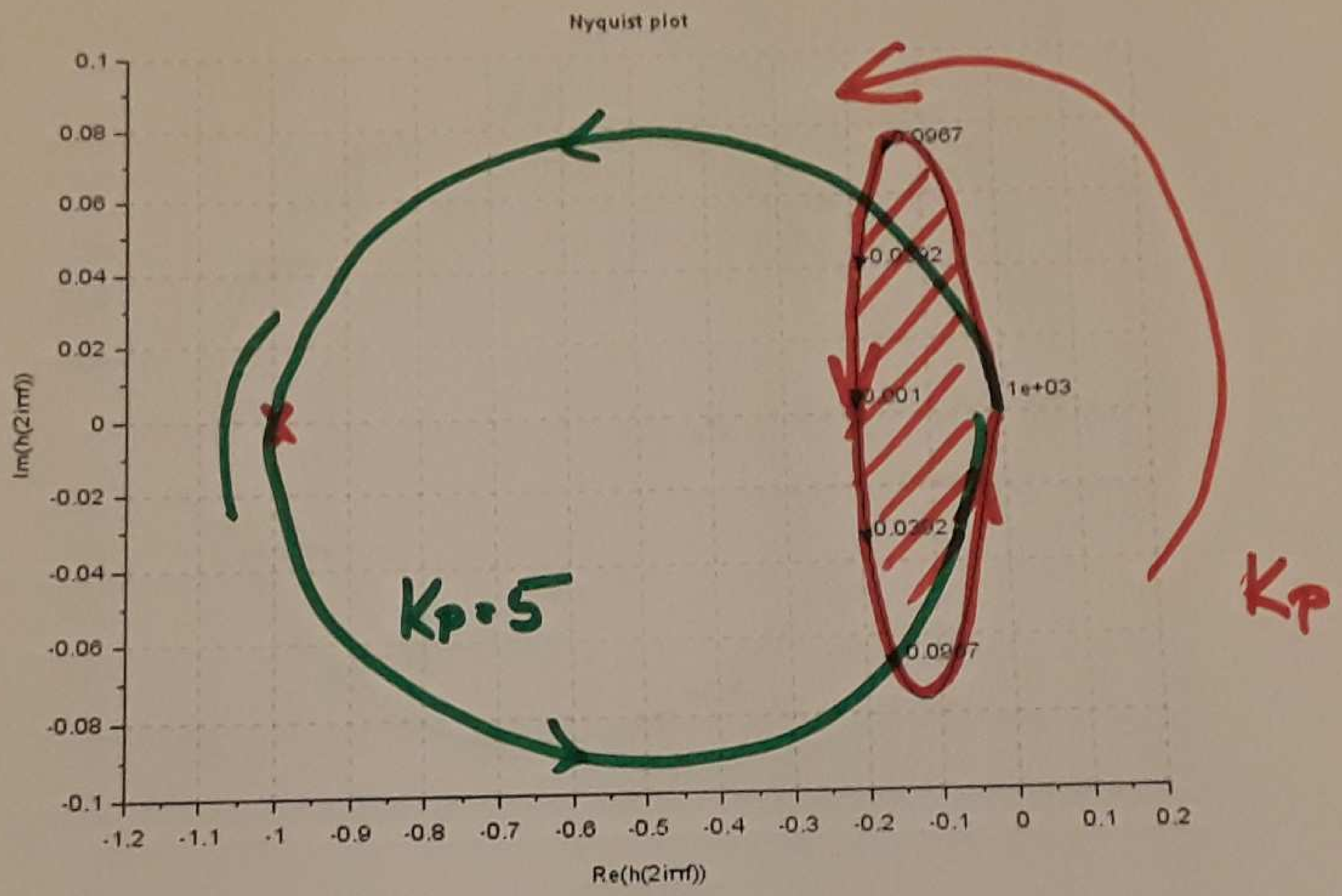
$K > 28,125$

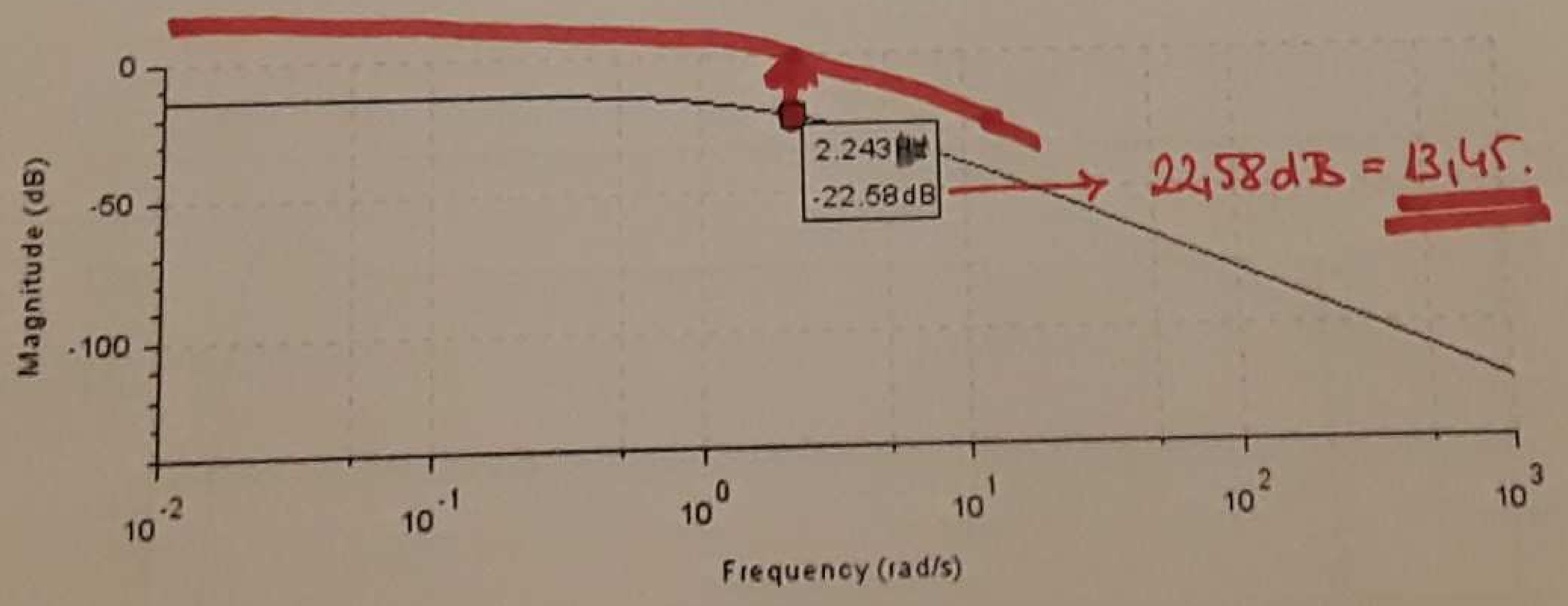
PID

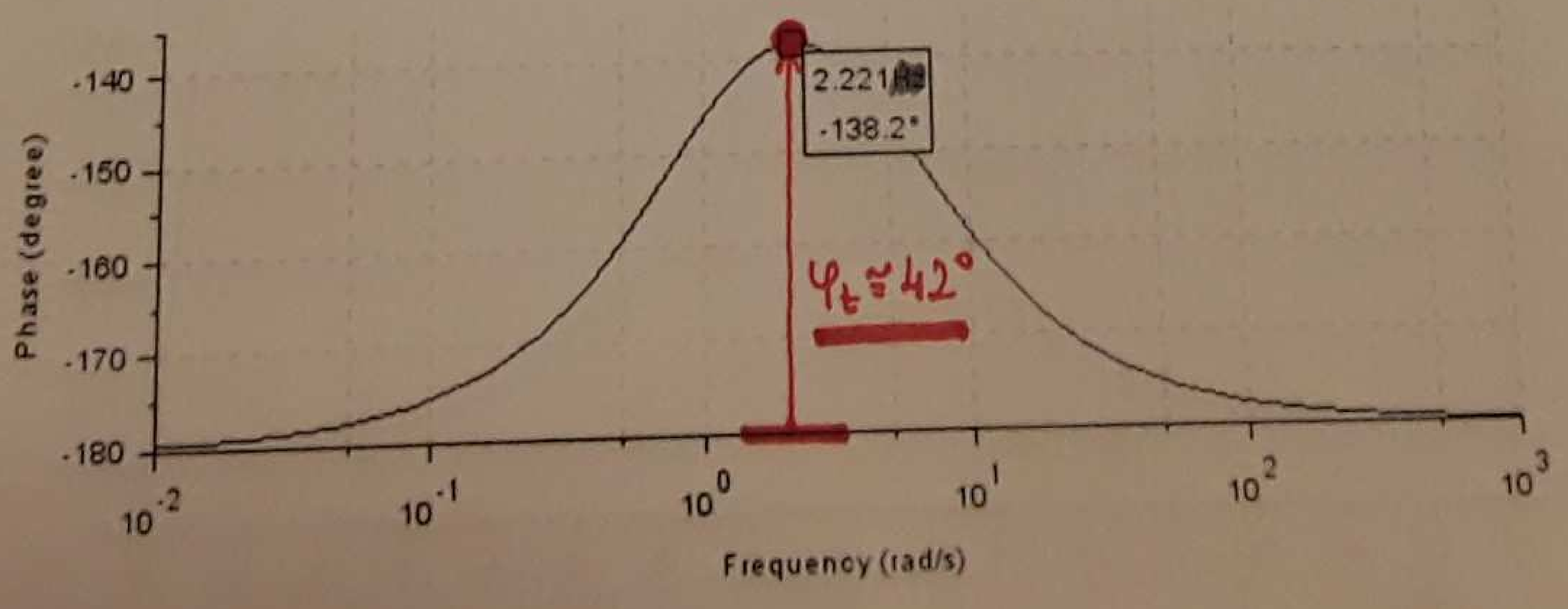


$$W_o = K_P \cdot \frac{-0{,}2}{(1-s)(1+s0{,}2)}$$

P|



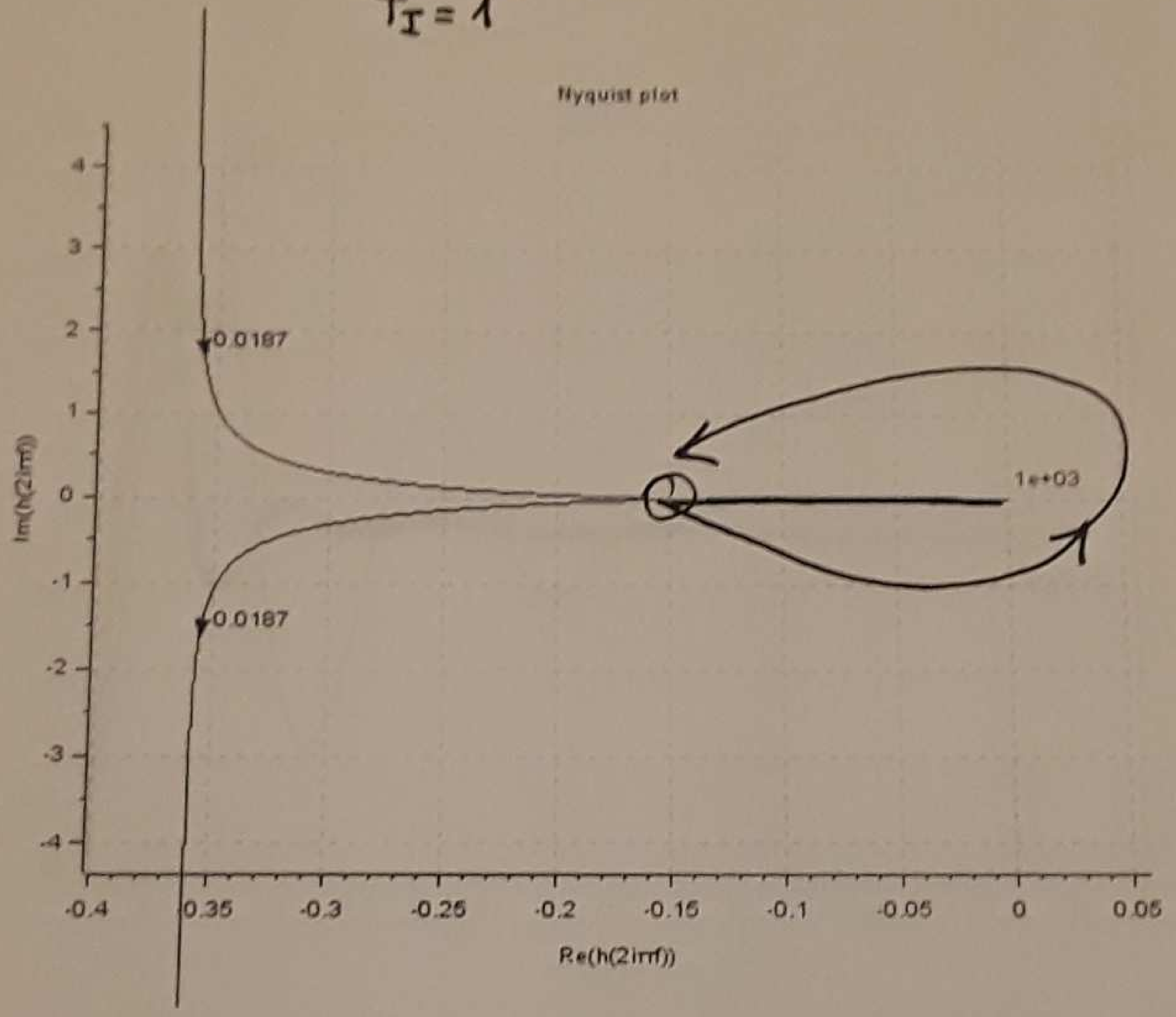


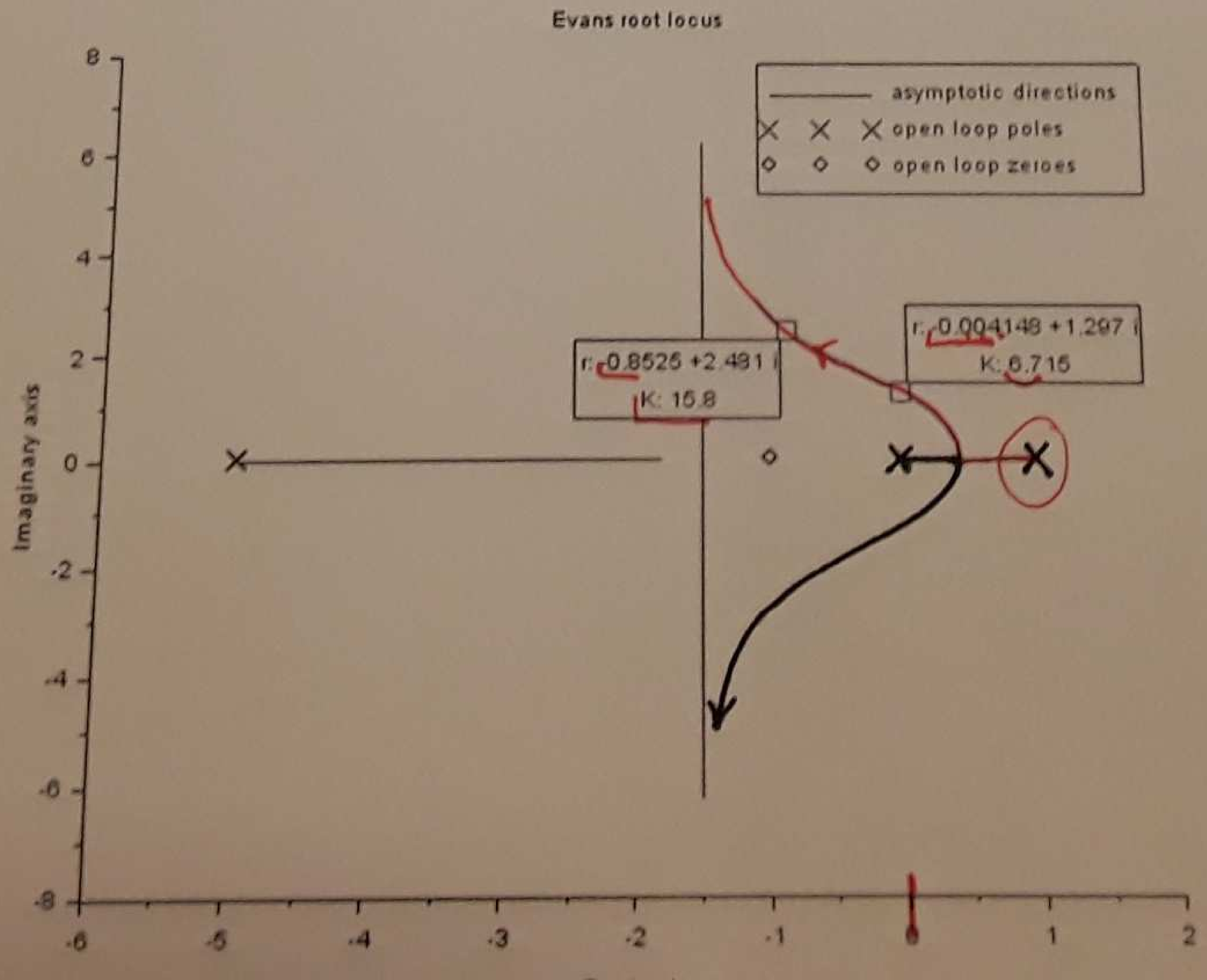


$$W_0 = K_{PI}\,\frac{1+sT_I}{sT_I}\,\frac{-0{,}2}{\underline{(1-s)}\,(1+s0{,}2)}$$

PI

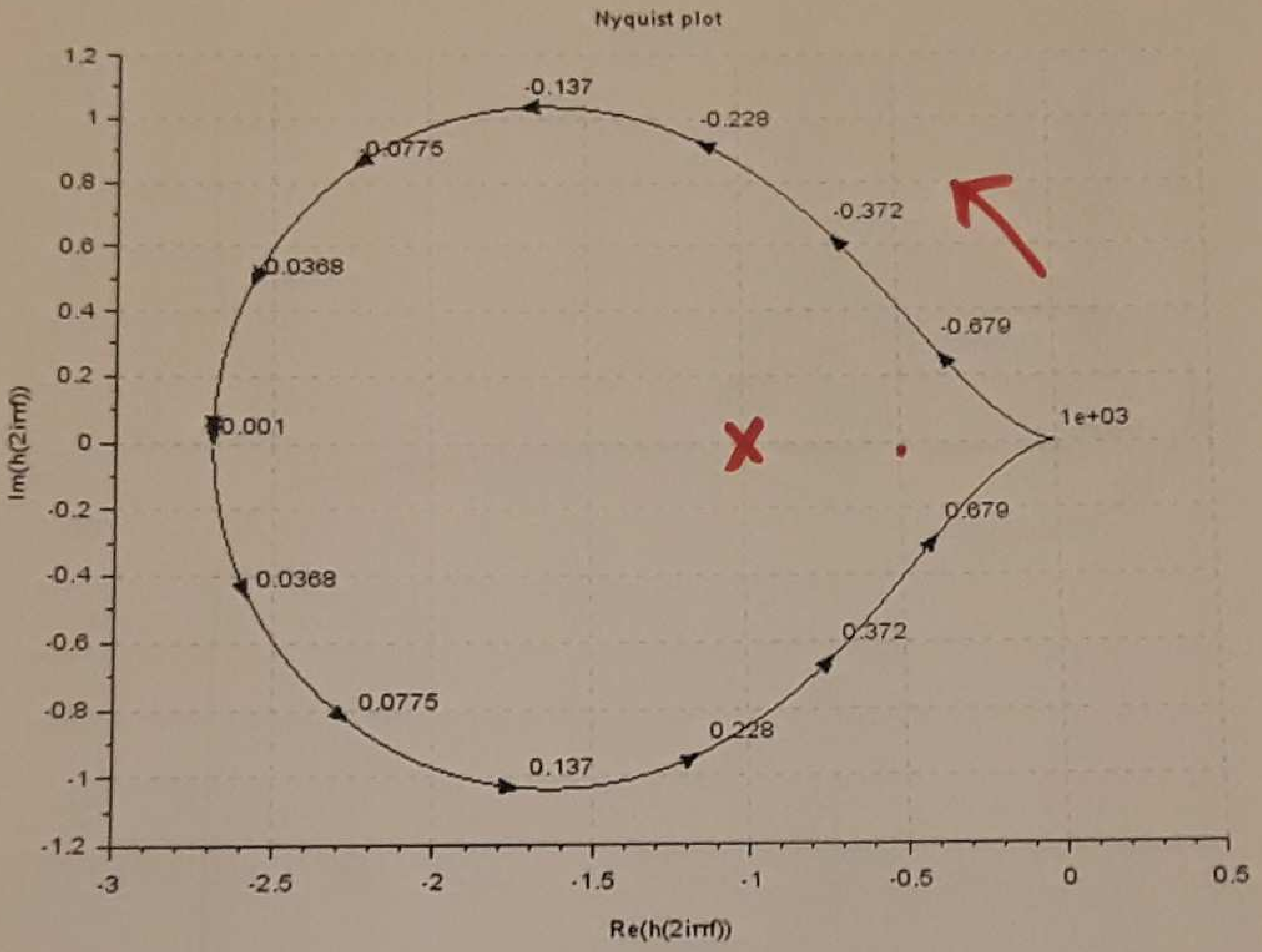
$T_I = 1$



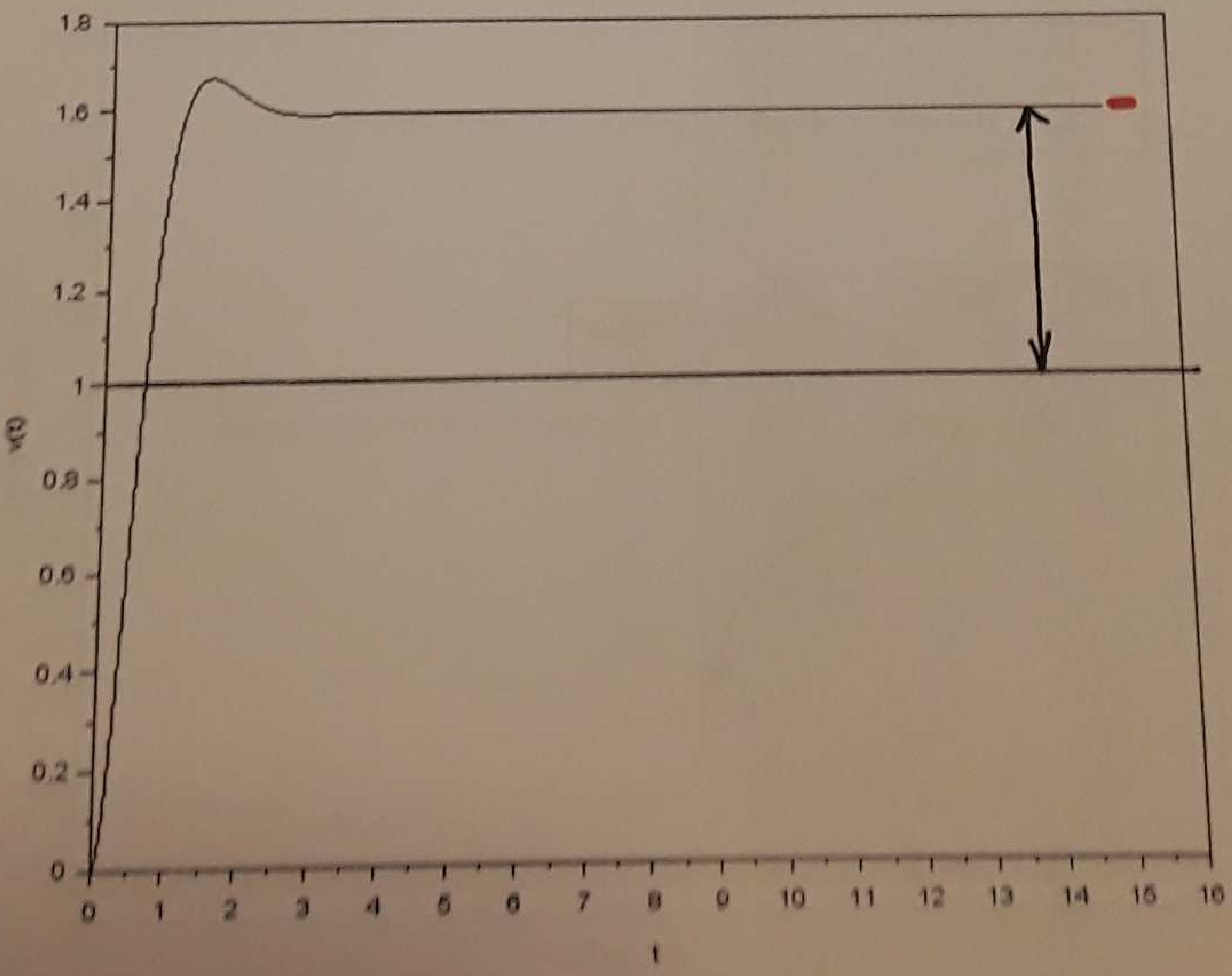


K>6,67

P)

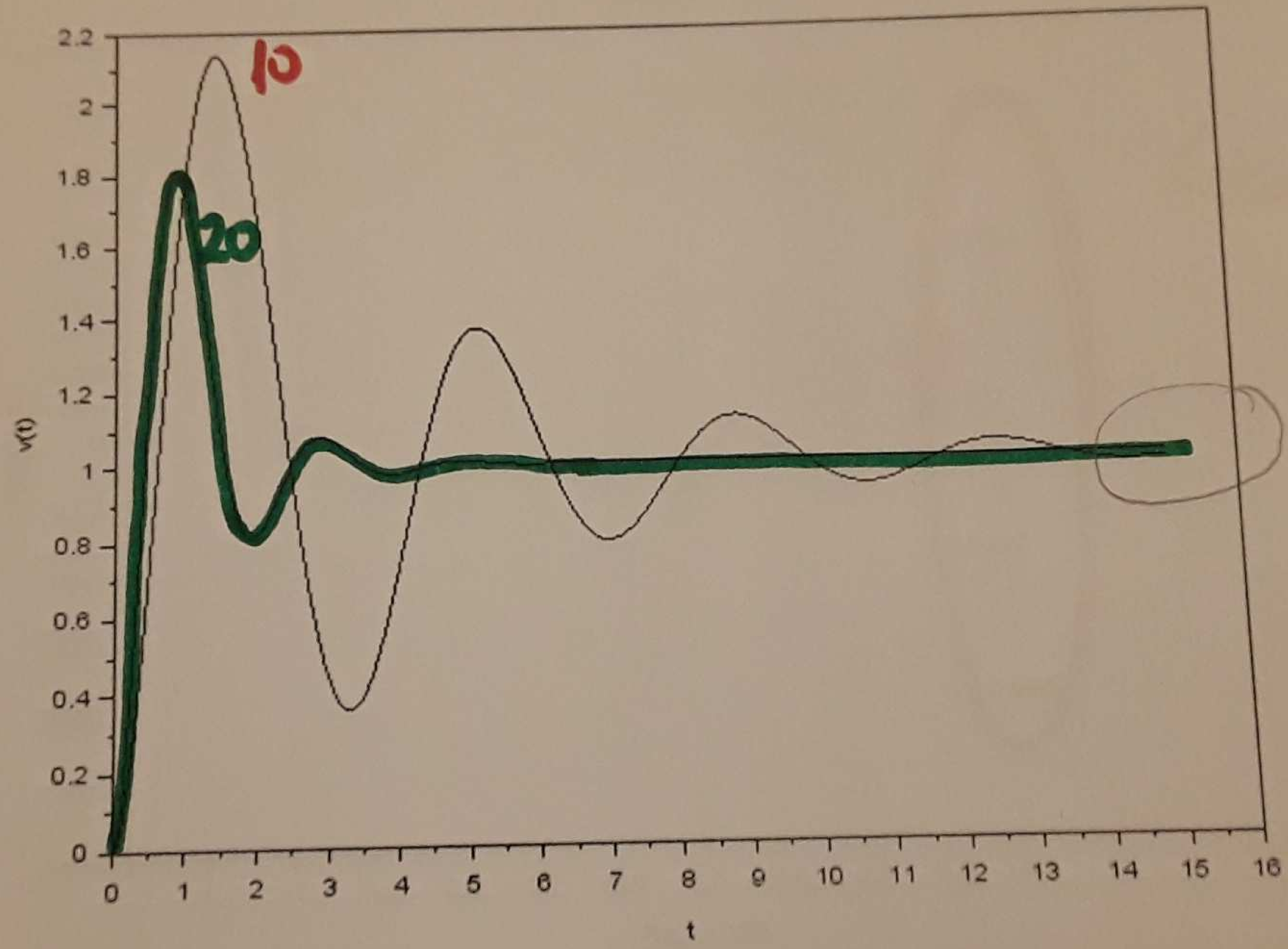


Nyquist plot
Im(h(2iπf))
Re(h(2iπf))
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
-0.137
-0.228
-0.0775
-0.372
-0.0368
-0.679
-0.001
1e+03
0.679
0.0368
0.372
0.0775
0.228
0.137



y(t)
t
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16

PI



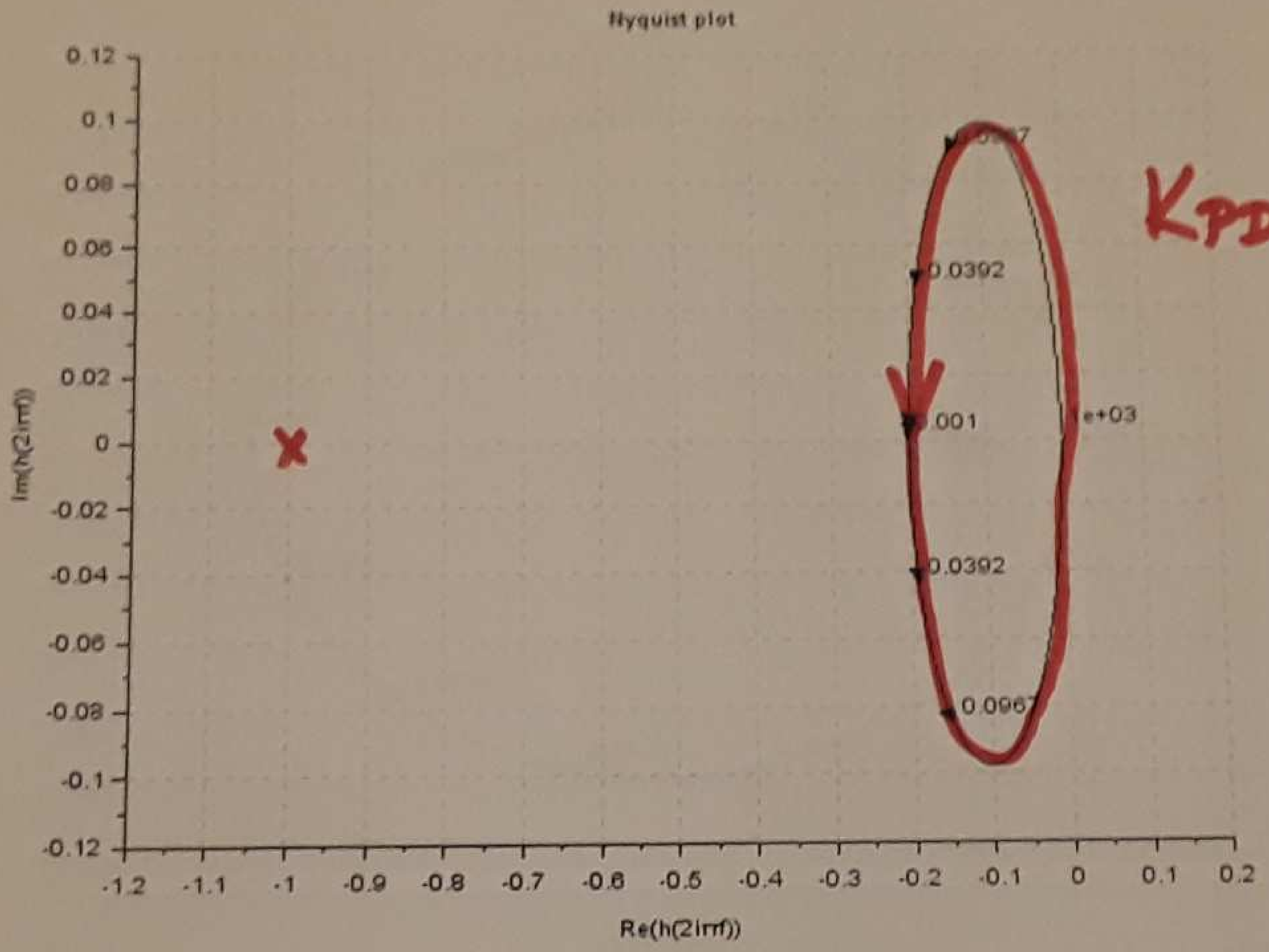
10
20
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
y(t)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
t

$$W_0 = \underline{K_{PD}} \frac{\cancel{1+sT_D}}{1+sT_D'} \frac{-0,2}{(1-s)\cancel{(1+s0,2)}}$$

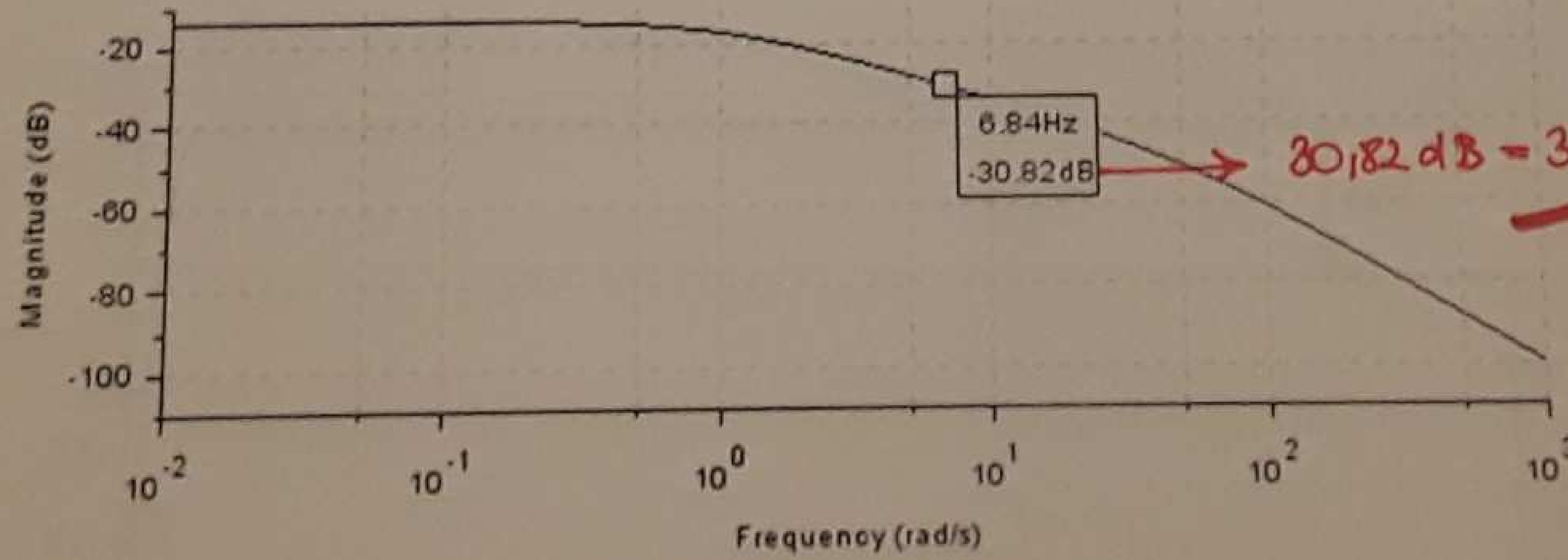
$T_D = 0,2$

$T_D' = 0,02$

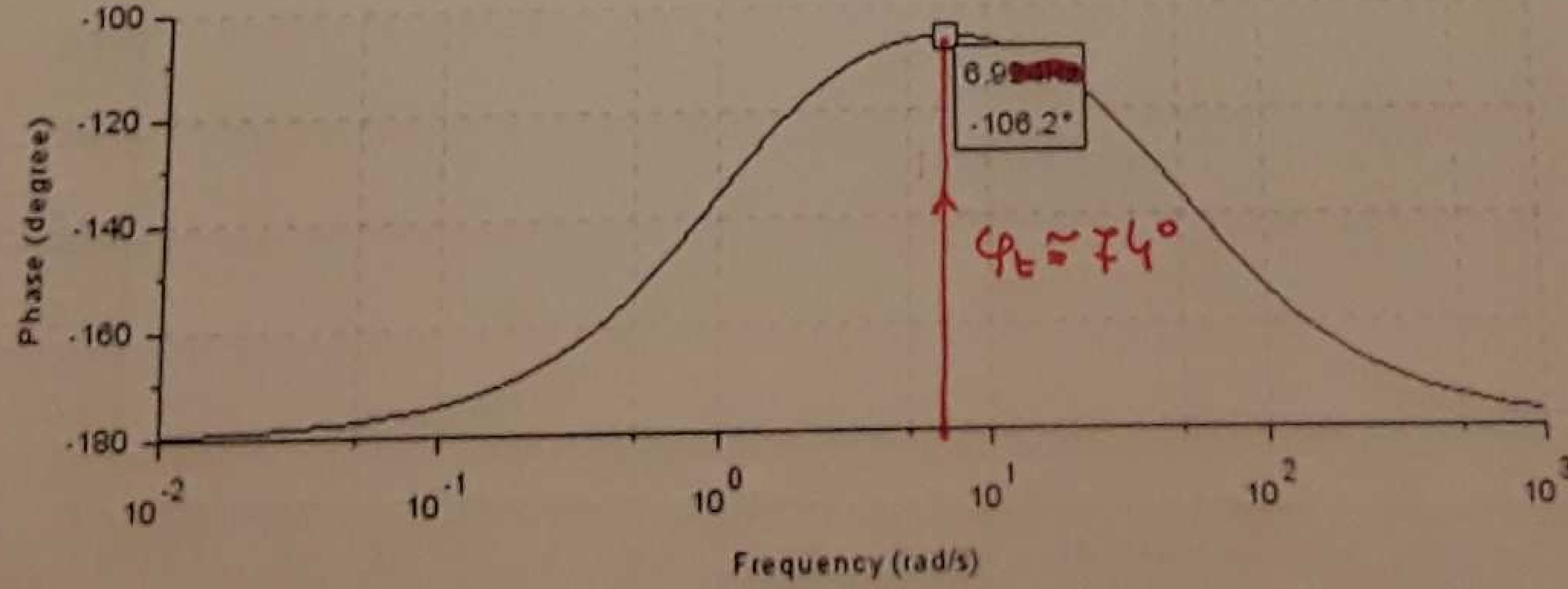
PD



$K_{PD} = 1$



30,82 dB = 34,67



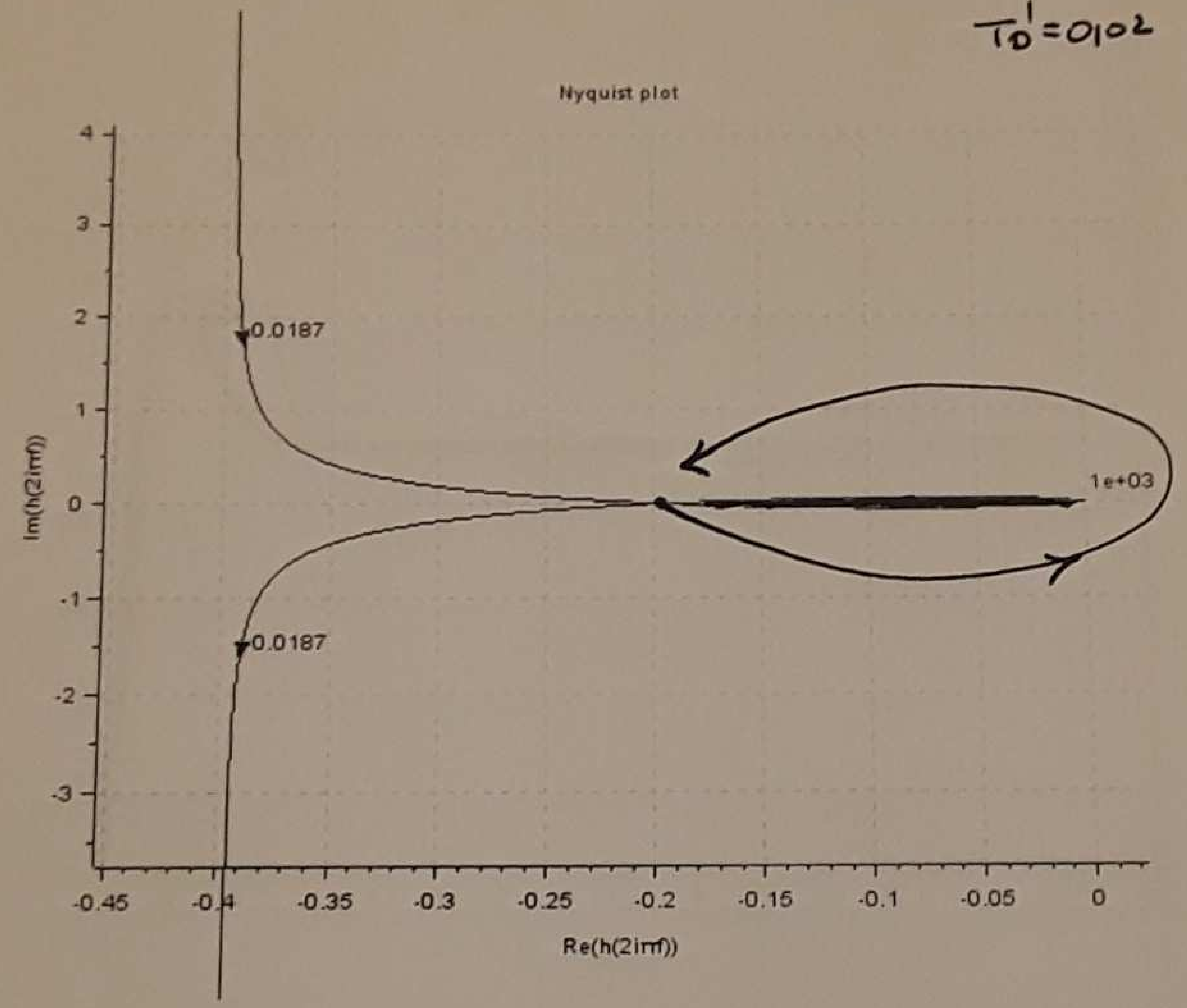
$\varphi_t \approx 74°$

$$W_o = K_{PID}\,\frac{1+sT_I}{sT_I}\,\frac{\cancel{1+sT_D}}{1+sT_D'}\,\frac{-0{,}2}{(1-s)\cancel{(1+s0{,}2)}}$$

