VEKTORANALÍZIS

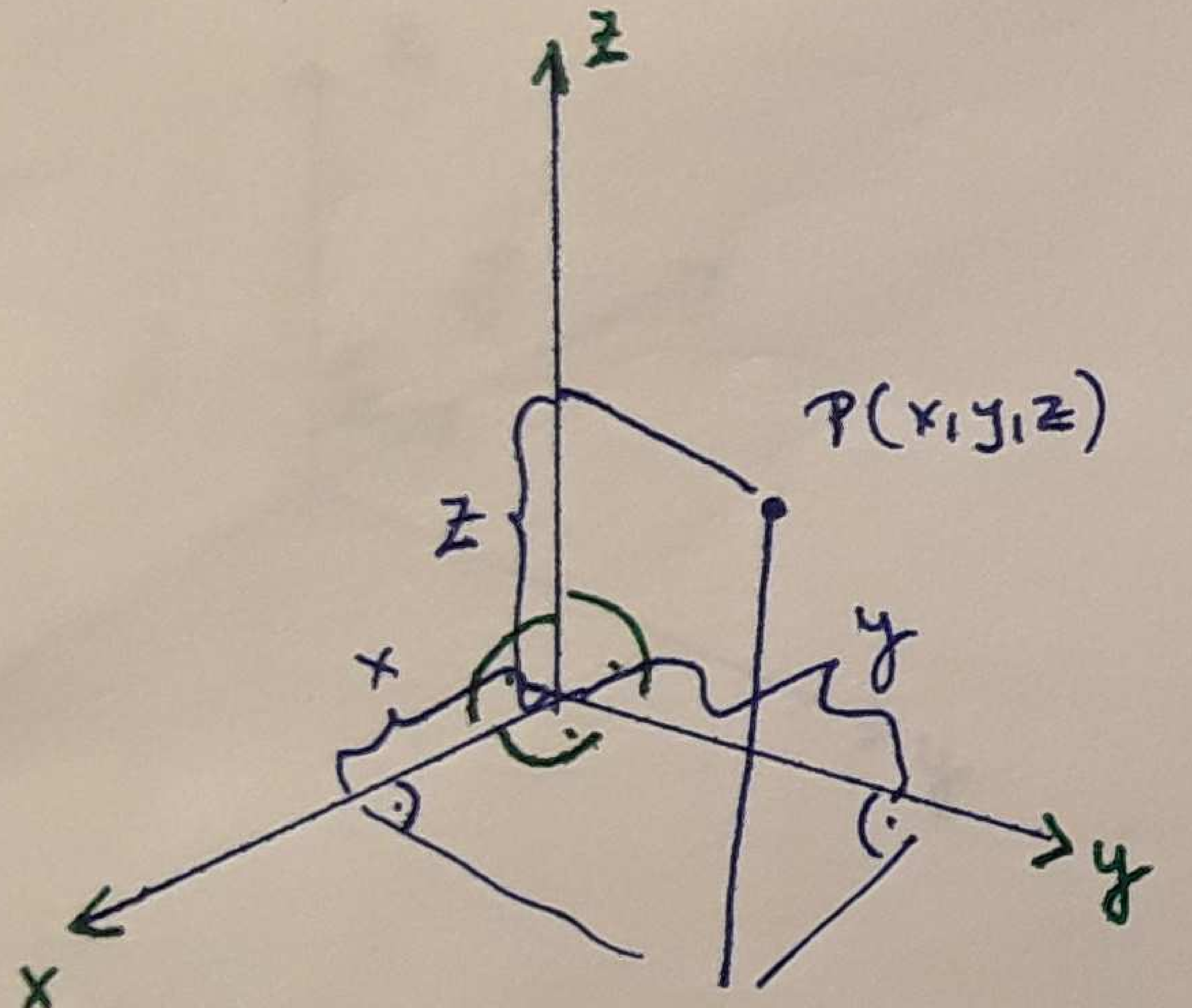
bizonyítások nélkül, a legszükségesebb ismeretanyag, ami nélkül nem tudunk továbbmenni.

Koord

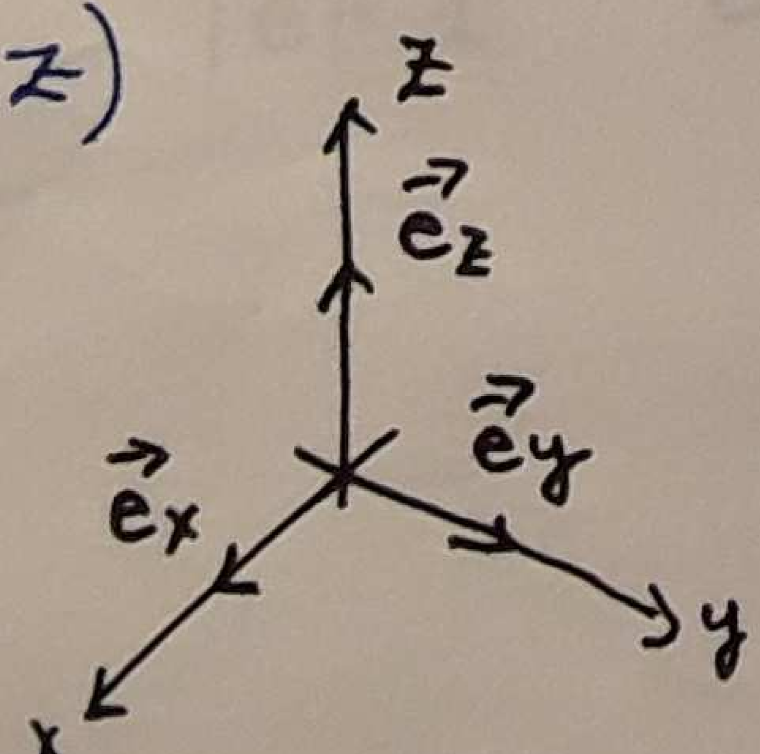
- Koordinátarendszer : Descartes-féle, henger, gömbi.
- Vektor
- Vektoralgebra röviden : összeadás, kivonás, skalárral szorzás egységvektor, normálvektor, skalár-, vektorszorzat, vegyesszorzat.
- Függvényeknél pár szó : skalár-skalár; vektor-skalár, skalár-vektor, vektor-vektor.
- Derivált függvény a fenti 4 esetben.
- Integrálok : vonalmenti, felületi, térfogati.
- Vektormező.
- Vektortér jellemzői : divergencia és rotáció.
- Integráltételek : Gauss-Osztrogradszkij-tétel, Stokes-tétel, Gradiens-tétel.
- Nabla-operátor.
- Ismételt műveletek.
- Néhány példa.

KOORDINÁTARENDSZER

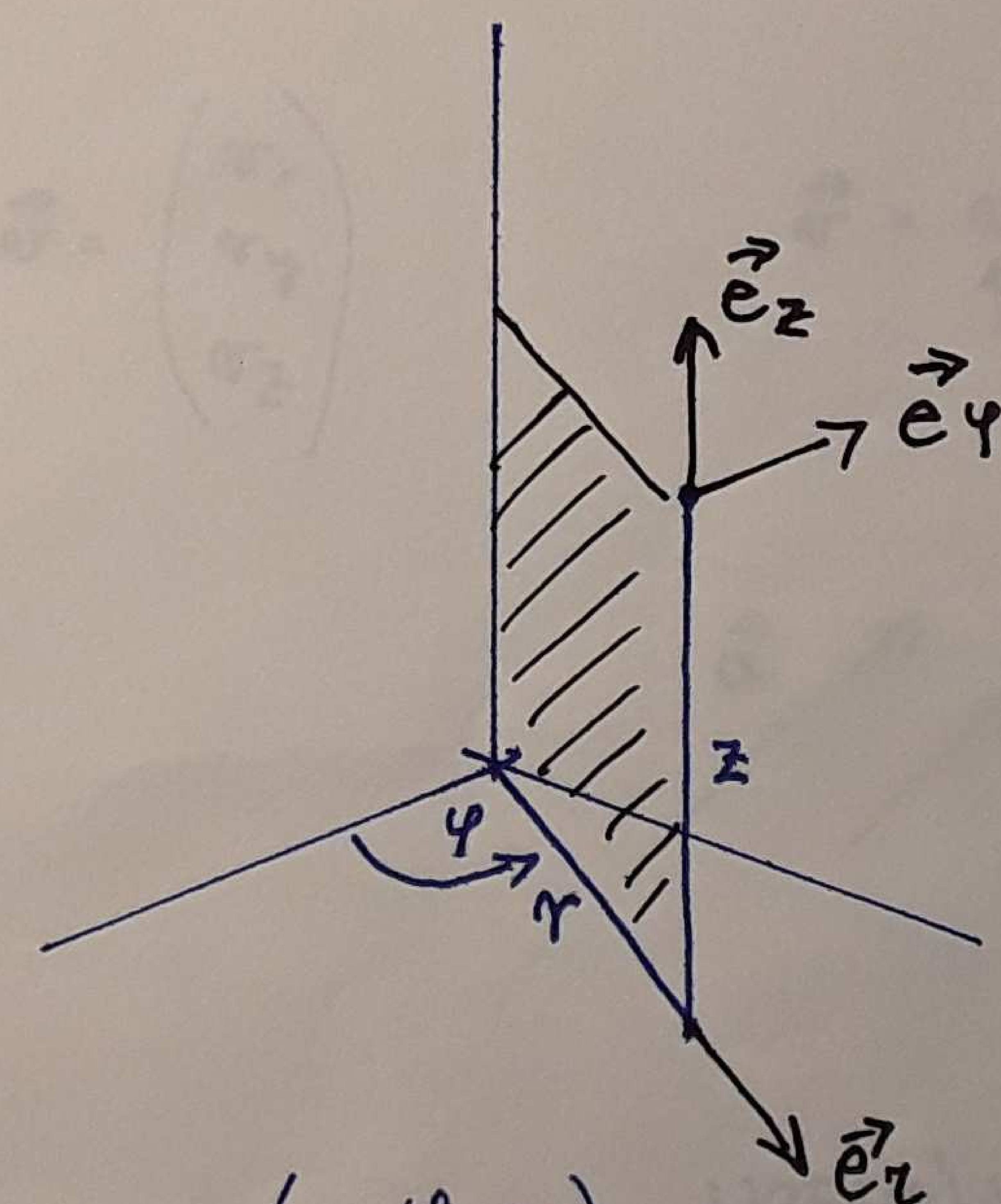
DESCARTES-FÉLE DERÉKSZÖGŰ



(x, y, z)

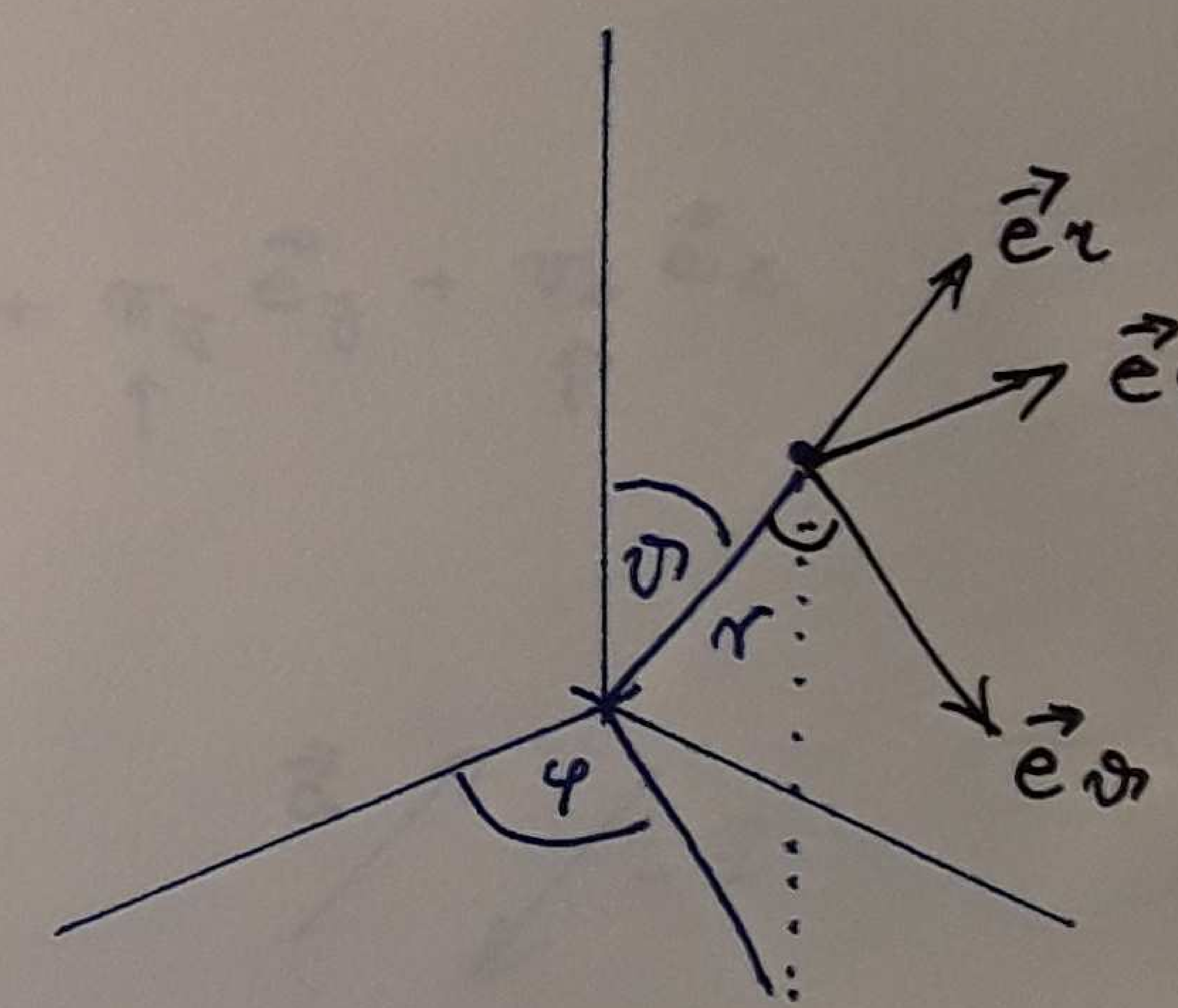


HENGER



(r, φ, z)

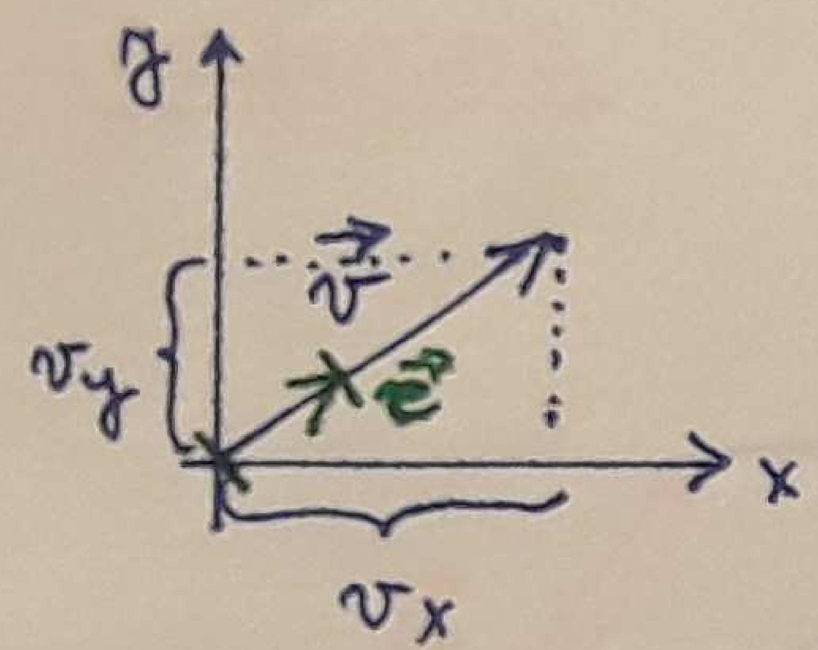
GÖMBI



(r, φ, ϑ)

VEKTOR

2D:



$$\vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

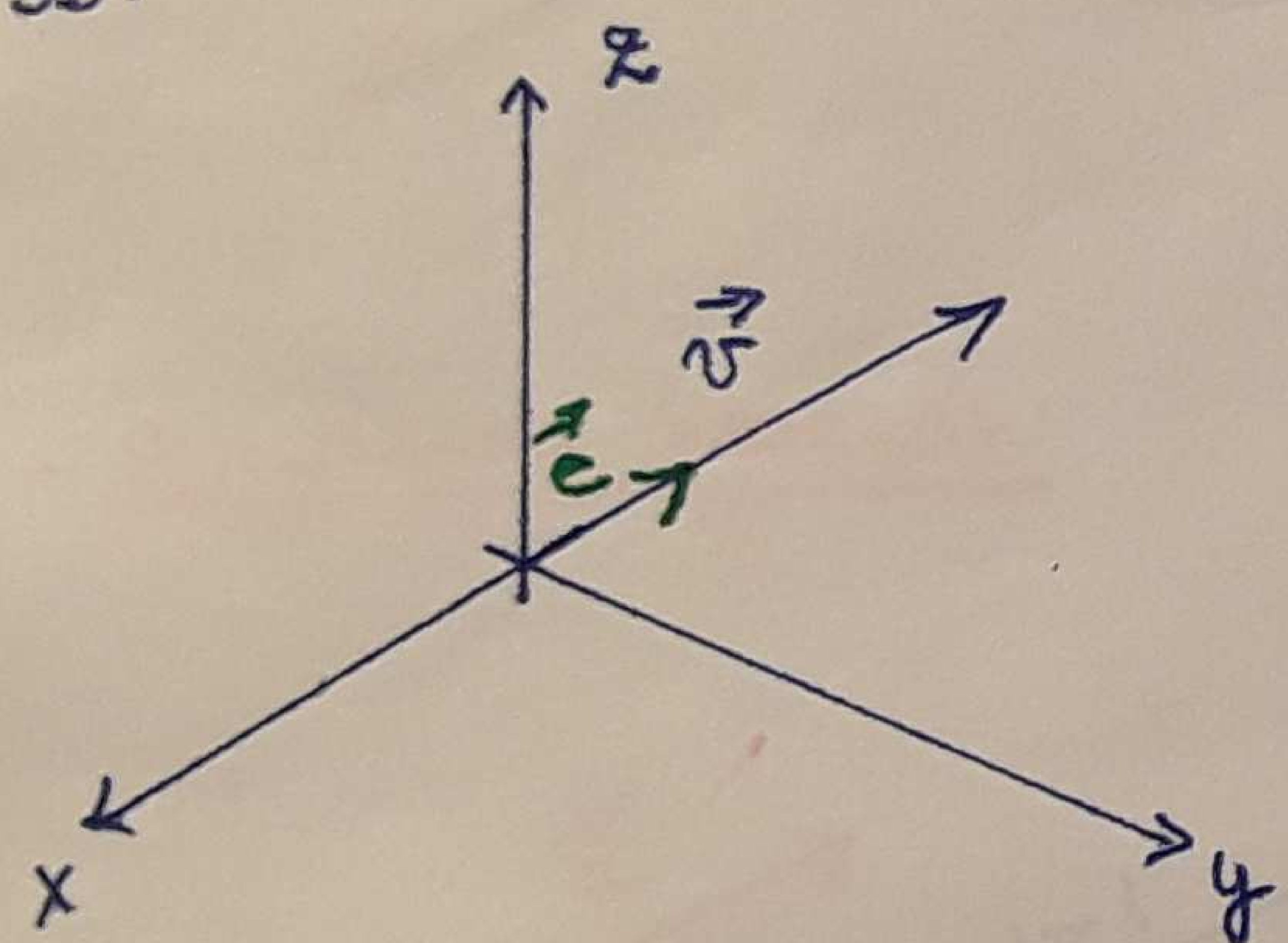
NAGYSÁG

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} \geq 0$$

$$= \sqrt{v_x^2 + v_y^2 + v_z^2}$$

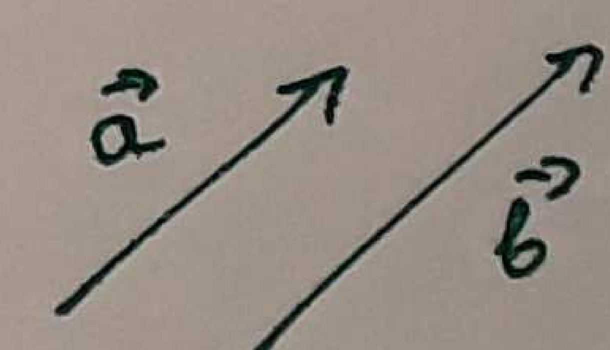
IRÁNY

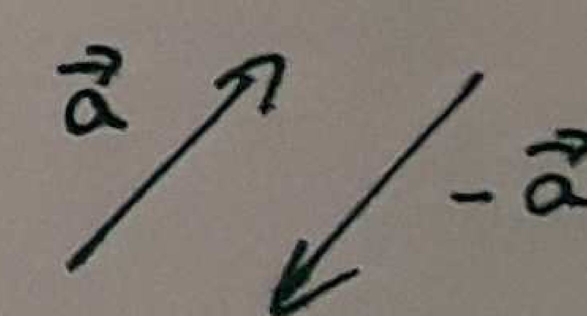
3D:



$$\vec{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

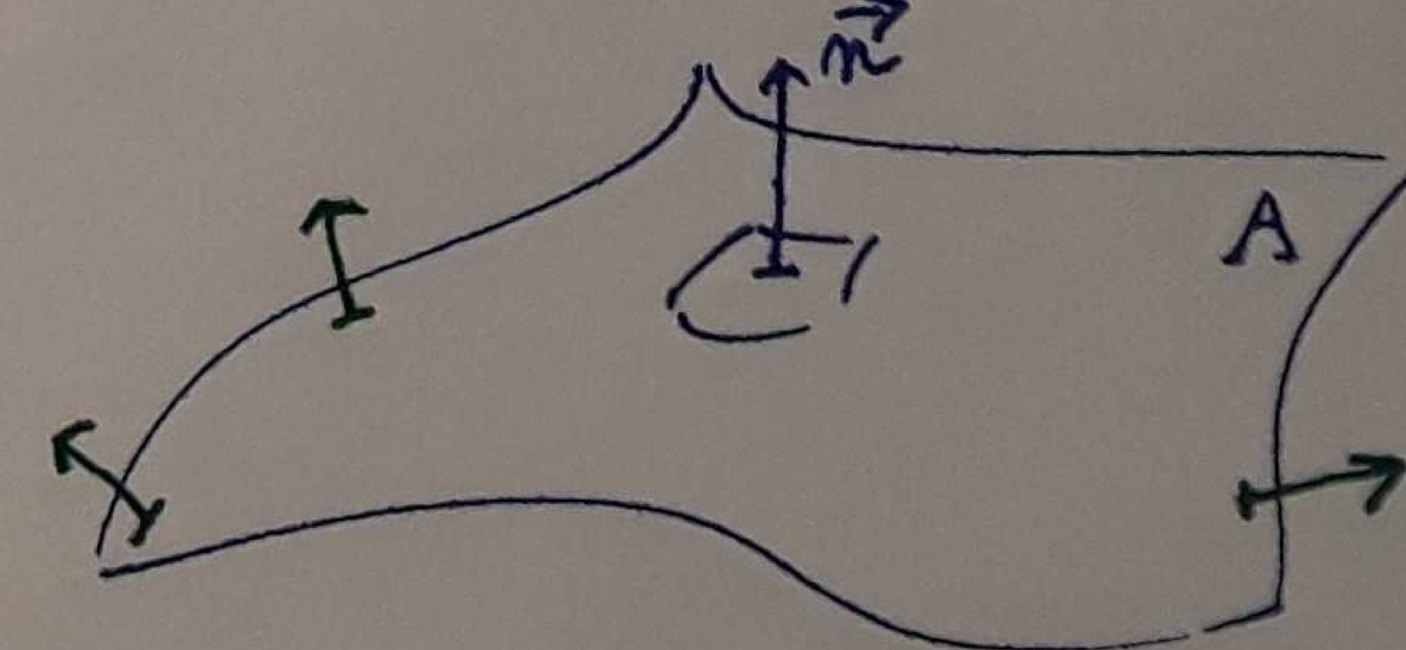
$$\vec{v} = \underset{\uparrow}{v_x} \vec{e}_x + \underset{\uparrow}{v_y} \vec{e}_y + \underset{\uparrow}{v_z} \vec{e}_z$$





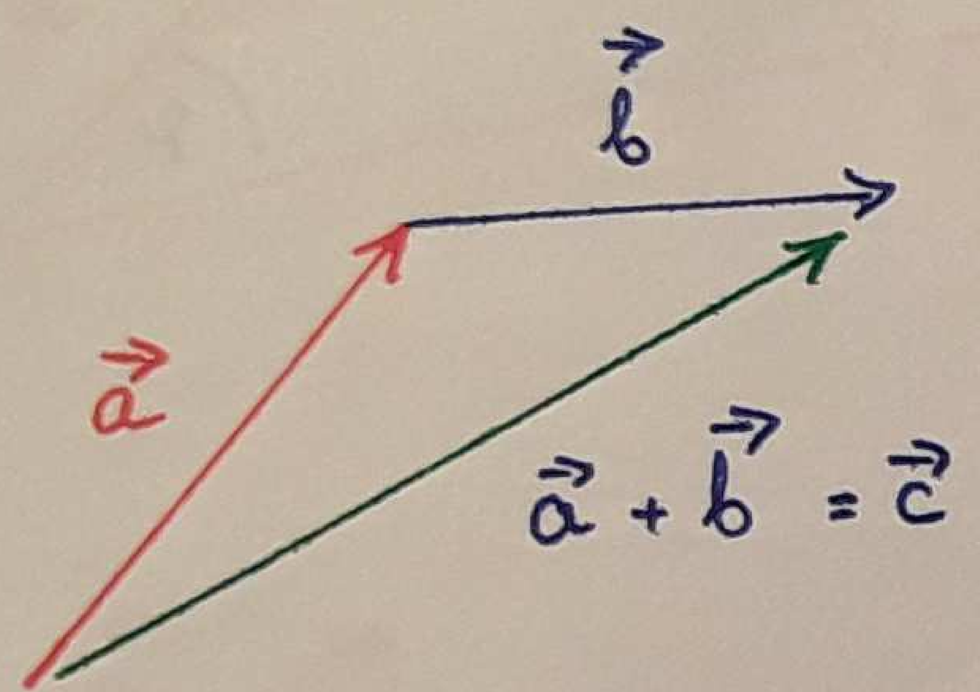
EGYSÉGVEKTOR : $|\vec{e}| = 1$ $\quad \vec{e} = \dfrac{\vec{v}}{|\vec{v}|}$

NORMÁLVEKTOR : $|\vec{n}| = 1$



VEKTORALGEBRA

- ÖSSZEADÁS:



$c_x = a_x + b_x$

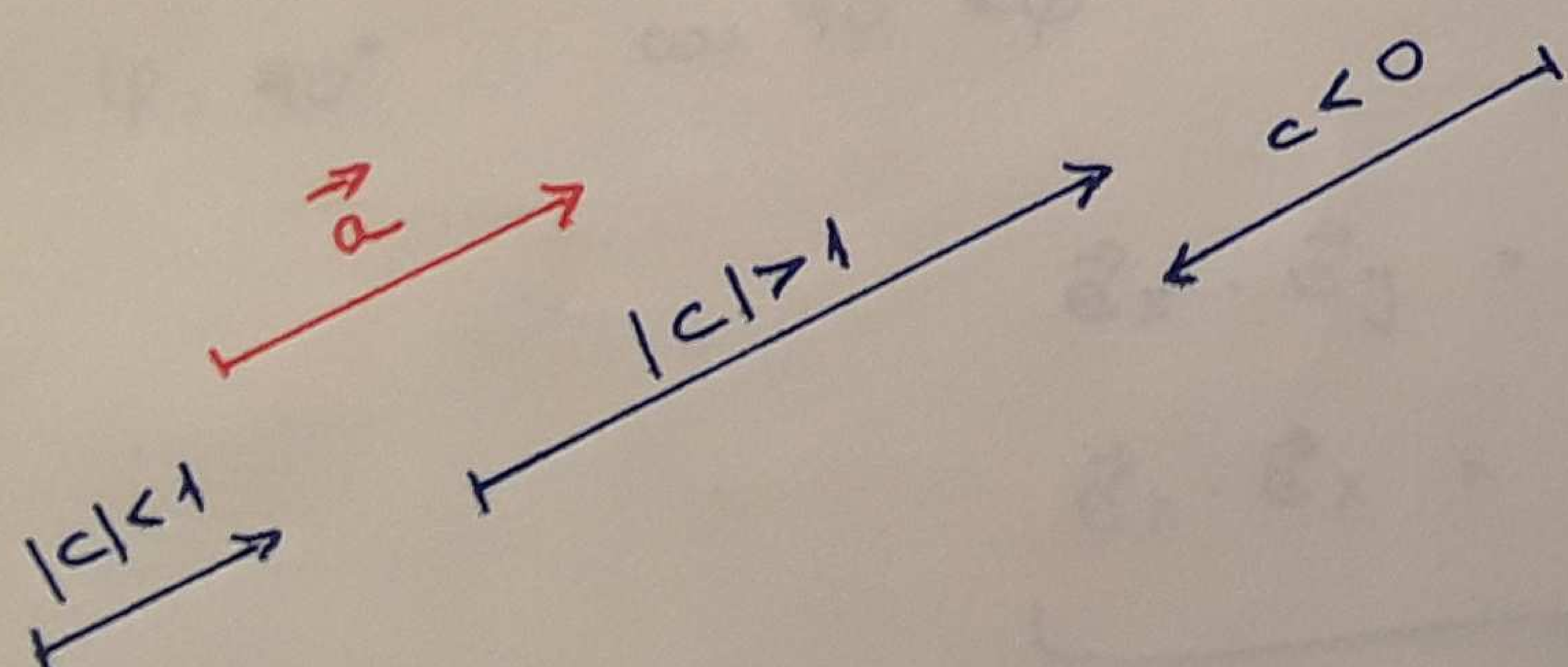
$c_y = a_y + b_y$

$c_z = a_z + b_z$

$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

- SKALÁRRAL SZORZÁS:

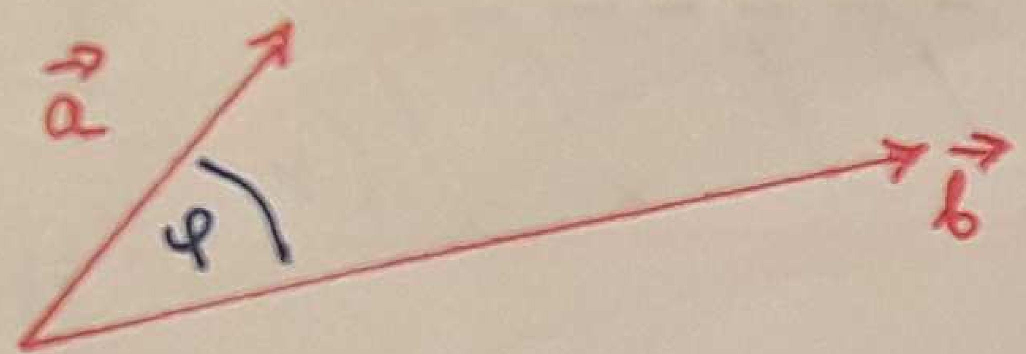
$c\,\vec{a}$



- SKALÁRSZORZAT:

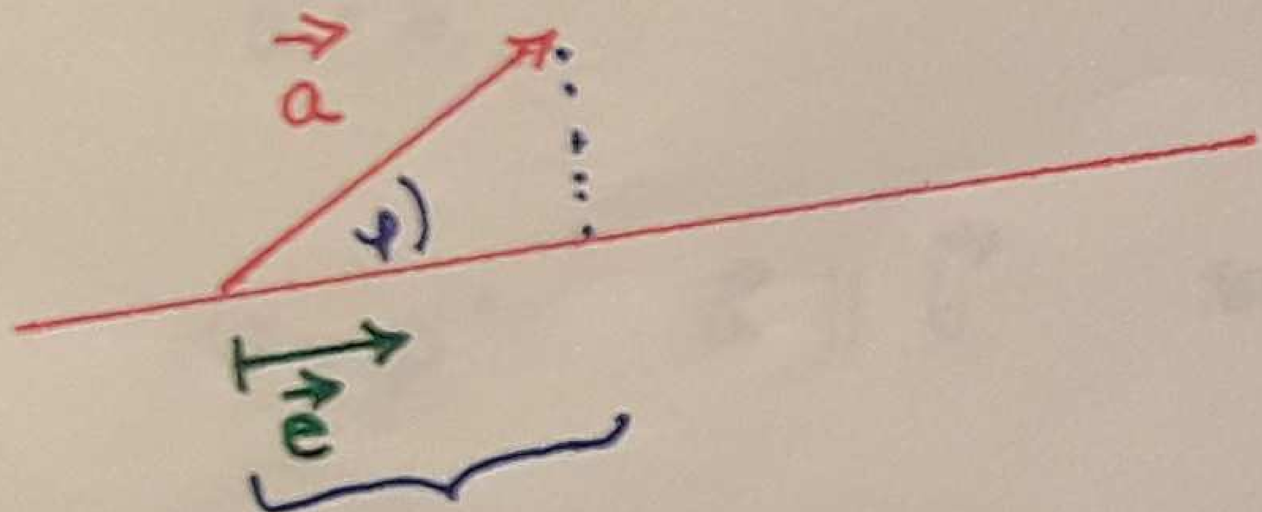
{jele !!!}

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\varphi$$



$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$



$$\vec{a} \cdot \vec{e} = |\vec{a}| \underbrace{|\vec{e}|}_{1} \cos\varphi$$

$\varphi = 90^\circ$ $\quad \cos 90^\circ = \emptyset \quad \vec{a} \cdot \vec{b} = \emptyset$

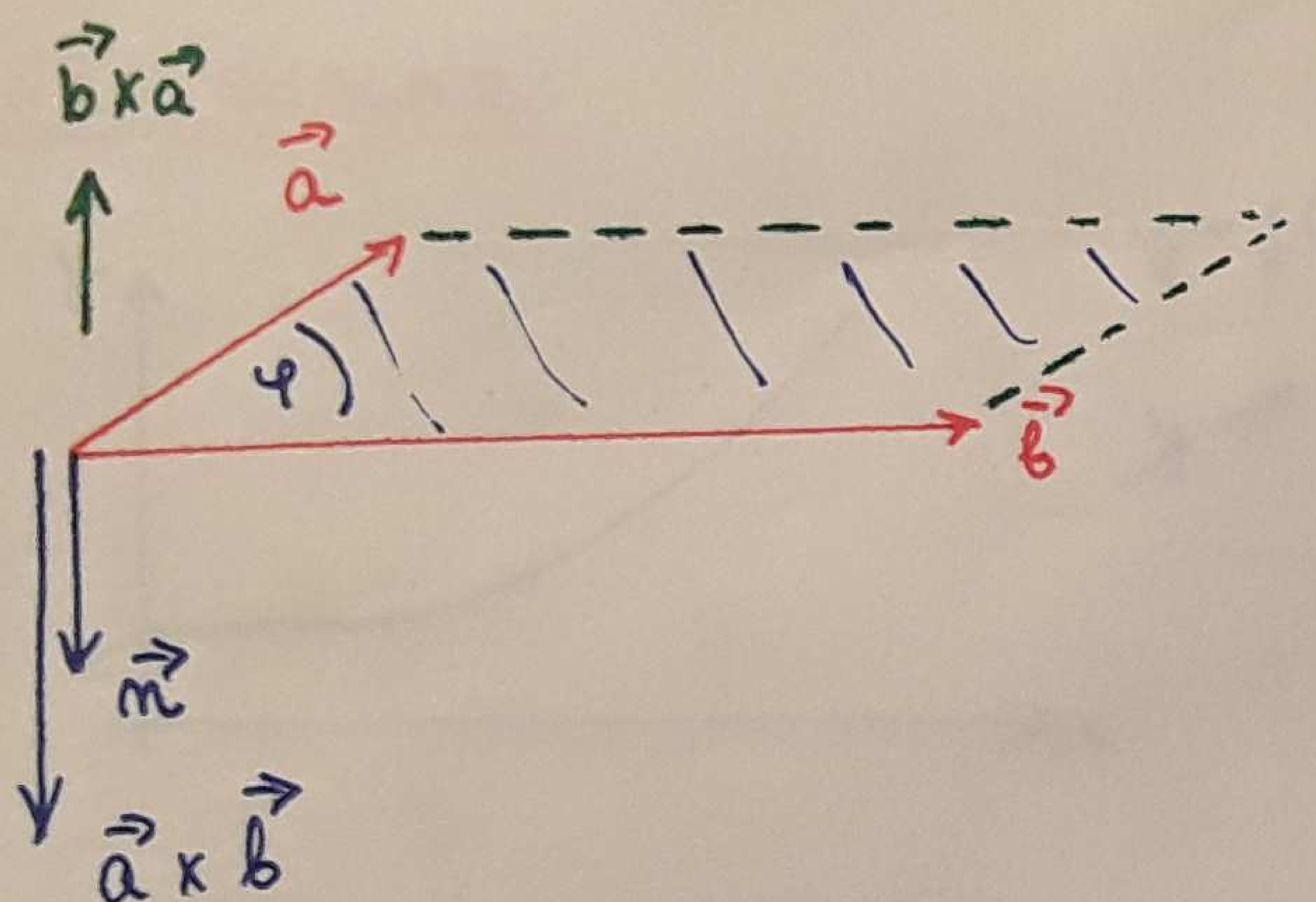
$$\left.\begin{aligned} \vec{e}_x \cdot \vec{e}_y &= \vec{e}_y \cdot \vec{e}_z = \vec{e}_z \cdot \vec{e}_x = \emptyset \\ \vec{e}_x \cdot \vec{e}_x &= \vec{e}_y \cdot \vec{e}_y = \vec{e}_z \cdot \vec{e}_z = 1 \end{aligned}\right.$$

$$\vec{a} \cdot \vec{a} = a^2$$

$$\vec{a} \cdot \vec{b} = (\underline{a_x \vec{e}_x} + a_y \vec{e}_y + a_z \vec{e}_z) \cdot (\underline{b_x \vec{e}_x} + \underline{b_y \vec{e}_y} + \underline{b_z \vec{e}_z}) = \underline{a_x b_x} + a_y b_y + a_z b_z$$

$$a_x \vec{e}_x \cdot b_x \vec{e}_x \; ; \quad a_x \vec{e}_x \cdot b_y \vec{e}_y = \emptyset$$

- VEKTORIÁLIS SZORZAT:



(jele!!!)

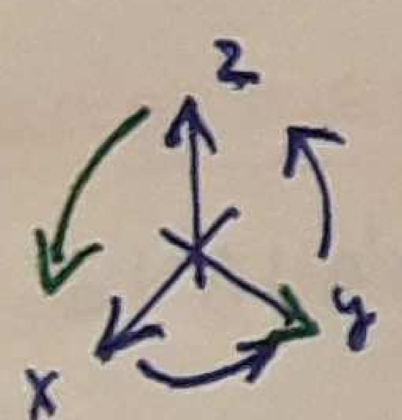
$$\vec{a} \times \vec{b} = \left(|\vec{a}||\vec{b}| \sin\varphi\right) \vec{n}$$

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

$\varphi = 0°$ $\quad \vec{a} \parallel \vec{b}$ $\quad \sin\varphi = 0$ $\quad \vec{a} \times \vec{b} = 0$



$$\vec{e}_x \times \vec{e}_x = \vec{e}_y \times \vec{e}_y = \vec{e}_z \times \vec{e}_z = 0$$

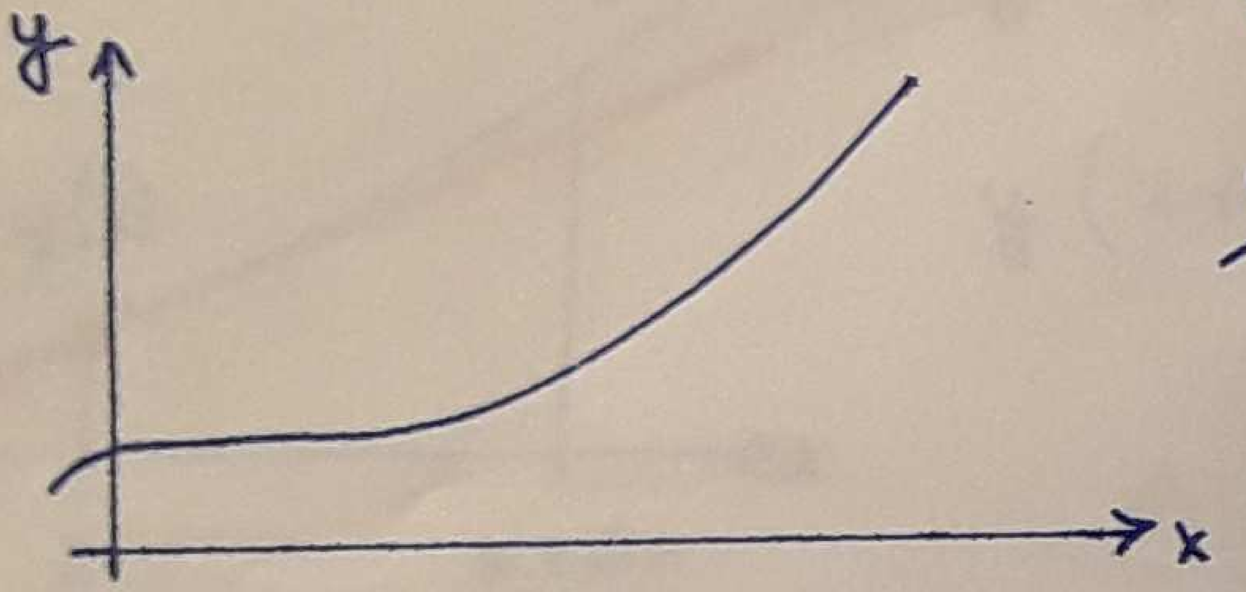
$$\vec{e}_x \times \vec{e}_y = \vec{e}_z \qquad \vec{e}_y \times \vec{e}_z = \vec{e}_x \qquad \vec{e}_z \times \vec{e}_x = \vec{e}_y$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \vec{e}_x (a_y b_z - a_z b_y) + \vec{e}_y (a_z b_x - a_x b_z) + \vec{e}_z (a_x b_y - a_y b_x)$$

$$\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}$$

FÜGGVÉNYEK

- SKALÁR - SKALÁR:



$y = f(x)$

$\cancel{y =}\ z = f(x, y)$

- VEKTOR - SKALÁR:

$\vec{v} = \vec{v}(t) = v_x(t)\,\vec{e}_x + v_y(t)\,\vec{e}_y + v_z(t)\,\vec{e}_z$

$\vec{a}$

- SKALÁR - VEKTOR:

SKALÁRTÉR

$T = T(\vec{r}) = T(x, y, z)$

- VEKTOR - VEKTOR:

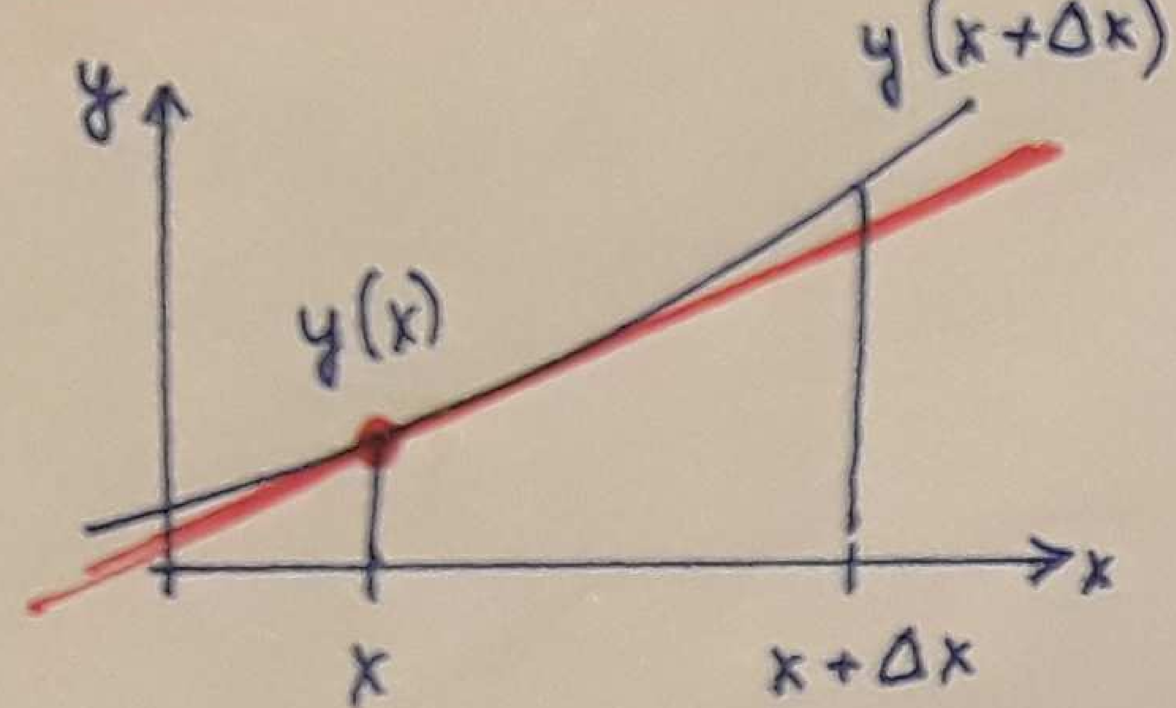
$\vec{a} = \vec{a}(\vec{r})$

$= a_x(x, y, z)\,\vec{e}_x + a_y(x, y, z)\,\vec{e}_y + a_z(x, y, z)\,\vec{e}_z$

$\cancel{\vec{v}(v}$ $\quad \vec{v} = \vec{v}(\vec{r}, t)$

DERIVÁLT FÜGGVÉNY

– SKALÁR – SKALÁR:



$$y = f(x)$$

$$y(x+\Delta x) - y(x) = y' \Delta x$$

$$\Delta y = y' \Delta x$$

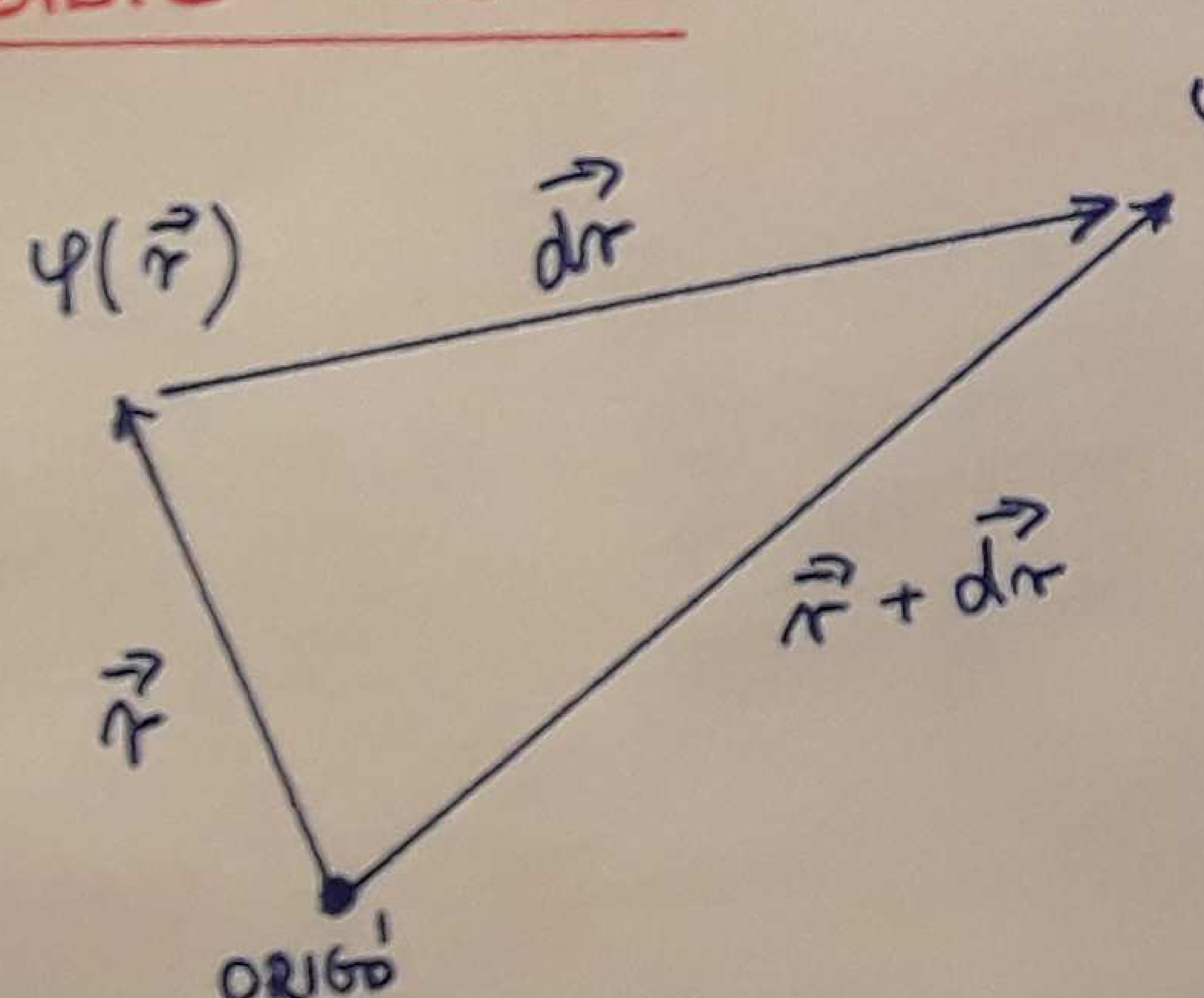
↑

– VEKTOR – SKALÁR:

$$\bar{y} = f(x) \qquad \Delta \bar{y} = y' \Delta x$$

$$\vec{v} = \underbrace{\frac{dv_x}{dt}} \vec{e}_x + \underbrace{\frac{dv_y}{dt}} \vec{e}_y + \underbrace{\frac{dv_z}{dt}} \vec{e}_z$$

– SKALÁR – VEKTOR:



$$\varphi = \varphi(x, y, z)$$

$$\varphi(\vec{r} + d\vec{r}) - \varphi(\vec{r}) = \underbrace{\operatorname{grad}\varphi} \cdot d\vec{r}$$

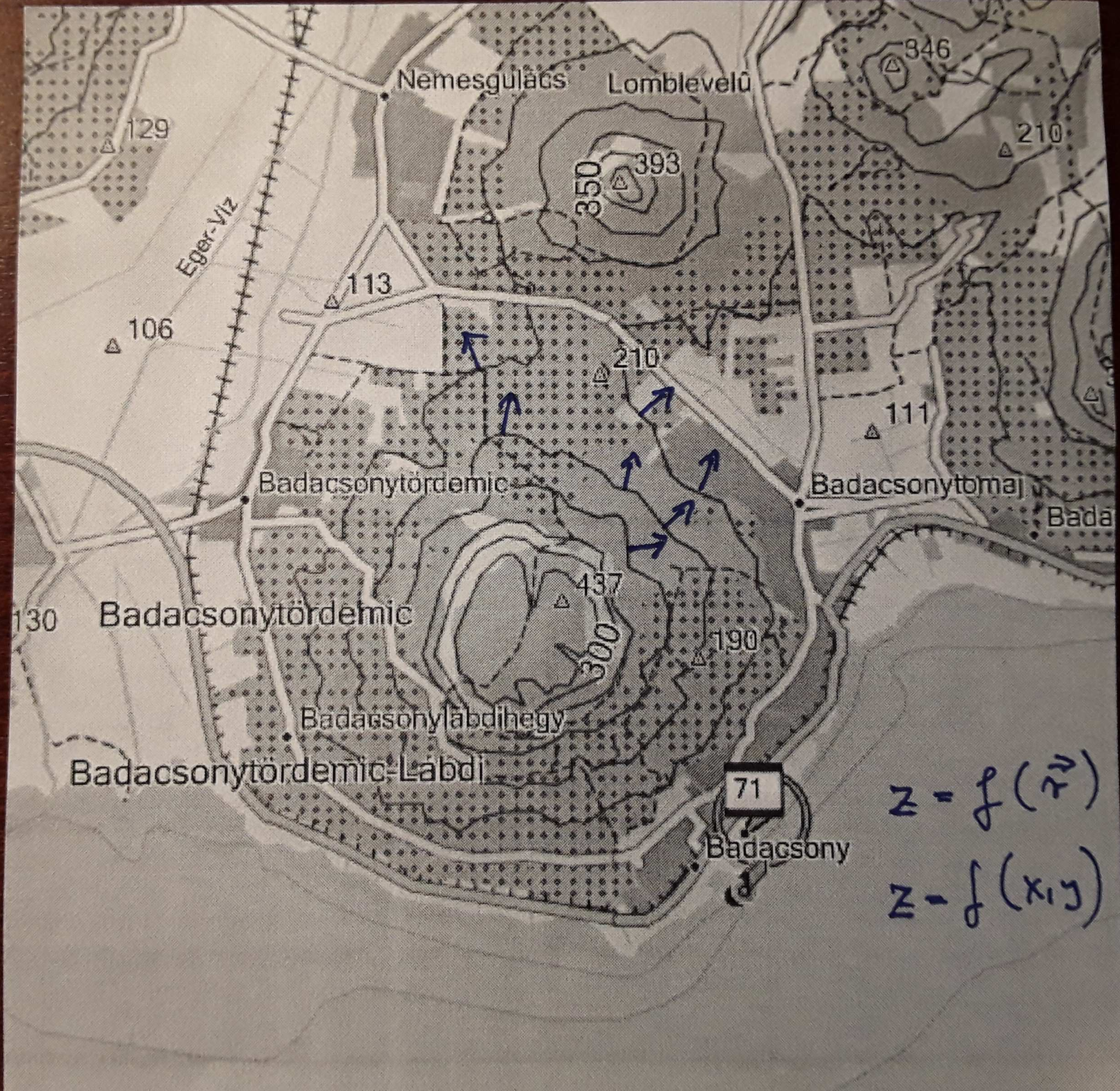
$$\operatorname{grad}\varphi = \frac{\partial \varphi}{\partial x} \vec{e}_x + \frac{\partial \varphi}{\partial y} \vec{e}_y + \frac{\partial \varphi}{\partial z} \vec{e}_z$$

$$\partial \qquad \frac{\partial \varphi}{\partial x}$$

– VEKTOR – VEKTOR:

$$\Delta \bar{y} = \underbrace{\bar{y}'}_{\text{TENZOR}} \cdot \Delta \bar{x} \qquad v$$

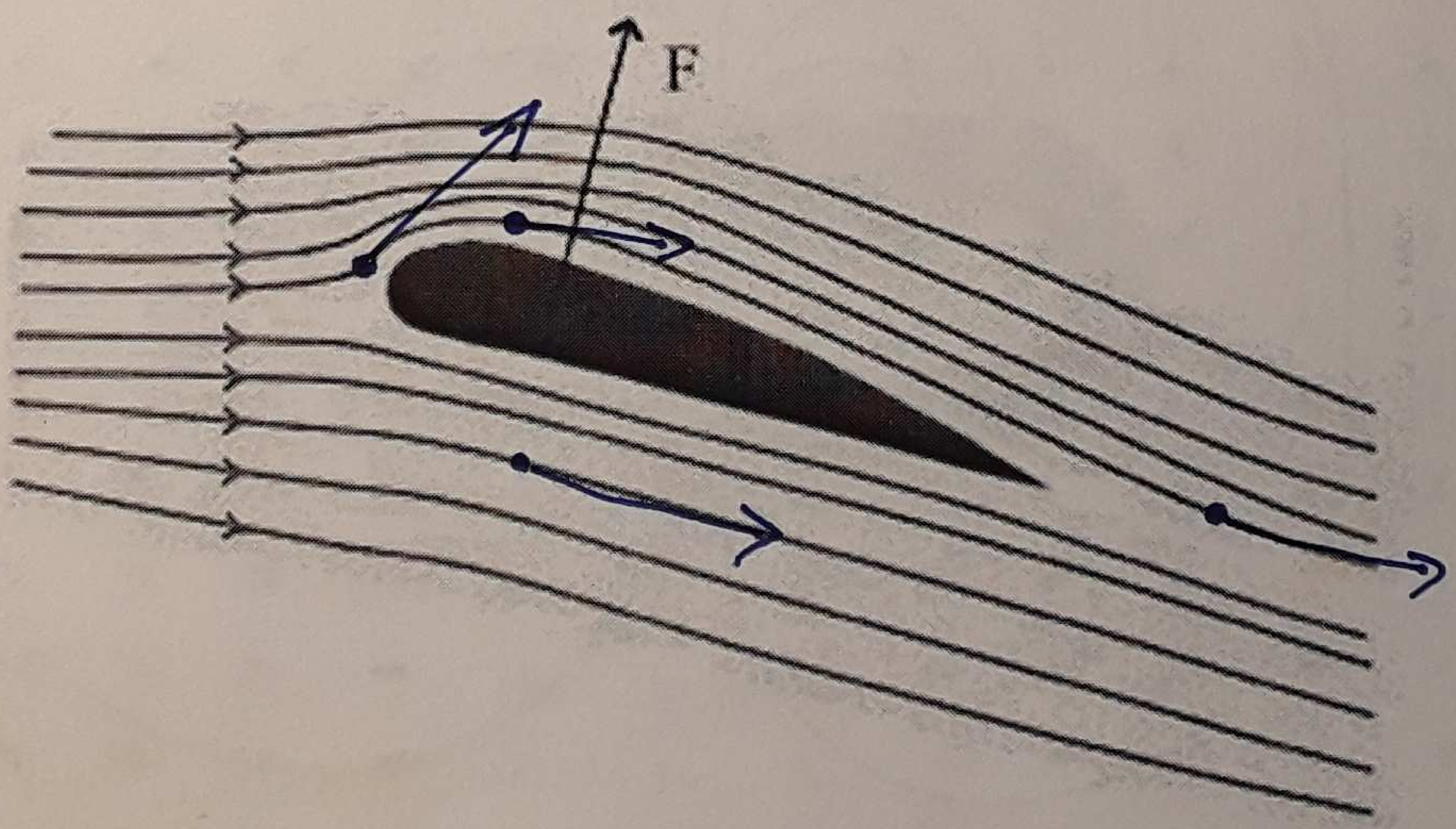
$$\begin{pmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{pmatrix}$$


Nemesgulács
Lomblevelű
346
129
393
350
210
Eger-Víz
113
106
210
111
Badacsonytördemic
Badacsonytomaj
437
130
Badacsonytördemic
300
190
Badacsonylábdihegy
Badacsonytördemic-Lábdi
71
Badacsony

$z = f(\vec{r})$

$z = f(x, y)$

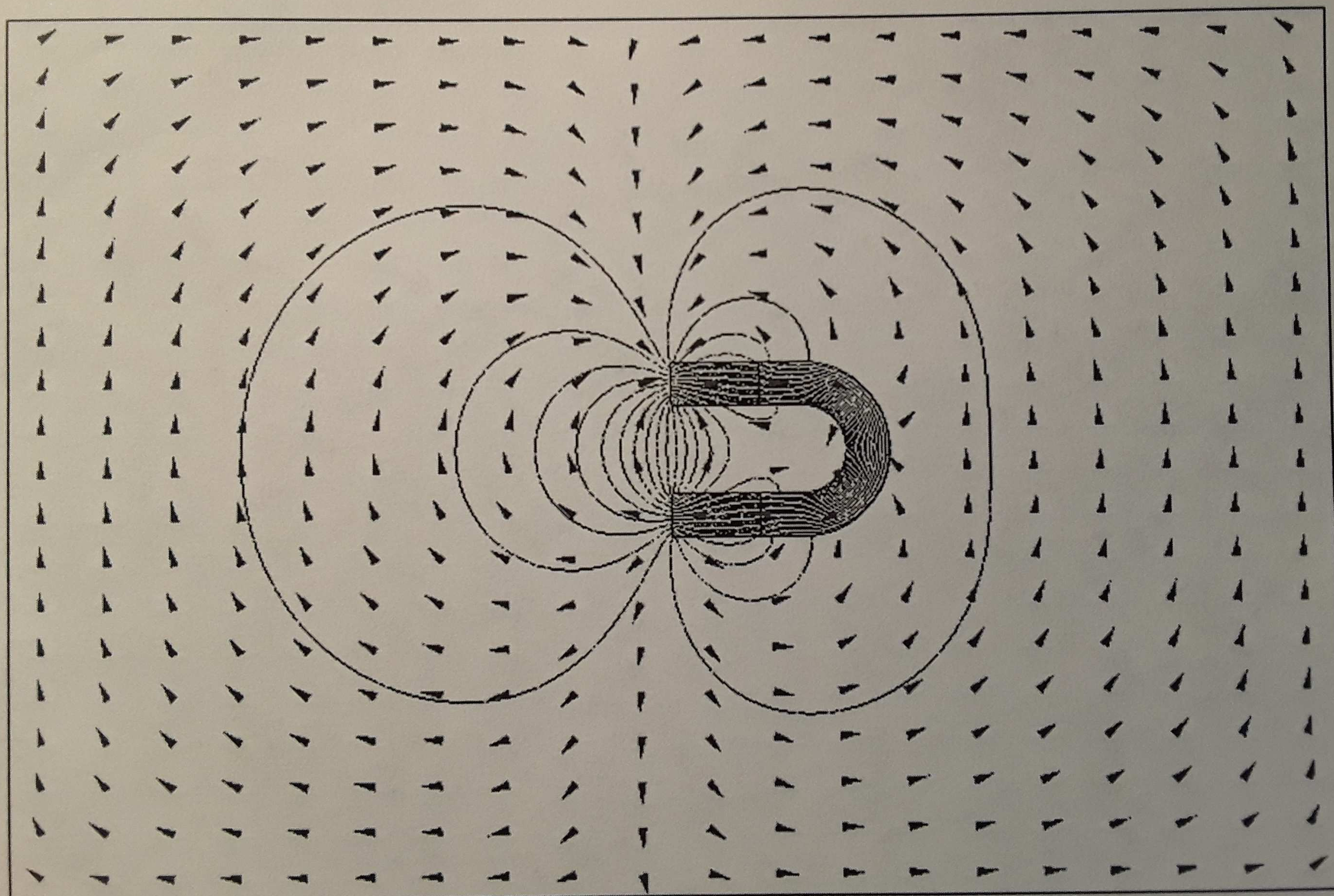
Sebességvektorok – vektormező



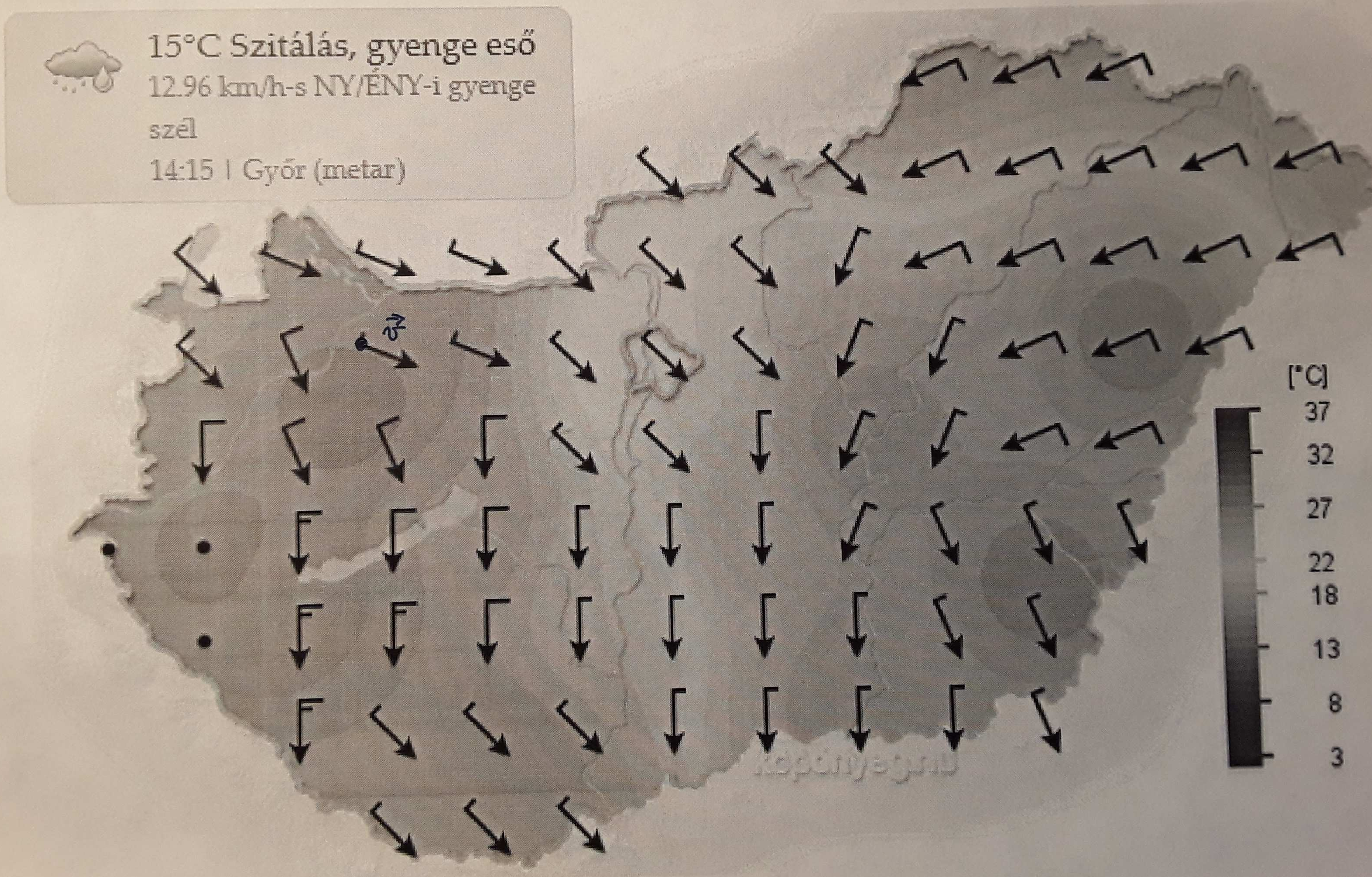
Minden helyen más és más a sebesség nagysága és iránya, ami pillanatról, pillanatra változik: $\mathbf{v}(\boldsymbol{r},t)$

Ez egy vektormező. A vektormező a helyvektorhoz rendel vektort, azaz vektor-vektor függvényt definiál.