$T_I = 1$ $T_D = 0{,}2$

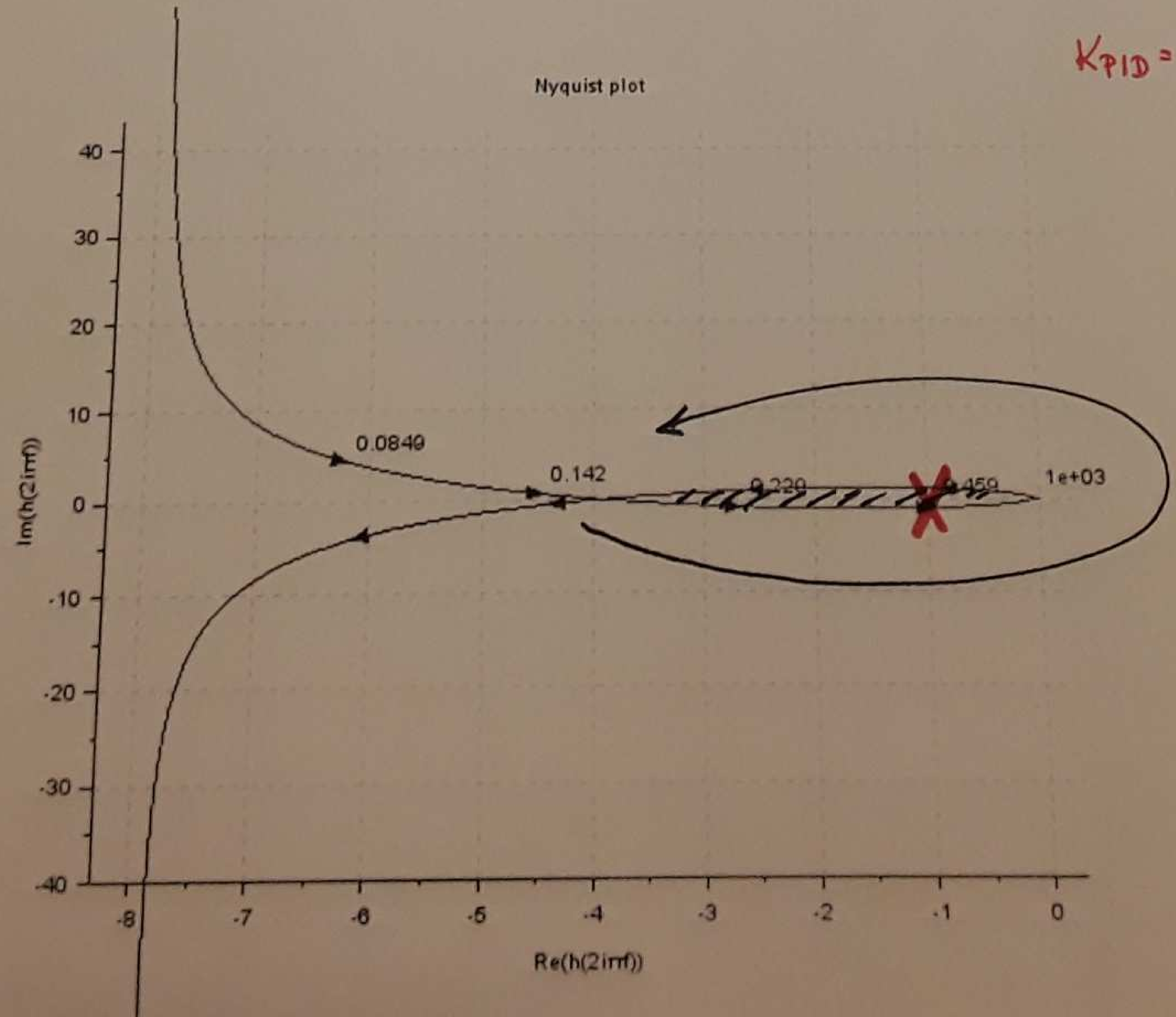
$T_D' = 0{,}02$

PID

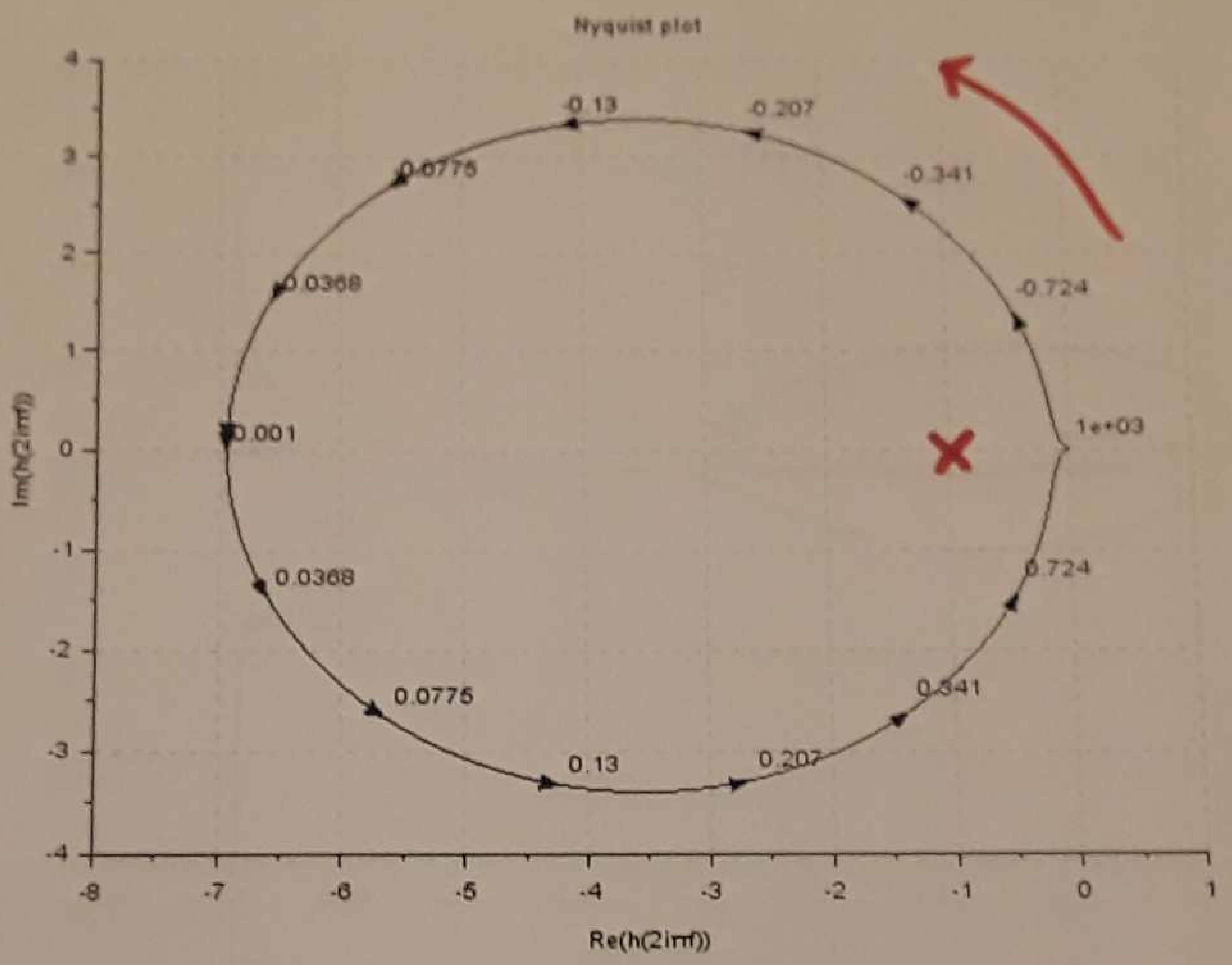


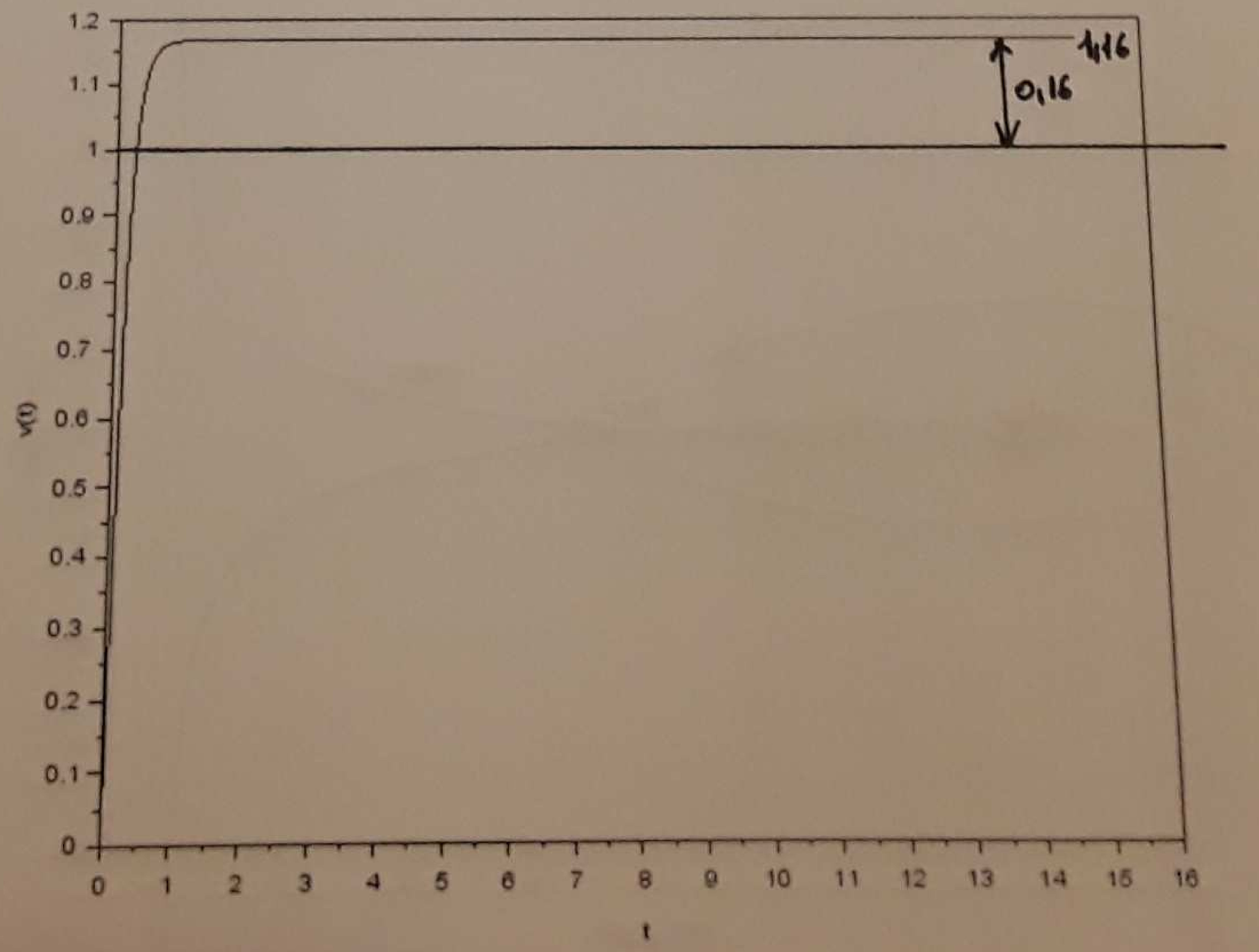
K_{PID}

$K_{PID} = 5{,}1$

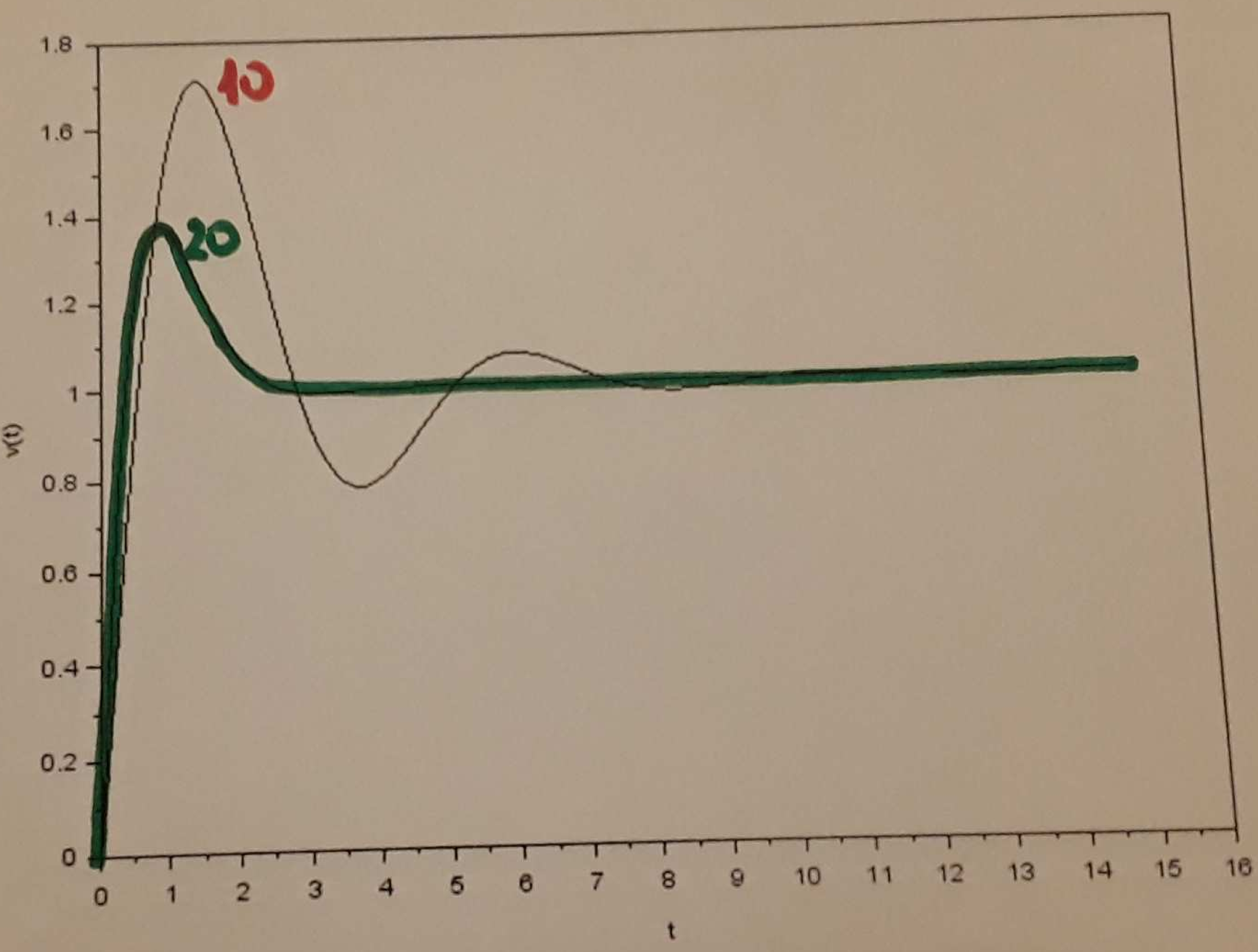


PD


Nyquist plot
Im(h(2iπf))
Re(h(2iπf))
4
3
2
1
0
-1
-2
-3
-4
-8
-7
-6
-5
-4
-3
-2
-1
0
1
-0.13
-0.207
-0.0775
-0.341
-0.0368
-0.724
0.001
1e+03
0.0368
0.724
0.0775
0.341
0.13
0.207


1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y(t)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
t
1,16
0,16

PID



Hol van a rendszerhez tartozó $\underbrace{\text{vágási körfrekvencia}}_{|W_0(j\omega_c)| = 1 \ (0\,dB)}$, és mekkora a fázistartalék,

ha $W_0(j\omega) = \dfrac{1{,}5}{j\omega(1 + j\omega\,1{,}05)}$?

$$|W_0(j\omega_c)| = \frac{1{,}5}{\omega_c\sqrt{1+\omega_c^2\,1{,}05^2}} = 1$$

$$2{,}25 = \omega_c^2(1 + \omega_c^2\,1{,}1)$$

$$2{,}25 = \omega_c^2 + \omega_c^4\,1{,}1$$

$$0 = 1{,}1\underbrace{\omega_c^4}_{x^2} + \underbrace{\omega_c^2}_{x} - 2{,}25$$

$$0 = 1{,}1x^2 + x - 2{,}25$$

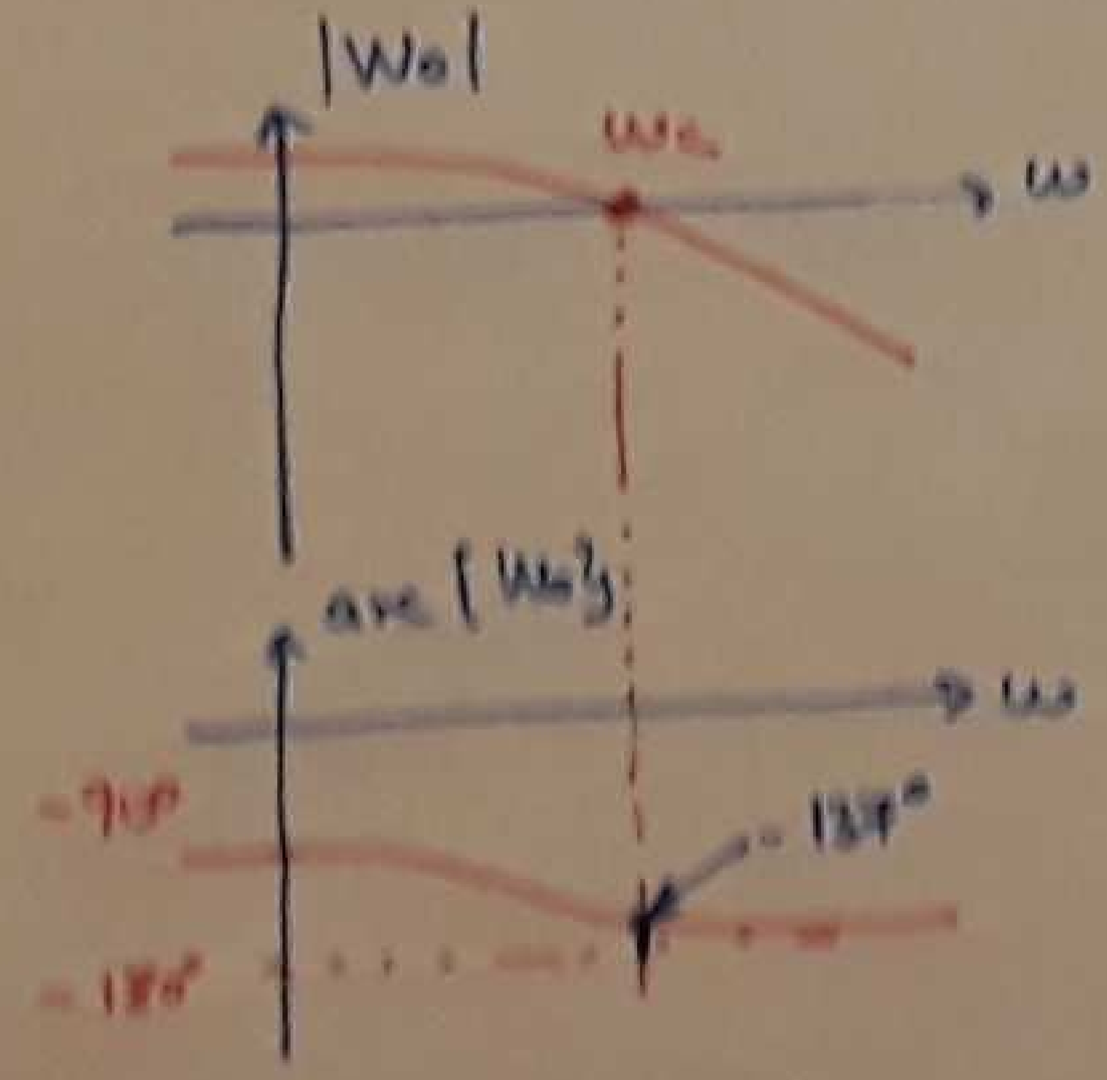
$$x_{1,2} = \frac{-1 \pm \sqrt{1 + 4 \cdot 1{,}1 \cdot 2{,}25}}{2 \cdot 1{,}1} = \frac{-1 \pm 3{,}3}{2{,}2}$$

$x_1 = \overset{1{,}045}{\cancel{1{,}15}} \rightarrow \omega_c = \sqrt{x} = \overset{1{,}02}{\cancel{1{,}07}}$

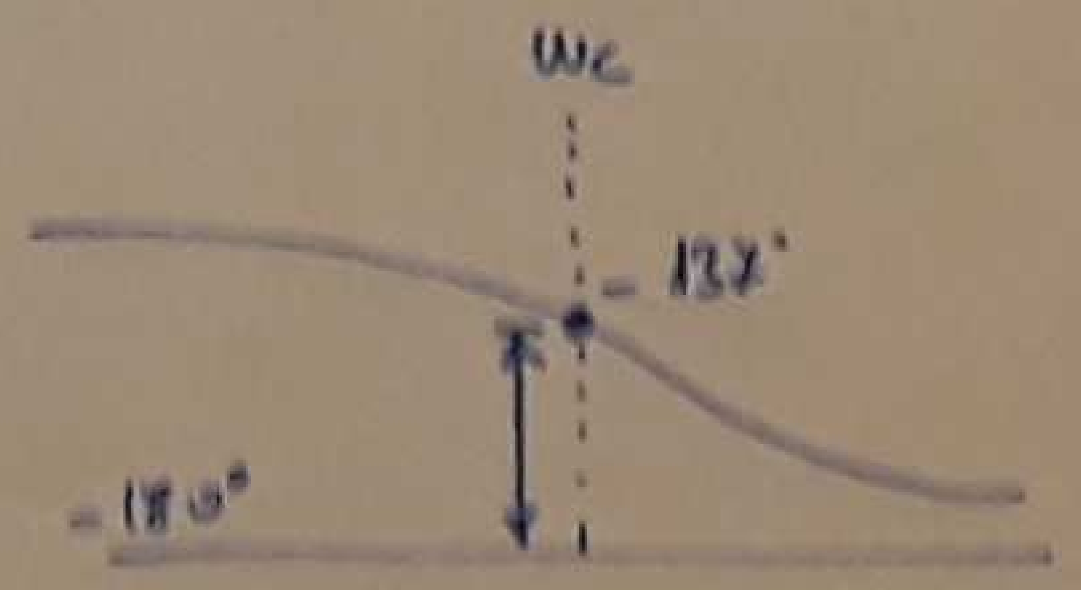
$\boxed{\omega_c = 1{,}02}$

~~$x_2 = -1{,}95$~~ $\rightarrow$? fizikai tartalma nincs!

$$\text{arc}\,W_0(j\omega_c) = 0° - 90° - \underbrace{\text{arctg}\,1{,}05\overset{1{,}02}{\omega_c}}_{46{,}36° \approx 47°} = -90° - 47° = -137°$$

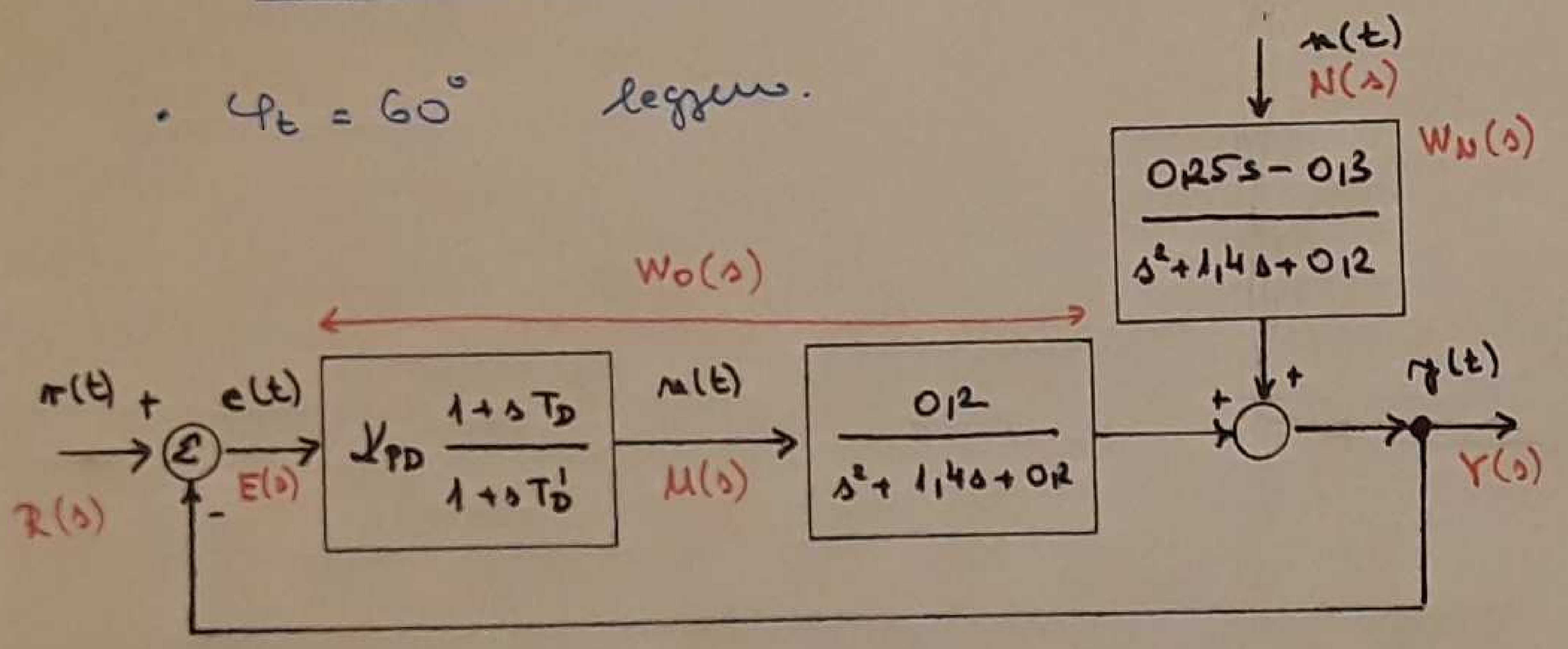


$\varphi_t = 180° - 137° = 43°$



Egy külső jellel zavart rendszer szabályozását PD-szabályozó végzi. Állítsuk be a szabályozó paramétereit:

- az 1(t) zavarójel hatása az y szabályozott jellemzőben $\Delta y = 0{,}1$ változást eredményezzen stacionárius állapotban,
- $\varphi_t = 60°$ legyen.



$E = R - Y$

$Y = W_0 E + W_N N$

$Y = W_0 (R - Y) + W_N N$

$Y = W_0 R \underline{- W_0 Y} + W_N N$

$Y (1 + W_0) = W_0 R + W_N N$

$$Y = \underbrace{\frac{W_0}{1+W_0} R}_{} + \boxed{\underbrace{\frac{W_N}{1+W_0} Ⓝ}_{\Delta y(t)}}$$

$$\lim_{s\to 0} s \frac{W_N}{1+W_0} \overset{1/s}{N} = \lim_{s\to 0} \frac{W_N}{1+W_0} = \frac{-\frac{0{,}3}{0{,}2}}{K_{PD}+1} = \underbrace{\frac{+1{,}5}{1+K_{PD}} = 0{,}1}_{}$$

$15 = 1 + K_{PD}$

$\boxed{K_{PD} = 14}$ MIN!

$$W_o = K_{PD} \frac{\cancel{1+sT_D}}{1+sT_D'} \frac{1}{(1+\underbrace{s6{,}19}_{\frac{1}{6{,}19} = 0{,}16})\cancel{(1+\underbrace{s0{,}181}_{\frac{1}{0{,}181} = 1{,}23})}}$$

$$\boxed{T_D = 0{,}181}$$

$$\frac{1}{T_D'} > 1{,}23 \checkmark$$

$$W_o = 14 \frac{1}{(1+s6{,}19)(1+sT_D')}$$

$20\lg 14 = 23dB$

$23 - 20\lg \omega_c 6{,}19 = 0$

$6{,}19\omega_c = 10^{23/20} = 14{,}12$

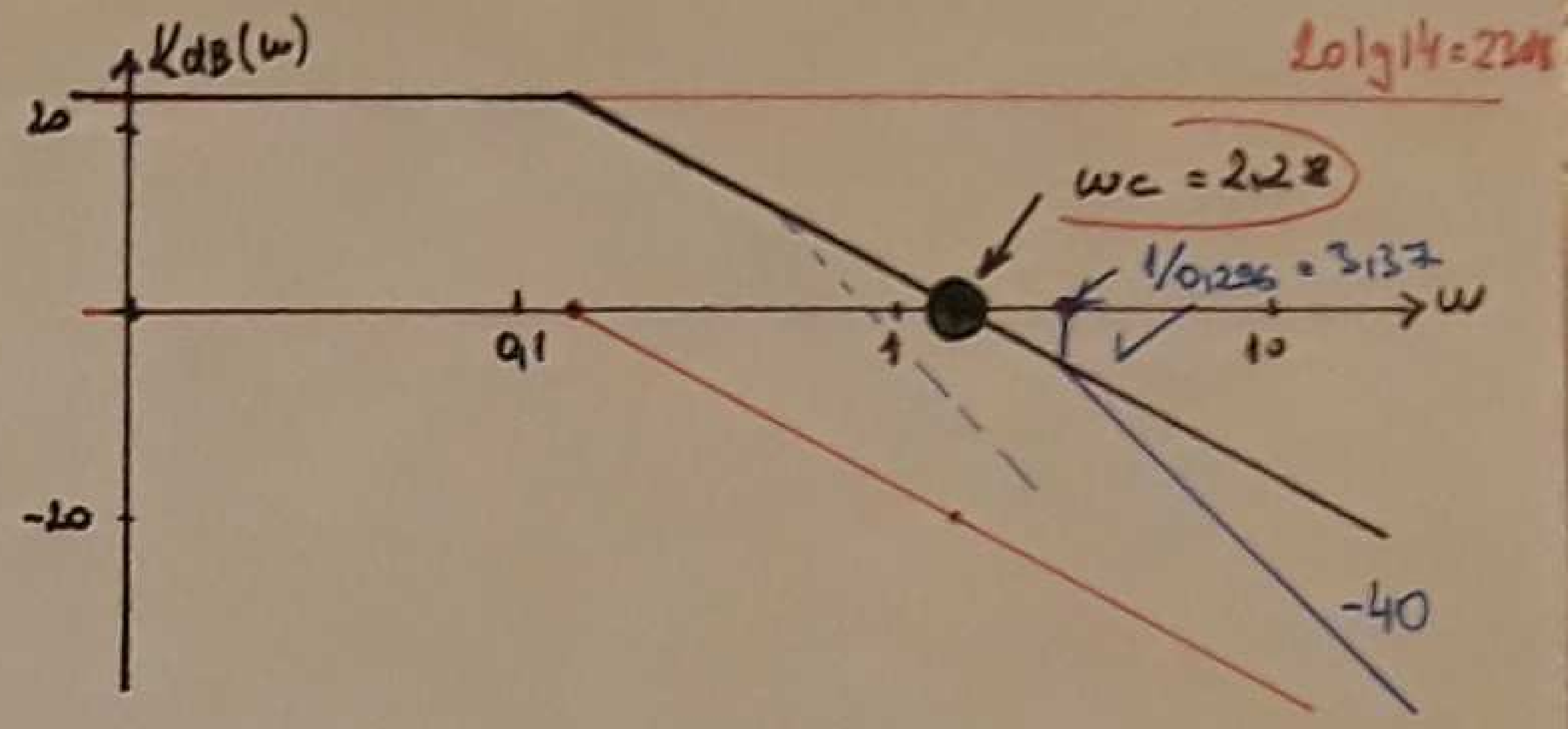
$\underline{\underline{\omega_c - 2{,}28}}$

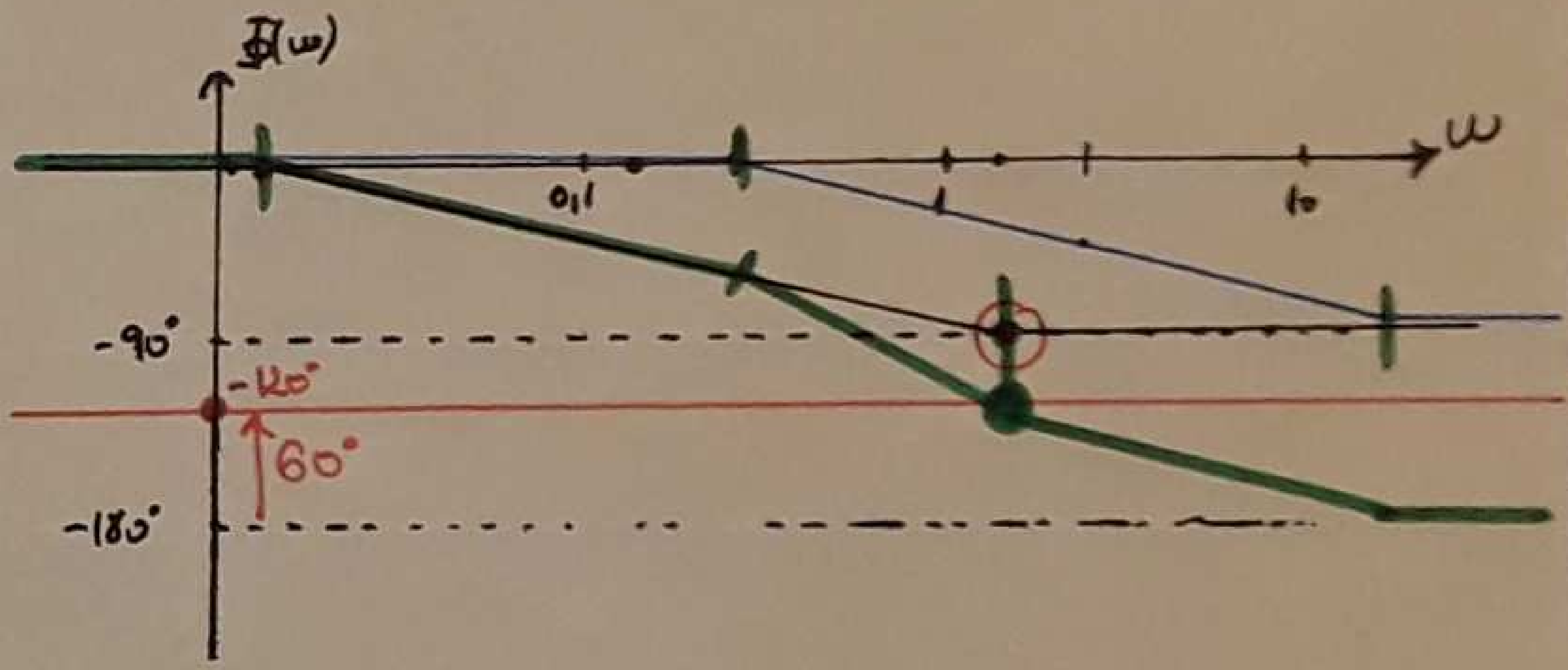
$$\underbrace{-120° = \underbrace{-\operatorname{arctg}(2{,}28 \cdot 6{,}19)}_{-85{,}94°} - \operatorname{arctg}(2{,}28 \cdot T_D')}_{34{,}05° = \operatorname{arctg}(2{,}28 \cdot T_D')}$$

$0{,}676 = 2{,}28 \cdot T_D'$

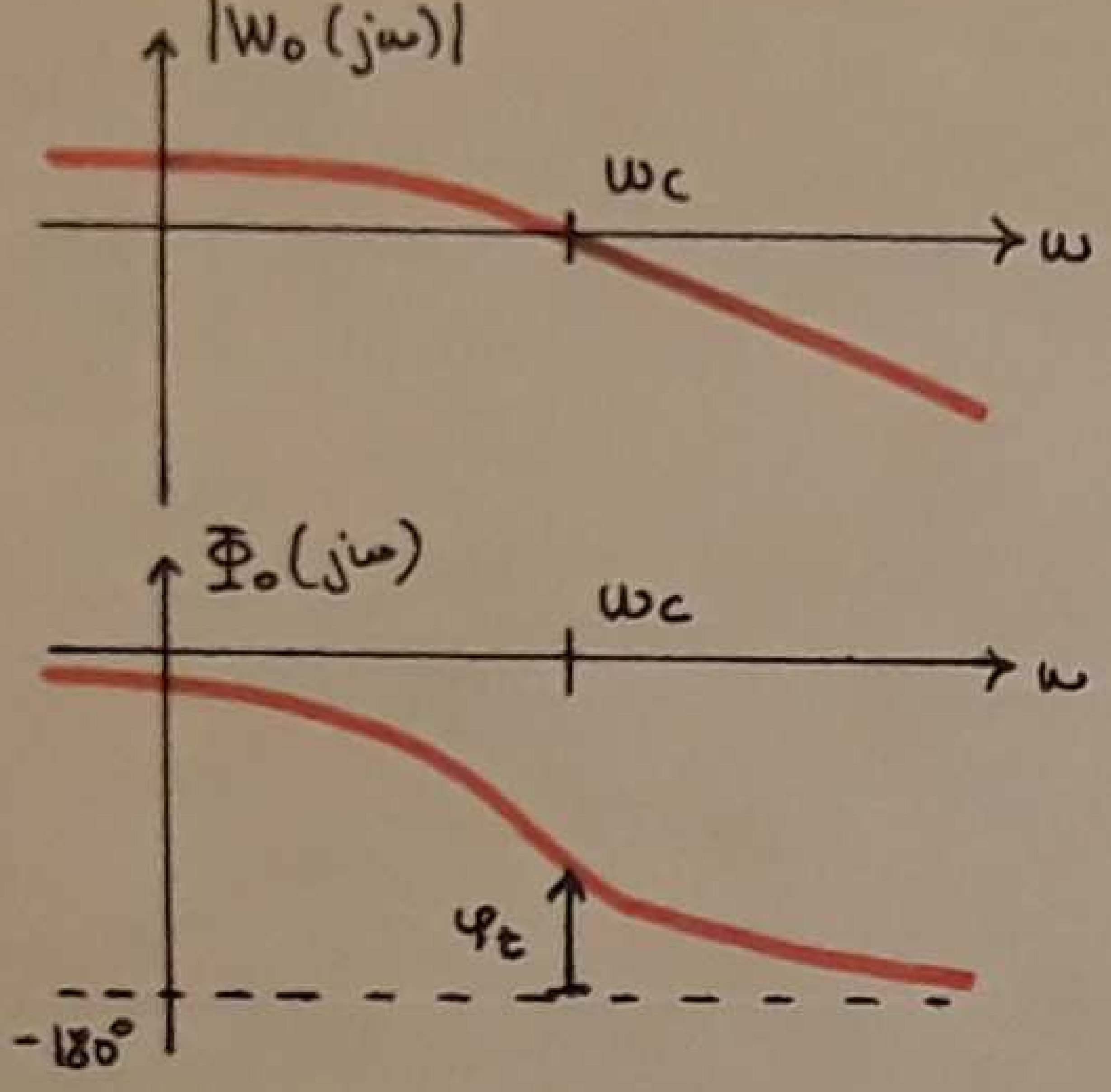
$$\boxed{T_D' = 0{,}296}$$

$$\boxed{W_{PD} = 14 \frac{1+s0{,}181}{1+s0{,}296}}$$





A TERVEZÉSI FELTÉTELEK MEGFOGALMAZÁSA NEMLINEÁRIS EGYENLETRENDSZER ALAKJÁBAN



$$W_o(j\omega) = \frac{K}{s(1+sT_1)(1+sT_2)} e^{-sT_h}$$

$$|W_o| = \frac{K}{\omega_c \sqrt{1+\omega_c^2 T_1^2} \sqrt{1+\omega_c^2 T_2^2}} \; 1$$

$$\Phi_o = -90° - \mathrm{arctg}\, \omega_c T_1 - \mathrm{arctg}\, \omega_c T_2 - \omega T_h$$

1.) stratégia P PI PD PID

$W_p(s) \rightarrow W_c(s) \rightarrow W_o(s)$

2.) feltétel u_{max}-ra ?

✓

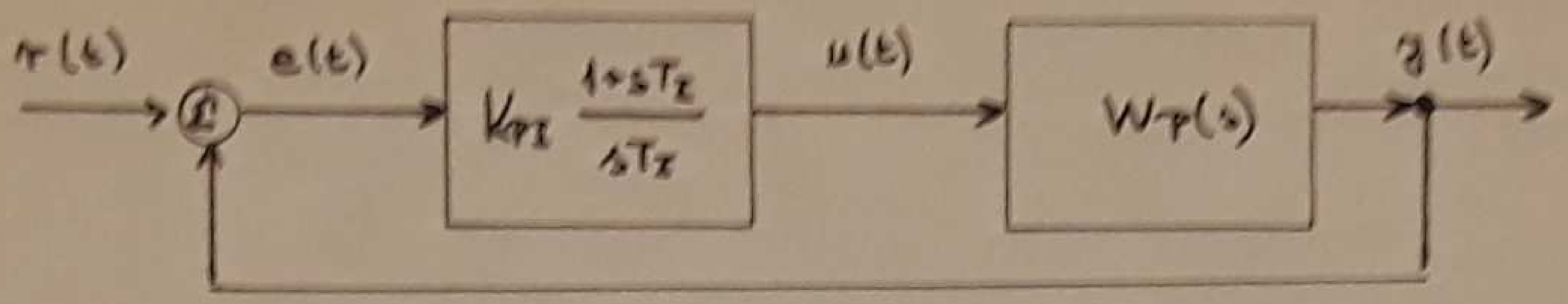
3.) $W_o(j\omega)$

4.) Nemlineáris egyenletek, induló érték

$$\boxed{\begin{array}{l} |W_o(j\omega_c)| = 1 \\ -180° + \varphi_t = \Phi_o(j\omega_c) \\ + \text{feltétel} \end{array}}$$

$$\left.\begin{array}{l} |W_o(j\omega_c)| - 1 = 0 \\ -180° + \varphi_t - \Phi_o(j\omega_c) = 0 \\ f = 0 \end{array}\right\}$$

Tervezzünk PI-szabályozót $\varphi_t = 60°$-ra az alábbi elrendezésben!



a.) $W_P(s) = \dfrac{1}{(1+s10)(1+s20)}$

$T_I = 20 \qquad W_{PI} = K_{PI} \dfrac{1+s20}{s20}$

$W_o(s) = K_{PI} \dfrac{1}{s20\,(1+s10)}$

$-180° + 60° = -90° - \operatorname{arctg} 10\omega_c$

$\underline{\underline{\omega_c = 0{,}0577}}$

$|W_o(j\omega_c)| = 1 = K_{PI} \dfrac{1}{|j\omega_c 20| \sqrt{1+100\omega_c^2}}$

$\underline{\underline{K_{PI} = 1{,}33}}$

$W_{PI}(s) = 1{,}33 \dfrac{1+s20}{s20}$

b.) $W_P(s) = \dfrac{1}{(1+s10)(1+s20)} e^{-4s}$

$W_o(s) = K_{PI} \dfrac{1}{s20(1+s10)} e^{-4s}$

RAD.

$-180° + 60° = -90° - \operatorname{arctg} 10\omega_c - \boxed{4\omega_c}$

$\underline{\underline{\omega_c = 0{,}0387}}$

$|W_o(j\omega_c)| = 1 = K_{PI} \dfrac{1}{20\omega_c \sqrt{1+100\omega_c^2}}$

$\underline{\underline{K_{PI} = 0{,}1829}}$

$W_{PI}(s) = 0{,}1829 \dfrac{1+s20}{s20}$

TAPASZTALATI HANGOLÁSI MÓDSZEREK

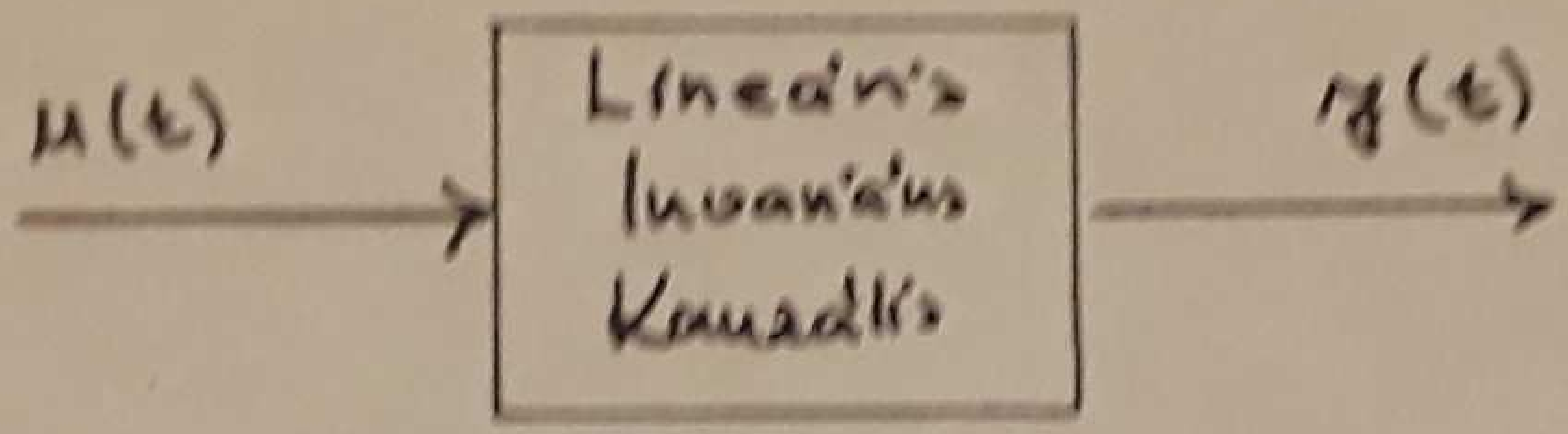
- Modell alapú tervezés

- Tapasztalati módszerek: receptek a folyamaton végzett néhány mérésre alapozva

  például:
  - Ziegler - Nichols - szabályok
  - Oppelt módszere
  - Åström módszere

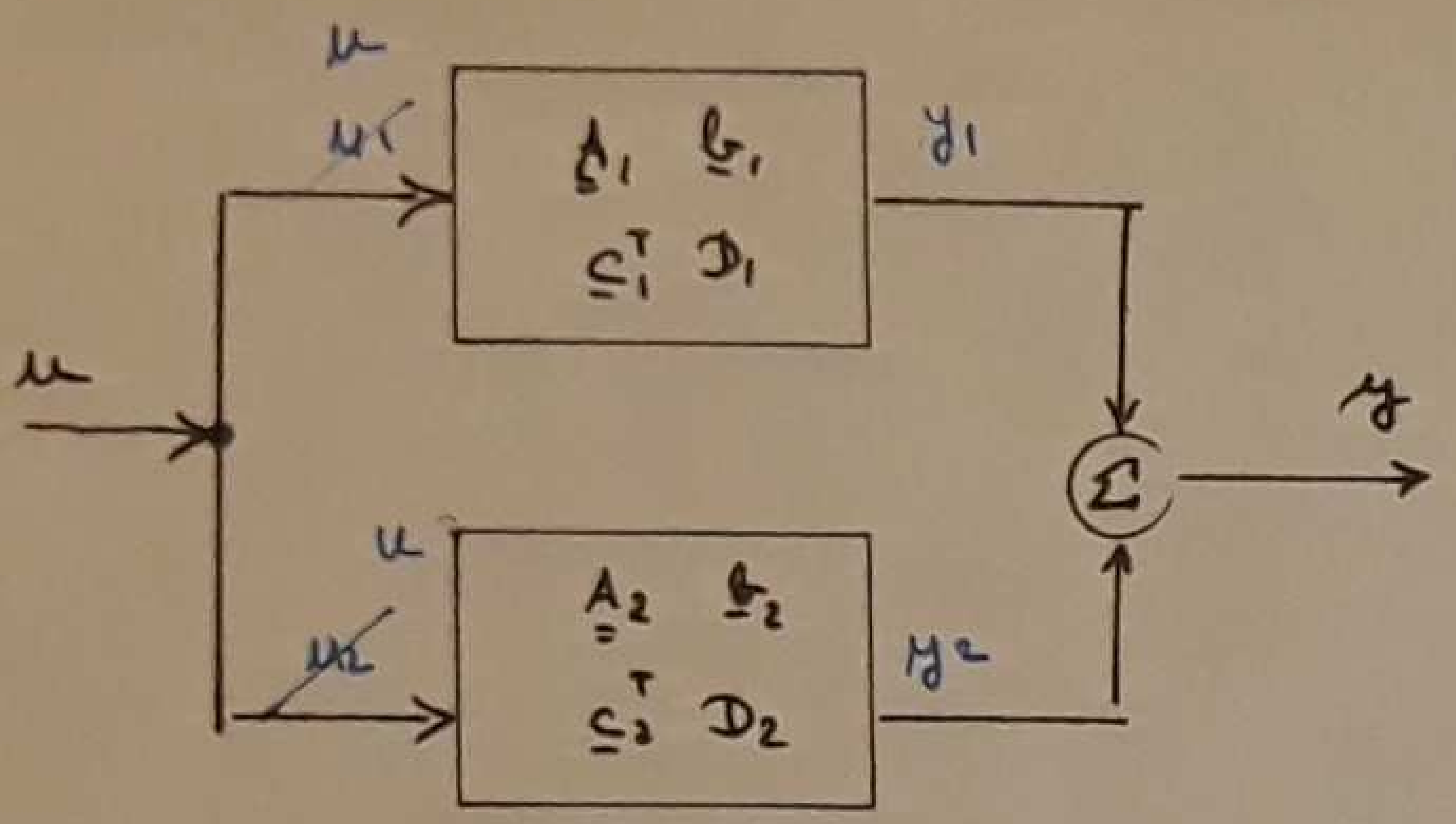
  stb.

AZ ÁLLAPOTTÉR MÓDSZER



$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}\,u$$

$$y = \underline{c}^T\underline{x} + D\,u$$



$$\dot{\underline{x}}_1 = \underline{\underline{A}}_1 \underline{x}_1 + \underline{b}_1 u \checkmark$$

$$y_1 = \underline{c}_1^T \underline{x}_1 + D_1 u$$

$$\dot{\underline{x}}_2 = \underline{\underline{A}}_2 \underline{x}_2 + \underline{b}_2 u \checkmark$$

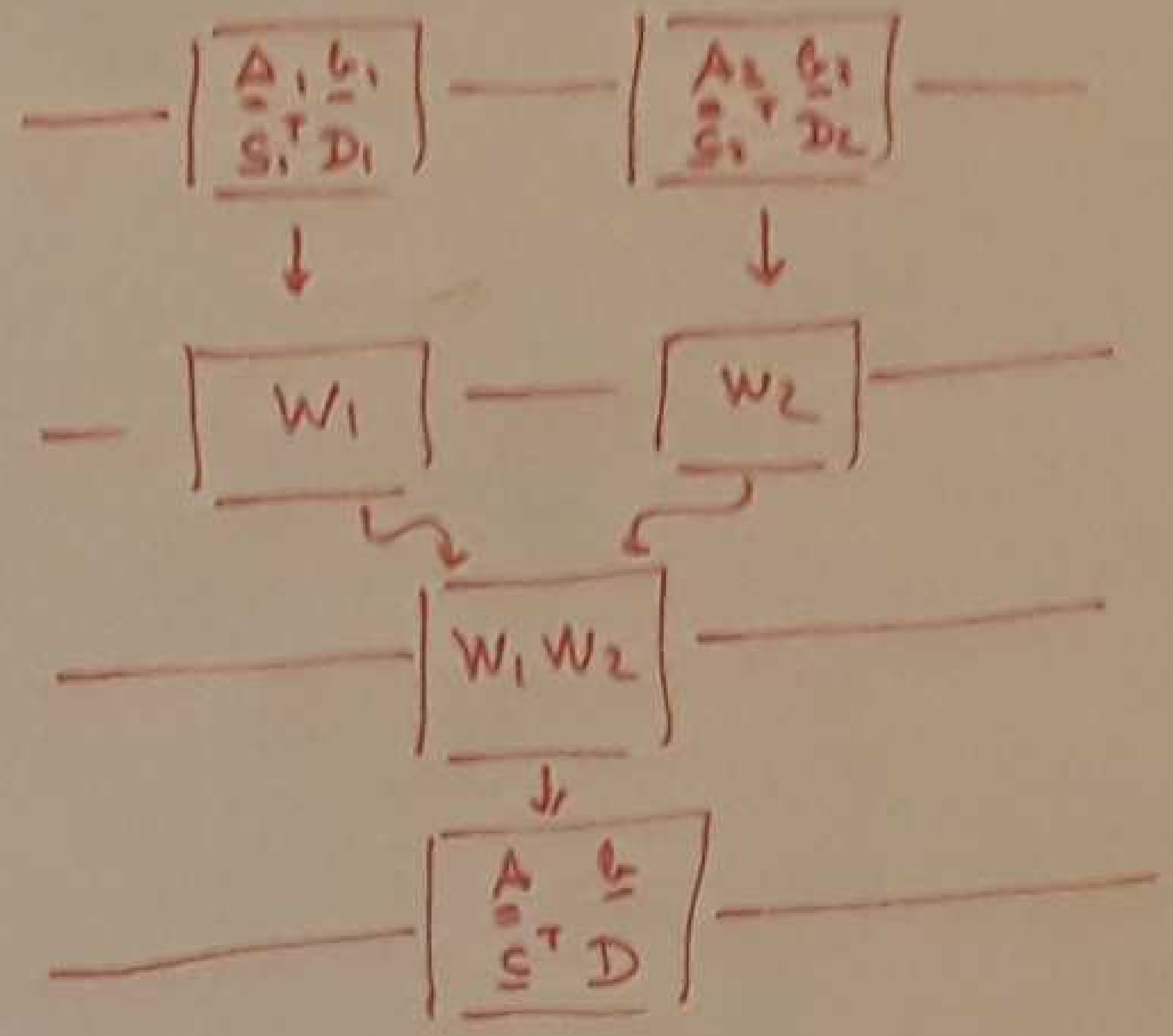
$$y_2 = \underline{c}_2^T \underline{x}_2 + D_2 u$$

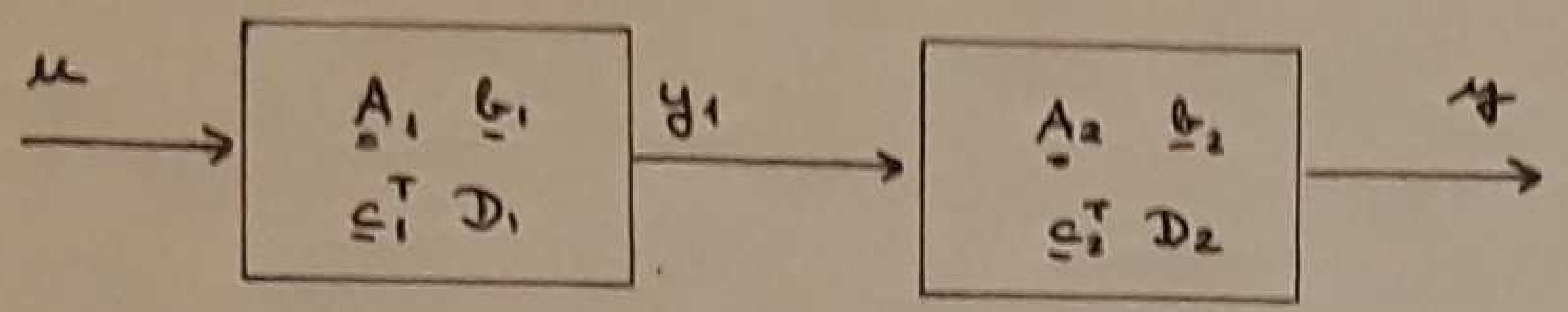
$$\begin{aligned} y &= y_1 + y_2 \\ &= \underline{c}_1^T \underline{x}_1 + D_1 u + \\ &\quad \underline{c}_2^T \underline{x}_2 + D_2 u \\ &= \underline{c}_1^T \underline{x}_1 + \underline{c}_2^T \underline{x}_2 + (D_1 + D_2) u \checkmark \end{aligned}$$

$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}}_1 & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{A}}_2 \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} \underline{b}_1 \\ \underline{b}_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} \underline{c}_1^T & \underline{c}_2^T \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + (D_1 + D_2) u$$

Határozzuk meg két, sorosan, illetve párhuzamosan kapcsolt SISO-rendszer eredő állapotváltozós leírását!

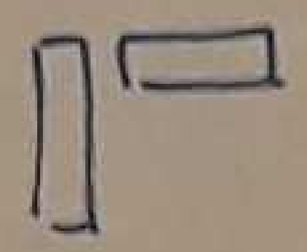




$$\begin{cases} \boxed{\dot{\underline{x}}_1 = \underline{\underline{A}}_1 \underline{x}_1 + \underline{b}_1 u} \\ y_1 = \underline{c}_1^T \underline{x}_1 + D_1 u \end{cases} \qquad \begin{cases} \dot{\underline{x}}_2 = \underline{\underline{A}}_2 \underline{x}_2 + \underline{b}_2 y_1 \\ y = \underline{c}_2^T \underline{x}_2 + D_2 y_1 \end{cases}$$

$$\dot{\underline{x}}_2 = \underline{\underline{A}}_2 \underline{x}_2 + \underline{b}_2 \left(\underline{c}_1^T \underline{x}_1 + D_1 u \right) = \underline{\underline{A}}_2 \underline{x}_2 + \underline{b}_2 \underline{c}_1^T \underline{x}_1 + \underline{b}_2 D_1 u \checkmark$$

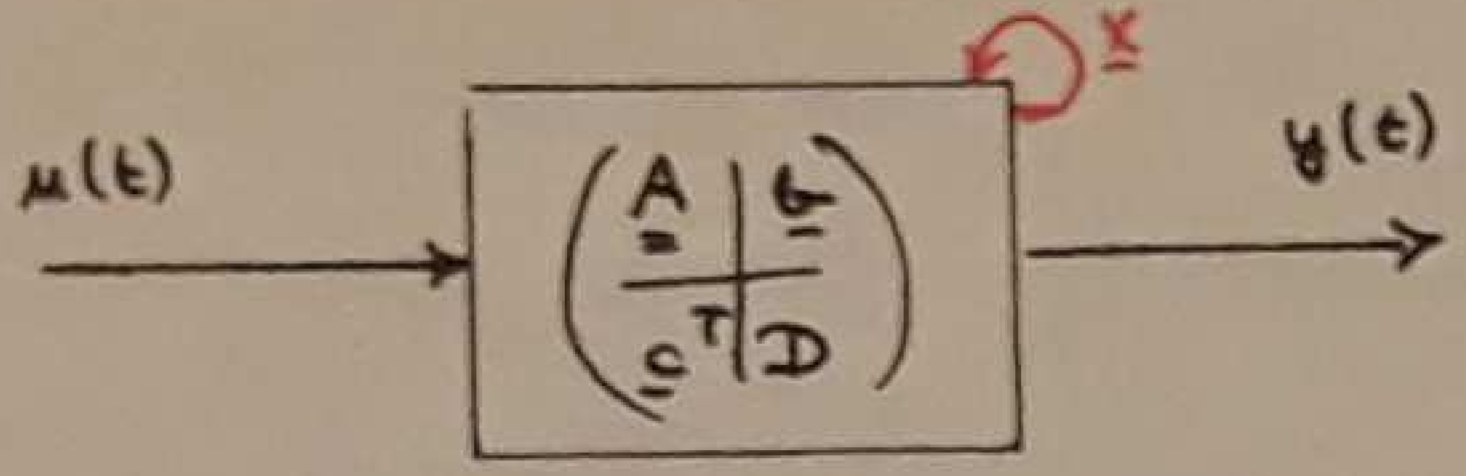
$$y = \underline{c}_2^T \underline{x}_2 + D_2 \left(\underline{c}_1^T \underline{x}_1 + D_1 u \right) = \underline{c}_2^T \underline{x}_2 + \underbrace{D_2 \underline{c}_1^T}_{} \underline{x}_1 + D_2 D_1 u \checkmark$$



$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}}_1 & \underline{\underline{0}} \\ \underline{b}_2 \underline{c}_1^T & \underline{\underline{A}}_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \underline{b}_1 \\ \underline{b}_2 D_1 \end{bmatrix} u$$

$$y = \begin{bmatrix} D_2 \underline{c}_1^T & \underline{c}_2^T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + D_2 D_1 u$$

AZ ÁLLAPOTEGYENLETEK TRANSZFORMÁCIÓJA



$$\dot{\underline{x}} = \tilde{\underline{\underline{A}}}\,\underline{x} + \tilde{\underline{b}}\,u$$

$$y = \tilde{\underline{c}}^T\underline{x} + \tilde{D}u$$

$$\tilde{\underline{x}} = \underline{\underline{T}}\,\underline{x}$$

$$\underline{x} = \underline{\underline{T}}^{-1}\tilde{\underline{x}} \qquad \exists\,\underline{\underline{T}}^{-1}\ !$$

$$\underline{\underline{T}}^{-1}\dot{\tilde{\underline{x}}} = \underline{\underline{A}}\,\underline{\underline{T}}^{-1}\tilde{\underline{x}} + \underline{b}\boxed{u} \qquad /\ \underline{\underline{T}}$$

$$\boxed{y} = \underline{c}^T\underline{\underline{T}}^{-1}\tilde{\underline{x}} + D\boxed{u}$$

$$\boxed{\begin{aligned}\dot{\tilde{\underline{x}}} &= \boxed{\underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1}}\,\tilde{\underline{x}} + \boxed{\underline{\underline{T}}\,\underline{b}}\,u \\ y &= \boxed{\underline{c}^T\underline{\underline{T}}^{-1}}\,\tilde{\underline{x}} + \boxed{D}\,u\end{aligned}}$$

$$\tilde{\underline{\underline{A}}} = \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1} \qquad \tilde{\underline{b}} = \underline{\underline{T}}\,\underline{b}$$

$$\tilde{\underline{c}}^T = \underline{c}^T\underline{\underline{T}}^{-1} \qquad \tilde{D} = D$$

$$w(t) = 1(t)\,\tilde{\underline{c}}^T e^{\tilde{\underline{\underline{A}}}t}\,\tilde{\underline{b}} + \tilde{D}\,\delta(t)$$

$$W(s) = \frac{\tilde{\underline{c}}^T \operatorname{adj}(s\underline{\underline{E}} - \tilde{\underline{\underline{A}}})\,\tilde{\underline{b}} + \tilde{D}\,|s\underline{\underline{E}} - \tilde{\underline{\underline{A}}}|}{|s\underline{\underline{E}} - \tilde{\underline{\underline{A}}}|}$$

$$w(t) = 1(t)\, \tilde{c}^T e^{\tilde{A} t} \tilde{b} + \tilde{D}\, \delta(t)$$

$$\tilde{A} = T A T^{-1} \qquad \tilde{b} = T b$$

$$\tilde{c}^T = c^T T^{-1} \qquad \tilde{D} = D$$

$$e^{\tilde{A}t} = E + \frac{\tilde{A}}{1!} t + \frac{\tilde{A}^2}{2!} t^2 + \ldots + \frac{\tilde{A}^n}{n!} t^n + \ldots$$

$$e^{\tilde{A}t} = E + \frac{TAT^{-1}}{1!} t + \frac{TA\cancel{T^{-1}T}AT^{-1}}{2!} t^2 + \ldots\ldots + \frac{TA\cancel{T^{-1}T}A\cancel{T^{-1}T}AT^{-1}\ldots}{n!} t^n + \ldots\ldots$$

$$= \underbrace{E}_{TT^{-1}} + \frac{TAT^{-1}}{1!} t + \frac{TA^2T^{-1}}{2!} t^2 + \ldots\ldots + \frac{TA^nT^{-1}}{n!} t^n + \ldots\ldots$$

$$= T \underbrace{\left[E + \frac{A}{1!} t + \frac{A^2}{2!} t^2 + \ldots + \frac{A^n}{n!} t^n + \ldots\ldots \right]}_{e^{At}} T^{-1}$$

$$\boxed{e^{\tilde{A}t} = T e^{At} T^{-1}}$$

$$w(t) = 1(t)\, c^T \underbrace{T^{-1}\, T}\, e^{At} \underbrace{T^{-1}\, T}\, b + D\,\delta(t)$$

$$= 1(t)\, c^T e^{At} b + D\,\delta(t).$$

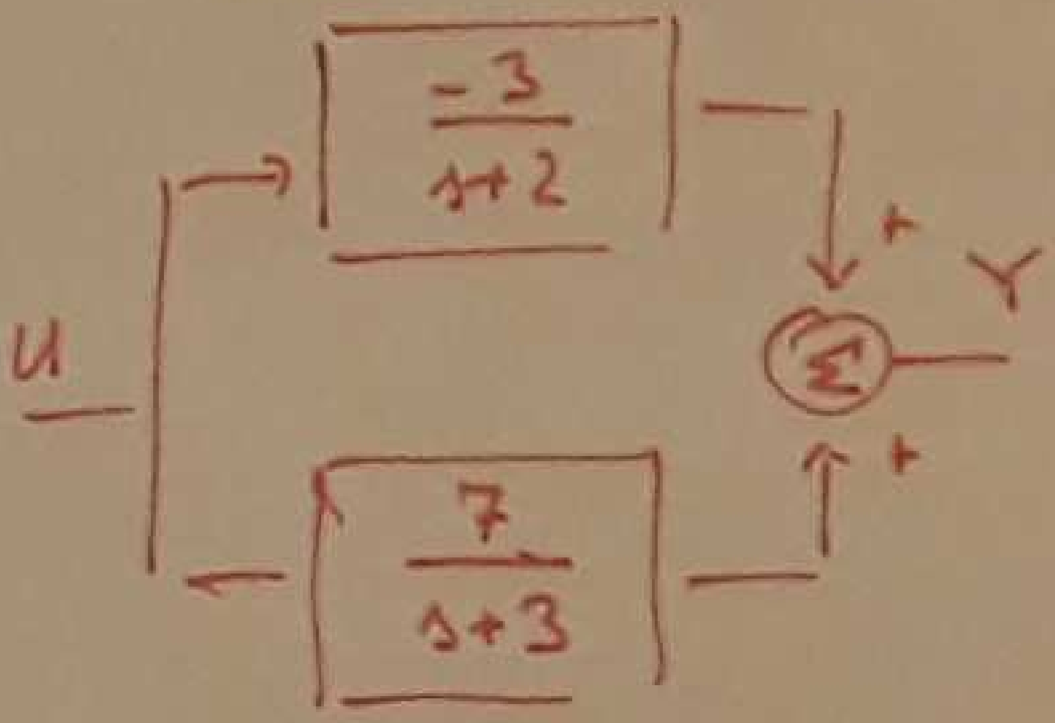
$$w(t) = \underbrace{1(t)\, c^T e^{At} b + D\,\delta(t)}_{\begin{pmatrix} A & b \\ c^T & D \end{pmatrix}} = \underbrace{1(t)\, \tilde{c}^T e^{\tilde{A}t} \tilde{b} + \tilde{D}\,\delta(t)}_{\begin{pmatrix} \tilde{A} & \tilde{b} \\ \tilde{c}^T & \tilde{D} \end{pmatrix}}$$

$$\begin{pmatrix} A & b \\ c^T & D \end{pmatrix} \xleftrightarrow{T} \begin{pmatrix} \tilde{A} & \tilde{b} \\ \tilde{c}^T & \tilde{D} \end{pmatrix}$$

Rekonstruáljuk a rendszer állapotváltozós leírását (1/3)!

$$W(s) = \frac{4s+5}{s^2+5s+6} = \frac{4s+5}{(s+2)(s+3)} = \boxed{\frac{-3}{s+2}} + \boxed{\frac{7}{s+3}}$$

$$\frac{4(-2)+5}{-2+3} = \frac{-8+5}{1} = -3 \qquad \frac{4(-3)+5}{-3+2} = \frac{-12+5}{-1} = 7$$

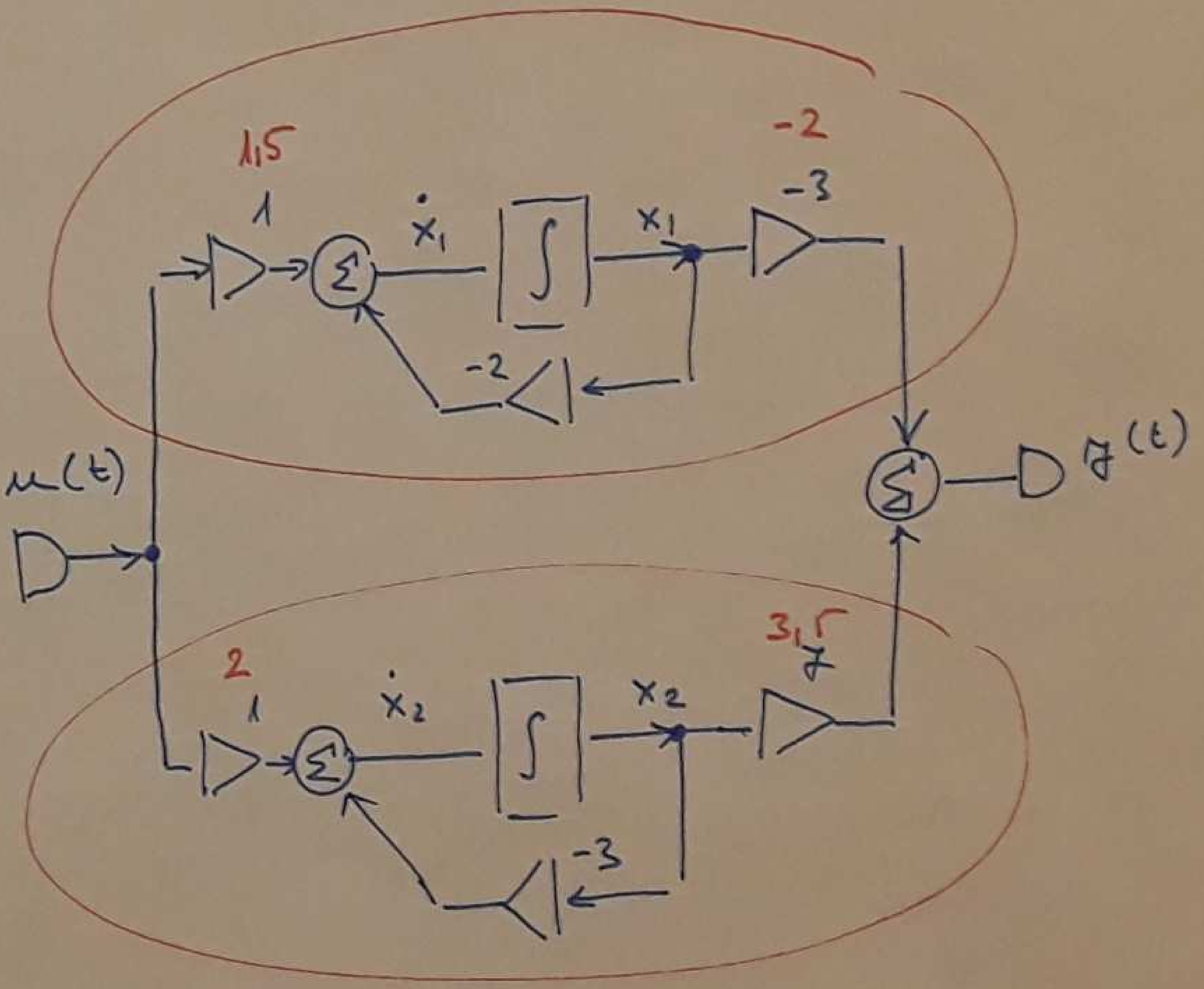


$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-2 & 0\\ 0 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\textcircled{1}\\ \boxed{1}\end{bmatrix} u$$

$$y = \left[\textcircled{-3} \mid \boxed{7}\right]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + 0 \; u$$

Diagonális alak.

$$W(s) = \frac{\underline{c}^T \operatorname{adj}(s\underline{\underline{E}} - \underline{\underline{A}})\underline{b} + D|s\underline{\underline{E}} - \underline{\underline{A}}|}{|s\underline{\underline{E}} - \underline{\underline{A}}|}$$



$$W(s) = \tilde{\underline{c}}^T (s\underline{\underline{E}} - \tilde{\underline{\underline{A}}})^{-1} \tilde{\underline{b}} + \tilde{D}$$

$\tilde{\underline{\underline{A}}} = \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1}$ $\qquad \tilde{\underline{b}} = \underline{\underline{T}}\,\underline{b}$

$\tilde{\underline{c}}^T = \underline{c}^T \underline{\underline{T}}^{-1}$ $\qquad \tilde{D} = D$

$$= \underline{c}^T \underbrace{\underline{\underline{T}}^{-1} (s\underline{\underline{E}} - \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1})}^{-1} \underline{\underline{T}}\,\underline{b} + D$$

$(\underline{\underline{A}}\,\underline{\underline{B}})^{-1} = \underline{\underline{B}}^{-1}\underline{\underline{A}}^{-1}$

$[(\ )\underline{\underline{T}}]^{-1} = \underline{\underline{T}}^{-1}(\ )^{-1}$

$$= \underline{c}^T \underbrace{\left[(s\underline{\underline{E}} - \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1})\underline{\underline{T}}\right]^{-1} \underline{\underline{T}}\,\underline{b}} + D$$

$(A B)^{-1} = B^{-1} A^{-1}$

$(T^{-1}[\ ])^{-1}\ [\ ]^{-1} T$

$$= \underline{c}^T \left(\underline{\underline{T}}^{-1}\left[(s\underline{\underline{E}} - \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1})\underline{\underline{T}}\right]\right)^{-1} \underline{b} + D$$

$$= \underline{c}^T \left(\underline{\underline{T}}^{-1}\left[(s\underline{\underline{E}} - \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1})\underline{\underline{T}}\right]\right)^{-1} \underline{b} + D$$

$$= \underline{c}^T \left(s\,\underline{\underline{T}}^{-1}\underline{\underline{E}}\,\underline{\underline{T}} - \underline{\underline{T}}^{-1}\underbrace{\underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1}}\underline{\underline{T}}\right)^{-1} \underline{b} + D$$

$$= \underline{c}^T (s\underline{\underline{E}} - \underline{\underline{A}})^{-1} \underline{b} + D$$

$$W(s) = \underbrace{\underline{c}^T (s\underline{\underline{E}} - \underline{\underline{A}})^{-1} \underline{b} + D} = \tilde{\underline{c}}^T (s\underline{\underline{E}} - \tilde{\underline{\underline{A}}})^{-1} \tilde{\underline{b}} + \tilde{D}$$

$$\begin{tikzcd} \begin{pmatrix} \underline{\underline{A}} , & \underline{b} \\ \underline{c}^T & D \end{pmatrix} \arrow[r, leftrightarrow, "\underline{\underline{T}}"] & \begin{pmatrix} \tilde{\underline{\underline{A}}} & \tilde{\underline{b}} \\ \tilde{\underline{c}}^T & \tilde{D} \end{pmatrix} \end{tikzcd}$$

Rekonstruáljuk a rendszer állapotváltozós leírását (2/3)!

$$W(s)=\frac{4s+5}{(s+2)(s+3)}=\underbrace{\frac{4s+5}{s^2+5s+6}}=\frac{Y}{U}$$

$$Y=(4s+5)\overset{Z}{\boxed{\frac{1}{s^2+5s+6}U}}$$

$$Y=(4s+5)Z=4\underset{X_1}{\boxed{Zs}}+5\underset{X_2}{\boxed{Z}} \quad \longrightarrow \quad \boxed{Y=4X_1+5X_2 \;\Big|\; +0U}$$

$$\begin{array}{l|l} \dot{x}_1=-5x_1-6x_2 & +\textcircled{1u} \\ \dot{x}_2=\;\;1x_1+0x_2 & +0u \\ \hline y=\;\;4x_1+5x_2 & +0u \end{array}$$