Patkómágnes mágneses tere – vektormező

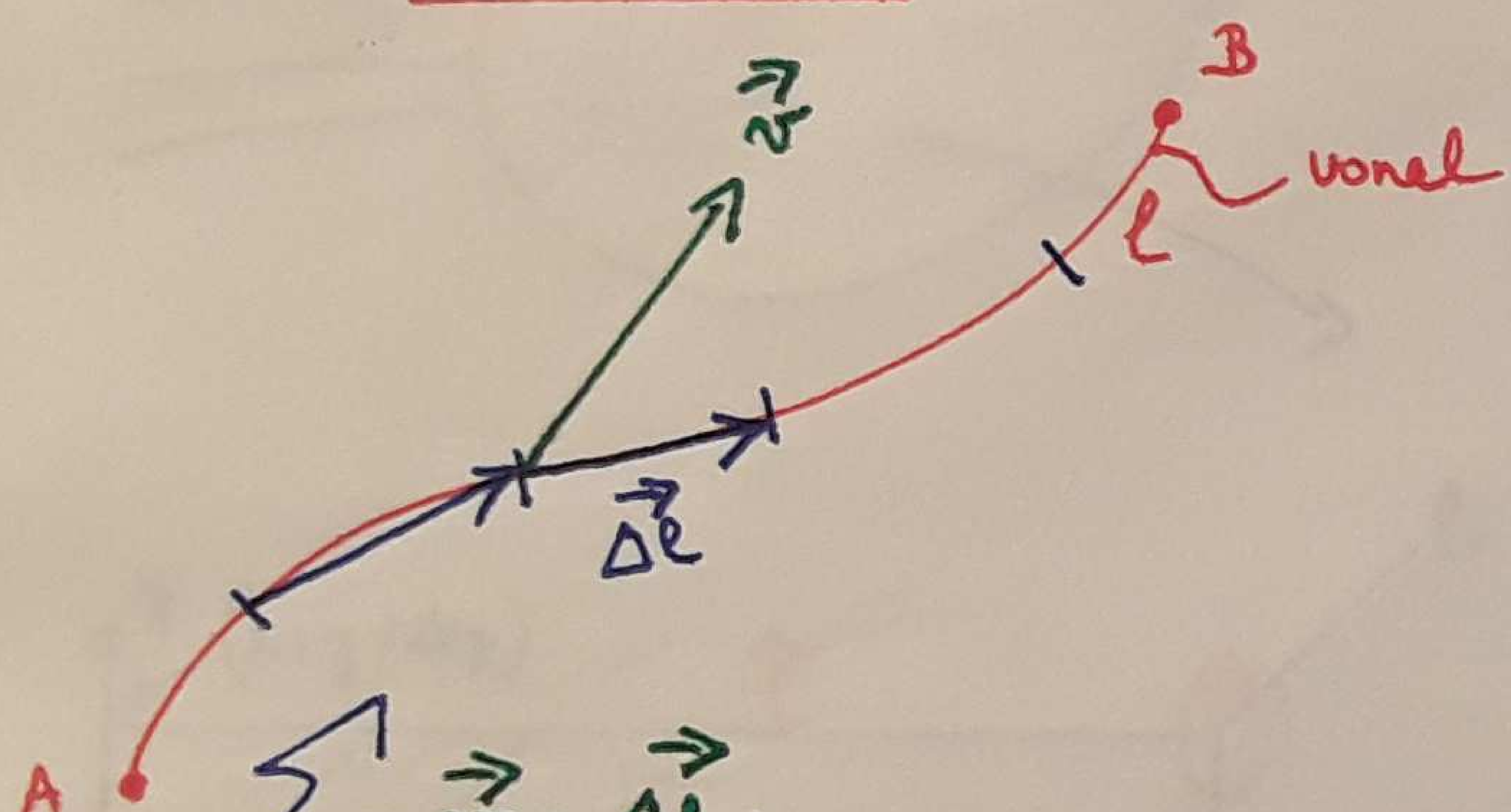


MAGYARORSZÁG SZÉLTÉRKÉPE



INTEGRÁLOK

VONALMENTI

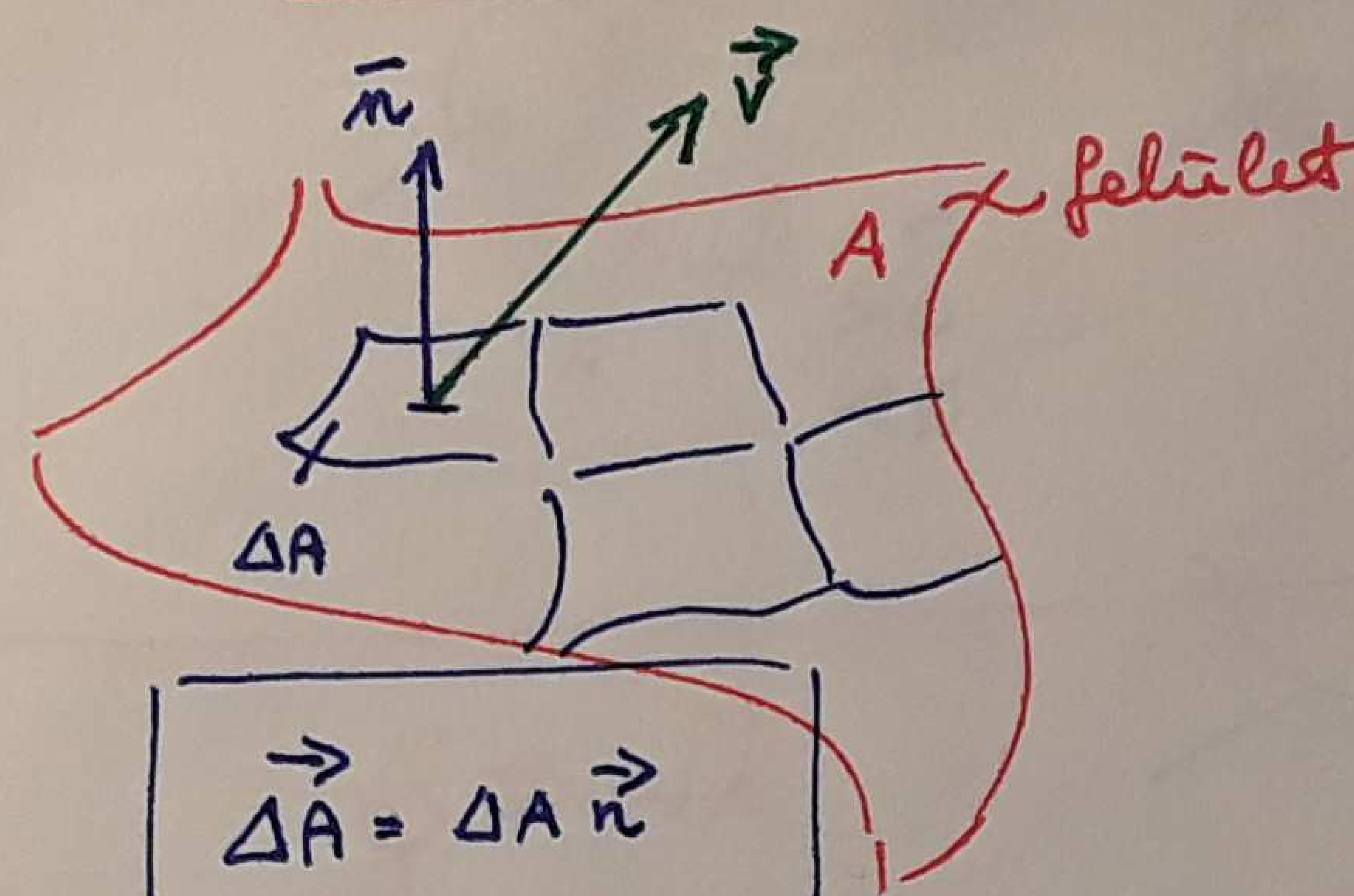


$$\sum_i \vec{v}_i \cdot \vec{\Delta l}_i$$

$$\lim_{\substack{\Delta l \to 0 \\ N \to \infty}} \sum_{i=1}^{N} \vec{v}_i \cdot \vec{\Delta l}_i = \int_A^B \vec{v} \cdot \vec{dl}$$

$$W = \int_A^B \vec{F} \cdot \vec{ds}$$

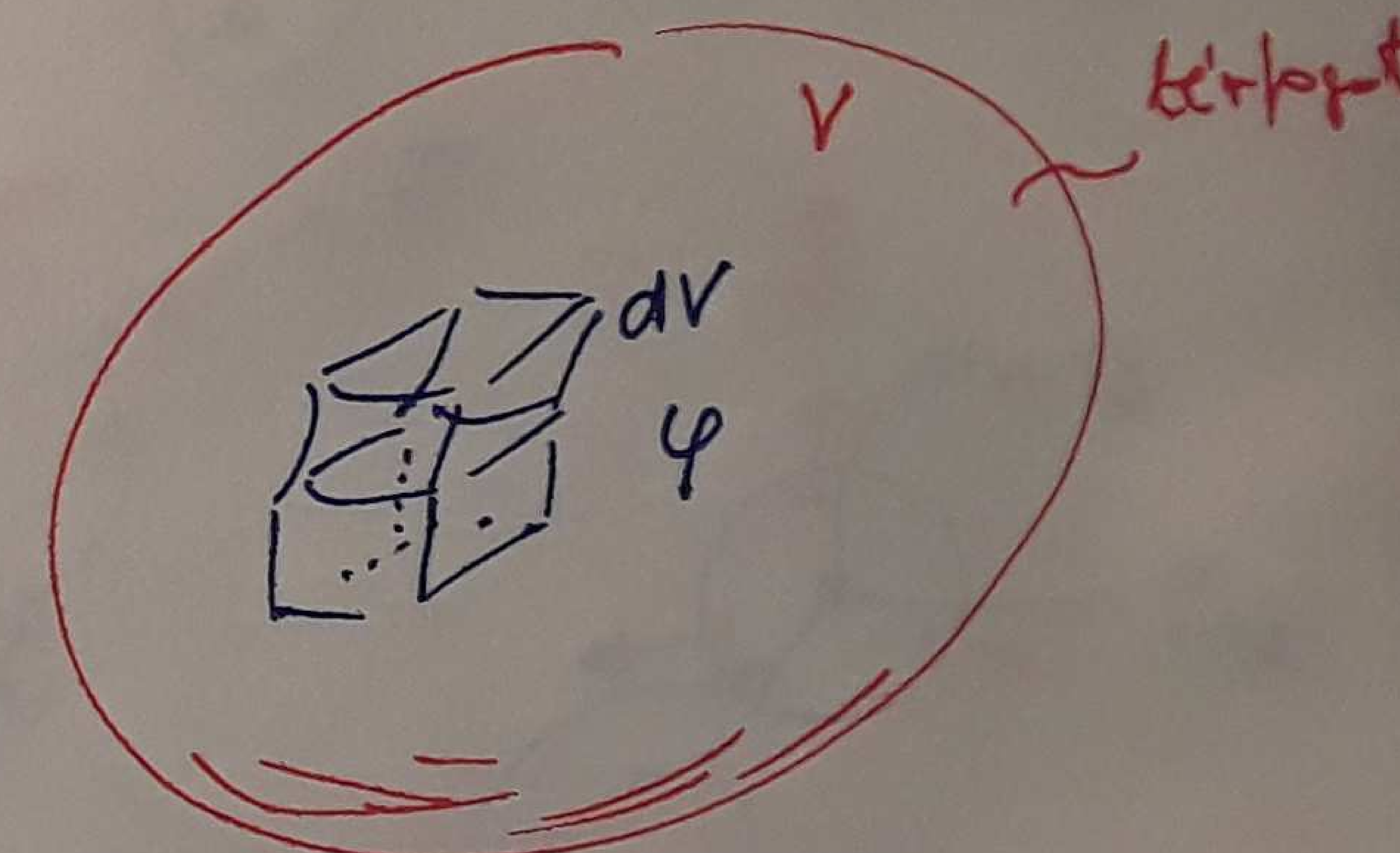
FELÜLETI



$$\boxed{\vec{\Delta A} = \Delta A \, \vec{n}}$$

$$\lim_{\substack{\Delta A \to 0 \\ N \to \infty}} \sum_{i=1}^{N} \vec{v}_i \cdot \vec{\Delta A}_i = \int_A \vec{v} \cdot \vec{dA}$$

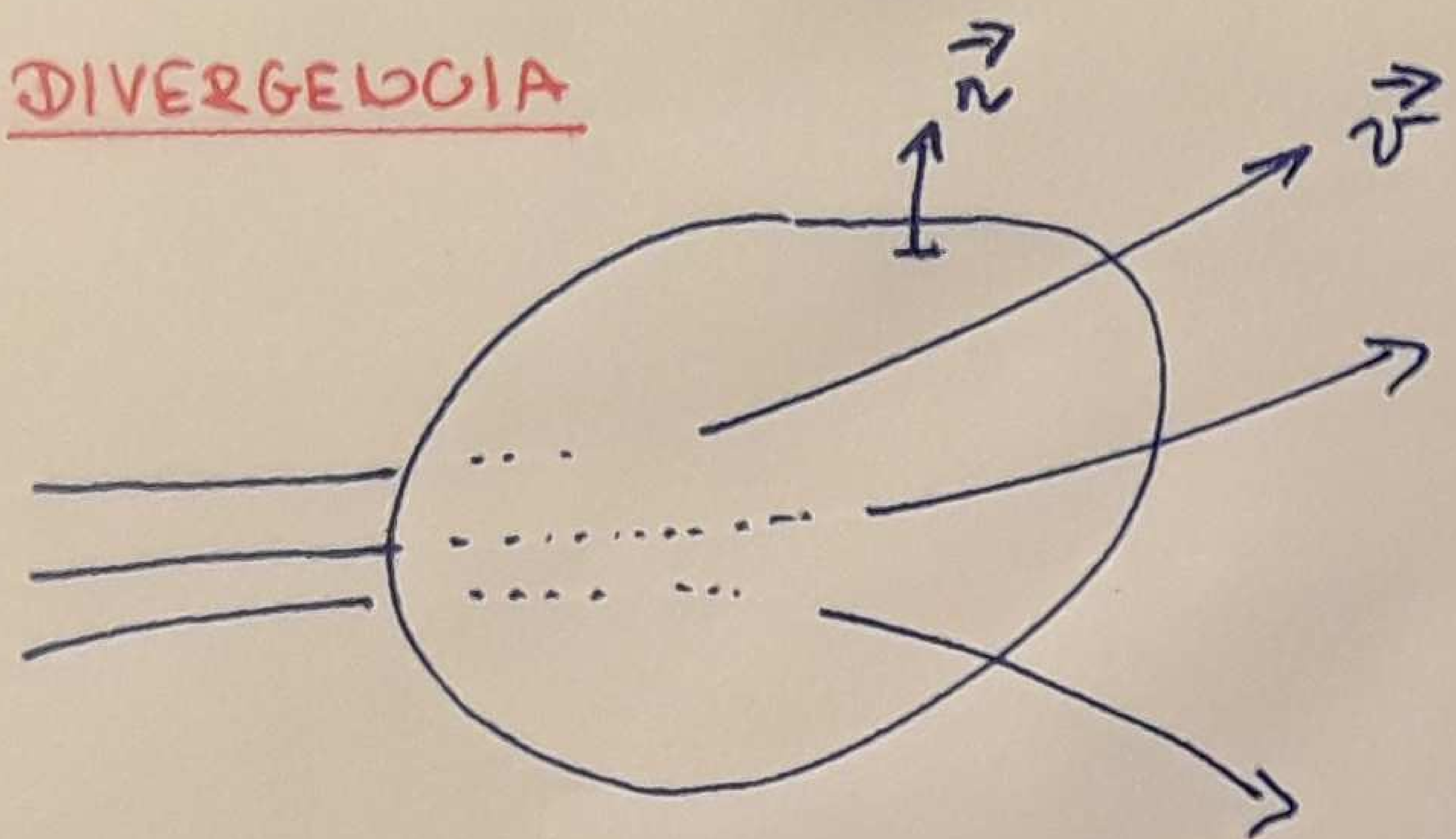
TÉRFOGATI



$$\lim_{\substack{\Delta V \to 0 \\ N \to \infty}} \sum_{i=1}^{N} \varphi_i \Delta V_i = \int_V \varphi \, dV$$

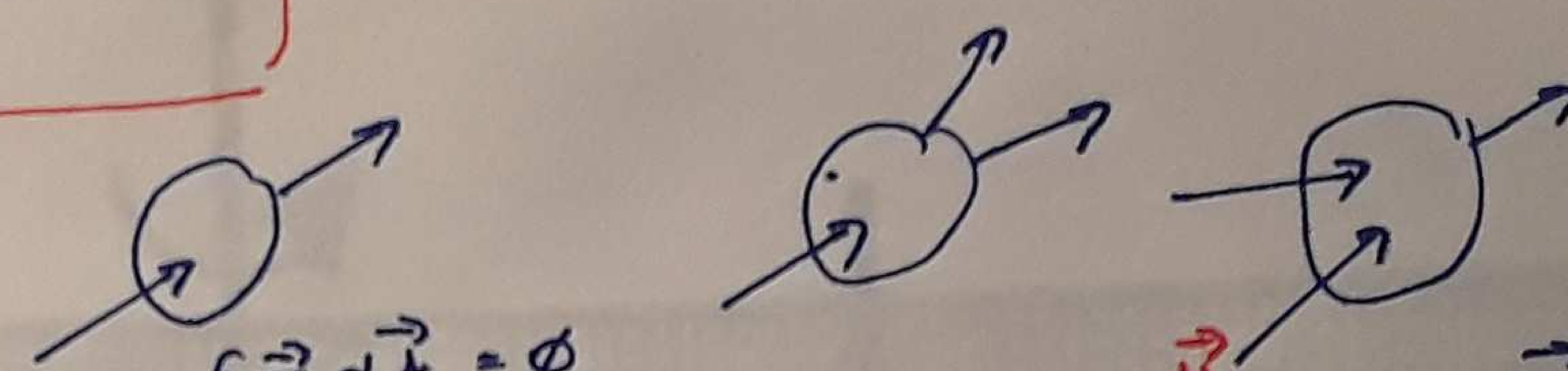
VEKTORTÉR JELLEMZŐI

- DIVERGENCIA

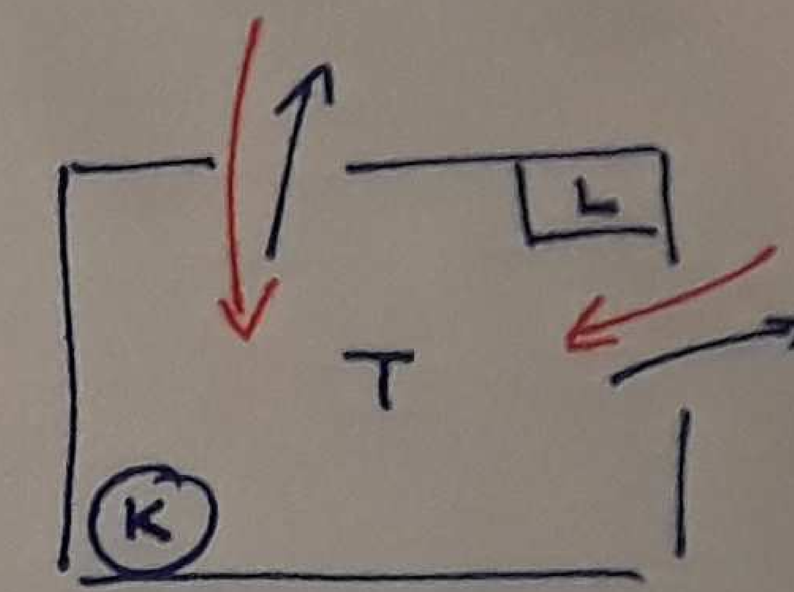


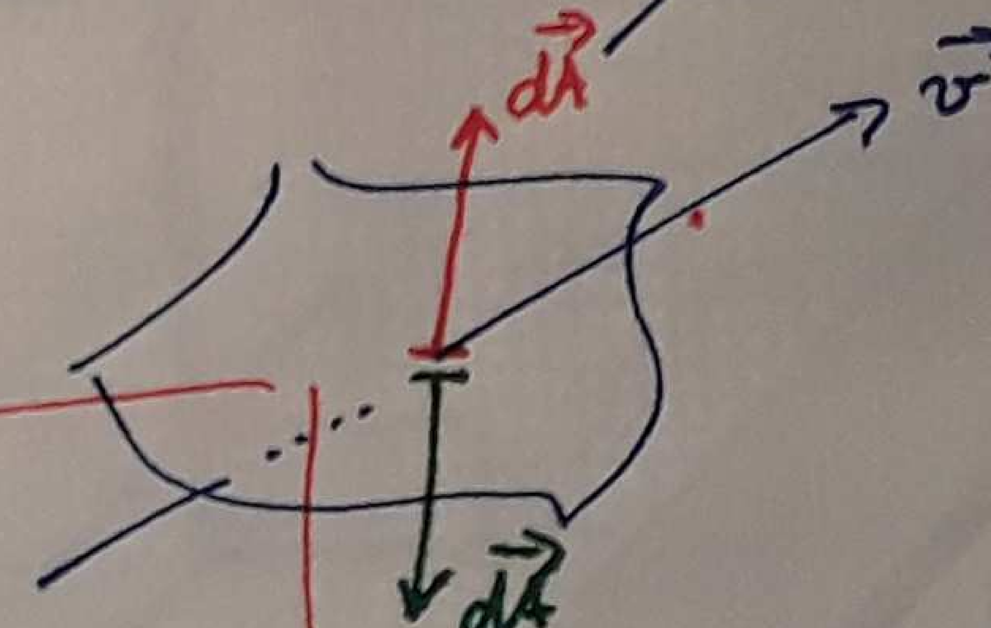
FLUXUS (forrássűrűség)

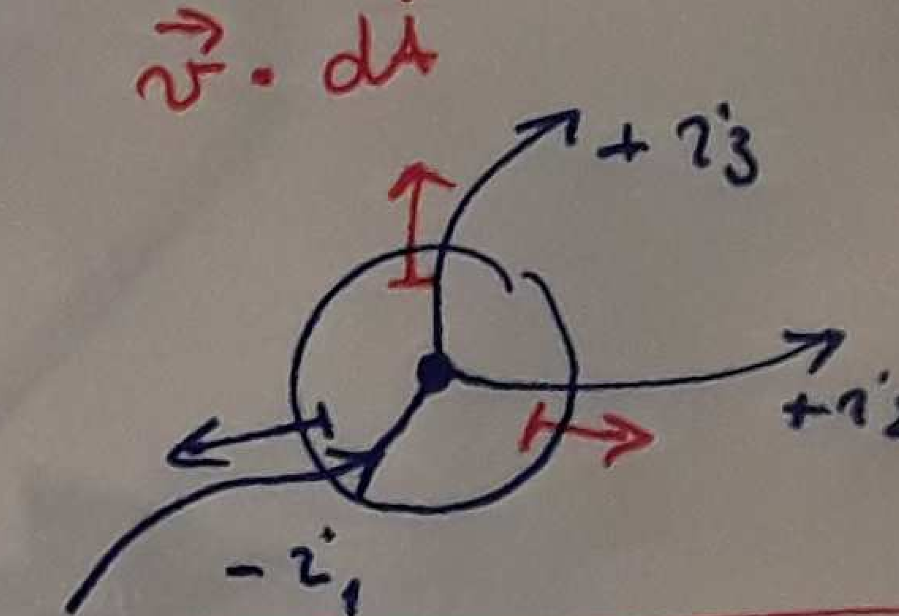
$$\boxed{\oint_A \vec{v}\cdot d\vec{A}}$$

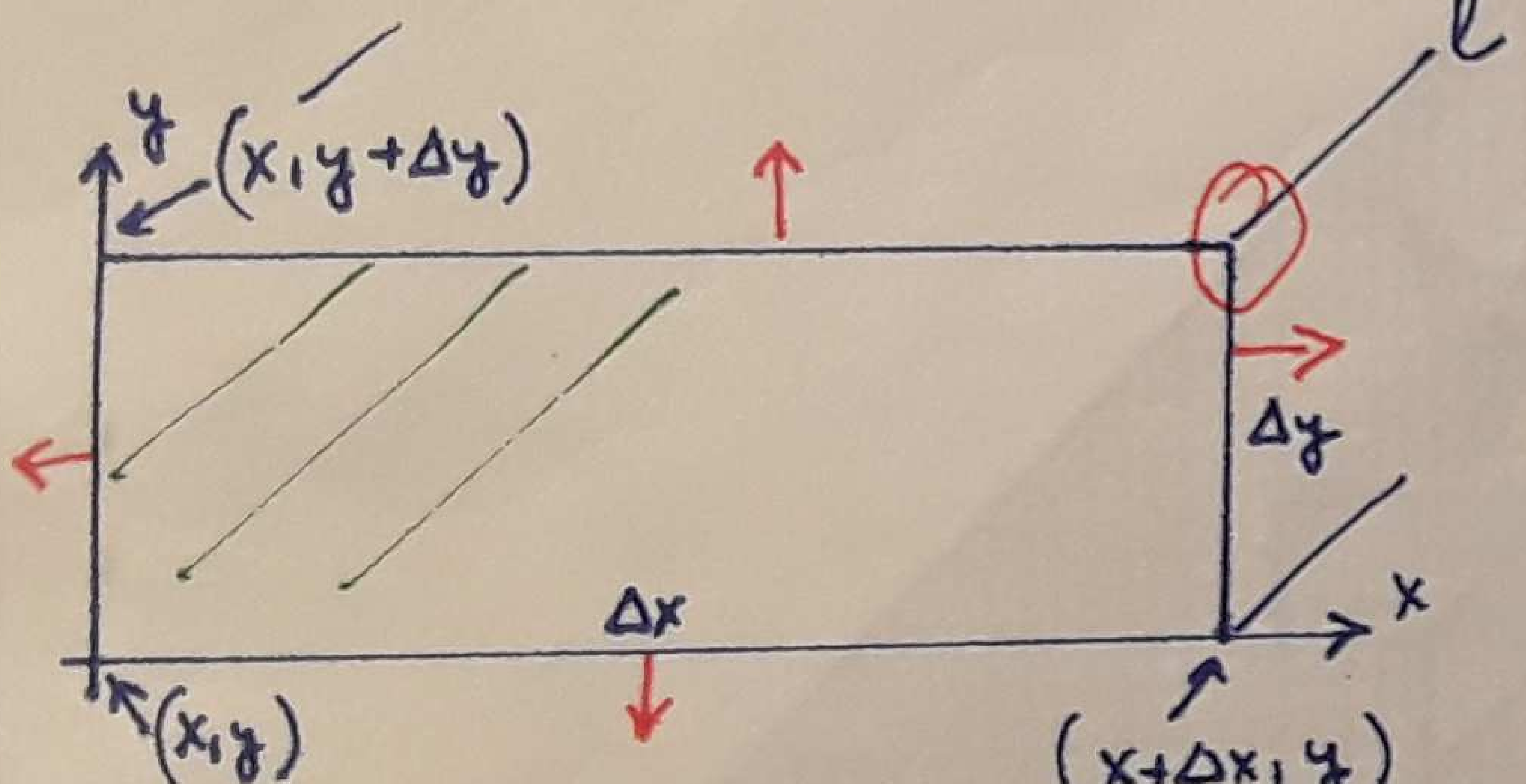


$$\oint_A \vec{v}\cdot d\vec{A} \begin{cases} = \emptyset \\ > 0 \\ < \emptyset \end{cases}$$









$$\boxed{\lim_{\Delta V \to \emptyset} \frac{\oint_A \vec{v}\cdot d\vec{A}}{\Delta V} = \operatorname{div}\vec{v}}$$

$$\boxed{\operatorname{div}\vec{v} = \left| \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right| + \frac{\partial v_z}{\partial z}}$$

$\vec{v} = v_x \vec{e}_x + v_y \vec{e}_y$

Alul: $\vec{v}(x,y)\,\Delta x \cdot (-\vec{e}_y)\, l$

Jobb: $\vec{v}(x+\Delta x, y)\,\Delta y\, l \cdot (\vec{e}_x)$

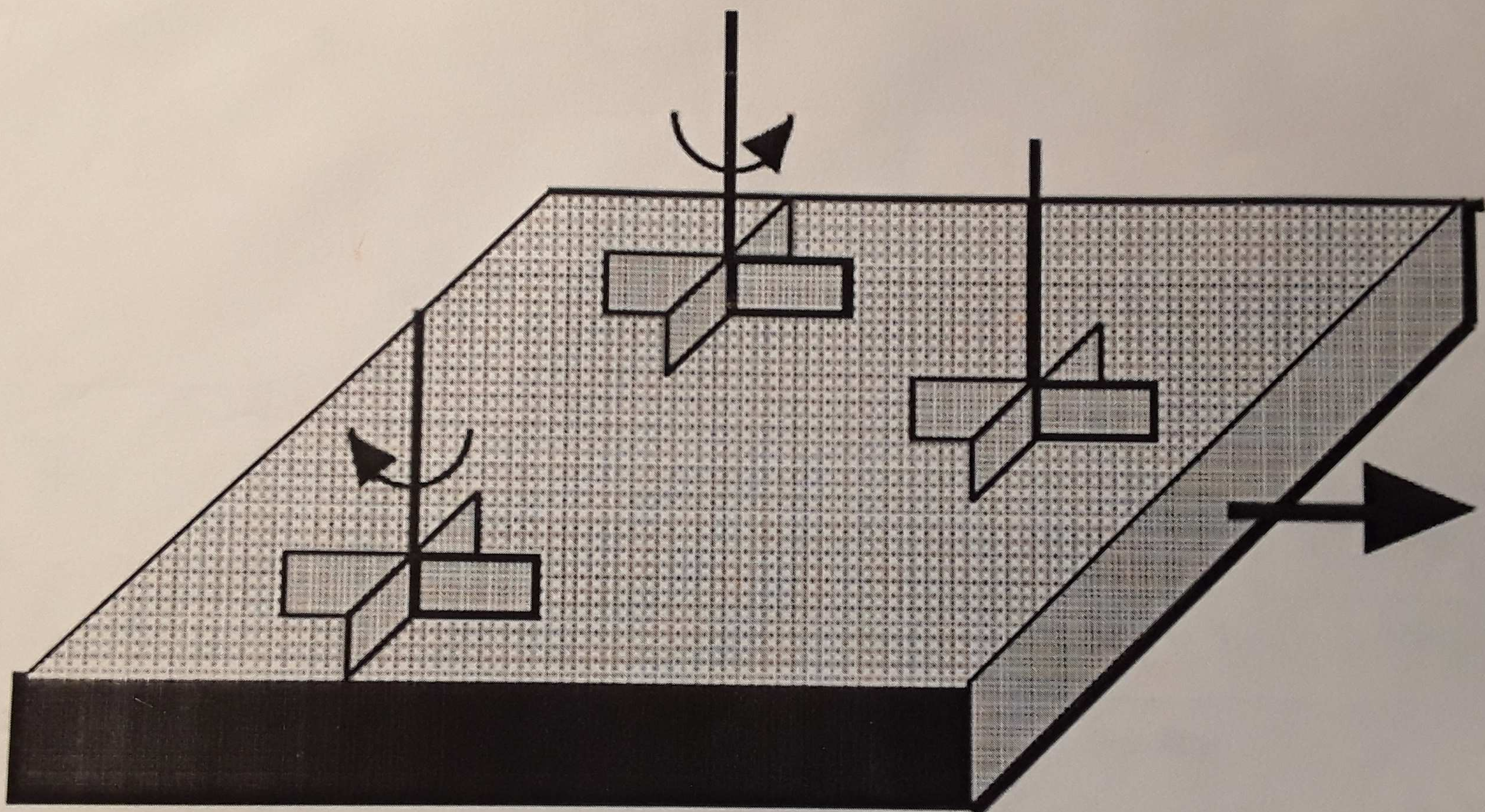
Felül: $\vec{v}(x, y+\Delta y)\,\Delta x\, l \cdot (\vec{e}_y)$

Bal: $\vec{v}(x,y)\,\Delta y\, l \cdot (-\vec{e}_x)$

$$\frac{-v_y(x,y)\Delta x\, l + v_x(x+\Delta x, y)\Delta y\, l + v_y(x, y+\Delta y)\Delta x\, l - v_x(x,y)\Delta y\, l}{\Delta x \Delta y\, l}$$

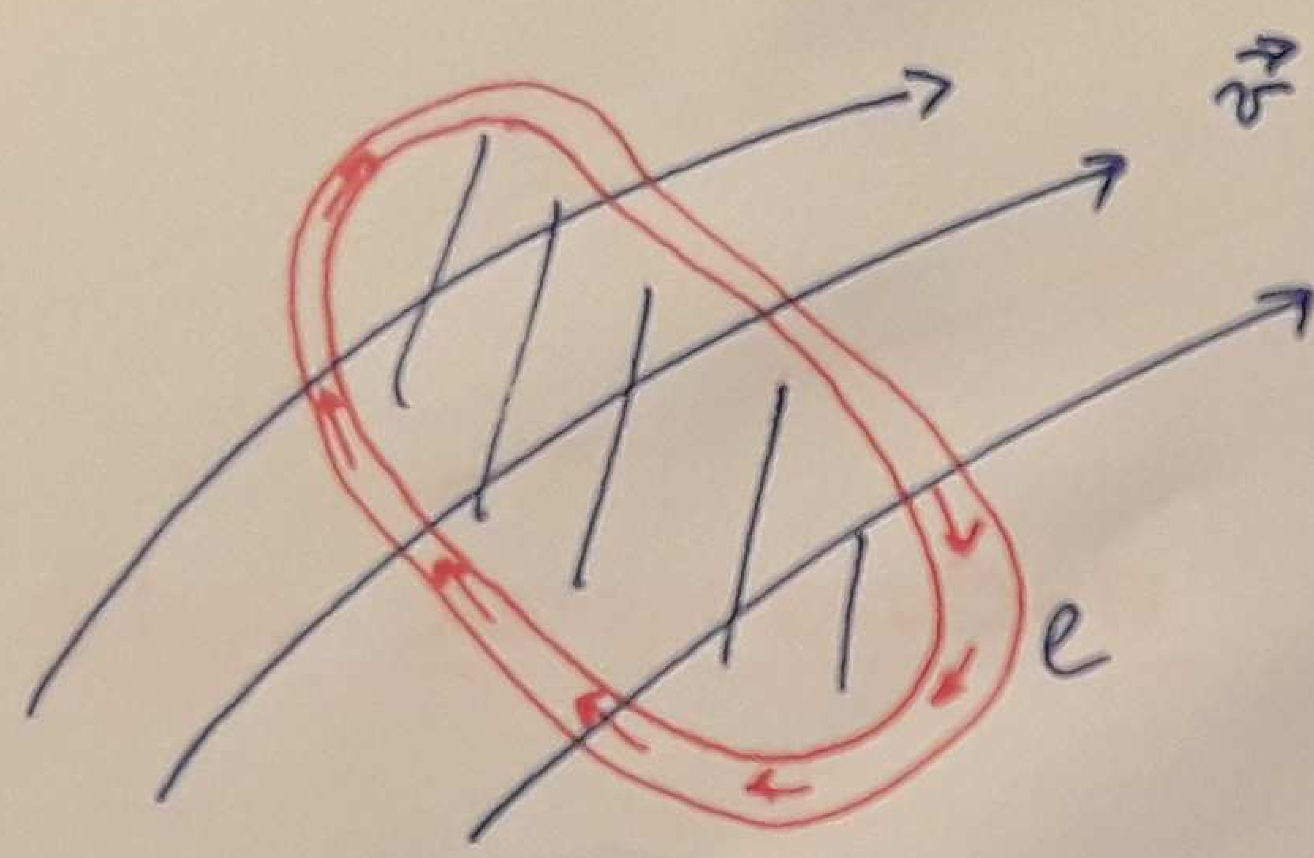
$$\frac{-v_y(x,y)}{\Delta y} + \frac{v_x(x+\Delta x, y)}{\Delta x} + \frac{v_y(x, y+\Delta y)}{\Delta y} - \frac{v_x(x,y)}{\Delta x}$$

$$\underset{\Delta x \to \emptyset}{\frac{v_x(x+\Delta x, y) - v_x(x,y)}{\Delta x}} \searrow \frac{\partial v_x}{\partial x} + \underset{\Delta y \to \emptyset}{\frac{v_y(x, y+\Delta y) - v_y(x,y)}{\Delta y}} \searrow \frac{\partial v_y}{\partial y}$$



A kis lapátkerék azt méri, milyen gyorsan folyik a folyadék és milyen irányba, azaz milyen gyorsan forog a kis lapátkerék és milyen irányban.

- ROTÁCIÓ



$$\boxed{\oint_e \vec{v}\cdot d\vec{l}}$$ ÖRVÉNYERŐSSÉG.

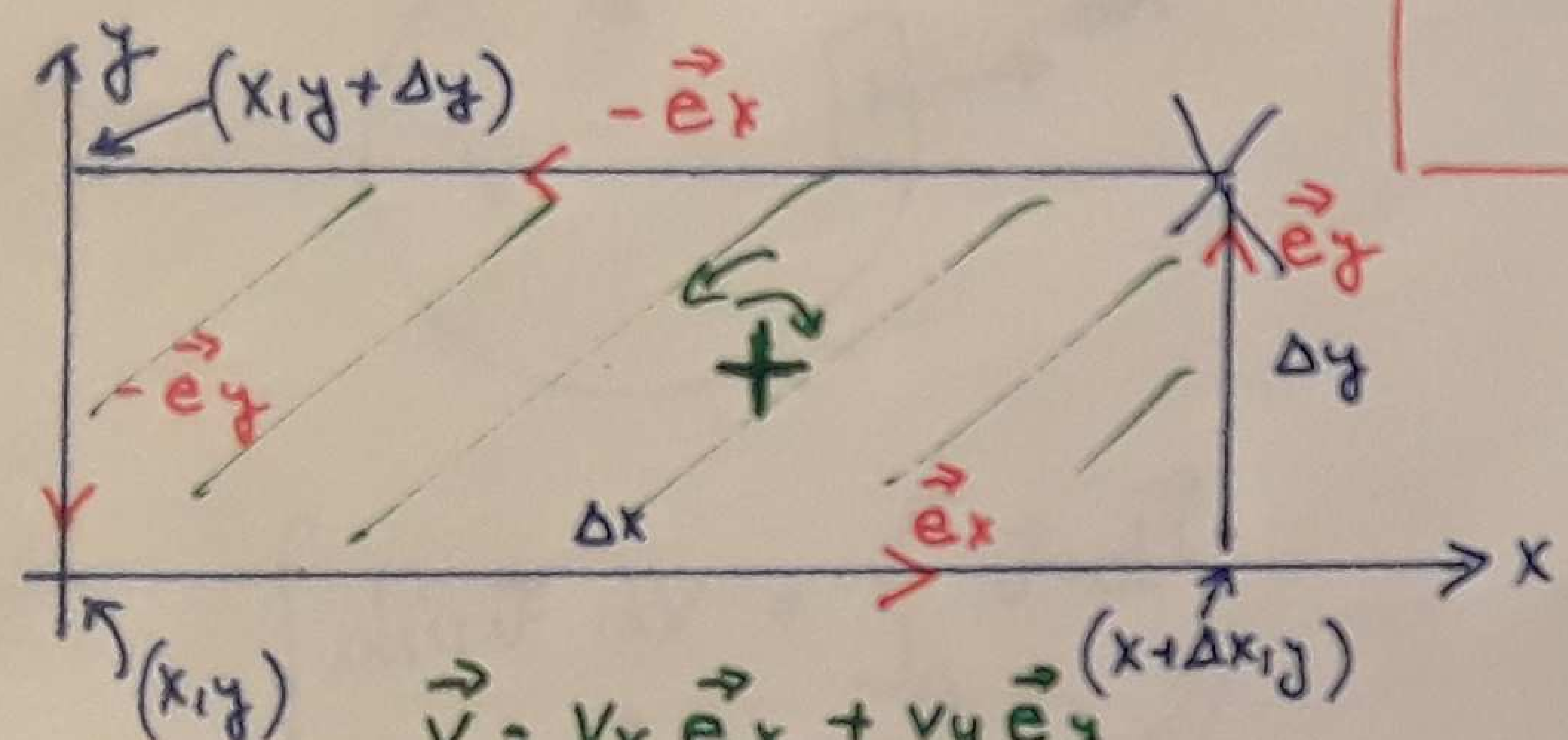
ÖRV. ERŐSSÉGSŰRŰSÉG

ROTÁCIÓ

$$\boxed{\operatorname{rot}\vec{v} = \lim_{\Delta A \to 0} \frac{\oint_e \vec{v}\cdot d\vec{l}}{\Delta A}}$$

$$\operatorname{rot}\vec{v} \overset{?}{=} \vec{0}$$

$$\operatorname{rot}\vec{v} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix}$$



$$\vec{v} = v_x\vec{e}_x + v_y\vec{e}_y$$

Alul: $\vec{v}(x,y)\cdot\Delta x\,\vec{e}_x$

Jobb: $\vec{v}(x+\Delta x,y)\cdot\Delta y\,\vec{e}_y$

Felső: $\vec{v}(x,y+\Delta y)\cdot\Delta x\,(-\vec{e}_x)$

Bal: $\vec{v}(x,y)\,\Delta y\cdot(-\vec{e}_y)$

$$\frac{v_x(x,y)\Delta x + v_y(x+\Delta x,y)\Delta y - v_x(x,y+\Delta y)\Delta x - v_y(x,y)\Delta y}{\Delta x\Delta y}$$

$$\frac{v_x(x,y)}{\Delta y} + \frac{v_y(x+\Delta x,y)}{\Delta x} - \frac{v_x(x,y+\Delta y)}{\Delta y} - \frac{v_y(x,y)}{\Delta x}$$

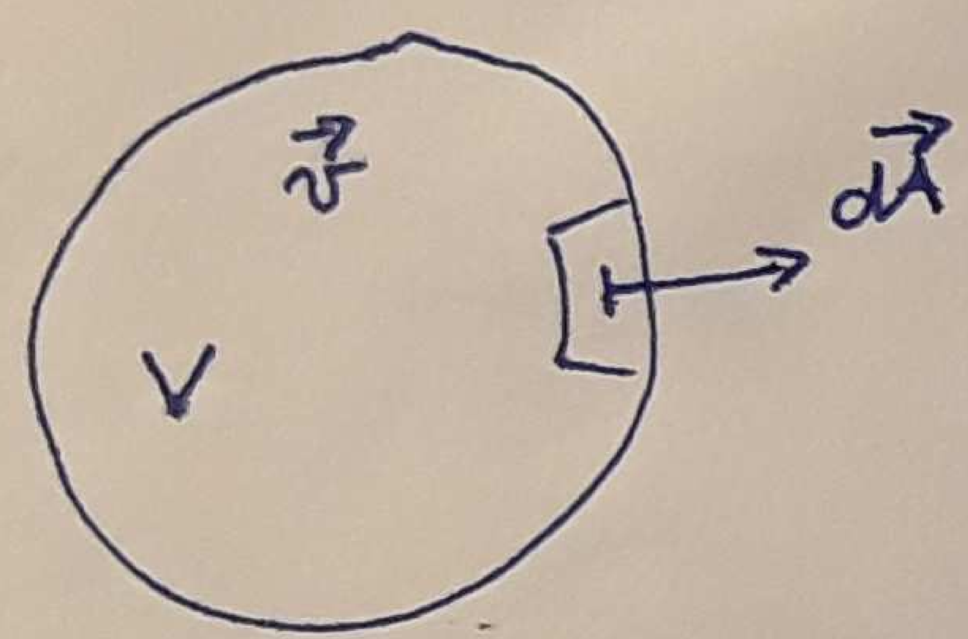
$$\frac{v_y(x+\Delta x,y) - v_y(x,y)}{\Delta x} - \frac{v_x(x,y+\Delta y) - v_x(x,y)}{\Delta y}$$

$\Delta x \to 0$ $\qquad$ $\Delta y \to 0$

$$\boxed{\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}}$$

INTEGRÁLTÉTELEK

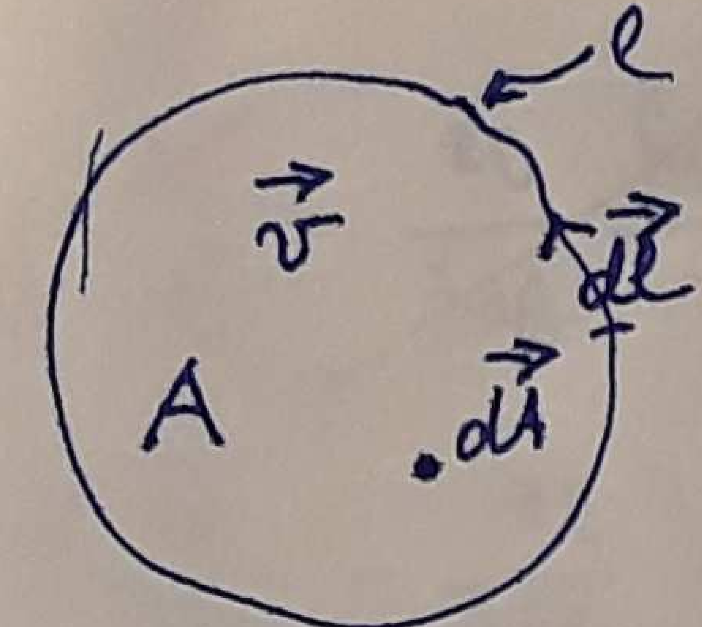
GAUSS-OSZTROGRADSZKIJ



$$\int_V \operatorname{div}\vec{v}\, dV = \oint_A \vec{v}\cdot d\vec{A}$$

$$\operatorname{div}\vec{v} = \lim_{\Delta V \to 0} \frac{\oint_A \vec{v}\cdot d\vec{A}}{\Delta V}$$

STOKES

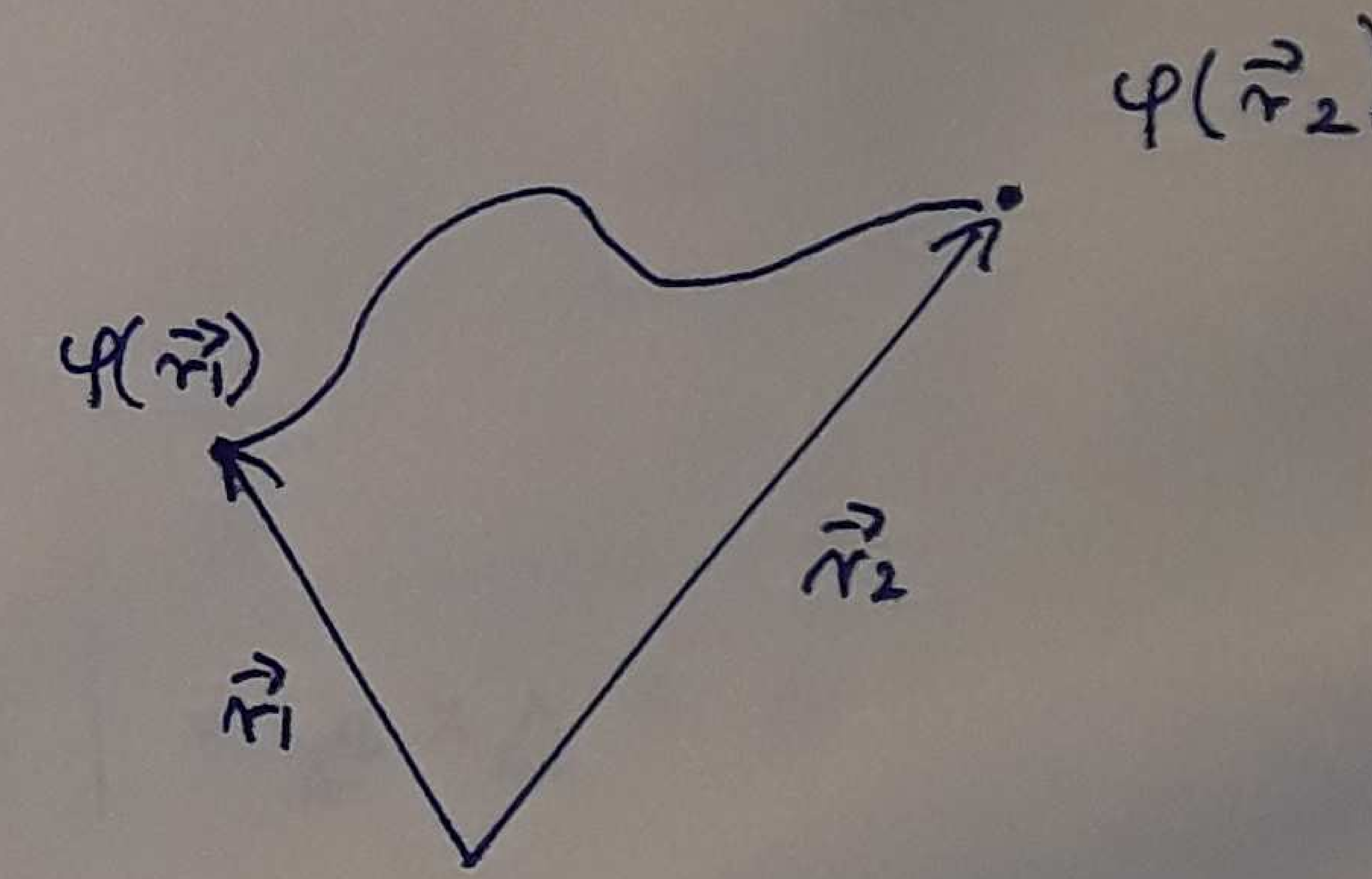


$$\int_A \operatorname{rot}\vec{v}\cdot d\vec{A} = \oint_l \vec{v}\cdot d\vec{l}$$

$$\operatorname{rot}\vec{v} = \lim_{\Delta A \to 0} \frac{\oint_l \vec{v}\cdot d\vec{l}}{\Delta A}$$

GRADIENS

$$\int_{\vec{r}_1}^{\vec{r}_2} \operatorname{grad}\varphi \cdot d\vec{r} = \varphi(\vec{r}_2) - \varphi(\vec{r}_1)$$



NABLA-OPERÁTOR   Hamilton

$$\nabla = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z$$

$$\operatorname{grad}\varphi = \nabla\varphi = \left(\frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z\right)\varphi = \frac{\partial\varphi}{\partial x}\vec{e}_x + \frac{\partial\varphi}{\partial y}\vec{e}_y + \frac{\partial\varphi}{\partial z}\vec{e}_z$$

$$\operatorname{div}\vec{v} = \nabla\cdot\vec{v} = \left(\frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z\right)\cdot\left(v_x\vec{e}_x + v_y\vec{e}_y + v_z\vec{e}_z\right) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\operatorname{rot}\vec{v} = \nabla\times\vec{v} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix} = \vec{e}_x\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \vec{e}_y\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \vec{e}_z\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

$$\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ v_x & v_z \end{vmatrix}$$

NÉHÁNY ISMÉTELT MŰVELET

$$\operatorname{div}\boxed{\operatorname{rot}\vec{v}} \equiv \emptyset \qquad \frac{\partial}{\partial x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

$$\cancel{\frac{\partial}{\partial x}\frac{\partial v_z}{\partial y}} - \cancel{\frac{\partial}{\partial x}\frac{\partial v_y}{\partial z}} + \cancel{\frac{\partial}{\partial y}\frac{\partial v_x}{\partial z}} - \cancel{\frac{\partial}{\partial y}\frac{\partial v_z}{\partial x}} + \cancel{\frac{\partial}{\partial z}\frac{\partial v_y}{\partial x}} - \cancel{\frac{\partial}{\partial z}\frac{\partial v_x}{\partial y}} = \emptyset$$

$$\operatorname{rot}\underbrace{\operatorname{grad}\varphi}_{} \equiv \emptyset$$

$$\frac{\partial\varphi}{\partial x}\vec{e}_x + \frac{\partial\varphi}{\partial y}\vec{e}_y + \frac{\partial\varphi}{\partial z}\vec{e}_z$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\varphi}{\partial x} & \frac{\partial\varphi}{\partial y} & \frac{\partial\varphi}{\partial z} \end{vmatrix} = \bar{e}_x\underbrace{\left(\frac{\partial^2\varphi}{\partial y\,\partial z} - \frac{\partial^2\varphi}{\partial y\,\partial z}\right)}_{=\emptyset} + \bar{e}_y\underbrace{\left(\frac{\partial^2\varphi}{\partial x\,\partial z} - \frac{\partial^2\varphi}{\partial x\,\partial z}\right)}_{=\emptyset}$$

Példák:

- $\varphi(x,y) = 3(x^2 - y^2)$; $\operatorname{grad}\varphi = ?$

$$\operatorname{grad}\varphi = \underbrace{\frac{\partial\varphi}{\partial x}}_{6x}\vec{e}_x + \underbrace{\frac{\partial\varphi}{\partial y}}_{-6y}\vec{e}_y = 6x\,\vec{e}_x - 6y\,\vec{e}_y .$$

- $\vec{F}(x,y) = \underbrace{(x^2 - y)}_{F_x}\vec{e}_x + \underbrace{(xy - y^2)}_{F_y}\vec{e}_y$; $\operatorname{div}\vec{F} = ?$

$$\operatorname{div}\vec{F} = \underbrace{\frac{\partial F_x}{\partial x}}_{2x} + \underbrace{\frac{\partial F_y}{\partial y}}_{x-2y} = 3x - 2y .$$

- $\vec{F}(x,y) = \underbrace{(x^2 - y)}_{F_x}\vec{e}_x + \underbrace{(xy - y^2)}_{F_y}\vec{e}_y$; $\operatorname{rot}\vec{F} = ?$

$$\operatorname{rot}\vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \emptyset \\ F_x & F_y & \emptyset \end{vmatrix} = \vec{e}_z\left(\underbrace{\frac{\partial F_y}{\partial x}}_{y} - \underbrace{\frac{\partial F_x}{\partial y}}_{-1}\right) = \vec{e}_z\,(y+1).$$

ELEKTRODINAMIKA TÖRVÉNYEI → MAXWELL-EGYENLETEK

Induktív úton

Deduktív úton.

VILLAMOS ; ELEKTROMOS TÉR

GERJESZTŐ MENNYISÉGEK

MÁGNESES TÉR

Az elektromos ; mágneses, elektromágneses teret hozzák létre.

TÖLTÉS

ÁRAM

TÉRINTENZITÁSOK

ELEKTROMOS TÉRERŐSSÉG

(feszültség)

Erőhatás

MÁGNESES INDUKCIÓ

(fluxus)

GERJESZTETTSÉGI MENNYISÉGEK

ELEKTROMOS ELTOLÁS

(EL. GERJESZTETTSÉG)

Anyag (közeg) jelenléte

MÁGNESES TÉRERŐSSÉG

ELEKTROMOS TÉR

ÁRAM: TÖLTÉSSEL BÍRÓ ELEMI RÉSZECSKÉK EGYIRÁNYÚ MOZGÁSA, ÁRAMLÁSA.

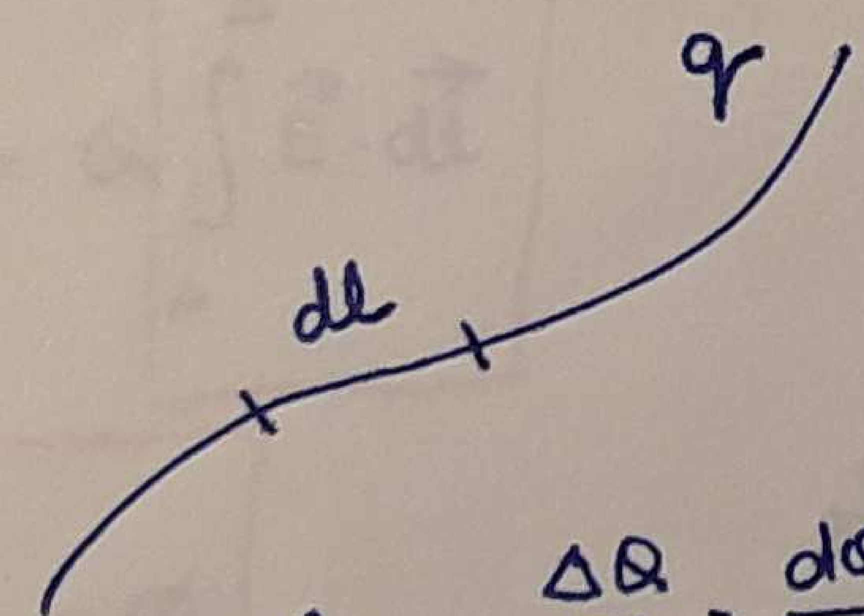
$\rightarrow I = \dfrac{Q}{t} \quad \mapsto \quad Q = I \cdot t$ $\qquad [Q] = As = C.$ coulomb.

$\xrightarrow{\text{dlt.}} i(t) = \dfrac{dQ}{dt} \quad \left(\lim\limits_{\Delta t \to 0} \dfrac{\Delta Q}{\Delta t}\right) \quad \mapsto \quad Q(t) = \int_{-\infty}^{t} i(\tau)\, d\tau$

ELŐFORDULÁSI FORMÁK:

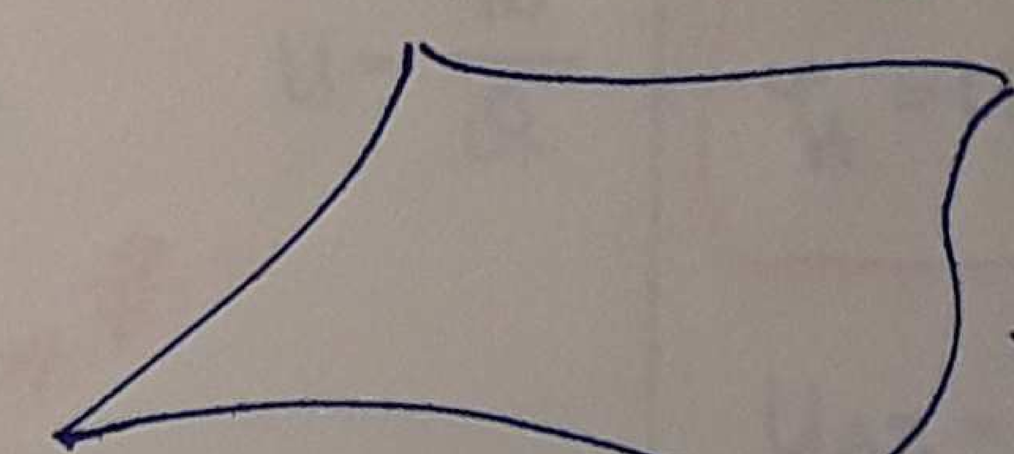
Q

C



$q = \lim\limits_{\Delta l \to 0} \dfrac{\Delta Q}{\Delta l} = \dfrac{dQ}{dl}$

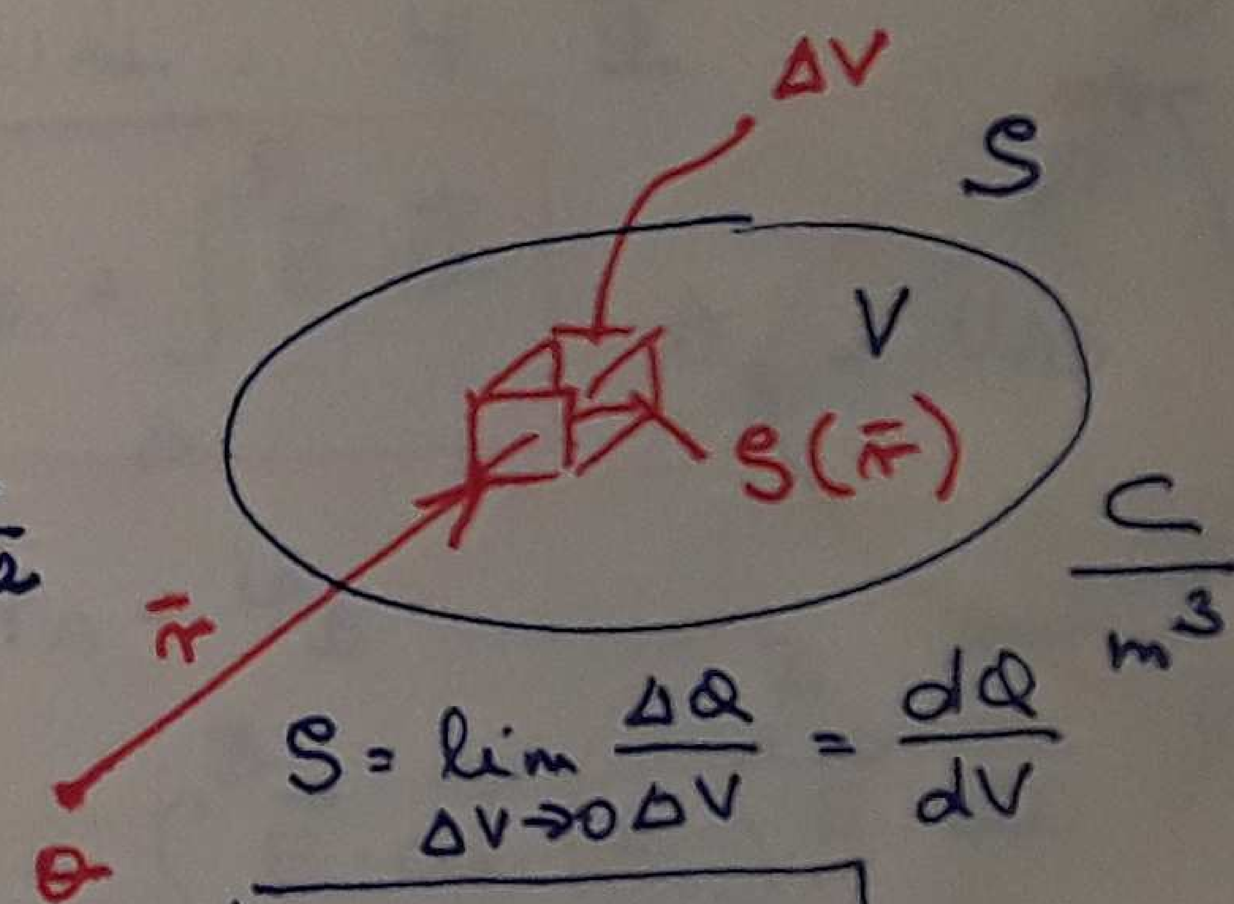
$\boxed{Q = \int_l q\, dl}$ $\quad \dfrac{C}{m}$



$\dfrac{C}{m^2}$

$\sigma = \lim\limits_{\Delta A \to 0} \dfrac{\Delta Q}{\Delta A} = \dfrac{dQ}{dA}$

$\boxed{Q = \int_A \sigma\, dA}$



$\dfrac{C}{m^3}$

$\rho = \lim\limits_{\Delta V \to 0} \dfrac{\Delta Q}{\Delta V} = \dfrac{dQ}{dV}$

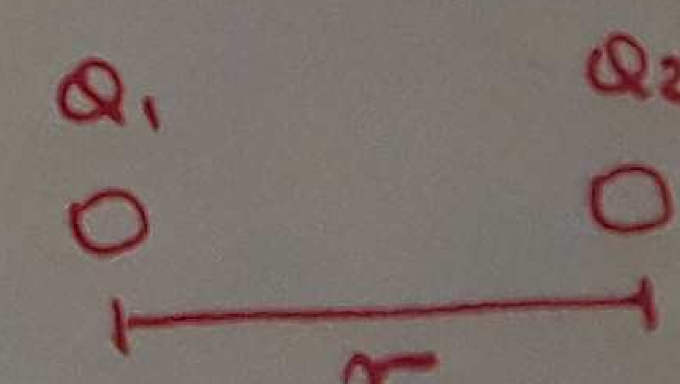
$\boxed{Q = \int_V \rho\, dV}$

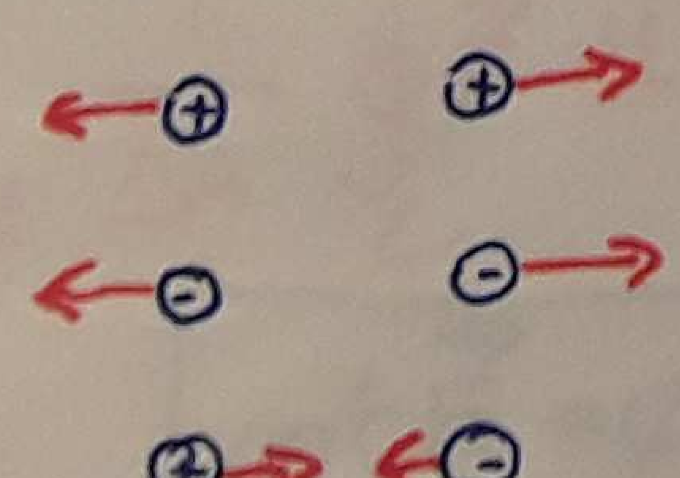
! MILLIKAN-KÍSÉRLET : $Q = Ne$

$\boxed{-1{,}602 \cdot 10^{-19}\, C}$

! COULOMB-TÖRVÉNY :

$\bar{F} = \boxed{\dfrac{1}{4\pi\varepsilon}} \dfrac{Q_1 Q_2}{r^2}\, \bar{e}$





TÖLTÉS $\longrightarrow$ ELEKTROMOS ERŐTÉR.

$W = Fs \qquad W = Pt$

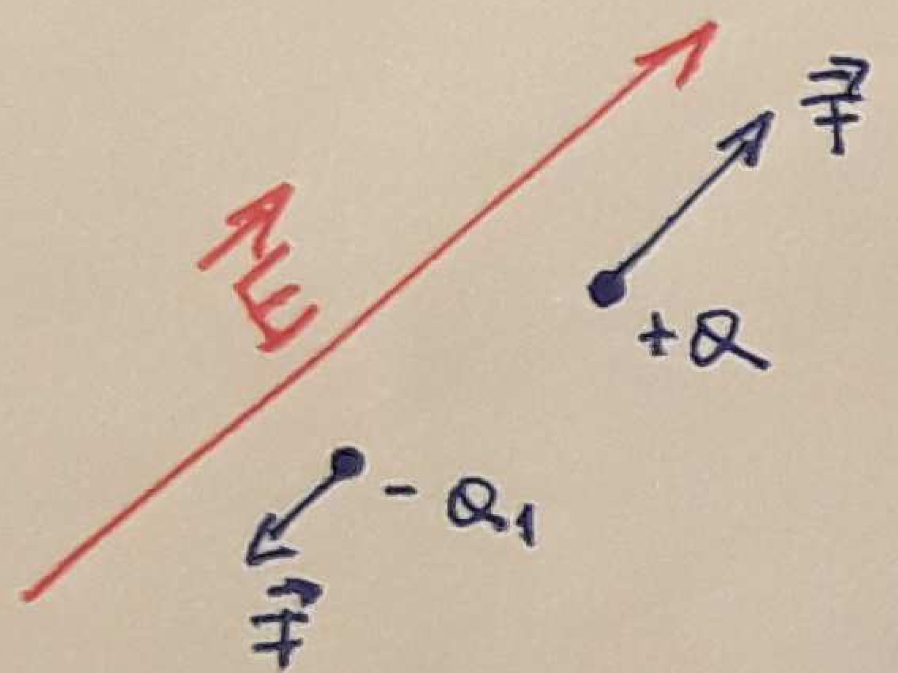
ELEKTROMOS TÉRERŐSSÉG.

$\vec{F} = Q\boxed{\vec{E}} \qquad \vec{E} = \frac{\vec{F}}{Q}$

$E = \frac{F}{Q} \rightarrow \frac{N}{C} = \frac{F/m}{As} = \frac{VAs}{Asm} = \boxed{\frac{V}{m}} \qquad \frac{kV}{cm}$

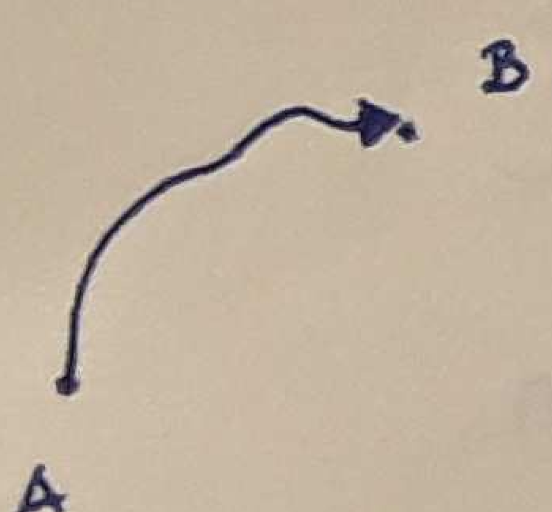
$1\frac{kV}{cm} = 10^5\frac{V}{m}$

$10\frac{kV}{cm} \qquad 100\frac{V}{m}$

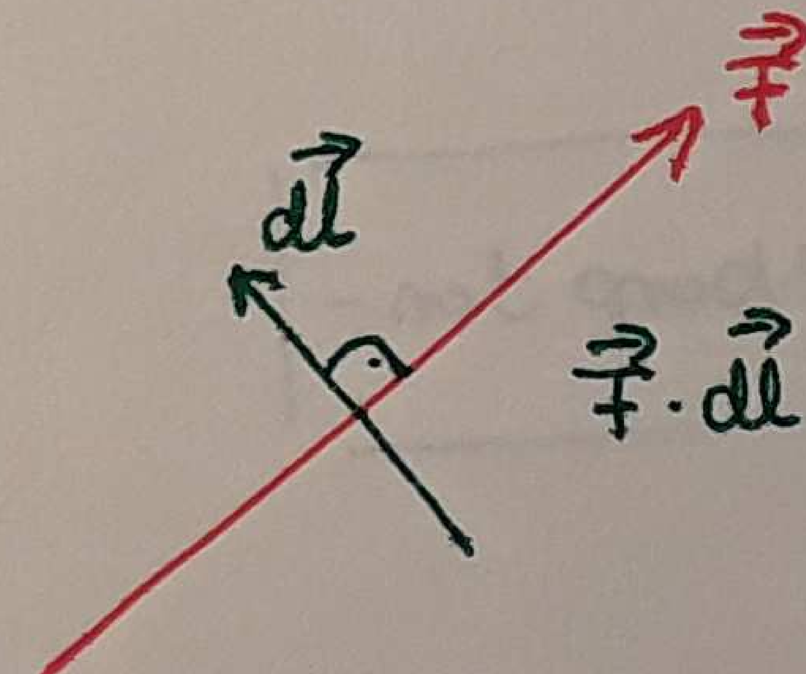


MUNKA:

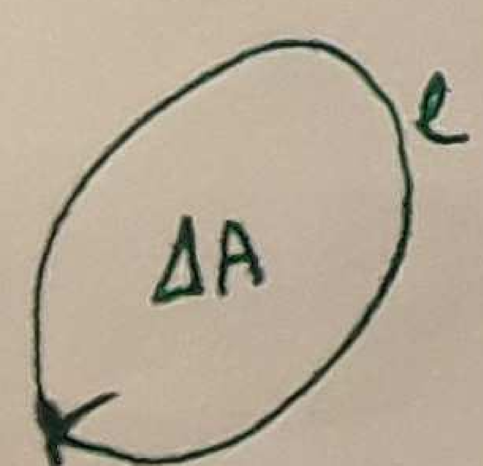
$W = \int_A^B \vec{F}\cdot d\vec{l} = Q\boxed{\int_A^B \vec{E}\cdot d\vec{l}}\; U \qquad W = QU \qquad U = \frac{W}{Q}$



$$\boxed{W = \oint_l \vec{F}\cdot d\vec{l} = \emptyset}$$



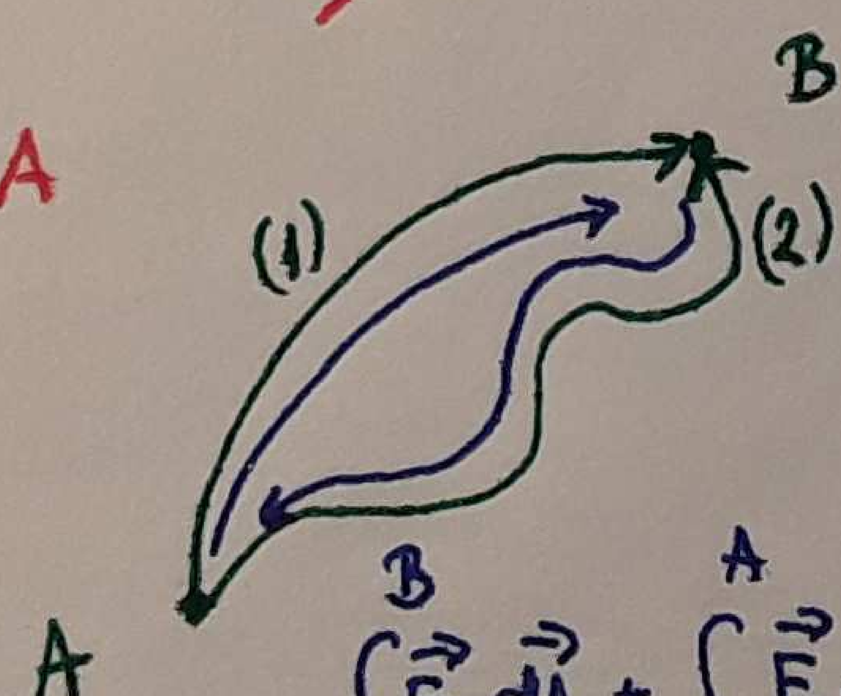
$\vec{F}\cdot d\vec{l} = \emptyset$



$$\boxed{\oint_l \vec{E}\cdot d\vec{l} = \emptyset} \quad :/\Delta A$$

$$\lim_{\Delta A \to \emptyset} \frac{1}{\Delta A}\oint_l \vec{E}\cdot d\vec{l} = \emptyset$$

$$\boxed{\mathrm{rot}\,\vec{E} = \emptyset}$$

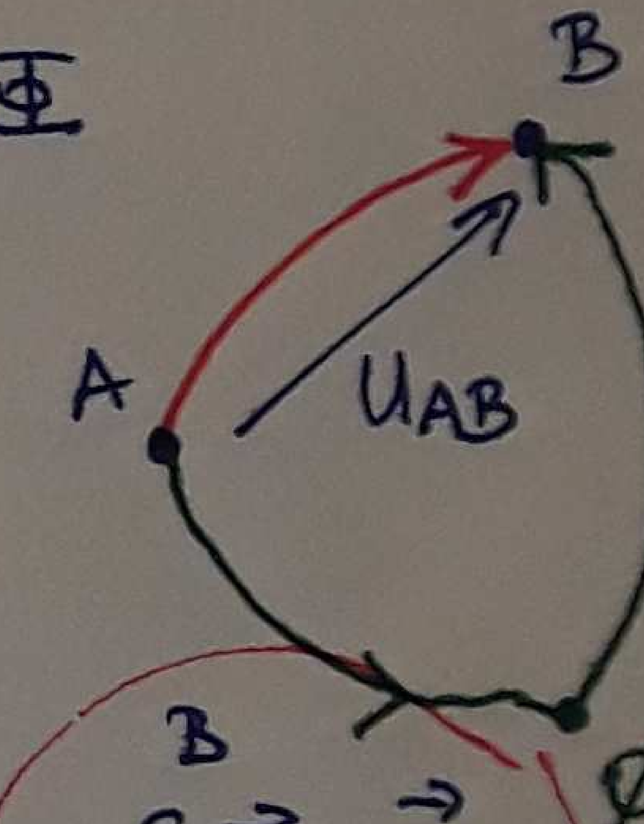


$\int_A^B \vec{E}\cdot d\vec{l} + \int_B^A \vec{E}\cdot d\vec{l} = \emptyset$

$\int_{A(1)}^B \vec{E}\cdot d\vec{l} = \int_{A(2)}^B \vec{E}\cdot d\vec{l}\; \cancel{=\emptyset}$

POTENCIÁL: $\varphi \quad \Phi$

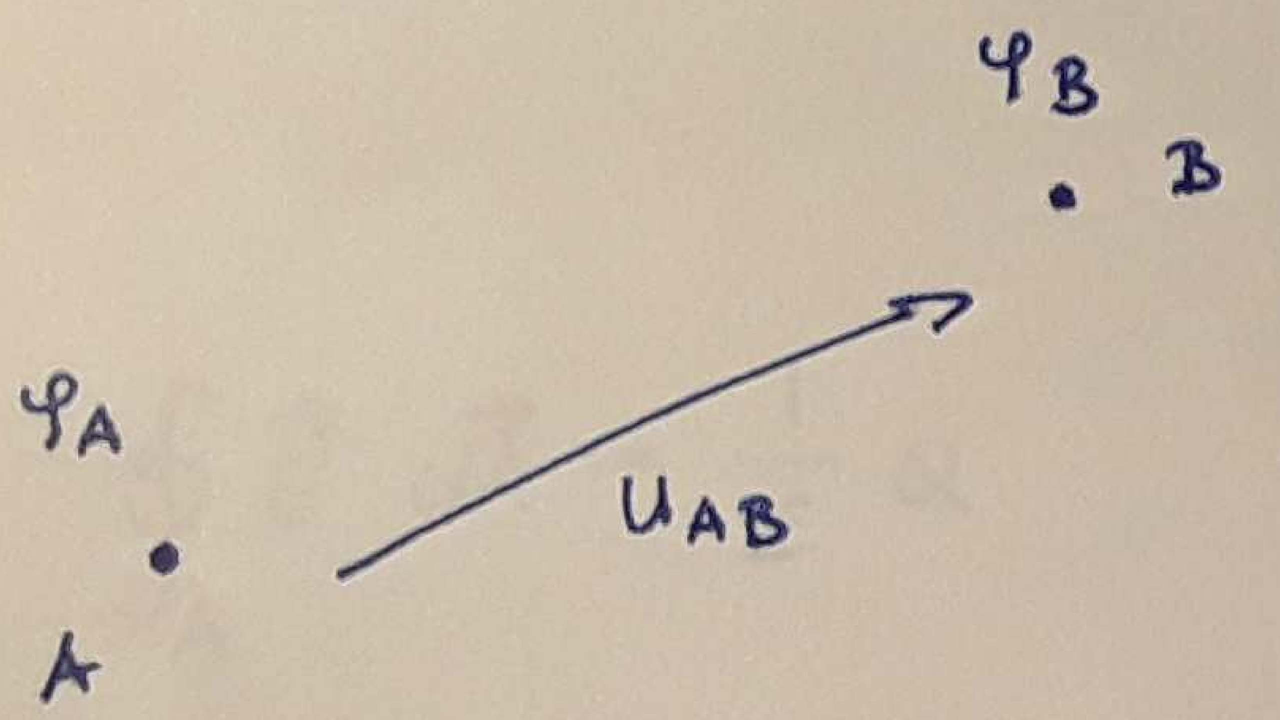
$$\boxed{\varphi_A = U_{A\emptyset} = \int_A^\emptyset \vec{E}\cdot d\vec{l}}$$



$U_{AB} = \varphi_A - \varphi_B$

$\int_A^\emptyset \vec{E}\cdot d\vec{l} + \int_\emptyset^B \vec{E}\cdot d\vec{l} = \int_A^B \vec{E}\cdot d\vec{l}$

$\int_A^\emptyset \vec{E}\cdot d\vec{l} - \int_B^\emptyset \vec{E}\cdot d\vec{l} = \varphi_A - \varphi_B$



$$\varphi_A = \int_A^{\varnothing} \vec{E}\cdot d\vec{l}$$

$$U_{AB} = \varphi_A - \varphi_B = \int_A^B \vec{E}\, d\vec{l}$$

$$\varphi_B - \varphi_A = \int_A^B \operatorname{grad}\varphi \cdot d\vec{l}$$

$$\varphi_A - \varphi_B = -\int_A^B \operatorname{grad}\varphi \cdot d\vec{l}$$

$$\boxed{\vec{E} = -\operatorname{grad}\varphi}$$

$$\oint_l \vec{E}\cdot d\vec{l} = \varnothing$$

$$\operatorname{rot}\vec{E} = \varnothing \qquad \underset{\substack{\uparrow \\ 3}}{\vec{E}} = -\operatorname{grad}\underset{\substack{\uparrow \\ 1}}{\varphi}$$

$$\boxed{-\operatorname{rot}\operatorname{grad}\varphi \equiv \varnothing}$$

AZ ELEKTROSZTATIKA GAUSS-TÖRVÉNYE:

$$\oint_A \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon} Q$$

permittivitás; dielektromos állandó

$$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$$

$$\boxed{\varepsilon_0 = 8{,}85 \cdot 10^{-12}\ F/m}$$

$$\varepsilon = \frac{Q}{EA} \Rightarrow \frac{As}{\frac{V}{m}\,m^2} = \frac{As}{Vm} = \frac{F}{m}$$

$$\oint_A \varepsilon\vec{E}\cdot d\vec{A} = Q$$

$$\vec{D} = \varepsilon\vec{E}$$

$$\boxed{\oint_A \vec{D}\cdot d\vec{A} = Q}$$

$$\boxed{\oint_A \vec{D}\cdot d\vec{A} = \int_V \rho\, dV}$$

$$\int_V \operatorname{div}\vec{D}\, dV = \int_V \rho\, dV$$

$$\boxed{\operatorname{div}\vec{D} = \rho}$$

ÖSSZEFOGLALÁS

$Q \quad \sigma$

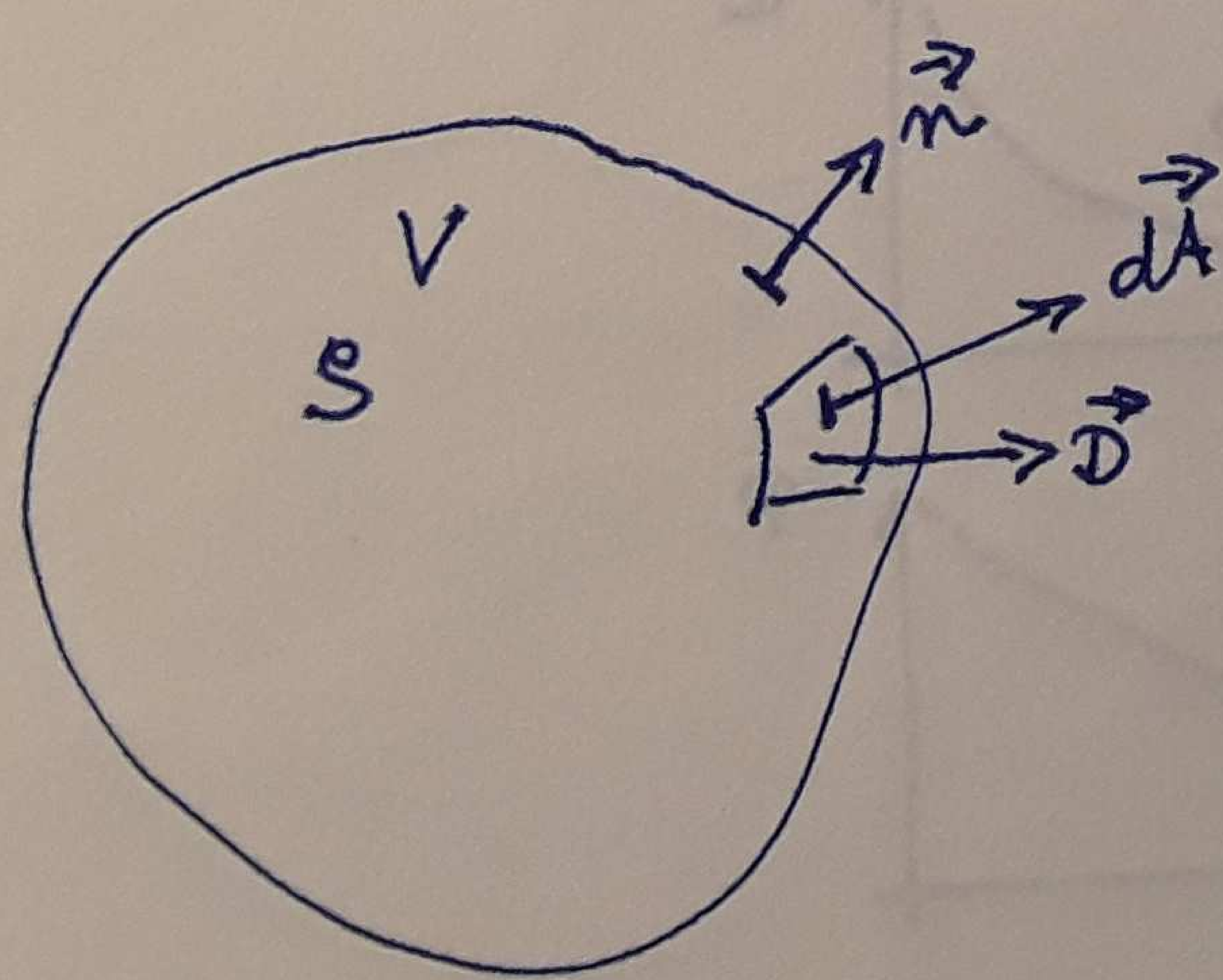
$q \quad \rho \quad \longmapsto \quad \vec{E} \qquad \vec{F} = \vec{E} Q$

$$\boxed{\begin{array}{ll} \oint_l \vec{E} \cdot d\vec{l} = 0 & \mathrm{rot}\, \vec{E} = 0 \\ \oint_A \vec{D} \cdot d\vec{A} = \int_V \rho \, dV & \mathrm{div}\, \vec{D} = \rho \\ \vec{D} = \varepsilon \vec{E} & \end{array}}$$

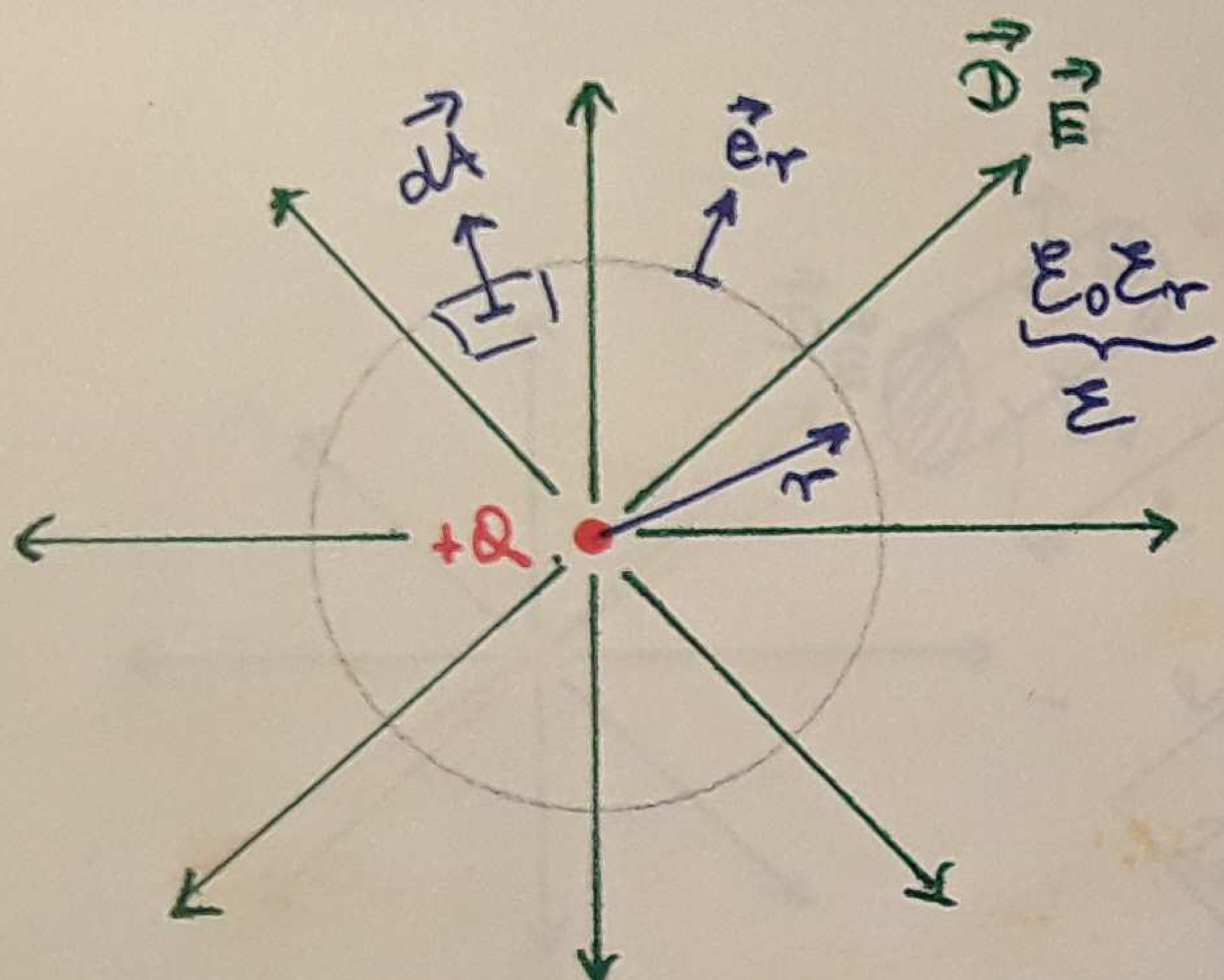
$$\vec{E} = -\,\mathrm{grad}\, \varphi$$

$$C = \frac{Q}{U} \qquad W = QU$$

$$U_{AB} = \varphi_A - \varphi_B = \int_A^B \vec{E} \cdot d\vec{l}$$



Pontszerű töltés:



$\vec{D} \parallel \vec{E}$

$$\boxed{\oint_A \underbrace{\vec{D}\cdot d\vec{A}} = \int_V \rho\, dV}$$

GAUSS-TÖRVÉNY

$$\left.\begin{aligned}\vec{D} &= \vec{e}_r D(r)\\ d\vec{A} &= \vec{e}_r\, dA\end{aligned}\right\} D(r)\,dA\,\underbrace{\vec{e}_r\cdot\vec{e}_r}_{1}$$

$$\oint_A D\,dA = \int_V \rho\, dV \qquad D\underbrace{\oint_A dA}_{A} = \int_V \rho\, dV$$

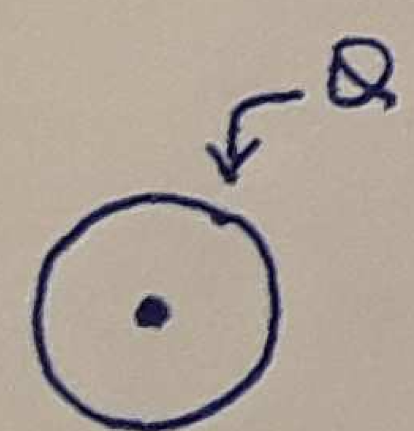
$$D\,4\pi r^2 = Q \rightarrow \boxed{D = \frac{Q}{4\pi r^2}}$$

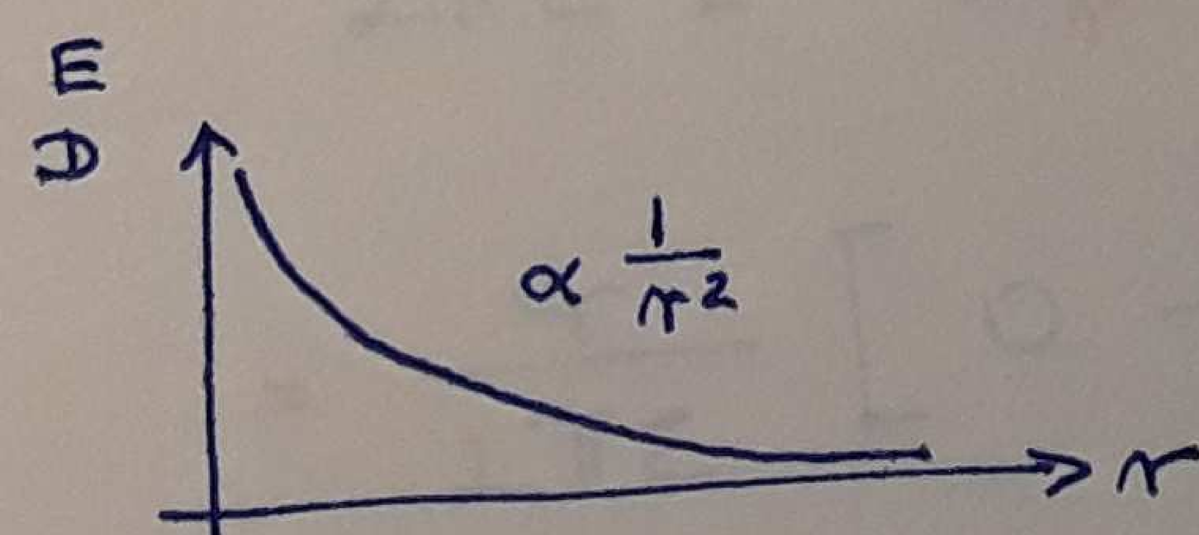
$\vec{D} = \varepsilon\vec{E}$ $\qquad$ $\vec{E} = \vec{e}_r E(r)$

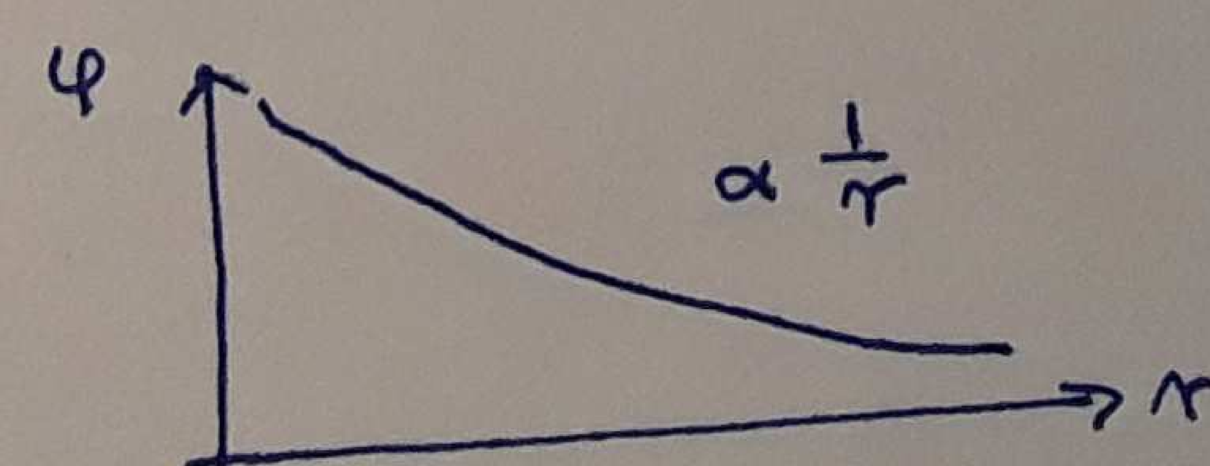
$$\boxed{E = \frac{Q}{4\pi\varepsilon r^2}}$$

$$\varphi(r) = \int_r^{r_0} E\,dr = \int_r^{\infty}\frac{Q}{4\pi\varepsilon r^2}\,dr = \frac{Q}{4\pi\varepsilon}\int_r^{\infty}\frac{1}{r^2}\,dr = \frac{Q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_r^{r_0\to\infty} = \boxed{\frac{Q}{4\pi\varepsilon}\,\frac{1}{r}}$$

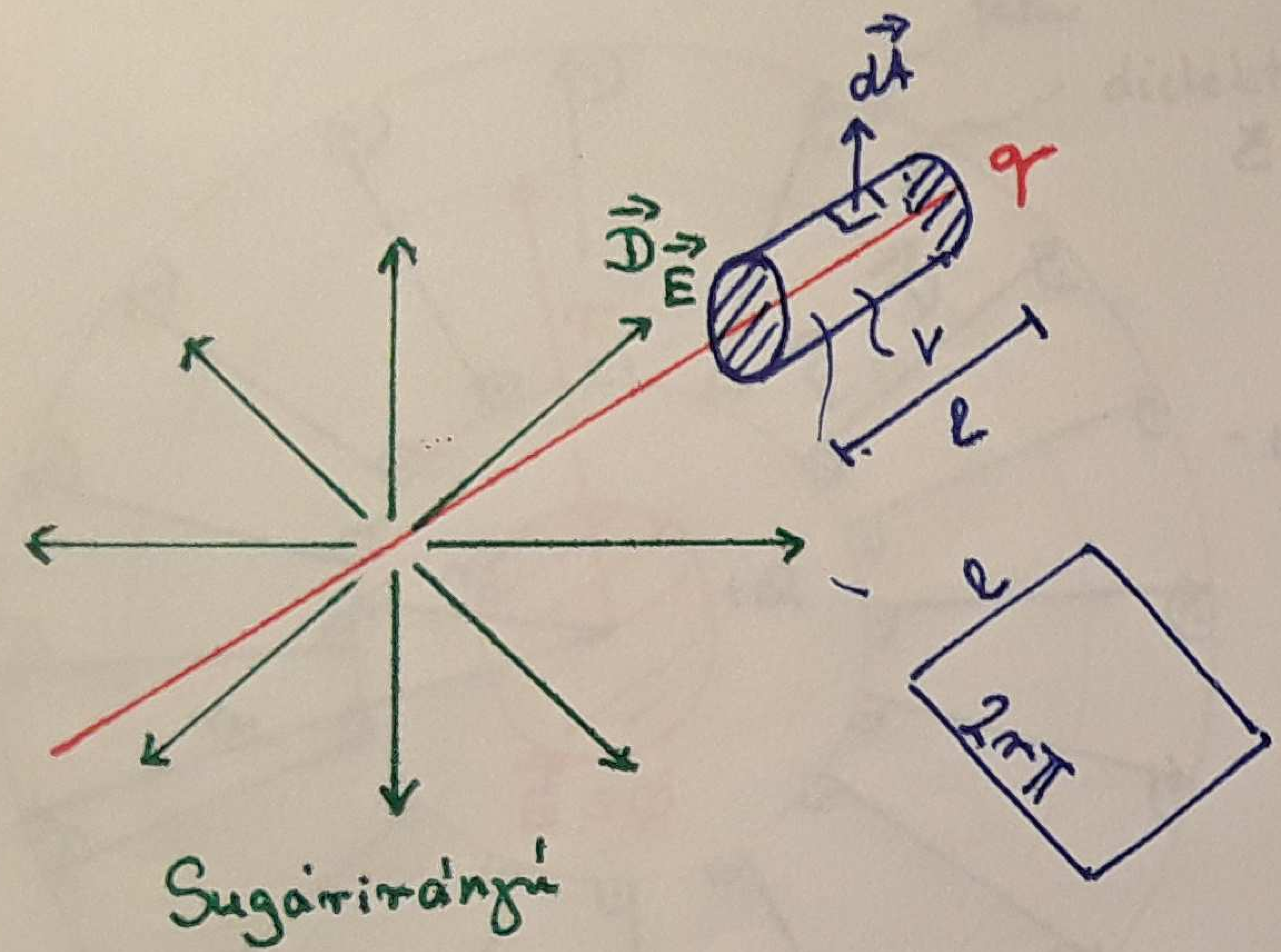
$$-\cancel{\frac{1}{\infty}} + \frac{1}{r}$$







Hosszú egyenes vezető



$$\oint_A \vec{D}\cdot d\vec{A} = \int_V \rho\, dV$$

$$\vec{D} \parallel d\vec{A}$$

$$D \underbrace{\oint_A dA} = ql$$

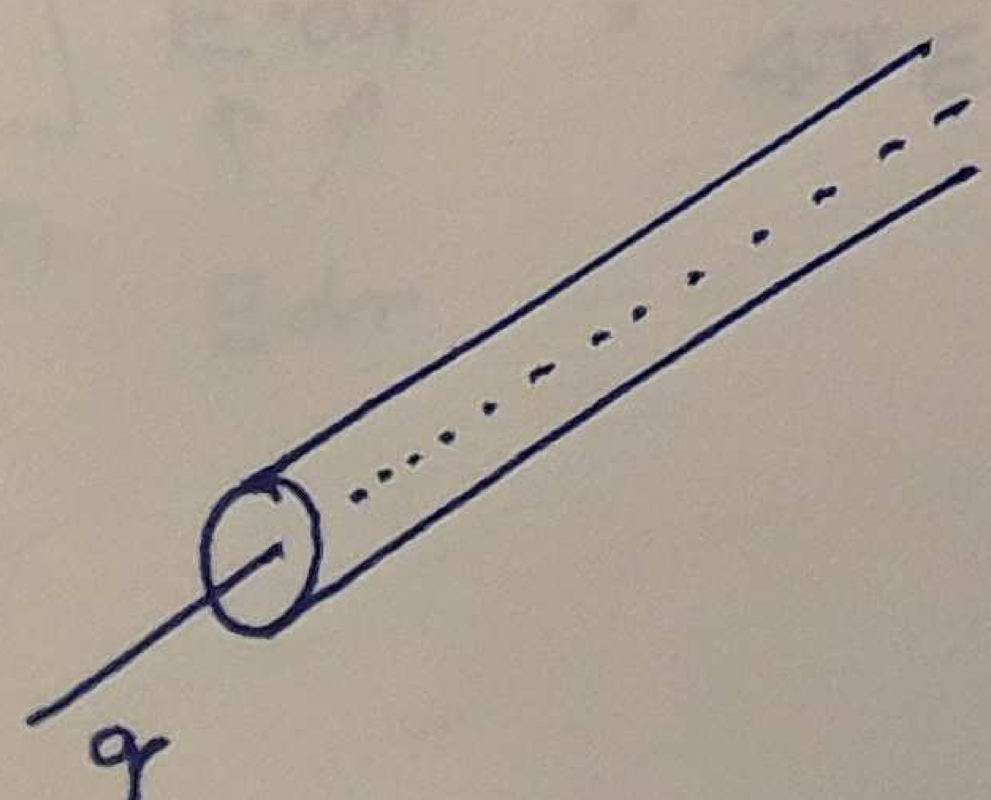
$$D\, 2r\pi \cancel{l} = q\cancel{l} \longrightarrow \boxed{D = \frac{q}{2r\pi}}$$

$$\boxed{E = \frac{q}{2r\pi\varepsilon}}$$

$\ln 1 = \emptyset$

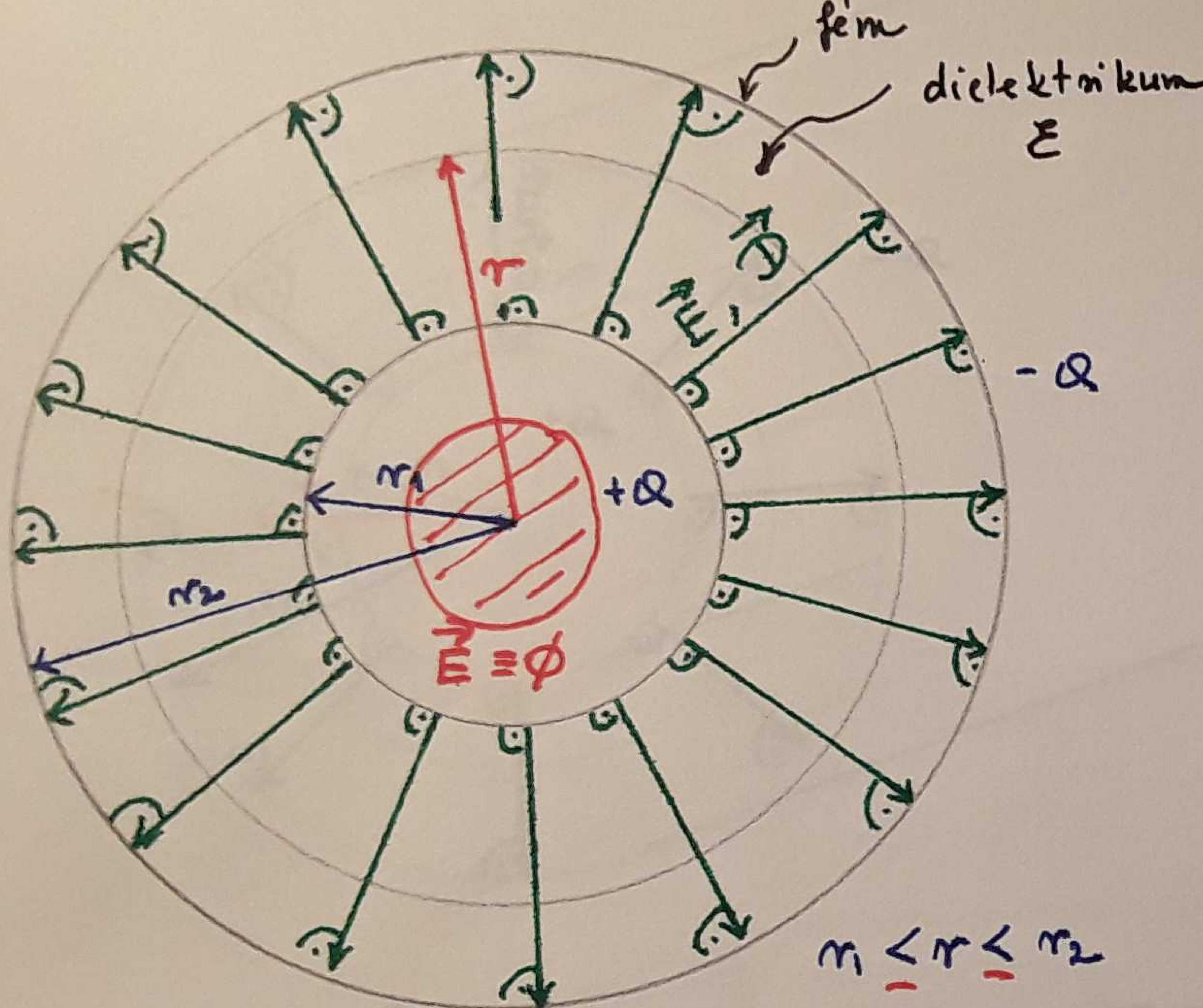
$$\varphi(r) = \int_r^{r_0} E\,dr = \frac{q}{2\pi\varepsilon}\int_r^{1m}\frac{1}{r}\,dr = \frac{q}{2\pi\varepsilon}\Big[\ln r\Big]_r^{r_0 = 1m} =$$

$$= \frac{q}{2\pi\varepsilon}\Big[0 - \ln r\Big] = \boxed{\frac{q}{2\pi\varepsilon}\ln\frac{1}{r}}$$



Gömbkondenzátor

Két, koncentrikus fémgömb, másnéven fegyverzet.



$$\left.\begin{aligned} \vec{D} &= \vec{e}_r \frac{Q}{4\pi r^2} \\ &\downarrow \\ \vec{E} &= \vec{e}_r \frac{Q}{4\pi \varepsilon r^2} \end{aligned}\right\} \quad r_1 \leq r \leq r_2$$

$$\downarrow$$

$$U = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{Q}{4\pi\varepsilon} \frac{r_2 - r_1}{r_1 r_2}$$

$$\downarrow$$

$$C = \frac{Q}{U} = 4\pi\varepsilon \frac{r_1 r_2}{r_2 - r_1}$$

$$\oint_A \vec{D} \cdot d\vec{A} = \int_V \varrho \, dV$$

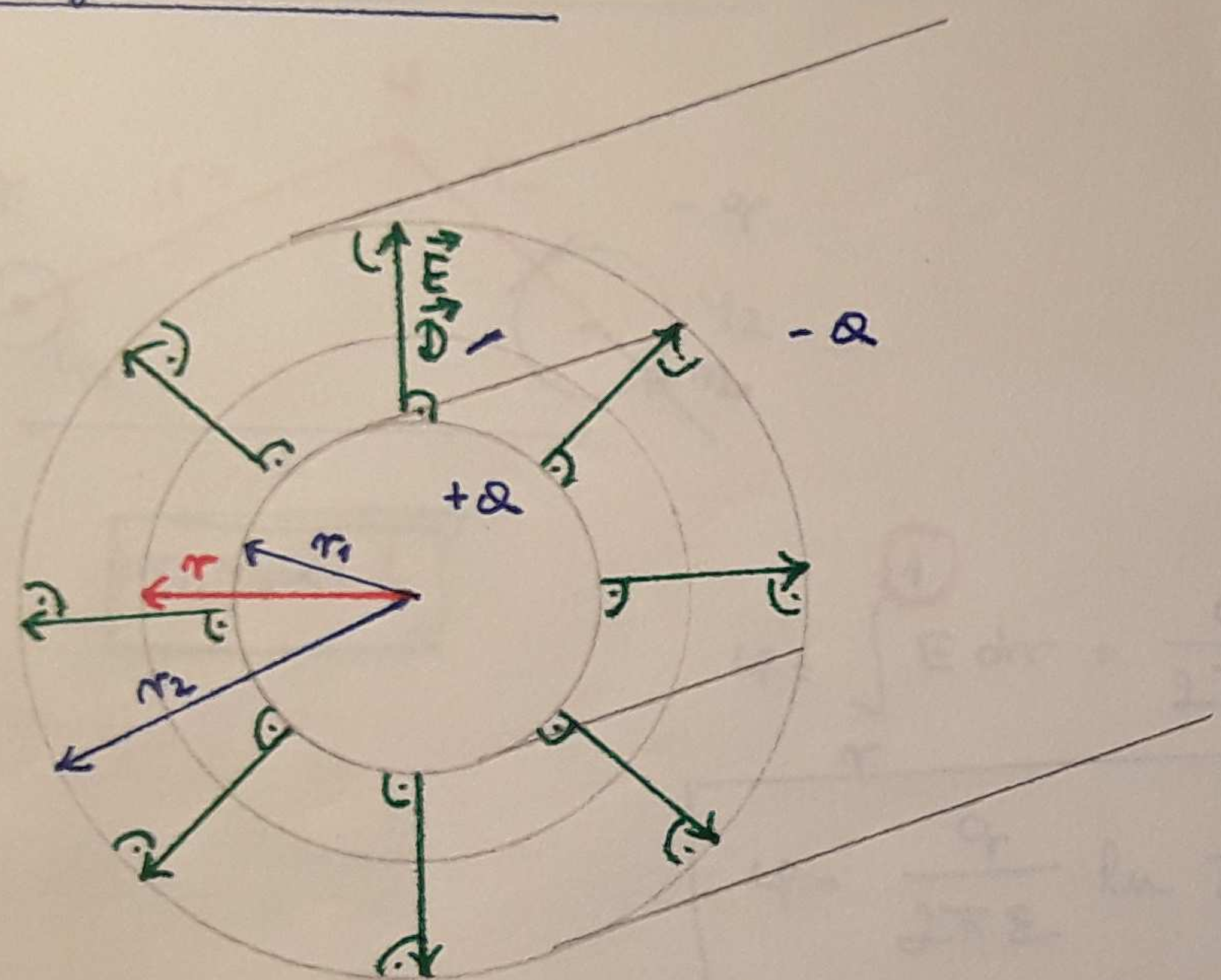
$$D 4\pi r^2 = Q \longrightarrow \boxed{D = \frac{Q}{4\pi r^2}} \qquad \boxed{E = \frac{Q}{4\pi\varepsilon r^2}}$$

$$U = \int_{r_1}^{r_2} \underset{\substack{\uparrow\ \ \nearrow \\ E\,dr}}{\vec{E} \cdot d\vec{r}} = \frac{Q}{4\pi\varepsilon} \int_{r_1}^{r_2} \frac{1}{r^2} dr = \frac{Q}{4\pi\varepsilon} \left[-\frac{1}{r}\right]_{r_1}^{r_2} = \frac{Q}{4\pi\varepsilon}\left(-\frac{1}{r_2} + \frac{1}{r_1}\right) = \boxed{\frac{Q}{4\pi\varepsilon}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

Belül: $\oint_A \vec{D} \cdot d\vec{A} = \underbrace{\int_V \varrho \, dV}_{\varnothing}$

Kívül: $\oint_A \vec{D} \cdot d\vec{A} = \underbrace{\int_V \varrho \, dV}_{\varnothing}$

Hengerkondenzátor



Két koaxiális hengeres vezető.

$$\vec{D} = \vec{e}_r \frac{q}{2r\pi}$$
$$\downarrow$$
$$\vec{E} = \vec{e}_r \frac{q}{2r\pi\varepsilon}$$

$r_1 \leq r \leq r_2$

$$\downarrow$$
$$U = \frac{q}{2\pi\varepsilon} \ln \frac{r_2}{r_1}$$
$$\downarrow$$
$$C = \frac{Q}{U} = \frac{2\pi\varepsilon l}{\ln r_2/r_1}$$

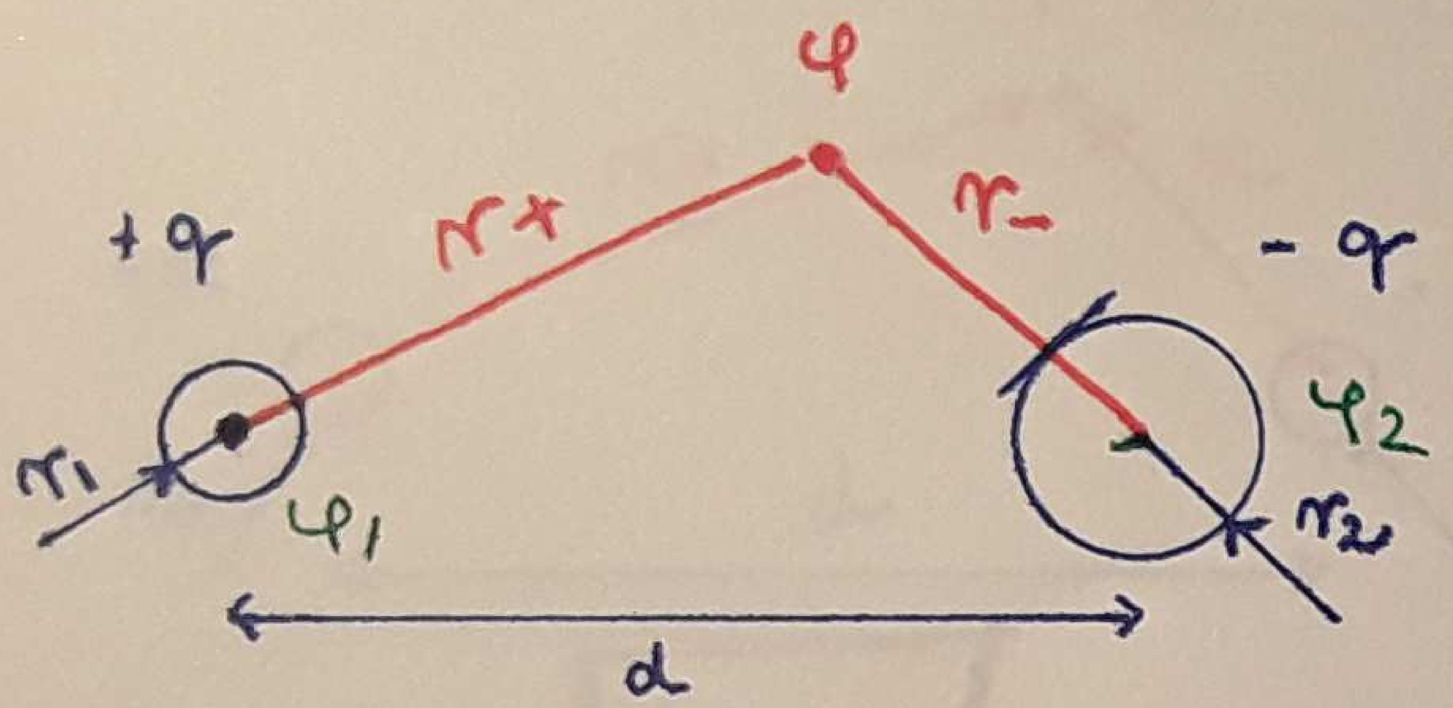
$$\oint_A \vec{D} \cdot d\vec{A} = \int_V \rho \, dV \qquad D \underbrace{2r\pi \cancel{l}}_{} = q\cancel{l} \rightarrow \boxed{D = \frac{q}{2r\pi}}$$

$$U = \int_{r_1}^{r_2} E \, dr = \frac{q}{2\pi\varepsilon} \int_{r_1}^{r_2} \frac{1}{r} dr = \frac{q}{2\pi\varepsilon} \left[\ln r\right]_{r_1}^{r_2} = \frac{q}{2\pi\varepsilon} \left(\ln r_2 - \ln r_1\right) = \boxed{\frac{q}{2\pi\varepsilon} \ln \frac{r_2}{r_1}}$$

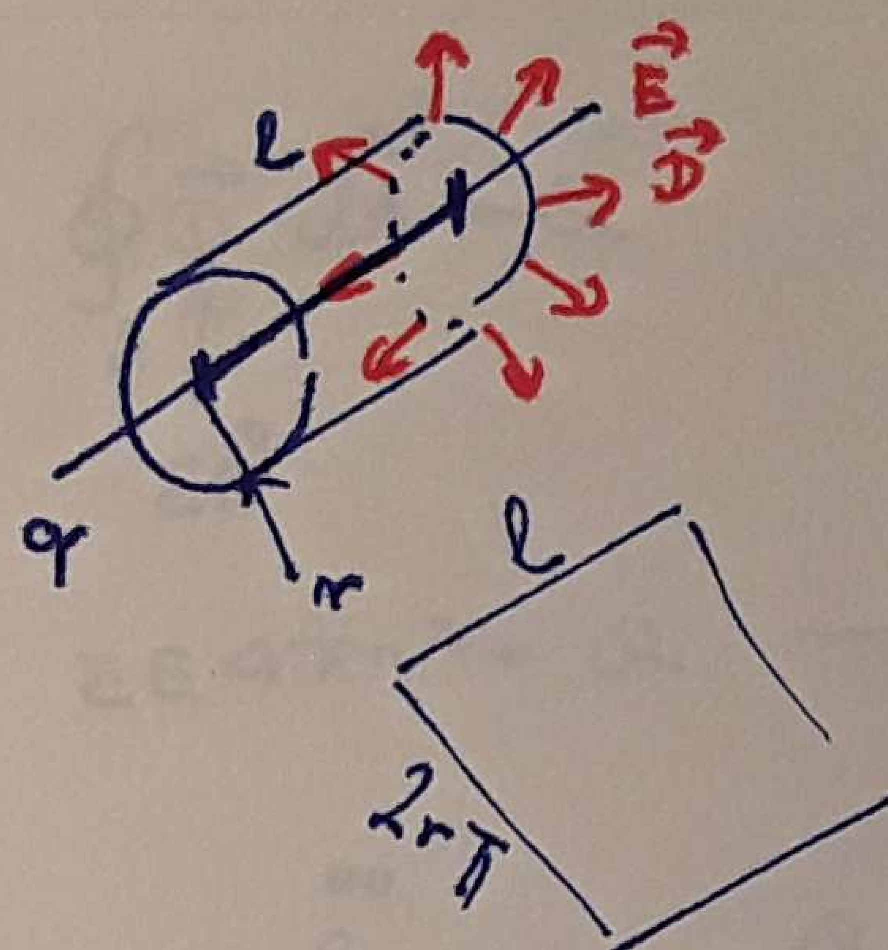
$$C = \frac{Q}{U} = \frac{ql}{U} = \frac{2\pi\varepsilon l}{\ln r_2/r_1}$$

Párhuzamos vezetékek:

$C = ?$



$\boxed{r_1, r_2 \ll d}$



$$\oint_A \vec{D} \cdot d\vec{A} = Q$$

$$\varepsilon E\, 2r\pi \cancel{l} = q \cancel{l}$$

$$\boxed{E = \frac{q}{2r\pi\varepsilon}}$$

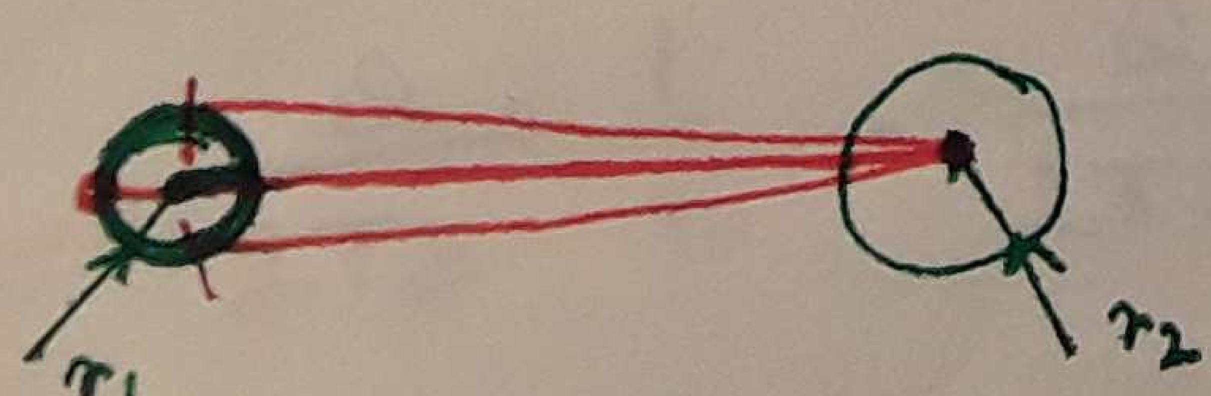
$$\varphi = \int_r^{1} E\, dr = \frac{q}{2\pi\varepsilon} \int_r^{1} \frac{1}{r}\, dr = \frac{q}{2\pi\varepsilon} \left[\ln r\right]_r^{\cancel{\infty}1} = \frac{q}{2\pi\varepsilon} \left(\underbrace{\ln 1}_{\emptyset} - \underbrace{\ln r}_{\ln\frac{1}{r}}\right) \Rightarrow$$

$$\boxed{\varphi = \frac{q}{2\pi\varepsilon} \ln \frac{1}{r}}$$

$$\left.\begin{aligned} \varphi_+ &= \frac{+q}{2\pi\varepsilon} \ln \frac{1}{r_+} \\ \varphi_- &= \frac{-q}{2\pi\varepsilon} \ln \frac{1}{r_-} \end{aligned}\right\} \quad \varphi = \frac{q}{2\pi\varepsilon} \underbrace{\ln \frac{1}{r_+}} - \frac{q}{2\pi\varepsilon} \underbrace{\ln \frac{1}{r_-}} = \frac{q}{2\pi\varepsilon} \ln \frac{r_-}{r_+}$$

$$\ln\left(\frac{1}{r_+}\right) - \ln\left(\frac{1}{r_-}\right) = \ln \frac{r_-}{r_+}$$

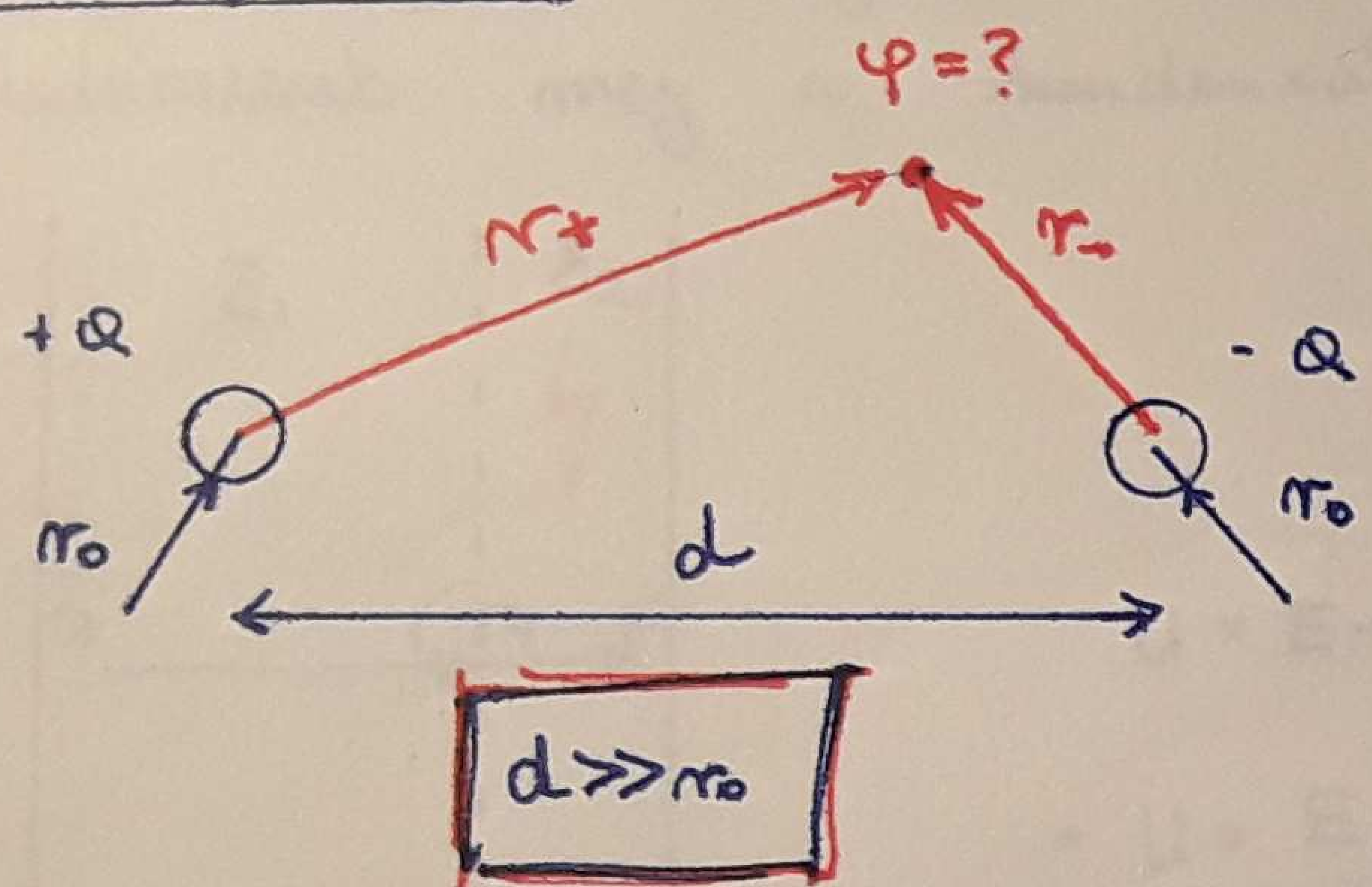
$$U = \varphi_1 - \varphi_2 = \frac{q}{2\pi\varepsilon} \ln \overset{!}{\frac{d}{r_1}} - \frac{q}{2\pi\varepsilon} \ln \frac{r_2}{d} = \boxed{\frac{q}{2\pi\varepsilon} \ln \frac{d^2}{r_1 \cdot r_2}}$$



$$\frac{\frac{d}{r_1}}{\frac{r_2}{d}} = \frac{d}{r_1} \cdot \frac{d}{r_2}$$

$$C = \frac{Q}{U} = \frac{ql}{U} = \boxed{\frac{2\pi\varepsilon l}{\ln \frac{d^2}{r_1 r_2}}}$$

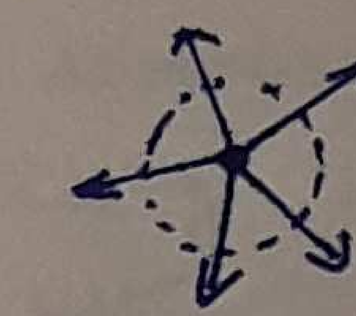
Két gömb: $C = ?$



$$\oint_A \vec{D}\cdot d\vec{A} = Q$$

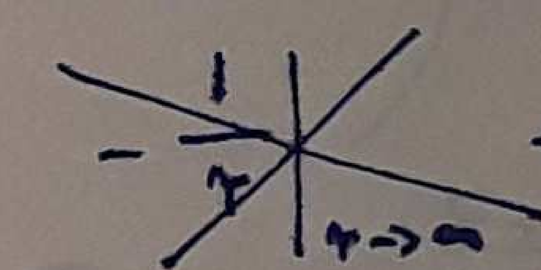
$\uparrow$

$\varepsilon \vec{E}$



$$\varepsilon E 4\pi r^2 = Q \longrightarrow \boxed{E = \frac{Q}{4\pi\varepsilon r^2}} \qquad r > r_0$$

$$\varphi = \int_r^\infty E\,dr = \frac{Q}{4\pi\varepsilon}\int_r^\infty \frac{1}{r^2}\,dr = \frac{Q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_r^\infty = \boxed{\frac{Q}{4\pi\varepsilon}\frac{1}{r}}$$

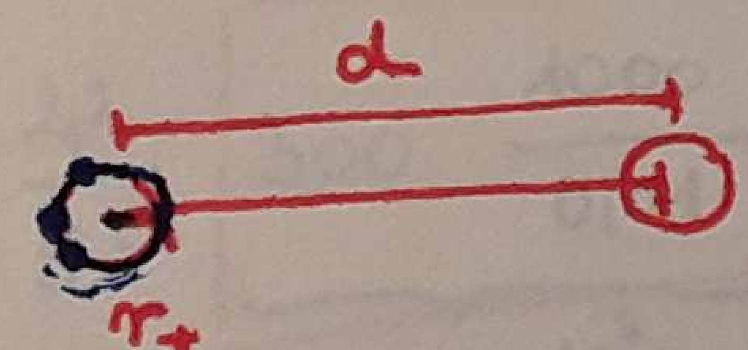


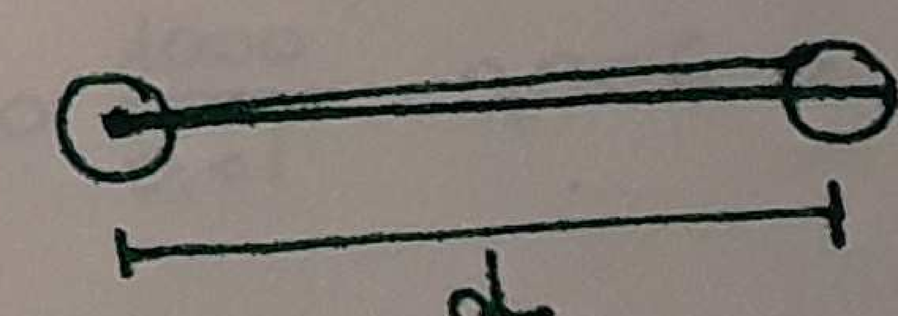
$+\frac{1}{r}$

Szuperpozíció:

$$\varphi = \frac{Q}{4\pi\varepsilon}\frac{1}{r_+} - \frac{Q}{4\pi\varepsilon}\frac{1}{r_-} \qquad !$$

$$\boxed{U = \varphi_1 - \varphi_2} = \frac{Q}{4\pi\varepsilon}\Big(\underbrace{\frac{1}{r_0}}_{r_+} - \underbrace{\frac{1}{d}}_{r_-}\Big) - \frac{Q}{4\pi\varepsilon}\Big(\underbrace{\frac{1}{d}}_{r_+} - \underbrace{\frac{1}{r_0}}_{r_-}\Big) =$$



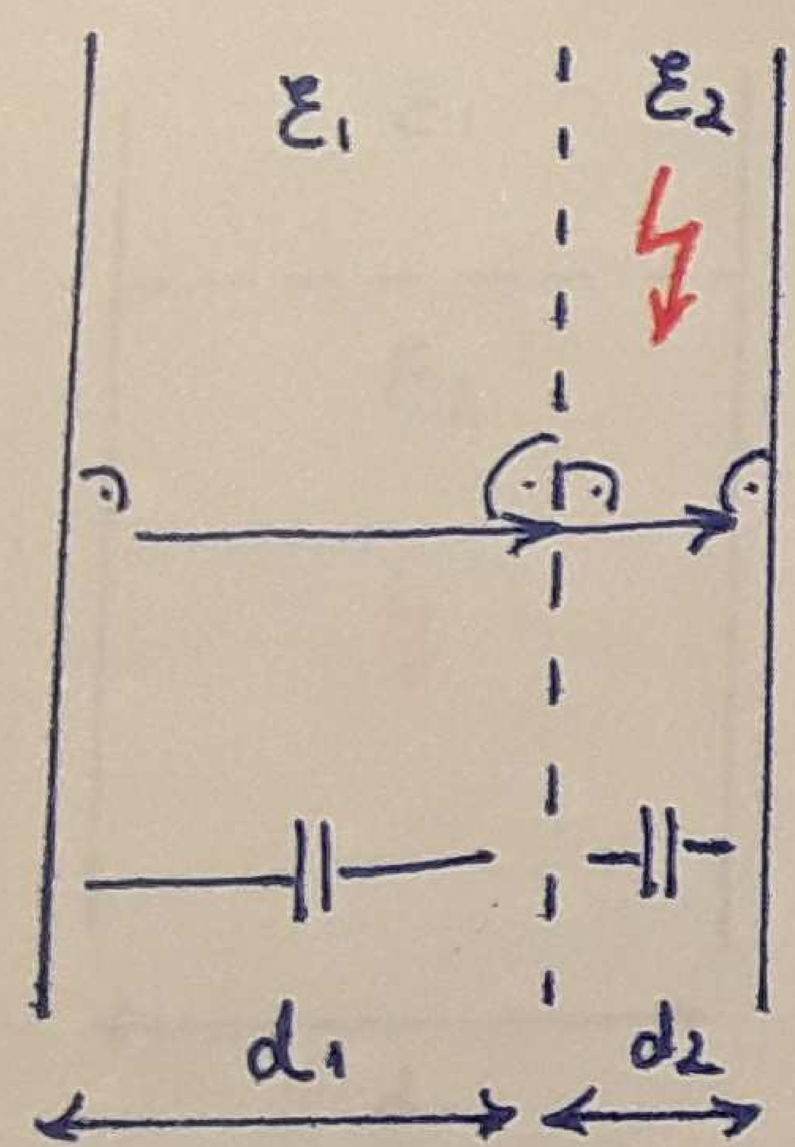


$$= \underbrace{\frac{Q}{4\pi\varepsilon}\frac{1}{r_0}} - \frac{Q}{4\pi\varepsilon}\frac{1}{d} - \frac{Q}{4\pi\varepsilon}\frac{1}{d} + \underbrace{\frac{Q}{4\pi\varepsilon}\frac{1}{r_0}}$$

$$= \frac{Q}{2\pi\varepsilon}\frac{1}{r_0} - \frac{Q}{2\pi\varepsilon}\frac{1}{d} = \boxed{\frac{Q}{2\pi\varepsilon}\left(\frac{1}{r_0} - \frac{1}{d}\right) = U}$$

$$C = \frac{Q}{U} = \frac{2\pi\varepsilon}{\frac{1}{r_0} - \frac{1}{d}}$$

Keresztirányban rétegzett síkkondenzátor dielektrikumainak átütési térerőssége ismert. Határozzuk meg a kondenzátorra kapcsolható maximális feszültség értékét!