Irányítható alak.

$$Z=\frac{1}{s^2+5s+6}U$$

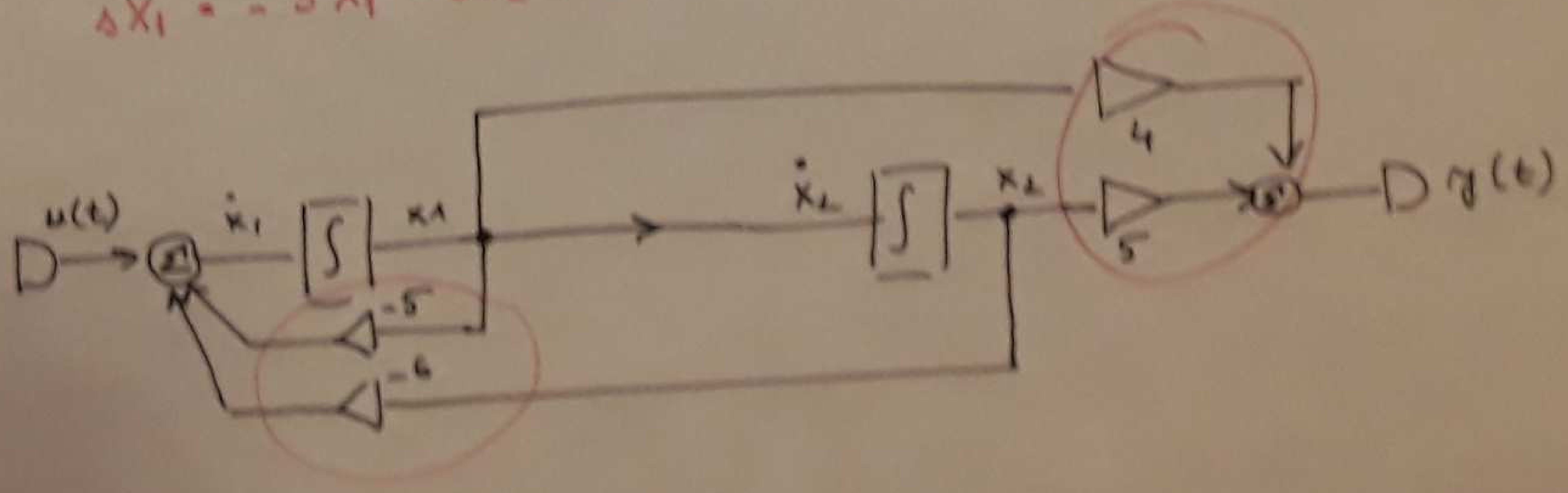
$$Zs^2+5Zs+6Z=U$$

$$Zs^2=-5Zs-6Z+U$$

$$sX_1=-5X_1-6X_2+U \quad \longrightarrow \quad \boxed{sX_1=-5X_1-6X_2+U}$$

$$Zs=X_1$$

$$\boxed{X_2s=X_1}$$



Rekonstruáljuk a rendszer állapotváltozós leírásait (3/3)!

$$W(s) = \frac{4s+5}{(s+2)(s+3)} = \frac{4s+5}{s^2+5s+6} \nearrow \frac{Y}{U}$$

$$4Us + 5U = \underbrace{Ys^2} + 5Ys + 6Y$$

$$Ys^2 = -5Ys - 6Y + 4Us + 5U$$

$$Y = \underbrace{-5Ys^{-1} - 6Ys^{-2} + 4Us^{-1} + 5Us^{-2}}_{X_1} \Rightarrow \boxed{Y = X_1}$$

$$X_1 = -5Ys^{-1} - 6Ys^{-2} + 4Us^{-1} + 5Us^{-2}$$

$$sX_1 = \underbrace{-5Y}_{X_1} - 6Ys^{-1} \underline{+4U} + 5Us^{-1} = -5X_1 + 4U \underbrace{\boxed{-6Ys^{-1} + 5Us^{-1}}}_{X_2}$$

$$\boxed{sX_1 = -5X_1 + X_2 + 4U}$$

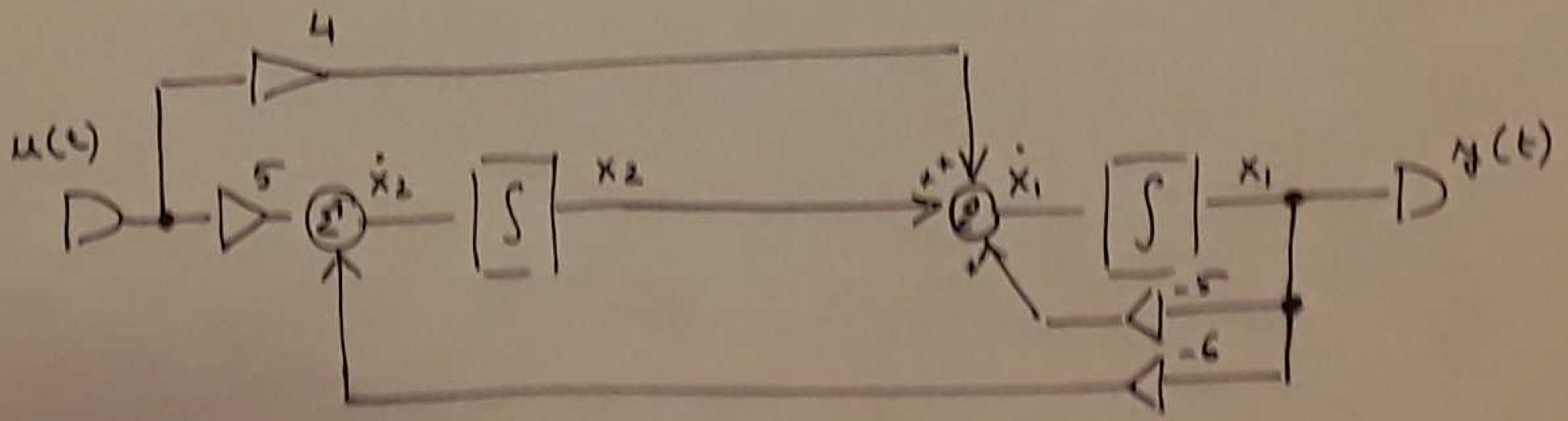
$$X_2 = -6Ys^{-1} + 5Us^{-1}$$

$$sX_2 = -6Y + 5U$$

$$\boxed{sX_2 = -6X_1 + 5U}$$

$$\begin{array}{l|l}
\dot{x}_1 = -5x_1 + 1x_2 + & 4u \\
\dot{x}_2 = -6x_1 + 0x_2 + & 5u \\
\hline
\boxed{y = 1x_1} + 0x_2 + & 0u
\end{array}$$

Megfigyelhető alak.



AZ ÁLLAPOTEGYENLET KANONIKUS ALAKJAI

DIAGONÁLIS ALAK

$$\dot{\underline{\tilde{x}}} = \begin{bmatrix} \lambda_1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_1 & & & \\ 0 & & \lambda_3 & & \\ \vdots & & & \ddots & \\ 0 & & & & \lambda_n \end{bmatrix} \underline{\tilde{x}} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} u$$

$$y = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_n \end{bmatrix} \underline{\tilde{x}} + d u$$

$$\dot{\tilde{x}}_1 = \lambda_1 x_1 + \beta_1 u \qquad y = \gamma_1 x_1 + \gamma_2 x_2 + \ldots$$

$$\dot{\tilde{x}}_2 = \lambda_2 x_2 + \beta_2 u$$

$$\vdots$$

$$\dot{\tilde{x}}_n = \lambda_n x_n + \beta_n u$$

$$W(s) = \frac{\beta_1\gamma_1}{s-\lambda_1} + \frac{\beta_2\gamma_2}{s-\lambda_2} + \cdots$$

$$w(t) = 1(t)\left[\beta_1\gamma_1 e^{\lambda_1 t} + \beta_2\gamma_2 e^{\lambda_2 t} + \ldots\right]$$

IRÁNYÍTHATÓ ALAK

$$W(s) = \frac{\cancel{b_0 s^n} + b_1 s^{n-1} + b_2 s^{n-2} + \ldots \; b_n}{s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n}$$

$$\Big\downarrow \; b_0 = 0$$

$$\dot{\underline{x}} = \begin{array}{c|cccc} & x_1 & x_2 & \cdots & \\ \hline x_1 & -a_1 & -a_2 & \cdots & -a_n \\ x_2 & 1 & 0 & \cdots & 0 \\ x_3 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ x_n & 0 & \cdots & 1 & 0 \end{array} \; \underline{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} \underline{x} + 0\, u$$

MEGFIGYELHETŐ ALAK

$$\dot{\underline{x}} = \begin{array}{c|cccccc} & x_1 & x_2 & x_3 & \cdots & & x_n \\ \hline \dot{x}_1 & -a_1 & 1 & 0 & \cdots & & 0 \\ \dot{x}_2 & -a_2 & 0 & 1 & \cdots & & 0 \\ \vdots & \vdots & \vdots & & \ddots & & \vdots \\ & & & & & & 1 \\ \dot{x}_n & -a_n & 0 & 0 & - & & 0 \end{array} \; \underline{x} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} u$$

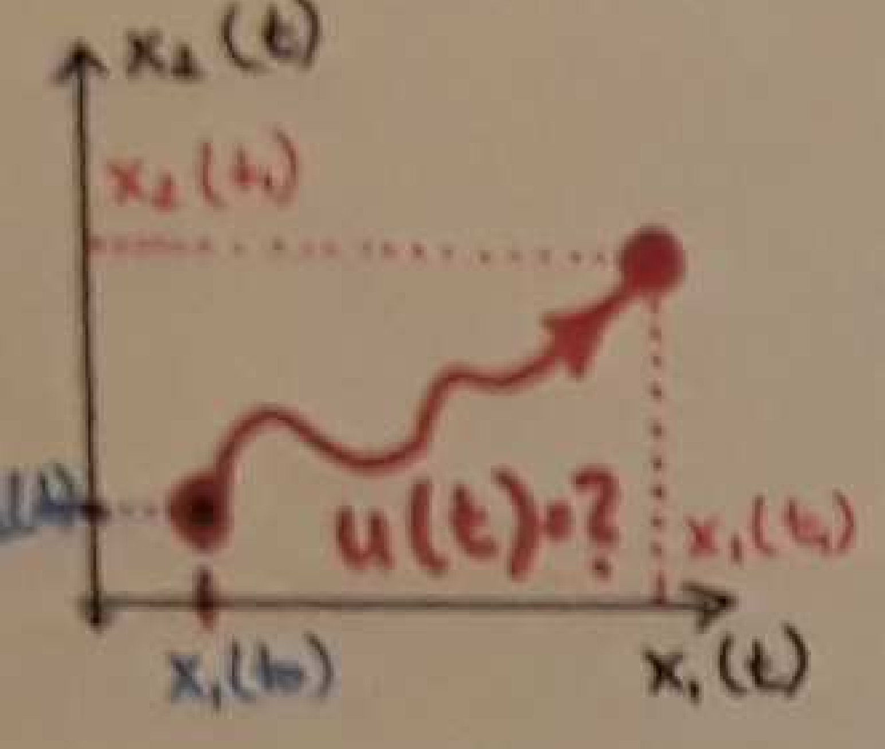
$$y = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \underline{x} + 0\, u$$

$y = x_1$

KÉT FOGALOM

IRÁNYÍTHATÓSÁG

Kérdés: a bemenőjellel valamennyi állapotváltozó tetszőlegesen befolyásolható?



A rendszer állapotirányítható, ha $\underline{x}(t)$ állapotvektora tetszőleges $\underline{x}(t_0)$ állapotból egy másik, tetszőleges $\underline{x}(t_1)$ állapotba mozgatható az $u(t)$ irányítással.

Lineáris, invariáns rendszernél $t_0 = 0$ választható. Az irányíthatóság a rendszer tulajdonsága.

pl.

$$\dot{x}_1 = \lambda_1 x_1 + b_1 u$$

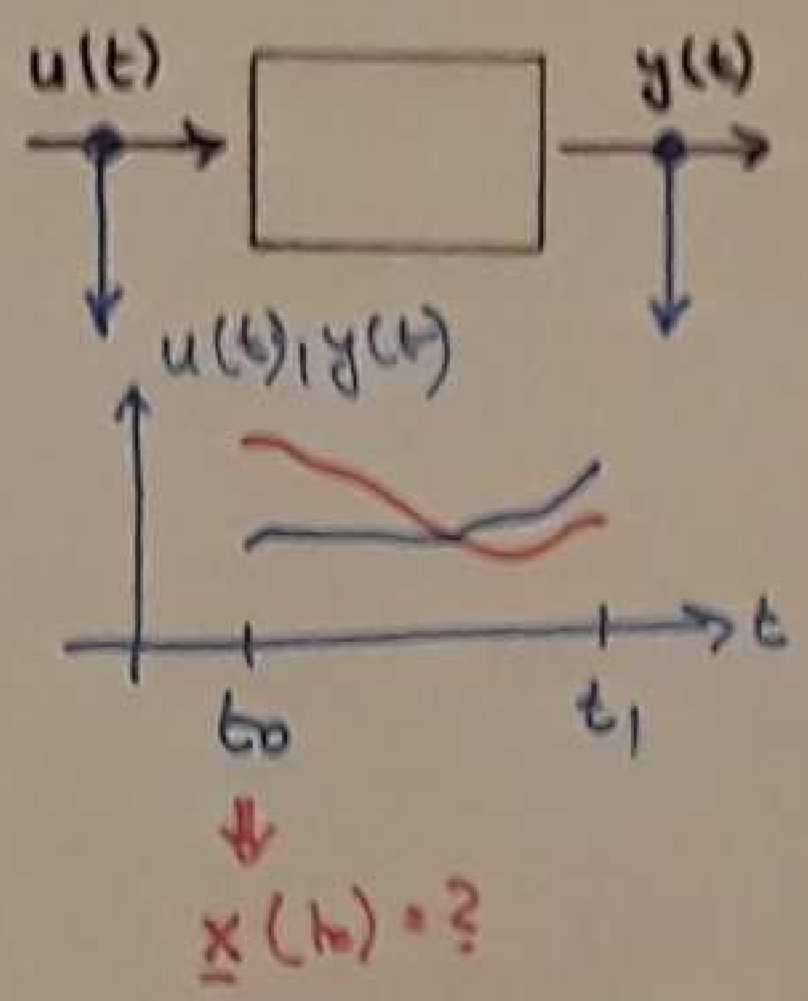
$$\dot{x}_2 = \lambda_2 x_2 + b_2 u$$

$$\dot{x}_3 = \lambda_3 x_3 + 0u \qquad x_3(0)$$

$$y = \underset{\smile}{c_1} x_1 + \underset{\smile}{c_2} x_2 + \underset{\smile}{c_3} x_3 \qquad c_1, c_2, c_3 \neq 0$$

MEGFIGYELHETŐSÉG

Kérdés: a ki- és bemenőjelek mérése által rekonstruálható a mérés kezdetekor az $\underline{x}$ állapot?



A rendszer megfigyelhető, ha $u(t)$ és $y(t)$ $t_0 \leq t \leq t_1$ intervallumban történő méréséből $\underline{x}(t_0)$ meghatározható.

$$\underline{\underline{A}}^{n} = -a_1 \underline{\underline{A}}^{n-1} - a_2 \underline{\underline{A}}^{n-2} - \dots\dots - a_n \underline{\underline{E}} \qquad / \underline{\underline{A}}$$

$$\underline{\underline{A}}^{n+1} = -a_1 \underline{\underline{A}}^{n} - a_2 \underline{\underline{A}}^{n-1} - \dots\dots - a_n \underline{\underline{A}} \qquad / \underline{\underline{A}}$$

$$= -a_1 \left(-a_1 \underline{\underline{A}}^{n-1} - a_2 \underline{\underline{A}}^{n-2} - \dots\dots \right) - a_2 \underline{\underline{A}}^{n-1} - \dots\dots - a_n \underline{\underline{A}}$$

$$\underline{\underline{A}}^{n+2} = -a_1 \underline{\underline{A}}^{\underline{n+1}} - a_2 \underline{\underline{A}}^{\underline{n}} - \dots\dots - a_n \underline{\underline{A}}^{2} \qquad n-1 \dots\dots$$

$$\vdots$$

$$e^{\underline{\underline{A}}t} = \underline{\alpha_0(t)}\, \underline{\underline{E}} + \underline{\alpha_1(t)}\, \underline{\underline{A}} + \underline{\alpha_2(t)}\, \underline{\underline{A}}^{2} + \dots\dots + \underline{\alpha_{n-1}(t)}\, \underline{\underline{A}}^{n-1}$$

A KÈT FOGALOMHOZ EGY-EGY MÁTRIX

(és egy-egy kritérium)

KÁLMÁN-FÉLE IRÁNYÍTHATÓSÁGI MÁTRIX:

A rendszer állapotirányítható, ha az

$$\underline{\underline{M}}_c = [\,\underline{b}\quad \underline{\underline{A}}\,\underline{b}\quad \underline{\underline{A}}^2\underline{b}\quad \dots\quad \underline{\underline{A}}^{n-1}\underline{b}\,]$$

irányíthatósági mátrix rangja maximális, azaz n, vagyis:

- $\underline{\underline{M}}_c$ invertálható
- $\underline{M}_c$ determinánsa nem zérus
- n számú lineárisan független oszlopa van.

KÁLMÁN-FÉLE MEGFIGYELHETŐSÉGI MÁTRIX:

A rendszer megfigyelhető, ha az

$$\underline{\underline{M}}_o = \begin{bmatrix} \underline{c}^T \\ \underline{c}^T\underline{\underline{A}} \\ \underline{c}^T\underline{\underline{A}}^2 \\ \vdots \\ \underline{c}^T\underline{\underline{A}}^{n-1} \end{bmatrix}$$

megfigyelhetőségi mátrix rangja maximális, azaz n, vagyis

- $\underline{\underline{M}}_o$ invertálható
- $\underline{\underline{M}}_o$ determinánsa nem zérus
- n számú lineárisan független sora van.

A KÁLMÁN-FÉLE MEGFIGYELHETŐSÉGI KRITÉRIUM IGAZOLÁSA

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}\,u$$

$$\boxed{y = \underline{c}^T \underline{x} + D\,u}$$

$$\dot{y} = \underline{c}^T \dot{\underline{x}} + D\dot{u} = \underline{c}^T(\underline{\underline{A}}\,\underline{x} + \underline{b}\,u) + D\dot{u} = \underline{c}^T \underline{\underline{A}}\,\underline{x} + \underline{c}^T \underline{b}\,u + D\dot{u}$$

$$\ddot{y} = \underline{c}^T \underline{\underline{A}}\,\dot{\underline{x}} + \underline{c}^T \underline{b}\,\dot{u} + D\ddot{u} = \underline{c}^T \underline{\underline{A}}(\underline{\underline{A}}\,\underline{x} + \underline{b}\,u) + \underline{c}^T \underline{b}\,\dot{u} + D\ddot{u} = \underline{c}^T \underline{\underline{A}}^2 \underline{x} + \underline{c}^T \underline{\underline{A}}\,\underline{b}\,u + \underline{c}^T \underline{b}\,\dot{u} + D\ddot{u}$$

⋮

: n-1. deriváltig.

$$\underbrace{\begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \\ \vdots \end{bmatrix}}_{\underline{Y}} = \underbrace{\begin{bmatrix} \underline{c}^T \\ \underline{c}^T \underline{\underline{A}} \\ \underline{c}^T \underline{\underline{A}}^2 \\ \vdots \\ \underline{c}^T \underline{\underline{A}}^{n-1} \end{bmatrix}}_{\underline{\underline{M}}_o} \underline{x} + \underbrace{\begin{bmatrix} D & 0 & 0 & \cdots \\ \underline{c}^T \underline{b} & D & 0 & \cdots \\ \underline{c}^T \underline{\underline{A}}\,\underline{b} & \underline{c}^T \underline{b} & D & \cdots \\ & \ddots & \ddots & \\ & & & D \end{bmatrix} \begin{array}{c|l} u & 1 \\ \dot{u} & 2 \\ \ddot{u} & \vdots \\ \vdots & \\ & n. \end{array}}_{\underline{U}}$$

$$\underline{Y} = \underline{\underline{M}}_o \underline{x} + \underline{U}$$

$$\underline{Y} - \underline{U} = \underline{\underline{M}}_o \underline{x}$$

$$\boxed{\underline{x} = \underline{\underline{M}}_o^{-1} (\underline{Y} - \underline{U})}$$

A KÁLMÁN-FÉLE IRÁNYÍTHATÓSÁGI KRITÉRIUM IGAZOLÁSA

$$\underset{\uparrow}{\underline{x}(t_1)} = \int_{t_0}^{t_1} \boxed{e^{\underline{\underline{A}}(t-\tau)}}\, \underline{b}\, \underbrace{u(\tau)}_{?}\, d\tau$$

$$\rightarrow e^{\underline{\underline{A}}t} = \underline{\underline{E}} + \frac{t}{1!}\underline{\underline{A}} + \frac{t^2}{2!}\underline{\underline{A}}^2 + \dots + \frac{t^N}{N!}\underline{\underline{A}}^N + \dots$$

CAYLEY-HAMILTON-TÉTEL $(\varphi(\lambda) = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_n = 0)$

$$\varphi(\underline{\underline{A}}) = \underline{\underline{A}}^n + a_1\underline{\underline{A}}^{n-1} + a_2\underline{\underline{A}}^{n-2} + \dots + a_n\underline{\underline{E}} = \underline{\underline{\Phi}}$$

$$\boxed{e^{\underline{\underline{A}}t} = \alpha_0(t)\underline{\underline{E}} + \alpha_1(t)\underline{\underline{A}} + \alpha_2(t)\underline{\underline{A}}^2 + \dots + \alpha_{n-1}(t)\underline{\underline{A}}^{n-1}}$$

$$\underline{x}(t_1) = \int_{t_0}^{t_1} \left[\underline{\alpha_0(t-\tau)\underline{\underline{E}}} + \underline{\alpha_1(t-\tau)\underline{\underline{A}}} + \alpha_2(t-\tau)\underline{\underline{A}}^2 + \dots + \alpha_{n-1}(t-\tau)\underline{\underline{A}}^{n-1} \right] \underline{\underline{\underline{b}\, u(\tau)}}\, d\tau$$

$$\underline{x}(t_1) = \underbrace{\left[\underline{b} \quad \underline{\underline{A}}\,\underline{b} \quad \underline{\underline{A}}^2\underline{b} \quad \dots \quad \underline{\underline{A}}^{n-1}\underline{b} \right]}\ \underbrace{\begin{bmatrix} \int_{t_0}^{t_1} \alpha_0(t-\tau)\, \underline{u(\tau)}\, d\tau \\ \int_{t_0}^{t_1} \alpha_1(t-\tau)\, \underline{u(\tau)}\, d\tau \\ \vdots \\ \int_{t_0}^{t_1} \alpha_{n-1}(t-\tau)\, \underline{u(\tau)}\, d\tau \end{bmatrix}}$$

$$\underset{\uparrow}{\underline{x}(t_1)} = \underset{\uparrow}{\underline{\underline{M}}_c} \cdot \underset{\uparrow ?}{\underline{\mathcal{U}}} \Rightarrow \boxed{\underline{\mathcal{U}} = \underline{\underline{M}}_c^{-1}\, \underline{x}(t_1)}$$

A $\lambda_1, \lambda_2, r_1$ és r_2 paraméterek mely értékei mellett irányítható és megfigyelhető az alábbi rendszer?

$$\dot{\underline{x}} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \underline{x} + \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \underline{x} + 1u$$

$$\underline{\underline{A}}\,\underline{b} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 r_1 \\ \lambda_2 r_2 \end{bmatrix}$$

$$\underline{c}^T \underline{\underline{A}} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & \lambda_2 \end{bmatrix}$$

$$\underline{\underline{M}}_c = \begin{bmatrix} \underline{b} & \underline{\underline{A}}\,\underline{b} \end{bmatrix} = \left[\begin{array}{c:c} r_1 & \lambda_1 r_1 \\ r_2 & \lambda_2 r_2 \end{array}\right]$$

$$\text{rang}\,\underline{\underline{M}}_c \overset{?}{=} 2 \qquad |\underline{\underline{M}}_c| \overset{?}{\neq} 0$$

$$\begin{vmatrix} r_1 & \lambda_1 r_1 \\ r_2 & \lambda_2 r_2 \end{vmatrix} = r_1 \lambda_2 r_2 - \lambda_1 r_1 r_2 = \underbrace{= r_1 r_2 (\lambda_2 - \lambda_1) \neq 0}$$

$$\boxed{\begin{array}{c} r_1 \neq 0 \\ r_2 \neq 0 \\ \lambda_1 \neq \lambda_2 \end{array}}$$

$$\underline{\underline{M}}_o = \begin{bmatrix} \underline{c}^T \\ \underline{c}^T \underline{\underline{A}} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix} \qquad \text{rang}\,\underline{\underline{M}}_o \overset{?}{=} 2 \qquad |\underline{\underline{M}}_o| \neq 0$$

$$\begin{vmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{vmatrix} = \lambda_2 - \lambda_1 \neq 0 \Rightarrow \boxed{\lambda_1 \neq \lambda_2}$$

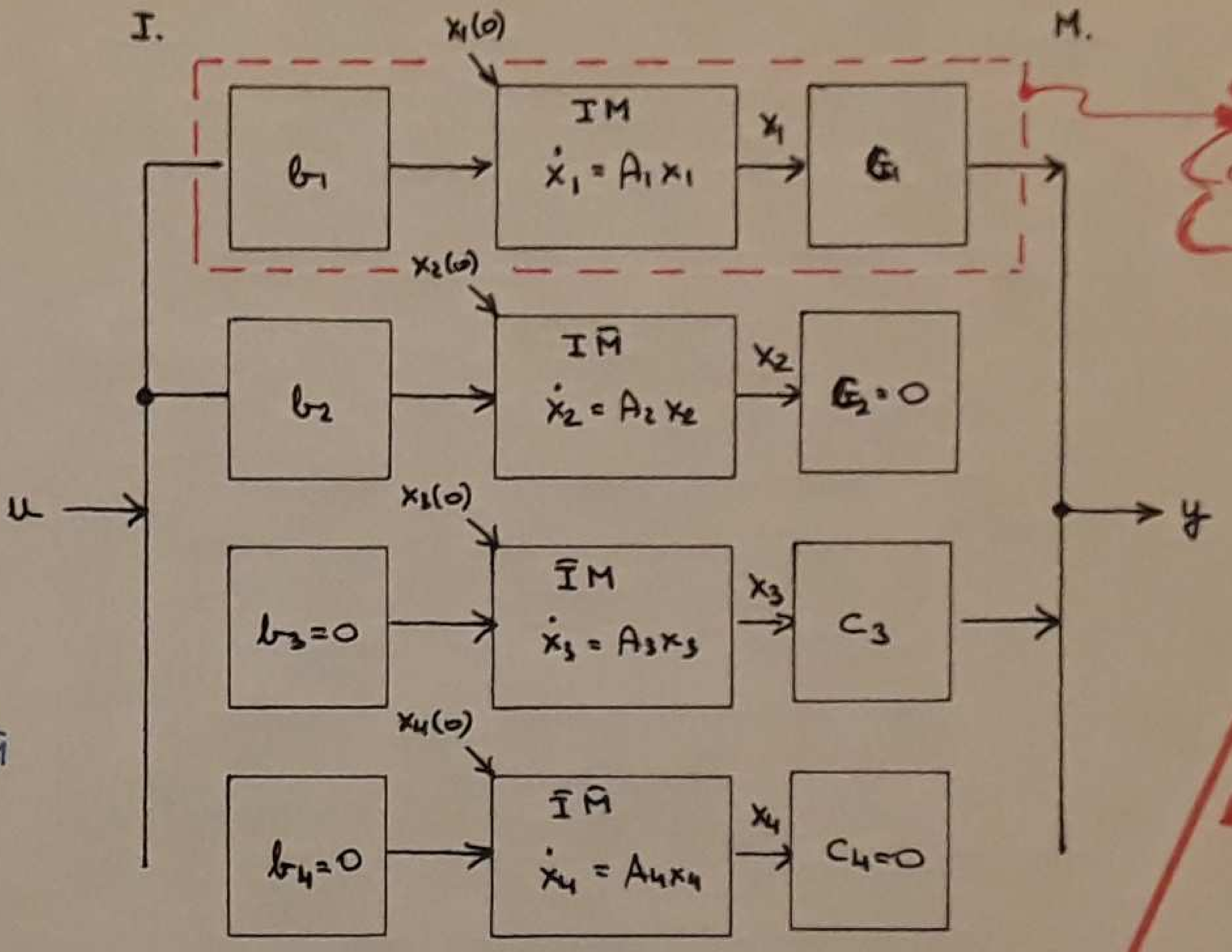
RENDSZEREK KÁLMÁN-FÉLE DEKOMPOZÍCIÓJA

Irányítható És Megfigyelhető : IM

Irányítható DE Nem megfigyelhető : $I\bar{M}$

Nem irányítható És Megfigyelhető : $\bar{I}M$

Nem irányítható És Nem megfigyelhető : $\bar{I}\bar{M}$



W(s) csak IM alrendszer!

karakterisztikus egyenlet: A_1, A_2, A_3, A_4 sajátértékeit adja!

mind a négy alrendszerét!

1. G-V- stabilitás és aszimptotikus stabilitás !

Vizsgáljuk meg az alábbi rendszer irányíthatóságát és megfigyelhetőségét!

$$W(s) = \frac{0s + 8}{s^2 + \underset{\llcorner}{6}s + \underset{\llcorner}{8}}$$

$$\dot{\underline{x}} = \left[\begin{array}{c:c} -6 & -8 \\ \hdashline 1 & 0 \end{array}\right] \underline{x} + \left[\begin{array}{c} 1 \\ \hdashline 0 \end{array}\right] u$$

$$y = \left[\begin{array}{c:c} 0 & 8 \end{array}\right] \underline{x} + 0\, u$$

$$\underline{\underline{M}}_c = \left[\underline{b} \quad \underline{\underline{A}}\,\underline{b}\right] = \begin{bmatrix} 1 & -6 \\ 0 & 1 \end{bmatrix}$$

$$|\underline{\underline{M}}_c| = 1 \neq 0$$

$$\dot{\underline{x}} = \begin{bmatrix} -6 & 1 \\ -8 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 8 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x} + 0\, u$$

$$\underline{\underline{M}}_o = \begin{bmatrix} \underline{c}^T \\ \underline{c}^T \underline{\underline{A}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -6 & 1 \end{bmatrix}$$

$$|\underline{\underline{M}}_o| = 1 \neq 0$$

$$\underline{c}^T \underline{\underline{A}} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} -6 & 1 \\ -8 & 0 \end{bmatrix}$$

Vizsgáljuk meg az alábbi rendszer irányíthatóságát és megfigyelhetőségét!

$$W(s) = \frac{4s+38}{(s+1)(s+2)(2s+6)} = \frac{0s^2+2s+19}{s^3+\underset{\llcorner}{6}s^2+\underset{\llcorner}{11}s+\underset{\llcorner}{6}}$$

$$\dot{\underline{x}} = \begin{bmatrix} -6 & -11 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 2 & 19 \end{bmatrix} \underline{x} + 0 \ u$$

$$\dot{\underline{x}} = \begin{bmatrix} -6 & 1 & 0 \\ -11 & 0 & 1 \\ -6 & 0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 2 \\ 19 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \underline{x} + 0 \ u$$

$$\underline{M}_c = [\ \underline{b} \quad \underline{A}\,\underline{b} \quad \underline{A}^2\underline{b}\ ] = \begin{bmatrix} 1 & -6 & 25 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

$$\underline{M}_o = \begin{bmatrix} \underline{c}^T \\ \underline{c}^T\underline{A} \\ \underline{c}^T\underline{A}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -6 & 1 & 0 \\ 25 & -6 & 1 \end{bmatrix} \checkmark$$

$$\begin{pmatrix} -6 & -11 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} -6 & -11 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 25 & 60 & 36 \\ -6 & -11 & -6 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\underline{A}^2 = \begin{pmatrix} 25 & -6 & 1 \\ 60 & -11 & 0 \\ 36 & -6 & 0 \end{pmatrix}$$

Döntsük el, hogy az alábbi felírás minimális vagy sem!

$$\dot{\underline{x}} = \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \underline{x}$$

$$\underline{\underline{M}}_c = \begin{bmatrix} \underline{b} & \underline{\underline{A}}\,\underline{b} \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix} \quad |\underline{\underline{M}}_c| = -1 \neq 0 \rightarrow \text{irányítható.}$$

$$\underline{\underline{M}}_o = \begin{bmatrix} \underline{c}^T \\ \underline{c}^T \underline{\underline{A}} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -3 \end{bmatrix} \quad |\underline{\underline{M}}_o| = -3 - (-3) = 0$$

nem megfigyelhető.

$$\underline{c}^T \underline{\underline{A}}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -3 \end{bmatrix}$$

$$W(s) = \frac{\underline{c}^T \operatorname{adj}(s\underline{\underline{E}} - \underline{\underline{A}})\,\underline{b}}{|s\underline{\underline{E}} - \underline{\underline{A}}|}$$

$$|s\underline{\underline{E}} - \underline{\underline{A}}| = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix} \right| = \begin{vmatrix} s+2 & 1 \\ 1 & s+2 \end{vmatrix} = (s+2)^2 - 1 = s^2 + 4s + 4 - 1 = \underline{\underline{s^2 + 4s + 3}} = 0$$

$p_1 = -3$ $(\lambda_1 = -3)$

$p_2 = -1$ $(\lambda_2 = -1)$ = Asz. st.

$$s^2 + 4s + 3 = \underline{\underline{(s+3)(s+1)}}$$

$$|\lambda \underline{\underline{E}} - \underline{\underline{A}}|$$

$$\begin{bmatrix} s+2 & 1 \\ 1 & s+2 \end{bmatrix} \Rightarrow \begin{bmatrix} s+2 & \vdots & 1 \\ \cdots & & \cdots \\ 1 & \vdots & s+2 \end{bmatrix} \overset{\substack{+\,- \\ -\,+}}{\Rightarrow} \begin{bmatrix} s+2 & -1 \\ -1 & s+2 \end{bmatrix} \overset{T}{\Rightarrow} \begin{bmatrix} s+2 & -1 \\ -1 & s+2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} s+2 & -1 \\ -1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

$$\underbrace{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} s+2 \\ -1 \end{bmatrix}}$$

$$(s+2) - 1 = \underline{\underline{s+1}}$$

$$W(s) = \frac{\cancel{s+1}}{(s+3)\cancel{(s+1)}} = \underline{\underline{\frac{1}{s+3}}}$$

GV st.

$$\frac{Y}{U} = \frac{1}{s+3}$$

$$Ys + 3Y = U$$

$$\dot{y} + 3y = u \rightarrow \dot{y} = -3y + u$$

$$\begin{cases} \dot{x} = -3x + u \\ y = x \end{cases}$$

Írjuk fel az alábbi rendszert diagonális alakban!

$$\dot{\underline{x}} = \begin{bmatrix} -6 & -4 \\ 2 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 4 \\ 0 \end{bmatrix} u \qquad \underline{\underline{A}} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$y = [0 \quad 1] \underline{x} + 0u$$

$$W(s) = \frac{\underline{c}^T \operatorname{adj}(s\underline{\underline{E}} - \underline{\underline{A}}) \underline{b}}{|s\underline{\underline{E}} - \underline{\underline{A}}|} =$$

$$|s\underline{\underline{E}} - \underline{\underline{A}}| = \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -6 & -4 \\ 2 & 0 \end{bmatrix} \right| = \begin{vmatrix} s+6 & 4 \\ -2 & s \end{vmatrix} = s(s+6) + 8 = \underline{s^2 + 6s + 8} = (s+2)(s+4)$$

$$\begin{bmatrix} s+6 & 4 \\ -2 & s \end{bmatrix} \Rightarrow \begin{bmatrix} s & \vdots & -2 \\ 4 & \vdots & s+6 \end{bmatrix} \overset{\substack{+- \\ -+}}{\Rightarrow} \begin{bmatrix} s & 2 \\ -4 & s+6 \end{bmatrix} \overset{T}{\Rightarrow} \begin{bmatrix} s & -4 \\ 2 & s+6 \end{bmatrix}$$

$$\underbrace{[0 \quad 1] \begin{bmatrix} s & -4 \\ 2 & s+6 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix}}_{[0 \quad 1] \begin{bmatrix} 4s \\ 8 \end{bmatrix} = \underline{8}}$$

$$W(s) = \frac{8}{(s+2)(s+4)} = \frac{4}{s+2} + \frac{-4}{s+4}$$

$$\frac{8}{-2+4} \qquad \frac{8}{-4+2}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 4^{(2)} \\ \hline -4^{(1)} \end{bmatrix} u$$

$$y = [1^{(2)} \mid 1^{(1)}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

TRANSZFORMÁCIÓS MÁTRIXOK

Irányíthatósági transzformációs mátrix

$$\boxed{\underline{\underline{T}} = \left(\underline{\underline{M}}_c \, \underline{\underline{T}}(\underline{a})\right)^{-1}}$$

Diagonális alakra transzformáló mátrix

$$\boxed{\underline{\underline{T}} = \left(\underline{\underline{M}}_c \, \underline{\underline{T}}(\underline{a}) \, \underline{\underline{P}}\right)^{-1}}$$

$$\underline{\underline{M}}_c = \left[\underline{b} \quad \underline{\underline{A}}\,\underline{b} \quad \underline{\underline{A}}^2 \underline{b} \quad \dots \quad \right]$$

$$|s\underline{\underline{E}} - \underline{\underline{A}}| = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n$$

$$\underline{\underline{T}}(\underline{a}) = \begin{bmatrix} 1 & a_1 & a_2 & \dots & a_{n-1} \\ 0 & 1 & a_1 & a_2 \dots & a_{n-2} \\ \vdots & & \ddots & & \\ 0 & \dots & & & 1 \end{bmatrix}$$

Töplitz-mátrix

$$\boxed{\begin{array}{ll} \tilde{\underline{\underline{A}}} = \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1} & \tilde{\underline{b}} = \underline{\underline{T}}\,\underline{b} \\ \tilde{\underline{c}}^T = \underline{c}^T \underline{\underline{T}}^{-1} & \tilde{D} = D \end{array}}$$

$$\underline{\underline{P}} = \begin{bmatrix} \lambda_1^{n-1} & \lambda_2^{n-1} & \dots & \lambda_n^{n-1} \\ \lambda_1^{n-2} & \vdots & & \lambda_n^{n-2} \\ \vdots & & & \vdots \\ \lambda_1^2 & \lambda_2^2 & \dots & \lambda_n^2 \\ \vdots & \lambda_2 & \dots & \lambda_n \\ 1 & 1 & \dots & 1 \end{bmatrix}$$