$d_1 = 0{,}01\,m$ $\quad \varepsilon_{1r} = 5{,}5$ $\quad E_{1kr} = \cancel{350\,\frac{kV}{cm}}$

$d_2 = 0{,}006\,m$ $\quad \varepsilon_{2r} = 2{,}2$ $\quad E_{2kr} = 300\,\frac{kV}{cm} = 3\cdot 10^7\,V/m$

$U = E\cdot d$

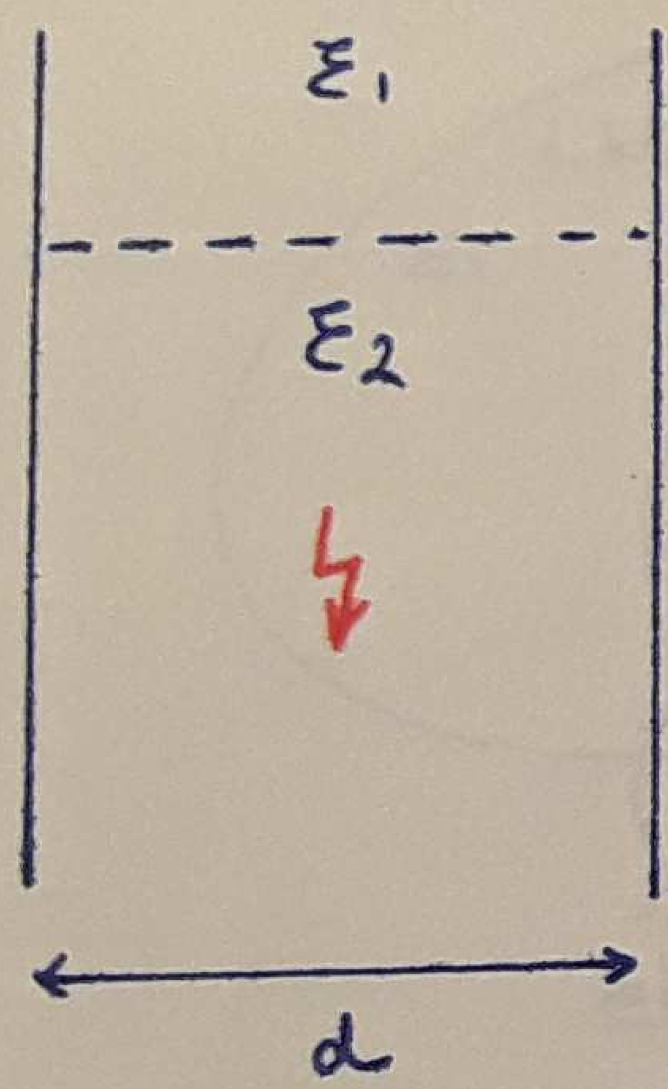
- $U = E_1 d_1 + E_2 d_2$
- $D_{normális} \longrightarrow D_{1N} = D_{2N}$

$\cancel{\varepsilon_0}\,\varepsilon_{1r} E_{1kr} = \cancel{\varepsilon_0}\,\varepsilon_{2r} E_{2kr} \longrightarrow E_{1kr} = \frac{\varepsilon_{2r}}{\varepsilon_{1r}} E_{2kr}$

$$U = \frac{\varepsilon_{2r}}{\varepsilon_{1r}} E_{2kr}\, d_1 + E_{2k}\cdot d_2$$

$$= \frac{2{,}2}{5{,}5}\underbrace{\boxed{300\,\frac{1000}{0{,}01}}}_{3\cdot 10^2\,\frac{10^3}{10^{-2}} = 3\cdot 10^7}\, 0{,}01 + 300\,\frac{1000}{0{,}01}\cdot 0{,}006 = \underline{\underline{300\,kV}}$$

Hosszirányban rétegelt síkkondenzátor dielektrikumainak átütési térerőssége ismert. Határozzuk meg a kondenzátorra kapcsolható maximális feszültség értékét!



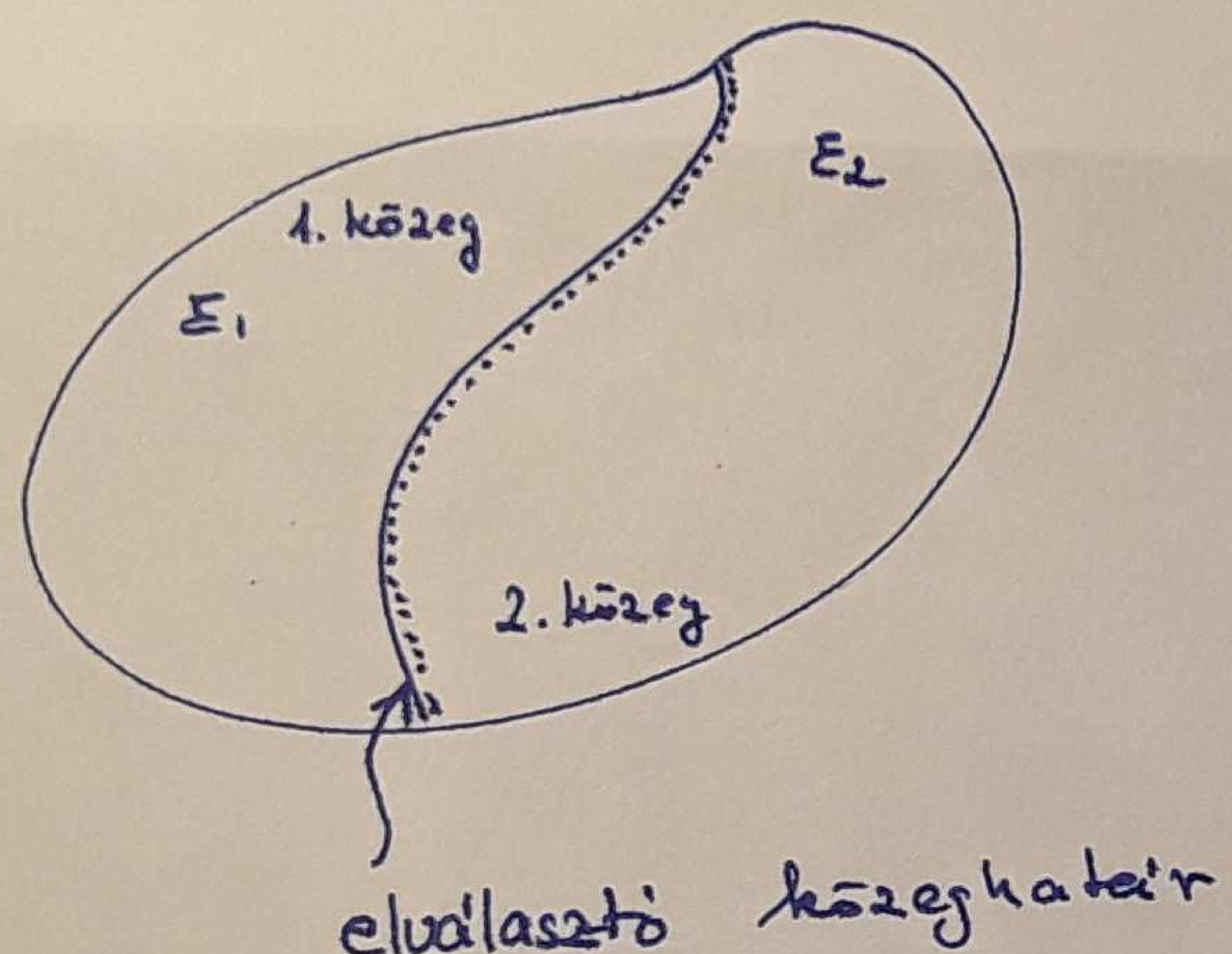
$d = 0{,}016\ m$

$\varepsilon_{1r} = 5{,}5$ $\qquad$ $\cancel{E_{1kr} = 350\ \frac{kV}{cm}}$

$\varepsilon_{2r} = 2{,}2$ $\qquad$ $E_{2kr} = 300\ \frac{kV}{cm}$

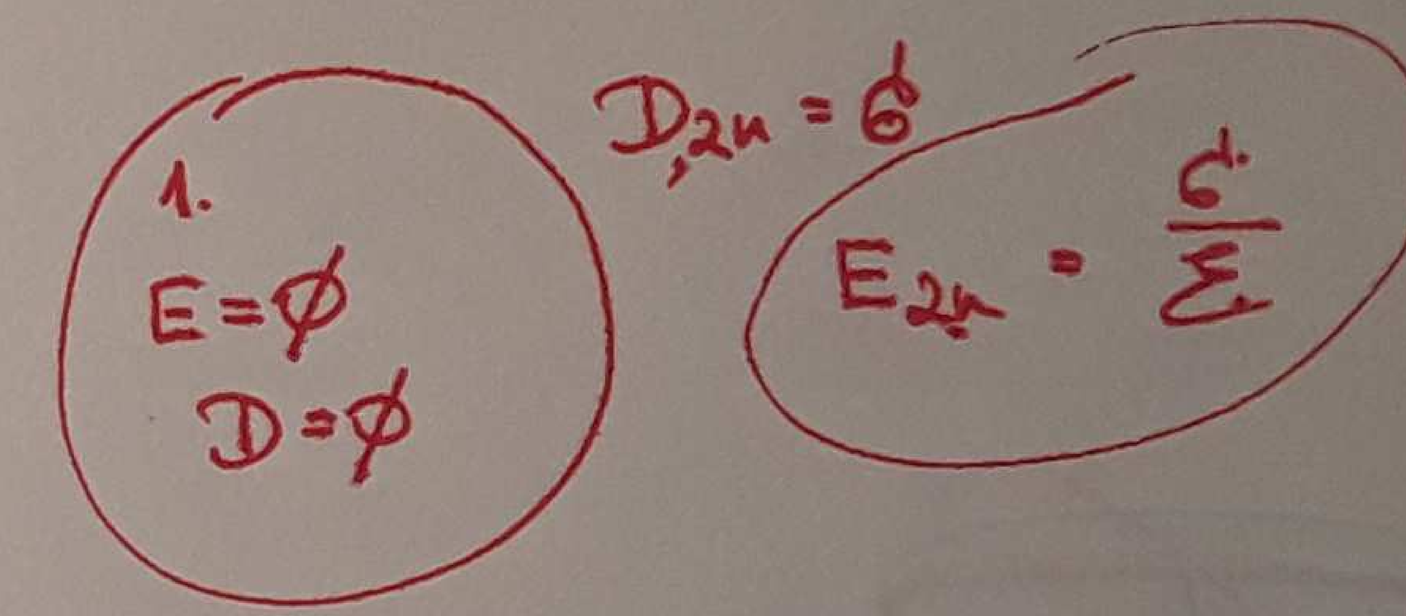
$U = E\,d$ $\qquad$ $U_{max} = E_{2kr} \cdot d = 3 \cdot 10^7 \cdot 0{,}016 = \underline{\underline{480\ kV}}$

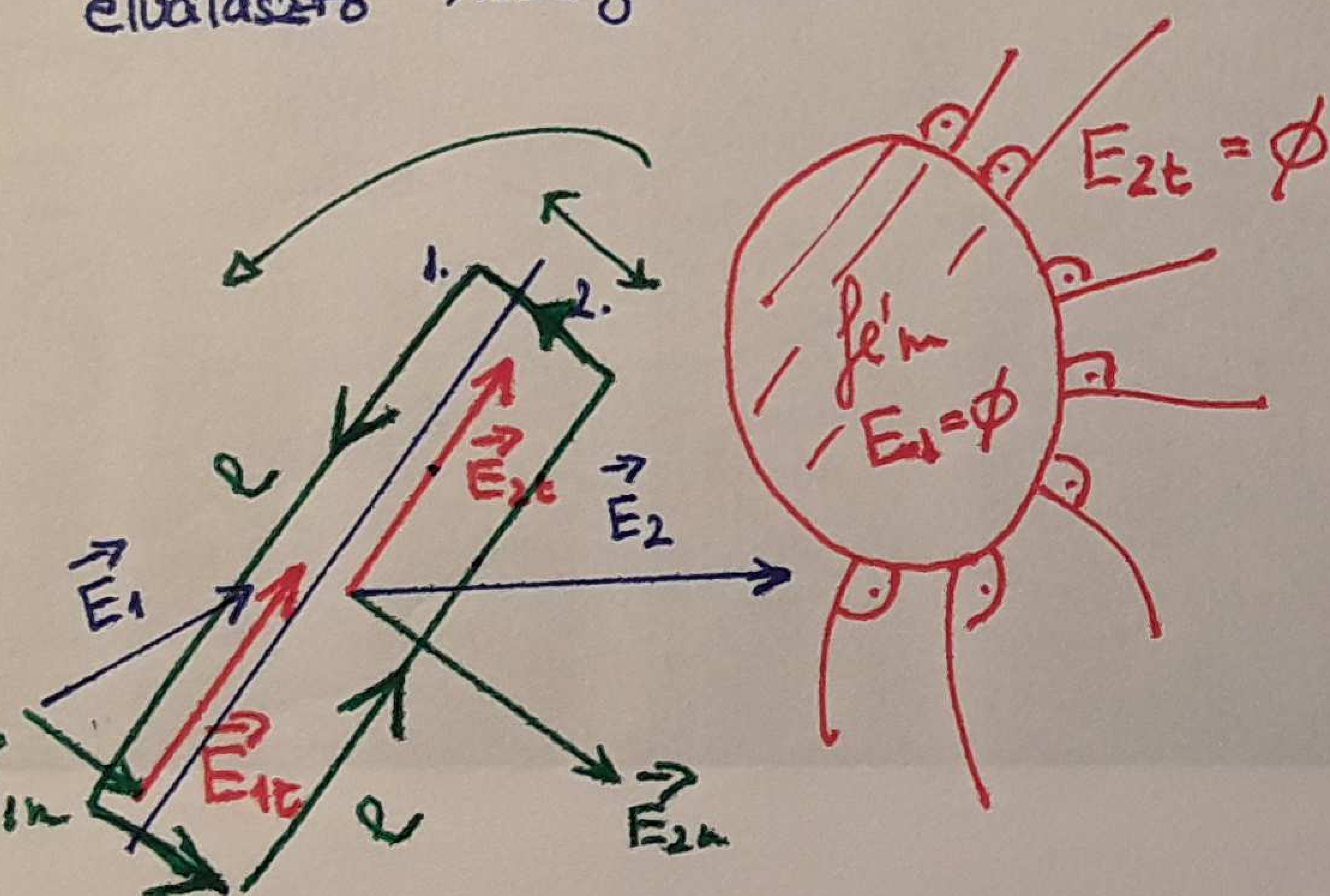
Határfeltételek elektrosztatikus térben

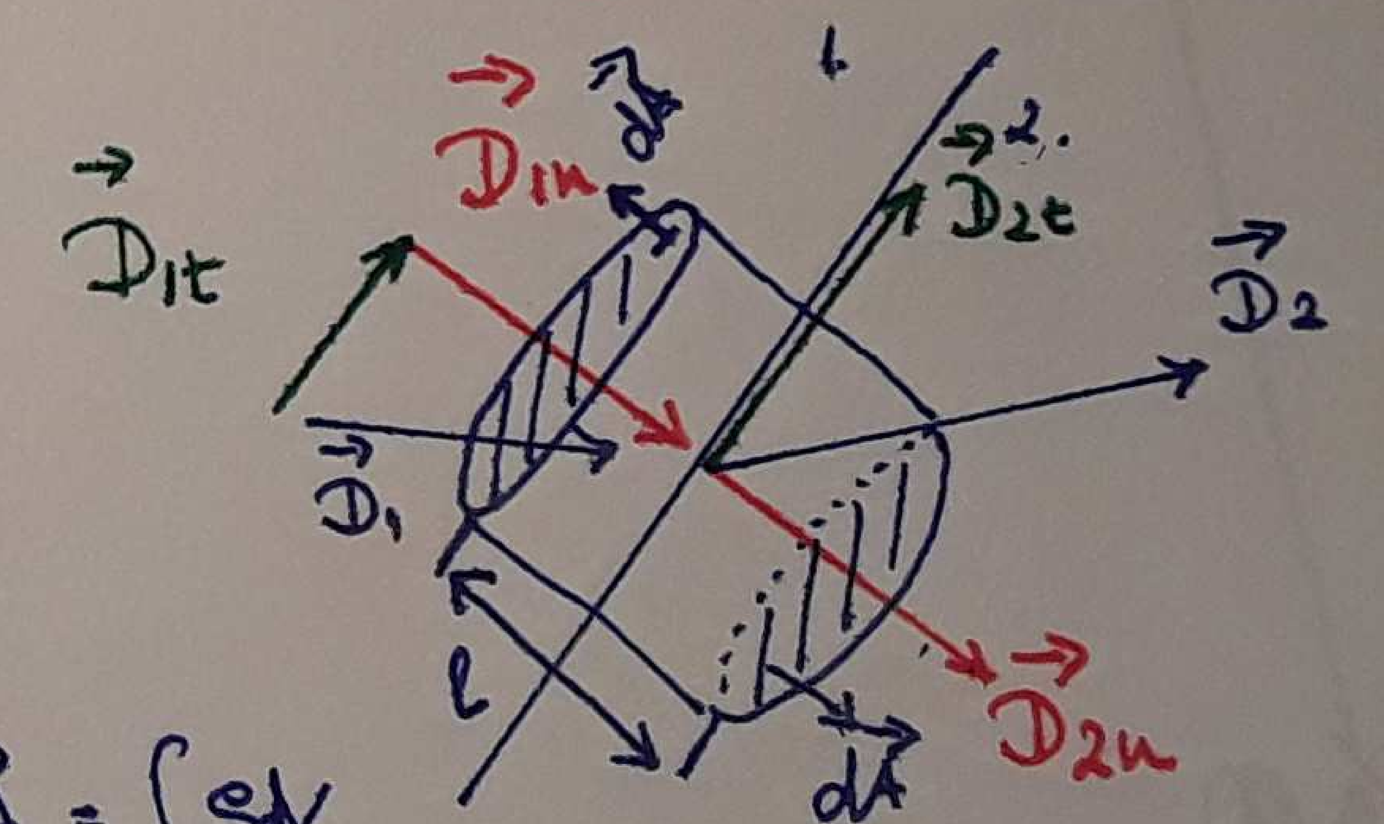


FOLYTONOSSÁGI FELTÉTELEK

E TANGENCIÁLIS ÉS D NORMÁLIS







$$\oint_l \vec{E} \cdot d\vec{l} = 0$$

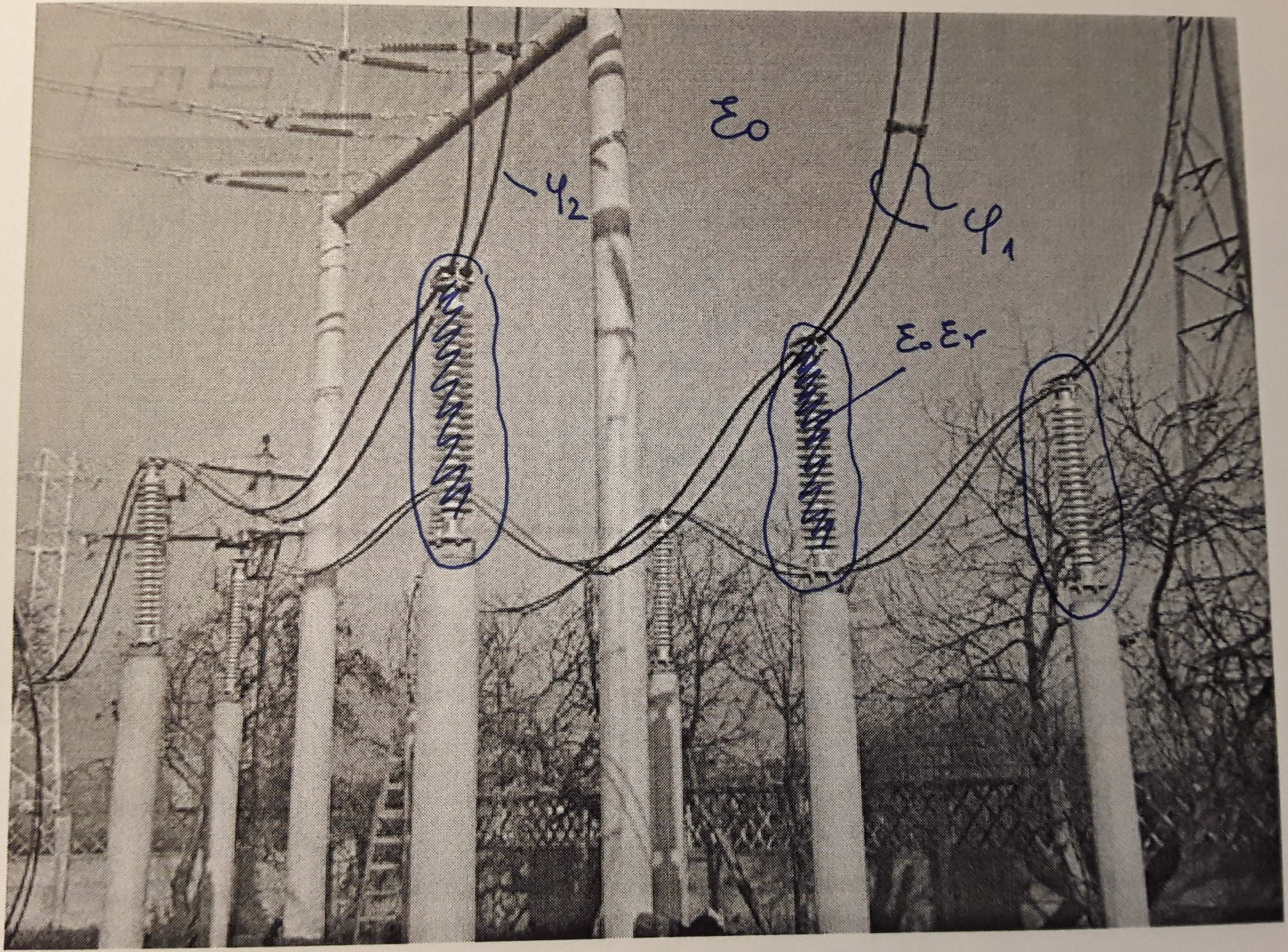
$$E_{2t}\,\cancel{l} - E_{1t}\,\cancel{l} = 0$$

$$\boxed{E_{2t} = E_{1t}}$$

$$\oint_A \vec{D} \cdot d\vec{A} = \int_V \rho\, dV$$

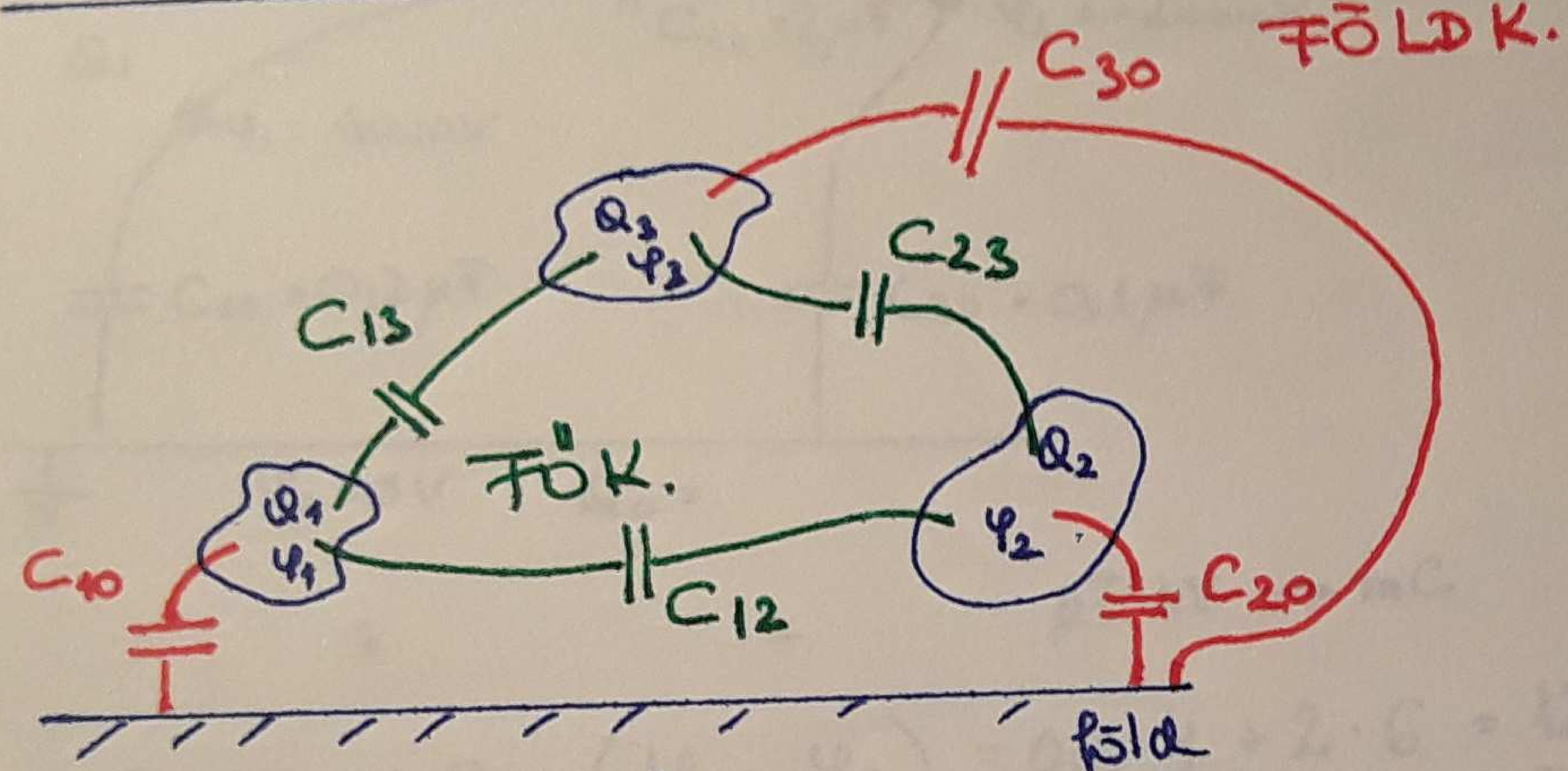
$$-D_{1n}A + D_{2n}A = \sigma A + \cancel{\rho A l}$$

$$\boxed{D_{2n} = D_{1n} + \sigma} \qquad \boxed{D_{2n} = D_{1n}}$$



ε_0
φ_2
φ_1
$\varepsilon_0 \varepsilon_r$

A részkapacitás fogalma



φ_0 $Q_0 = -Q_1 - Q_2 - Q_3$

0V

- φ ismert, $Q = ?$

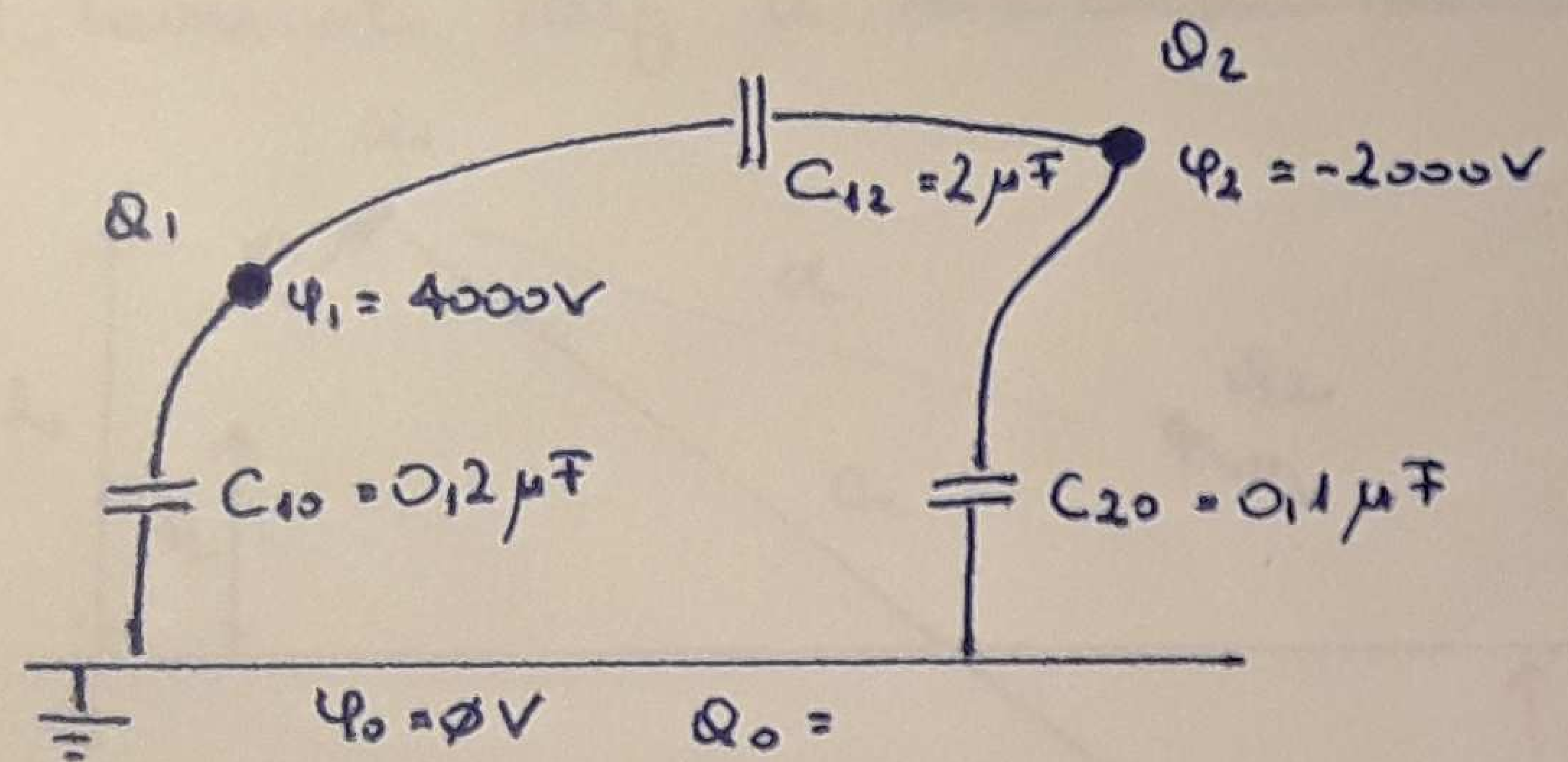
$$Q_1 = C_{10}\varphi_1 + C_{13}(\varphi_1 - \varphi_3) + C_{12}(\varphi_1 - \varphi_2)$$

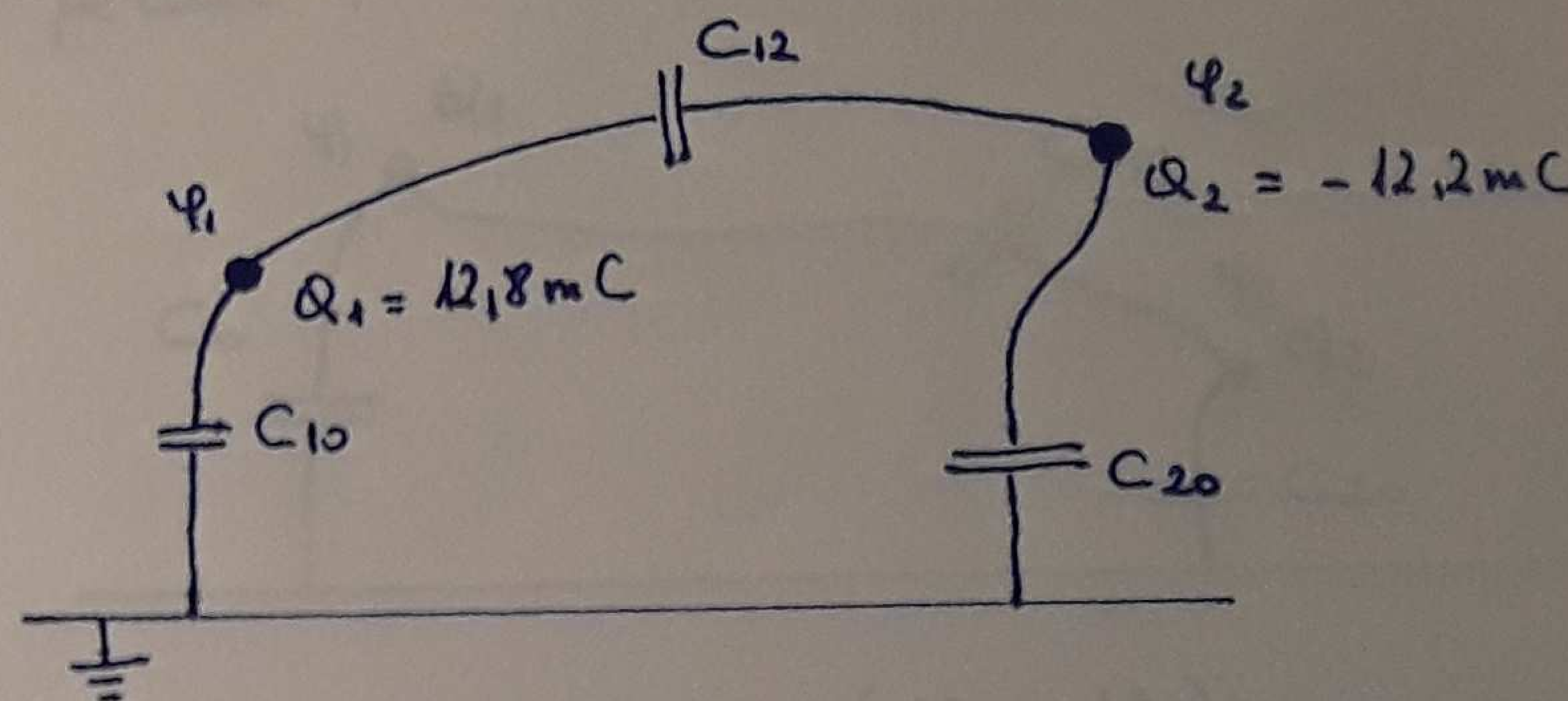
$$Q_2 = C_{20}\varphi_2 + C_{12}(\varphi_2 - \varphi_1) + C_{23}(\varphi_2 - \varphi_3)$$

$$Q_3 = \underbrace{C_{30}\varphi_3} + C_{13}(\varphi_3 - \varphi_1) + C_{23}(\varphi_3 - \varphi_2)$$

- Q ismert, $\varphi = ?$

- C_{ij} !





$\mu F \cdot kV \rightarrow mC$

$$Q_1 = C_{10}\varphi_1 + C_{12}(\varphi_1 - \varphi_2) = 0{,}2 \cdot 4 + 2 \cdot 6 = \underline{\underline{12{,}8mC}}$$

$$Q_2 = C_{20}\varphi_2 + C_{12}(\varphi_2 - \varphi_1) = 0{,}1 \cdot (-2) + 2(-6) = \underline{\underline{-12{,}2mC}}$$

$$Q_0 = -Q_1 - Q_2 = -12{,}8 + 12{,}2 = \underline{\underline{-0{,}6\ mC}}$$

$$Q_1 = C_{10}\varphi_1 + C_{12}(\varphi_1 - \varphi_2)$$

$$Q_2 = C_{20}\varphi_2 + C_{12}(\varphi_2 - \varphi_1)$$

$$Q_1 = (C_{10} + C_{12})\varphi_1 - C_{12}\varphi_2$$

$$Q_2 = -C_{12}\varphi_1 + (C_{12} + C_{20})\varphi_2$$

$$12{,}8 = 2{,}2\varphi_1 - 2\varphi_2$$

$$-12{,}2 = -2\varphi_1 + 2{,}1\varphi_2$$

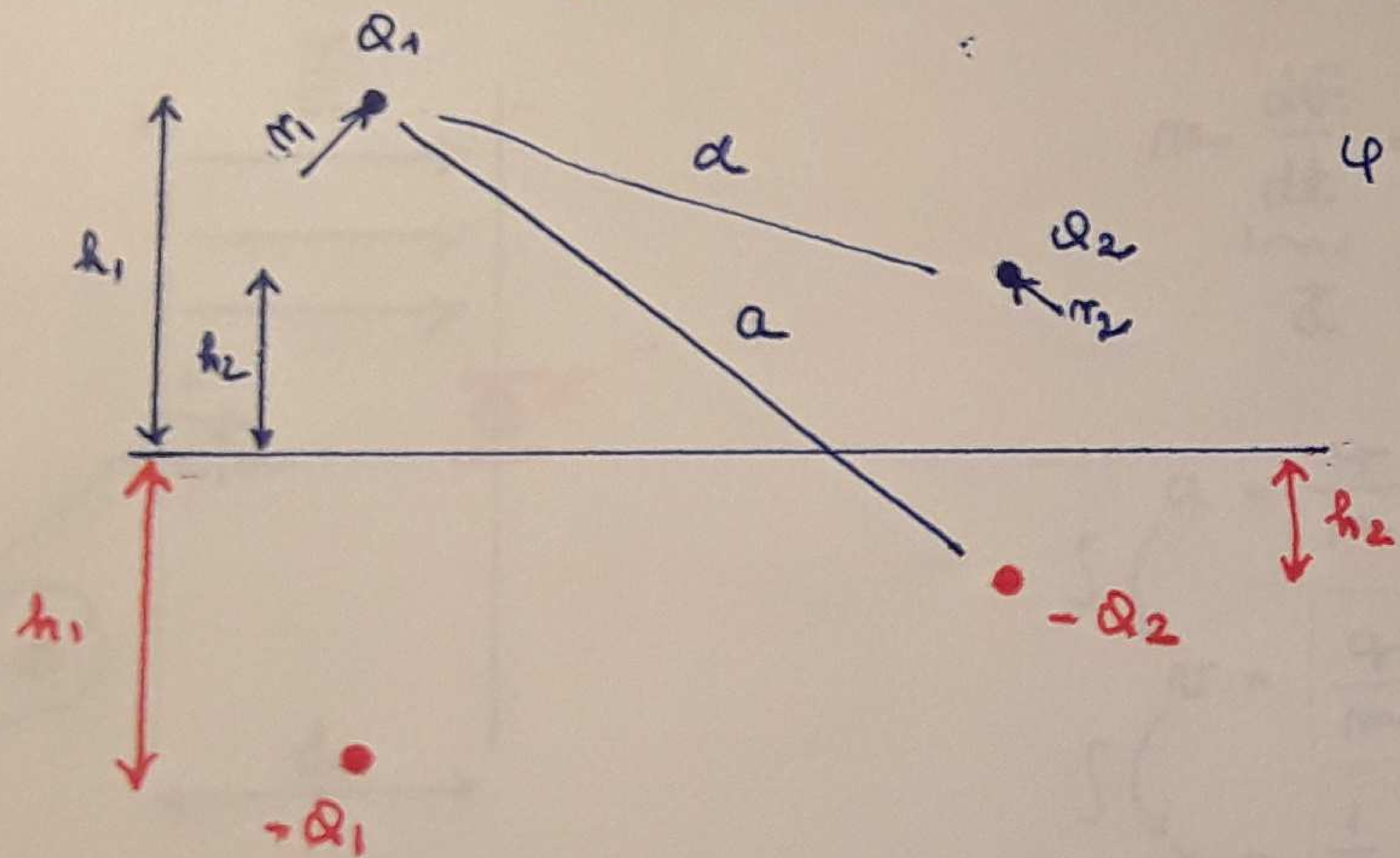
$$C = \frac{Q}{U}$$

$$\underbrace{mC;\ kV;\ \mu F}_{\text{Koherens e.r.}}$$

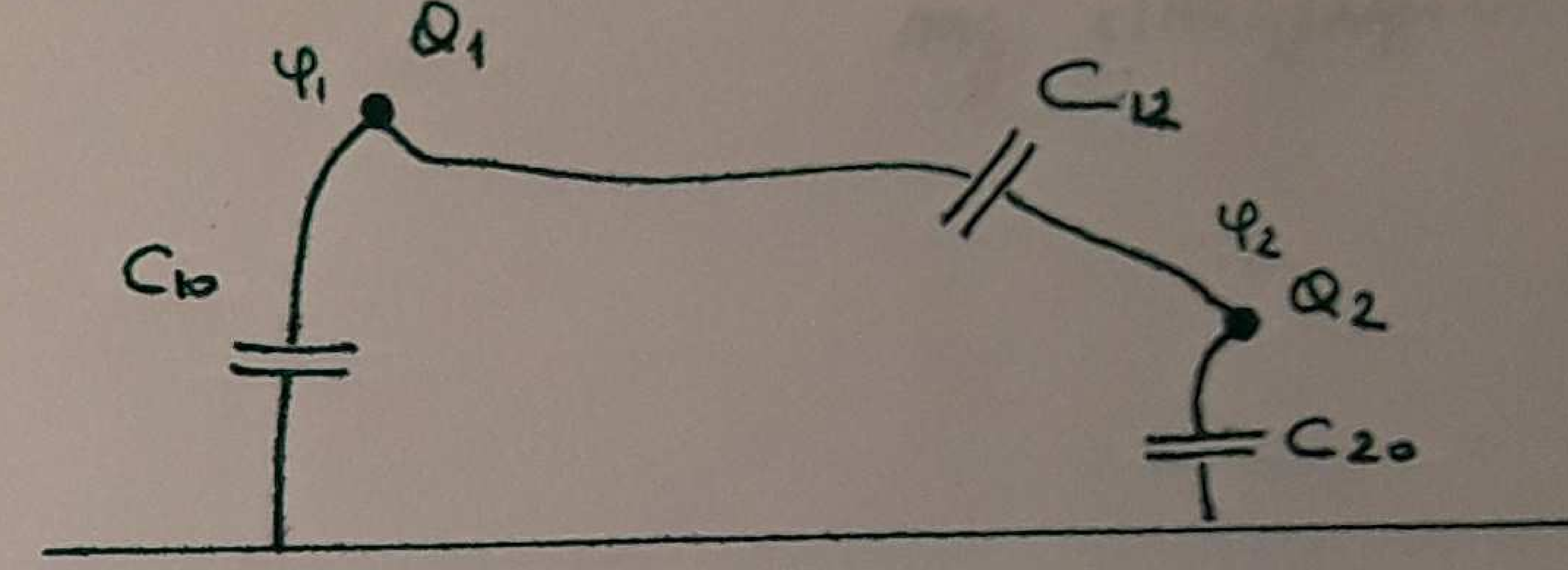
$$\varphi_1 = \underline{\underline{4kV}}$$

$$\varphi_2 = \underline{\underline{-2kV}}$$

Határozzuk meg a részkapacitásokat – illusztratív példa!



$$\varphi = \frac{Q}{4\pi\varepsilon r}$$



$$Q_1 = C_{10}\varphi_1 + C_{12}(\varphi_1 - \varphi_2)$$

$$Q_2 = C_{20}\varphi_2 + C_{12}(\varphi_2 - \varphi_1)$$

$$Q_1 = (C_{10} + C_{12})\varphi_1 \boxed{-C_{12}}\varphi_2$$

$$Q_2 = \boxed{-C_{12}}\varphi_1 + (C_{20} + C_{12})\varphi_2$$

$$\varphi_1 = \boxed{\frac{Q_1}{4\pi\varepsilon_0 r_1} - \frac{Q_1}{4\pi\varepsilon_0 2h_1}} + \frac{Q_2}{4\pi\varepsilon_0 d} - \frac{Q_2}{4\pi\varepsilon_0 a}$$

$$\varphi_2 = \frac{Q_2}{4\pi\varepsilon_0 r_2} - \frac{Q_2}{4\pi\varepsilon_0 2h_2} \boxed{-\frac{Q_1}{4\pi\varepsilon_0 d} + \frac{Q_1}{4\pi\varepsilon_0 a}}$$

$$\varphi_1 = Q_1\underbrace{\left(\frac{1}{4\pi\varepsilon_0 r_1} - \frac{1}{4\pi\varepsilon_0 2h_1}\right)}_{A} + Q_2\underbrace{\left(\frac{1}{4\pi\varepsilon_0 d} - \frac{1}{4\pi\varepsilon_0 a}\right)}_{B}$$

$$\varphi_2 = Q_1\underbrace{\left(\frac{1}{4\pi\varepsilon_0 a} - \frac{1}{4\pi\varepsilon_0 d}\right)}_{-B} + Q_2\underbrace{\left(\frac{1}{4\pi\varepsilon_0 r_2} - \frac{1}{4\pi\varepsilon_0 2h_2}\right)}_{C}$$

$$\begin{cases}\varphi_1 = AQ_1 + BQ_2 \\ \varphi_2 = -BQ_1 + CQ_2\end{cases}$$

$$\begin{cases}Q_1 = A'\varphi_1 + B'\varphi_2 \\ Q_2 = B'\varphi_1 + C'\varphi_2\end{cases}$$

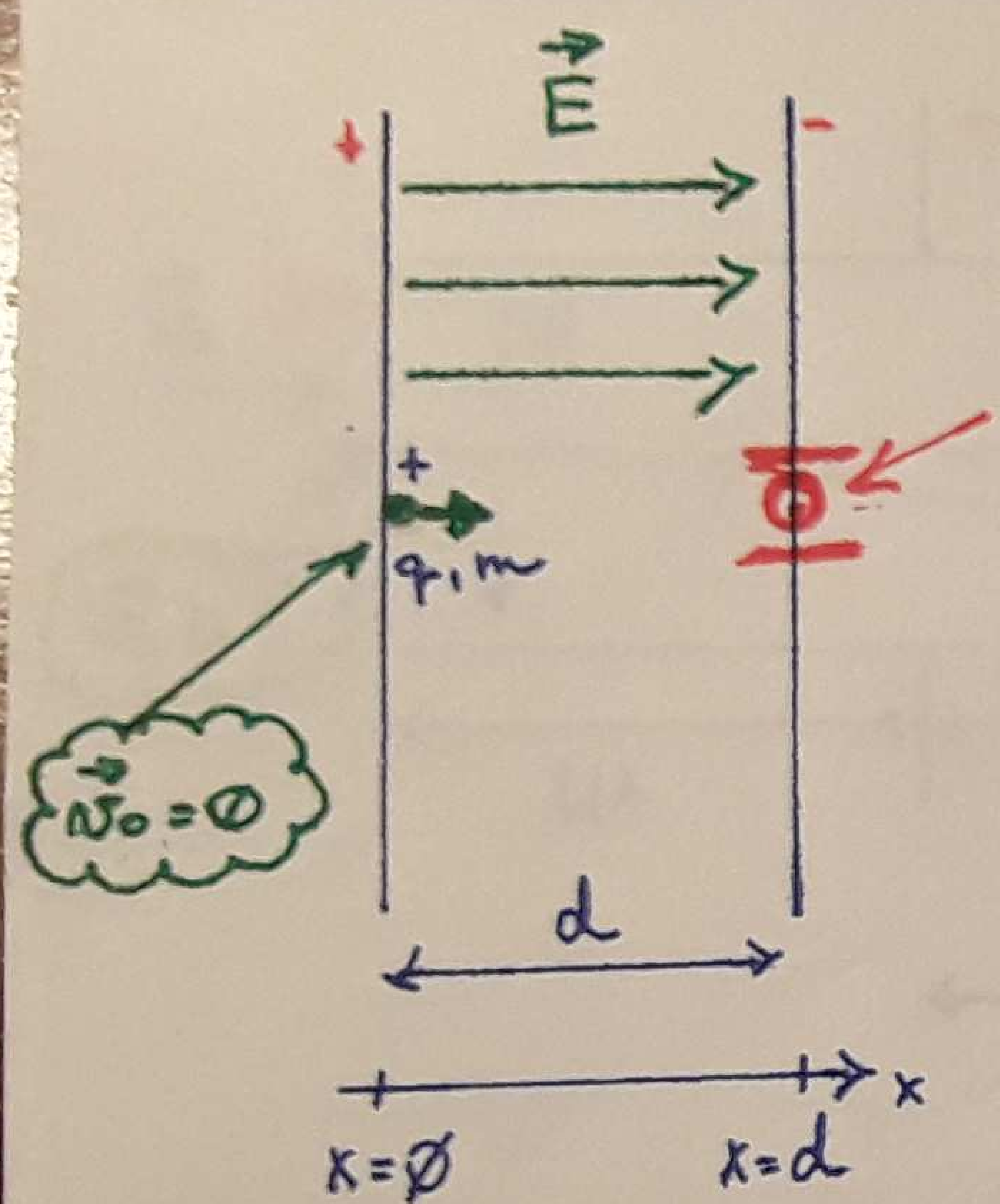
$$B' = -C_{12}$$

$$A' = C_{10} + C_{12}$$

$$C' = C_{20} + C_{12}$$

Töltött részecske mozgása homogén elektrosztatikus térben

vákuum.
mg elhanyagolt.



$$m \underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}} = q\vec{E}$$

$$a = \frac{q}{m}E \quad \text{konstans}$$

$$v = \boxed{\frac{q}{m}E}\, t + \cancel{v_0}$$

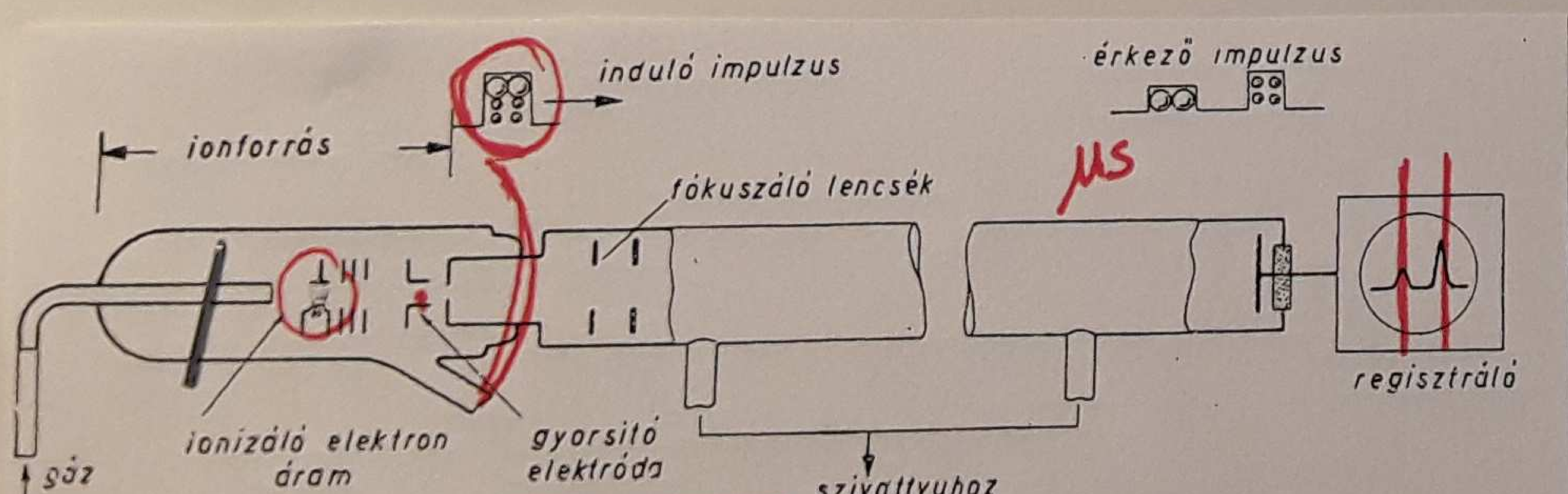
$$x = \frac{1}{2}\frac{q}{m}E t^2$$

$$d = \frac{1}{2}\frac{q}{m}E t^2 \longrightarrow t = \sqrt{2d\frac{m}{q}\frac{1}{E}}$$

fajlagos töltés

$$v = \frac{q}{m}E t = \sqrt{\frac{q^2}{m^2}E^2\, 2d\,\frac{m}{q}\frac{1}{E}} = \sqrt{2dE\frac{q}{m}} = \sqrt{\frac{2qU}{m}}$$

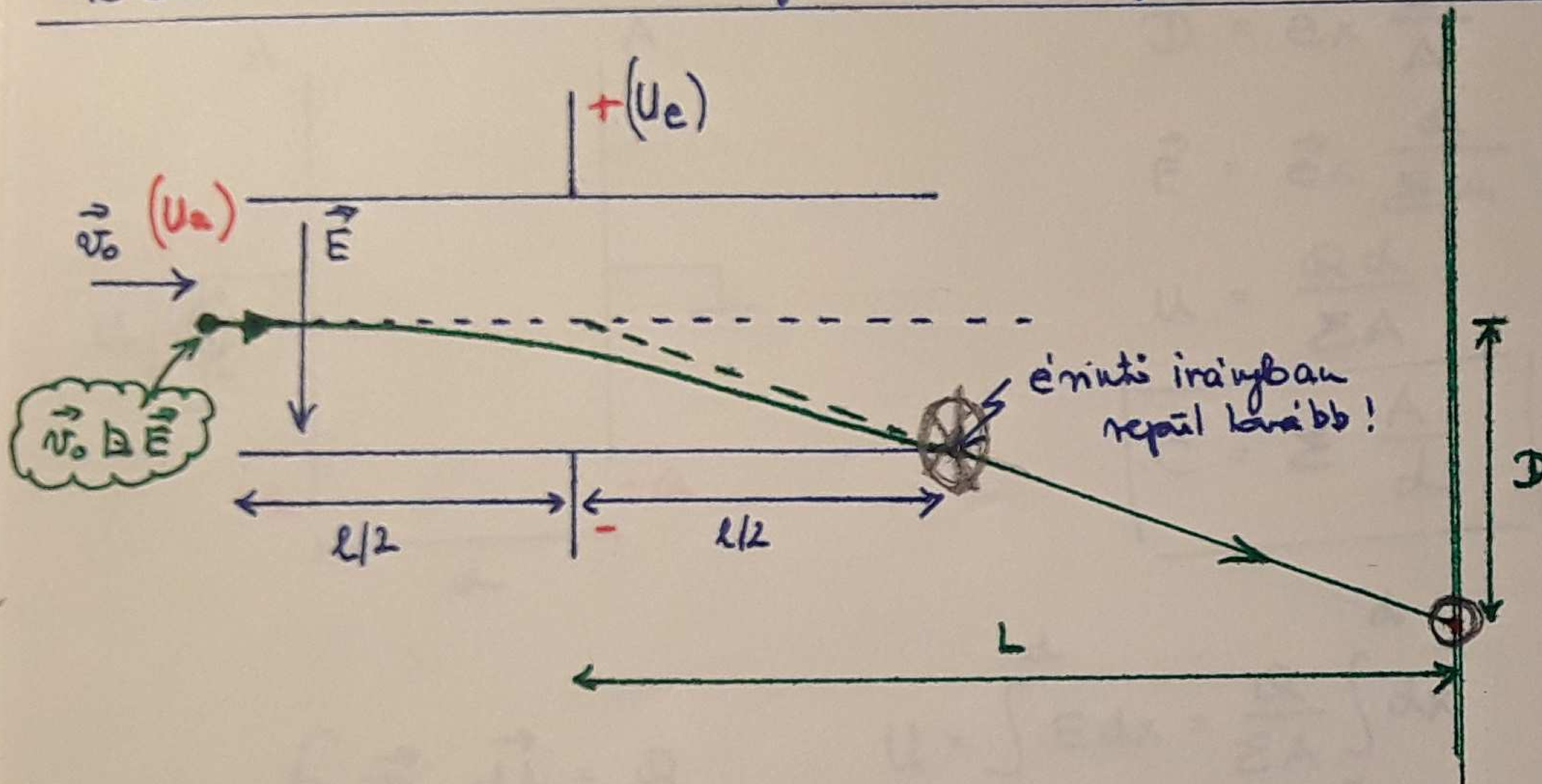
$$E = \frac{U}{d}$$



8. ábra. Az impulzus-tömegspektrográf. Az egész cső hossza körülbelül 3,5 m

Töltött részecske mozgása homogén elektrosztatikus térben

vákuumban.
mg elhanyagolt.



$$v_{0x} = \sqrt{\frac{2qU_a}{m}}$$

$$\begin{cases} x = v_{0x} \cdot t \\ y = \frac{1}{2}\frac{q}{m} E t^2 \end{cases} \rightarrow y = \frac{1}{2}\frac{q}{m}\frac{E}{v_{0x}^2} x^2$$

$$\left.\frac{dy}{dx}\right|_{x=l} = \left.\frac{qEx}{m v_{0x}^2}\right|_{x=l} = \frac{qEl}{m v_{0x}^2}$$

$$e = mx + b$$

$$\boxed{\frac{1}{2}\frac{q}{m}\frac{E}{v_{0x}^2} l^2} = \boxed{\frac{qEl}{m v_{0x}^2}} l + b \rightarrow b = -\frac{1}{2}\frac{q}{m}\frac{E}{v_{0x}^2} l^2$$

$$e = \frac{qEl}{m v_{0x}^2} x - \frac{1}{2}\frac{q}{m}\frac{E}{v_{0x}^2} l^2$$

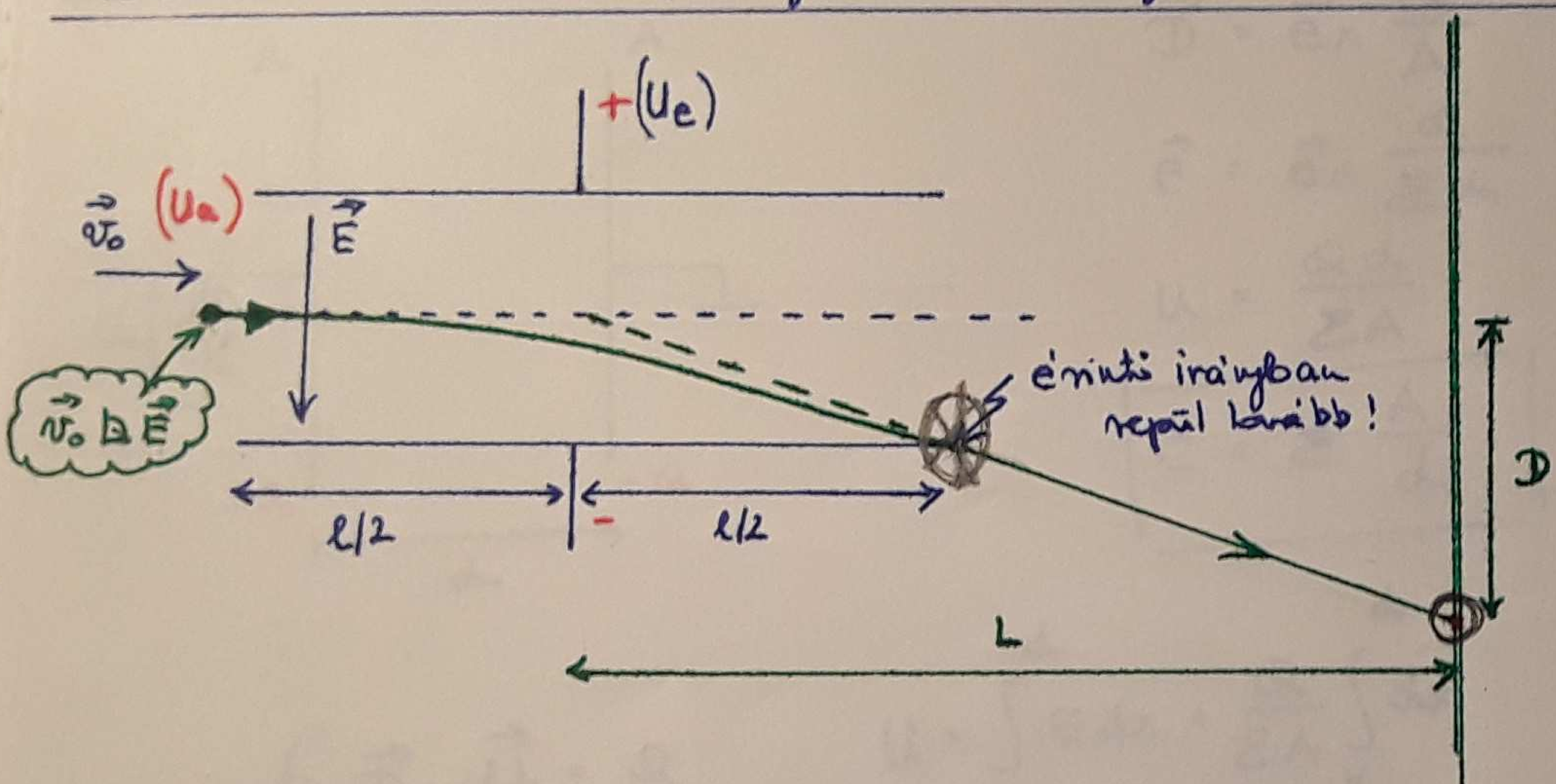
$$D = \frac{qEl}{m v_{0x}^2}\left(l/2 + L\right) - \frac{1}{2}\frac{qEl^2}{m v_{0x}^2}$$

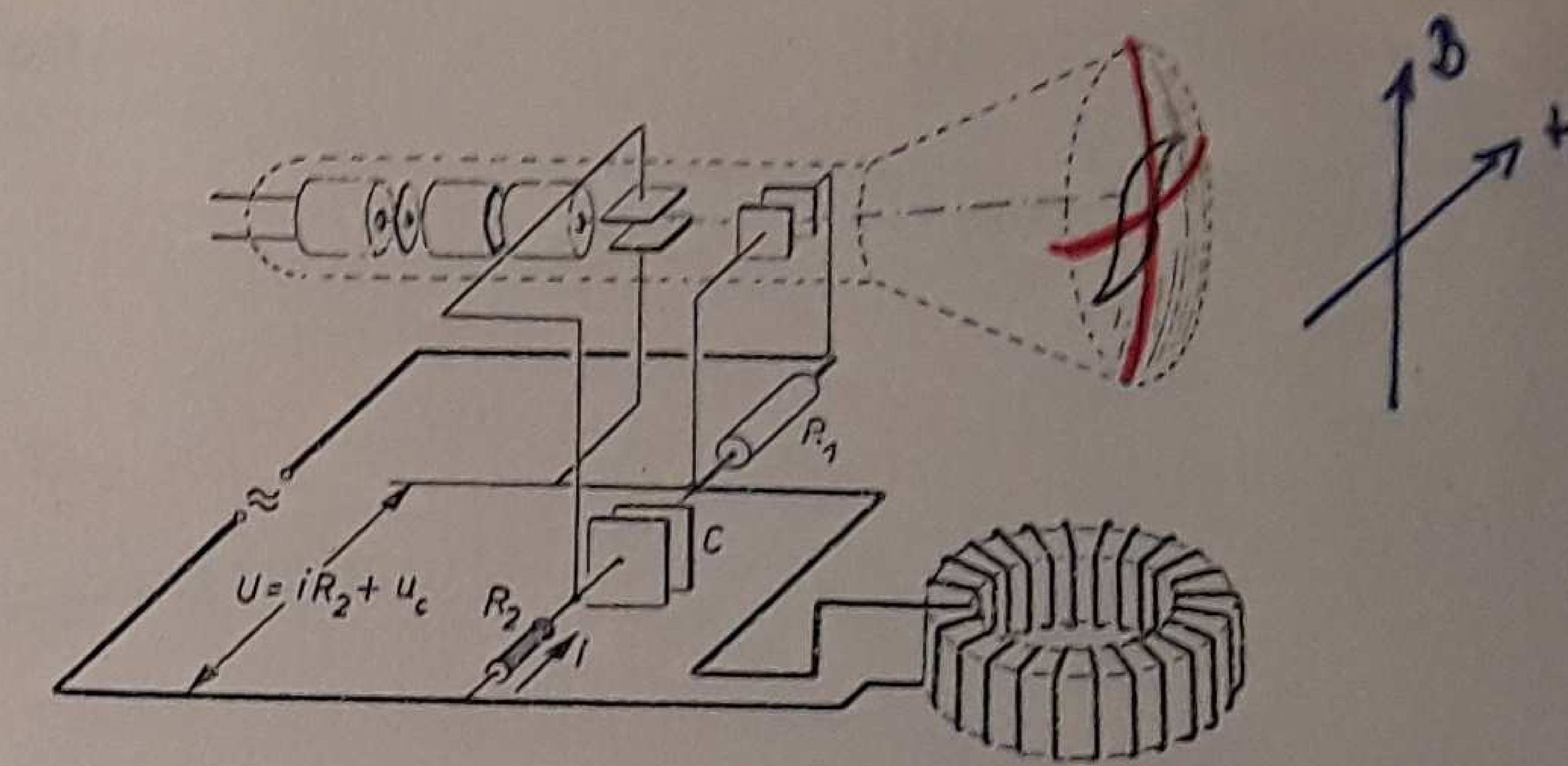
$$\boxed{D = \frac{qElL}{m v_{0x}^2}}$$

$$= \frac{\cancel{q} U_e l L}{\cancel{m} d \frac{2\cancel{q}U_a}{\cancel{m}}}$$

$$\boxed{D = \frac{lL}{2d}\frac{U_e}{U_a}}$$

Töltött részecske mozgása homogén elektrosztatikus

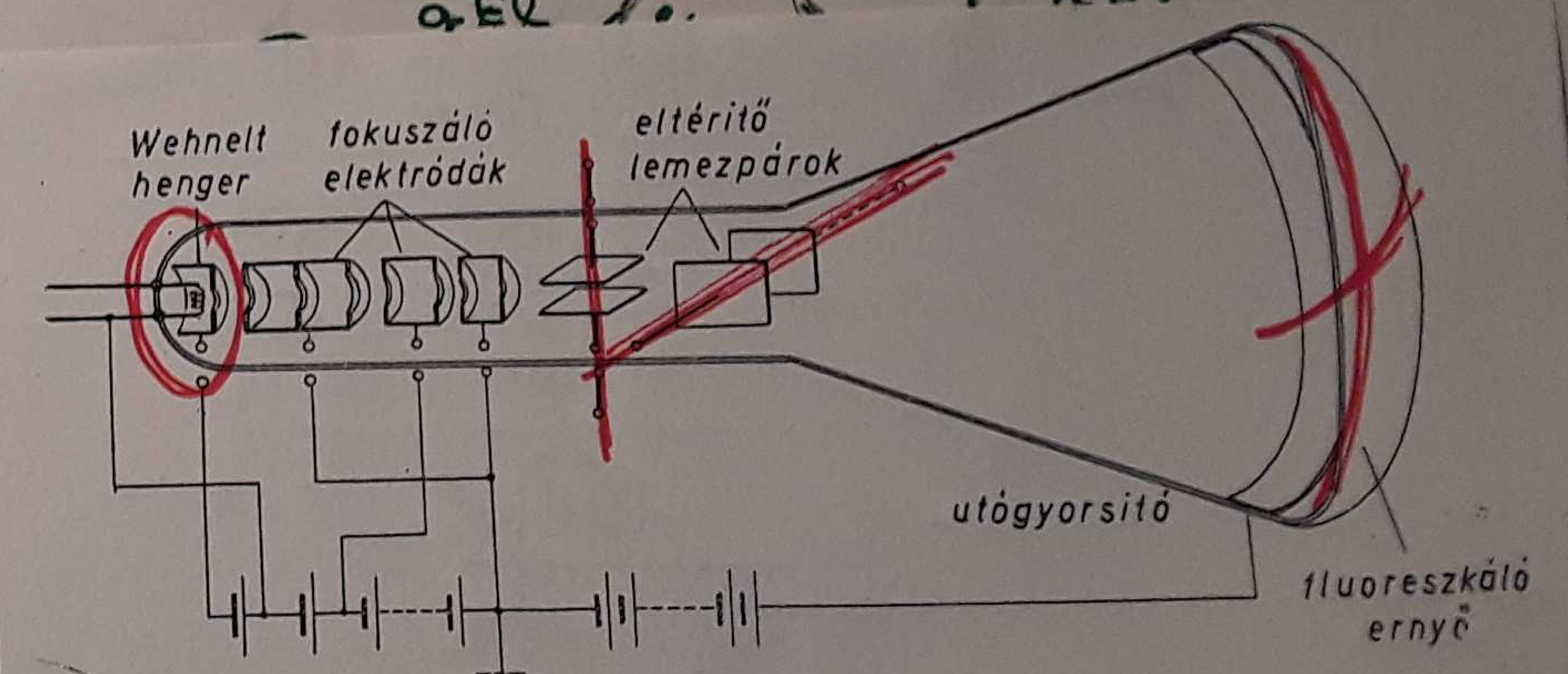




12. ábra. Az oszcilloszkóp felhasználása mágnesezési görbe felvételére

$$\begin{cases} x = v_{0x} \cdot t \\ y = \frac{1}{2} \frac{q}{m} E t^2 \end{cases} \rightarrow y = \frac{1}{2} \frac{q}{m} \frac{E}{v_{0x}^2} x^2$$

$$\left.\frac{dy}{dx}\right|_{x=l} = \left.\frac{qEx}{m v_{0x}^2}\right|_{x=l} = \frac{qEl}{m v_{0x}^2}$$

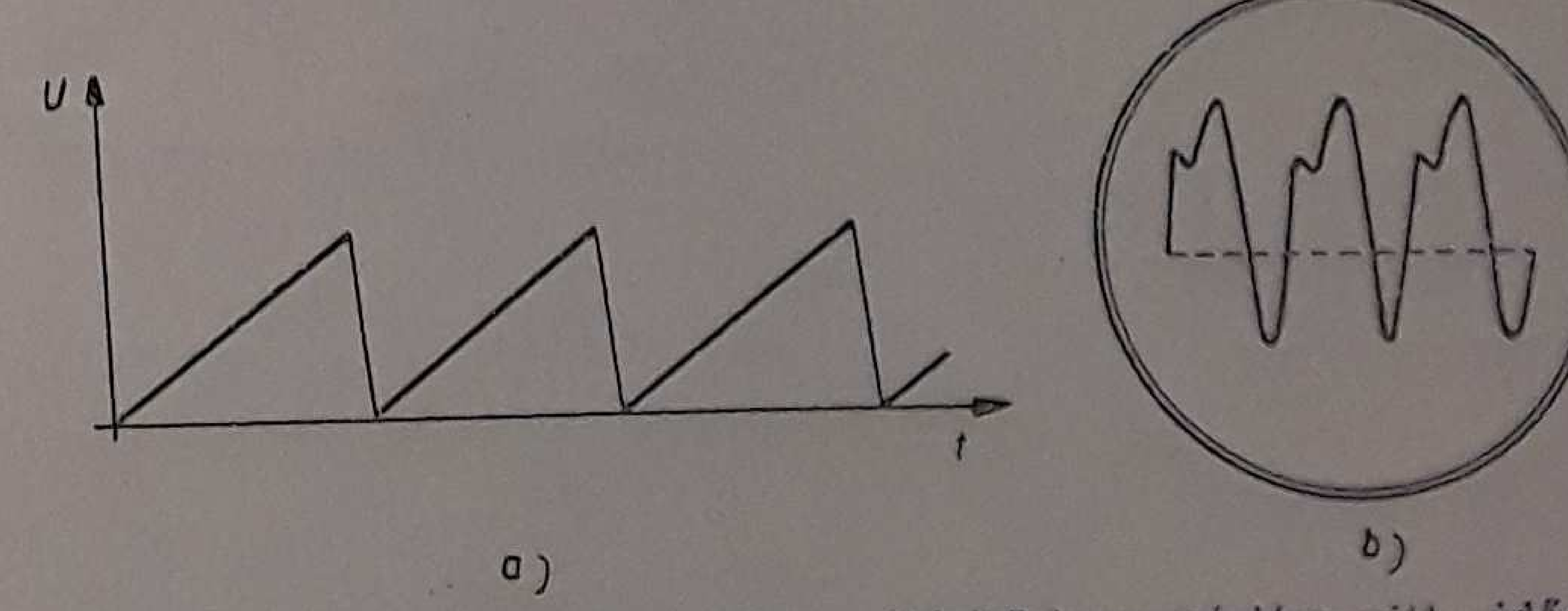


10. ábra. Katódsugárcső elektrosztatikus eltérítéssel

$$e = mx + b$$

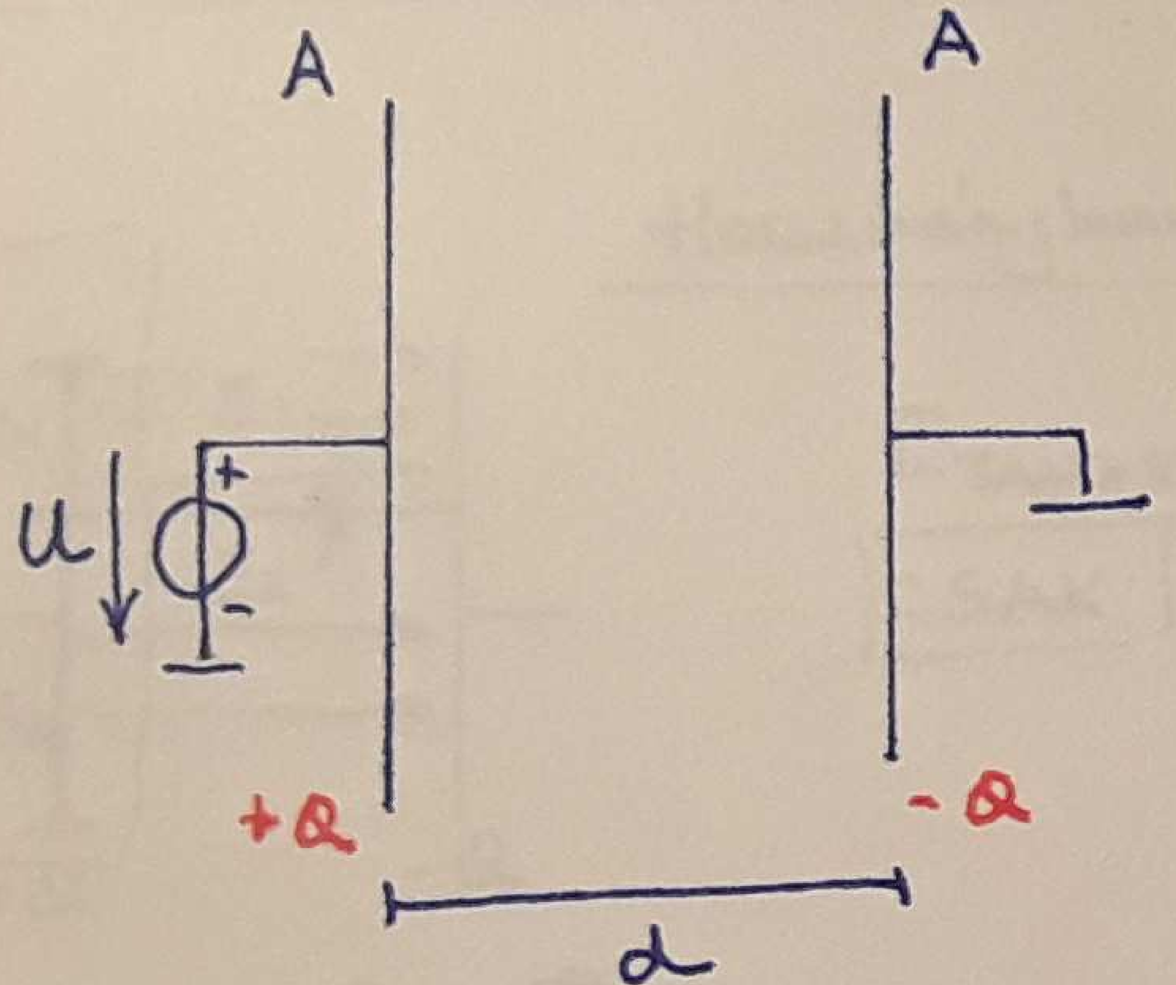
$$\boxed{\frac{1}{2} \frac{q}{m} \frac{E}{v_{0x}^2} l^2} = \boxed{\frac{qEl}{m v_{0x}^2}} l + b \rightarrow b = -\frac{1}{2} \frac{q}{m} \frac{E}{v_{0x}^2} l^2$$

$$e = \frac{qEl}{m v_{0x}^2} x - \frac{1}{2} \frac{q}{m} \frac{E}{v_{0x}^2} l^2$$



11. ábra. a) A katódsugárcső vízszintes eltérítő lemezpárjára vitt „időeltérítés”
b) A katódsugár ernyőjén látható kép. A szaggatott vonal az időeltérítés visszaugró ágának felel meg

Síkkondenzátor



$$\vec{D} = \vec{e}_x \frac{Q}{A}$$

$$\vec{E} = \vec{e}_x \frac{Q}{\varepsilon A}$$

$$U = \frac{Qd}{\varepsilon A}$$

$$\boxed{C = \varepsilon \frac{A}{d}}$$

$$\oint_A \underbrace{\vec{D} \cdot d\vec{A}} = Q$$

$$\int_A D\,dA = Q$$

$$DA = Q$$

$$\boxed{D = \frac{Q}{A}}$$

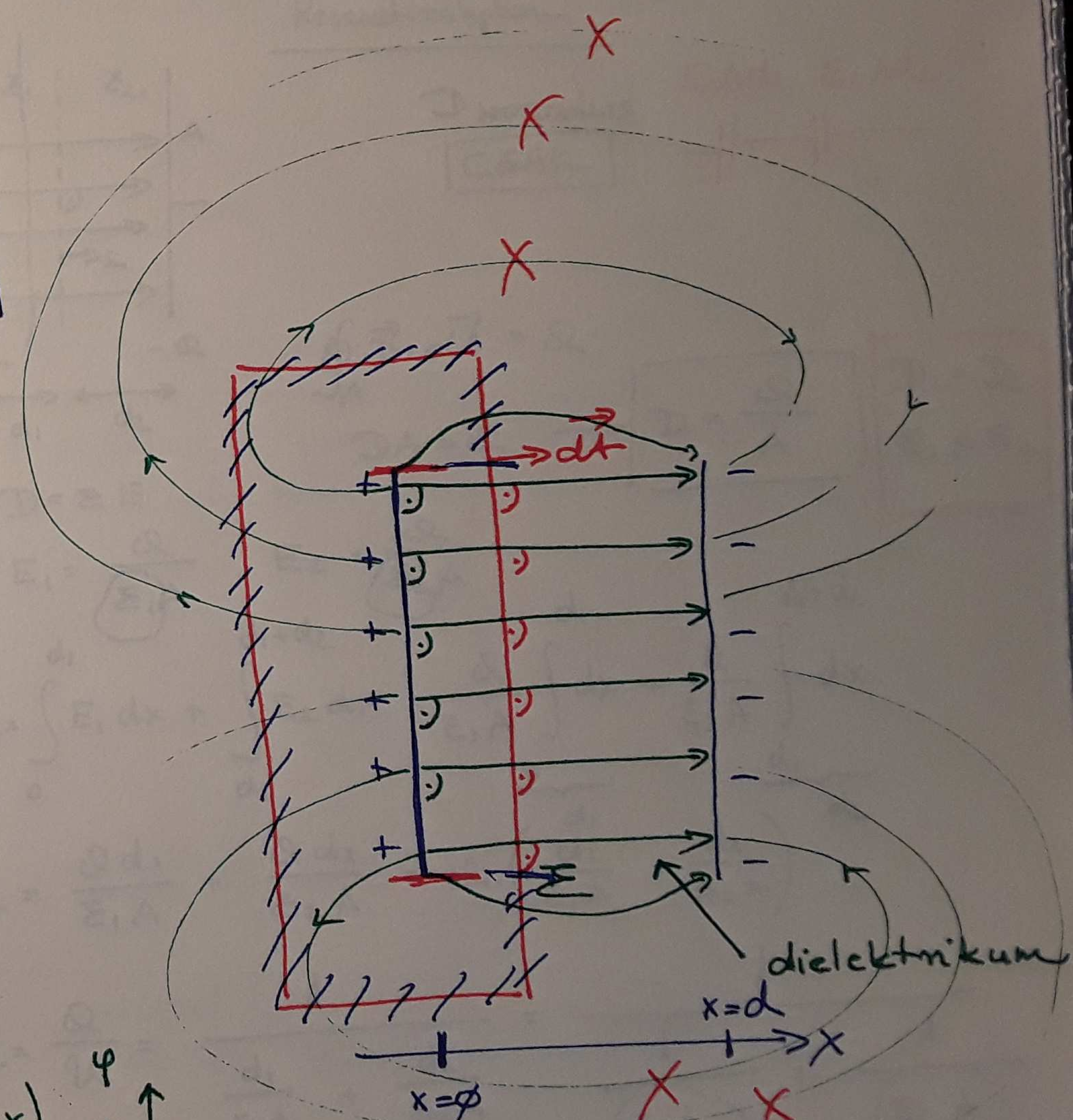
$$\vec{E} = \frac{\vec{D}}{\varepsilon}$$

$$U = \int_0^d E\,dx = \frac{Q}{\varepsilon A} \underbrace{\int_0^d dx}_{d}$$

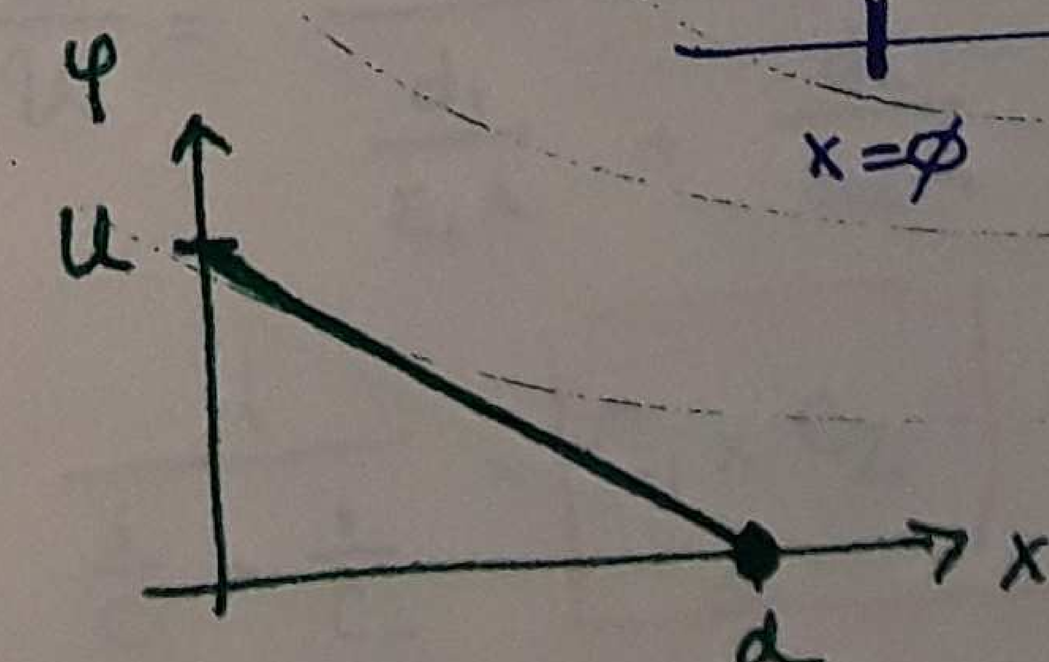
$$= \frac{Q}{\varepsilon A} [x]_0^d =$$

$$= \frac{Q}{\varepsilon A}(d-0)$$

$$C = \frac{Q}{U} = \varepsilon \frac{A}{d}$$

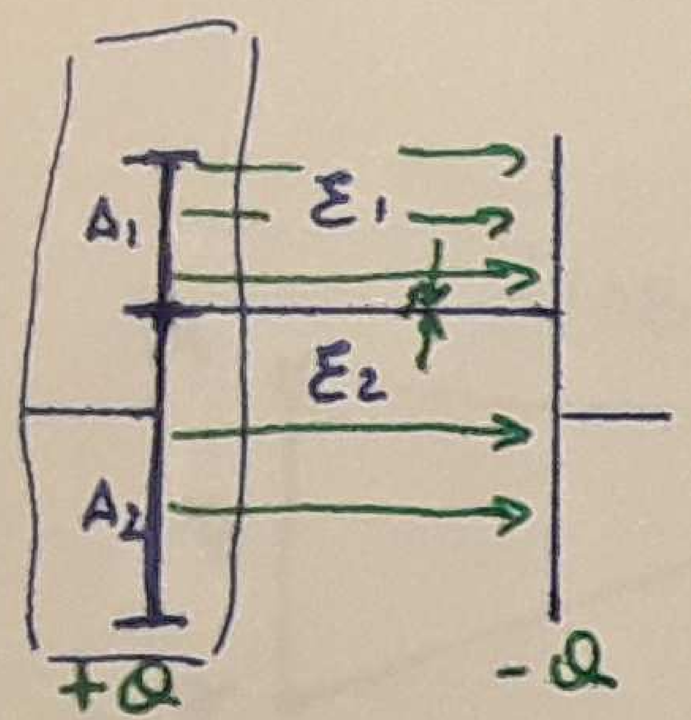


$$\varphi = \int_{r\,(x)}^{r_0\,(x_0)} E\,dx = \int_x^d E\,dx = \frac{Q}{\varepsilon A} \int_x^d dx = \frac{Q}{\varepsilon A}(d-x)$$



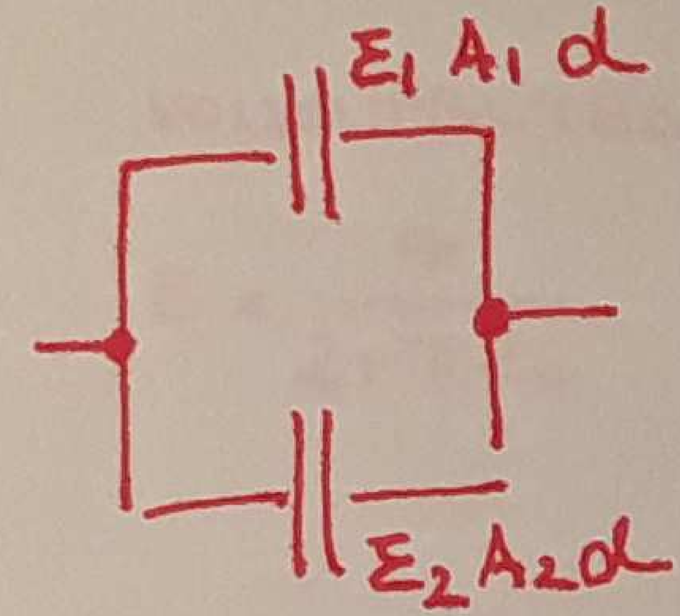
Rétegzett síkkondenzátor:

Hosszirányban



$E_{\text{TANGENCIÁLIS}}$

$\boxed{\text{CSAK}}$



$$\oint_A \vec{D} \cdot d\vec{A} = Q$$

$$D_1 A_1 + D_2 A_2 = Q$$

$$\varepsilon_1 E_1 A_1 + \varepsilon_2 E_2 A_2 = Q$$

$$\boxed{\begin{matrix} D_1 \neq D_2 \\ E_1 = E_2 \end{matrix}}$$

$$E(\varepsilon_1 A_1 + \varepsilon_2 A_2) = Q$$

$$\boxed{E = \frac{Q}{\varepsilon_1 A_1 + \varepsilon_2 A_2}}$$

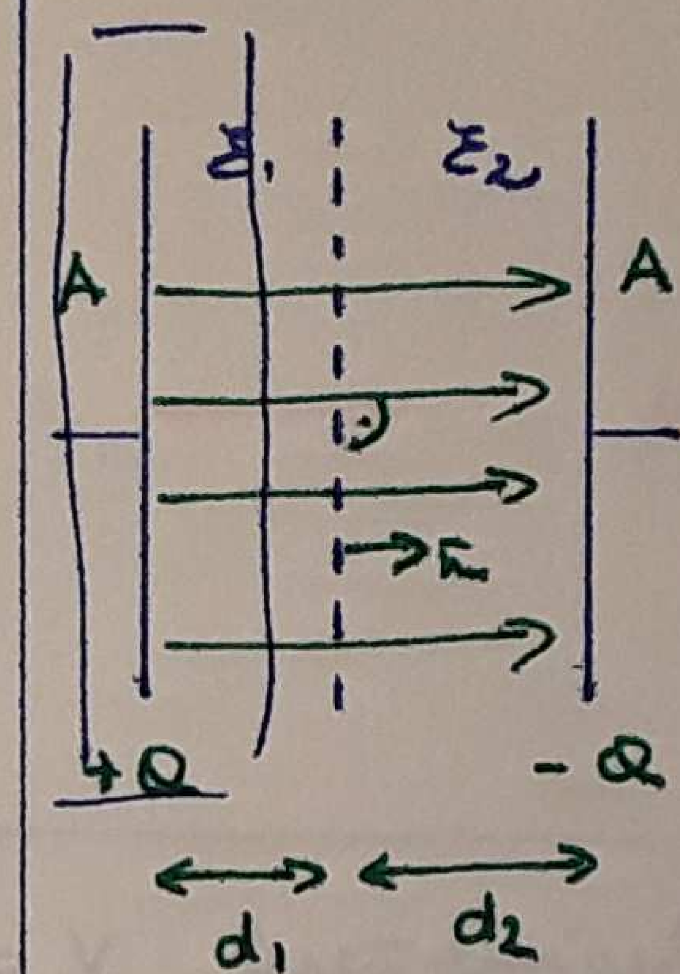
$\rightarrow D_1 = \varepsilon_1 E$

$\rightarrow D_2 = \varepsilon_2 E$

$$U = \int_0^d E\,dx = \frac{Qd}{\varepsilon_1 A_1 + \varepsilon_2 A_2}$$

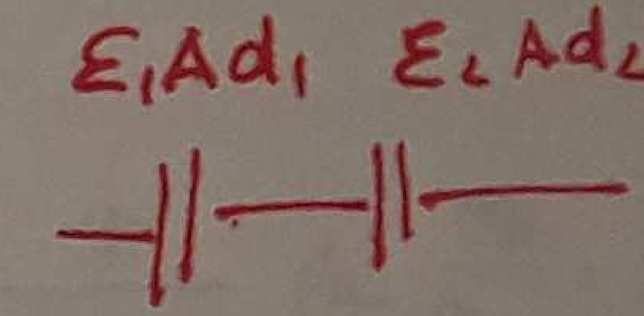
$$C = \frac{Q}{U} = \frac{\varepsilon_1 A_1 + \varepsilon_2 A_2}{d} = \boxed{\underbrace{\varepsilon_1 \frac{A_1}{d}}_{C_1} + \underbrace{\varepsilon_2 \frac{A_2}{d}}_{C_2}}$$

Keresztirányban



$D_{\text{NORMÁLIS}}$

$\boxed{\text{CSAK}}$



$$\oint_A \vec{D} \cdot d\vec{A} = Q$$

$$DA = Q \longrightarrow \boxed{D = \frac{Q}{A}}$$

$$\boxed{\begin{matrix} D_1 = D_2 \\ E_1 \neq E_2 \end{matrix}}$$

$$D = \varepsilon E$$

$$E_1 = \frac{Q}{\varepsilon_1 A} \qquad E_2 = \frac{Q}{\varepsilon_2 A}$$

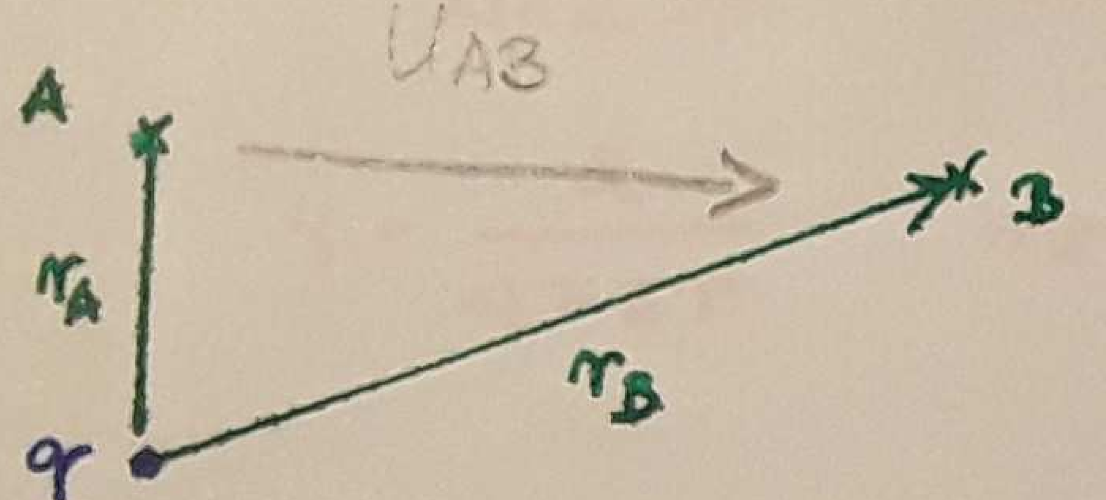
$$U = \int_0^{d_1} E_1\,dx + \int_{d_1}^{d_1+d_2} E_2\,dx = \frac{Q}{\varepsilon_1 A} \underbrace{\int_0^{d_1} dx}_{d_1} + \frac{Q}{\varepsilon_2 A} \underbrace{\int_{d_1}^{d_1+d_2} dx}_{d_2}$$

$$U = \frac{Q d_1}{\varepsilon_1 A} + \frac{Q d_2}{\varepsilon_2 A} = Q\left(\frac{d_1}{\varepsilon_1 A} + \frac{d_2}{\varepsilon_2 A}\right)$$

$$C = \frac{Q}{U} = \frac{1}{\frac{d_1}{\varepsilon_1 A} + \frac{d_2}{\varepsilon_2 A}} = \frac{1}{\frac{1}{\underbrace{\frac{\varepsilon_1 A}{d_1}}_{C_1}} + \frac{1}{\underbrace{\frac{\varepsilon_2 A}{d_2}}_{C_2}}}$$

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \boxed{C_1 \times C_2}$$

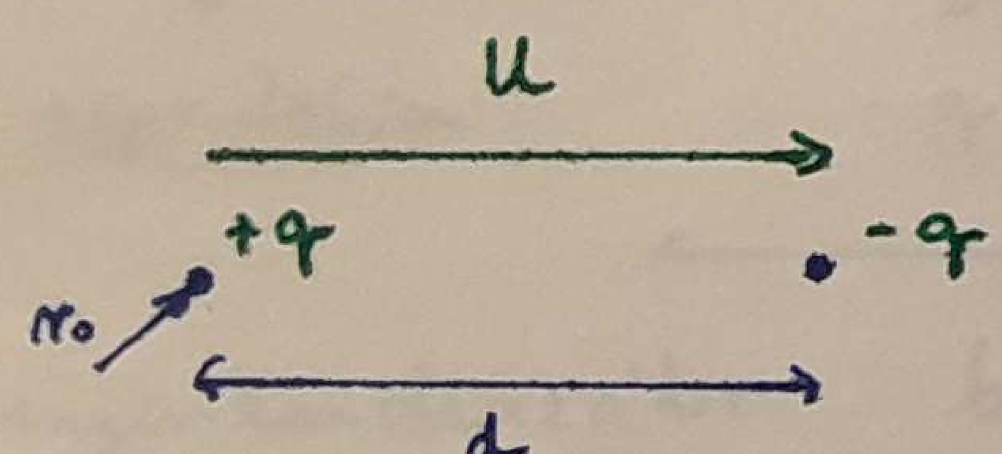
Számítsuk ki a $q = 0{,}5\ nC/m$ vonaltöltés tengelyétől $r_A = 4m$ és $r_B = 10m$ távolságra lévő két pont között a feszültséget! A vonaltöltés levegőben van.



VONALTÖLTÉS

$$E = \frac{q}{2r\pi\varepsilon_0} \longrightarrow U_{AB} = \int_{r_A}^{r_B} \frac{q}{2\pi\varepsilon_0}\frac{1}{r}\,dr = \frac{q}{2\pi\varepsilon_0}\ln\frac{r_B}{r_A} = \underline{\underline{8{,}235\,V}}$$

Két, egymással párhuzamos vezetőre $U = 200\ V$ feszültséget kapcsolunk. Határozzuk meg a vezetők 1m hosszú darabján a töltést, ha a vezetők levegőben helyezkednek el, sugaruk $r_0 = 2mm$, távolságuk pedig $d = 5cm$!



VONALTÖLTÉSEK (L. PÁRHUZAMOS VEZETÉKEK PÉLDA).

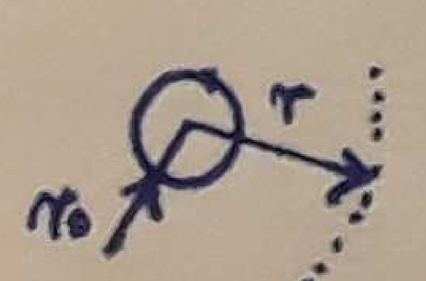
$$U = \frac{q}{2\pi\varepsilon_0}\ln\frac{d^2}{r_1 r_2}\bigg|_{r_1 = r_2 = r_0} = \frac{q}{\pi\varepsilon_0}\ln\frac{d}{r_0}$$

$$q = \frac{U\pi\varepsilon_0}{\ln d/r_0} = 1{,}728\cdot 10^{-9}\ C/m = \underline{\underline{1{,}728\ nC/m}}$$

Egy levegőben elhelyezett $r_0 = 10cm$ sugarú, igen hosszú töltött fémhenger tengelyétől $r = 20\,cm$ távolságra az 1m sugarú hengerre vonatkoztatott potenciál $\varphi = 578\,kV$. Számítsuk ki a henger töltését!

VONALTÖLTÉS POTENCIÁLJA:

$$\varphi = \frac{q}{2\pi\varepsilon_0}\ln\frac{1}{r} \rightarrow q = \frac{\varphi\, 2\pi\varepsilon_0}{\ln 1/r} \cong 2\cdot 10^{-5}\ C/m = \underline{\underline{20\ \frac{\mu C}{m}}}$$



Mekkora a Föld kapacitása, ha sugara $R = 6000\ km$?

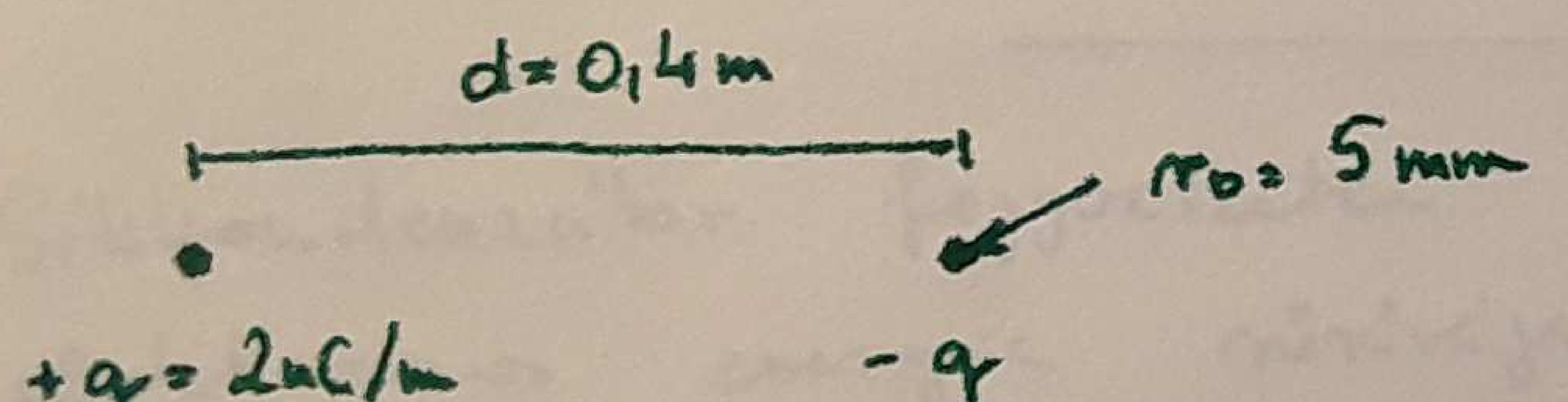
GÖMB

$$E = \frac{Q}{4\pi\varepsilon r^2} \qquad r \geq R$$

$$\varphi = \frac{Q}{4\pi\varepsilon r} \qquad r \geq R \longrightarrow U = \frac{Q}{4\pi\varepsilon_0 R} \longrightarrow C = \frac{Q}{U} = 4\pi\varepsilon_0 R = \underline{\underline{667{,}6\ \mu F}}$$

($C = 4\pi\varepsilon R$ a gömb kapacitása)

Határozzuk meg az alábbi vonaltöltések között a potenciálkülönbséget!



$$U = \frac{q}{\pi\varepsilon_0} \ln\frac{d}{r_0} = \underline{\underline{315\ V}}$$

Hengerkondenzátor belső és külső elektródájának sugara $r_1 = 1cm$ és $r_2 = 3cm$, a dielektrikum relatív permittivitása 3. A belső vezetőn fellépő elektromos térerősség 50 kV/cm. Mekkora vonaltöltéssel modellezhető a kondenzátor?

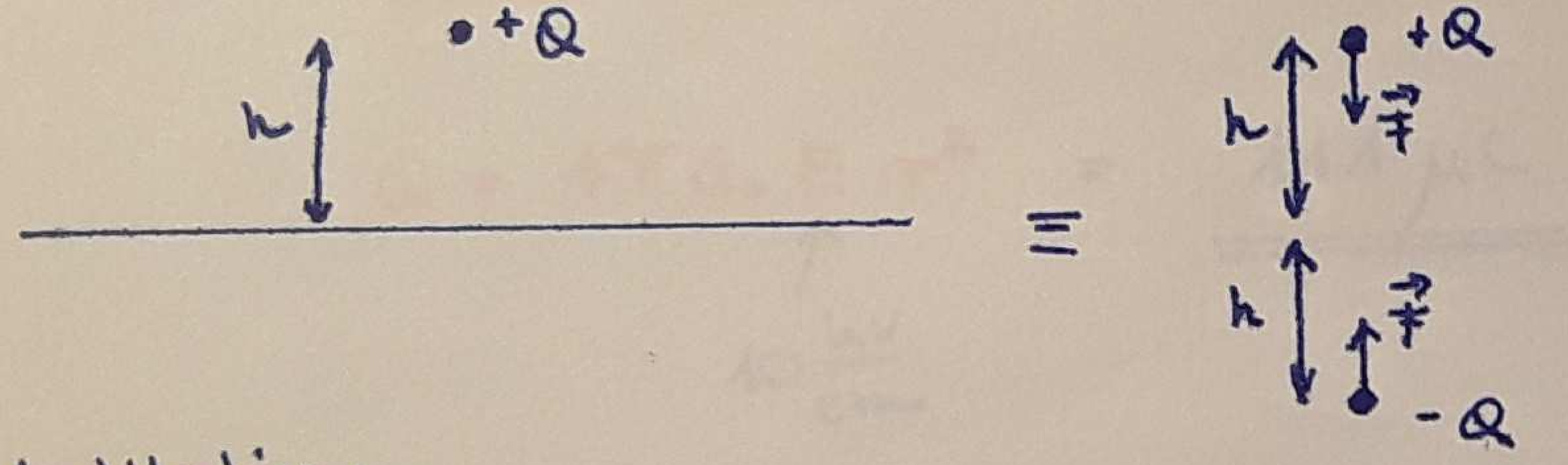
VONALTÖLTÉS

$$q = \varepsilon E 2 r \pi = \underline{\underline{8{,}35\ \frac{\mu C}{m}}}$$

r_1 !

$$\left(8{,}854 \cdot 10^{-12} \cdot 3 \cdot 50 \cdot \frac{1000}{0{,}01} \cdot 2 \cdot 0{,}01 \cdot 3{,}14\right)$$

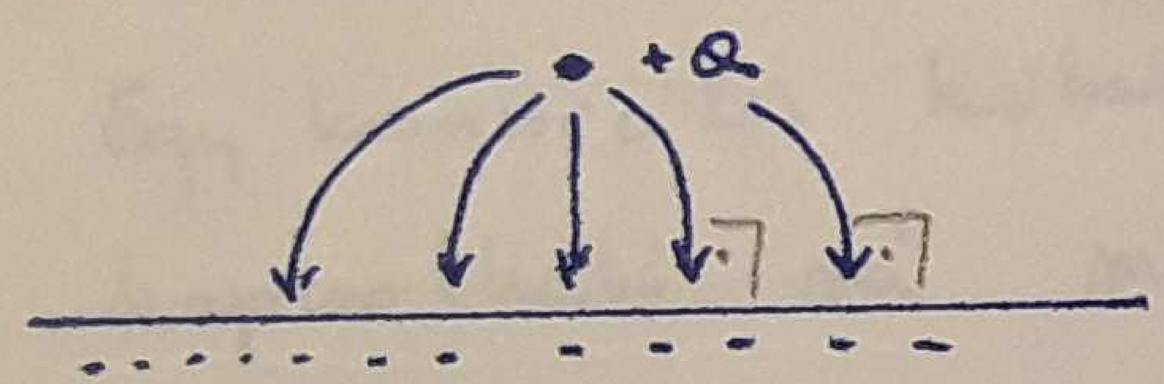
Végtelen kiterjedésűnek tekinthető fémsík felett $h = 20\,cm$ magasságban kis sugarú vezető gömb helyezkedik el, töltése $Q = 3\,nC$. Határozzuk meg a töltésre ható erőt!



$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(2h)^2} = \underline{\underline{5{,}056 \cdot 10^{-7}\,N}}$$

vonzóerő lép fel.

Kvalitatíve:



Síkkondenzátor fegyverzetei $d = 1\,cm$-re vannak egymástól, $\varepsilon_r = 20$. A tárolt elektromos energia sűrűsége $1\,mJ/m^3$. Mekkora a kapocsfeszültség?

$$w_e = \frac{1}{2}\varepsilon E^2 \longrightarrow \quad w_e = \frac{1}{2}\varepsilon\frac{U^2}{d^2} \longrightarrow U = \sqrt{\frac{2 w_e d^2}{\varepsilon_0 \varepsilon_r}} = \underline{\underline{33{,}61\,V}}$$

$$E = \frac{U}{d}$$

W_e-ről később beszélünk!

Mekkora töltést lehet felvinni egy $r_0 = 1m$ sugarú gömbre, ha a levegőben a kritikus térerősség 30 kV/cm, s ennek harmadát engedélyezzük?

GÖMB

$$Q = 4\pi\varepsilon_0 E r^2 = \underline{\underline{111\ \mu C}}$$

E: $10 \frac{kV}{cm}$

Egy koaxiális kábel adatai: $r_1 = 3mm$, $r_2 = 12mm$, $\varepsilon_r = 4$. Mekkora feszültség kapcsolható rá maximum, ha a kritikus térerősség 280 kV/cm, s négyszeres biztonságra törekszünk?

VONALTÖLTÉS

$$q = 2r\pi\varepsilon_0\varepsilon_r E$$

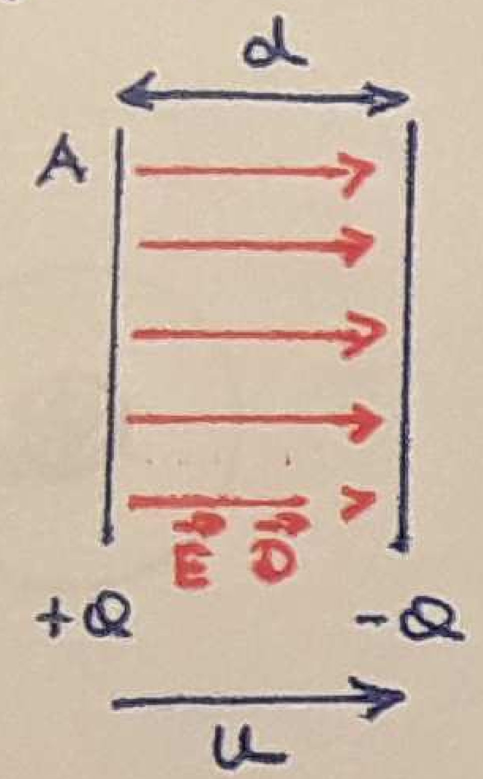
$$U = \int_{r_1}^{r_2} E\,dr = \frac{q}{2\pi\varepsilon_0\varepsilon_r} \ln\frac{r_2}{r_1}$$

$$U = r_1 E \ln\frac{r_2}{r_1} \cong \underline{\underline{29\,kV}}$$

r_1: !

E: $\frac{280}{4}$ kV/cm

Igazoljuk síkkondenzátor esetén az $E = U/d$ összefüggést!



$$\oint_A \vec{D}\cdot d\vec{A} = Q$$

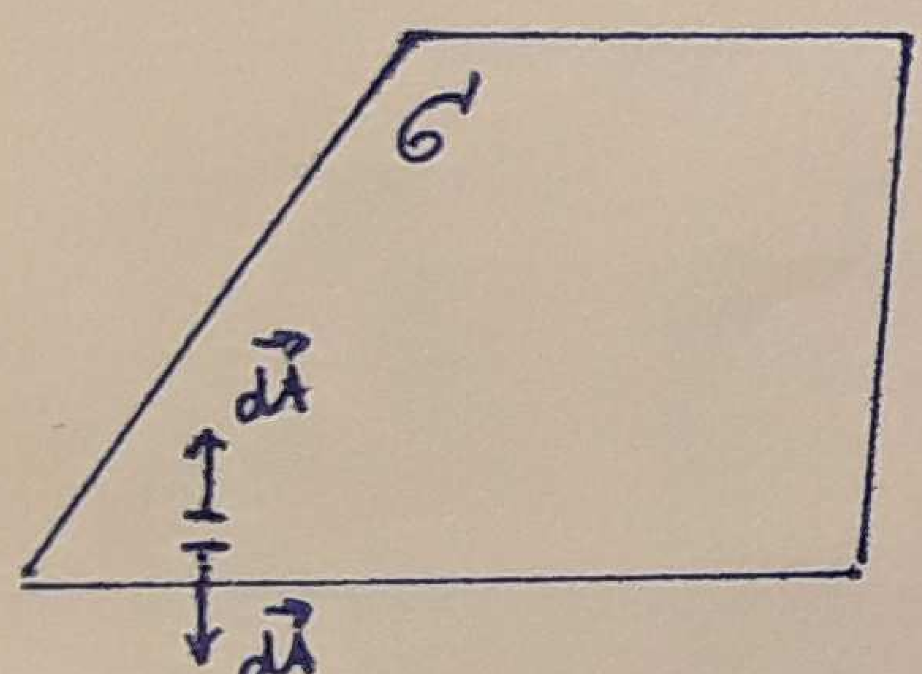
$$\varepsilon E A = Q$$

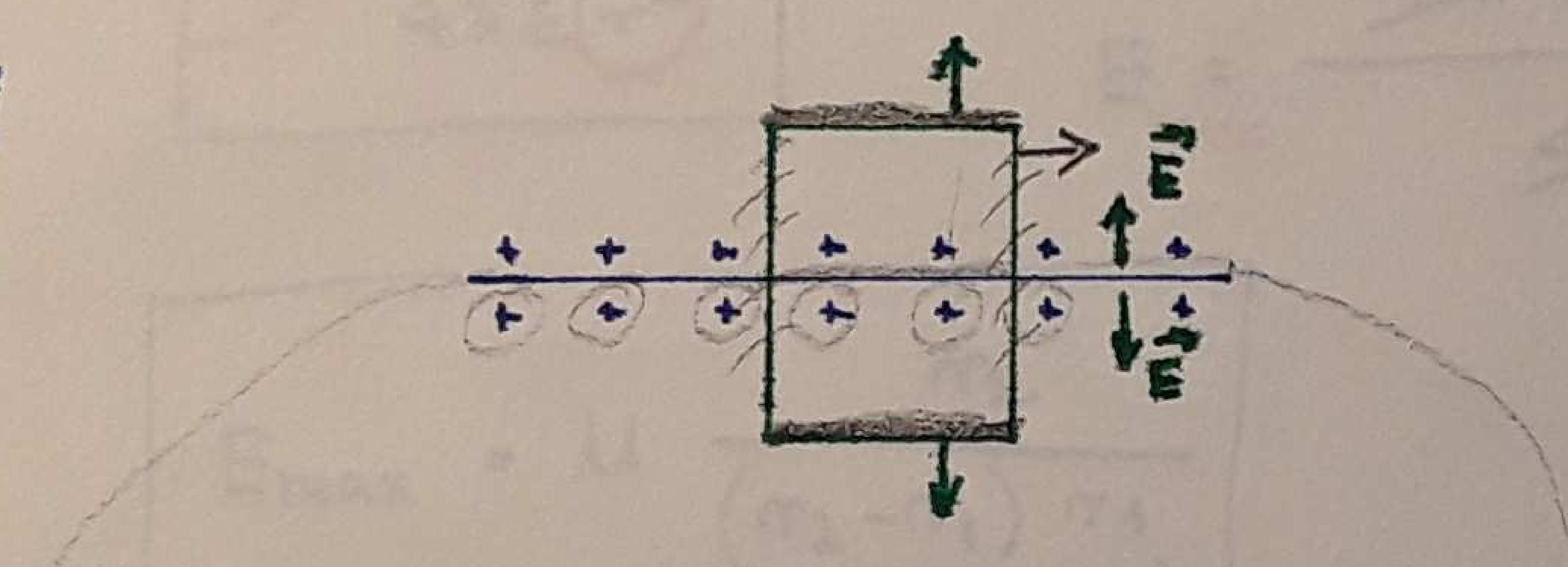
$$E = \frac{Q}{\varepsilon A}$$

$$U = \int_0^d E\,dx = \frac{dQ}{\varepsilon A}$$

$$U = \underbrace{\frac{Q}{\varepsilon A}}_{E}\, d \qquad U = Ed$$

Határozzuk meg egy végtelen nagy kiterjedésű sík elektromos terét!

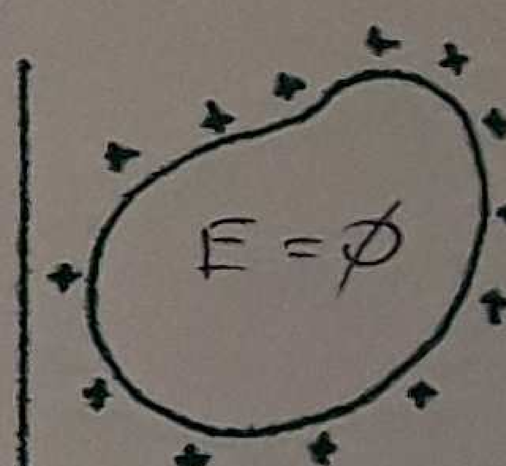




$$\oint_A \vec{D}\cdot d\vec{A} = \int \rho\, dV$$

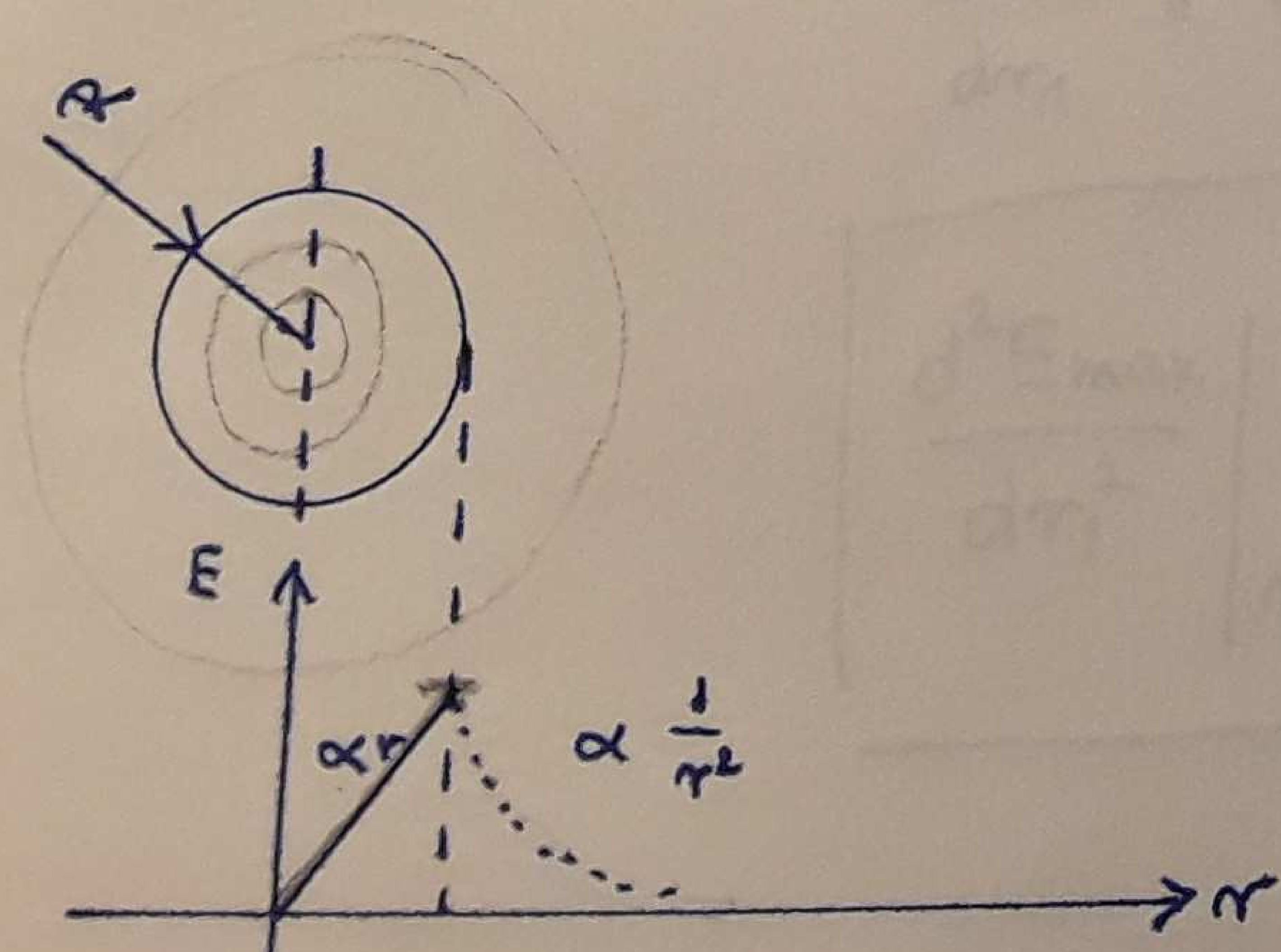
$$2\cdot \boxed{\varepsilon E A} = \sigma A$$

$$E = \frac{\sigma}{2\varepsilon}$$



$$E = \frac{\sigma}{\varepsilon}$$

Határozzuk meg a ρ térfogati töltéssűrűséggel töltött gömb elektromos terét!



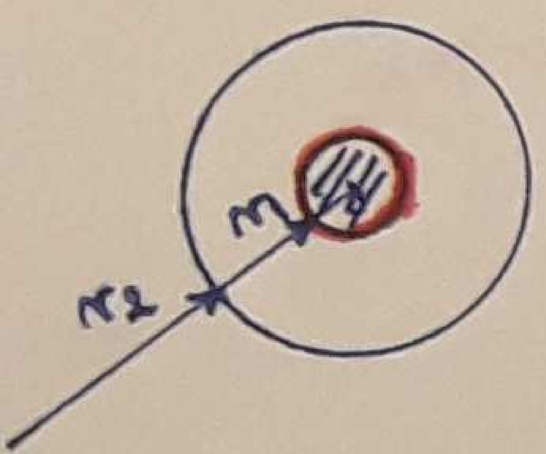
Belül: $\underbrace{\varepsilon E}_{D}\underbrace{4\pi r^2}_{A} = \underbrace{\frac{4}{3} r^3 \pi \rho}_{V} \longrightarrow E = \frac{\rho r}{3\varepsilon}$

Kívül: $\varepsilon E 4\pi r^2 = Q \longrightarrow E = \frac{Q}{4\pi\varepsilon r^2}$

$Q = \frac{4}{3} R^3 \pi \rho$

R

Gömbkondenzátor: belső sugara r_1, külső sugara r_2, a rákapcsolt feszültség U. Határozzuk meg r_1 azon értékét, ahol a maximális térerősség a legkisebb!



$E_{max}\ (r = r_1)$

$$\oint_A \vec{D}\,d\vec{A} = Q \qquad U = \int_{r_1}^{r_2} E\,dr = \dots = \frac{Q}{4\pi\varepsilon}\,\frac{r_2 - r_1}{r_1 r_2}$$

$$\varepsilon E 4\pi r^2 = Q \qquad Q = 4\pi\varepsilon U \frac{r_1 r_2}{r_2 - r_1}$$

$$\boxed{E = \frac{\boxed{Q}}{4\pi\varepsilon r^2}}$$

$$E_{max} = \frac{\cancel{4\pi\varepsilon} U \frac{\cancel{r_1} r_2}{r_2 - r_1}}{\cancel{4\pi\varepsilon}\, r_1^{\cancel{2}}}$$

$$\boxed{E_{max} = U \frac{r_2}{\underbrace{(r_2 - r_1)\, r_1}_{r_1 r_2 - r_1^2}}}$$

E_{max} legkisebb $\longrightarrow$ $r_1 = ?$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

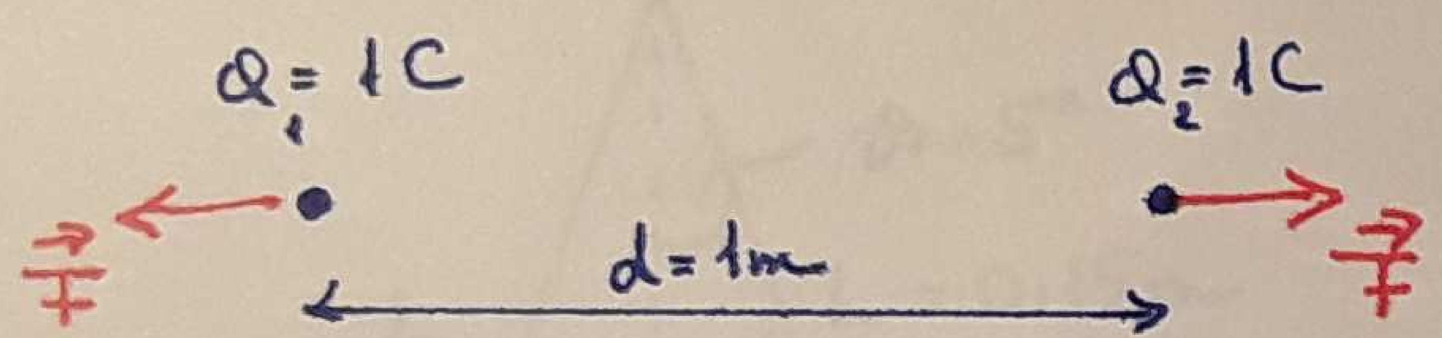
$$\frac{dE_{max}}{dr_1} = U \frac{\cancel{0 \cdot (r_2 - r_1) r_1} - r_2 (r_2 - 2r_1)}{(r_2 - r_1)^2 r_1^2} = -U \frac{r_2 (r_2 - 2r_1)}{(r_2 - r_1)^2 r_1^2} \stackrel{?}{=} 0$$

$$r_2 - 2r_1 = 0$$

$$r_2 = 2r_1 \longrightarrow \boxed{r_1 = \frac{r_2}{2}}$$

$$\boxed{\left.\frac{d^2 E_{max}}{dr_1^2}\right|_{r_1 = \frac{r_2}{2}} > 0}$$ H.F.

Határozzuk meg a két töltés közti erő nagyságát!

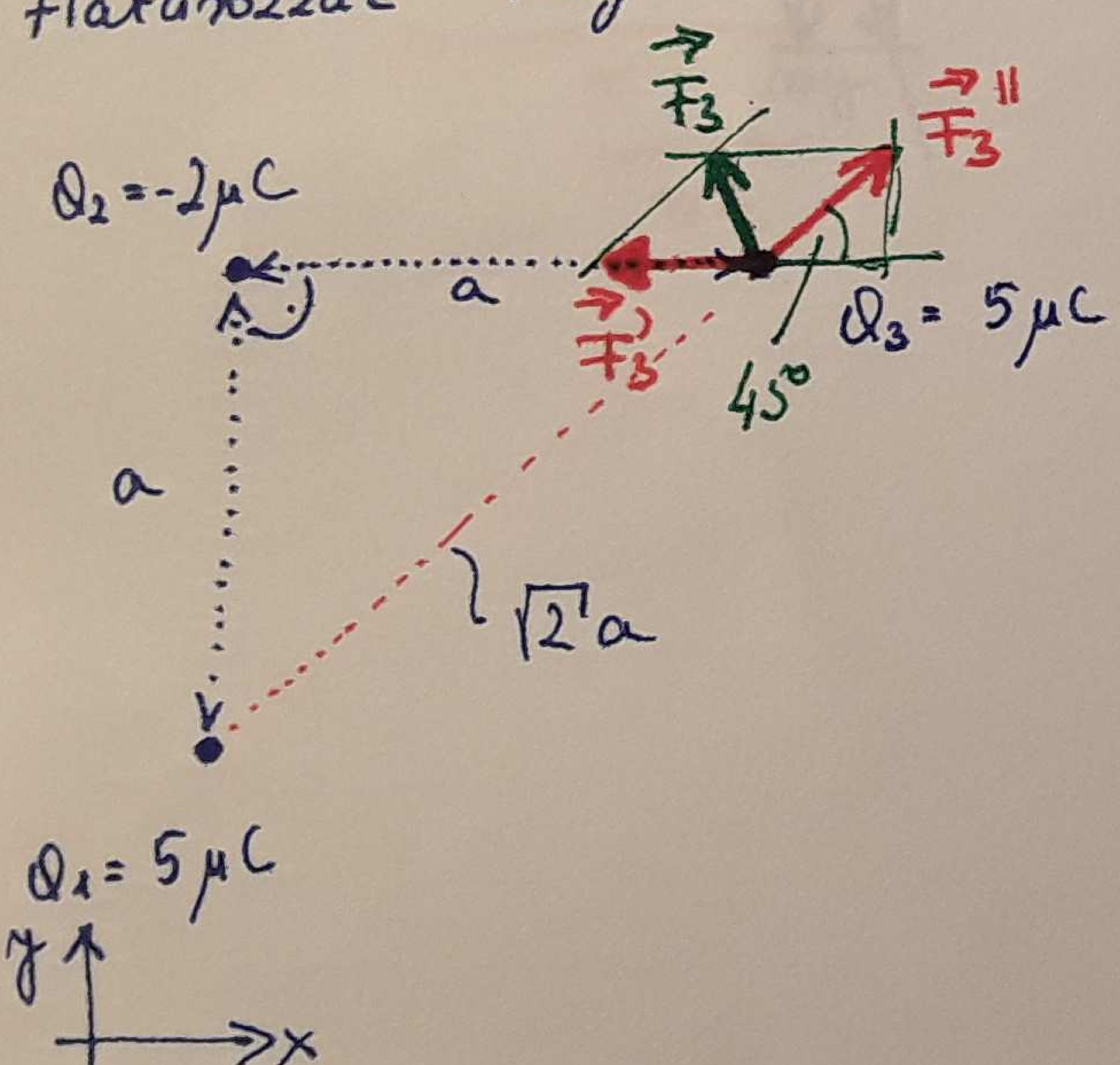


$$F = \frac{1}{4\pi\varepsilon_0}\frac{|Q_1 Q_2|}{d^2} \approx 9\cdot 10^9 \,\frac{1}{1} = \underline{\underline{9\cdot 10^9\, N}}$$

ÓRIÁSI ÉRTÉK!

$$F = mg \rightarrow m = \frac{F}{g} = 9\cdot 10^8\, kg \quad (900.000 \text{ tonna})$$

Határozzuk meg az $\vec{F}_3$ erőt! ($a = 0,1m$)



$$\vec{F}_3 = \vec{F}_3' + \vec{F}_3''$$

$$\boxed{\varepsilon_0 = 8{,}854\cdot 10^{-12}\,\frac{F}{m}}$$

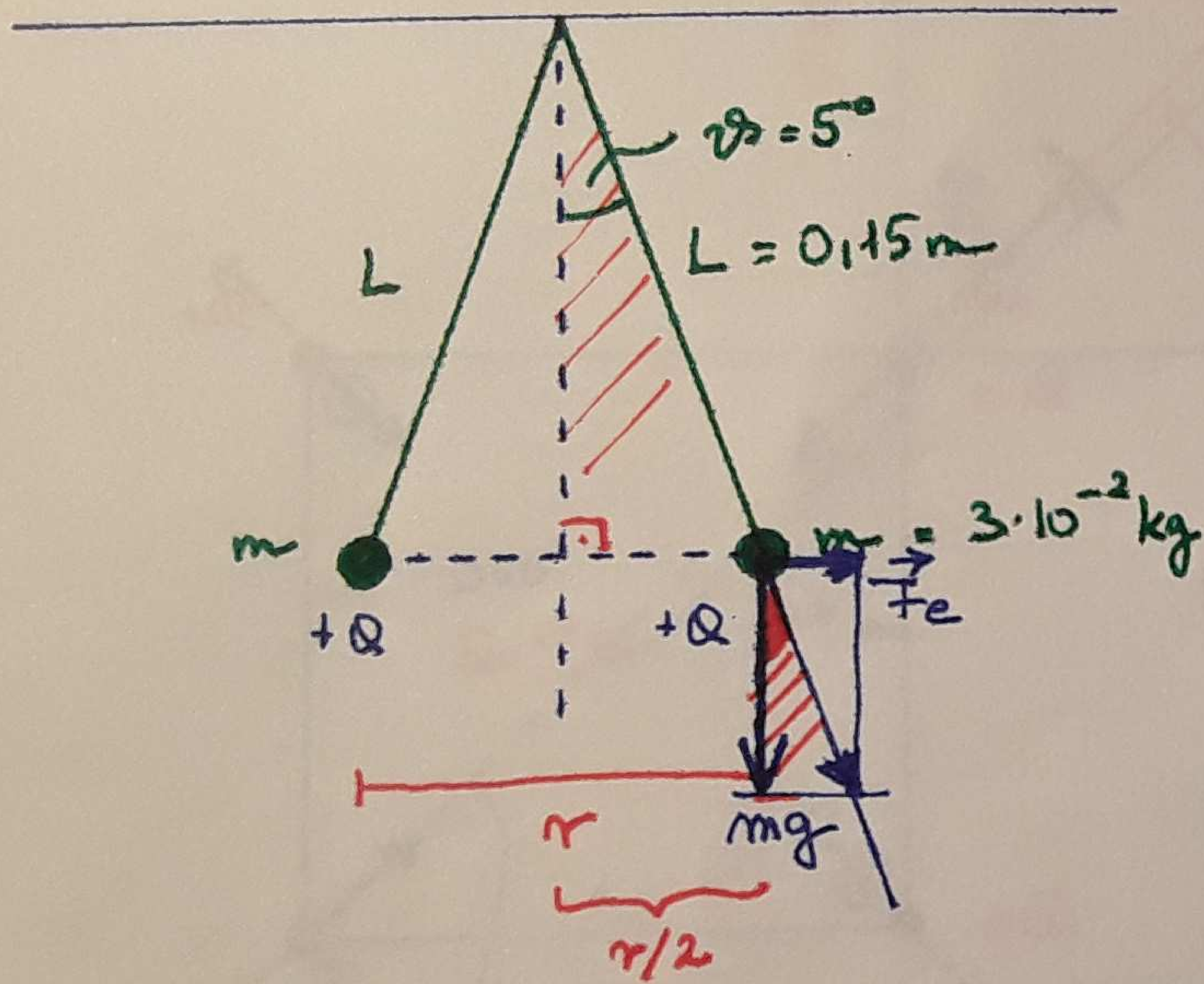
$$F_3' = \frac{1}{4\pi\varepsilon_0}\frac{|Q_2\cdot Q_3|}{a^2} = \underline{\underline{8{,}99N}}$$

$$\vec{F}_3' = (-8{,}99)\,\vec{e}_x\; N$$

$$F_3'' = \frac{1}{4\pi\varepsilon_0}\frac{|Q_1 Q_3|}{2a^2} = \underline{\underline{11{,}23N}} \qquad /\sqrt{2}$$

$$\vec{F}_3'' = (7{,}94\,\vec{e}_x + 7{,}94\,\vec{e}_y)\,N$$

$$\vec{F}_3 = (-1{,}05\,\vec{e}_x + 7{,}94\,\vec{e}_y)\,N$$



Határozzuk meg a gömbök töltését!

$$\sin\vartheta = \frac{r/2}{L} \longrightarrow r = 2L\sin\vartheta = \underline{\underline{0{,}02615\,m}}$$