Vandermonde-mátrix

Példa

$$\underline{\underline{A}} = \begin{bmatrix} -6 & -4 \\ 2 & 0 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\underline{\underline{T}} = \left(\underline{\underline{M}}_c \, \underline{\underline{T}}(a)\right)^{-1}$$

$$\underline{\underline{T}} = \left(\underline{\underline{M}}_c \, \underline{\underline{T}}(\underline{a}) \, \underline{\underline{P}}\right)^{-1}$$

$$|\lambda \underline{\underline{E}} - \underline{\underline{A}}| = \left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -6 & -4 \\ 2 & 0 \end{bmatrix} \right| = \begin{vmatrix} \lambda+6 & 4 \\ -2 & \lambda \end{vmatrix} = \lambda(\lambda+6) + 8$$

$$= \underline{\lambda^2 + \underset{a_1}{6}\lambda + \underset{a_2}{8}} = 0$$

$$\underline{\underline{M}}_c = [\, \underline{b} \quad \underline{\underline{A}}\,\underline{b} \,] = \begin{bmatrix} 4 & -24 \\ 0 & 8 \end{bmatrix}$$

$$\begin{bmatrix} -6 & -4 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4 \cdot 8}}{2} = \frac{-6 \pm 2}{2} < \begin{matrix} \lambda_1 = -2 \\ \lambda_2 = -4 \end{matrix}$$

$$\underline{\underline{T}}(a) = \begin{bmatrix} 1 & a_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$$

$$\underline{\underline{M}}_c \, \underline{\underline{T}} = \begin{bmatrix} 4 & -24 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} = \boxed{\underset{\underline{\underline{T}}^{-1}}{\begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}}} \quad \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}^{-1} = \boxed{\begin{bmatrix} 0{,}125 & 0 \\ 0 & 0{,}125 \end{bmatrix} = \underline{\underline{T}}}$$

$$\underline{\underline{P}} = \begin{bmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ 1 & 1 \end{bmatrix}$$

$$\underline{\underline{M}}_c \, \underline{\underline{T}} \, \underline{\underline{P}} = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} -2 & -4 \\ 1 & 1 \end{bmatrix} = \boxed{\overset{\underline{\underline{T}}^{-1}}{\begin{bmatrix} -8 & -16 \\ 8 & 8 \end{bmatrix}}} \quad \begin{bmatrix} -8 & -16 \\ 8 & 8 \end{bmatrix}^{-1} = \boxed{\begin{bmatrix} 0{,}125 & 0{,}25 \\ -0{,}125 & -0{,}125 \end{bmatrix} = \underline{\underline{T}}}$$

$$\begin{bmatrix} -8 & -16 \\ 8 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 8 & 8 \\ -16 & -8 \end{bmatrix} \overset{\substack{+ - \\ - +}}{\Rightarrow} \begin{bmatrix} 8 & -8 \\ 16 & -8 \end{bmatrix} \overset{T}{\Rightarrow} \cancel{\begin{bmatrix} 8 & -8 \\ 16 & -8 \end{bmatrix}} \begin{bmatrix} 8 & 16 \\ -8 & -8 \end{bmatrix}$$

$$\begin{vmatrix} -8 & -16 \\ 8 & 8 \end{vmatrix} = -8 \cdot 8 + 8 \cdot 16 = \underline{\underline{64}}$$

$$\underline{\underline{A}} = \begin{bmatrix}-6 & -4\\ 2 & 0\end{bmatrix} \qquad \underline{b} = \begin{bmatrix}4\\ 0\end{bmatrix}$$

$$\underbrace{\underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1} = \begin{bmatrix}0{,}25 & 0\\ 0 & 0{,}125\end{bmatrix}\underbrace{\begin{bmatrix}-6 & -4\\ 2 & 0\end{bmatrix}\begin{bmatrix}4 & 0\\ 0 & 8\end{bmatrix}}_{\begin{bmatrix}-24 & -32\\ 8 & 0\end{bmatrix}}}$$

$$\tilde{\underline{\underline{A}}} = \begin{bmatrix}-6 & -8\\ 1 & 0\end{bmatrix} \checkmark$$

$$\tilde{\underline{b}} = \underline{\underline{T}}\,\underline{b} = \begin{bmatrix}0{,}25 & 0\\ 0 & 0{,}125\end{bmatrix}\begin{bmatrix}4\\ 0\end{bmatrix} = \begin{bmatrix}1\\ 0\end{bmatrix} \checkmark$$

$$\underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1} = \underbrace{\begin{bmatrix}0{,}125 & 0{,}25\\ -0{,}125 & -0{,}125\end{bmatrix}\underbrace{\begin{bmatrix}-6 & -4\\ 2 & 0\end{bmatrix}\begin{bmatrix}-8 & -16\\ 8 & 8\end{bmatrix}}_{\begin{bmatrix}16 & 64\\ -16 & -32\end{bmatrix}}}$$

$$\begin{array}{c|c} +6\cdot 8 - 4\cdot 8 & 6\cdot 16 - 4\cdot 8\\ \hline -2\cdot 8 + 0\cdot 8 & -2\cdot 16 + 0\cdot 8\end{array}$$

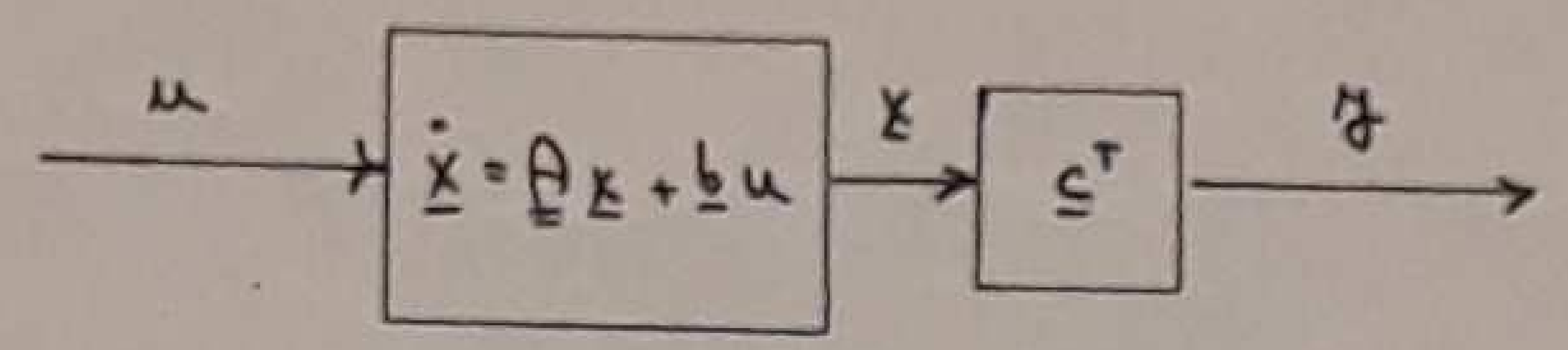
$$\tilde{\underline{\underline{A}}} = \left[\begin{array}{c|c}-2 & 0\\ \hline 0 & -4\end{array}\right]$$

$$\tilde{\underline{b}} = \underline{\underline{T}}\,\underline{b} = \begin{bmatrix}0{,}125 & 0{,}25\\ -0{,}125 & -0{,}125\end{bmatrix}\begin{bmatrix}4\\ 0\end{bmatrix} = \begin{bmatrix}0{,}5\\ -0{,}5\end{bmatrix}$$

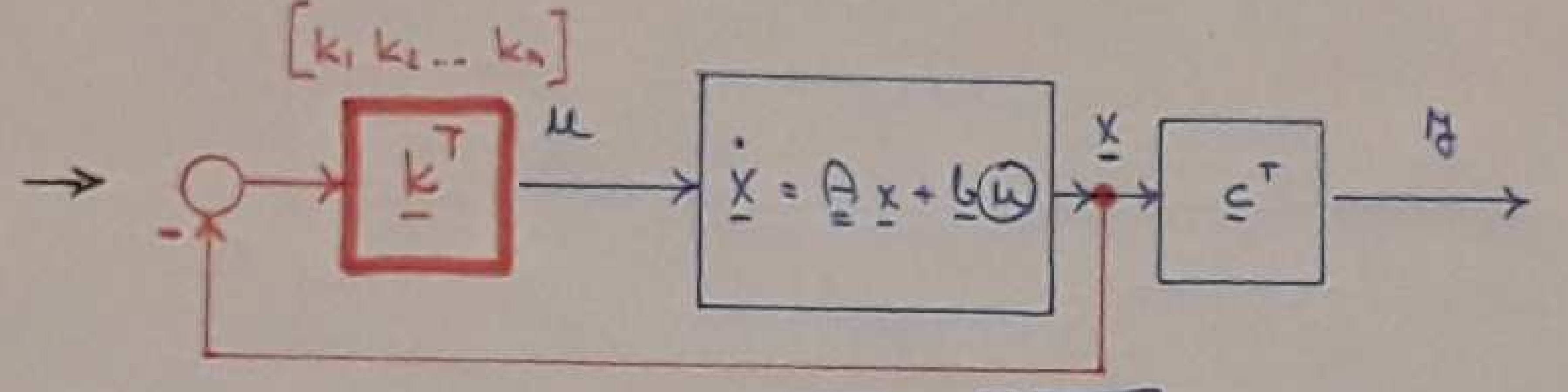
$$\boxed{c_i\ b_i}$$

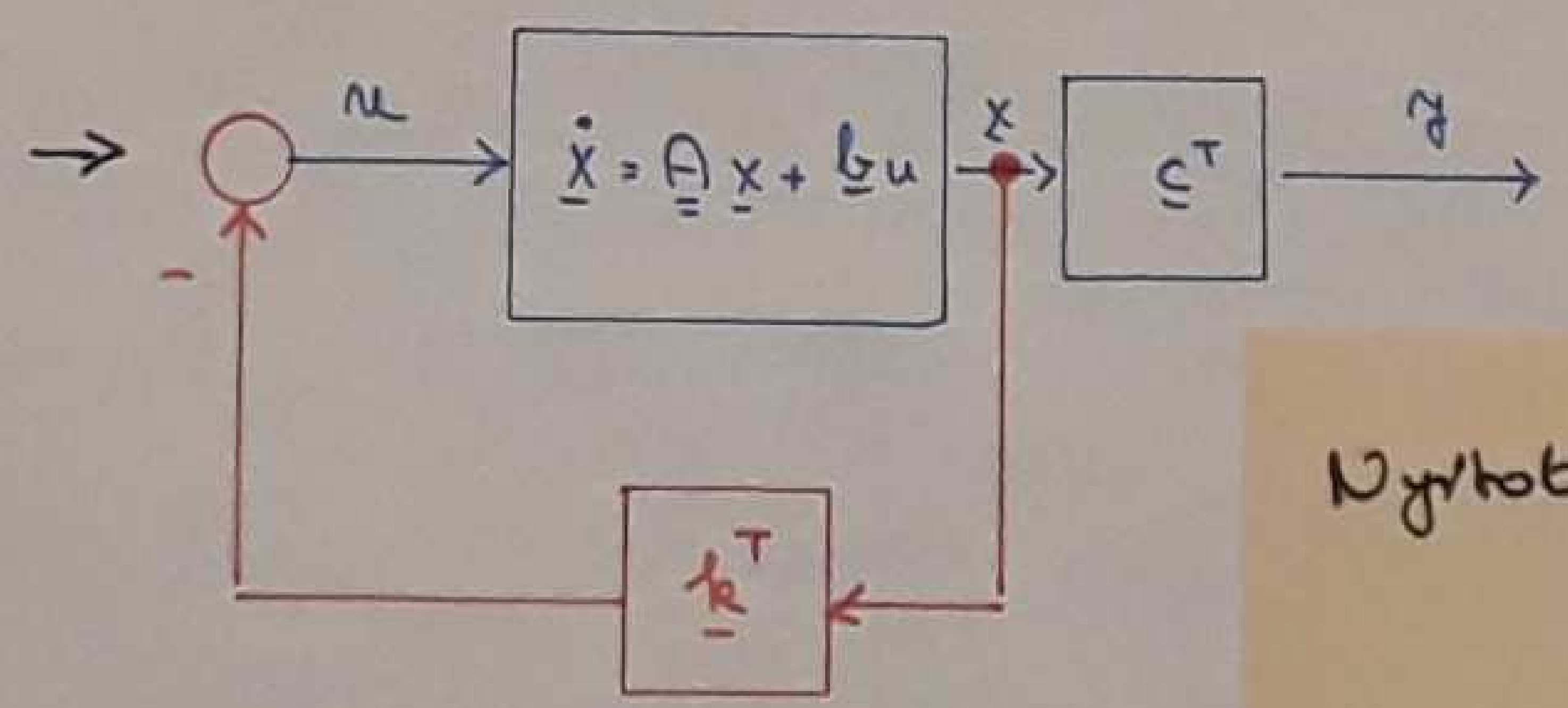
PÓLUSÁTHELYEZÉS ÁLLAPOT – VISSZACSATOLÁSSAL

NYITOTT :



ZÁRT:





$\underline{\dot{x}} = \boxed{\underline{\underline{A}}}\,\underline{x} + \underline{b}u$

$y = \underline{c}^T\underline{x}$

$\varphi(\lambda) = |\lambda\underline{\underline{E}} - \underline{\underline{A}}| = 0$

$\lambda^n + a_1\lambda^{n-1} + \dots + a_n = 0$ ■

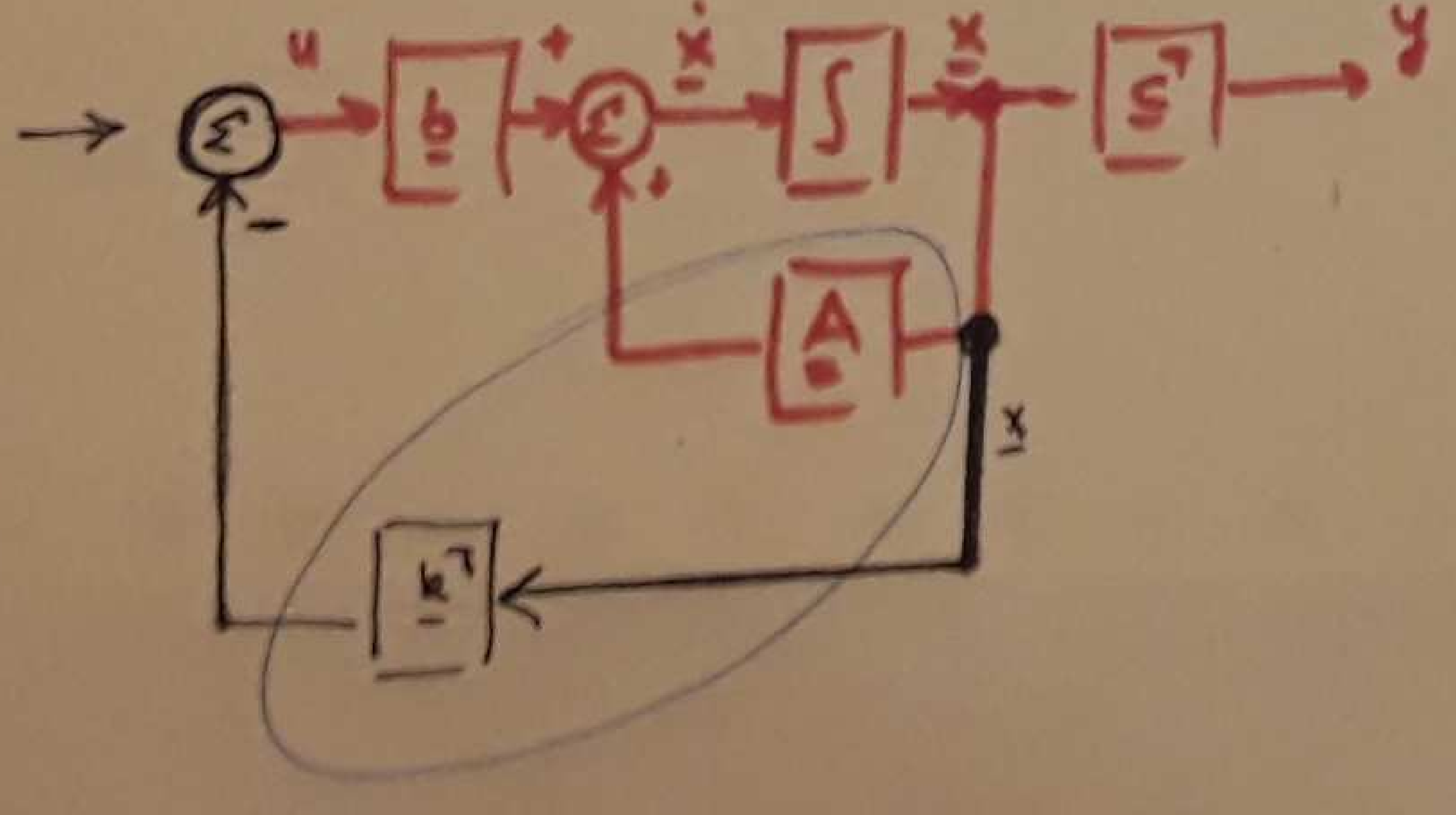
$u = -\underline{k}^T\underline{x}$ ▭▯

$= -(k_1x_1 + k_2x_2 + \dots + k_nx_n)$

$\underline{\dot{x}} = \underline{\underline{A}}\,\underline{x} - \underline{b}\,\underline{k}^T\underline{x} = \boxed{(\underline{\underline{A}} - \underline{b}\,\underline{k}^T)}\,\underline{x}$

CLOSED: $\varphi_{cl}(\lambda) = |\lambda\underline{\underline{E}} - (\underline{\underline{A}} - \underline{b}\,\underline{k}^T)| = 0$

→ $\lambda^n + \underline{p_1}\lambda^{n-1} + \underline{p_2}\lambda^{n-2} + \dots + \underline{p_n} = 0$



Nyitott : $\underline{\underline{A}}$ $|\lambda\underline{\underline{E}} - \underline{\underline{A}}| = 0$

$\lambda_1 = -1 \leftarrow$ $\tilde{\lambda}_1 = -2$

$\lambda_2 = \textcircled{+1} \leftarrow$ $\tilde{\lambda}_2 = -1$

$\varphi_{cl}(\lambda) = (\lambda - \tilde{\lambda}_1)(\lambda - \tilde{\lambda}_2)$

$= (\lambda + 2)(\lambda + 1) = \lambda^2 + \underset{p_1}{3}\lambda + \underset{p_2}{2}$

$|\lambda\underline{\underline{E}} - (\underline{\underline{A}} - \underline{b}\,\underline{k}^T)|$

$\underbrace{p_1 \quad p_2}_{\underline{k}^T = ?}$

Bevezető példa

1.) $\underline{\underline{A}} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\varphi(\lambda) = |\lambda \underline{\underline{E}} - \underline{\underline{A}}| = \left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \right| = \begin{vmatrix} \lambda-1 & -1 \\ -1 & \lambda-2 \end{vmatrix} = (\lambda-1)(\lambda-2) - 1 =$$

$$= \lambda^2 - 3\lambda + 2 - 1$$

$$= \boxed{\lambda^2 - 3\lambda + 1} = 0$$

$$\lambda_{1,2} = \frac{3 \pm \sqrt{9-4}}{2} = \begin{cases} \lambda_1 = 2{,}62 & \tilde{\lambda}_1 = -5 \\ \lambda_2 = 0{,}38 & \tilde{\lambda}_2 = -6 \end{cases}$$

$$\varphi_{cl}(\lambda) = (\lambda - \tilde{\lambda}_1)(\lambda - \tilde{\lambda}_2) = (\lambda+5)(\lambda+6) = \boxed{\lambda^2 + 11\lambda + 30}$$

$u = -\underline{k}^T \underline{x} \qquad \underline{\underline{A}} - \underline{b}\,\underline{k}^T \to |\lambda \underline{\underline{E}} - (\underline{\underline{A}} - \underline{b}\,\underline{k}^T)|$

$$\left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \left[\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} - \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix} (k_1 \; k_2)}_{\begin{pmatrix} k_1 & k_2 \\ 0 & 0 \end{pmatrix}} \right] \right| = \left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1-k_1 & 1-k_2 \\ 1 & 2 \end{pmatrix} \right| = \begin{vmatrix} \lambda - (1-k_1) & -(1-k_2) \\ -1 & \lambda-2 \end{vmatrix} =$$

$$(\lambda - (1-k_1))(\lambda-2) - (1-k_2) = \lambda^2 - 2\lambda - (1-k_1)\lambda + 2(1-k_1) - (1-k_2)$$

$$= \lambda^2 \underbrace{- (3-k_1)\lambda}_{11} + \underbrace{2(1-k_1) - (1-k_2)}_{30}$$

$$\begin{cases} k_1 - 3 = 11 \longrightarrow \boxed{k_1 = 14} \\ 2 - 2k_1 - 1 + k_2 = 30 \end{cases}$$

$$1 - 2 \cdot 14 + k_2 = 30 \longrightarrow \boxed{k_2 = 57}$$

2.) $\underline{\underline{A}} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{vmatrix} \lambda-1 & -1 \\ 0 & \lambda-2 \end{vmatrix}$

$\underline{\underline{A}} - \underline{b}\,\underline{k}^T$

$$|\lambda \underline{\underline{E}} - (\underline{\underline{A}} - \underline{b}\,\underline{k}^T)| = \left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \left[\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix} - \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix} (k_1 \; k_2)}_{\begin{pmatrix} k_1 & k_2 \\ 0 & 0 \end{pmatrix}} \right] \right| = \left| \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} - \begin{pmatrix} 1-k_1 & 1-k_2 \\ 0 & 2 \end{pmatrix} \right| =$$

$$\begin{vmatrix} \lambda - (1-k_1) & -(1-k_2) \\ 0 & \lambda-2 \end{vmatrix} = \underbrace{(\lambda - (1-k_1))(\lambda-2)}_{} + \cancel{\boxed{0\,(1-k_2)}}$$

$$\underline{(\lambda - (1-k))}\;\underline{(\lambda-2)} = 0$$

$$\left[\, \underline{\underline{A}} \;\middle|\; \underline{b} \,\right]$$

IRÁNYÍTHATÓSÁG

PÓLUSÁTHELYEZÉS ÁLLAPOT - VISSZACSATOLÁSSAL

FOLYTATÁS

IRÁNYÍTHATÓSÁGI ALAK

$$\begin{pmatrix} \boxed{\underline{\underline{A}}} & \underline{b} \\ \underline{c}^T & D \end{pmatrix} \longrightarrow \underline{\underline{T}} = \left(\underline{\underline{M}}_c \underline{\underline{T}}(a)\right)^{-1} \longrightarrow \begin{pmatrix} \boxed{\underline{\underline{\tilde{A}}}} & \underline{\tilde{b}} \\ \underline{\tilde{c}}^T & \tilde{D} \end{pmatrix}$$

$$W(s) = \frac{b_1 s^{n-1} + \dots \quad + b_n}{s^n + \underline{a_1} s^{n-1} + \underline{a_2} s^{n-2} + \dots + a_n}$$

$$\tilde{\underline{\underline{A}}} = \begin{bmatrix} -\underline{a_1} & -\underline{a_2} & -a_3 & \cdots & -a_n \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & \cdots & & \end{bmatrix} \qquad \tilde{\underline{b}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\boxed{|\lambda \underline{\underline{E}} - \underline{\underline{A}}| = |\lambda \underline{\underline{E}} - \tilde{\underline{\underline{A}}}|}$$

$$\lambda^n + a_1 \lambda^{n-1} + \dots \quad + a_n = \emptyset$$

$$\varphi_{cl}(\lambda) = (\lambda - \underset{\uparrow}{\tilde{\lambda}_1})(\lambda - \underset{\uparrow}{\tilde{\lambda}_2}) \dots \quad (\lambda - \underset{\uparrow}{\tilde{\lambda}_n}) = \lambda^n + \underline{p_1} \lambda^{n-1} + p_2 \lambda^{n-2} + \dots \quad + p_n$$

$$\tilde{\underline{\underline{A}}} - \tilde{\underline{b}} \tilde{\underline{k}}^T = \tilde{\underline{\underline{A}}} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} [\tilde{k}_1 \;\; \tilde{k}_2 \dots \tilde{k}_n] = \begin{bmatrix} \underline{-(a_1 + \tilde{k}_1)} & -(a_2 + \tilde{k}_2) & \cdots & -(a_n + \tilde{k}_n) \\ 1 & 0 & \cdots & 0 \\ \vdots & & & \\ & & & \end{bmatrix}$$

$$\boxed{\begin{aligned} p_1 &= a_1 + \tilde{k}_1 \\ p_2 &= a_2 + \tilde{k}_3 \\ &\vdots \\ p_n &= a_n + \tilde{k}_n \end{aligned}}$$

$$\boxed{\tilde{k}_i = p_i - a_i} \; !$$

$$\text{ⓤ} \; -\boxed{\underline{k}^T} \underline{x} = -\tilde{\underline{k}}^T \tilde{\underline{x}} = -\boxed{\tilde{\underline{k}}^T \underline{\underline{T}}} \underline{x}$$

$$\boxed{\underline{k}^T = \tilde{\underline{k}}^T \underline{\underline{T}}}$$

$$\boxed{\begin{aligned} &① \quad \tilde{k}_i = p_i - a_i \qquad i = 1 \dots n \\ &\cancel{\underline{k}_i} \\ &\tilde{\underline{k}}^T = \underline{p}^T - \underline{a}^T \\ &\underline{k}^T = \tilde{\underline{k}}^T \underline{\underline{T}} \end{aligned}}$$

BASS-GURA -ALGORITMUS: $\underline{k}^T = \tilde{\underline{k}}^T \underline{\underline{T}} \Rightarrow$ ② $\boxed{\underline{k}^T = (\underline{p}^T - \underline{a}^T) \, \underline{\underline{T}}^{-1}(a) \, \underline{\underline{M}}_c^{-1}}$

$$\underline{\underline{M}}_c = [\underline{b} \;\; \underline{\underline{A}} \underline{b} \dots]$$

ACKERMANN - KÉPLET: ③ $\boxed{\underline{k}^T = [0 \; 0 \dots \; 0 \; 1] \, \underline{\underline{M}}_c^{-1} \varphi_{cl}(\underline{\underline{A}})}$

$$\varphi_{cl}(\underline{\underline{A}}) = \underline{\underline{A}}^n + p_1 \underline{\underline{A}}^{n-1} + p_2 \underline{\underline{A}}^{n-2} + \dots + p_n \underline{\underline{E}}$$

Tervezzünk az alábbi rendszerhez állapot-visszacsatolást Bass-Gura- és Ackermann módszerével! Legyen a két új pólus: $\tilde{p}_1 = -5$ és $\tilde{p}_2 = -6$.

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\underline{k}^T = (\underline{p} - \underline{a})^T \underline{\underline{T}}^{-1}(a) \, \underline{\underline{M}}_c^{-1} \qquad \underline{k}^T = \begin{bmatrix} 14 & 57 \end{bmatrix}$$

$$\underline{k}^T = [0 \; 0 \ldots 0 \; 1] \, \underline{\underline{M}}_c^{-1} \, \varphi_{cl}(\underline{\underline{A}})$$

[redacted]

$$\underline{a}) \quad \varphi(\lambda) = |\lambda \underline{\underline{E}} - \underline{\underline{A}}| = \left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \right| = \begin{vmatrix} \lambda-1 & -1 \\ -1 & \lambda-2 \end{vmatrix} = (\lambda-1)(\lambda-2) - 1 = \lambda^2 \underbrace{-3}_{a_1}\lambda + \underbrace{1}_{a_2}$$

Nem stabil.

$$\underline{a} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \qquad \underline{p} = \begin{bmatrix} 11 \\ 30 \end{bmatrix}$$

$$\underline{p}) \quad (s - \tilde{p}_1)(s - \tilde{p}_2) = (s+5)(s+6) = s^2 + \underbrace{11}_{p_1}s + \underbrace{30}_{p_2}$$

$$\underline{\underline{T}}(a) = \begin{bmatrix} 1 & a_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \qquad \underline{\underline{T}}^{-1} = \frac{\operatorname{adj} T}{|T|} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{c:c} 1 & 0 \\ -3 & 1 \end{array}\right] \xrightarrow{\substack{+ - \\ - +}} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \xrightarrow{T} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$|T| = 1 - (-3) \cdot 0 = 1$$

$$\underline{p} - \underline{a} = \begin{bmatrix} 11 \\ 30 \end{bmatrix} - \begin{bmatrix} -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 29 \end{bmatrix}$$

$$\underline{\underline{M}}_c = [\underline{b} \quad A\,\underline{b}] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad \underline{\underline{M}}_c^{-1} = \frac{\operatorname{adj} \underline{\underline{M}}_c}{|\underline{\underline{M}}_c|} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \qquad |\underline{\underline{M}}_c| = 1$$

$$[14 \;\; 29] \cdot \underbrace{\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}}$$

$$\underbrace{[14 \;\; 29] \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}}$$

$$\underline{k}^T = [14 \cdot 1 + 29 \cdot 0 \;\vdots\; 2 \cdot 14 + 29]$$

A pólusáthelyezés helyben hagyja a zérusokat !

$$W(s)=\underline{c}^T\underbrace{\left(s\underline{\underline{E}}-\underline{\underline{A}}+\underline{b}\,\underline{k}^T\right)^{-1}}\underline{b}$$

$$(A+BCD)^{-1}=A^{-1}-A^{-1}B\left(C^{-1}+DA^{-1}B\right)^{-1}DA^{-1}$$ (Lemma)

$$\left[\underbrace{\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)}_{A}+\underbrace{\underline{b}}_{B}\,\underbrace{\underline{\underline{E}}}_{C}\,\underbrace{\underline{k}^T}_{D}\right]^{-1}=$$

$$=\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}-\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}\left(\underbrace{\underline{\underline{E}}}_{=1}+\underline{k}^T\underbrace{\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}}_{▭\;□\;▯}\underline{b}\right)^{-1}\underline{k}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}$$

SKALÁR

$$=\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}-\frac{\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}\cdot\underline{k}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}^{-1}\right)^{2}}{1+\underline{k}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}}$$

$$\underline{c}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}-\frac{\underline{c}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}\cdot\underline{k}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}}{1+\underline{k}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\cdot\underline{b}}$$

$$\frac{\underline{c}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}+\underline{c}^T\left(s\underline{\underline{E}}-\cancel{\underline{\underline{A}}\right)^{-1}\underline{b}\,\underline{k}^T}\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\cdot\underline{b}-\underline{c}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\cancel{\right)^{-1}\underline{b}\,\underline{k}^T}\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}}{1+\underline{k}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}}$$

$$W(s)=\underline{c}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}+\underline{b}\,\underline{k}^T\right)^{-1}\underline{b}=\frac{\boxed{\underline{c}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}}}{1+\underline{k}^T\left(s\underline{\underline{E}}-\underline{\underline{A}}\right)^{-1}\underline{b}}$$

$\underline{k}^T = [0\ 0 \dots\ 0\ 1]\ \underline{\underline{M}}_c^{-1}\ \underbrace{\varphi_{cl}}_{\text{CLOSE}}(\underline{A})$ $\qquad \underline{\tilde{p}_1 = -5} \qquad \underline{\tilde{p}_2 = -6}$ $\qquad \underline{\underline{M}}_c^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\underline{A} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\varphi_{cl}(\lambda) = (\lambda - \tilde{p}_1)(\lambda - \tilde{p}_2) = (\lambda+5)(\lambda+6) = \lambda^2 + 11\lambda + 30$$

$$\varphi_{cl}(\underline{A}) = \underline{A}^2 + 11 \cdot \underline{A} + 30\ \underline{\underline{E}}$$

$$\underline{A}^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$$

$1 \cdot 1 + 1 \cdot 1 \qquad 1 \cdot 1 + 1 \cdot 2$

$1 \cdot 1 + 2 \cdot 1 \qquad 1 \cdot 1 + 2 \cdot 2$

$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 11 & 11 \\ 11 & 22 \end{bmatrix} + \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} = \begin{bmatrix} 43 & 14 \\ 14 & 57 \end{bmatrix}$$

$$\underline{k}^T = [0\ \ 1]\ \underbrace{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 43 & 14 \\ 14 & 57 \end{bmatrix}}$$

$43 - 14 \qquad 14 - 57$

$14 \qquad 57$

$$\underline{k}^T = [14\ \ 57]$$

$$[0\ \ 1]\begin{bmatrix} 29 & -43 \\ \boxed{14} & \boxed{57} \end{bmatrix}$$

Rajzoljuk fel a visszacsatolt rendszer jelfolyam hálózatát!

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \underline{k}^T = \begin{bmatrix} 14 & 57 \end{bmatrix}$$

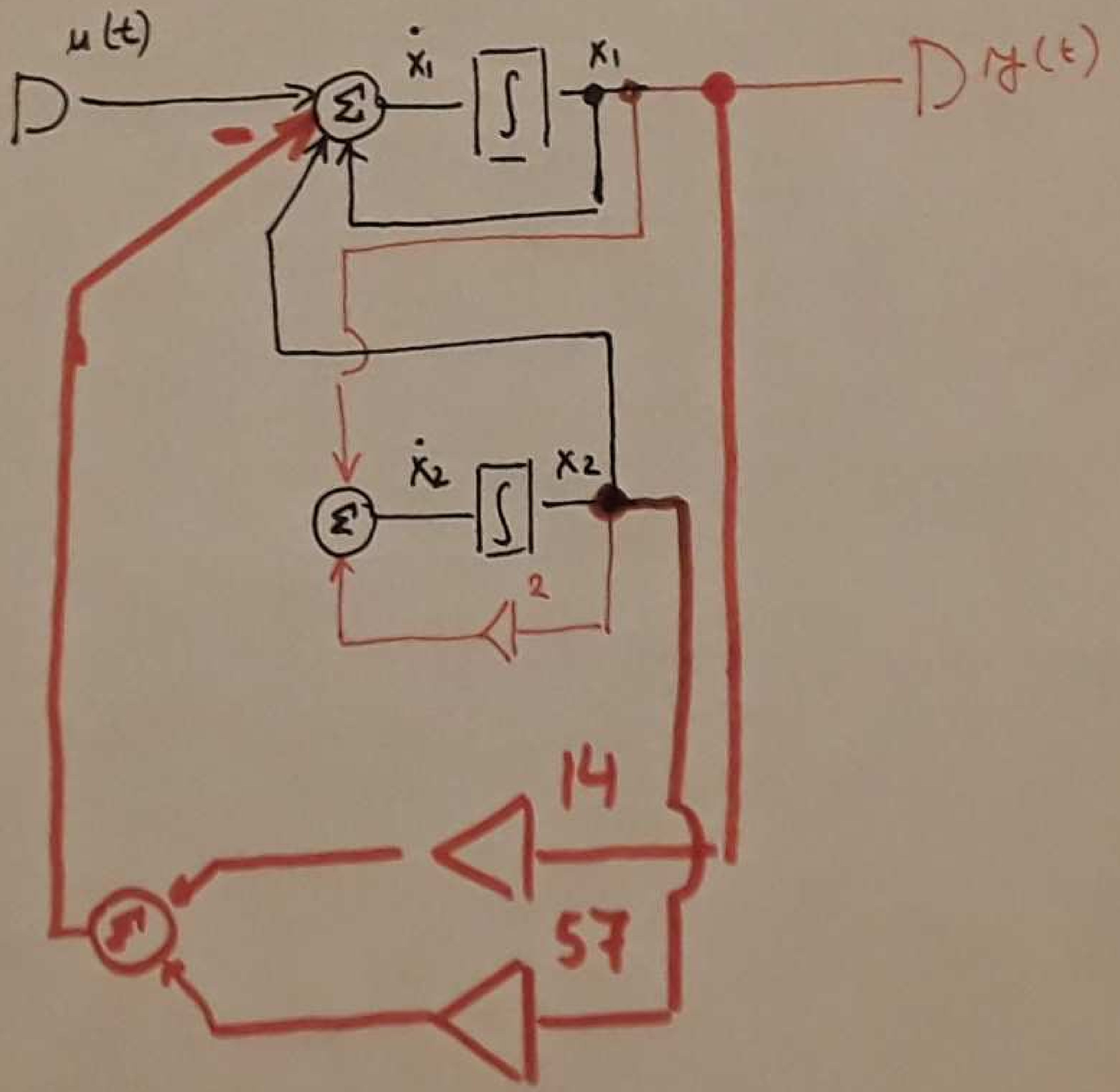
$$\underline{c}^T = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

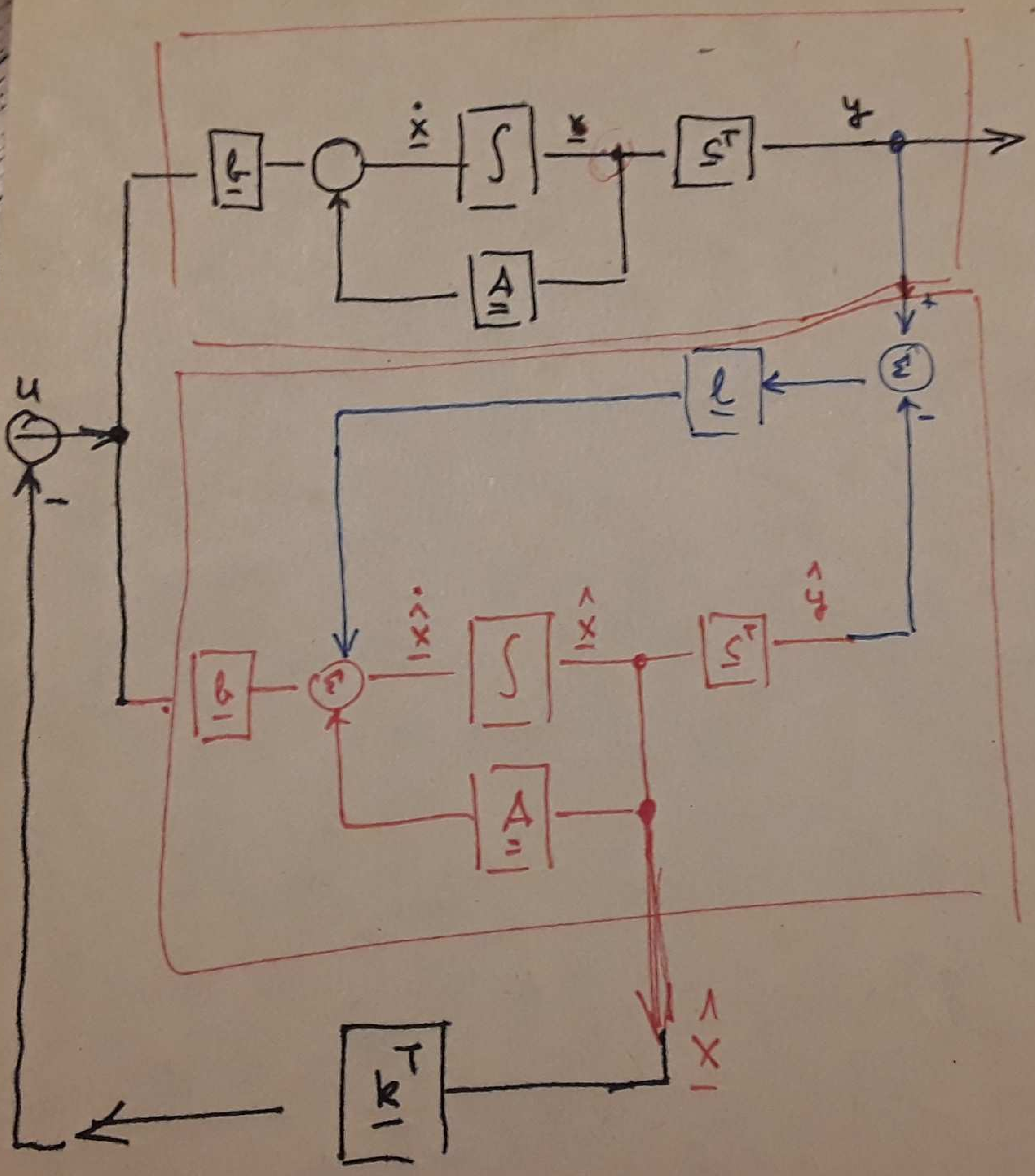
$$\dot{x}_1 = x_1 + x_2 + u$$

$$\dot{x}_2 = x_1 + 2x_2$$

$$y = x_1$$

$$u = -14\,x_1 - 57\,x_2 \qquad (u = -\underline{k}^T\underline{x})$$

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Hasonlítsuk össze $\underline{\underline{A}}$ és $\underline{\underline{A}}_{cl}$ sajátértékeit!