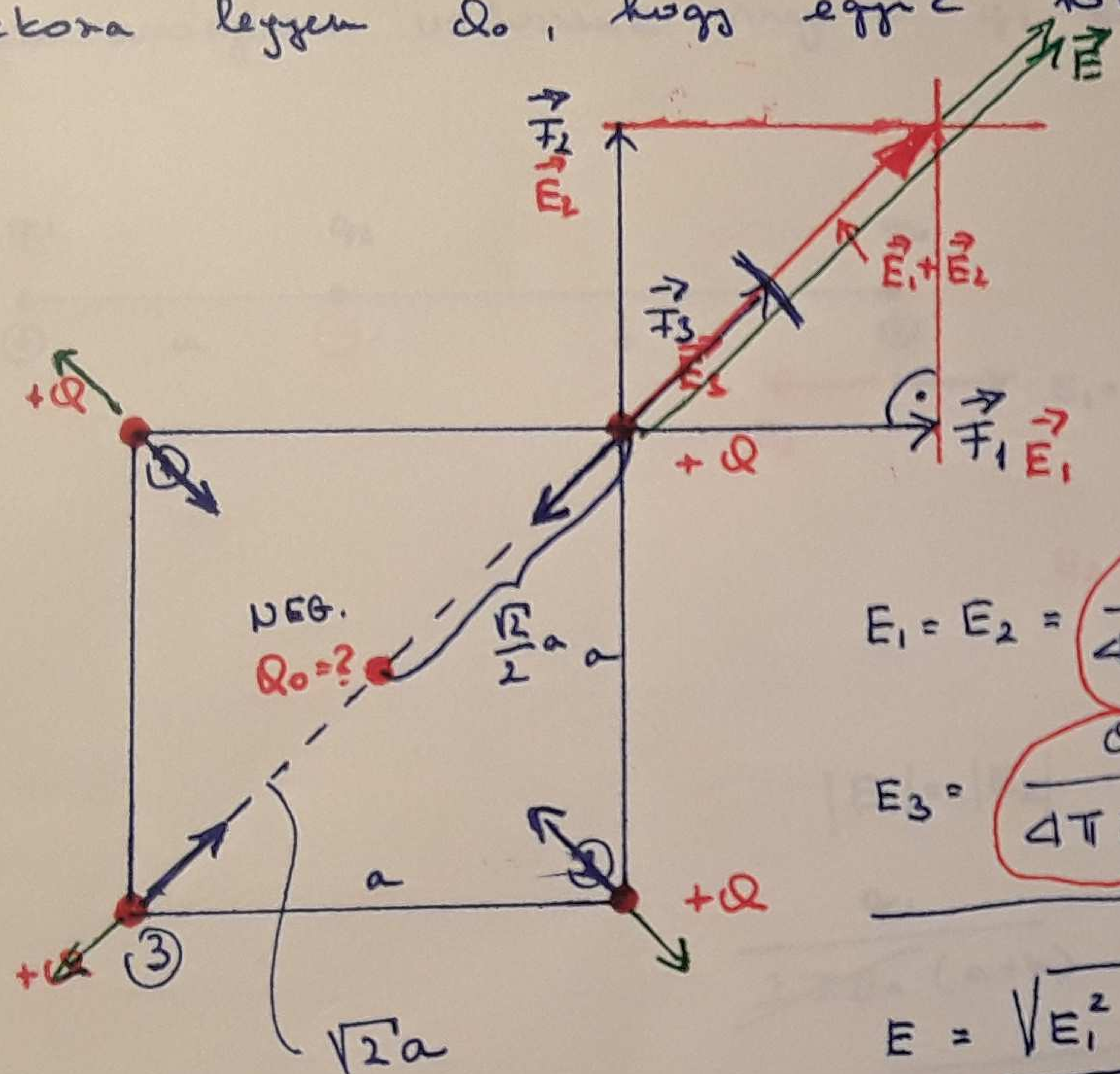
$$\operatorname{tg}\vartheta = \frac{F_e}{mg} \longrightarrow F_e = mg\operatorname{tg}\vartheta = \underline{\underline{0{,}02575\,N}}$$

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2}$$

$$Q = \sqrt{4\pi\varepsilon_0 r^2 F_e} = 4{,}426\cdot 10^{-8}\,C$$

$$\underline{\underline{44{,}26\,nC}}$$

Mekkora legyen Q_0, hogy egyik töltésre se hasson erő?



$$E_1 = E_2 = \frac{Q}{4\pi\varepsilon_0 a^2}$$

$$E_3 = \frac{Q}{4\pi\varepsilon_0 2a^2}$$

$$E = \sqrt{E_1^2+E_2^2} + E_3 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{\sqrt{2}}{a^2}+\frac{1}{2a^2}\right)$$

$$\sqrt{\left(\frac{1}{a^2}\right)^2+\left(\frac{1}{a^2}\right)^2}$$

$$\sqrt{\frac{1}{a^4}+\frac{1}{a^4}} = \sqrt{\frac{2}{a^4}} = \frac{\sqrt{2}}{a^2} \qquad \frac{\sqrt{2}+0{,}5}{a^2}$$

$$\frac{Q_0}{\cancel{4\pi\varepsilon_0}\left(\frac{\sqrt{2}}{2}a\right)^2} = \frac{Q}{\cancel{4\pi\varepsilon_0}}\underbrace{\left(\frac{\sqrt{2}}{a^2}+\frac{1}{2a^2}\right)}$$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2} + \text{irány.}\ (\underline{\underline{\text{IT}}})$$

$$\oint_A \vec{D}\cdot d\vec{A} = \int_V \varrho\, dV$$

$$\varepsilon E 4\pi r^2 = Q$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon r^2}}$$

ha $\vec{E}=\emptyset \longrightarrow \vec{F}=\emptyset$

$\vec{E} = \vec{F}/Q$

$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

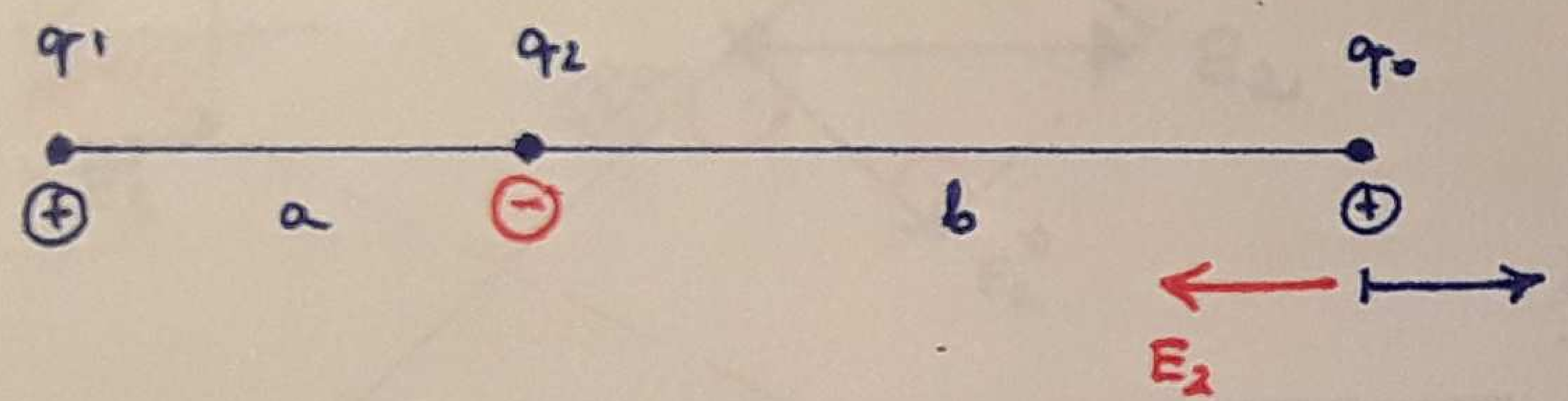
VEKTORIÁLIS!!!

$$\frac{Q_0}{\frac{2}{4}\cancel{a^2}} = Q\,\frac{\sqrt{2}+0{,}5}{\cancel{a^2}}$$

$$Q_0 = Q\underbrace{\frac{\sqrt{2}+0{,}5}{2}}_{0{,}96}$$

$$\boxed{Q_0 = -0{,}96\,Q.}$$

Milyen arányban válasszuk meg a q_1 és q_2 vonaltöltéseket, hogy q_0-ra ne hasson erő?



$$E_1 = \frac{q_1}{2\pi\varepsilon_0 (a+b)}$$

$$E_2 = \frac{-q_2}{2\pi\varepsilon_0 b}$$

$$\oint_A \vec{D}\cdot d\vec{A} = \int_V \rho\, dV$$

$$D\, 2r\pi l = q l$$

$$\varepsilon E\, 2r\pi \cancel{l} = q \cancel{l}$$

$$\boxed{E = \frac{q}{2 r \pi \varepsilon}}$$

ha $\vec{E} = 0 \longrightarrow \vec{F} = \emptyset$

ÁLTALÁNOS ...

$$|E_1| = |E_2|$$

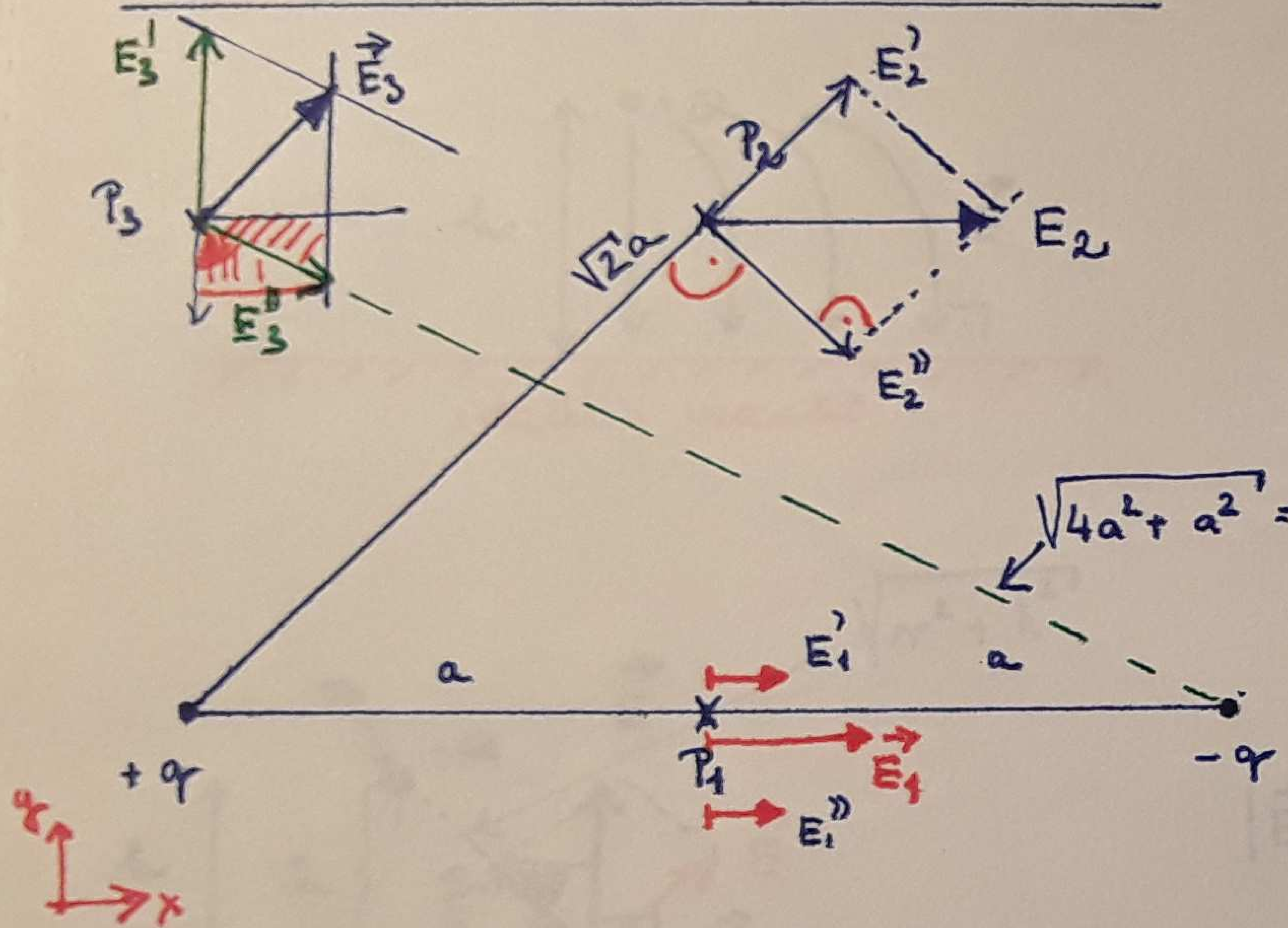
$$\frac{q_1}{\cancel{2\pi\varepsilon_0}(a+b)} = \frac{-q_2}{\cancel{2\pi\varepsilon_0}\, b}$$

$$\boxed{\frac{q_1}{q_2} = -\frac{a+b}{b}}$$

$E(P_1)$, $E(P_2)$, $E(P_3)$ = ?   vonaltöltések tere.

$q = 2\ \mu C/m$

$a = 1{,}2\ m$



$\varepsilon = \varepsilon_0$

$\sqrt{4a^2 + a^2} = \sqrt{5}a$

$$\boxed{E = \frac{q}{2\pi\varepsilon r}}$$

35.950,97

$$E_1' = E_1'' = \frac{q}{2\pi\varepsilon}\frac{1}{a} = \underline{29.959\ V/m}$$

$$\boxed{E_1 \cong 60.000\ V/m} \qquad \vec{E}_1 = (60000\,\vec{e}_x + 0\,\vec{e}_y)\ V/m.$$

$$E_2' = E_2'' = \frac{q}{2\pi\varepsilon}\frac{1}{\sqrt{2}a} = 21.183{,}7\ V/m$$

$$\boxed{E_2 = \sqrt{2}\,E_2' \cong 30.000\ V/m}$$

$$\vec{E}_2 = (30.000\,\vec{e}_x + 0\,\vec{e}_y)\ V/m.$$

$$E_3' = \frac{q}{2\pi\varepsilon}\frac{1}{a} = \underline{29.959\ V/m}$$

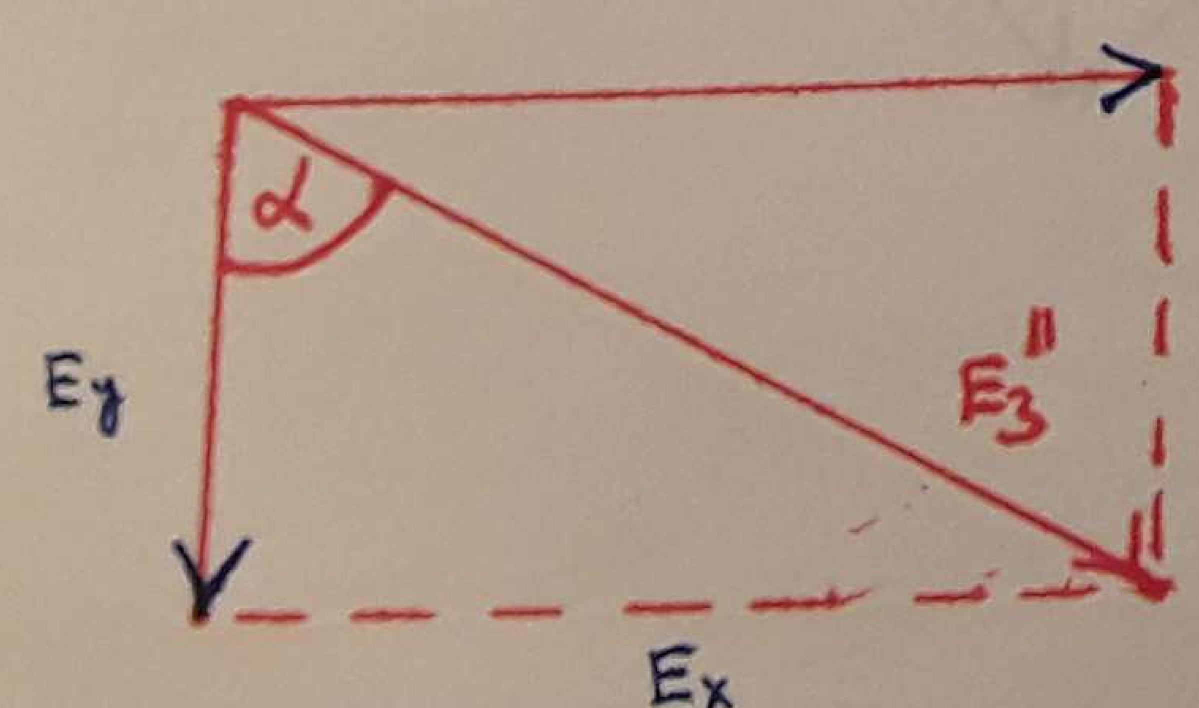
$$E_3'' = \frac{q}{2\pi\varepsilon}\frac{1}{\sqrt{5}a} = \underline{13.398\ V/m} \quad !$$

$$\mathrm{tg}\,\alpha = \frac{2a}{a} \Rightarrow \boxed{\alpha = 63{,}43^\circ}$$

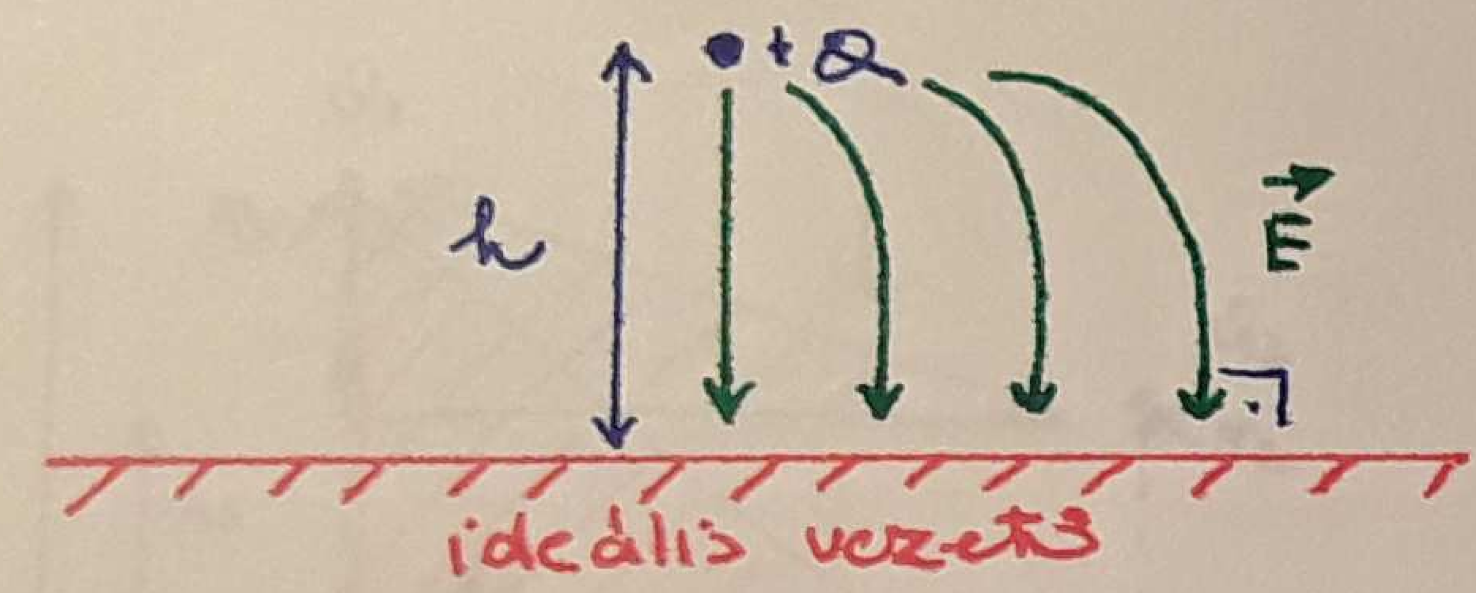
$$\cos 63{,}43^\circ = \frac{E_y}{E_3''} \longrightarrow E_y = \overset{!}{-}5992{,}8\ V/m$$

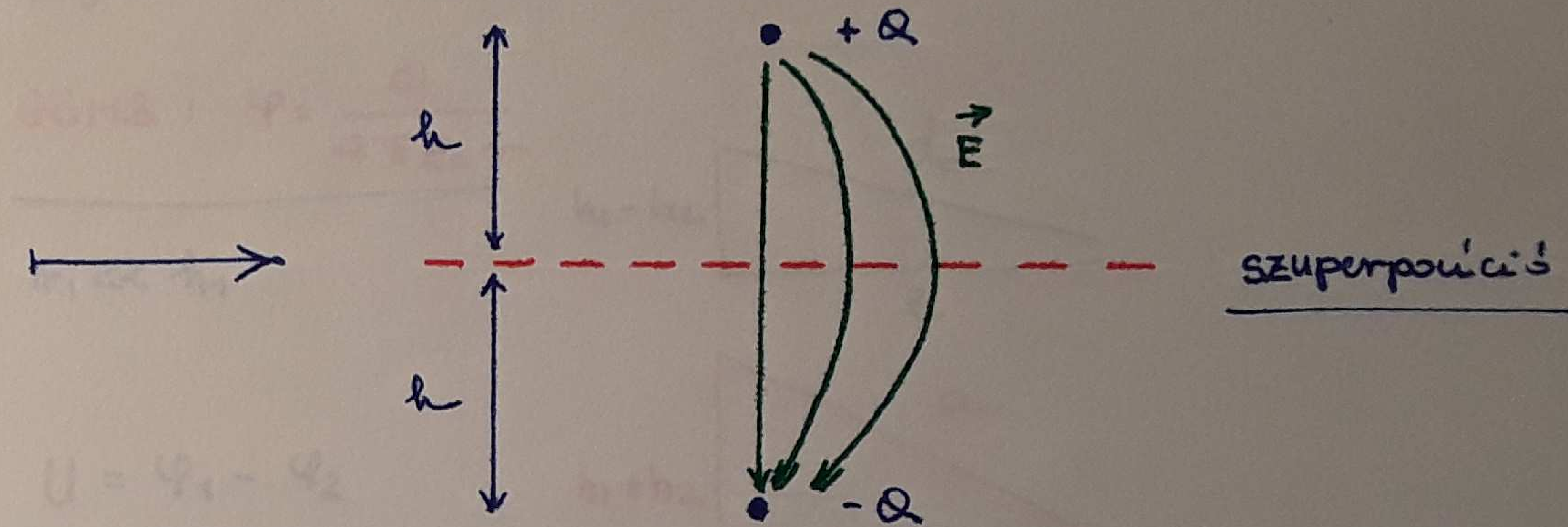
$$\sin 63{,}43^\circ = \frac{E_x}{E_3''} \longrightarrow E_x = 11.983\ V/m$$

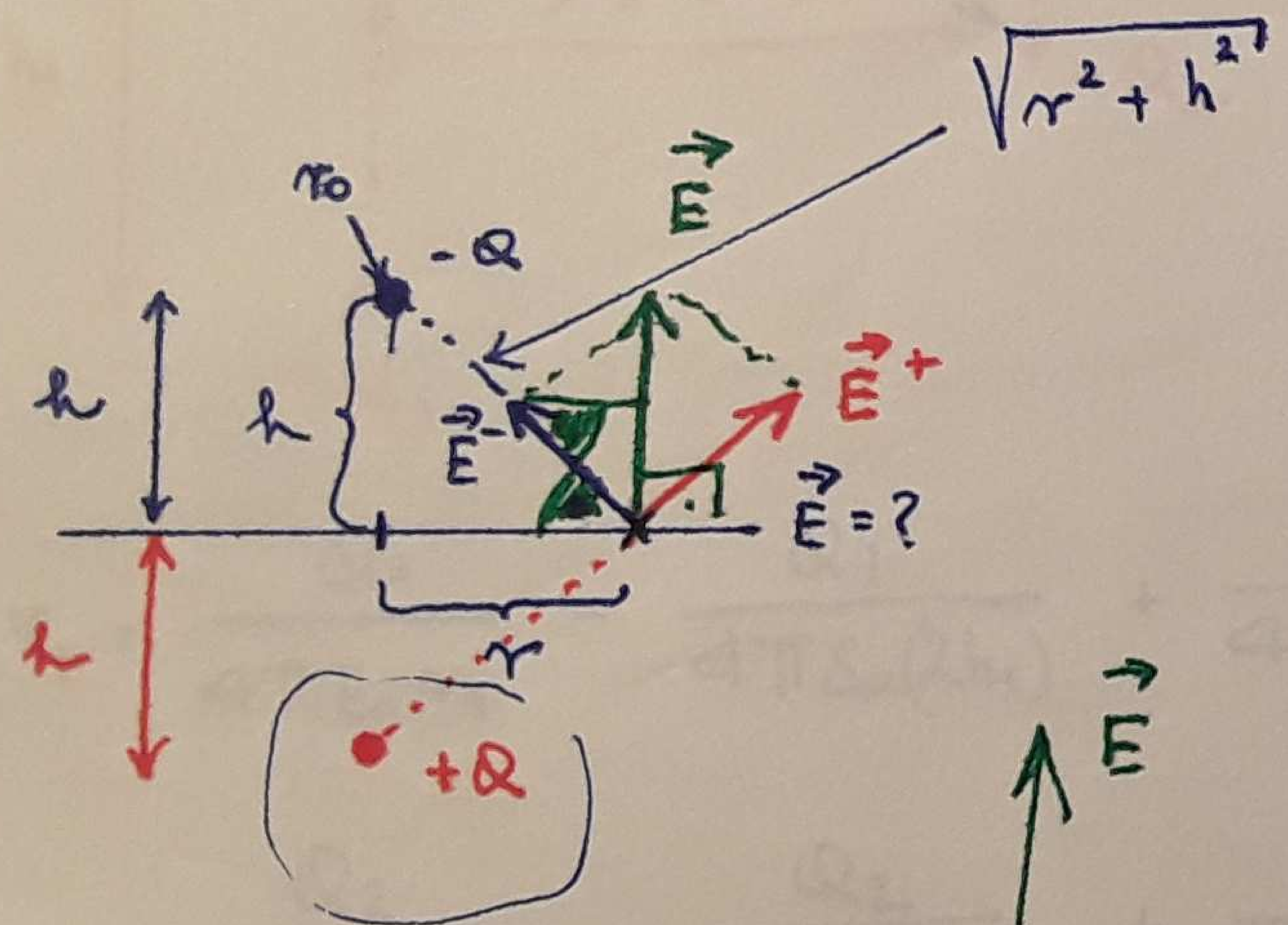
$$\vec{E}_3 = \left(11.983\,\vec{e}_x + \big(\underbrace{29.959 - 5992{,}8}_{23.966{,}2}\big)\vec{e}_y\right)\ V/m$$



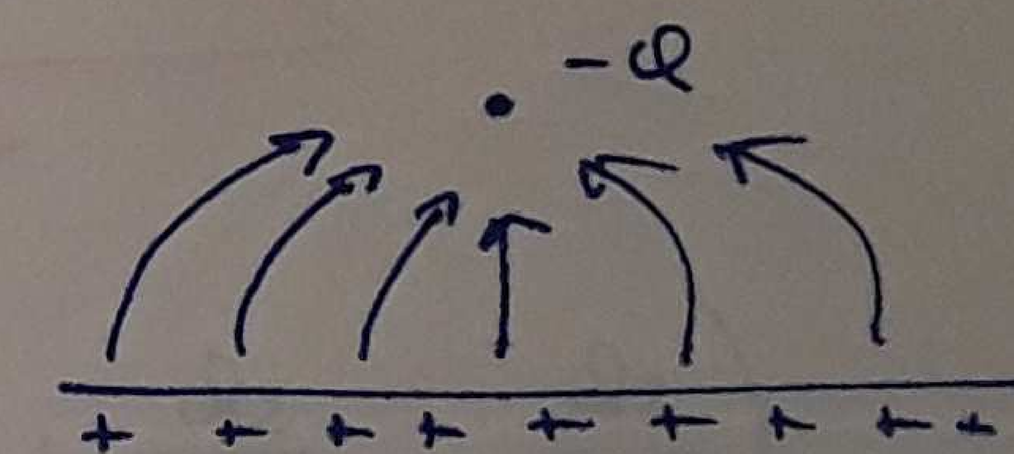
Tükrözés sík felületen





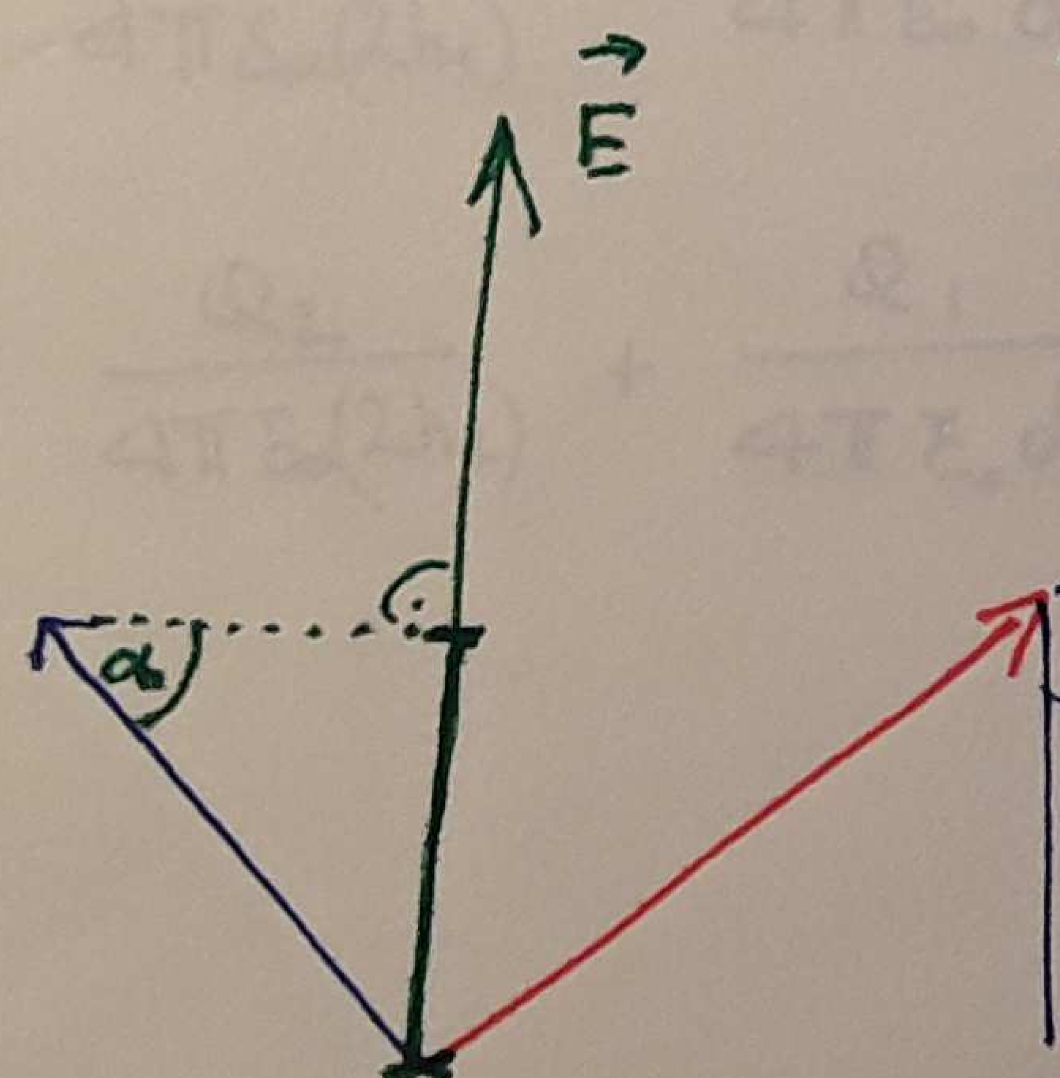


$$|\vec{E}^+| = |\vec{E}^-| = \frac{Q}{4\pi\varepsilon(r^2+h^2)}$$



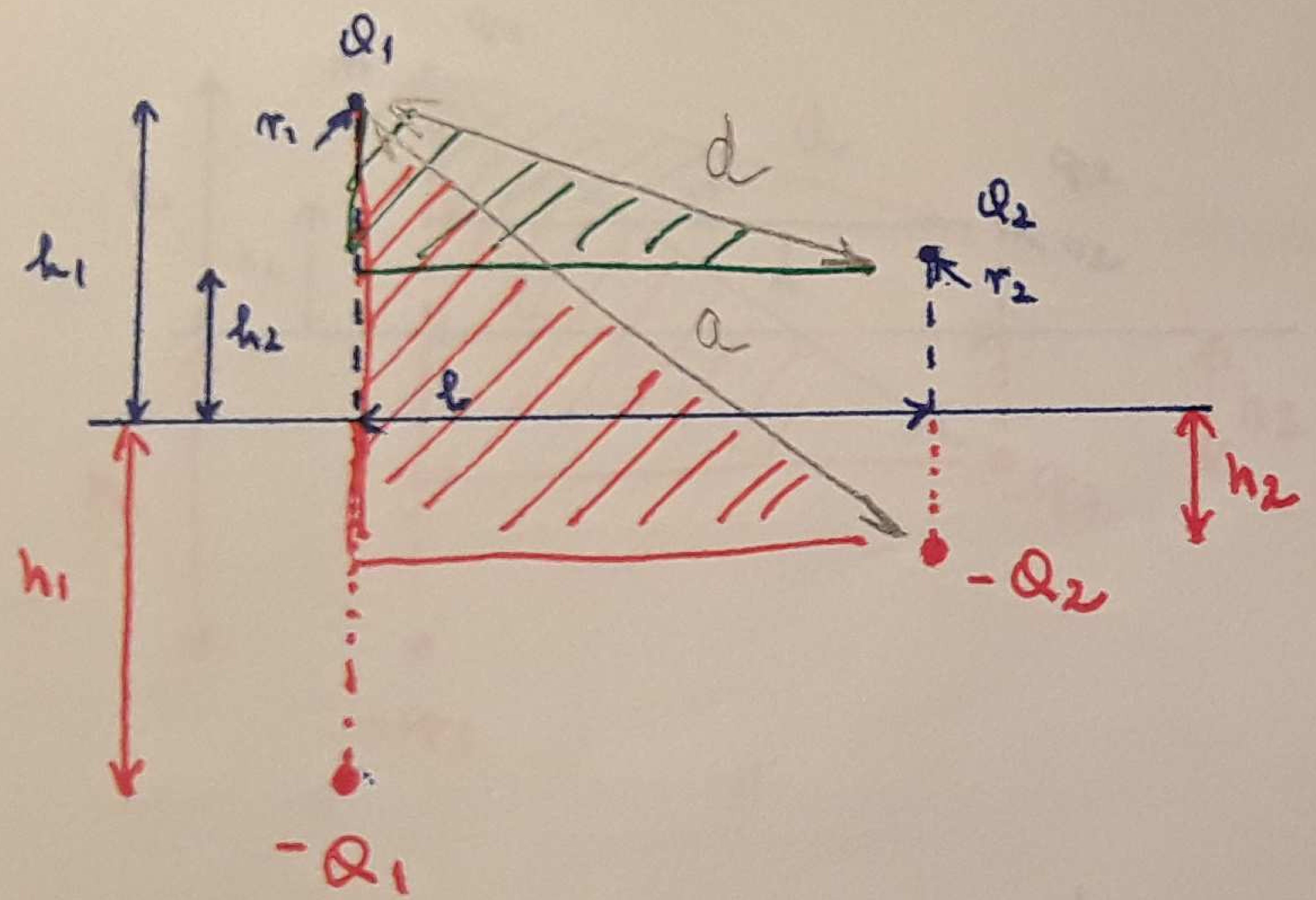
$$\frac{E}{2} = |\vec{E}^-| \sin\alpha$$

$$E = 2|\vec{E}^-| \sin\alpha = 2|\vec{E}^-| \frac{h}{\sqrt{r^2+h^2}}$$



$$\boxed{E = \frac{2Qh}{4\pi\varepsilon(r^2+h^2)^{3/2}}}$$

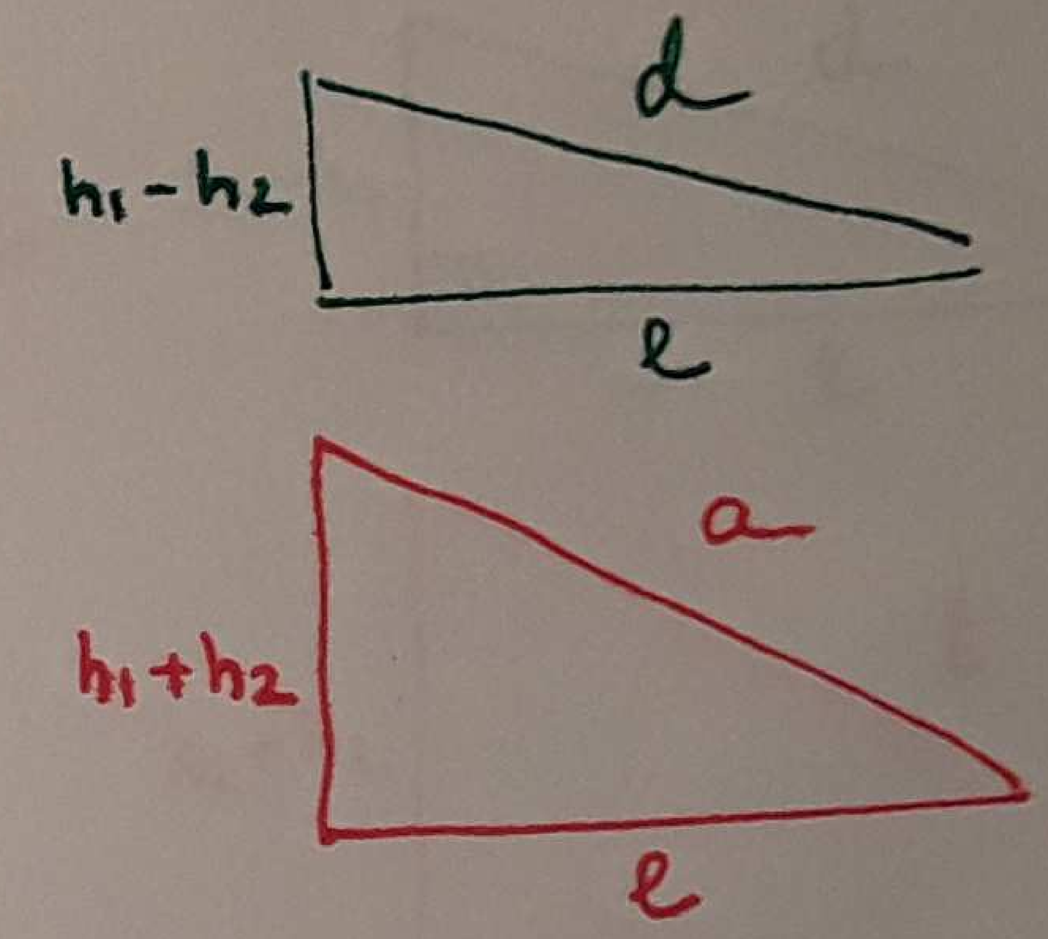
Két gömb helyezkedik el végtelen nagy kiterjedésűnek tekinthető fémlap felett. Határozzuk meg a gömbök potenciálját!



GÖMB : $\varphi = \dfrac{Q}{4\pi\varepsilon_0 r}$

$r_1 \ll h_1$

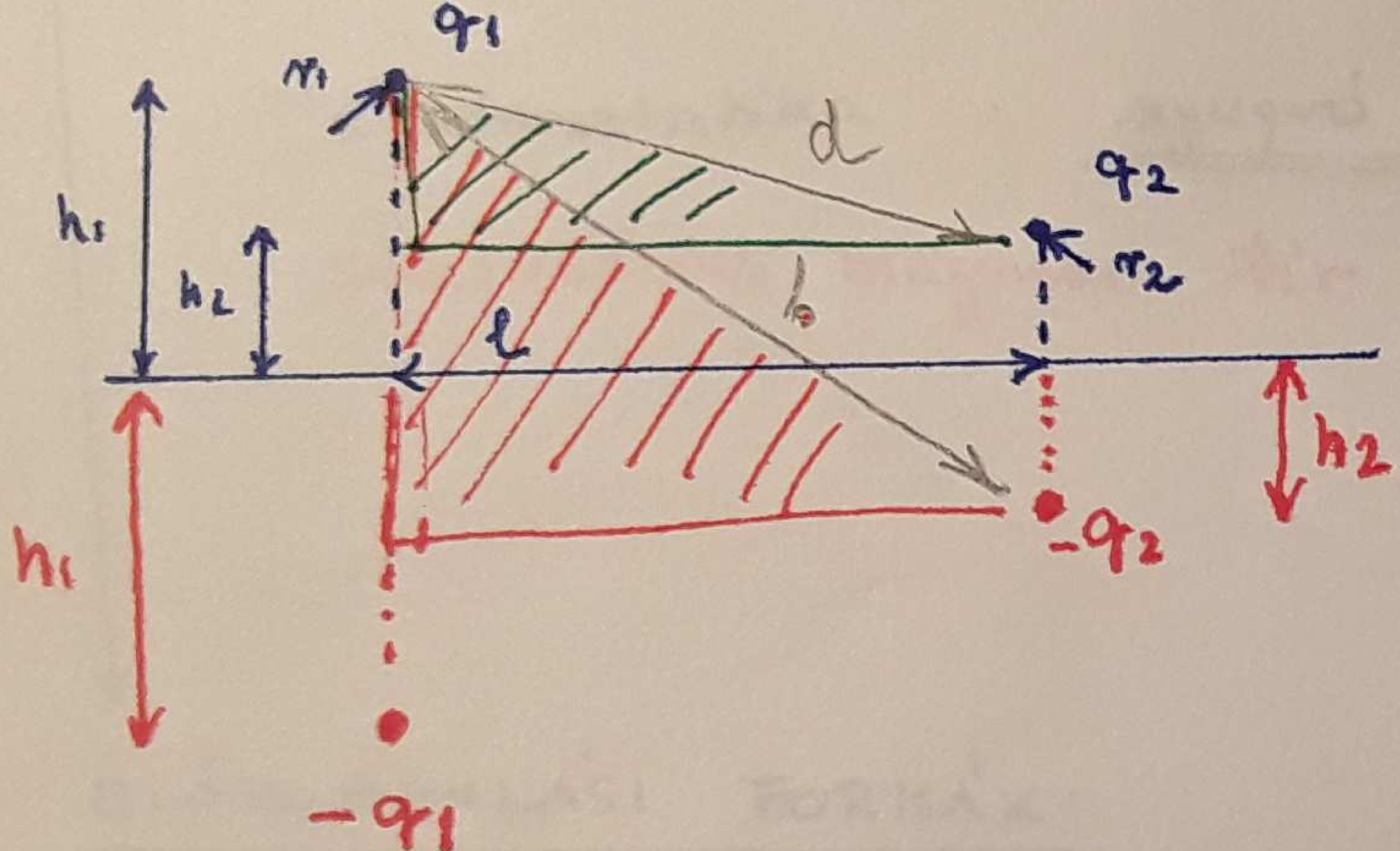
$\underline{U = \varphi_1 - \varphi_2}$



$$\varphi_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1} - \frac{Q_1}{4\pi\varepsilon_0 (2h_1)} + \frac{Q_2}{4\pi\varepsilon_0 d} - \frac{Q_2}{4\pi\varepsilon_0 a} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_1} - \frac{Q_1}{2h_1} + \frac{Q_2}{d} - \frac{Q_2}{a}\right)$$

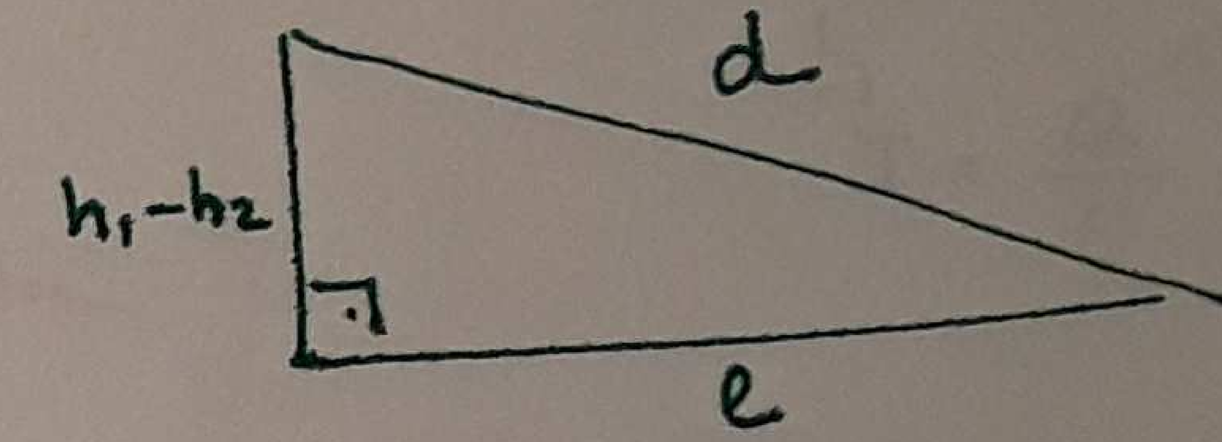
$$\varphi_2 = \frac{Q_2}{4\pi\varepsilon_0 r_2} - \frac{Q_2}{4\pi\varepsilon_0 (2h_2)} + \frac{Q_1}{4\pi\varepsilon_0 d} - \frac{Q_1}{4\pi\varepsilon_0 a} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_2}{r_2} - \frac{Q_2}{2h_2} + \frac{Q_1}{d} - \frac{Q_1}{a}\right)$$

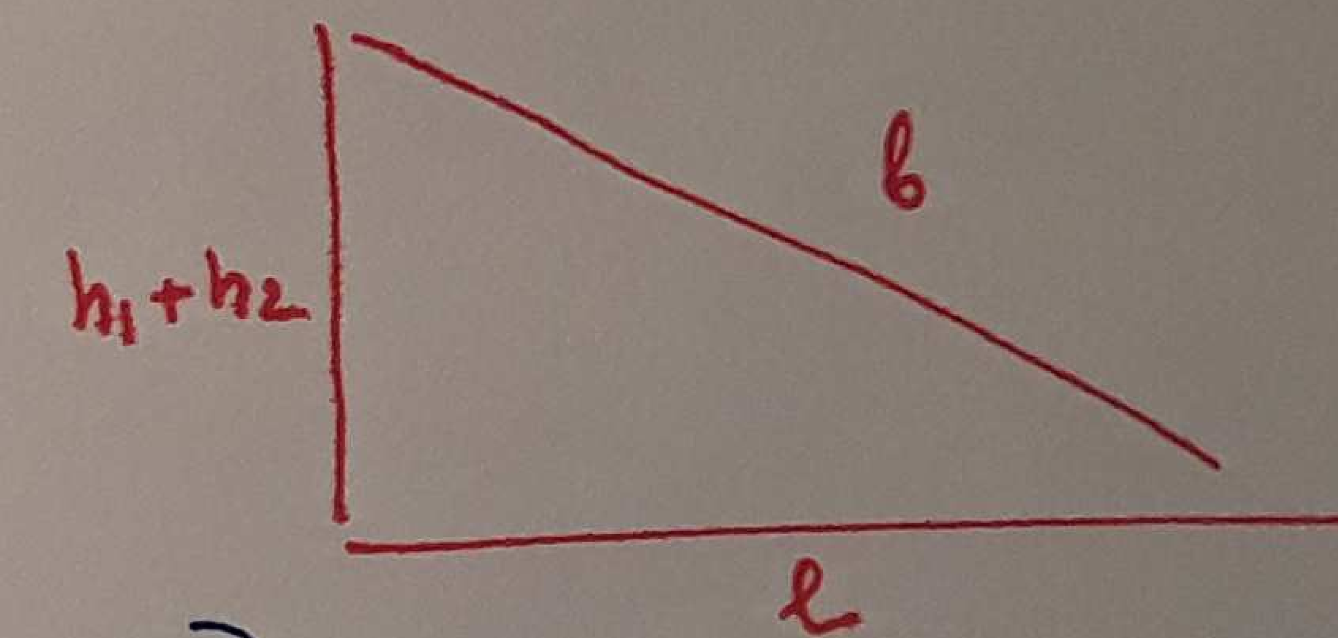
Földelt síkkal párhuzamosan két vonalszerű vezető fut. Határozzuk meg a vezetők potenciáljait, ha q_1 és q_2 ismert!



VONALTÖLTÉS

$$\varphi = \frac{q}{2\pi\varepsilon}\ln\frac{1}{r}$$





$$\varphi_1 = \frac{q_1}{2\pi\varepsilon_0}\ln\frac{1}{r_1} - \frac{q_1}{2\pi\varepsilon}\ln\frac{1}{2h_1} + \frac{q_2}{2\pi\varepsilon_0}\ln\frac{1}{d} - \frac{q_2}{2\pi\varepsilon_0}\ln\frac{1}{b}$$

$$\varphi_2 = \frac{q_2}{2\pi\varepsilon_0}\ln\frac{1}{r_2} - \frac{q_2}{2\pi\varepsilon_0}\ln\frac{1}{2h_2} + \frac{q_1}{2\pi\varepsilon_0}\ln\frac{1}{d} - \frac{q_1}{2\pi\varepsilon_0}\ln\frac{1}{b}$$

$$U = \varphi_1 - \varphi_2$$

STACIONÁRIUS MÁGNESES TÉR

ÁRAM: TÖLTÉSSEL BÍRÓ ELEMI RÉSZECSKÉK EGYIRÁNYÚ MOZGÁSA, ÁRAMLÁSA.

amper, A

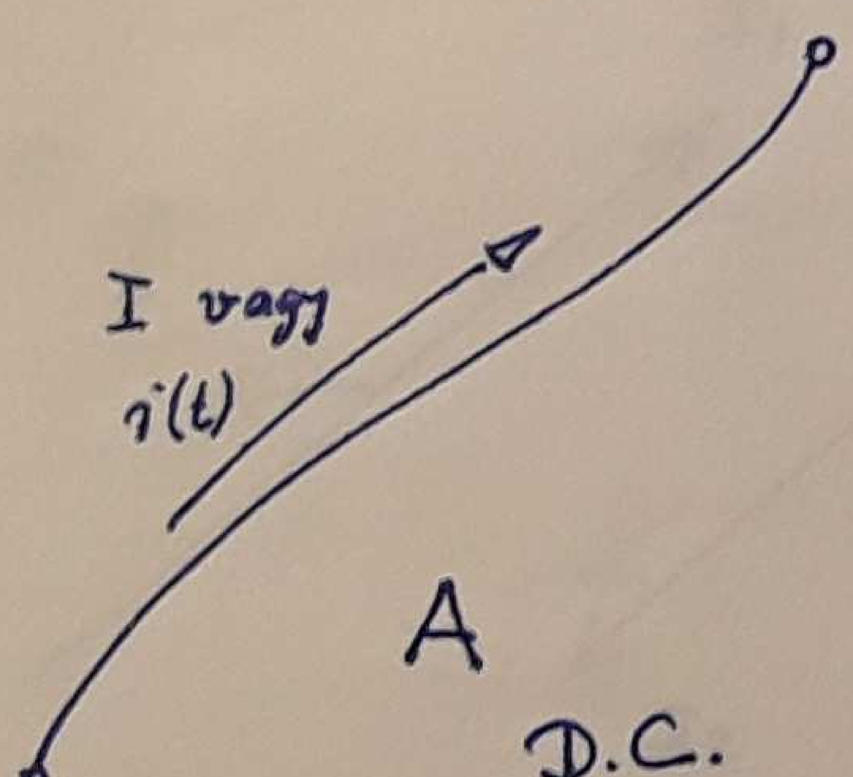
$I = \frac{Q}{t}$ — coulomb, C

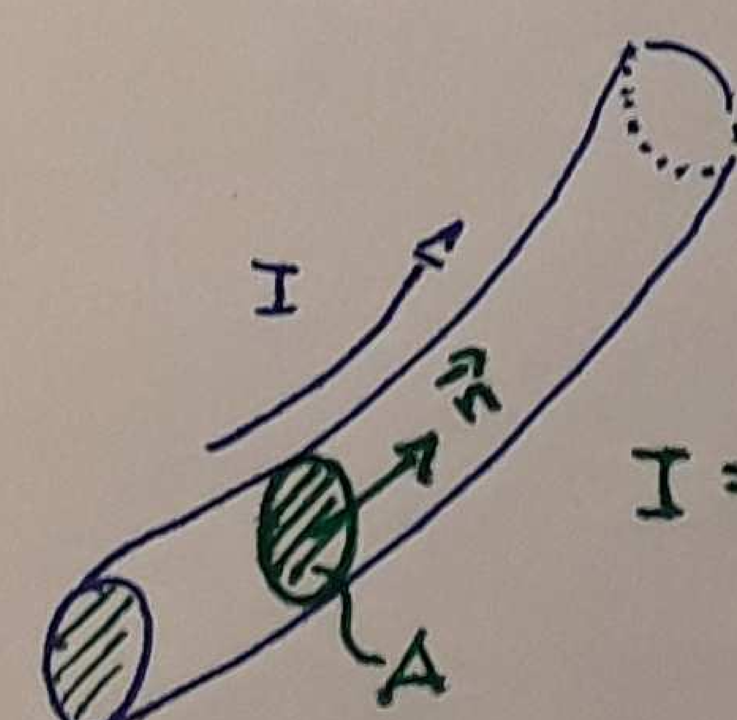
elektrosztatika : nyugvó töltések hozzák létre

stacionárius mágneses tér : egyenáram hozza létre

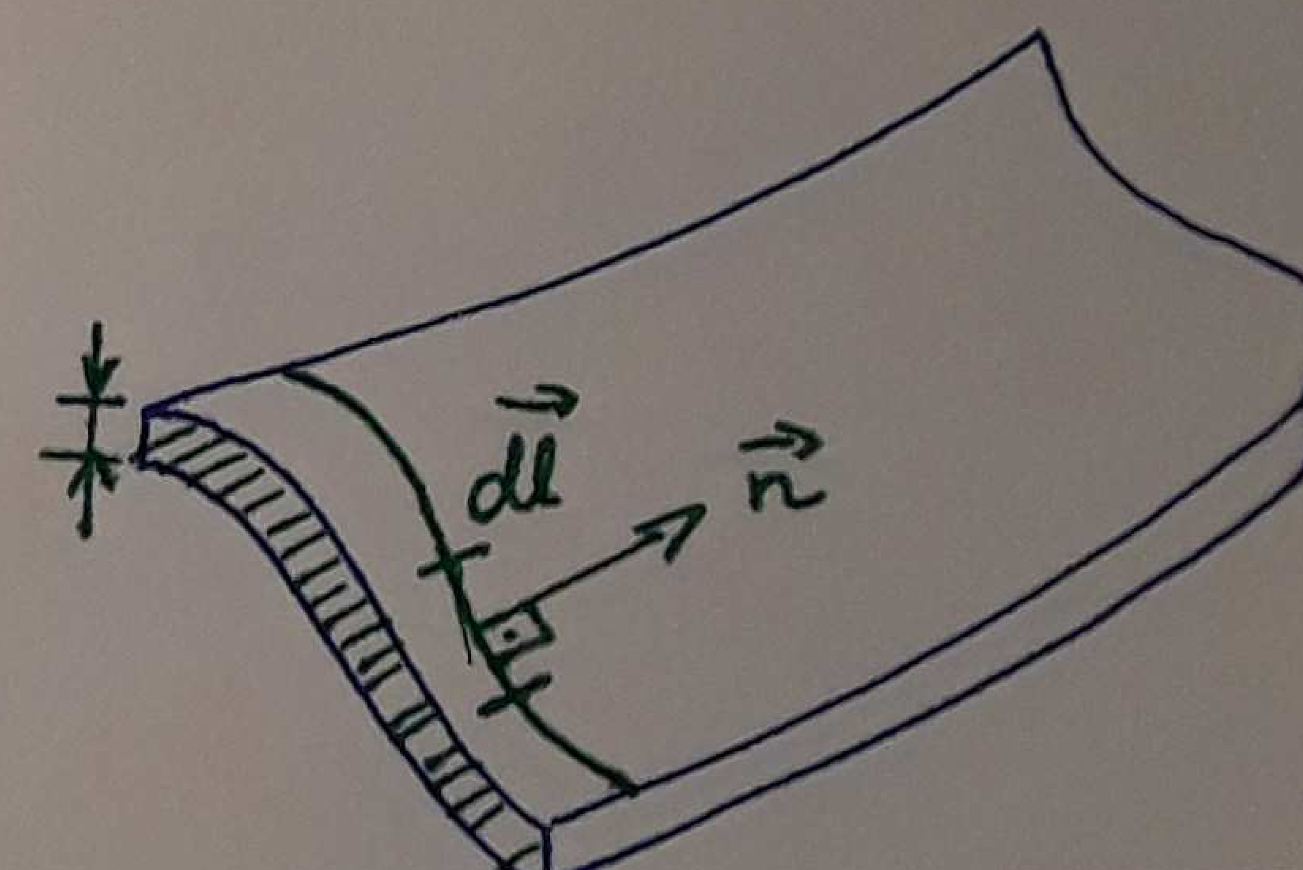
↑ mozgó töltés

ELŐFORDULÁSI FORMÁK:





$I = \int_A \vec{J} \cdot d\vec{A}$



$\frac{A}{m^2}$

$\lim_{\Delta A \to 0} \frac{\Delta I}{\Delta A} = \underbrace{\frac{dI}{dA}}$

$\vec{J} = \frac{dI}{dA}\vec{n}$ $\quad \vec{J} = \vec{J}(\vec{r}, t)$

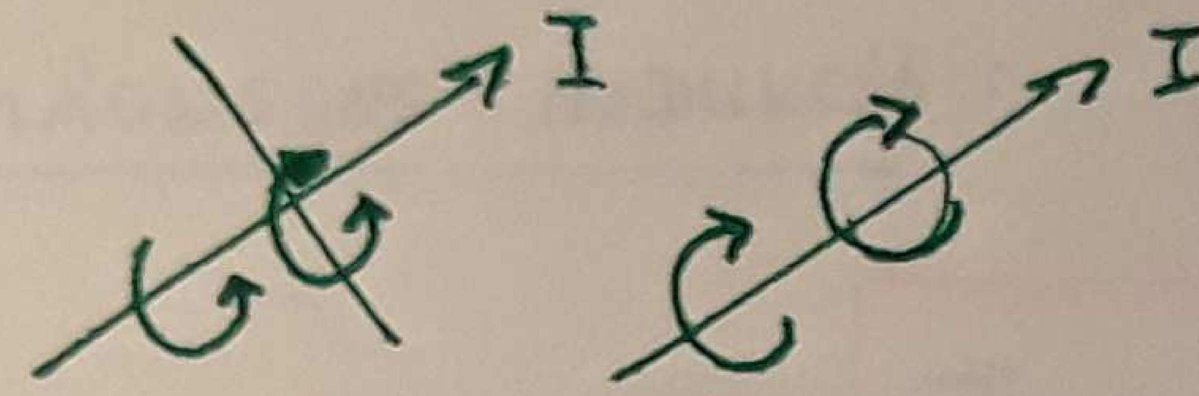
$\lim_{\Delta l \to 0} \frac{\Delta I}{\Delta l} = \frac{dI}{dl}$

$\boxed{\frac{dI}{dl}\vec{n} = \vec{K}}$

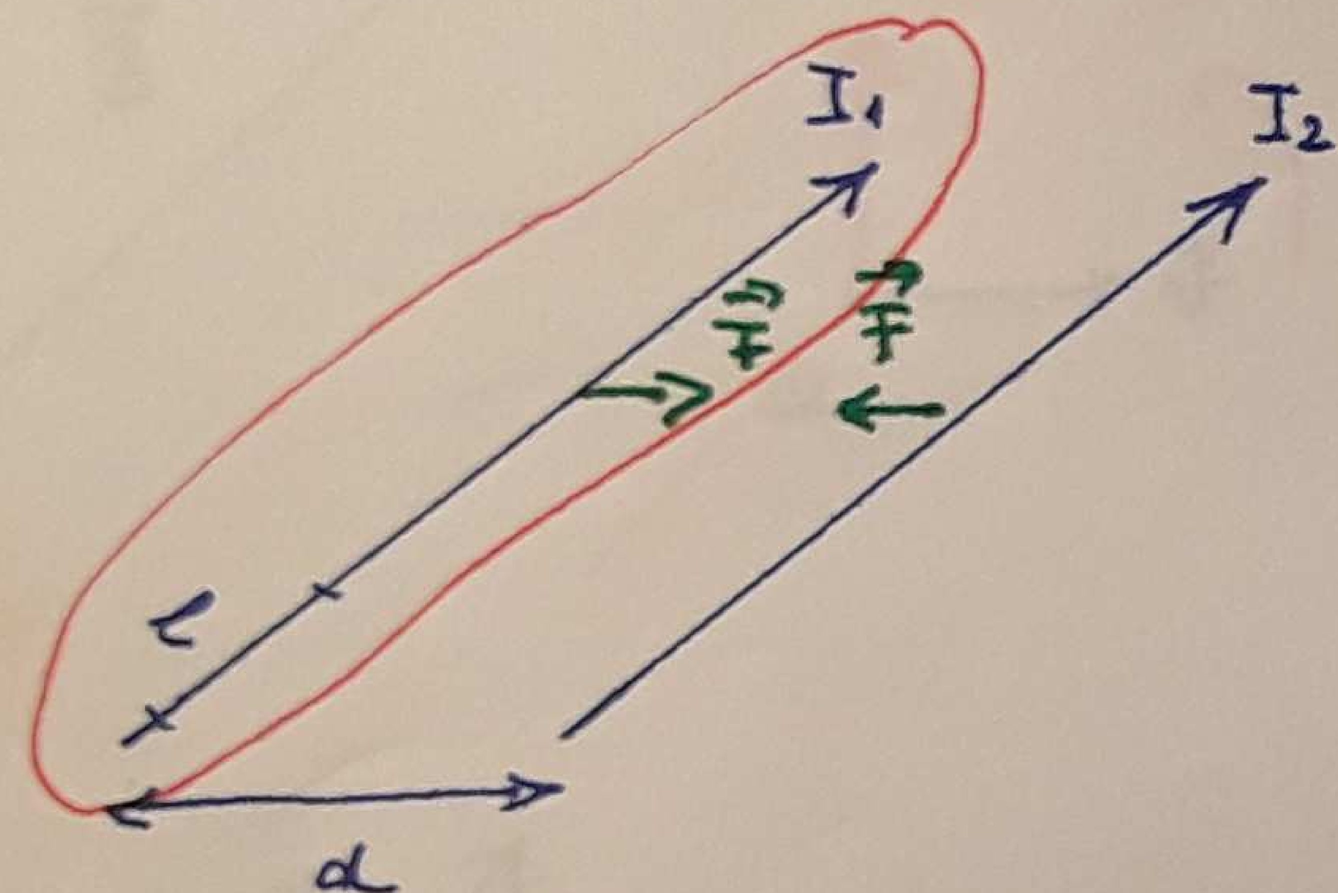
$I = \int_l \vec{K} \cdot d\vec{l}$!!!

$[K] = \frac{A}{m}$

! OERSTED - KÍSÉRLET



! AMPÈRE - KÍSÉRLET



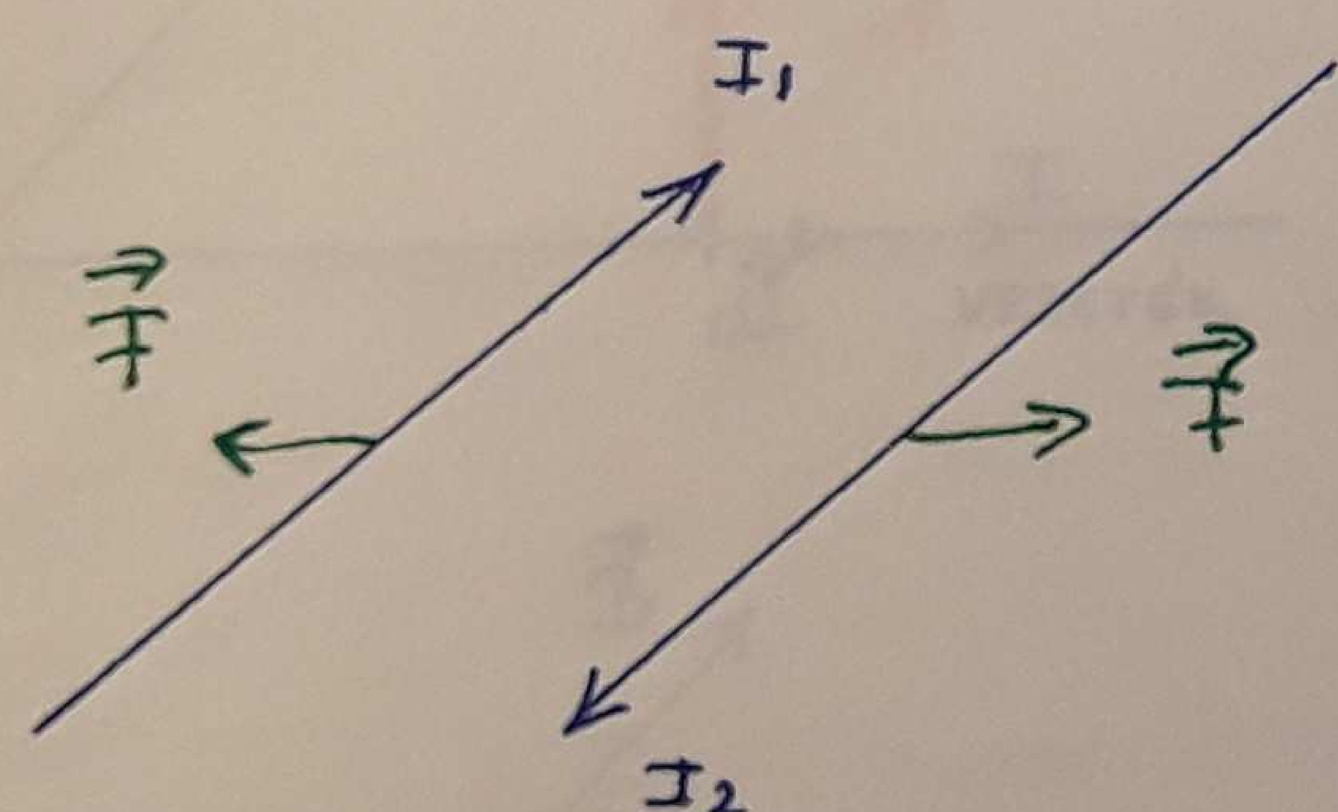
permeabilitás

$$\mu = \mu_0 = 4\pi\cdot10^{-7}\,\frac{H}{m}$$

$$\mu = \mu_0\,\mu_r$$

$$F = \boxed{\frac{\mu}{2\pi}}\,\frac{|I_1\,I_2|\,l}{d}$$

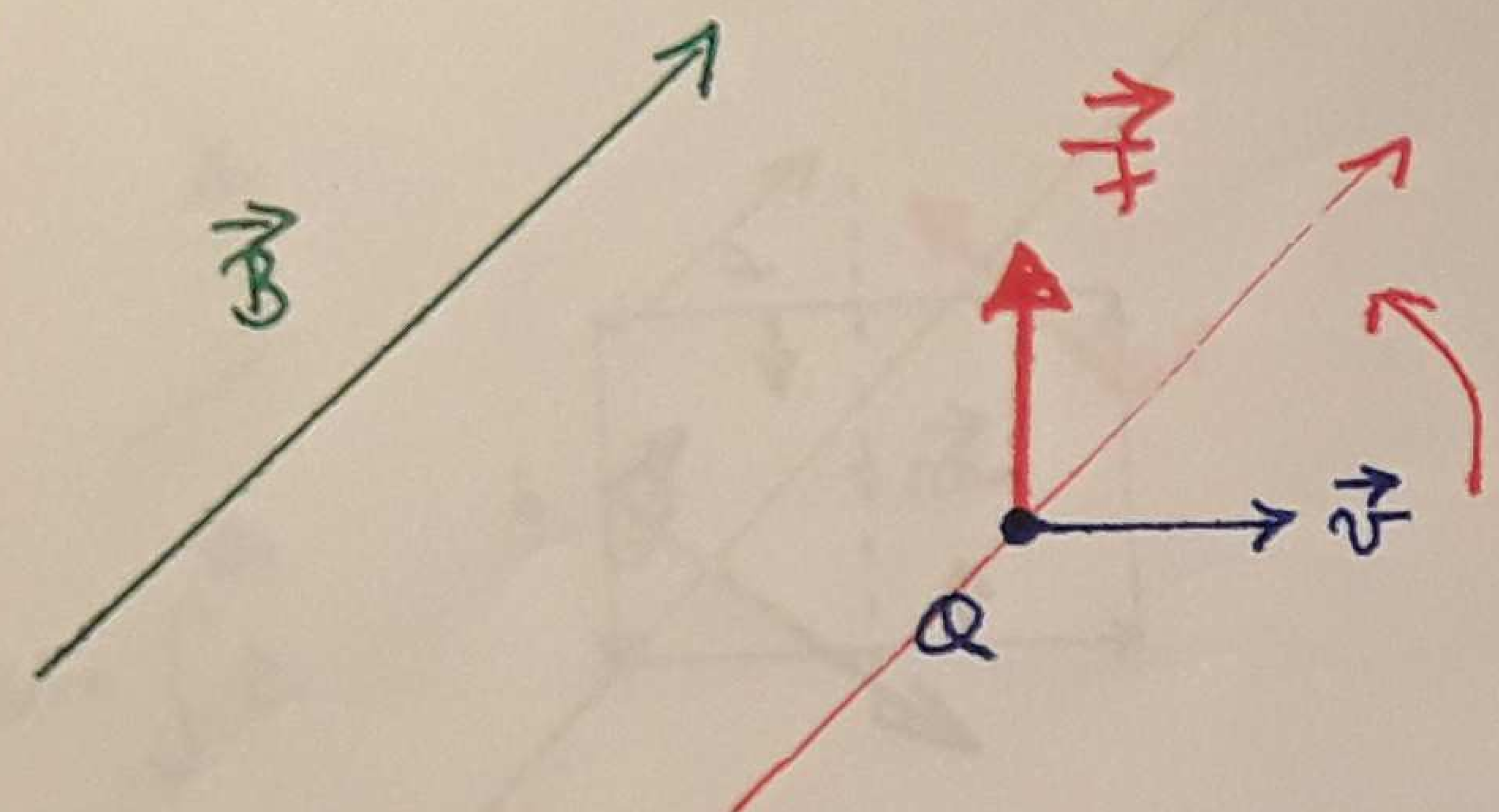
$$\left.\begin{aligned} I_1 &= I_2 = 1A \\ l &= 1m \\ d &= 1m \end{aligned}\right\}\quad F = 2\cdot10^{-7}N$$



Áram $\rightarrow$ Mágneses tér, Mágneses erőtér

Mágneses erőtér : <u>MÁGNESES INDUKCIÓ</u> :

a.)