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \qquad |\lambda \underline{\underline{E}} - \underline{\underline{A}}| = \begin{vmatrix} \lambda-1 & -1 \\ -1 & \lambda-2 \end{vmatrix} = (\lambda-1)(\lambda-2) - 1 = \lambda^2 - 3\lambda + 1 = 0$$

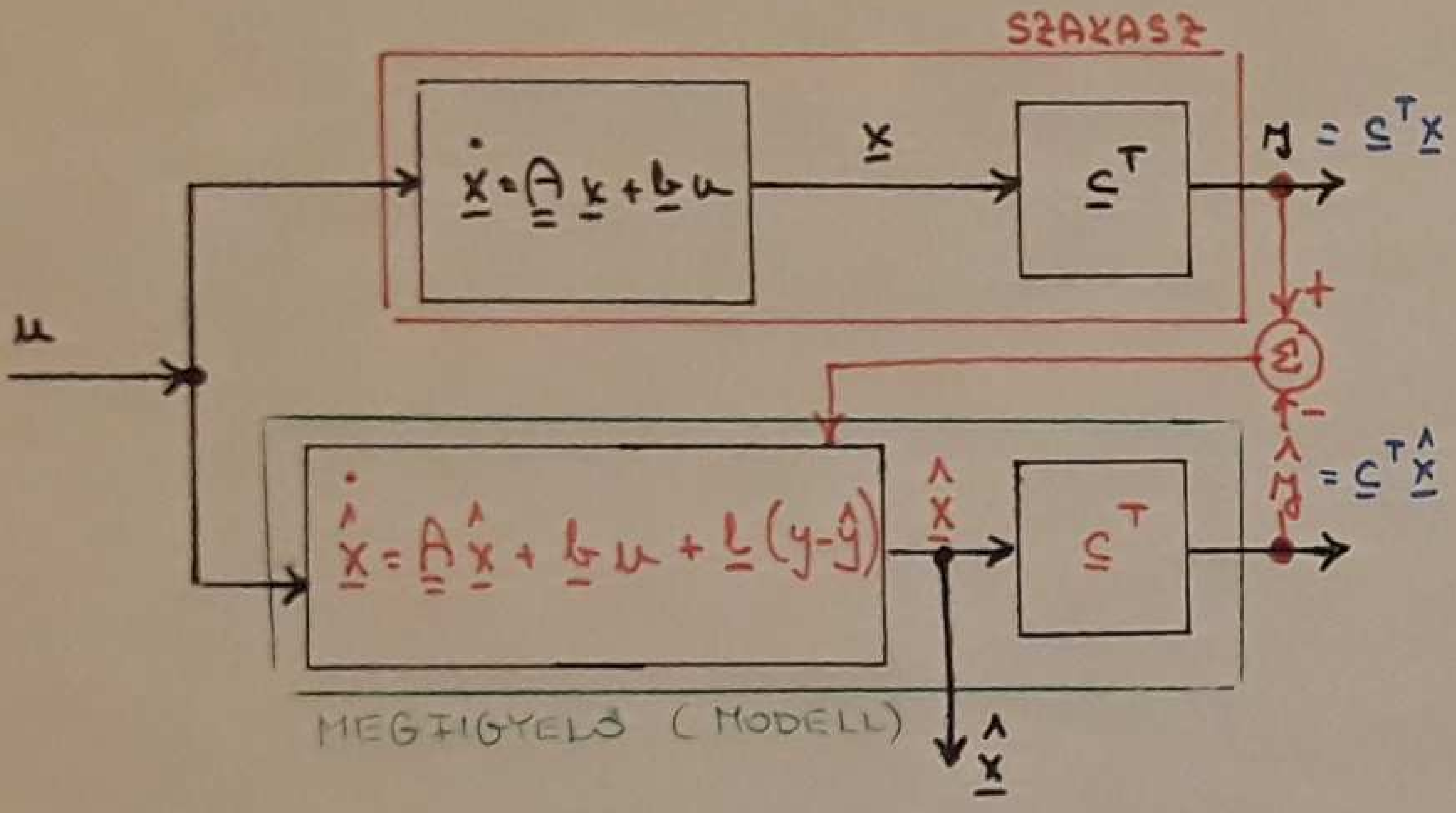
$$\lambda_{1,2} = \frac{3 \pm \sqrt{9-4\cdot 1}}{2} = \frac{3 \pm \sqrt{5}}{2} \begin{cases} \lambda_1 = \underline{2{,}62} \\ \lambda_2 = \underline{0{,}38} \end{cases}$$

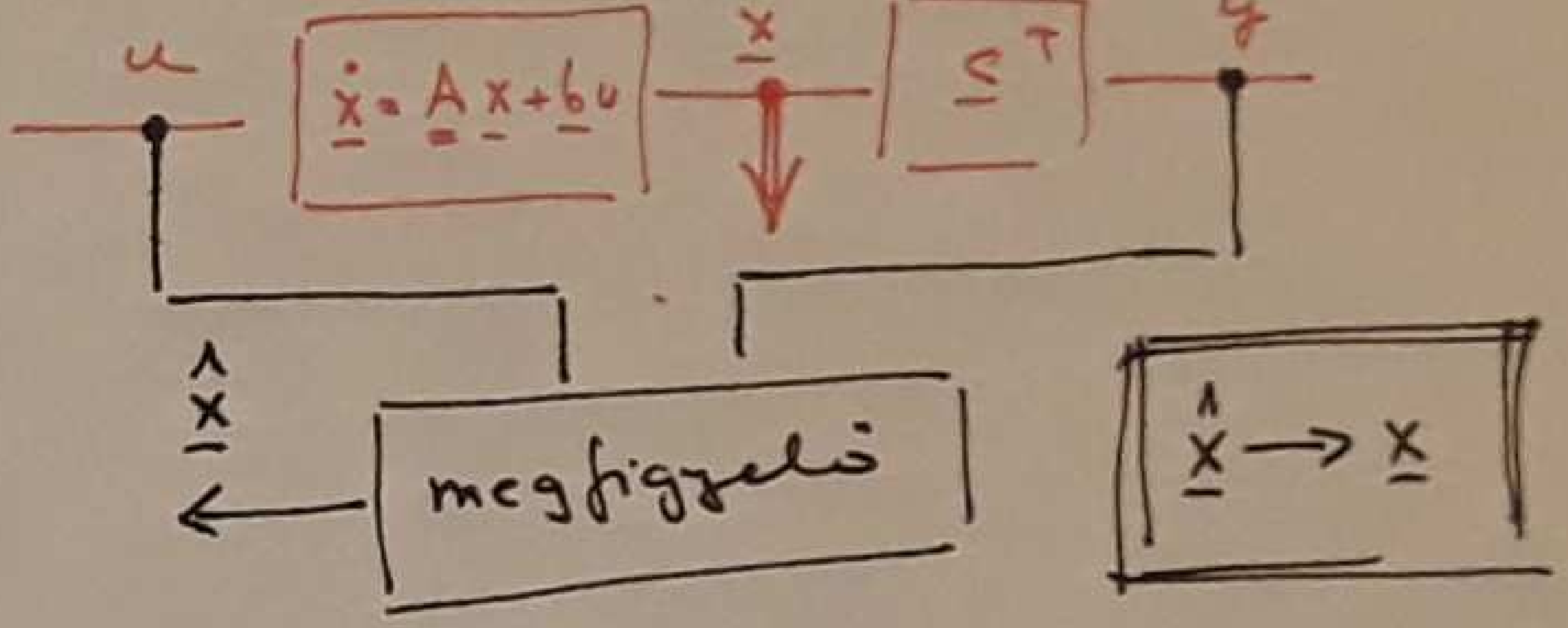
$$\underset{\overset{\nearrow}{\text{CLOSED}}}{\underline{\underline{A}}_{CL}} = \underline{\underline{A}} - \underline{b}\,\underline{k}^T = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 14 & 57 \end{bmatrix}}_{\begin{bmatrix} 14 & 57 \\ 0 & 0 \end{bmatrix}} = \begin{bmatrix} -13 & -56 \\ 1 & 2 \end{bmatrix}$$

$$|\lambda \underline{\underline{E}} - \underline{\underline{A}}_{CL}| = \begin{vmatrix} \lambda+13 & 56 \\ -1 & \lambda-2 \end{vmatrix} = (\lambda+13)(\lambda-2) + 56 = \underbrace{\lambda^2 + 11\lambda + 30 = 0}$$

$$\lambda_{1,2} = \frac{-11 \pm \sqrt{121-4\cdot 30}}{2} = \frac{-11 \pm 1}{2} \begin{cases} \lambda_1 = -5 \\ \lambda_2 = -6 \end{cases}$$

ÁLLAPOT-VISSZACSATOLÁS REALIZÁLÁSA MEGFIGYELŐVEL





$$\dot{\hat{x}} = A\hat{x} + bu + l(y-\hat{y})$$

$$\dot{\hat{x}} = A\hat{x} + bu + lc^Tx - lc^T\hat{x}$$

$$\dot{\hat{x}} = \underbrace{(A - lc^T)}_{F}\hat{x} + \underbrace{l\overset{y}{c^Tx}}_{ly} + bu$$

$$\boxed{\dot{\hat{x}} = F\hat{x} + Gy + Hu}$$

$F = A - Gc^T$

$G = l$

$H = b$

$l\ (G)$

$$\begin{cases} \dot{x} = Ax + bu \\ \dot{\hat{x}} = (A - lc^T)\hat{x} + lc^Tx + bu \end{cases}$$

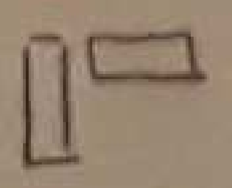
$x - \hat{x} \to 0$

$$\dot{x} - \dot{\hat{x}} = Ax + \cancel{bu} - (A - lc^T)\hat{x} - lc^Tx - \cancel{bu}$$

$$\dot{x} - \dot{\hat{x}} = (A - lc^T)x - (A - lc^T)\hat{x}$$

$$\dot{x} - \dot{\hat{x}} = (A - lc^T)(x - \hat{x}) \Rightarrow \dot{\varepsilon} = (A - lc^T)\varepsilon$$

$$x - \hat{x} = e^{(A - lc^T)t}(x(0) - \hat{x}(0))$$



bc^T

Tervezzünk megfigyelőt $p_1 = -1$ és $p_2 = -3$ pólusokkal, ha

$$\underline{\underline{A}} = \begin{bmatrix} 4 & -3 \\ 1 & 0 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \underline{c}^T = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$\underline{l} = \underline{\underline{M}}_0^{-1} \underline{\underline{T}}^{-T}(a) (\underline{f} - \underline{a})$$

$$\underline{l}^T = [0\ 0 \ldots 0\ 1]\ \underline{\underline{M}}_0^{-T} \varphi_0(\underline{\underline{A}}^T)$$

$$\underline{a})\ |\lambda \underline{\underline{E}} - \underline{\underline{A}}| = \left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 4 & -3 \\ 1 & 0 \end{bmatrix} \right| = \begin{vmatrix} \lambda-4 & 3 \\ -1 & \lambda \end{vmatrix} = \lambda(\lambda-4) + 3 = \lambda^2 \underbrace{-4}_{a_1}\lambda + \underbrace{3}_{a_2}$$

$$\underline{a} = \begin{bmatrix} -4 \\ 3 \end{bmatrix} \qquad \underline{f} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\underline{f})\ (s-p_1)(s-p_2) = (s+1)(s+3) = s^2 + \underbrace{4}_{f_1}s + \underbrace{3}_{f_2}$$

$$\underline{\underline{M}}_0 = \begin{bmatrix} \underline{c}^T \\ \underline{c}^T\underline{\underline{A}} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix}$$

$$[1\ 2]\begin{bmatrix} 4 & -3 \\ 1 & 0 \end{bmatrix} = \underset{1 \cdot 4 + 2 \cdot 1 \quad -3 \cdot 1 + 2 \cdot 0}{[6\ -3]}$$

$$\underline{\underline{M}}_0^{-1} = \frac{\text{adj}\ \underline{\underline{M}}_0}{|\underline{\underline{M}}_0|} = \begin{bmatrix} 0{,}2 & 0{,}133 \\ 0{,}4 & -0{,}066 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 6 \\ 2 & 1 \end{bmatrix} \xrightarrow[-+]{+-} \begin{bmatrix} -3 & -6 \\ -2 & 1 \end{bmatrix} \xrightarrow{T} \begin{bmatrix} -3 & -2 \\ -6 & 1 \end{bmatrix}$$

$$|\underline{\underline{M}}_0| = 1 \cdot (-3) - 6 \cdot 2 = -3 - 12 = -15$$

$$\underline{\underline{T}}(a) = \begin{bmatrix} 1 & a_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ +4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} = \underline{\underline{T}}^{-1} \qquad \underline{\underline{T}}^{-T} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$\underline{l} = \begin{bmatrix} 0{,}2 & 0{,}133 \\ 0{,}4 & -0{,}066 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} \overset{8}{4+4} \\ \overset{0}{3-3} \end{bmatrix}}_{\begin{bmatrix} 8 \\ 32 \end{bmatrix}}$$

$$\begin{bmatrix} 0{,}2 \cdot 8 + 0{,}133 \cdot 32 \\ 0{,}4 \cdot 8 - 0{,}066 \cdot 32 \end{bmatrix} = \begin{bmatrix} 1{,}6 + 4{,}26 \\ 3{,}2 - 2{,}13 \end{bmatrix} = \begin{bmatrix} 5{,}86 \\ 1{,}07 \end{bmatrix}$$

Írjuk fel a megfigyelő differenciálegyenletét, ha

$$\underline{\underline{A}} = \begin{bmatrix} 4 & -3 \\ 1 & 0 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \underline{c}^T = \begin{bmatrix} 1 & 2 \end{bmatrix} \qquad \underline{l} = \begin{bmatrix} 5,86 \\ 1,07 \end{bmatrix} \qquad \underline{\hat{x}} \longrightarrow \underline{x}$$

$$\hat{y} = \underline{c}^T \underline{\hat{x}}$$

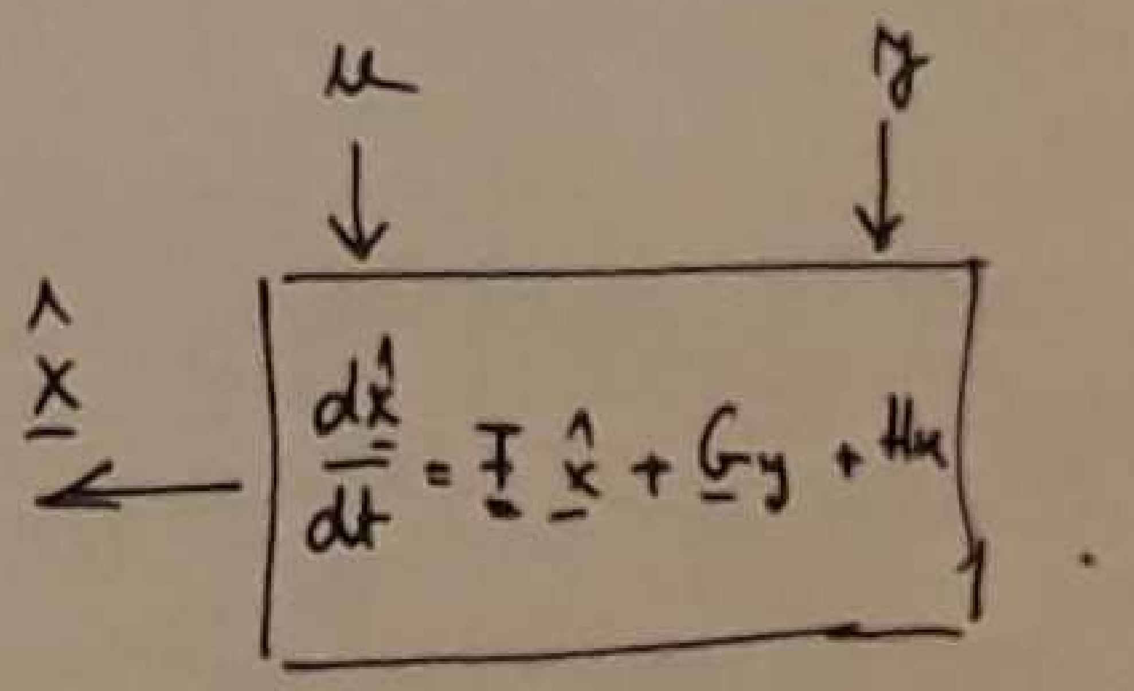
$$\dot{\underline{\hat{x}}} = \underline{\underline{A}}\,\underline{\hat{x}} + \underline{b}u + \underline{l}(y - \hat{y})$$

$$= \underbrace{\underline{\underline{A}}}\,\underline{\hat{x}} + \underline{b}u + \underline{l}y \underbrace{- \underline{l}\,\underline{c}^T}\underline{\hat{x}}$$

$$\boxed{\dot{\underline{\hat{x}}} = (\underline{\underline{A}} - \underline{l}\,\underline{c}^T)\underline{\hat{x}} + \underline{b}u + \underline{l}y}$$

$$\begin{bmatrix} 4 & -3 \\ 1 & 0 \end{bmatrix} - \underbrace{\begin{bmatrix} 5,86 \\ 1,07 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix}}_{\begin{bmatrix} 5,86 & 11,72 \\ 1,07 & 2,14 \end{bmatrix}} = \begin{bmatrix} -1,86 & -14,72 \\ -0,07 & -2,14 \end{bmatrix}$$

$$\boxed{\begin{aligned} \dot{\hat{x}}_1 &= -1,86\hat{x}_1 - 14,72\hat{x}_2 + 1u + 5,86y \\ \dot{\hat{x}}_2 &= \underbrace{-0,07\hat{x}_1 - 2,14\hat{x}_2} \underbrace{+ 0u} \underbrace{+ 1,07y} \end{aligned}}$$



$$\underline{k}^T = [0\ 0 \dots\ 0\ 1]\ \underline{\underline{M}}_0^{-T}\ \varphi_0(\underline{A}^T)$$

$$\underline{\underline{M}}_0^{-1} = \begin{bmatrix} 0{,}2 & 0{,}133 \\ 0{,}4 & -0{,}066 \end{bmatrix} \qquad \underline{A} = \begin{bmatrix} 4 & -3 \\ 1 & 0 \end{bmatrix} \qquad \boxed{\begin{matrix} p_1 = -1 \\ p_2 = -3 \end{matrix}}$$

$$\Downarrow$$

$$\varphi_0(\lambda) = (\lambda - p_1)(\lambda - p_2) = (\lambda + 1)(\lambda + 3) = \lambda^2 + 4\lambda + 3$$

$$\varphi_0(\underline{\underline{A}}^T) = (\underline{\underline{A}}^T)^2 + 4(\underline{\underline{A}})^T + 3\underline{\underline{E}} \qquad \underline{A}^T = \begin{bmatrix} 4 & 1 \\ -3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 4 \\ -12 & -3 \end{bmatrix}$$

$$\begin{array}{c|c} 4\cdot 4 - 1\cdot 3 & 4\cdot 1 + 1\cdot 0 \\ \hline -4\cdot 3 + 0(-3) & -1\cdot 3 + 0\cdot 0 \end{array}$$

$$\varphi_0(\underline{\underline{A}}^T) = \begin{bmatrix} 13 & 4 \\ -12 & -3 \end{bmatrix} + \begin{bmatrix} 16 & 4 \\ -12 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 32 & 8 \\ -24 & 0 \end{bmatrix}}}$$

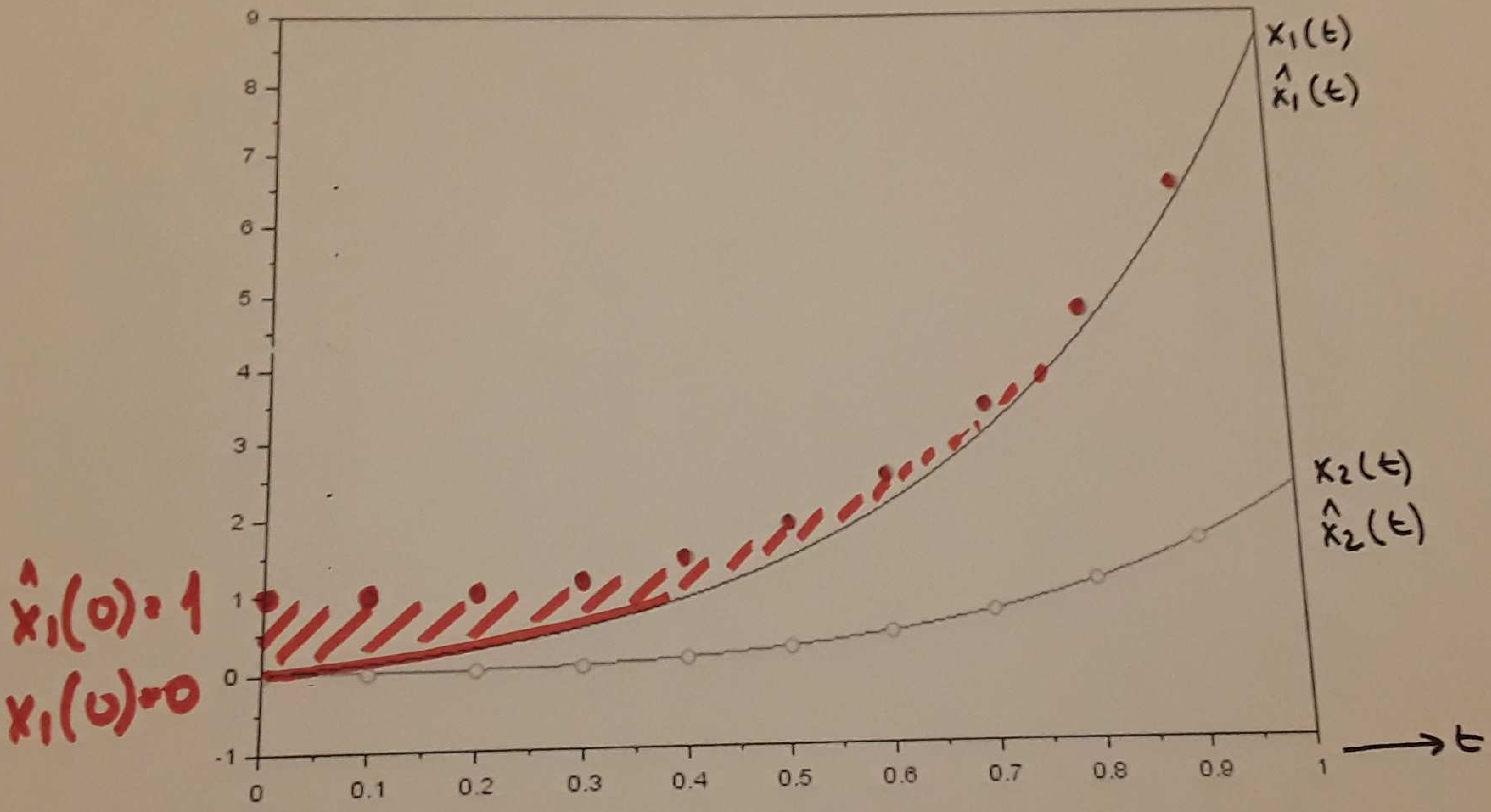
$$\underline{k}^T = [0\ 1]\underbrace{\begin{bmatrix} 0{,}2 & 0{,}4 \\ 0{,}133 & -0{,}066 \end{bmatrix}\begin{bmatrix} 32 & 8 \\ -24 & 0 \end{bmatrix}}$$

$$\begin{array}{c|c} 0{,}2\cdot 32 - 24\cdot 0{,}4 & 0{,}2\cdot 8 \\ \hline 32\cdot 0{,}133 + 24\cdot 0{,}066 & 0{,}133\cdot 8 \end{array}$$

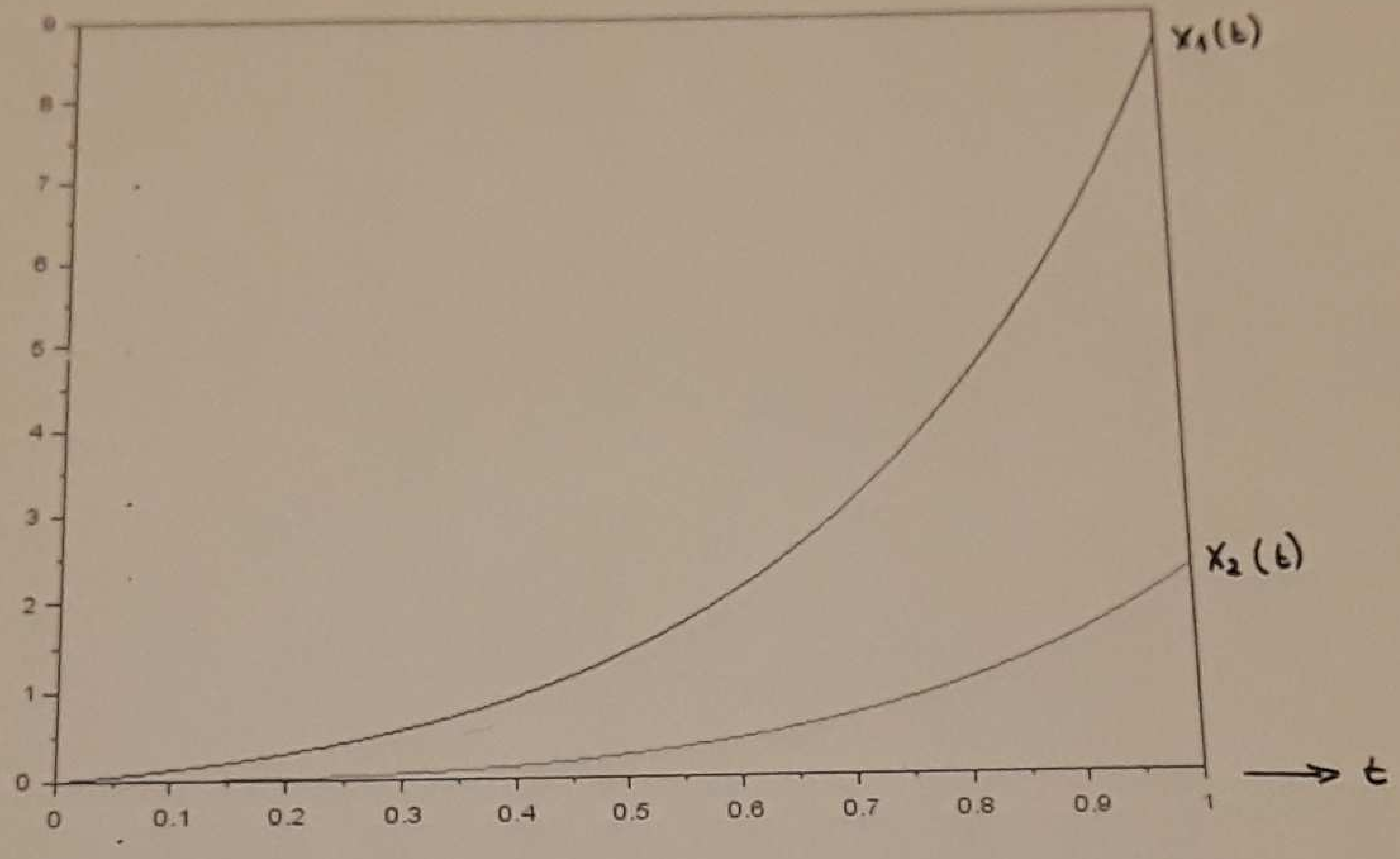
$$6{,}4 - 9{,}6$$
$$4{,}26 + 1{,}6$$

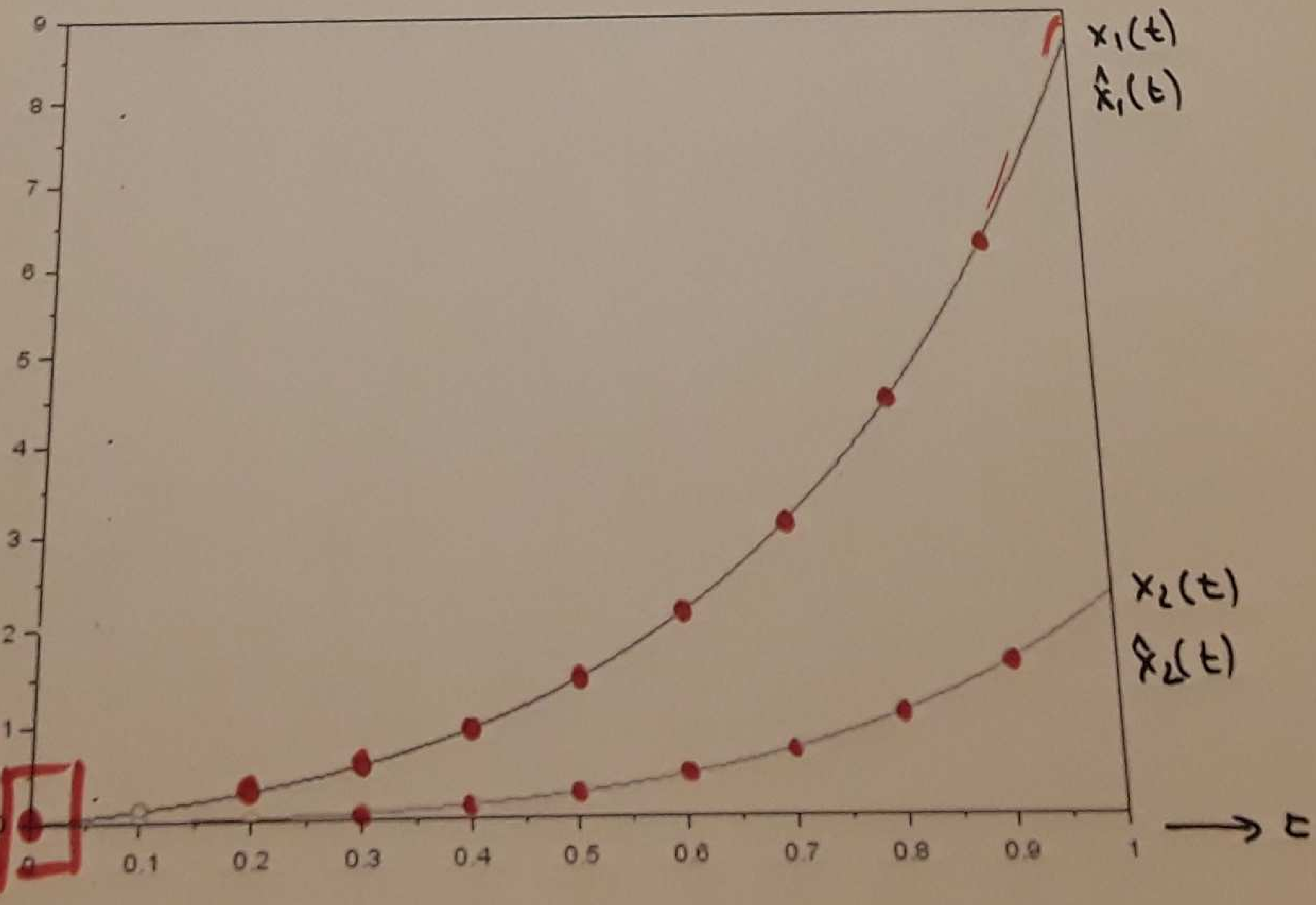
$$[0\ 1]\begin{bmatrix} -3{,}2 & 1{,}6 \\ \underbrace{\boxed{5{,}86}}_{k_1} & \underbrace{\boxed{1{,}066}}_{k_2} \end{bmatrix}$$

$$\boxed{\underline{k}^T = [5{,}86\ \ 1{,}066]}$$



$\hat{x}_1 \rightarrow x_1$





$x_1(0) = \hat{x}_1(0) = 0$

$x_2(0) = \hat{x}_2(0) = 0$

↑

MEGFIGYELHETŐSÉGI ALAK

$$\tilde{A} = \begin{bmatrix} -a_1 & 1 & 0 & 0 & \cdots \\ -a_2 & 0 & 1 & 0 & \cdots \\ \vdots & & & \ddots & \\ -a_n & & & & \end{bmatrix}$$

$$\tilde{A} = \begin{bmatrix} -a_1 & -a_2 & \cdots \\ 1 & 0 & \cdots \\ 0 & 1 & \\ \vdots & & \ddots \end{bmatrix} \qquad \tilde{b} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\tilde{c}^T = \begin{bmatrix} 1 & 0 & 0 & \cdots \end{bmatrix}$$

$\bigcirc\!\!\!\!\!\;k^T$

$$\varphi_o(\lambda) = (\lambda - \tilde{\lambda}_1)(\lambda - \tilde{\lambda}_2)\cdots = \lambda^n \underline{+ f_1}\lambda^{n-1} \underline{+ f_2}\lambda^{n-2} + \dots f_n$$

$\bigcirc\!\!\!\!\!\;l$

$$\tilde{A} - \tilde{l}\tilde{c}^T = \begin{bmatrix} -a_1 & 1 & 0 & 0 & \cdots \\ -a_2 & 0 & 1 & 0 & \cdots \\ -a_3 & 0 & 0 & 1 & \cdots \\ \vdots & & & & \\ -a_n & & & & \end{bmatrix} - \underbrace{\begin{bmatrix} \tilde{l}_1 \\ \tilde{l}_2 \\ \vdots \\ \tilde{l}_n \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix}} = \begin{bmatrix} \underline{-(a_1+\tilde{l}_1)} & 1 & 0 & \\ \underline{-(a_2+\tilde{l}_2)} & 0 & 1 & \ddots \\ \vdots & & & \\ -(a_n+\tilde{l}_n) & & & \end{bmatrix}$$

$$\left.\begin{aligned} a_1 + \tilde{l}_1 &= f_1 \\ a_2 + \tilde{l}_2 &= f_2 \\ &\vdots \\ a_n + \tilde{l}_n &= f_n \end{aligned}\right\} \boxed{\tilde{l}_i = f_i - a_i}$$

①

$$\tilde{A} - \tilde{l}\tilde{c}^T$$

Bass-Gura: $k^T = (p - a)^T T^{-1}(a) M_c^{-1}$ $/T$

$k = M_c^{-T} T^{-T}(a)(p - a)$

$A \to A^T$

$b \to c$

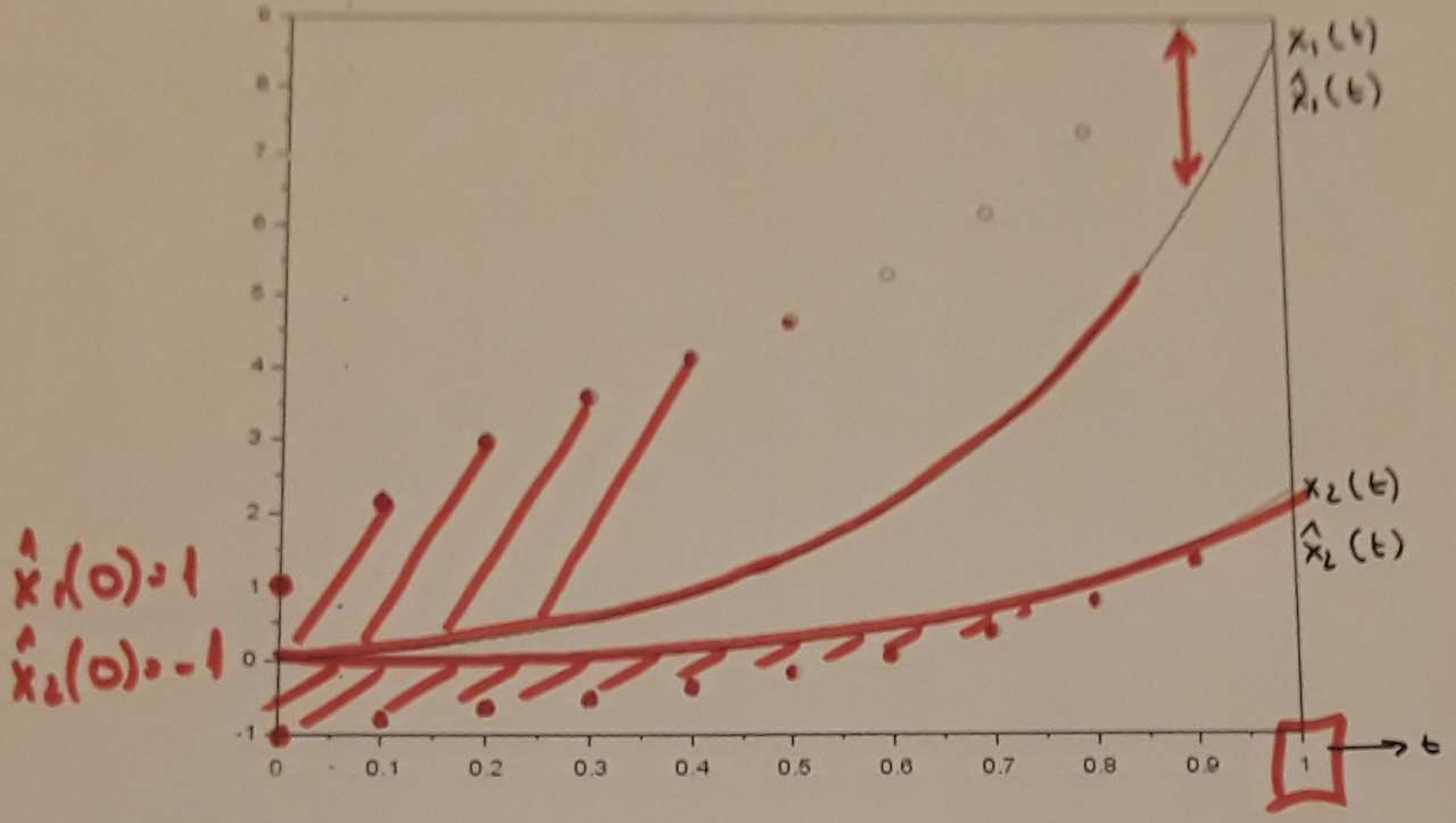
$k \to l$ duális

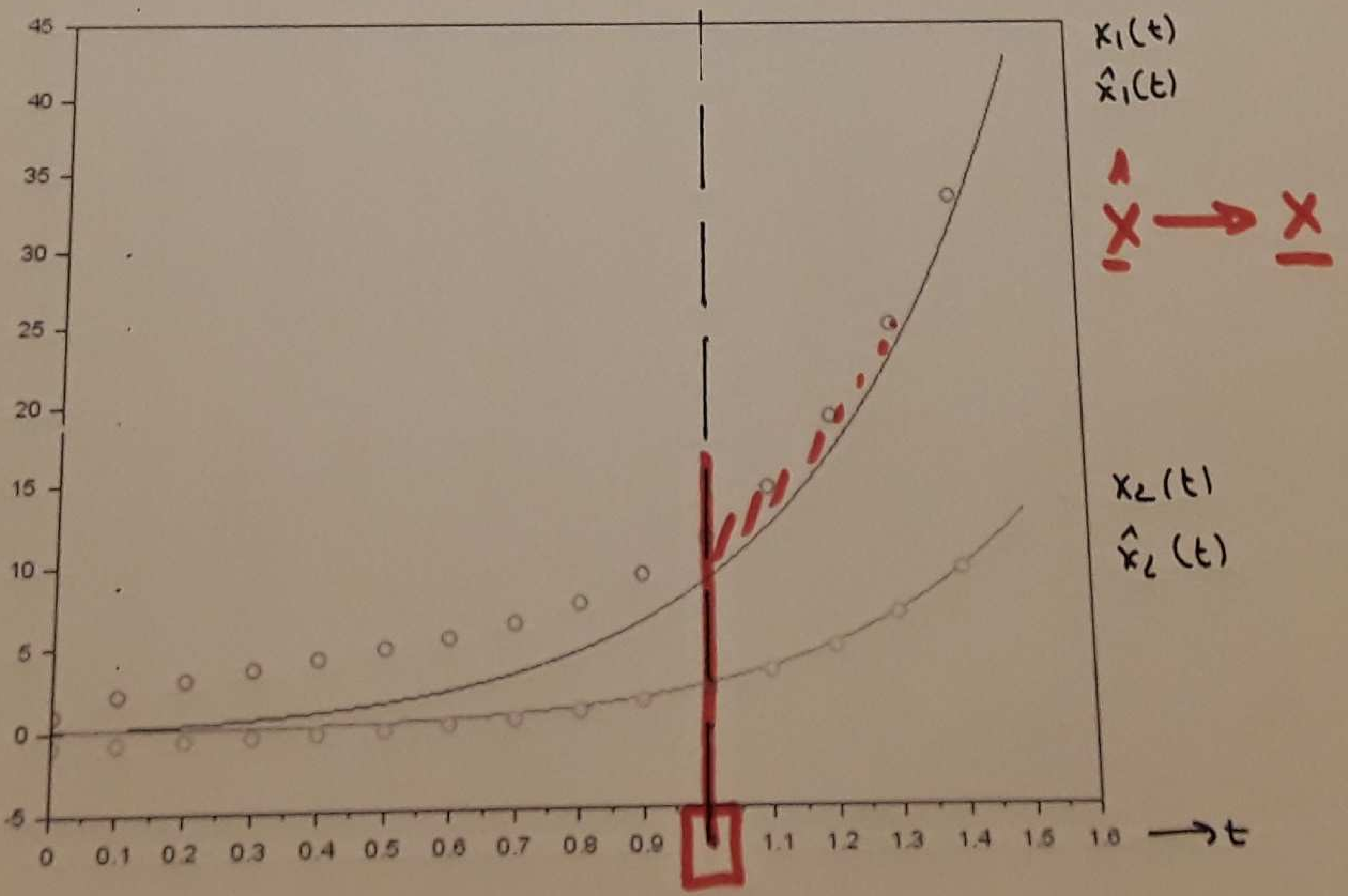
② $\boxed{l = M_o^{-1} T^{-T}(a)(f - a)}$

$$M_c = [\,b \;\; Ab \;\; A^2b \dots\,] \Rightarrow [\,c \;\; A^Tc \;\; A^{2T}c \dots\,] = \begin{bmatrix} c^T \\ c^TA \\ c^TA^2 \\ \vdots \end{bmatrix}^T = M_o^T \qquad (AB)^T = B^TA^T$$

Ackermann-képlet: $k^T = [0 \; 0 \dots 0 \; 1]\, M_c^{-1}\varphi_{cl}(A)$

③ $\boxed{l^T = [0 \; 0 \dots 0 \; 1]\, M_o^{-T}\varphi_o(A^T)}$ a videóban l^T-ben T lemaradt.


$x_1(t)$
$\hat{x}_1(t)$
$x_2(t)$
$\hat{x}_2(t)$
$\hat{x}_1(0)=1$
$\hat{x}_2(0)=-1$
9
8
7
6
5
4
3
2
1
0
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
t


$x_1(t)$
$\hat{x}_1(t)$
$\underline{\hat{x}} \rightarrow \underline{x}$
$x_2(t)$
$\hat{x}_2(t)$
45
40
35
30
25
20
15
10
5
0
-5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
t

A SZEPARÁCIÓS ELV

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{b}\,u = \underline{\underline{A}}\,\underline{x} - \underline{b}\,\underline{k}^T\hat{\underline{x}} = \underbrace{\underline{\underline{A}}\,\underline{x}} + \underline{b}\,\underline{k}^T\underline{x} \underbrace{- \underline{b}\,\underline{k}^T\underline{x}} - \underline{b}\,\underline{k}^T\hat{\underline{x}}$$

$$u = -\underline{k}^T\hat{\underline{x}}$$

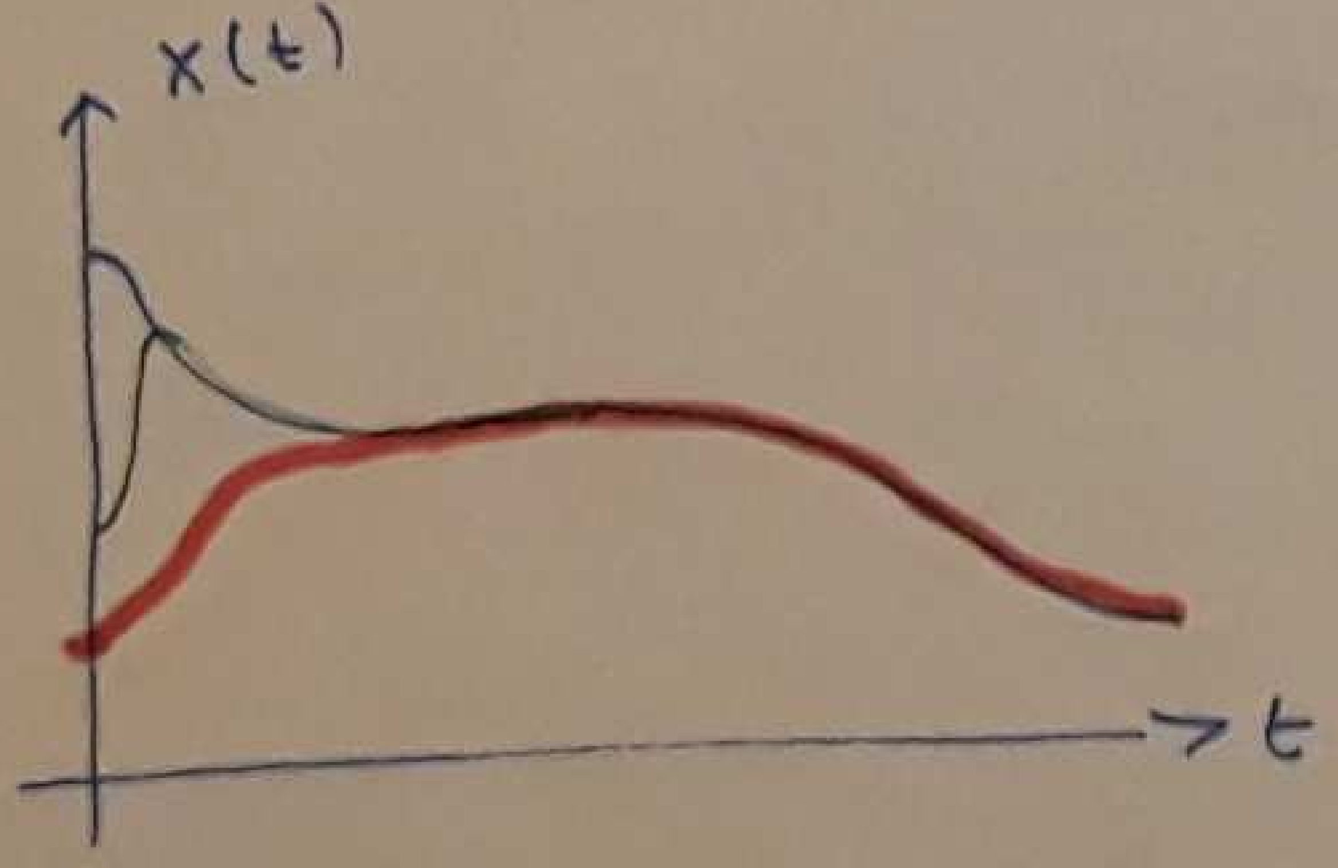
$$\left(\underline{\underline{A}} - \underline{b}\,\underline{k}^T\right)\underline{x} + \underline{b}\,\underline{k}^T\underbrace{\left(\underline{x} - \hat{\underline{x}}\right)}_{\underline{\varepsilon}}$$

$$\dot{\underline{\varepsilon}} = \left(\underline{\underline{A}} - \underline{l}\,\underline{c}^T\right)\underline{\varepsilon}$$

$$\dot{\underline{x}} = \left(\underline{\underline{A}} - \underline{b}\,\underline{k}^T\right)\underline{x} + \underline{b}\,\underline{k}^T\underline{\varepsilon}$$

$$\dot{\underline{\varepsilon}} = \left(\underline{\underline{A}} - \underline{l}\,\underline{c}^T\right)\underline{\varepsilon}$$

$$\Rightarrow \begin{bmatrix}\dot{\underline{x}}\\ \dot{\underline{\varepsilon}}\end{bmatrix} = \left[\begin{array}{c|c}\underline{\underline{A}} - \underline{b}\,\underline{k}^T & \underline{b}\,\underline{k}^T\\ \hline \boxed{\underline{\underline{0}}} & \underline{\underline{A}} - \underline{l}\,\underline{c}^T\end{array}\right]\begin{bmatrix}\underline{x}\\ \underline{\varepsilon}\end{bmatrix}$$



$$\left|\lambda\underline{\underline{E}} - \left(\underline{\underline{A}} - \underline{b}\,\underline{k}^T\right)\right| \cdot \left|\lambda\underline{\underline{E}} - \left(\underline{\underline{A}} - \underline{l}\,\underline{c}^T\right)\right| = 0$$

$$\Downarrow$$

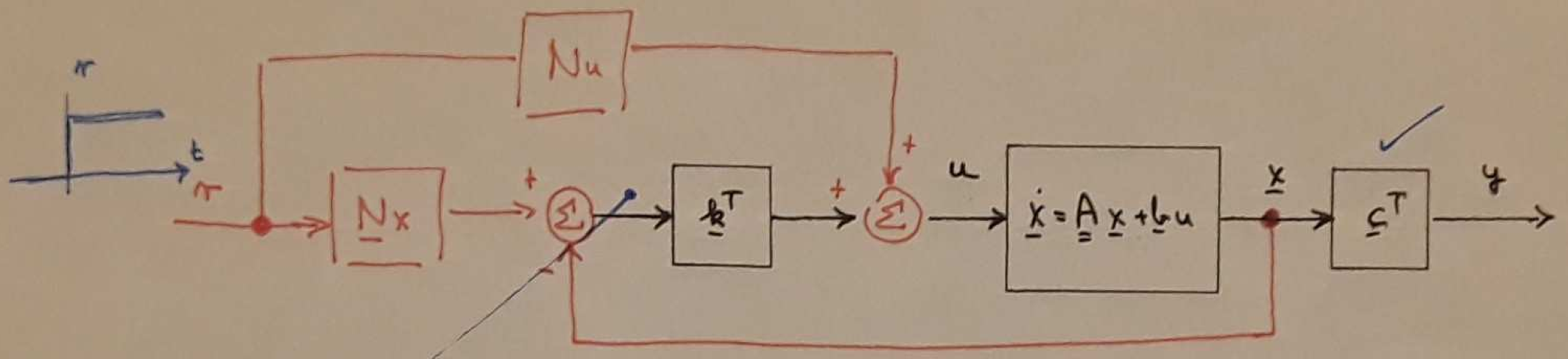
$$\left|\lambda\underline{\underline{E}} - \left(\underline{\underline{A}} - \underline{b}\,\underline{\underset{\uparrow}{k}}^T\right)\right| = 0 \qquad \left|\lambda\underline{\underline{E}} - \left(\underline{\underline{A}} - \underline{l}\,\underline{c}^T\right)\right| = 0$$

Áll. visszacsatolás

megfigyelő

FÜGGETLENEK!

AZ ALAPJEL FIGYELEMBEVÉTELE (1.)



$$\frac{\underline{N}_x}{N_u} = ?$$

STACIONÁRIUS

$$\left.\begin{aligned} \underline{N}_x \cdot r &= \underline{x}_{st} \\ y_{st} &= r \end{aligned}\right\} \qquad y_{st} = \underbrace{r} = \underline{c}^T \underline{x}_{st} = \underbrace{\underline{c}^T \underline{N}_x r} \Rightarrow \boxed{\underline{c}^T \underline{N}_x = 1}$$

▭ ▯ = □

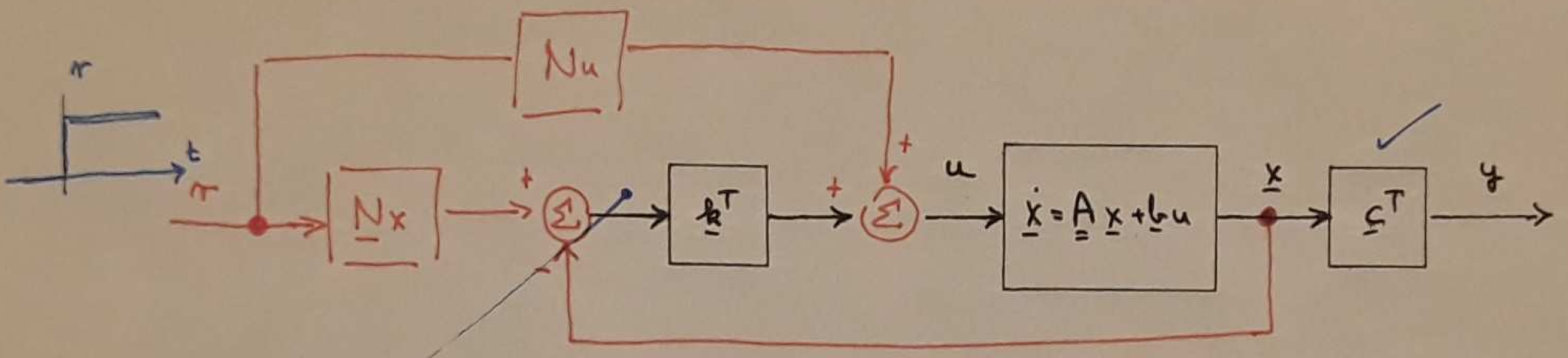
$$u_{st} = N_u r \longrightarrow \underset{\sim}{\dot{\underline{x}}_{st}} = \underline{\underline{A}}\,\underline{x}_{st} + \underline{b}\,u_{st}$$

$$\emptyset = \underline{\underline{A}}\,\underline{N}_x r + \underline{b} N_u r = \underbrace{(\underline{\underline{A}}\,\underline{N}_x + \underline{b} N_u)} r$$

$$\boxed{\underline{\underline{A}}\,\underline{N}_x + \underline{b} N_u = \underline{\emptyset}}$$

$$\begin{bmatrix} \underline{\underline{A}} & \underline{b} \\ \underline{c}^T & 0 \end{bmatrix} \begin{bmatrix} \underline{N}_x \\ N_u \end{bmatrix} = \begin{bmatrix} \underline{0} \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \underline{N}_x \\ N_u \end{bmatrix} = \underbrace{\begin{bmatrix} \underline{\underline{A}} & \underline{b} \\ \underline{c}^T & 0 \end{bmatrix}^{-1}}_{\text{utolsó oszlop}} \begin{bmatrix} \underline{0} \\ 1 \end{bmatrix}$$

AZ ALAPJEL FIGYELEMBEVÉTELE (1.)



$$\frac{\underline{N}_x}{N_u} = ?$$

STACIONÁRIUS

$$\left.\begin{aligned} \underline{N}_x \cdot r &= \underline{x}_{st} \\ y_{st} &= r \end{aligned}\right\}$$

$$y_{st} = \underbrace{r} = \underline{c}^T \underline{x}_{st} = \underbrace{\underline{c}^T \underline{N}_x r} \Rightarrow \boxed{\underline{c}^T \underline{N}_x = 1}$$

▭ ▯ = □

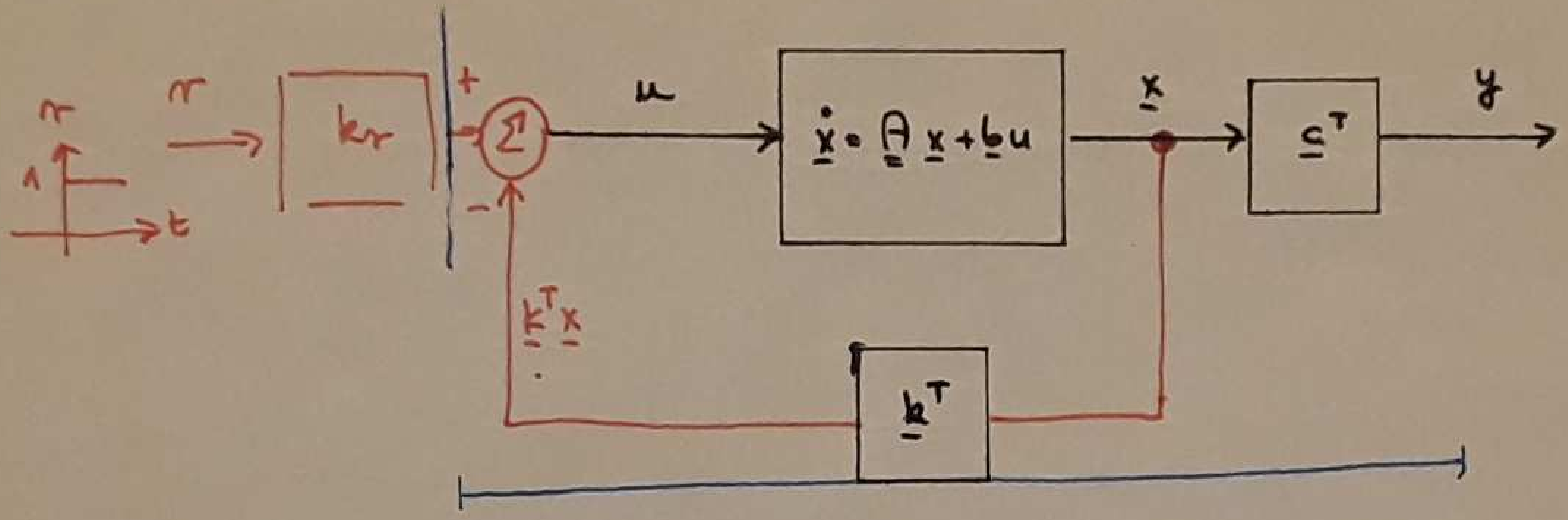
$$u_{st} = N_u r \longrightarrow \underbrace{\dot{\underline{x}}_{st}} = \underline{\underline{A}}\,\underline{x}_{st} + \underline{b}\,u_{st}$$

$$\emptyset = \underline{\underline{A}}\,\underline{N}_x r + \underline{b}\,N_u r = \underbrace{(\underline{\underline{A}}\,\underline{N}_x + \underline{b}\,N_u)}\, r$$

$$\boxed{\underline{\underline{A}}\,\underline{N}_x + \underline{b}\,N_u = \underline{0}}$$

$$\begin{bmatrix} \underline{\underline{A}} & \underline{b} \\ \underline{c}^T & 0 \end{bmatrix} \begin{bmatrix} \underline{N}_x \\ N_u \end{bmatrix} = \begin{bmatrix} \underline{0} \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \underline{N}_x \\ N_u \end{bmatrix} = \underbrace{\begin{bmatrix} \underline{\underline{A}} & \underline{b} \\ \underline{c}^T & 0 \end{bmatrix}^{-1}}_{\text{utolsó oszlop}} \begin{bmatrix} \underline{0} \\ 1 \end{bmatrix}$$

AZ ALAPJEL FIGYELEMBEVÉTELE (2.)



$u = -\underline{k}^T\underline{x} + k_r r$

$$W(s) = \underbrace{\underline{c}^T\left(s\underline{\underline{E}} - [\underline{\underline{A}} - \underline{b}\underline{k}^T\right)^{-1}\underline{b}}_{}\ \underbrace{k_r}_{}$$

$$Y = \frac{1}{s}W(s)$$

STAC.

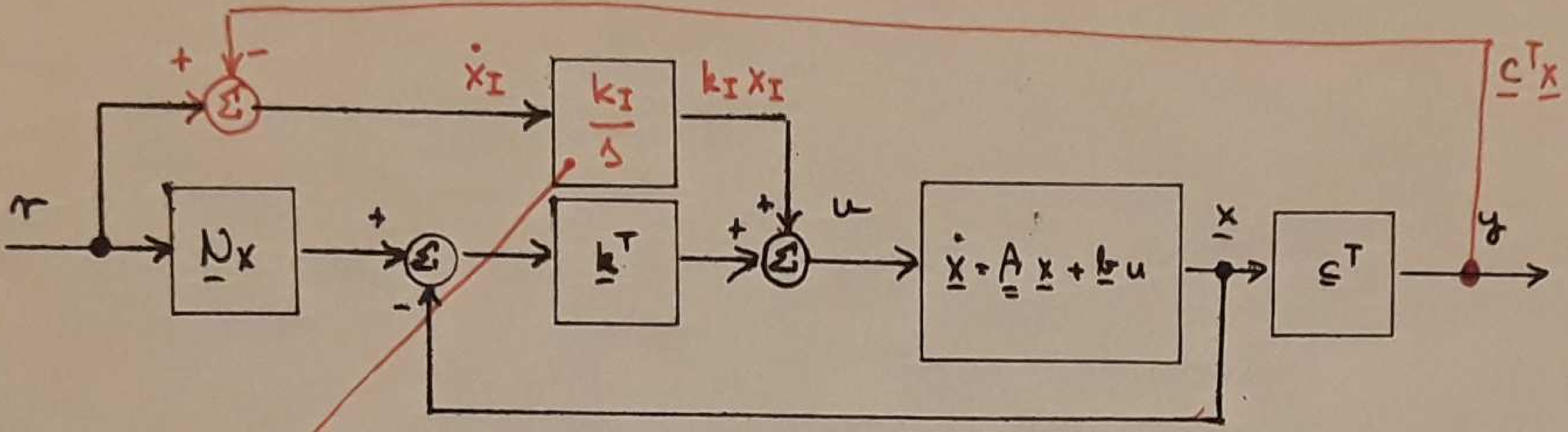
$$y_{st} = \lim_{s\to 0} sY = \lim_{s\to 0} s\frac{1}{s}W(s) = \boxed{\lim_{s\to 0} W(s) = 1}$$

$$\underline{c}^T\left(-\underline{\underline{A}} + \underline{b}\underline{k}^T\right)^{-1}\underline{b}\,k_r = 1$$

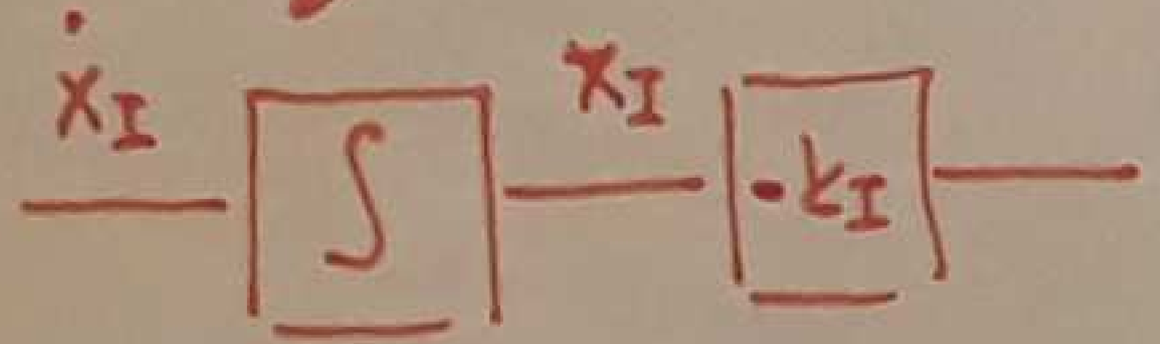
$$\hookrightarrow k_r = \frac{1}{\underline{c}^T\left(-\underline{\underline{A}} + \underline{b}\underline{k}^T\right)^{-1}\underline{b}}$$

$$\boxed{k_r = \frac{-1}{\underline{c}^T\left(\underline{\underline{A}} - \underline{b}\underline{k}^T\right)^{-1}\underline{b}}}$$

INTEGRÁLÓ SZABÁLYOZÁS (1.)



$$\begin{bmatrix} \underline{N}_x \\ N_u \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}} & \underline{b} \\ \underline{c}^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} \underline{0} \\ 1 \end{bmatrix}$$

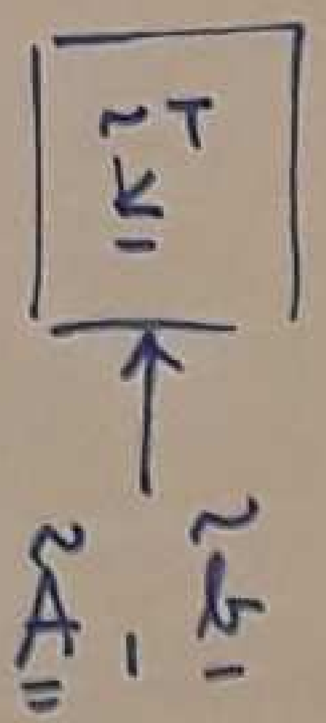


$\dot{x}_I = + \underline{c}^T \underline{x}$

$$\left.\begin{aligned} \begin{bmatrix} \dot{\underline{x}} \\ \dot{x}_I \end{bmatrix} &= \overset{\tilde{\underline{\underline{A}}}}{\begin{bmatrix} \underline{\underline{A}} & \underline{0} \\ \underline{c}^T & 0 \end{bmatrix}} \overset{\tilde{\underline{x}}}{\begin{bmatrix} \underline{x} \\ x_I \end{bmatrix}} + \overset{\tilde{\underline{b}}}{\begin{bmatrix} \underline{b} \\ 0 \end{bmatrix}} u \\ u &= -\overset{\tilde{\underline{k}}^T}{\begin{bmatrix} \underline{k}^T & k_I \end{bmatrix}} \begin{bmatrix} \underline{x} \\ x_I \end{bmatrix} \end{aligned}\right\}$$

$$\dot{\tilde{\underline{x}}} = \tilde{\underline{\underline{A}}}\,\tilde{\underline{x}} + \tilde{\underline{b}}\,u$$

$$u = -\tilde{\underline{k}}^T \tilde{\underline{x}}$$

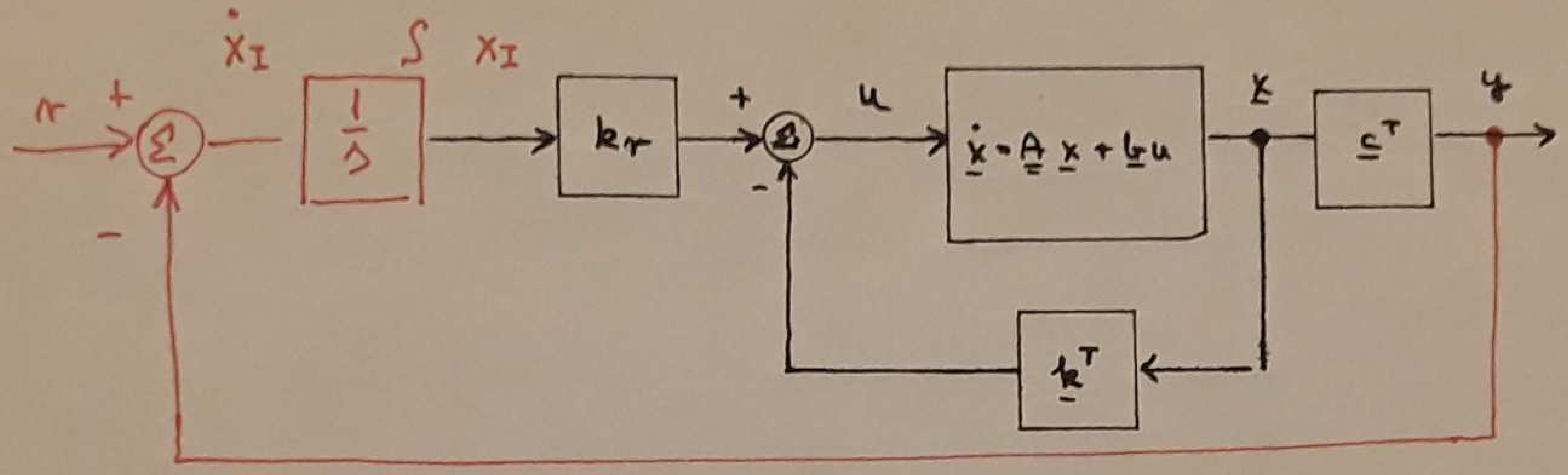


Ackermann - képlet

Bass - Gura - alg.

$\cancel{x}$ $\dot{x}_I = r - y$; $x_I = \int (r - y)\,dt$

INTEGRÁLÓ SZABÁLYOZÁS (2.)



$\dot{x}_I = -\underline{c}^T\underline{x}.$

$\dot{x}_I = r - y$

$x_I = \int (r-y)\,dt$

$$\begin{bmatrix} \dot{\underline{x}} \\ \dot{x}_I \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}} & 0 \\ +\underline{c}^T & 0 \end{bmatrix} \begin{bmatrix} \underline{x} \\ x_I \end{bmatrix} + \begin{bmatrix} \underline{b} \\ 0 \end{bmatrix} u$$

$$u = -\begin{bmatrix} \underline{k}^T & k_r \end{bmatrix} \begin{bmatrix} \underline{x} \\ x_I \end{bmatrix}$$

$$\dot{\tilde{\underline{x}}} = \tilde{\underline{\underline{A}}}\,\tilde{\underline{x}} + \tilde{\underline{b}}\,u$$

$$u = -\tilde{\underline{k}}^T\tilde{\underline{x}}$$

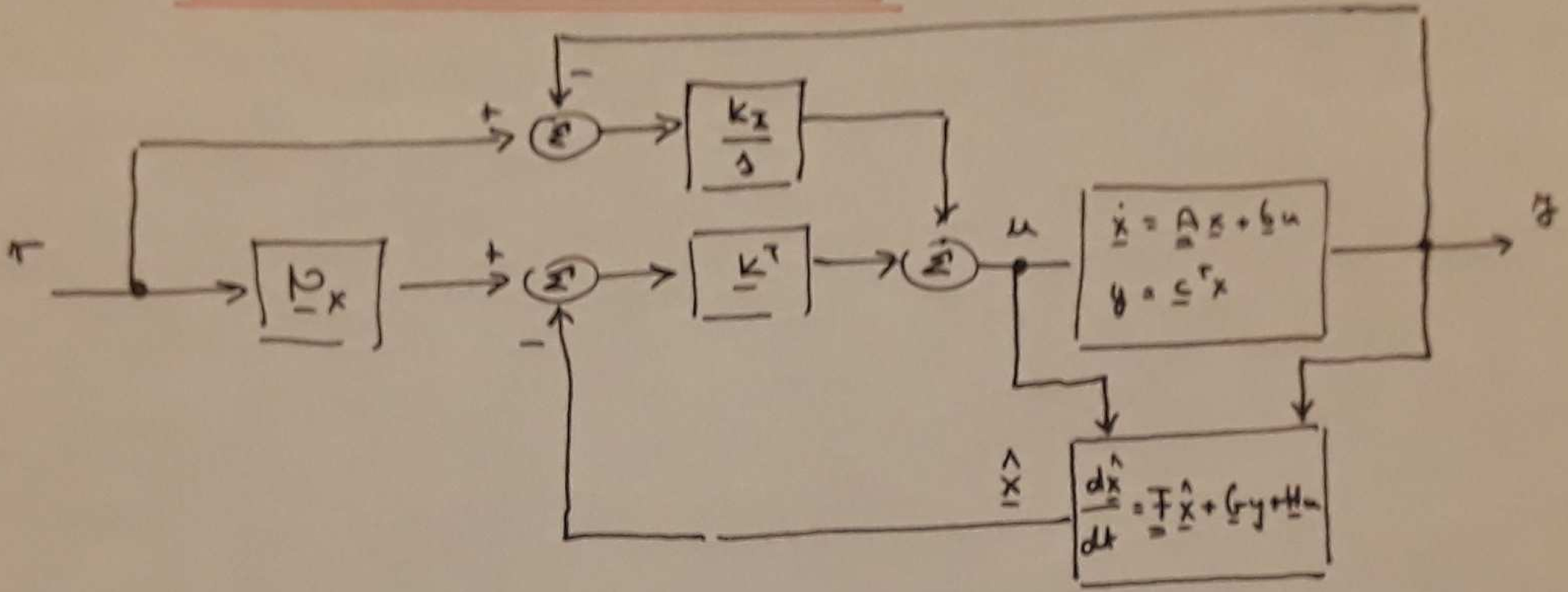
$$\begin{tikzcd} \tilde{\underline{k}}^T \\ \tilde{\underline{\underline{A}}},\ \tilde{\underline{b}} \arrow[u] \end{tikzcd}$$

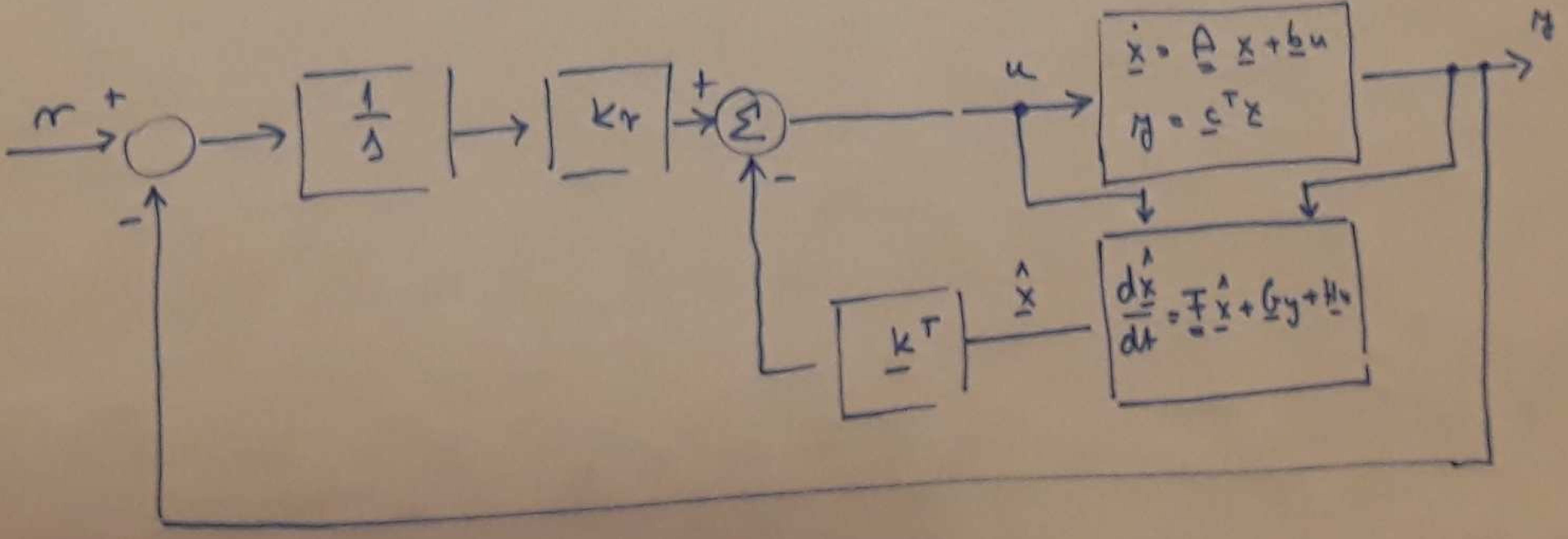
HOVA HELYEZZÜK ÁT A PÓLUSOKAT?