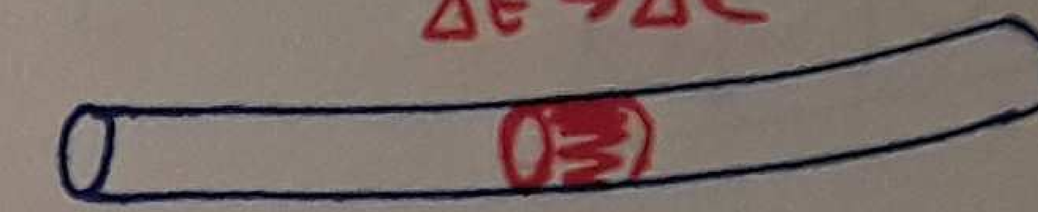
$$\boxed{\vec{F} = Q\vec{v}\times\vec{B}}$$

$$B = \frac{F}{Qv} = \frac{Ns}{Cm} = \frac{\frac{J}{m}s}{\frac{C}{m}m} =$$

W = Pt W = Fs

$$= \frac{J}{Am^2} = \frac{AVs}{Am^2} = \boxed{\frac{Vs}{m^2}} \quad \frac{Wb}{m^2} \quad T$$

$\boxed{1-2\,T}$ $6\cdot10^{-5}\,T$

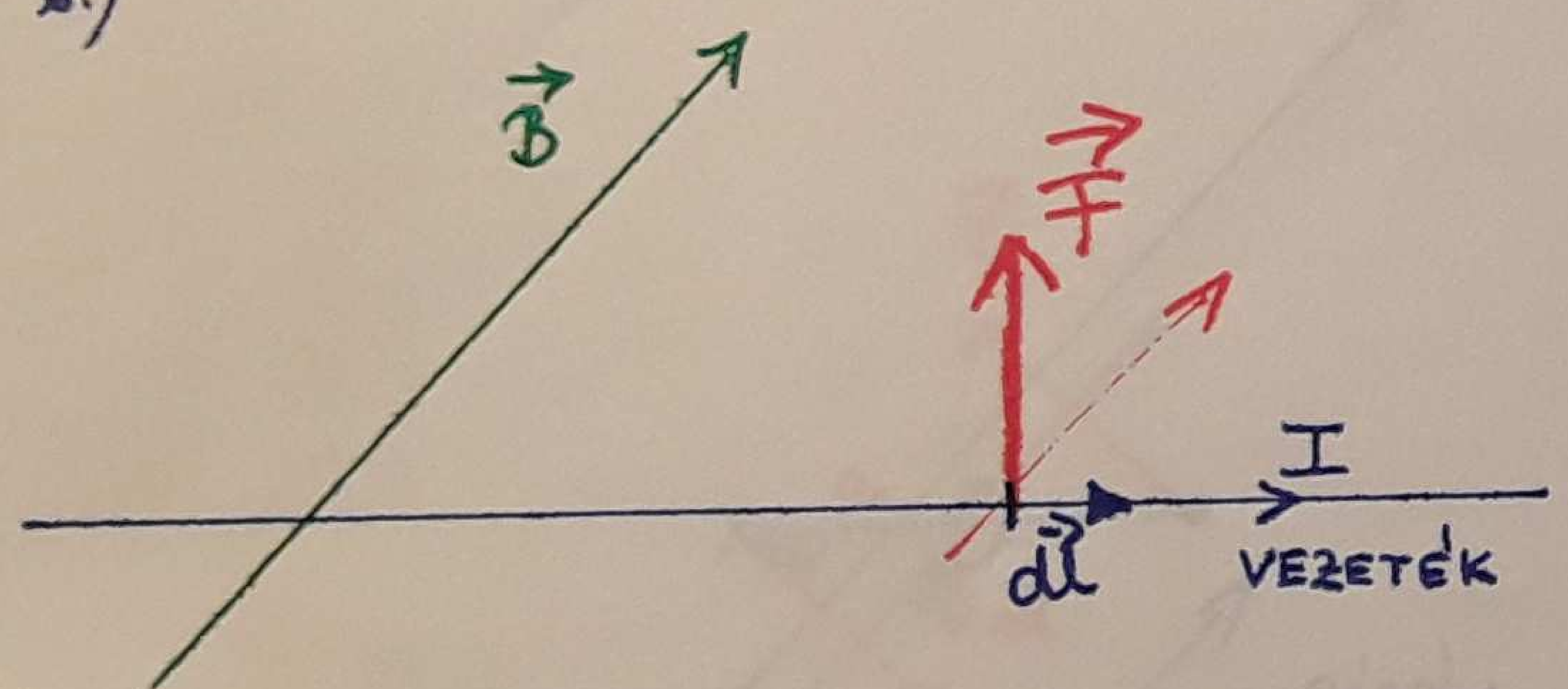


$Qv = I\underbrace{tv}_{l}$

$Q\vec{v}\times\vec{B}$

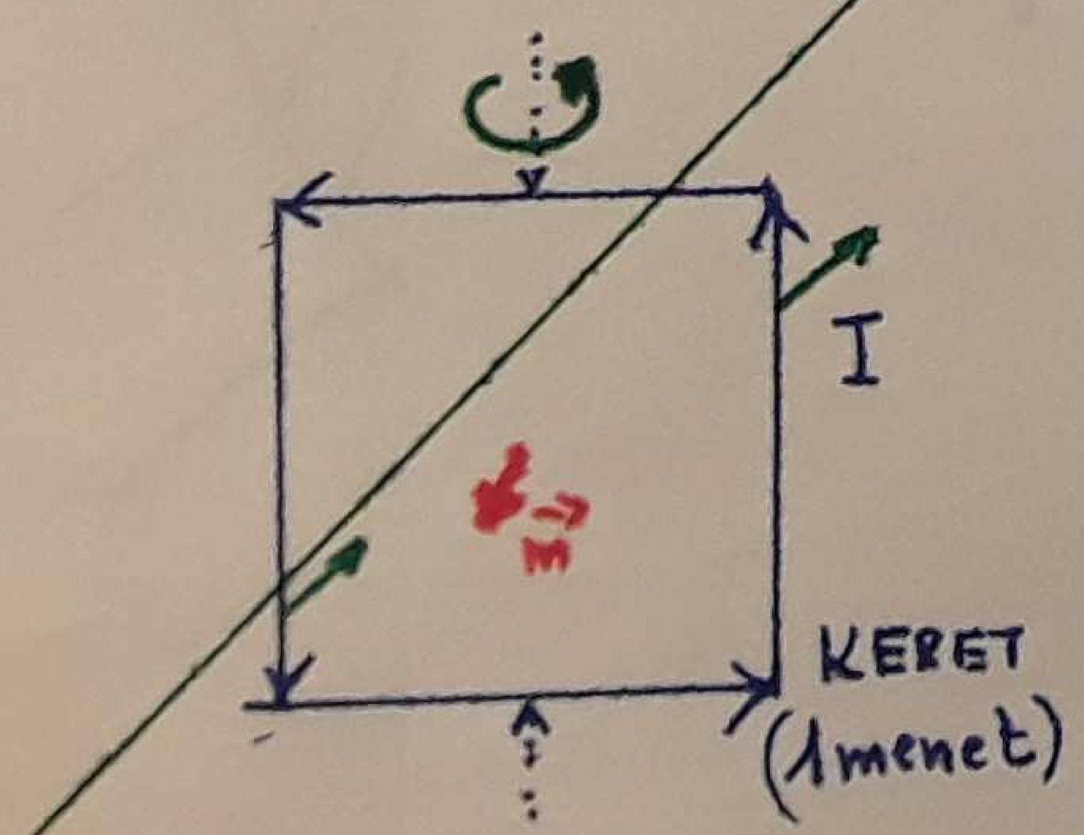
$I\vec{dl}\times\vec{B}$

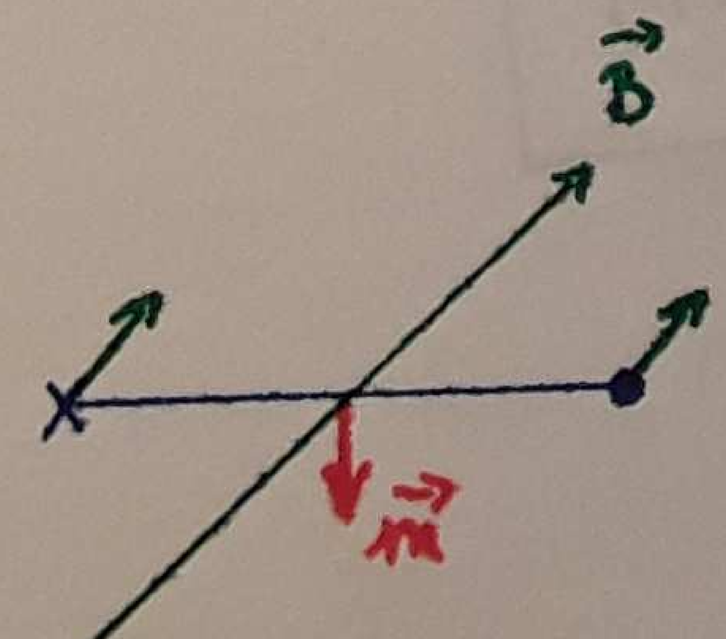
b.)



$$\boxed{\vec{F} = I\vec{dl}\times\vec{B}}$$

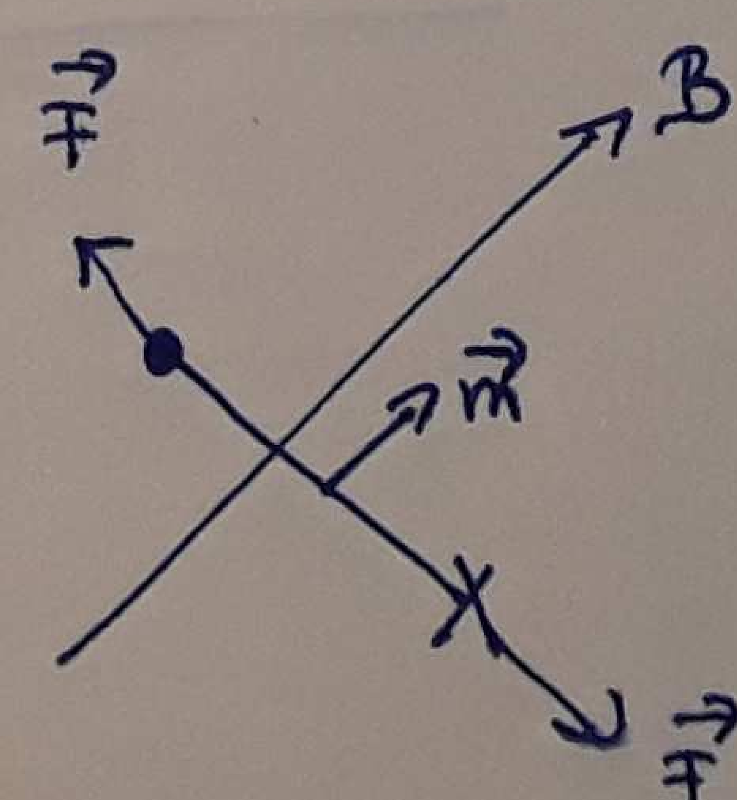
c.)

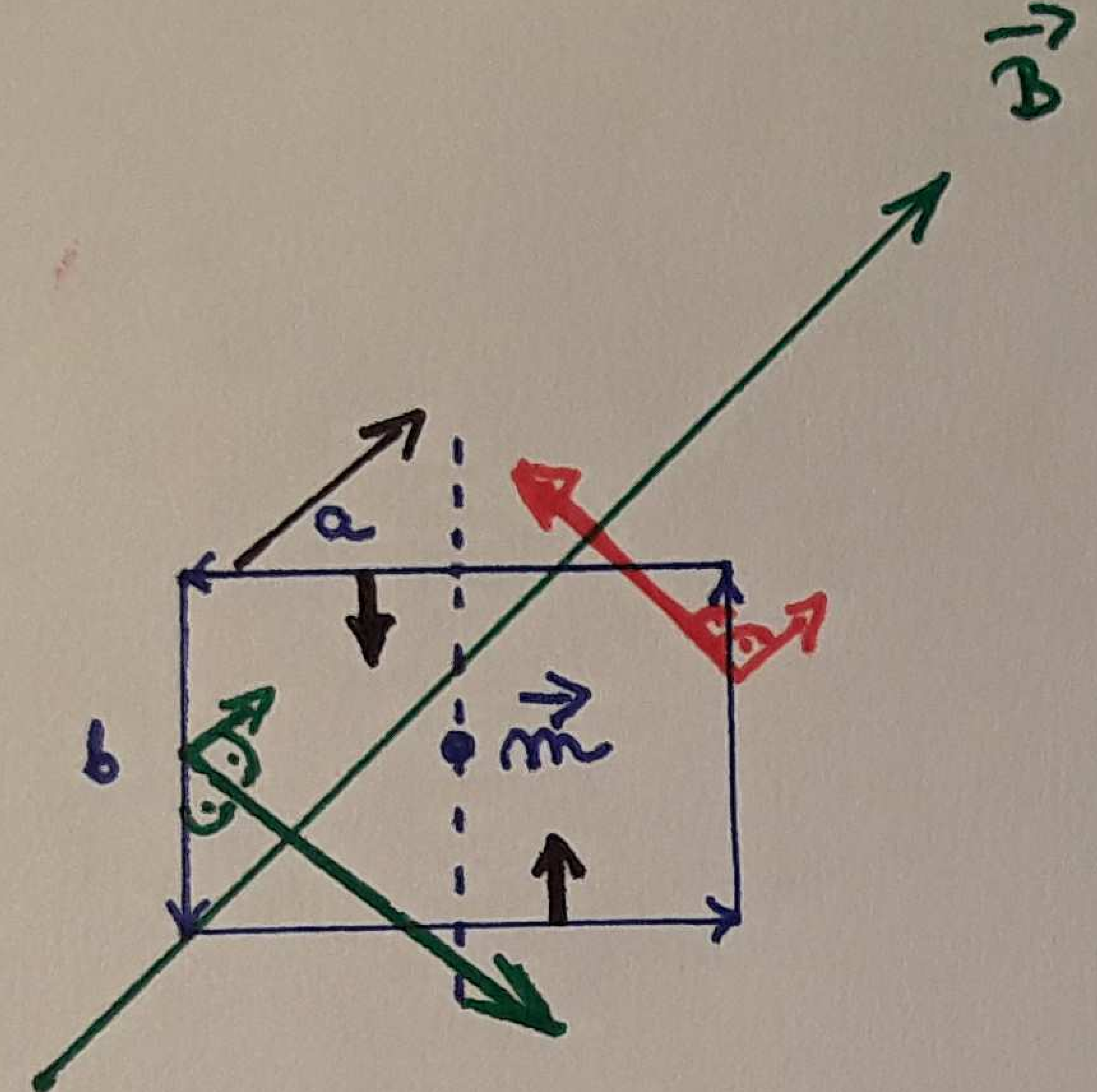




felülnézet.

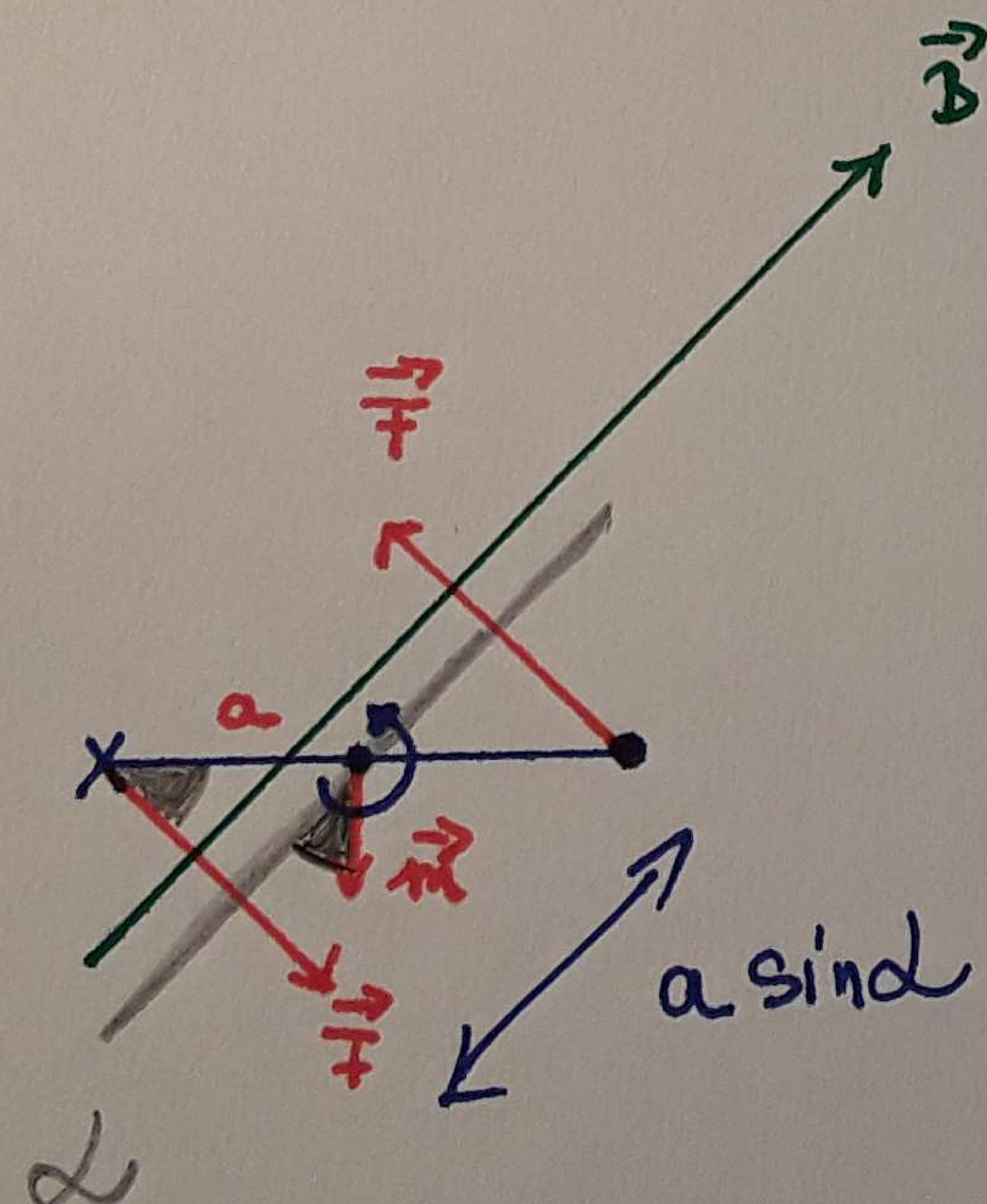
$$\boxed{\vec{M} = \vec{m}\times\vec{B}}$$





$$\vec{F} = I\, d\vec{l} \times \vec{B}$$

$$\boxed{F = IbB}$$



$$M = F a \sin\alpha$$

$$M = IbBa\sin\alpha$$

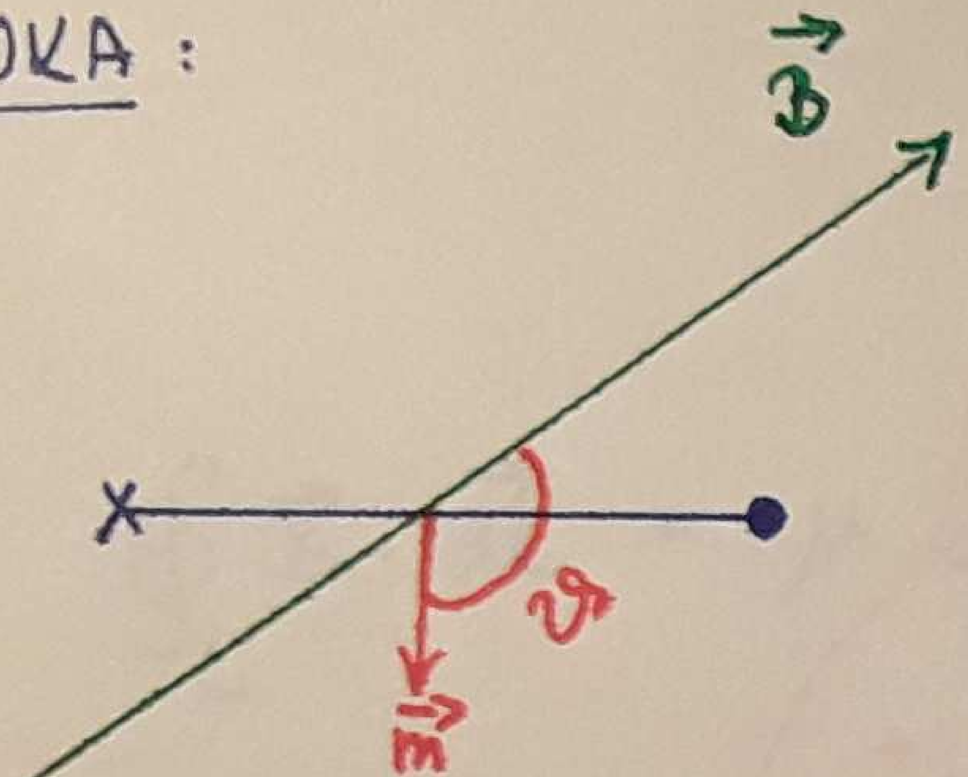
$$= I\underbrace{ab}_{A} B \sin\alpha$$

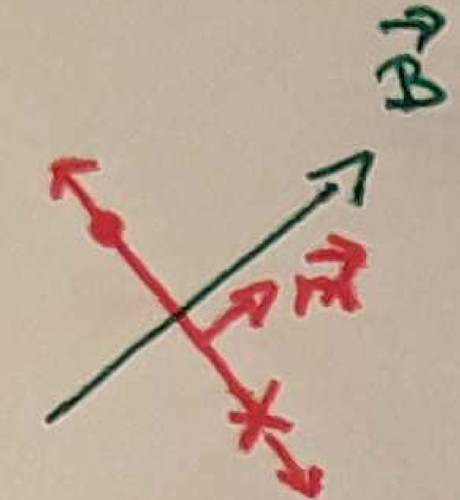
$$= \underbrace{I A}_{m} B \sin\alpha$$

mágn. momentum

$$\boxed{\vec{M} = \vec{m} \times \vec{B}}$$

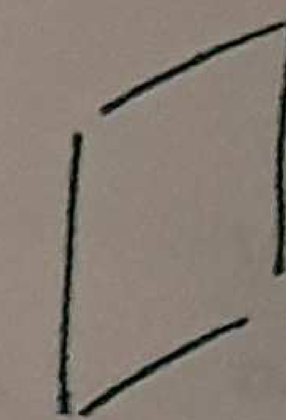
MUNKA:

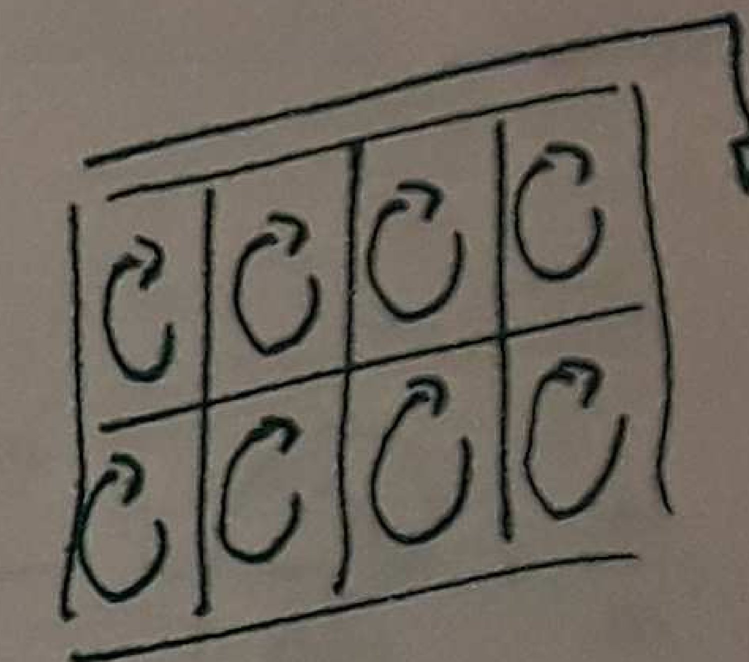




$\vec M = \vec m \times \vec B = I A \vec n \times \vec B$

$|\vec M| = IAB \sin\vartheta$





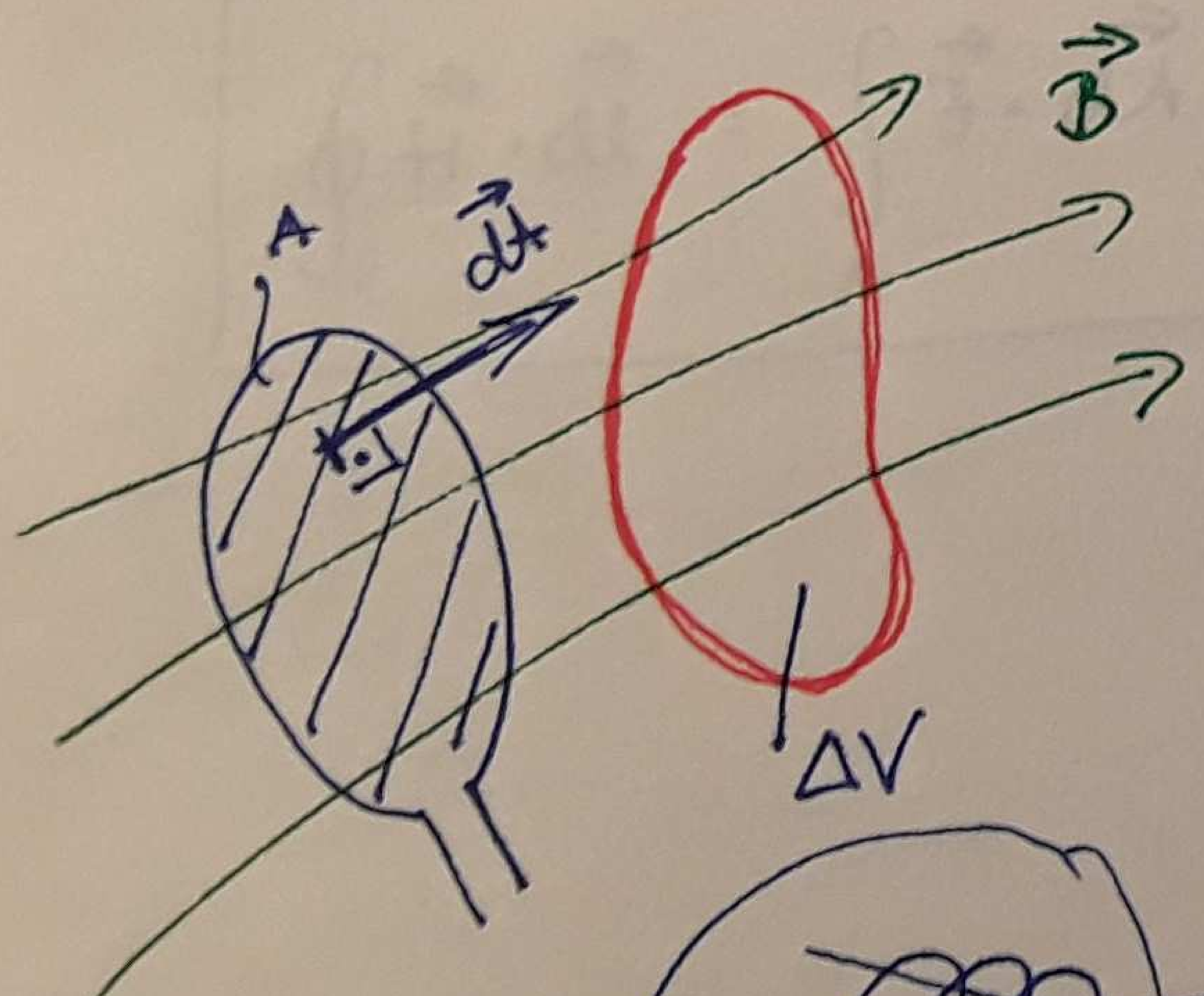
$$W = \int_0^{\vartheta} M\, d\vartheta = IAB \int_0^{\vartheta} \sin\vartheta\, d\vartheta = IAB\left[-\cos\vartheta\right]_0^{\vartheta} = IAB(-\cos\vartheta + 1) = IAB - IAB\cos\vartheta$$

$$W = -\underbrace{IAB\cos\vartheta}_{\Phi}$$

$$\boxed{W = -I\Phi}$$

$$\boxed{\Phi = \int_A \vec B \cdot d\vec A}$$

$$\boxed{\oint_A \vec B \cdot d\vec A = 0} \;!$$



$$\lim_{\Delta V \to 0} \frac{1}{\Delta V} \oint_A \vec B \cdot d\vec A = \boxed{\operatorname{div} \vec B = 0}$$

FORRÁSMENTES.

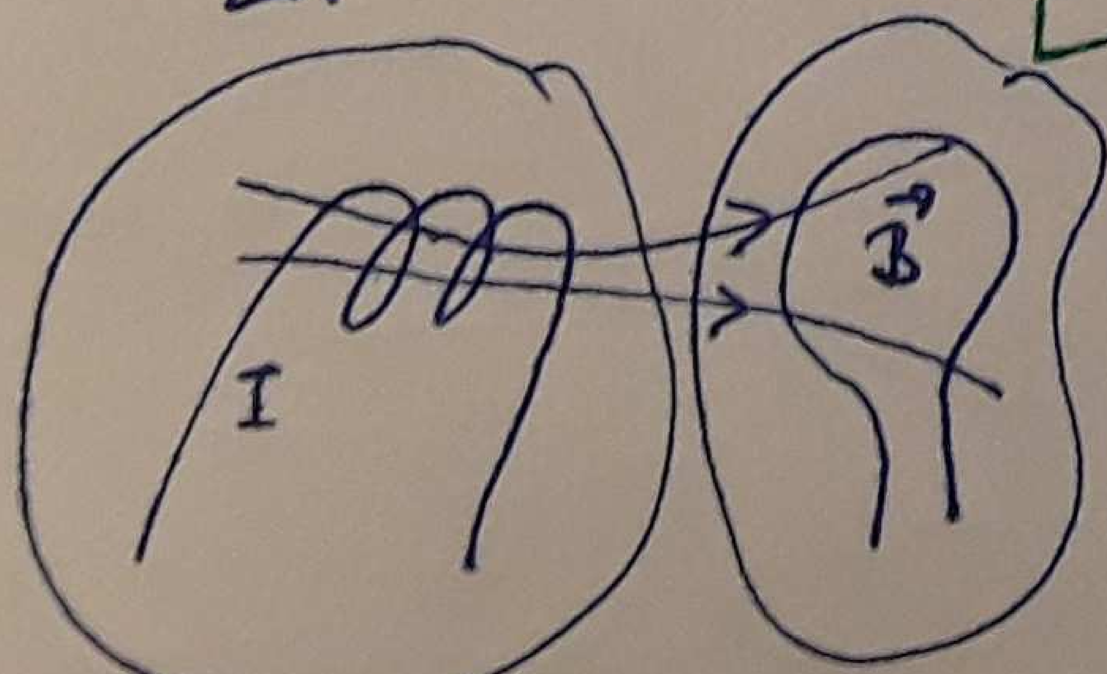
$\operatorname{div} \vec B = S$

$I \to B, \Phi$

$\frac{Vs}{A}$ $\quad [\Phi] = \frac{Vs}{\cancel{m^2}}\cancel{m^2}$ $\quad$ Wb

$$\boxed{\Phi = LI} \qquad L = \frac{\Phi}{I}$$

$[L] = H$ henry.

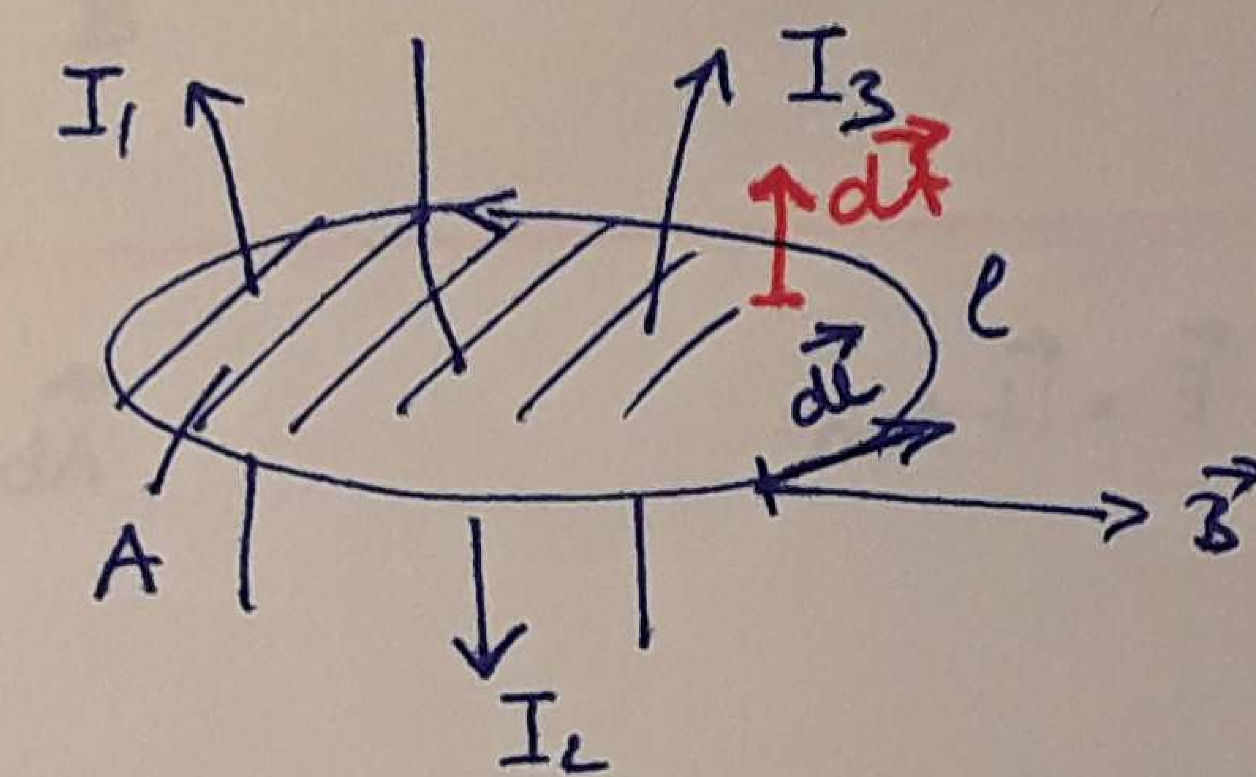


GERJESZTÉSI TÖRVÉNY

AMPÈRE-TÖRVÉNY

$$\oint_l \vec{B}\cdot d\vec{l} = \mu \sum_j I_j$$

$$\underbrace{+I_1 - I_2 + I_3}_{H}$$



$$\mu \rightarrow \frac{Bl}{I} \qquad \frac{\frac{Vs}{m^2}\cdot \cancel{m}}{A} = \frac{Vs}{Am} = \frac{H}{m}$$

μ permeabilitás

$$\mu = \mu_0 = 4\pi\cdot 10^{-7}\ H/m$$

$$\mu = \mu_0 \mu_r$$

$$\vec{H} = \frac{1}{\mu}\vec{B}$$

$$\oint_l \vec{H}\cdot d\vec{l} = \sum I = \int_A \vec{J}\cdot d\vec{A}$$

$$\boxed{\oint_l \vec{H}\cdot d\vec{l} = \int_A \vec{J}\cdot d\vec{A}}$$

STOKES $\displaystyle\int_A \mathrm{rot}\,\vec{H}\cdot d\vec{A} = \int_A \vec{J}\cdot d\vec{A}$

$$\boxed{\mathrm{rot}\,\vec{H} = \vec{J}}$$

ÖSSZEFOGLALVA

$I; \vec{J}; \vec{K} \longrightarrow \vec{B}$ $\qquad \vec{F} = Q\vec{v}\times\vec{B}$; $\vec{F} = I\,d\vec{l}\times\vec{B}$; $\vec{M} = \vec{m}\times\vec{B}$

STACIONÁRIUS MÁGNESES TÉR

$$\oint_l \vec{H}\cdot d\vec{l} = \int_A \vec{J}\cdot d\vec{A} \qquad \operatorname{rot}\vec{H} = \vec{J}$$

$$\oint_A \vec{B}\cdot d\vec{A} = 0 \qquad \operatorname{div}\vec{B} = 0$$

$$\vec{B} = \mu\vec{H}$$

$$L = \frac{\Phi}{I} \qquad W = -I\Phi$$

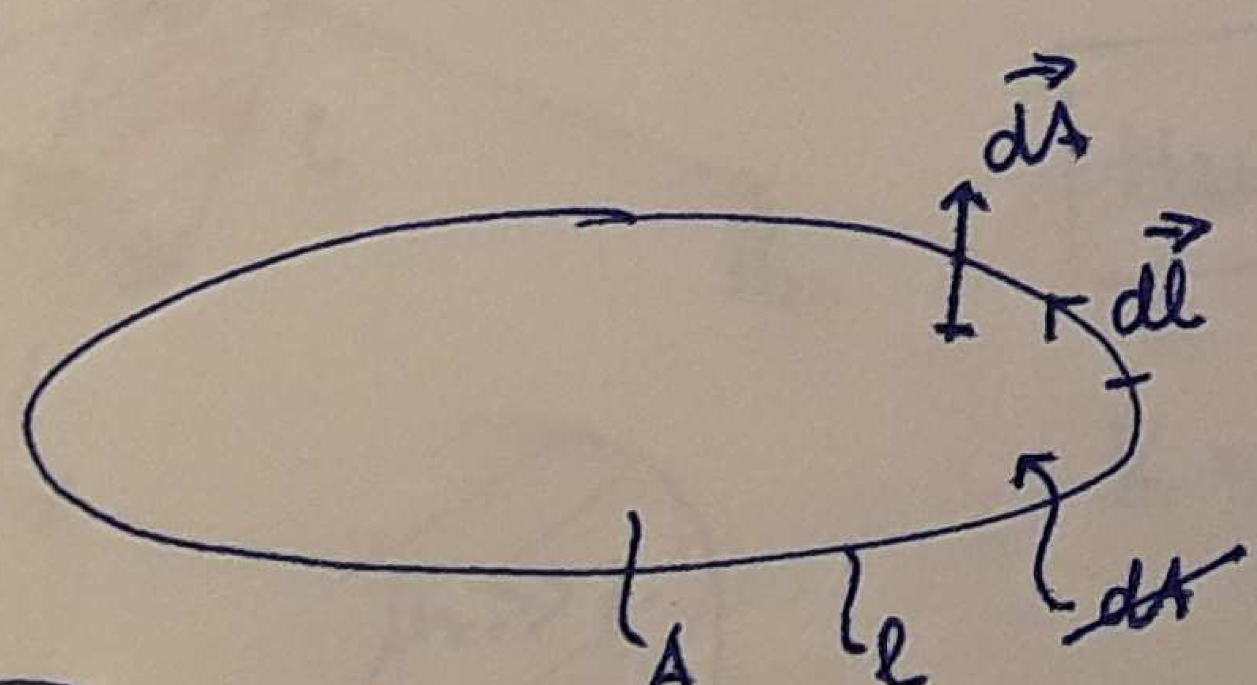
MAGNETOSZTATIKA

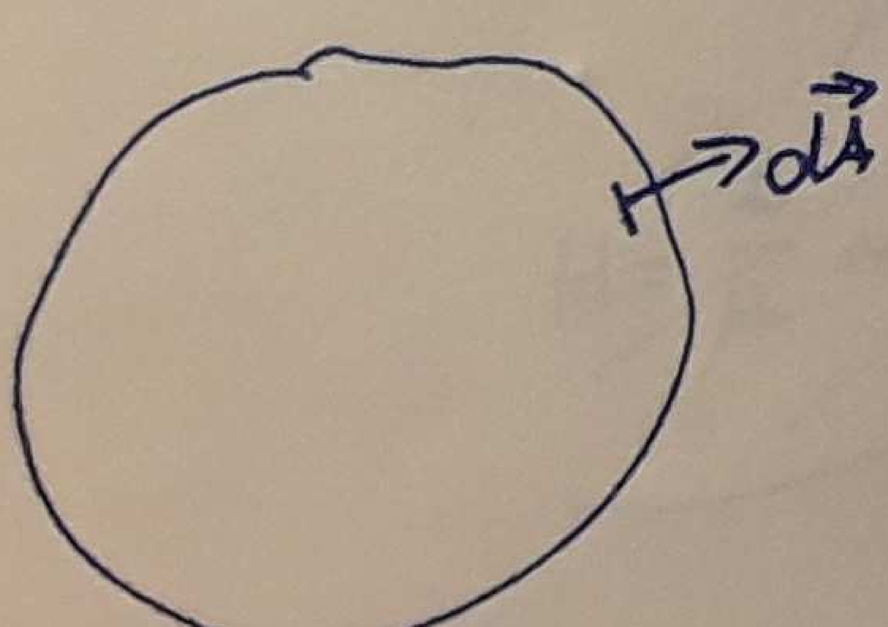
$$\oint_l \vec{H}\cdot d\vec{l} = 0 \qquad \operatorname{rot}\vec{H} = 0$$

$$\oint_A \vec{B}\cdot d\vec{A} = 0 \qquad \operatorname{div}\vec{B} = 0$$

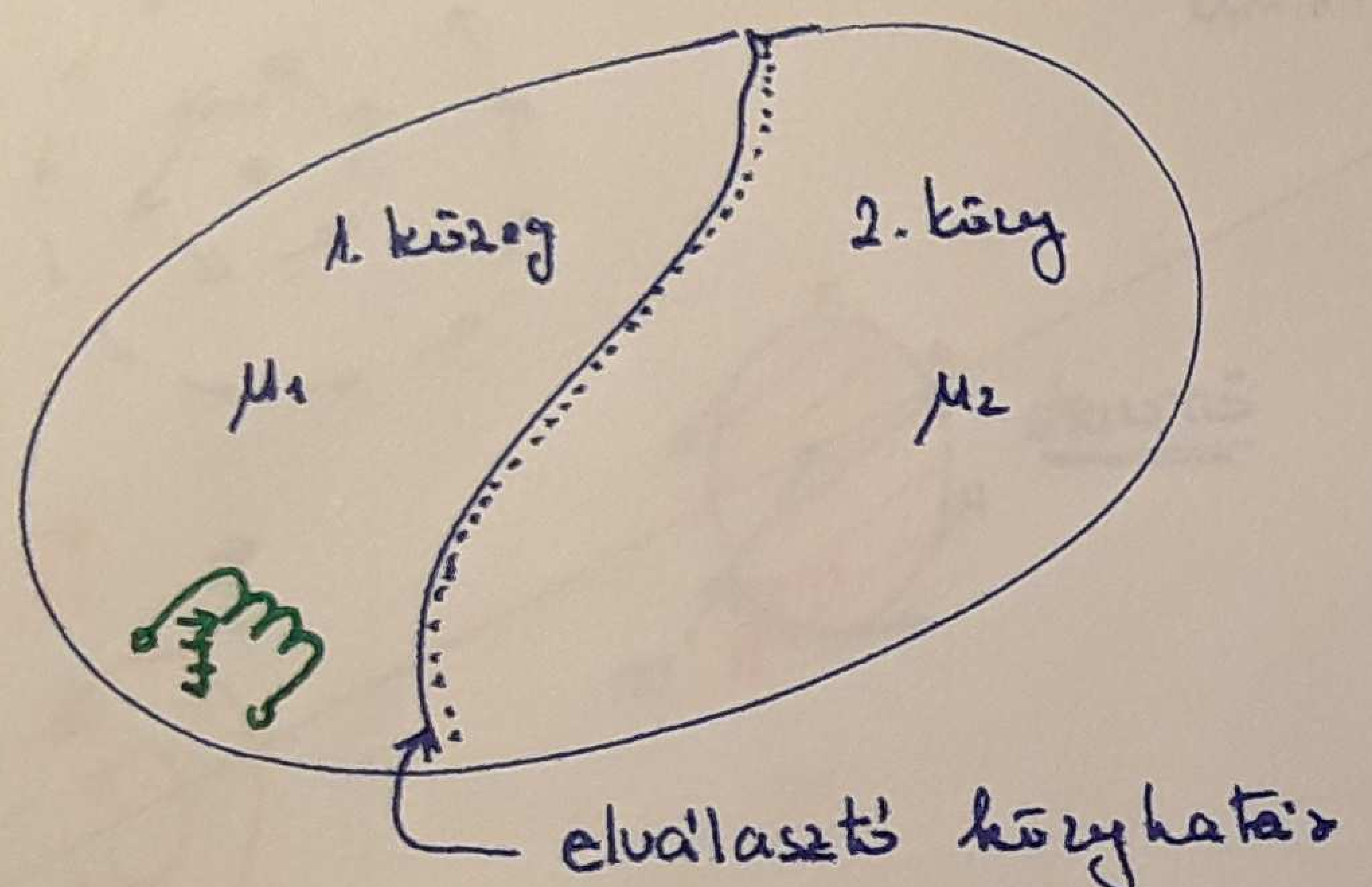
$$\vec{B} = \mu\vec{H}\ldots$$

Ha áram nincs.



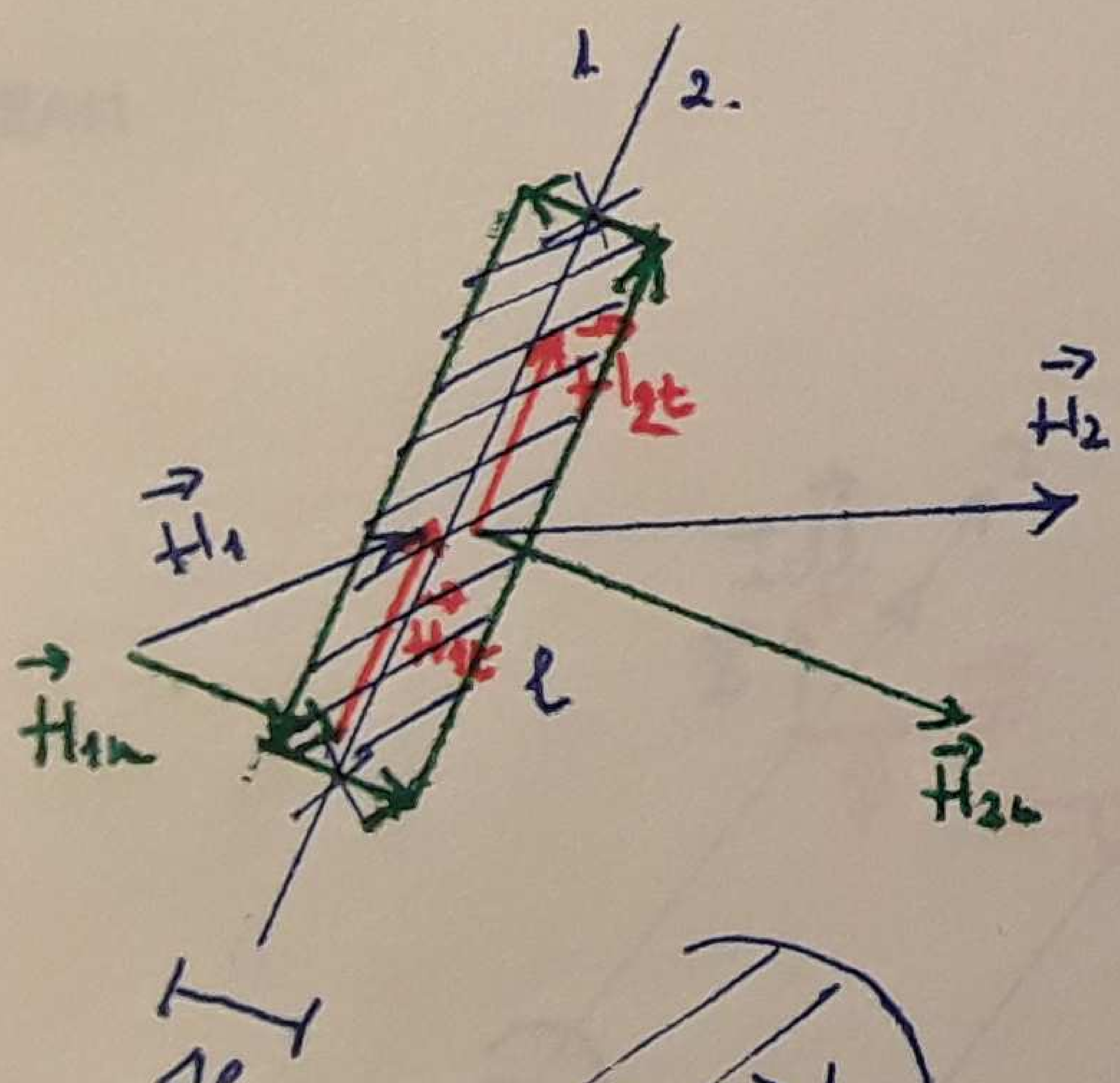


Határfeltételek stacionárius mágneses térben



FOLYTONOSSÁGI FELTÉTELEK

$H_{\text{TANGENCIÁLIS}}$ · $B_{\text{NORMÁLIS}}$

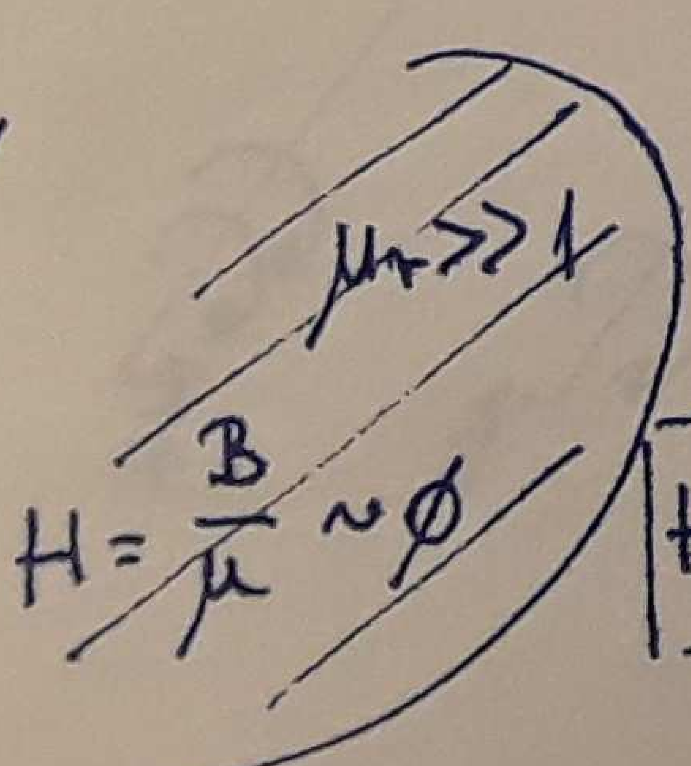


$$\oint_l \vec{H}\cdot d\vec{l} = \int_A \vec{J}\cdot d\vec{A} + \int_l \vec{K}\cdot d\vec{l}$$

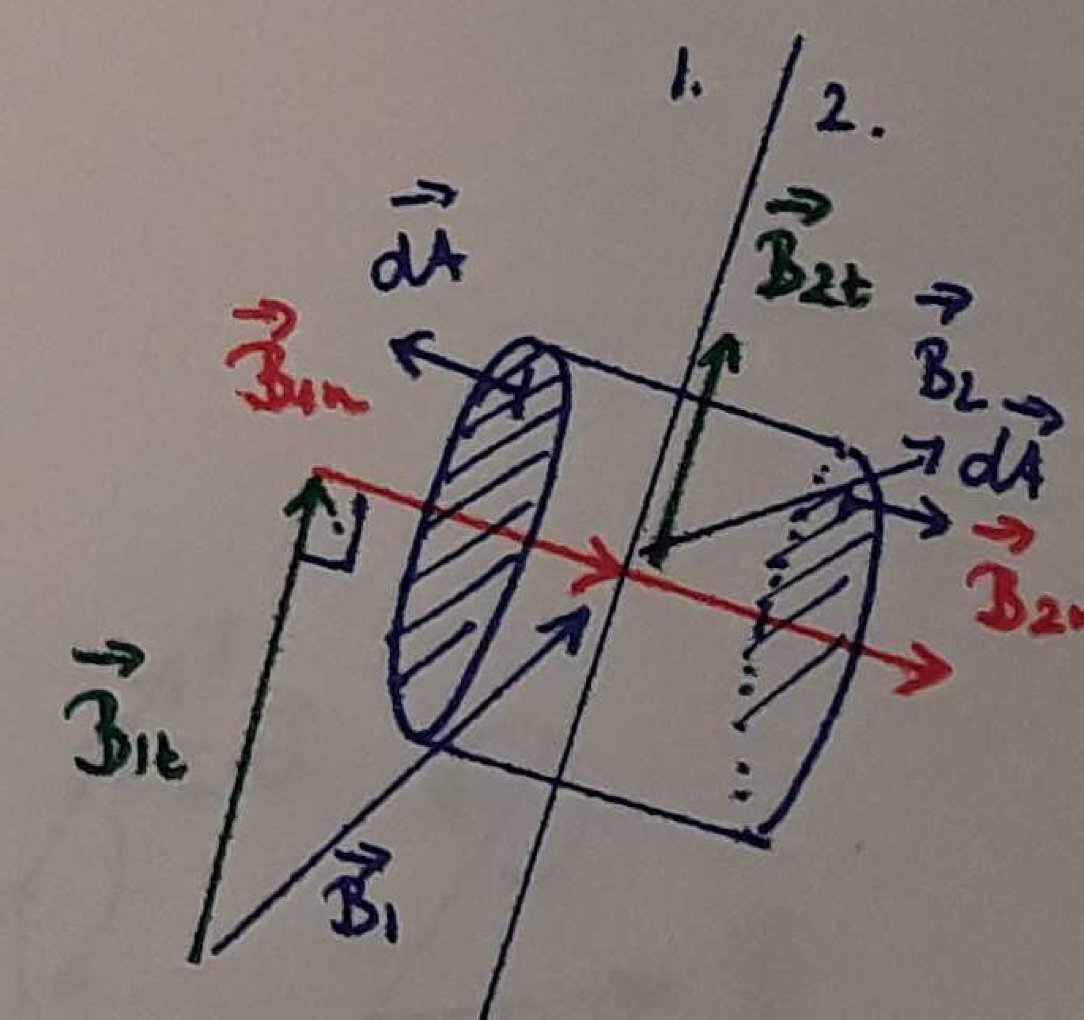
$$H_{2t}\,l - H_{1t}\,l = \cancel{J\,l\,\Delta l} + K l$$

$$\boxed{H_{2t} - H_{1t} = K}$$

$$K = 0 \longrightarrow \boxed{H_{1t} = H_{2t}}$$



$$H = \frac{B}{\mu} \sim 0 \qquad \boxed{H_t = 0}$$



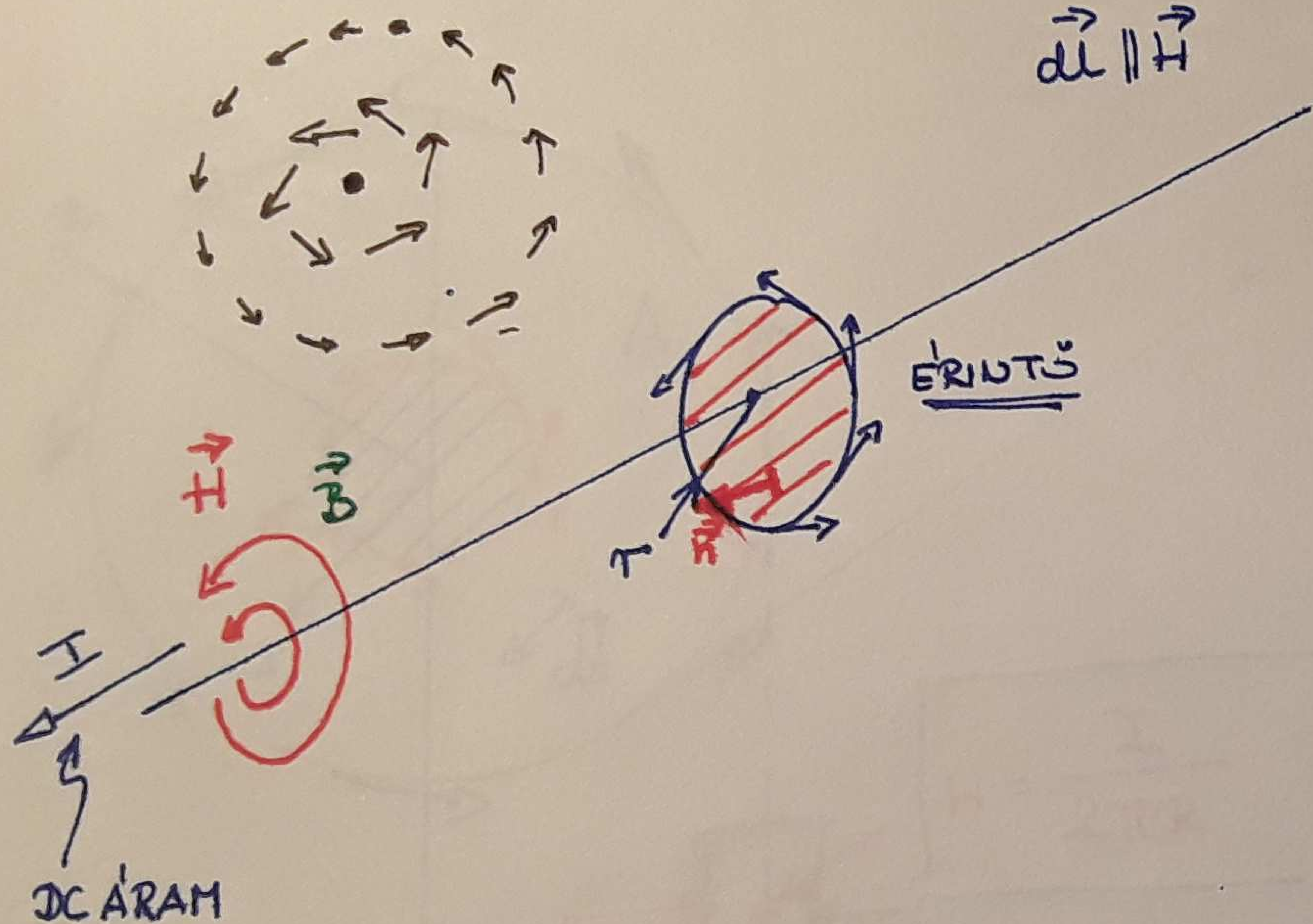
$$\oint_A \vec{B}\cdot d\vec{A} = 0$$

$$-B_{1n}\,dA + B_{2n}\,dA = 0$$

$$-B_{1n} + B_{2n} = 0$$

$$\boxed{B_{1n} = B_{2n}}$$

Hosszú, egyenes vezető stacionárius mágneses tere



$d\vec l \parallel \vec H$

$$\oint_l \vec H \cdot d\vec l = \underbrace{\int_A \vec J \cdot d\vec A}_{I}$$

$$\oint_l H\, dl$$

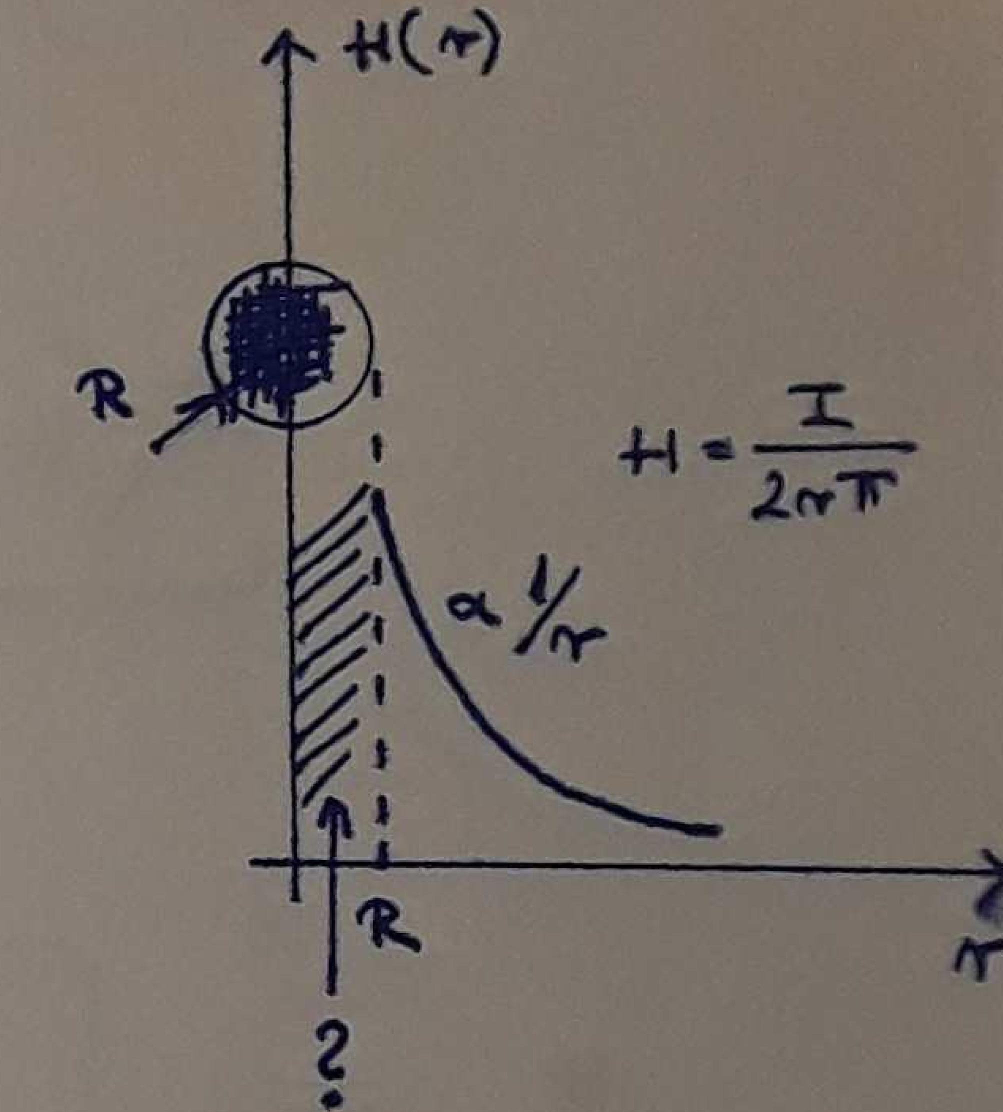
$$H \underbrace{\oint_l dl}_{l}$$

$$\boxed{H\, 2r\pi} = I$$

$$\boxed{H = \frac{I}{2r\pi}}$$

$H \propto I$

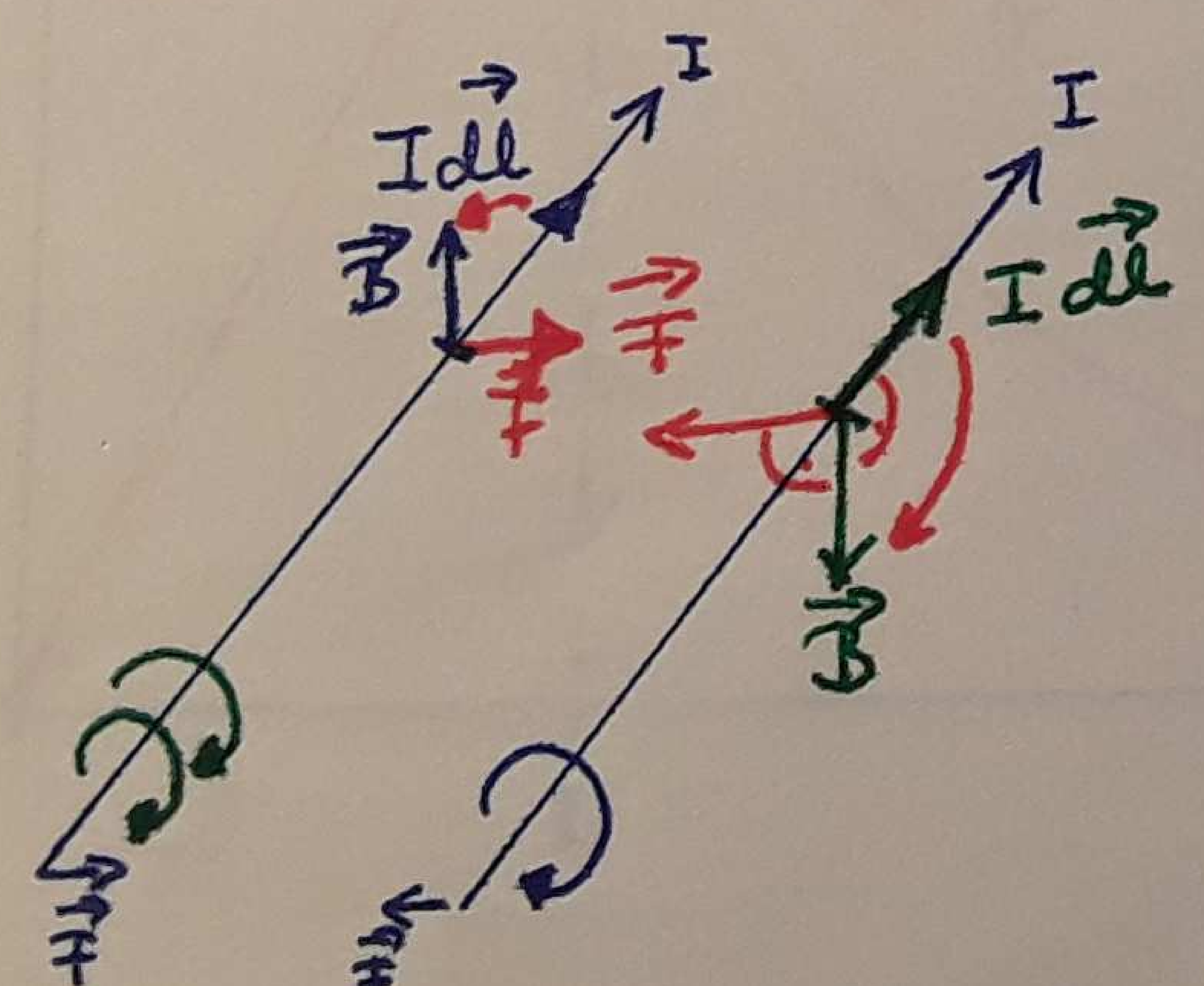
$H \propto \frac{1}{r}$

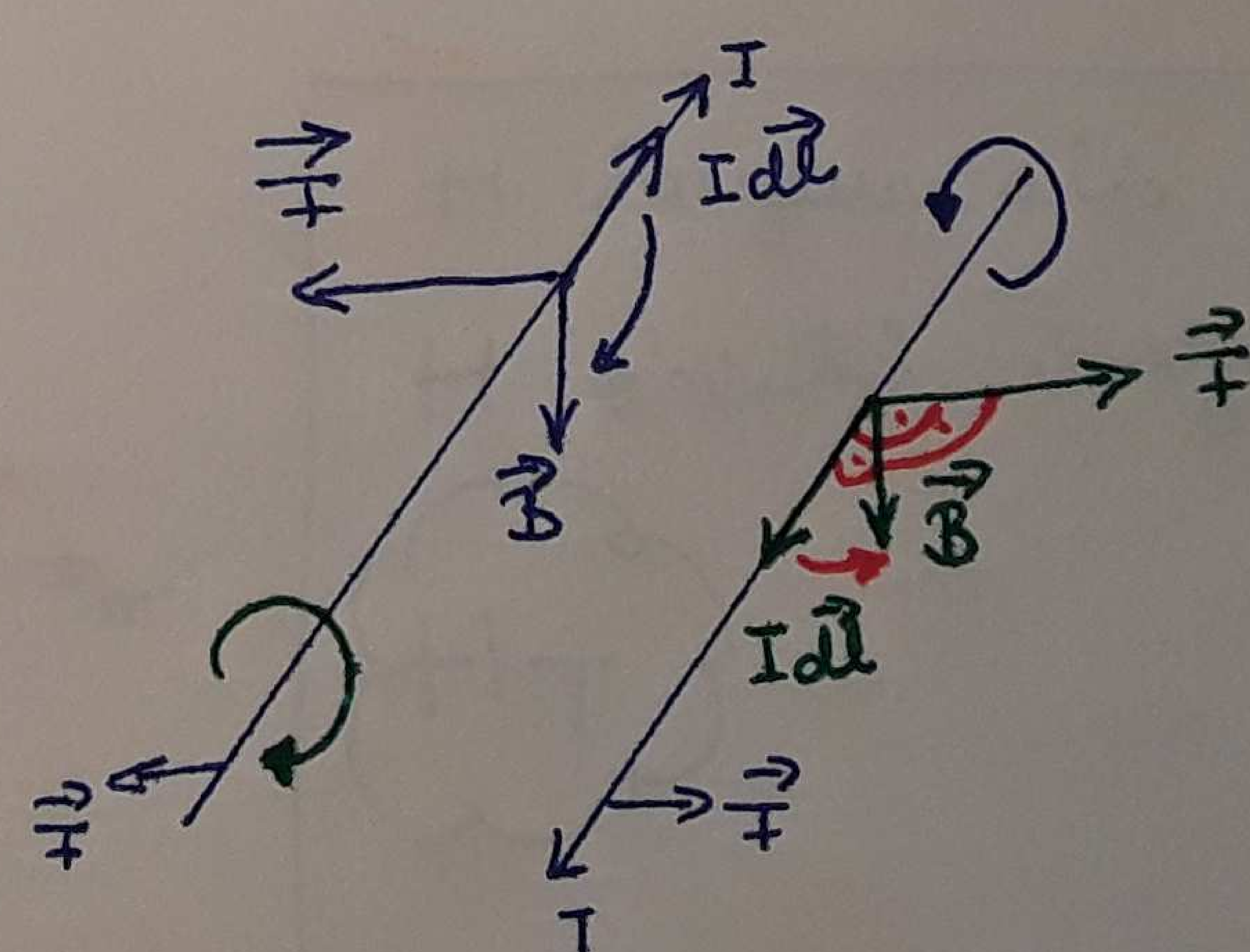


$$\vec B = \mu_0 \vec H$$

$$\vec B = \mu_0 \frac{I}{2r\pi} \vec e$$

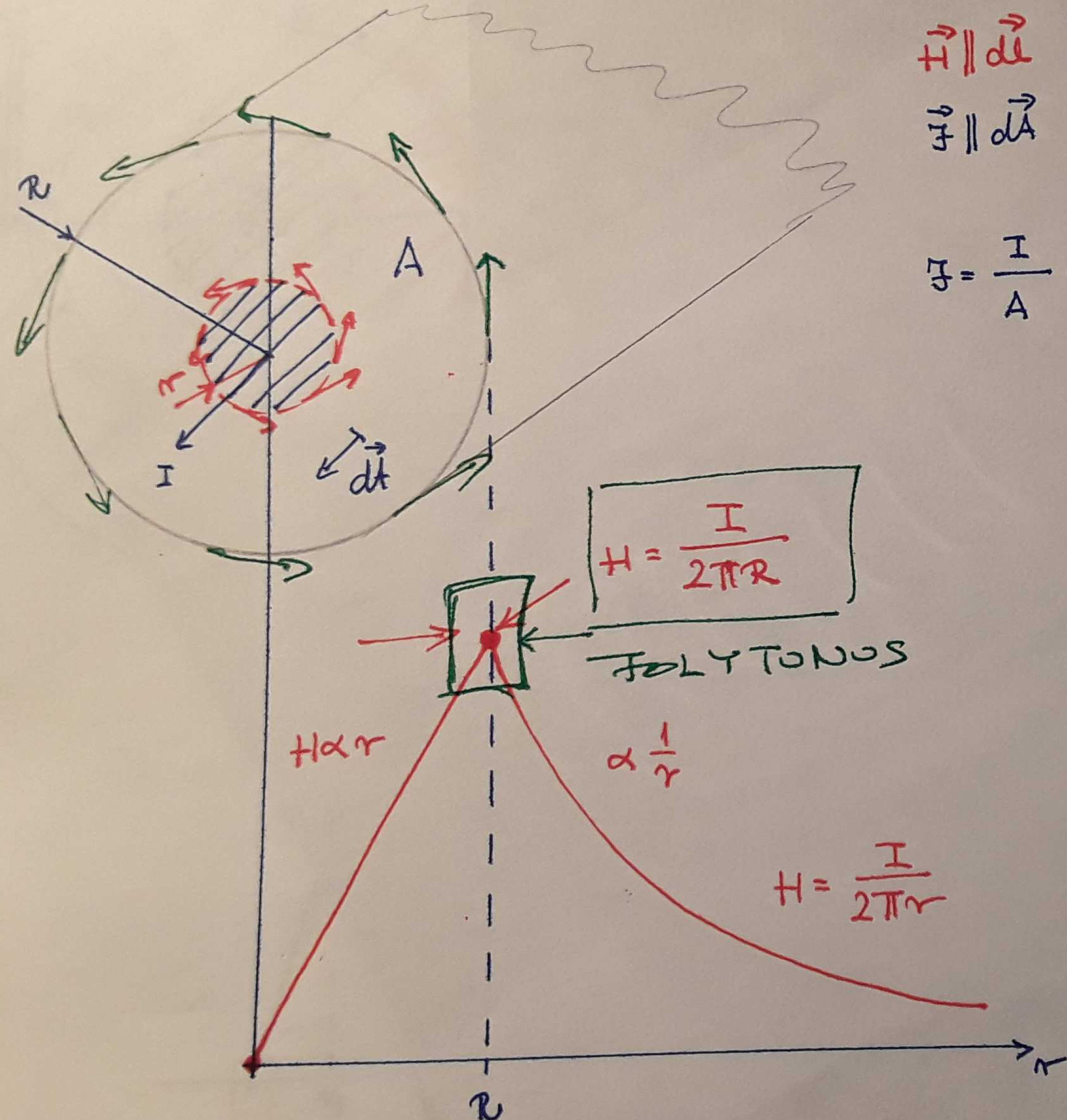
$$\vec F = \underline{I\, d\vec l} \times \vec B$$





Vezetékben belül kialakuló stacionárius mágneses tér

D.C.