- ahova szeretnénk, de nagy jelek lehetnek → szimuláció
- domináns póluspárral írjuk le a zárt rendszer

  a többit (ha van), jó messze, de nagy jelek lehetnek

ÖSSZEFOGLALÓ BLOKKVÁZLAT





$$\tilde{\underline{k}}^T = [0\ 0 \dots 0\ 1]\,\varphi_{cl}(\tilde{\underline{\underline{A}}})$$

$$\underline{k}^T = \tilde{\underline{k}}^T \underline{\underline{T}} = [0\ 0 \dots 0\ 1]\,\varphi_{cl}(\overset{\underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1}}{\tilde{\underline{\underline{A}}}})\cdot \underline{\underline{T}} = [0\,0 \dots 0\,1]\,\underline{\varphi_{cl}(\underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{T}}^{-1})}\,\underline{\underline{T}} = [0\,0 \dots 0\,1]\,\underline{\underline{T}}\,\varphi_{cl}(\underline{\underline{A}})\underbrace{\underline{\underline{T}}^{-1}\underline{\underline{T}}}_{\underline{\underline{E}}} =$$

$$= [0\ 0 \dots 0\ 1]\,\boxed{\underline{\underline{T}}}\,\varphi_{cl}(\underline{\underline{A}})$$

$$\underline{\underline{M}}_c = [\underline{b}\quad \underline{\underline{A}}\,\underline{b}\quad \underline{\underline{A}}^2\underline{b}\quad \dots\quad \underline{\underline{A}}^{n-1}\underline{b}]$$

$$\tilde{\underline{\underline{M}}}_c = [\tilde{\underline{b}}\quad \tilde{\underline{\underline{A}}}\tilde{\underline{b}}\quad \tilde{\underline{\underline{A}}}^2\tilde{\underline{b}}\quad \dots\quad \tilde{\underline{\underline{A}}}^{n-1}\tilde{\underline{b}}] = [\underline{\underline{T}}\,\underline{b}\quad \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{\underline{T}}^{-1}\underline{\underline{T}}}\,\underline{b}\quad \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{\underline{\underline{T}}^{-1}\underline{\underline{T}}}\,\underline{\underline{A}}\,\underline{\underline{\underline{T}}^{-1}\underline{\underline{T}}}\,\underline{b} \dots]$$

$$= [\underline{\underline{T}}\,\underline{b}\quad \underline{\underline{T}}\,\underline{\underline{A}}\,\underline{b}\quad \underline{\underline{T}}\,\underline{\underline{A}}^2\underline{b}\ \dots] = \underline{\underline{T}}\underbrace{[\underline{b}\quad \underline{\underline{A}}\,\underline{b}\quad \underline{\underline{A}}^2\underline{b} \dots]}_{\underline{\underline{M}}_c} = \underline{\underline{T}}\,\underline{\underline{M}}_c$$

$$\tilde{\underline{\underline{M}}}_c = \underline{\underline{T}}\,\underline{\underline{M}}_c \longrightarrow \boxed{\underline{\underline{T}} = \tilde{\underline{\underline{M}}}_c\,\underline{\underline{M}}_c^{-1}}$$

$$\tilde{\underline{\underline{M}}}_c = [\tilde{\underline{b}}\quad \tilde{\underline{\underline{A}}}\tilde{\underline{b}}\quad \tilde{\underline{\underline{A}}}^2\tilde{\underline{b}}] = \begin{bmatrix} 1 & -a_1 & a_1^2 - a_2 \\ 0 & 1 & -a_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{k}^T = \underbrace{[0\ 0 \dots 0\ 1]\boxed{\tilde{\underline{\underline{M}}}_c}}_{[0\ 0 \dots 0\ 1]}\underline{\underline{M}}_c^{-1}\,\varphi_{cl}(\underline{\underline{A}})$$

$$\boxed{\underline{k}^T = [0\ 0\ 0 \dots 0\ 1]\,\underline{\underline{M}}_c^{-1}\,\varphi_{cl}(\underline{\underline{A}})}$$

AZ ACKERMANN-KÉPLET LEVEZETÉSE

$$\boxed{\begin{array}{l} \underline{\underline{A}} \longrightarrow \underline{\underline{A}} - \underline{b}\,\underline{k}^T \\ \underline{k}^T = [0\ 0 \dots 0\ 1]\ \underline{\underline{M}}_c^{-1}\ \varphi_{cl}(\underline{\underline{A}}) \end{array}}$$

$$\boxed{\underline{k}^T = \underline{\tilde{k}}^T\ \underline{\underline{T}}}$$

$$\tilde{k}_i = p_i - a_i$$

$$\boxed{\underline{\tilde{k}}^T} = [\,(p_1 - a_1)\ \ (p_2 - a_2)\ \dots\ (p_n - a_n)\,]$$

Cayley-Hamilton-tétel:

$$\varphi(\lambda) = \lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \dots + a_n = 0$$

$$\varphi(\underline{\underline{\tilde{A}}}) = \boxed{\underline{\underline{\tilde{A}}}^n} + a_1 \underline{\underline{\tilde{A}}}^{n-1} + a_2 \underline{\underline{\tilde{A}}}^{n-2} + \dots + a_n \underline{\underline{E}} = 0$$

$$\hookrightarrow \underline{\underline{\tilde{A}}}^n = -a_1 \underline{\underline{\tilde{A}}}^{n-1} - a_2 \underline{\underline{\tilde{A}}}^{n-2} - \dots - a_n \underline{\underline{E}}$$

$$\varphi_{cl}(\lambda) = \lambda^n + p_1 \lambda^{n-1} + p_2 \lambda^{n-2} + \dots + p_n = 0$$

$$\varphi_{cl}(\underline{\underline{\tilde{A}}}) = \boxed{\underline{\underline{\tilde{A}}}^n} + p_1 \underline{\underline{\tilde{A}}}^{n-1} + p_2 \underline{\underline{\tilde{A}}}^{n-2} + \dots + p_n \underline{\underline{E}}$$

$$\boxed{\varphi_{cl}(\underline{\underline{\tilde{A}}})} = \underbrace{(p_1 - a_1)}_{[1\ 0\ 0]} \underline{\underline{\tilde{A}}}^{n-1} + \underbrace{(p_2 - a_2)}_{[0\ 1\ 0]} \underline{\underline{\tilde{A}}}^{n-2} + \dots \underbrace{(p_n - a_n)}_{[0\ 0\ 1]} \underline{\underline{E}}$$

$$\underline{\underline{E}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \boxed{0 \quad 0 \quad 1} \end{bmatrix} \qquad \underline{\underline{\tilde{A}}} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ \boxed{0 \quad 1 \quad 0} \end{bmatrix}$$

$$\underline{\underline{\tilde{A}}}^2 = \underline{\underline{\tilde{A}}}\,\underline{\underline{\tilde{A}}} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} a_1^2 - a_2 & a_1 a_2 - a_3 & a_1 a_3 \\ -a_1 & -a_2 & -a_3 \\ \boxed{1 \quad 0 \quad 0} \end{bmatrix}$$

$$\underline{\tilde{k}}^T = [0\ 0 \dots 0\ \boxed{1}]\ \boxed{\varphi_{cl}(\underline{\underline{\tilde{A}}})} = [\,(p_1 - a_1)\ \ (p_2 - a_2)\ \dots\ (p_n - a_n)\,]$$

LINEÁRIS KVADRATIKUS IRÁNYÍTÁS : LQR

(bevezetés)

Lagrange-függvény.

$$J(\underline{x}, \underline{u}) = \frac{1}{2}\int_0^T \left[\underline{x}^T \underline{\underline{Q}}\, \underline{x} + \underline{u}^T \underline{\underline{R}}\, \underline{u}\right] dt \longrightarrow \min$$

$\underline{x}^T \underline{x} = \underline{0}$

$\underline{u} \to \min.$

$$\underline{\dot{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$$

MIMO

$$\underline{y} = \underline{\underline{C}}\,\underline{x}$$

$\underline{\underline{Q}}^T = \underline{\underline{Q}}$ $\underline{\underline{R}}^T = \underline{\underline{R}}$ $\left(\frac{1}{2} Q x^2\right)' = Qx$

$\underline{\underline{Q}} \geq \underline{\underline{0}}$ $\underline{\underline{R}} > \underline{\underline{0}}$

pozitív semidefinit pozitív definit

$$\boxed{\underline{\Phi} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} - \underline{\dot{x}} = \underline{0}}$$

Állapotvisszacsatolás:

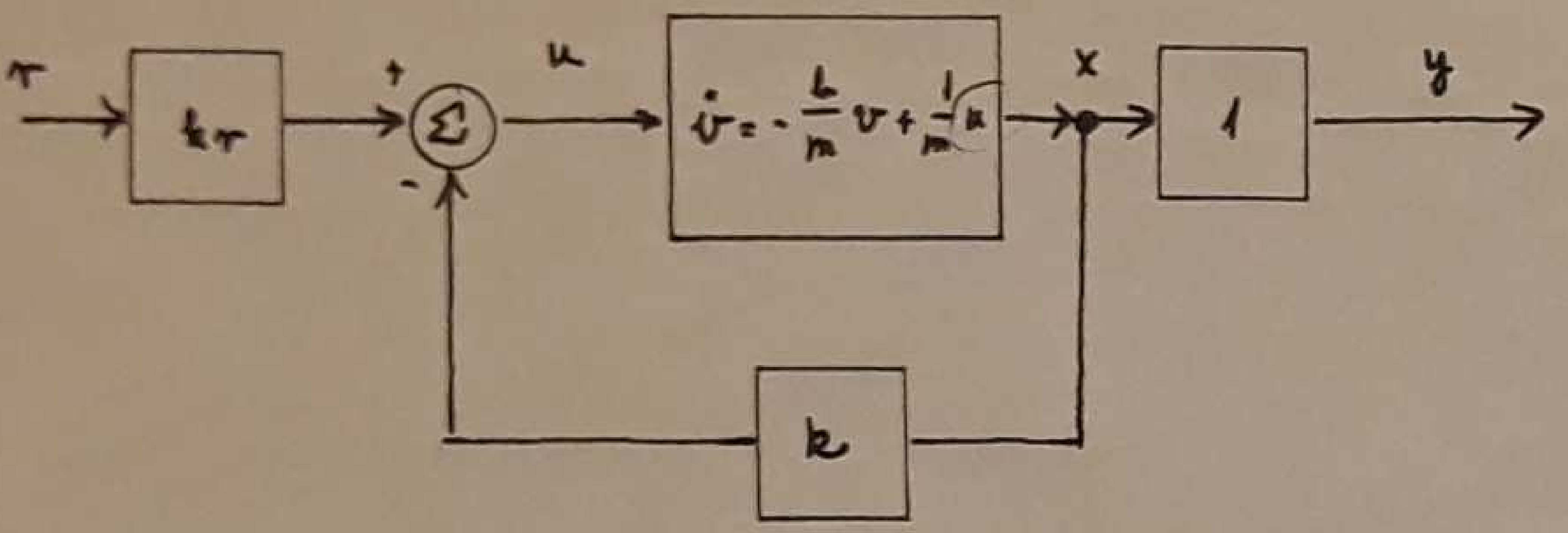
$$\underline{u} = -\underbrace{\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}}}_{\underline{\underline{K}}}\,\underline{x} = -\underline{\underline{K}}\,\underline{x}$$

$$\underline{\underline{P}}\,\underline{\underline{A}} + \underline{\underline{A}}^T\underline{\underline{P}} - \underline{\underline{P}}\,\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}} + \underline{\underline{Q}} = \underline{\underline{0}}$$

CARE : Control Algebraic Riccati Equatn

$\underline{\underline{P}} > \underline{\underline{0}}$ $\underline{\underline{P}}^T = \underline{\underline{P}}$

Sebességszabályozás állapot-visszacsatolással.



$m = 1000\,kg$

$b = 50\,\frac{Ns}{m}$

$r = 10\,\frac{m}{s}$

$$\dot{v} = \underbrace{-0{,}05}_{\downarrow\, -1{,}5}\,v + 0{,}001\,u$$

$$\underline{k}^T = [0\;0\;\ldots\;0\;1]\,\underline{\underline{M}}_c^{-1}\,\varphi_{cl}(\underline{\underline{A}})$$

$$\underline{\underline{M}}_c \Rightarrow \underline{M_c = b = 0{,}001}$$

$$\varphi_{cl}(A) = \underline{\underline{\lambda + 1{,}5}} \longrightarrow A + 1{,}5 = -0{,}05 + 1{,}5 = \underline{\underline{1{,}45}}$$

$$k = 1 \cdot M_c^{-1}\,\varphi_{cl}(A)$$

$$k = 1 \cdot \frac{1}{0{,}001} \cdot 1{,}45 = \underline{\underline{1450}}$$

$$k_r = \frac{-1}{\underline{c}^T(\underline{\underline{A}} - \underline{b}\,\underline{k}^T)^{-1}\underline{b}} = \frac{-1}{1 \cdot \underbrace{(-0{,}05 - 0{,}001 \cdot 1450)^{-1}}_{-1{,}5^{-1}}\,0{,}001} = \underline{\underline{1500}}$$

$$u = k_r r - k v$$

$$\dot{v} = -0{,}05\,v + 0{,}001\,(k_r \cdot r - 1450\,v) = \underline{-0{,}05\,v} + 0{,}001\,k_r \cdot r\,\underline{-1{,}45\,v} = \boxed{-1{,}5\,v} + 0{,}001\,\underline{k_r} \cdot r$$

$$0 = -1{,}5\,\underbrace{v_{st}}_{1} + 0{,}001 \cdot k_r \cdot \underbrace{r_{st}}_{1} \Rightarrow -1{,}5 + 0{,}001\,k_r = 0$$

$$1{,}5 = 0{,}001\,k_r \longrightarrow \underline{\underline{k_r = 1500}}$$

LQ - IRÁNYITÁS - LEVEZETÉS

$\underline{x}$ - áll. változó

$\underline{\lambda}$ - társ. áll. változó

$\underline{\lambda} = \underline{\lambda}(t)$

$$\left.\begin{aligned} L(\underline{x},\underline{u},t) &= \frac{1}{2}\left[\underline{x}^T\underline{\underline{Q}}\,\underline{x} + \underline{u}^T\underline{\underline{R}}\,\underline{u}\right] \\ \underline{\Phi} &= \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} - \dot{\underline{x}} \end{aligned}\right\}$$

▭ ▯ - ⓪

$$\tilde{L}(\overset{\downarrow}{\underline{x}},\dot{\underline{x}},\overset{\downarrow}{\underline{u}},\overset{\downarrow}{\underline{\lambda}},t) = L(\underline{x},\underline{u},t) + \underline{\lambda}^T\underline{\Phi}(\underline{x},\dot{\underline{x}},\underline{u})$$

$$\frac{1}{2}\underline{x}^T\underline{\underline{Q}}\,\underline{x} + \frac{1}{2}\underline{u}^T\underline{\underline{R}}\,\underline{u} + \underline{\lambda}^T\left[\underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} - \dot{\underline{x}}\right] =$$

$$\frac{1}{2}\underline{x}^T\underline{\underline{Q}}\,\underline{x} + \frac{1}{2}\underline{u}^T\underline{\underline{R}}\,\underline{u} + \left[\underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} - \dot{\underline{x}}\right]^T\underline{\lambda} =$$

$$\frac{1}{2}\underline{\underline{x}^T\underline{\underline{Q}}\,\underline{x}} + \frac{1}{2}\underline{u}^T\underline{\underline{R}}\,\underline{u} + \underline{\underline{x}^T\underline{\underline{A}}^T\underline{\lambda}} + \underline{u}^T\underline{\underline{B}}^T\underline{\lambda} - \dot{\underline{x}}^T\underline{\lambda}$$

SZABÁLYOK:

$$\frac{\partial}{\partial\underline{x}}\,\underline{x}^T\underline{\underline{Q}}\,\underline{x} = 2\underline{\underline{Q}}\,\underline{x}$$

$$\frac{\partial}{\partial\underline{u}}\,\underline{u}^T\underline{\underline{R}}\,\underline{u} = 2\underline{\underline{R}}\,\underline{u}$$

$$\frac{\partial}{\partial\underline{x}}\,\underline{x}^T\underline{\underline{A}}^T\underline{\lambda} = \underline{\underline{A}}^T\underline{\lambda}$$

$$\frac{\partial}{\partial\underline{u}}\,\underline{u}^T\underline{\underline{B}}^T\underline{\lambda} = \underline{\underline{B}}^T\underline{\lambda}$$

$$\frac{\partial}{\partial\dot{\underline{x}}}\,\dot{\underline{x}}^T\underline{\lambda} = \underline{\lambda}$$

$$\frac{d}{dt}\frac{\partial\tilde{L}}{\partial\dot{\underline{x}}} - \frac{\partial\tilde{L}}{\partial\underline{x}} = \underline{0} \quad\rightarrow$$

$$\cancel{\frac{d}{dt}\frac{\partial\tilde{L}}{\partial\dot{\underline{u}}}} - \frac{\partial\tilde{L}}{\partial\underline{u}} = \underline{0} \quad\rightarrow$$

$$\cancel{\frac{d}{dt}\frac{\partial\tilde{L}}{\partial\dot{\underline{\lambda}}}} - \frac{\partial\tilde{L}}{\partial\underline{\lambda}} = \underline{0} \quad\rightarrow$$

$$\boxed{\begin{aligned} -\dot{\underline{\lambda}} - \underline{\underline{Q}}\,\underline{x} - \underline{\underline{A}}^T\underline{\lambda} &= \underline{0} \\ -\underline{\underline{R}}\,\underline{u} - \underline{\underline{B}}^T\underline{\lambda} &= \underline{0} \end{aligned}}$$

$$\left.\begin{aligned} -\left(\underline{x}^T\underline{\underline{A}}^T + \underline{u}^T\underline{\underline{B}}^T - \dot{\underline{x}}^T\right)^T &= \underline{0}^T \\ \dot{\underline{x}} &= \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} \end{aligned}\right\} ✓$$

$$\left\{\begin{aligned} \underline{\underline{Q}}\,\underline{x} + \underline{\underline{A}}^T\underline{\lambda} + \dot{\underline{\lambda}} &= \underline{0} \\ \underline{\underline{R}}\,\underline{u} + \underline{\underline{B}}^T\underline{\lambda} &= \underline{0} \end{aligned}\right. \longrightarrow \boxed{\underline{u} = -\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\lambda}}$$

$$\underline{\lambda} = \overset{?}{\underline{\underline{P}}}\,\underline{x} \qquad \underline{\lambda}(t) = \underline{\underline{P}}(t)\,\underline{x}(t)$$

$$\underline{u} = -\underbrace{\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}}}_{\underline{\underline{K}}}\,\underline{x}$$

MIMO

$$\boxed{\underline{u} = -\underline{\underline{K}}\,\underline{x}}$$

SISO

$$\cancel{\underline{\underline{K}}} \qquad u = -\underset{\uparrow\;\frac{1}{r}\underline{b}^T\underline{\underline{P}}}{\underline{k}^T}\,\underline{x}$$

$\dot{\underline{\lambda}} = -\underline{\underline{Q}}\,\underline{x} - \underline{\underline{A}}^T\underline{\lambda}$ $\boxed{\underline{\lambda} = \underline{\underline{P}}\,\underline{x}}$ $\underline{\lambda}(t) = \underline{\underline{P}}(t)\,\underline{x}(t)$ $\underline{u} = -\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underset{\overset{\uparrow}{\underline{\underline{P}}\,\underline{x}}}{\underline{\lambda}}$

$$\dot{\underline{\lambda}} = (\underline{\underline{P}}\,\underline{x})' = \dot{\underline{\underline{P}}}\,\underline{x} + \underline{\underline{P}}\,\dot{\underline{x}}$$

$$\dot{\underline{\underline{P}}}\,\underline{x} + \underline{\underline{P}}\,\underset{\downarrow}{\dot{\underline{x}}} = -\underline{\underline{Q}}\,\underline{x} - \underline{\underline{A}}^T\underline{\underline{P}}\,\underline{x}$$

$$\dot{\underline{\underline{P}}}\,\underline{x} + \underline{\underline{P}}\left[\underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underset{\downarrow}{\underline{u}}\right] + \underline{\underline{Q}}\,\underline{x} + \underline{\underline{A}}^T\underline{\underline{P}}\,\underline{x} = \underline{\underline{0}}$$

$$\overset{\checkmark}{\dot{\underline{\underline{P}}}}\,\underline{x} + \underline{\underline{P}}\,\overset{\checkmark}{\underline{\underline{A}}}\,\underline{x} - \underline{\underline{P}}\,\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\overset{\checkmark}{\underline{\underline{B}}^T\underline{\underline{P}}}\,\underline{x} + \underline{\underline{Q}}\,\underline{x} + \overset{\checkmark}{\underline{\underline{A}}^T\underline{\underline{P}}}\,\underline{x} = \underline{\underline{0}}$$

$$\underbrace{\left(\dot{\underline{\underline{P}}} + \underline{\underline{P}}\,\underline{\underline{A}} + \underline{\underline{A}}^T\underline{\underline{P}} - \underline{\underline{P}}\,\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}} + \underline{\underline{Q}}\right)}\,\underline{\underline{x}} = \underline{0}$$

$$\boxed{\dot{\underline{\underline{P}}} + \underline{\underline{P}}\,\underline{\underline{A}} + \underline{\underline{A}}^T\underline{\underline{P}} - \underline{\underline{P}}\,\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}} + \underline{\underline{Q}} = \underline{\underline{0}}}$$

mátrix Riccati - differenciál-
egyenlet $\rightarrow \underline{\underline{P}}(t)$

$\underline{\lambda} = \underline{\underline{P}}\,\underline{x}$

$\underline{\underline{P}}(t) \rightarrow \underline{\underline{P}}$

$T \rightarrow \infty$ $\lim\limits_{t\to\infty} \underline{\underline{P}}(t) = \underline{\underline{P}}$ Konstans! $\dot{\underline{\underline{P}}} = \underline{\underline{0}}$

$$\boxed{\underline{\underline{P}}\,\underline{\underline{A}} + \underline{\underline{A}}^T\underline{\underline{P}} - \underline{\underline{P}}\,\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}} + \underline{\underline{Q}} = \underline{\underline{0}}}$$

CARE

Control
Algebraic
Riccati
Equation. $\longrightarrow \underline{\underline{P}}$

$\underline{\underline{P}}^T = \underline{\underline{P}}$

$\underline{\underline{P}} > \underline{\underline{0}}$

$\underline{\underline{K}} = -\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}}$

$\boxed{\underline{u} = -\underline{\underline{K}}\,\underline{x}}$

Tervezzünk LQ- optimális irányítójelet!

$\dot{x} = \underline{-2{,}5}x + u$ $\quad Q = 2{,}75$

$y = \underline{2{,}5}x$ $\quad R = 1$

$$\underline{\underline{P}} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

---

① $\underline{\underline{P}}\,\underline{\underline{A}} + \underline{\underline{A}}^T\underline{\underline{P}} - \underline{\underline{P}}\,\underline{\underline{B}}\,\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}} + \underline{\underline{Q}} = \underline{\underline{0}}$

$-2{,}5P - 2{,}5P - \underbrace{P\cdot 1\cdot 1\cdot 1\cdot P} + 2{,}75 = 0$

$p^2 + 5p - 2{,}75 = 0$ $\quad p_{1,2} = \dfrac{-5 \pm \sqrt{25 + 4\cdot 2{,}75}}{2}$

$\boxed{p_1 = 0{,}5} > 0$

$\cancel{p_2 = -5{,}5}$

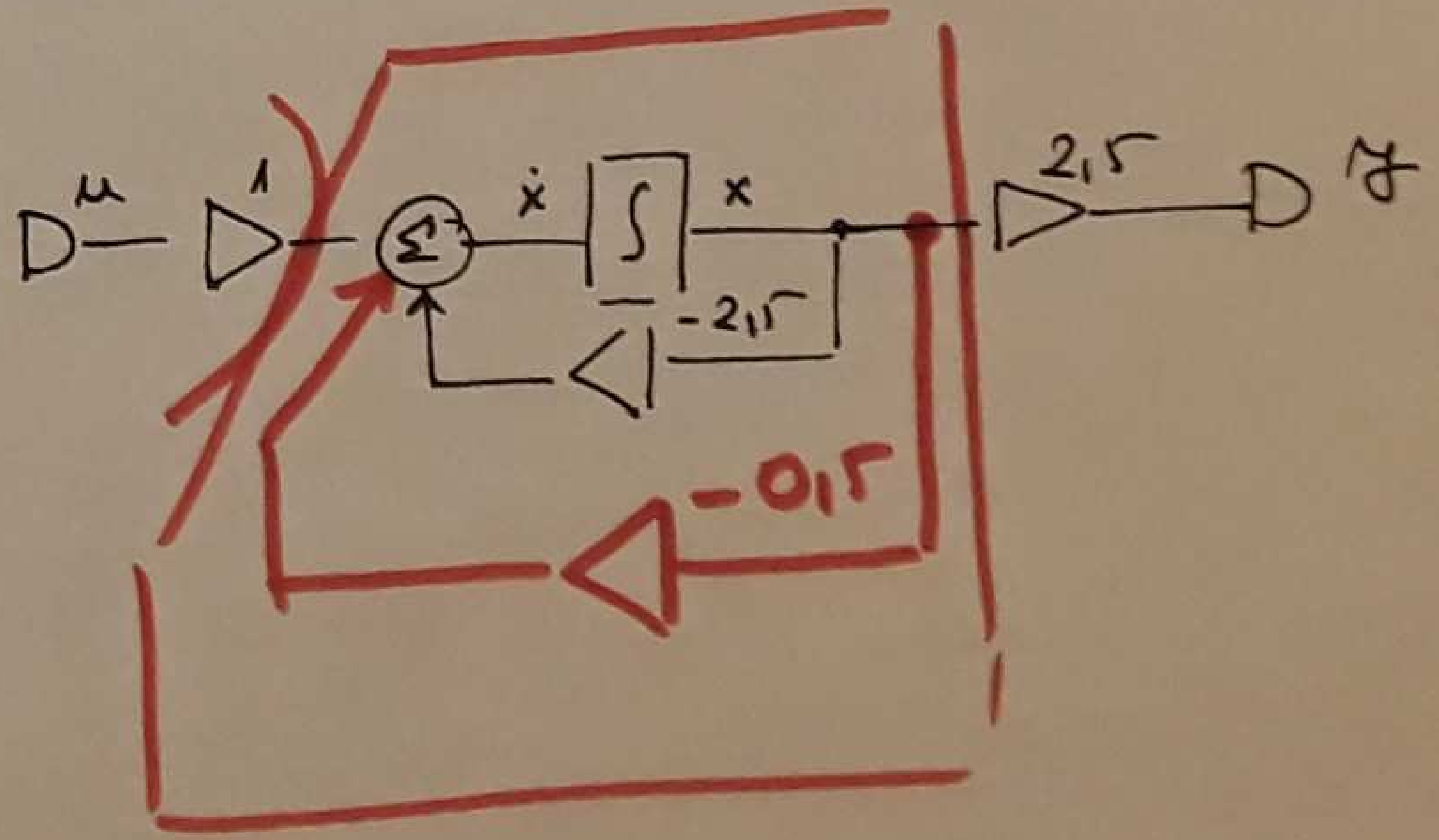
$\underline{\underline{P}}^T = \underline{\underline{P}}$

$\underline{\underline{P}} > \underline{\underline{0}}$

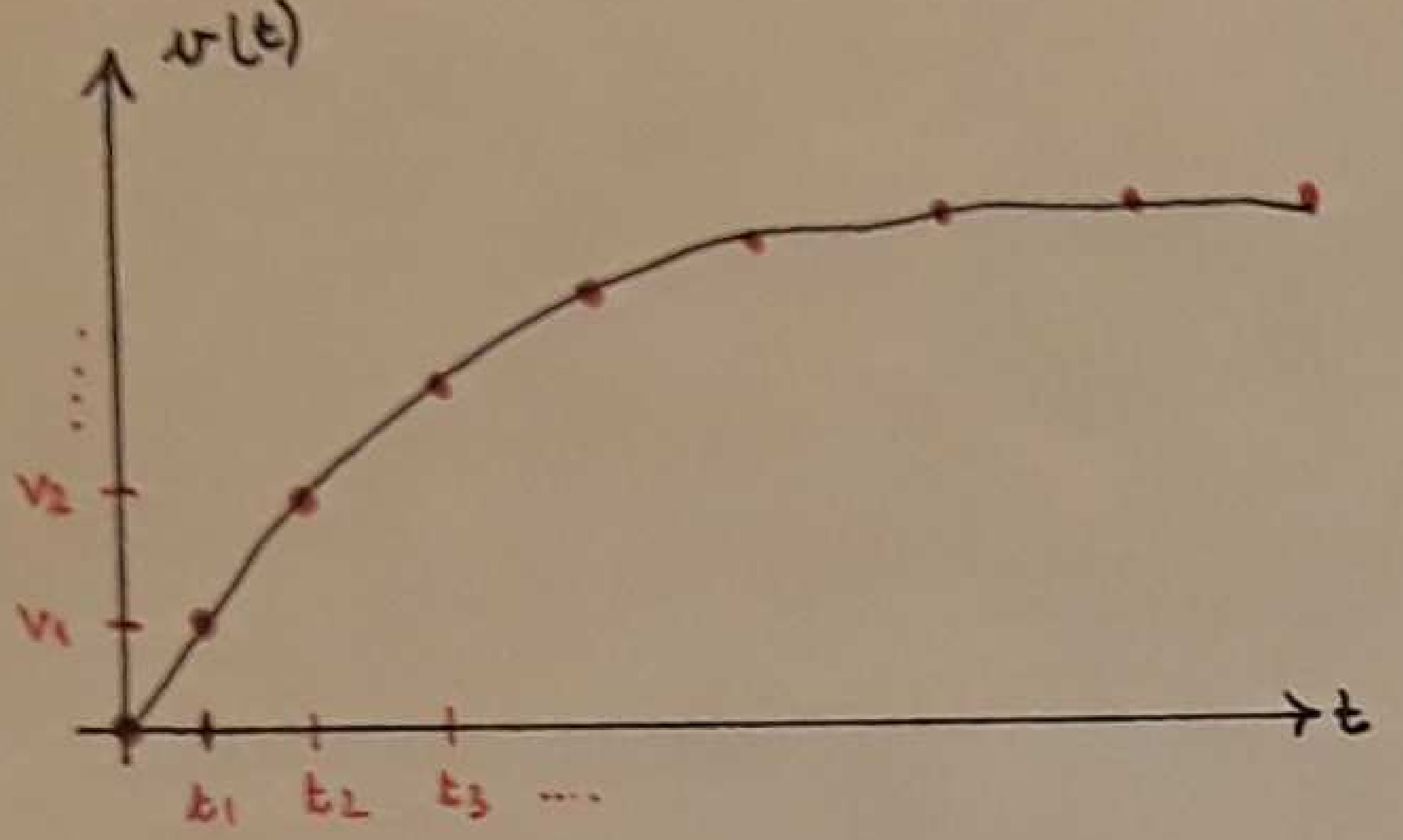
② $\underline{u} = -\underline{\underline{R}}^{-1}\underline{\underline{B}}^T\underline{\underline{P}}\,\underline{x}$ $\quad u = -1\cdot 1\cdot 0{,}5x = -0{,}5x$

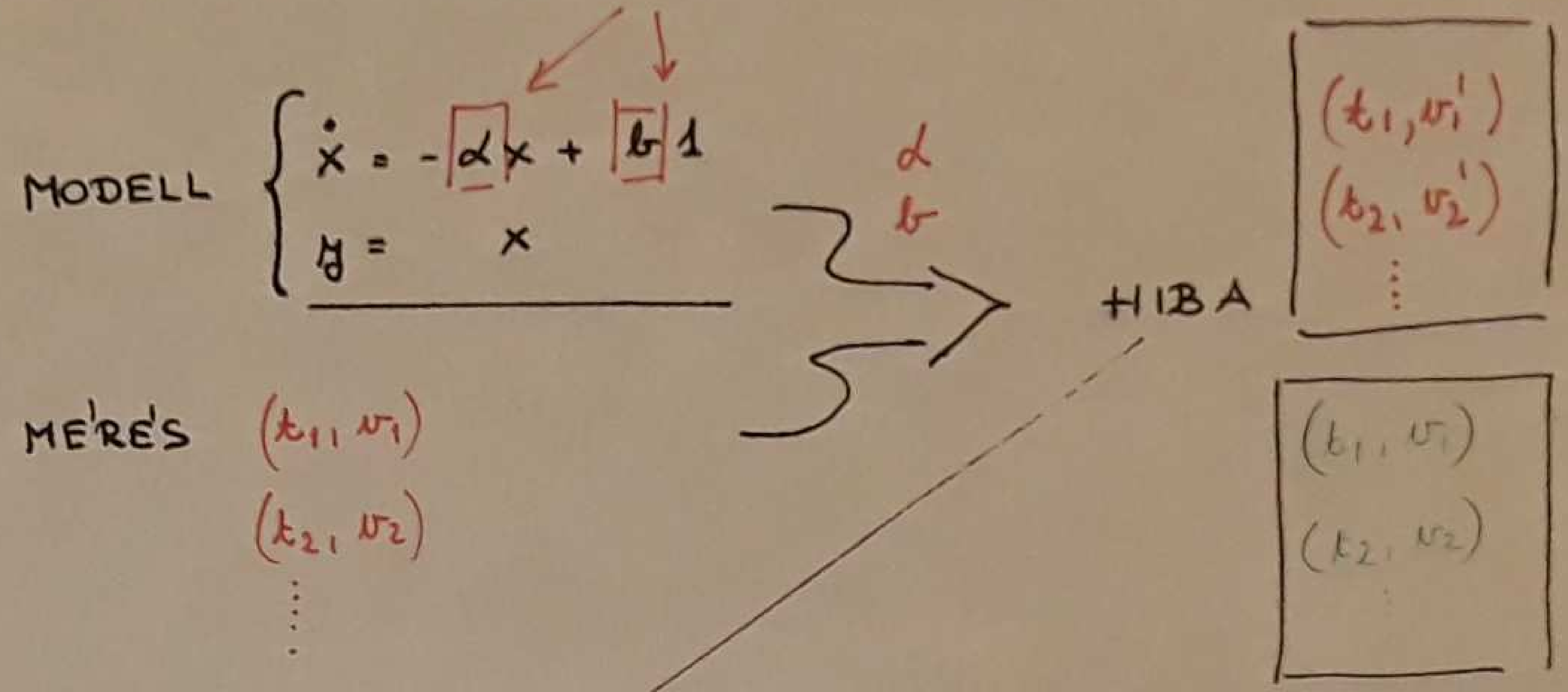
③ $\cancel{x}$ $\quad \dot{x} = -2{,}5x - 0{,}5x = -3x + u$

$y = 2{,}5x$



EGYSZERŰ IDENTIFIKÁCIÓ 1.





MODELL $\begin{cases} \dot{x} = -\boxed{\alpha}x + \boxed{b}1 \\ y = x \end{cases}$

α
b

HIBA

(t_1, v_1')
(t_2, v_2')
$\vdots$

MÉRÉS (t_1, v_1)
(t_2, v_2)
$\vdots$

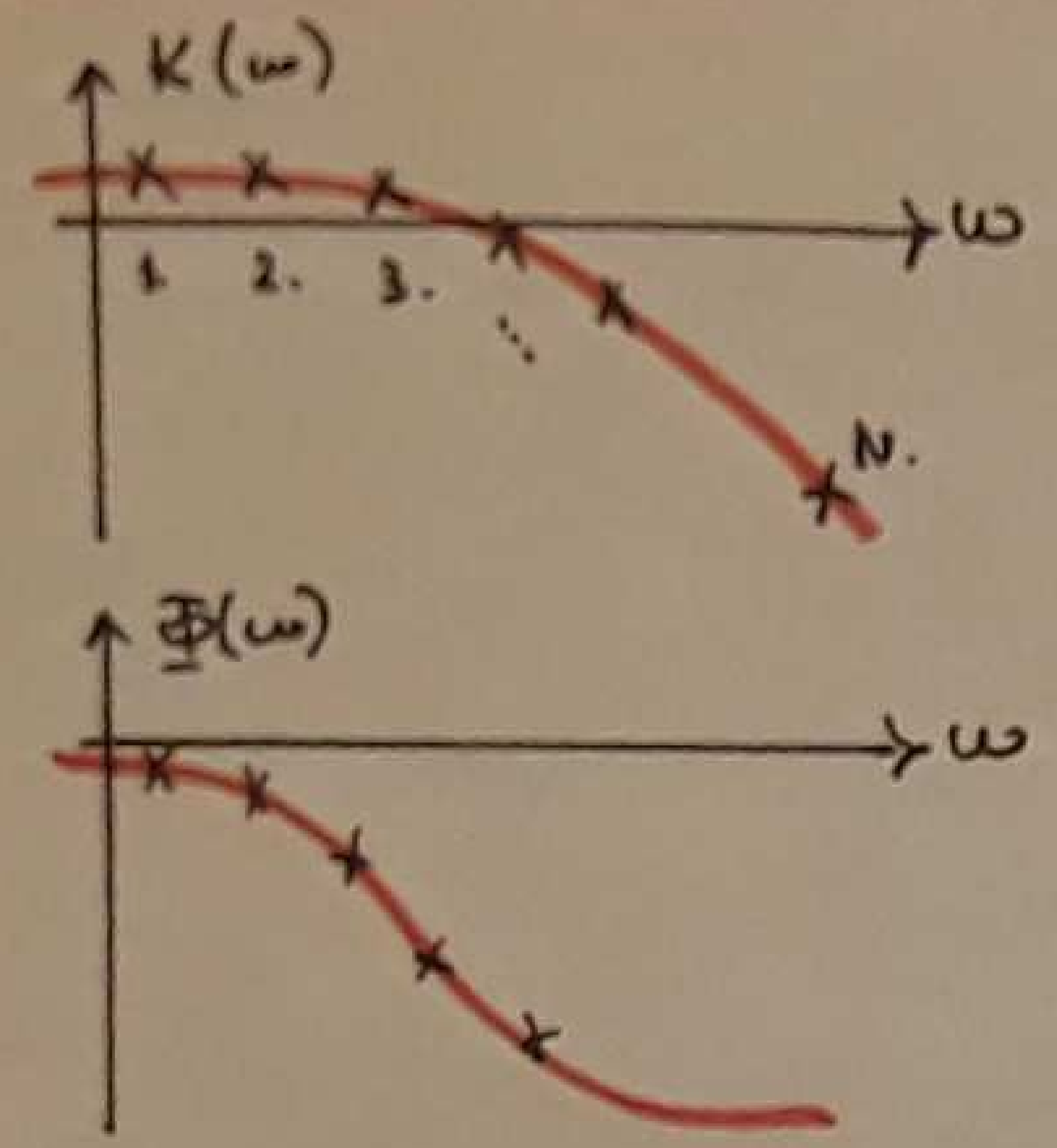
(t_1, v_1)
(t_2, v_2)

$$\text{HIBA} = \frac{1}{N}\sqrt{(v_1' - v_1)^2 + (v_2' - v_2)^2 + \ldots\ldots}$$

$$\Downarrow$$

α, b

EGYSZERŰ IDENTIFIKÁCIÓ II.

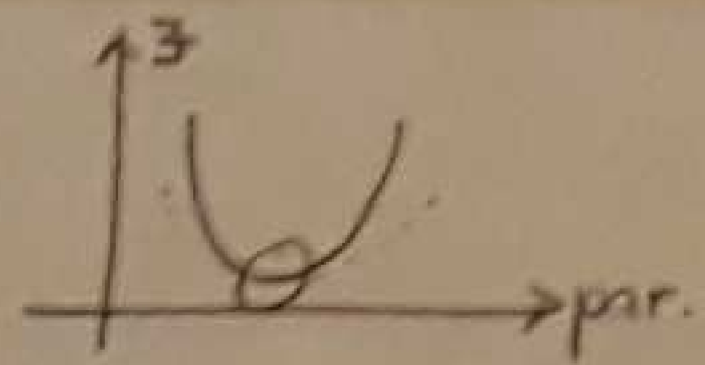


Mérés: $\hat{W}(j\omega_k)$ $k=1\ldots N$

Modell: $W(j\omega) = \dfrac{B(j\omega)}{A(j\omega)}$

$$J = \frac{1}{2}\sum_{k=1}^{N}\left[W(j\omega_k) - \hat{W}(j\omega_k)\right]^2 \rightarrow \text{min.}$$

$$\frac{B(j\omega_k)}{A(j\omega_k)}$$



$$W = \hat{W}$$

$$\frac{B}{A} = \hat{W}$$

$$B = A\hat{W}$$

$$\boxed{B - A\hat{W} = 0}$$

$$J = \frac{1}{2}\sum_{k=1}^{N}\left[B(j\omega_k) - A(j\omega_k)\cdot\hat{W}(j\omega_k)\right]^2 \rightarrow \text{min.}$$

$$\frac{B}{A} = \frac{b_1 j\omega + b_2}{(j\omega)^2 + a_1(j\omega) + a_2}$$

$$\boxed{b_1 \quad b_2 \quad a_1 \quad a_2}\ ?$$

$$J(b_1, b_2, a_1, a_2)$$

$$J(b_1, b_2, a_1, a_2) = \frac{1}{2}\sum_{k=1}^{N}\left[\boxed{(b_1 j\omega_k + b_2) - \left((j\omega_k)^2 + a_1 j\omega_k + a_2\right)\hat{W}(j\omega_k)}\right]^2 \longrightarrow \text{min}$$

$$\rightarrow b_1 j\omega_k + b_2 - (j\omega_k)^2\hat{W} - a_1 j\omega_k\hat{W} - a_2\hat{W} = 0$$

$$\frac{\partial J}{\partial b_1} = \boxed{\sum_k [\quad]}\, j\omega_k = 0$$

$$\frac{\partial J}{\partial b_2} = \boxed{\sum_k [\quad]}\, 1 = 0$$

$$\frac{\partial J}{\partial a_1} = \boxed{\sum_k [\quad]}\, j\omega_k\hat{W} = 0$$

$$\frac{\partial J}{\partial a_2} = \boxed{\sum_k [\quad]}\, \hat{W} = 0$$

$$\begin{array}{c|cccc}
 & b_1 & b_2 & a_1 & a_2 \\
\hline
1. & j\omega_k & 1 & -a_1 j\omega_k\hat{W} & -\hat{W} \\
2. & & & & \\
3. & & & & \\
\vdots & & & N\times 4 & \\
N. & & & &
\end{array}
\begin{bmatrix} b_1 \\ b_2 \\ a_1 \\ a_2 \end{bmatrix}
=
\begin{bmatrix} (j\omega_k)^2\hat{W} \\ \\ N\times 1 \\ \\ \end{bmatrix}$$

$$\underline{\underline{A}} \qquad \underline{p} \qquad \underline{b}$$

$$\boxed{\underline{\underline{A}}\,\underline{p} = \underline{b}}$$

Lineáris paraméter-becslés

$$\underbrace{\underset{4\times N}{\underline{\underline{A}}^T}\ \underset{N\times 4}{\underline{\underline{A}}}}_{4\times 4}\ \overset{4\times 1}{\underline{p}} = \underbrace{\underset{4\times N}{\underline{\underline{A}}^T}\ \underset{N\times 1}{\underline{b}}}_{4\times 1}$$

$$\boxed{\underline{p} = \left(\underline{\underline{A}}^T\underline{\underline{A}}\right)^{-1}\underline{\underline{A}}^T\underline{b}}$$