$\vec{H} \parallel d\vec{l}$

$\vec{J} \parallel d\vec{A}$

$$\oint_l \vec{H} \cdot d\vec{l} = \int_A \vec{J} \cdot d\vec{A} \quad (= JA)$$

$$J = \frac{I}{A} = \frac{I}{R^2\pi}$$

$\cancel{H 2 r \pi} =$

$$H 2 \cancel{r} \pi = \frac{I}{R^2 \cancel{\pi}} \cdot r^{\cancel{2}} \cancel{\pi}$$

$$H 2\pi = \frac{I r}{R^2}$$

$$\boxed{H = \frac{I}{2\pi R^2} r}$$

~~$H \propto r$~~

$H \propto r$

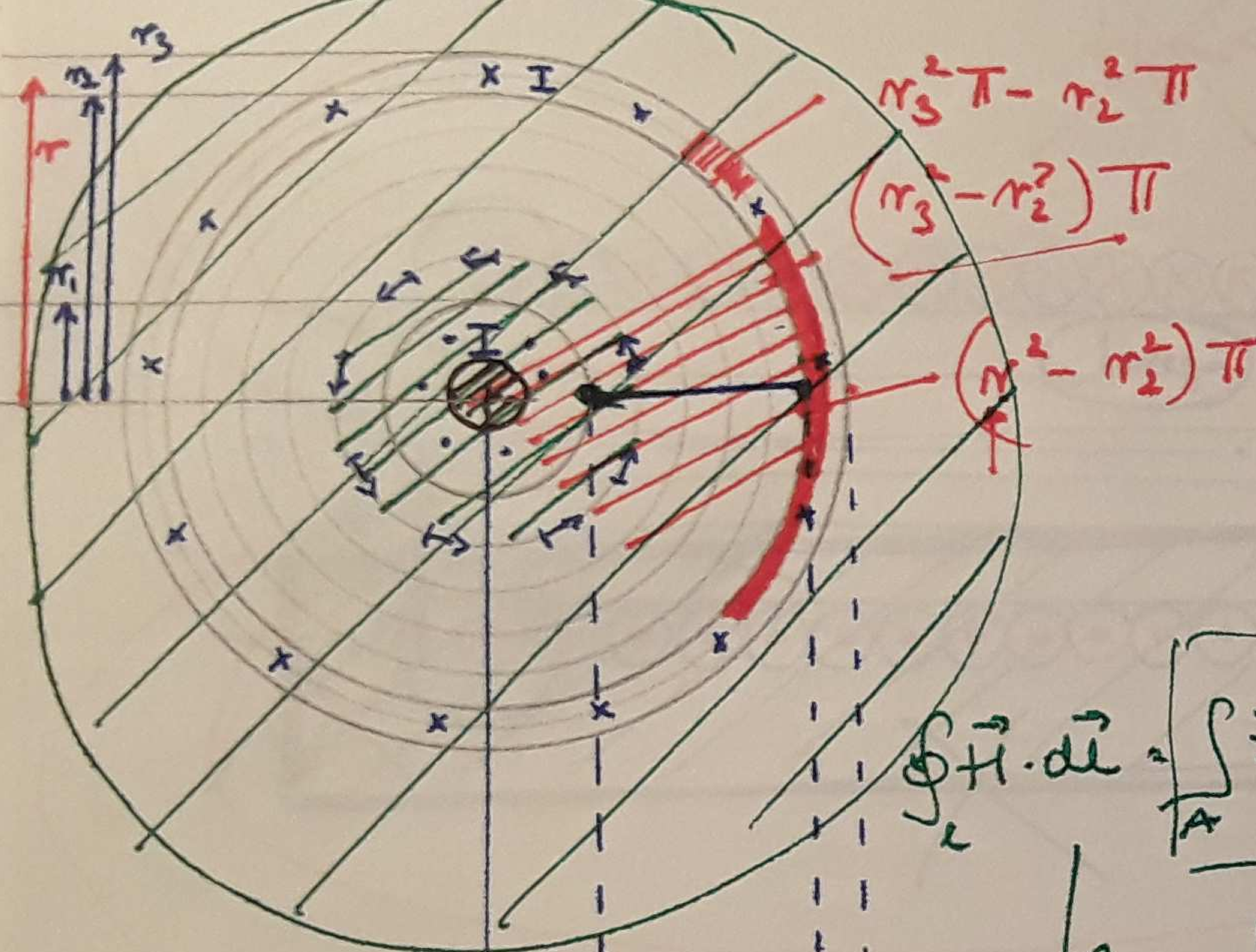
$$r = R \longrightarrow H = \frac{I}{2\pi R}$$

H tangenciális

H érintő irányú

H_T

Koaxiális kábel stacionárius mágneses tere



$$\underbrace{\oint_l \vec{H}\cdot d\vec{l}}_{Hl} = \underbrace{\int_A \vec{J}\cdot d\vec{A}}$$

$H2r\pi = ?$

ér: $0 \le r \le r_1$

$J = \dfrac{I}{r_1^2\pi}$ $A = r^2\pi$

$$\boxed{JA = \frac{I}{r_1^2\pi} r^2\pi}$$

$$H2\cancel{r}\pi = \frac{I}{r_1^2\cancel{\pi}} r^{\cancel{2}}\cancel{\pi}$$

$$H2\pi = \frac{Ir}{r_1^2} \longrightarrow \boxed{H = \frac{Ir}{2\pi r_1^2}}$$

dielektrikum: $r_1 \le r \le r_2$

$$H2r\pi = I \longrightarrow \boxed{H = \frac{I}{2r\pi}} \quad B = \frac{\mu I}{2r\pi}$$

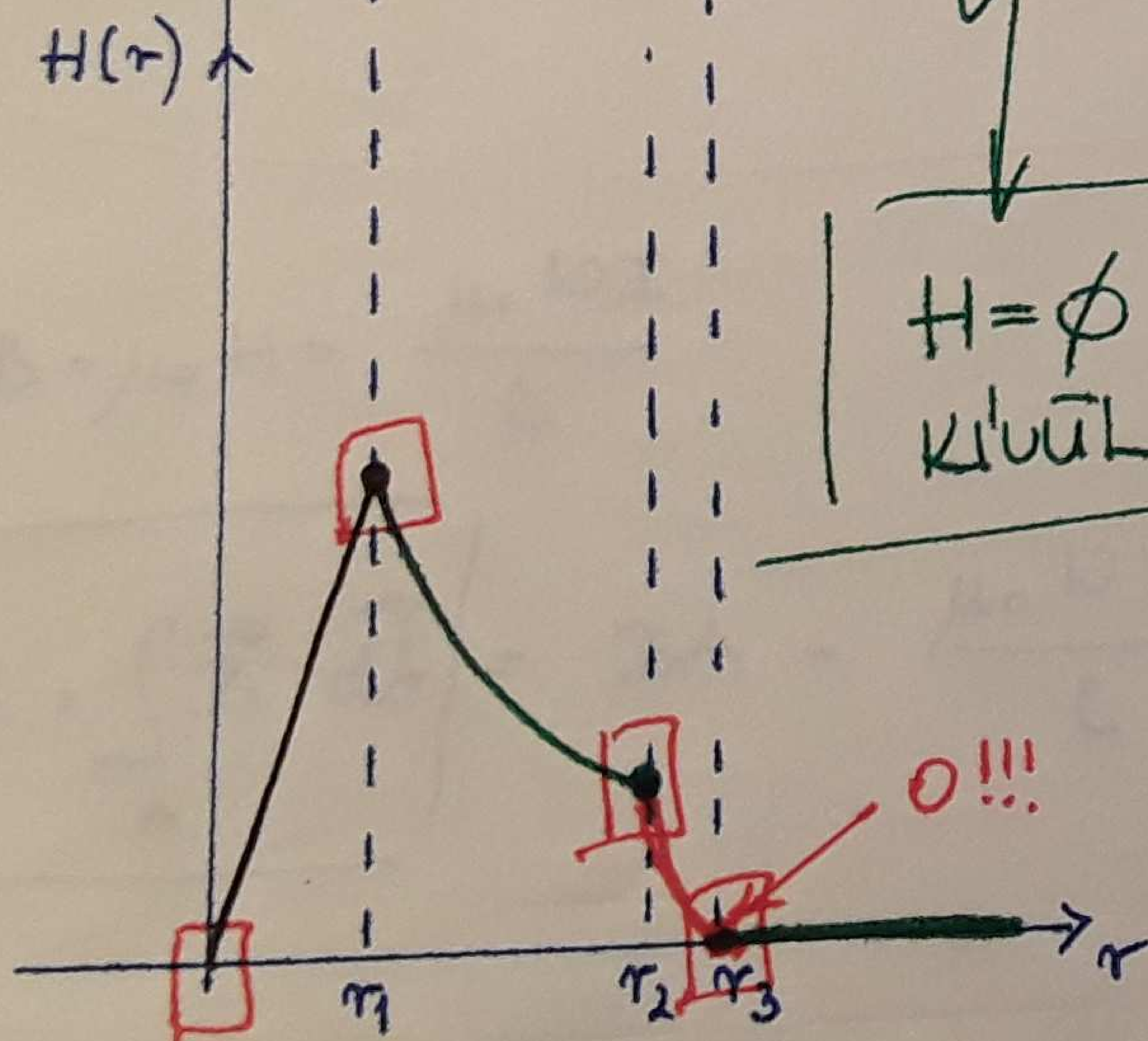
köpeny: $r_2 \le r \le r_3$

$$\int_A \vec{J}\cdot d\vec{A} = \underbrace{I - \frac{I}{(r_3^2 - r_2^2)\cancel{\pi}}(r^2 - r_2^2)\cancel{\pi}}$$

$$H2r\pi = I\frac{r_3^2 - r^2}{r_3^2 - r_2^2} \longrightarrow \boxed{H = I\frac{r_3^2 - r^2}{r_3^2 - r_2^2}\frac{1}{2r\pi}}$$

$$\oint_l \vec{H}\cdot d\vec{l} = \boxed{\int_A \vec{J}\cdot d\vec{A}} = \emptyset$$

$$\boxed{H = \emptyset \text{ D.C. KÍVÜL}}$$



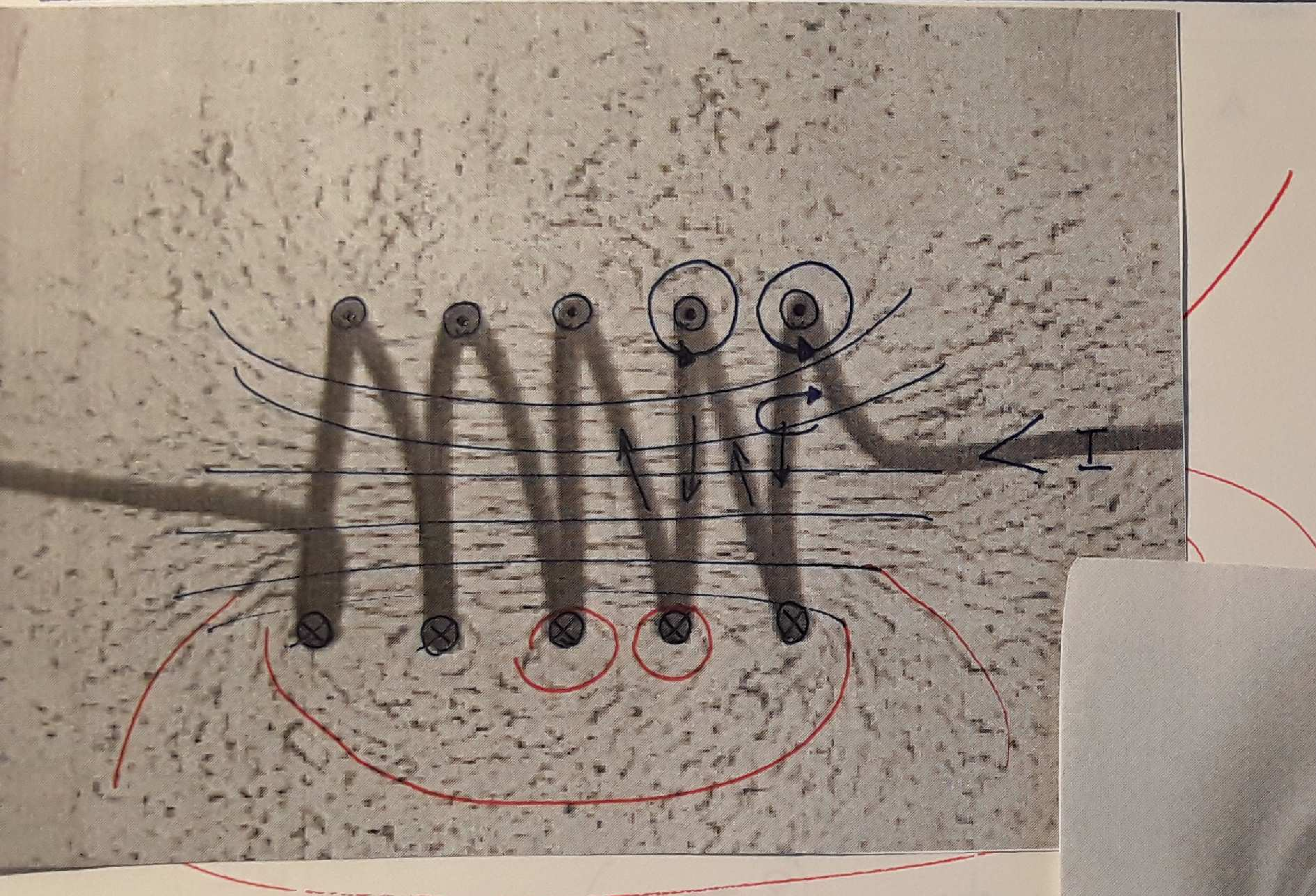
$L = \dfrac{\Phi}{I}$

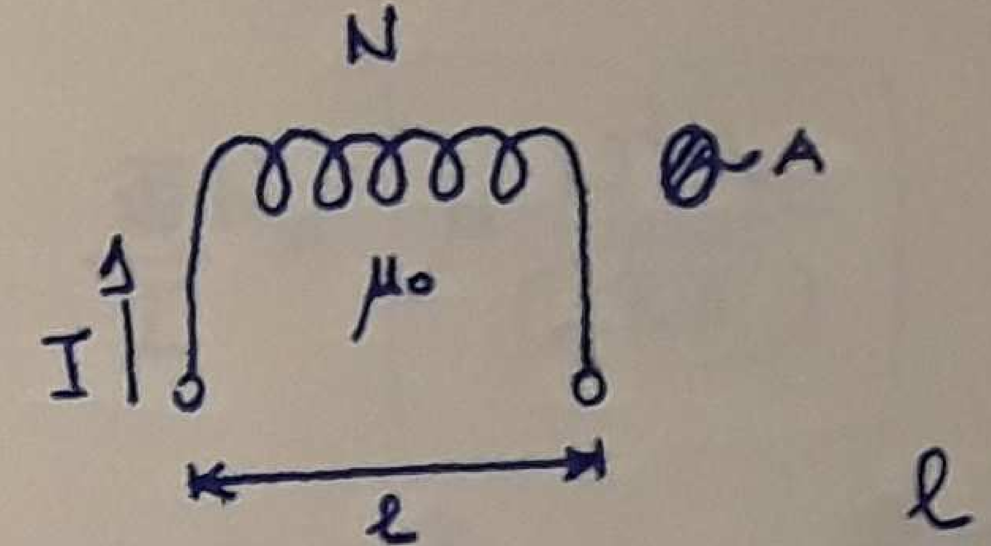
$$\Phi = \int_A \vec{B}\cdot d\vec{A} = \frac{\mu I}{2\pi}\int_{r_1}^{r_2}\frac{1}{r}\underbrace{\boxed{l\,dr}}_{dA} = \frac{\mu I l}{2\pi}\int_{r_1}^{r_2}\frac{1}{r}dr = \boxed{\frac{\mu I l}{2\pi}\ln\frac{r_2}{r_1}}$$

($\int \frac{1}{r}dr$: $\ln r$)

$$\boxed{L_k = \frac{\mu l}{2\pi}\ln\frac{r_2}{r_1}}$$

Szolenoid tekercs közelítő stacionárius mágneses tere

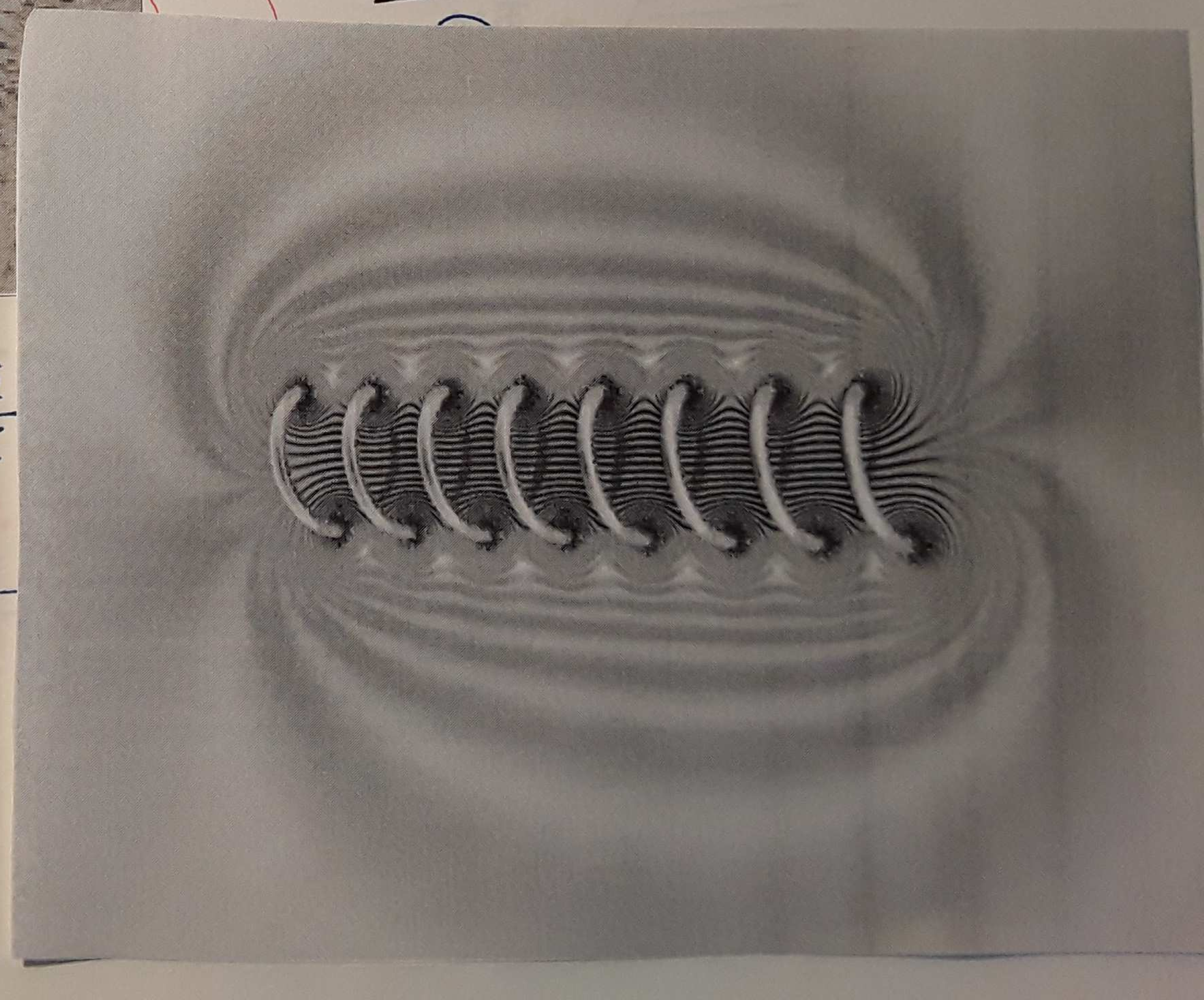




$l \gg D$

$$\oint_l \vec{H}\cdot d\vec{l} = \underbrace{\int_A \vec{J}\cdot d\vec{A}}_{IN}$$

BELSŐ

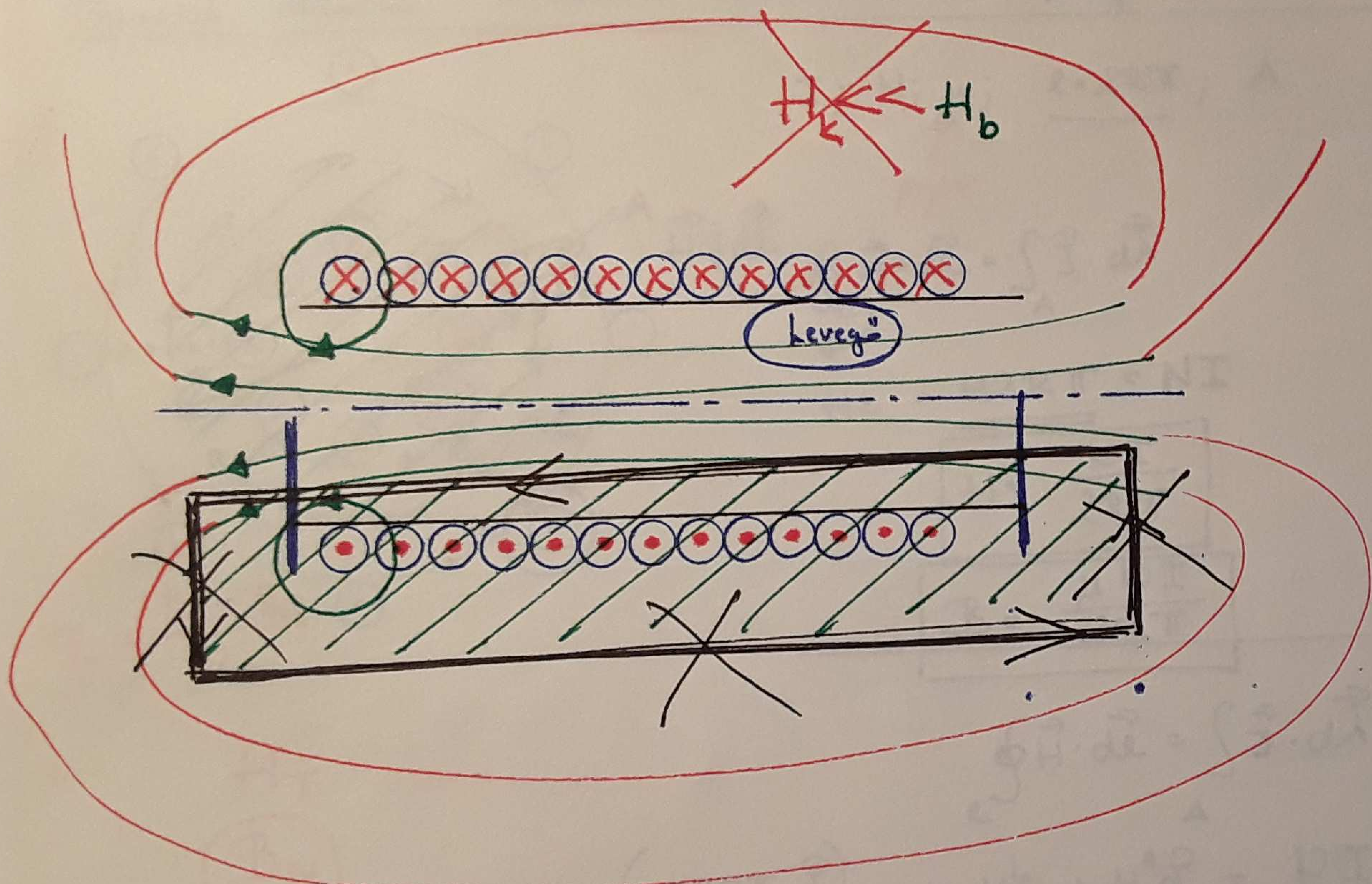


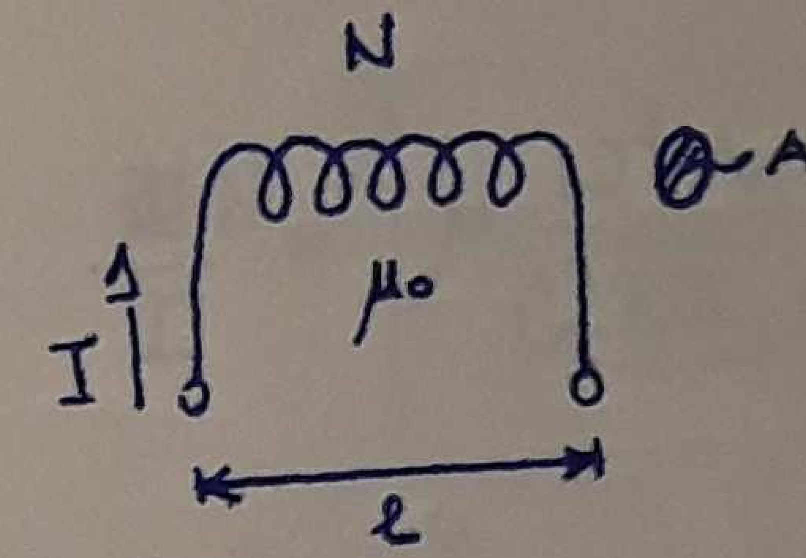
$$B = \mu_0 H = \frac{\mu_0 N I}{l}$$

$$L = \frac{\Phi}{I}$$

$$\boxed{\Phi = \int_A \vec{B}\cdot d\vec{A}} = BA = \frac{\mu_0 N I A}{l}$$

Szolenoid tekercs közelítő stacionárius mágneses tere





$l >> D$

$$\oint_l \vec{H} \cdot d\vec{l} = \underbrace{\int_A \vec{J} \cdot d\vec{A}}_{IN}$$

BELSŐ

$$\int_l \vec{H} \cdot d\vec{l}$$

$$Hl = NI$$

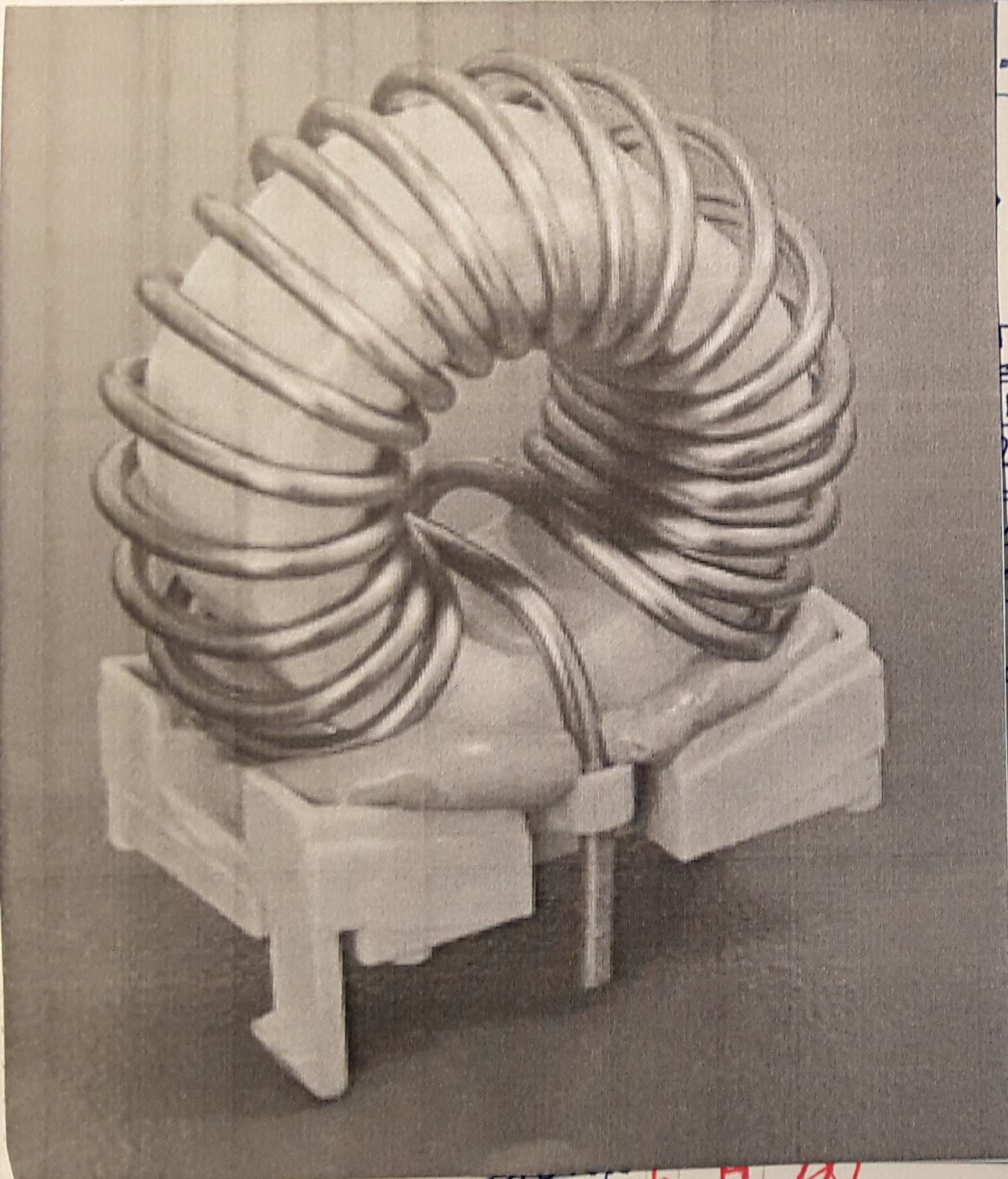
$$\boxed{H \cong \frac{NI}{l}}$$

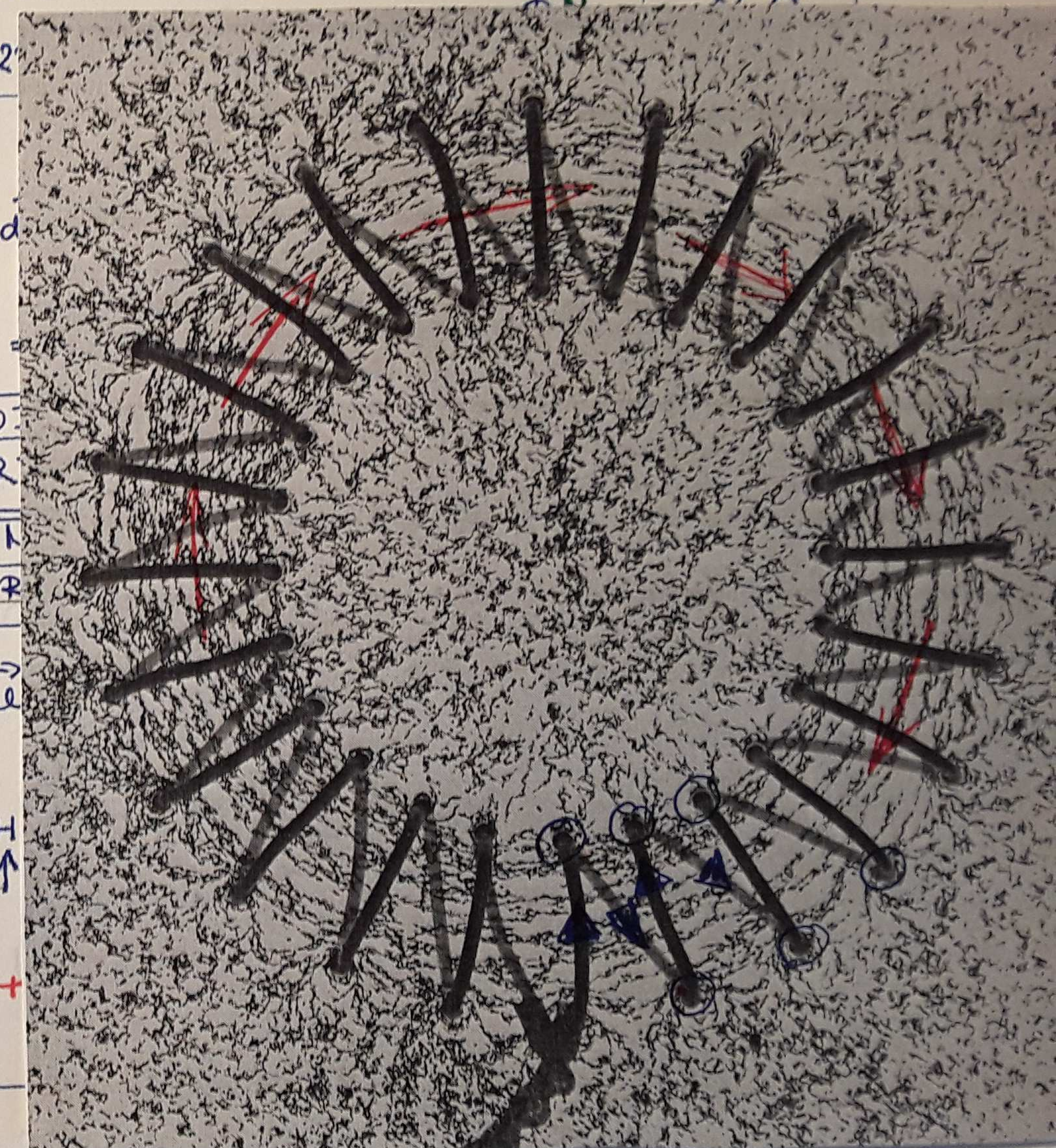
$$B = \mu_0 H = \frac{\mu_0 NI}{l}$$

$$L = \frac{\Phi}{I}$$

$$\boxed{\Phi = \int_A \vec{B} \cdot d\vec{A}} = BA = \frac{\mu_0 NIA}{l} \longrightarrow \boxed{L = \frac{\mu_0 N^2 A}{l}}$$

Toroid tekercs közelítő stacionárius mágneses tere



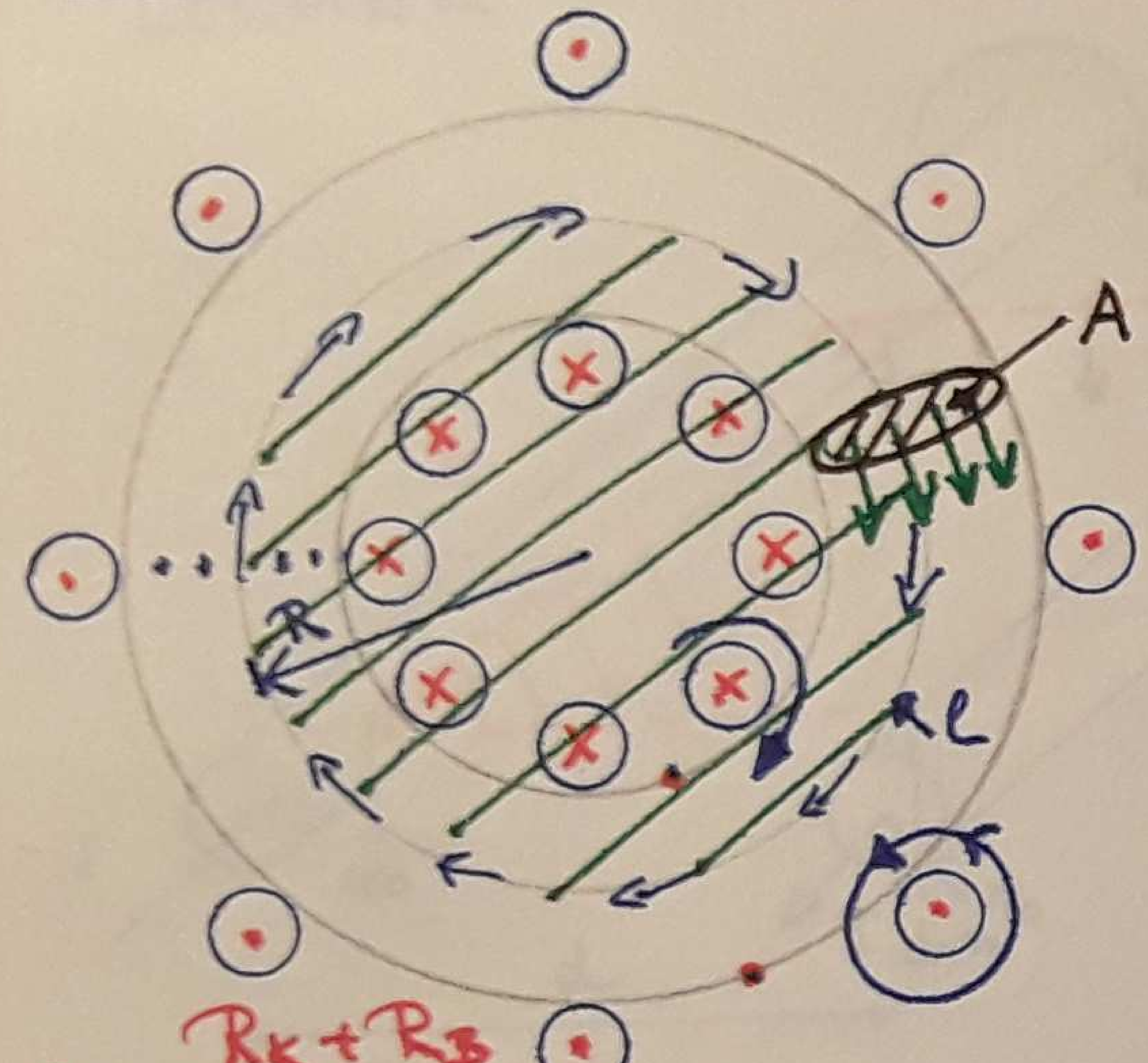


$$I = \frac{\quad}{\frac{l}{\mu_0 \mu_r} + \frac{\delta}{\mu_0}}$$

$$N\Phi = BA = \frac{\quad}{\frac{l}{\mu_0 \mu_r} + \frac{\delta}{\mu_0}}$$

Levegő

Toroid tekercs közelítő stacionárius mágneses tere



$R = \frac{R_K + R_B}{2}$

$I;\ N;\ \mu;\ l = 2R\pi;\ A$

$\mu_0 \mu_r$

$$\oint_l \vec{H} \cdot d\vec{l} = \int_A \vec{J} \cdot d\vec{A}$$

$$Hl = H2R\pi = NI$$

$$\boxed{H = \frac{NI}{2R\pi}}$$

$$\boxed{B = \frac{\mu NI}{2R\pi}}$$

$$L = \frac{\Phi N}{I} = \boxed{\mu \frac{N^2 A}{2R\pi}}$$

$$\Phi = \int_A \vec{B} \cdot d\vec{A} \qquad \vec{B} \parallel d\vec{A}$$

$$\Phi = BA = \frac{\mu NIA}{2R\pi}$$

$$N\Phi = \frac{\mu N^2 IA}{2R\pi}$$

H_T

B_N

$$\oint_l \vec{H} \cdot d\vec{l} = \int_A \vec{J} \cdot d\vec{A}$$

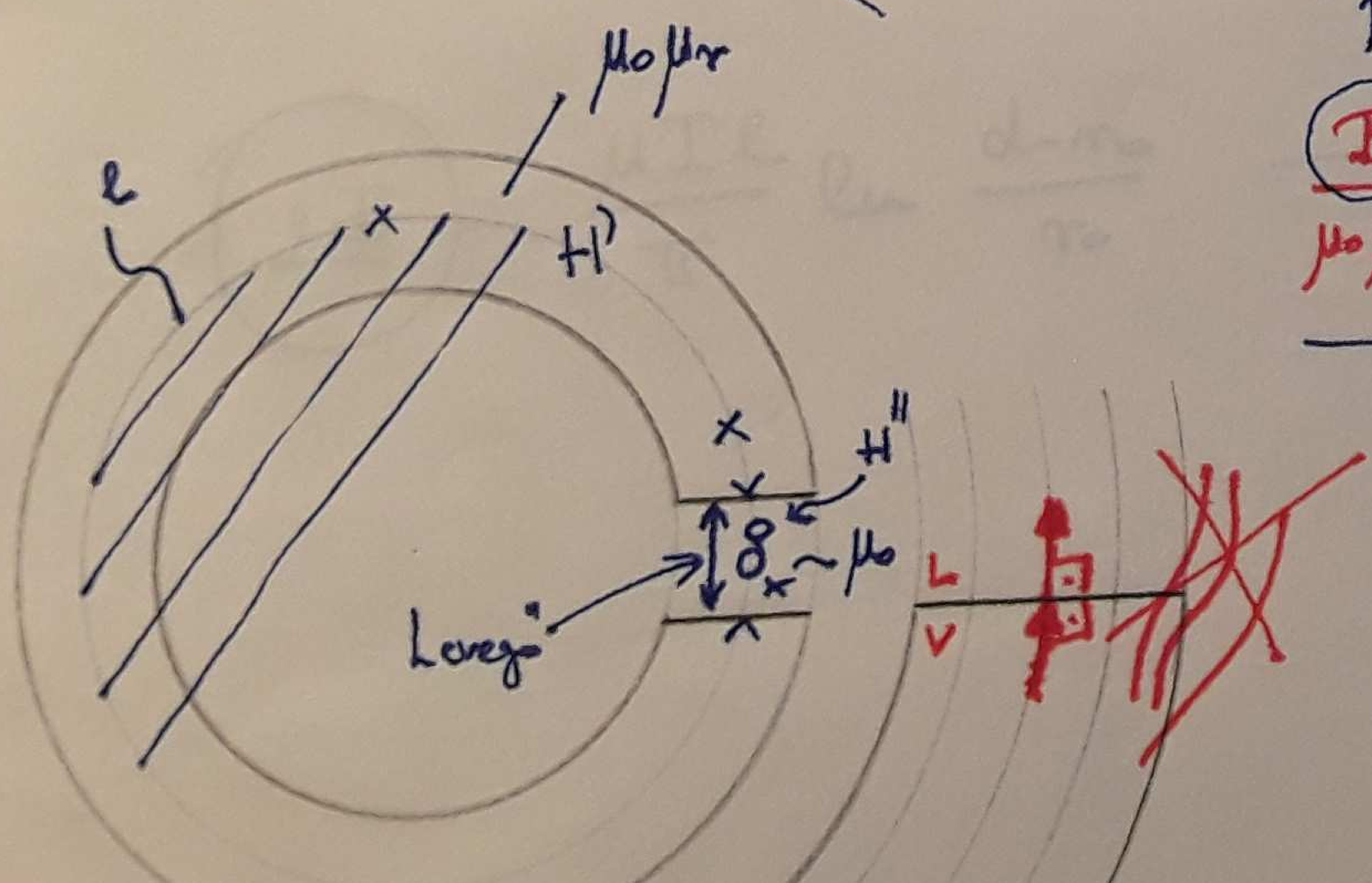
$(l = 2R\pi - \delta)$

$$H'l + H''\delta = NI$$

$$\frac{B}{\mu_0 \mu_r} l + \frac{B}{\mu_0} \delta = NI$$

$$B \left(\frac{l}{\mu_0 \mu_r} + \frac{\delta}{\mu_0} \right) = NI$$

$$\boxed{B = \frac{NI}{\frac{l}{\mu_0 \mu_r} + \frac{\delta}{\mu_0}}}$$

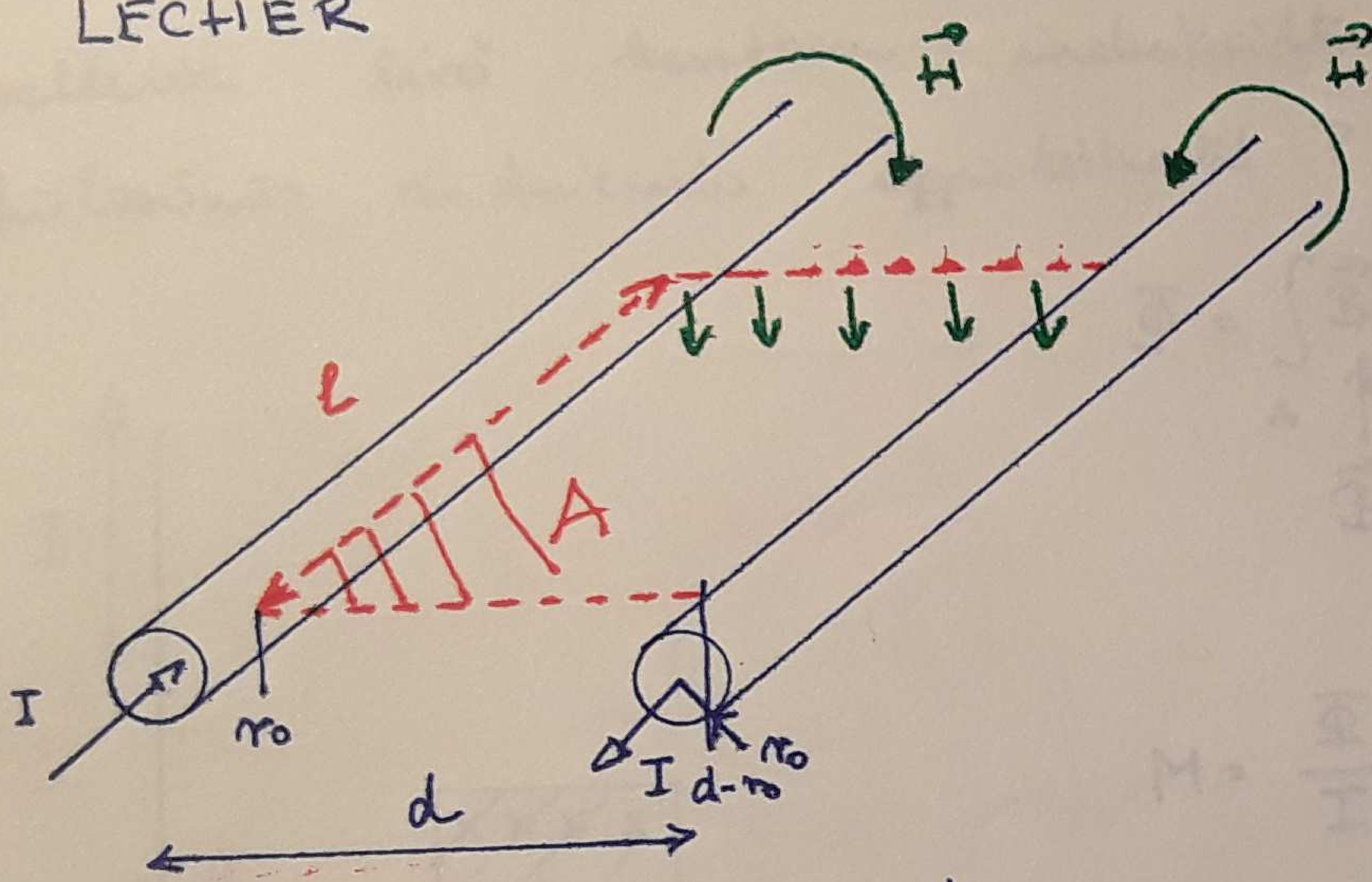


$$L = \frac{\Phi}{I} = \frac{N^2 A}{\frac{l}{\mu_0 \mu_r} + \frac{\delta}{\mu_0}}$$

$$N\Phi = BA = \frac{N^2 IA}{\frac{l}{\mu_0 \mu_r} + \frac{\delta}{\mu_0}}$$

Kettősvezeték stacionárius mágneses tere

LECHER



$$\oint_l \vec{H}\cdot d\vec{l} = \int_A \vec{J}\cdot d\vec{A}$$

1 vezető: $H = \frac{I}{2r\pi}$

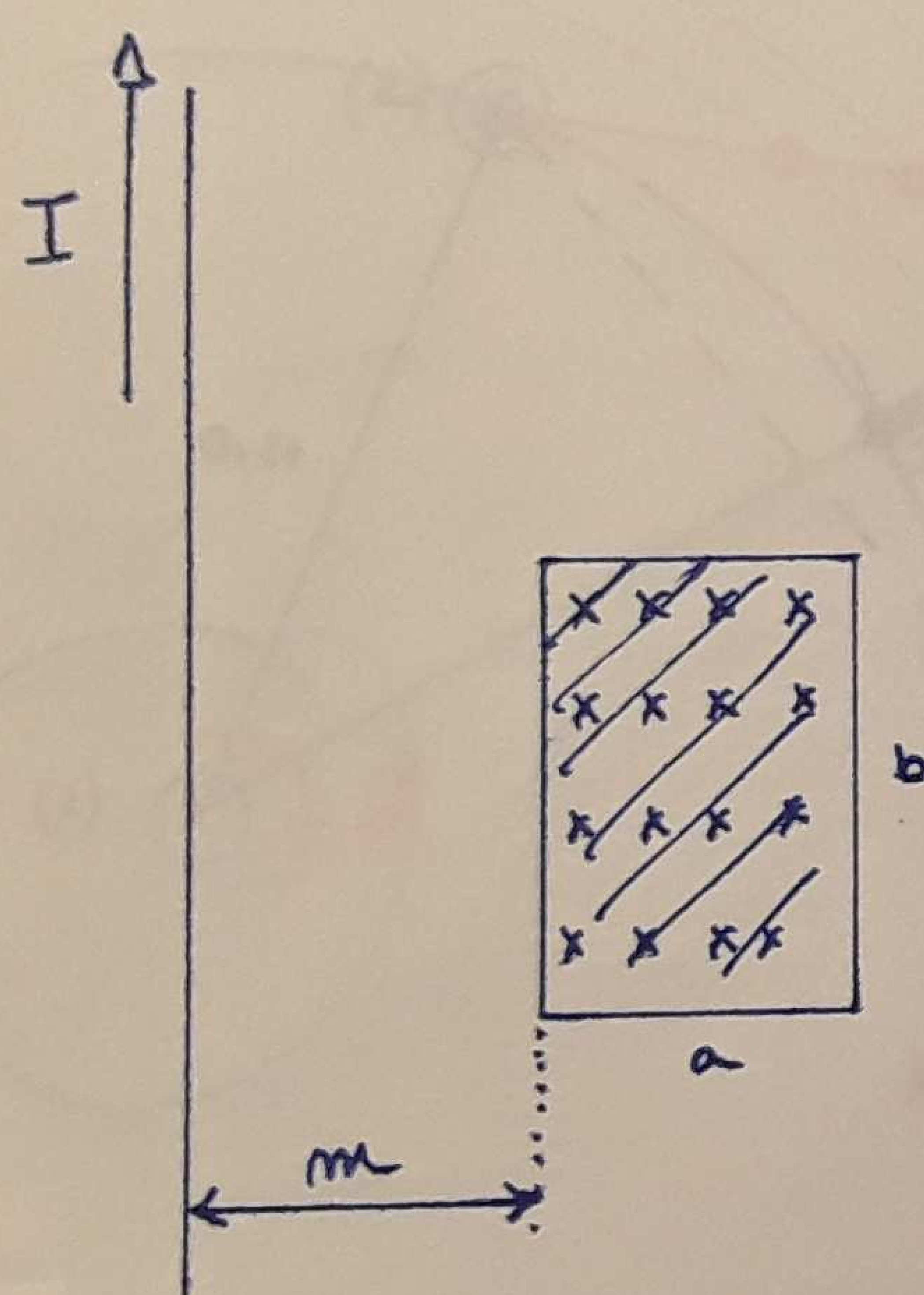
$$B = \frac{\mu I}{2r\pi}$$

$$\Phi = \int_A \vec{B}\cdot d\vec{A} = \int_{r_0}^{d-r_0} \frac{\mu I}{2r\pi}\underbrace{l\,dr} = \frac{\mu I l}{2\pi}\underbrace{\int_{r_0}^{d-r_0}\frac{1}{r}\,dr}_{[\ln r]_{r_0}^{d-r_0} = \ln\frac{d-r_0}{r_0}} = \frac{\mu I l}{2\pi}\ln\frac{d-r_0}{r_0}$$

$$2\Phi = \frac{\mu I l}{\pi}\ln\frac{d-r_0}{r_0} \longrightarrow L_K = \frac{\Phi}{I} = \frac{\mu l}{\pi}\ln\frac{d-r_0}{r_0}$$

ha $r_0 \ll d \rightarrow L_K = \frac{\mu l}{\pi}\ln\frac{d}{r_0}$

Határozzuk meg a végtelen hosszúnak tekinthető egyenes vezető által a mellette lévő keretben indukált fluxus nagyságát! Mekkora a kölcsönös indukciós együttható?



$$\Phi = \int_A \vec{B}\cdot d\vec{A} = \frac{\mu_0 I}{2\pi} b \int_m^{m+a} \frac{1}{r}\, dr = \frac{\mu_0 b I}{2\pi}\left[\ln r\right]_m^{m+a} = \frac{\mu_0 b I}{2\pi}\ln\frac{m+a}{m}$$

$$\vec{B} = \mu_0 \vec{H} \Rightarrow B = \mu_0 H = \mu_0 \frac{I}{2r\pi}$$

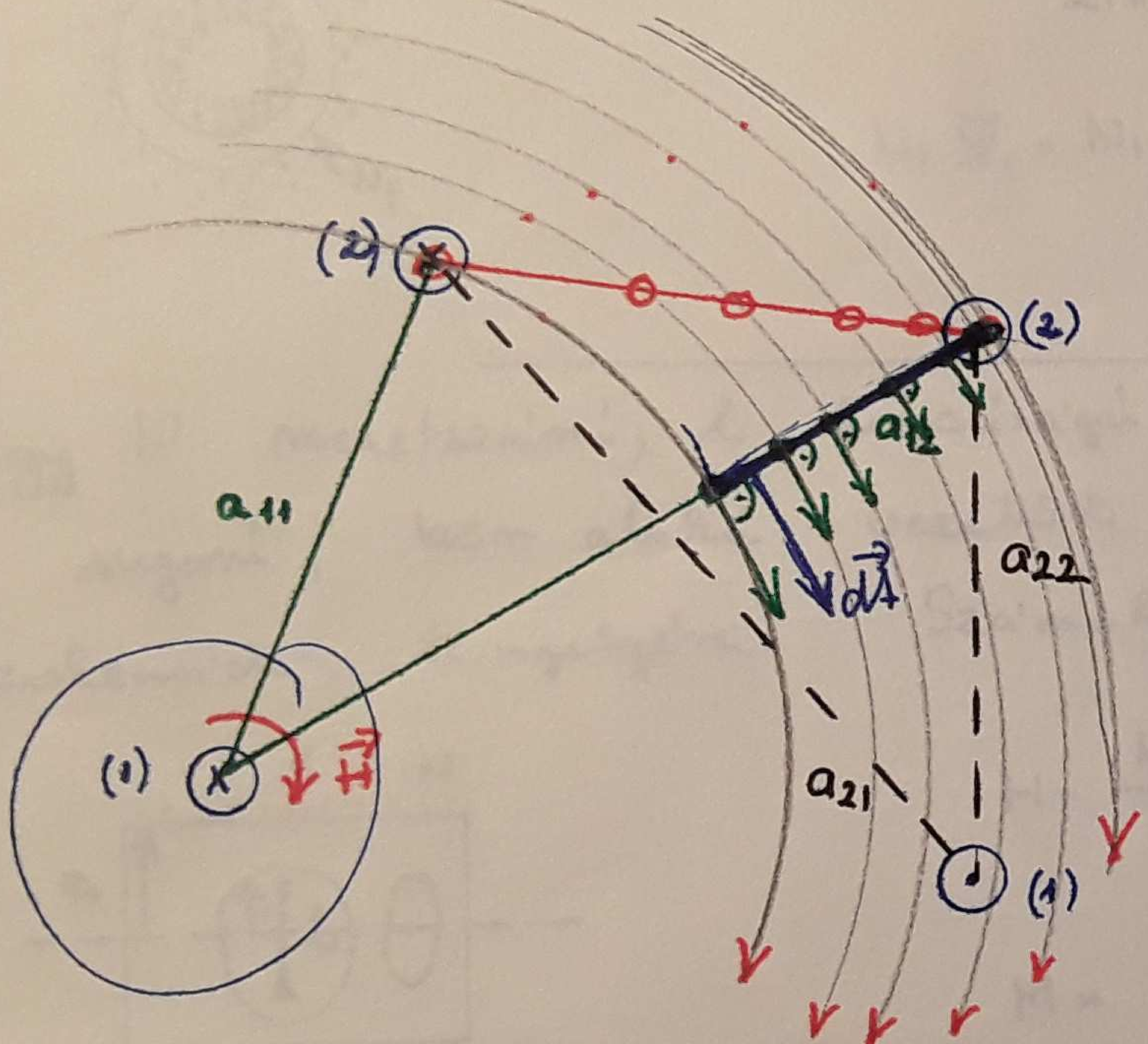
$$M = \frac{\Phi}{I} = \frac{\mu_0 b}{2\pi}\ln\frac{m+a}{m}$$

Határozzuk meg a téglalapon átmenő fluxust, ha $\vec{B}$ merőleges a síkra, és $B(x) = B_0 \sin\left(\frac{\pi}{a}x\right)$

$$\Phi = \int_A \vec{B}\cdot d\vec{A} = b\int_0^a B_0 \sin\left(\frac{\pi}{a}x\right) dx = bB_0\left[-\cos\frac{\pi}{a}x\right]_0^a \frac{a}{\pi} =$$

$$\vec{B} \parallel d\vec{A} \qquad = \frac{abB_0}{\pi}\left(\underbrace{-\cos\frac{\pi}{\cancel{a}}\cancel{a}}_{+1} + \underbrace{\cos\frac{\pi}{a}0}_{+1}\right) = \frac{2abB_0}{\pi}$$

Határozzuk meg az 1. vezetőpár által a 2. vezetőpárban létrehozott fluxust, és a kölcsönös induktivitást!



$$H = \frac{I}{2r\pi}$$

$$\Phi_1 = \int_A \vec{B} \cdot \vec{dA} = \frac{\mu_0 I}{2\pi} l \int_{a_{11}}^{a_{12}} \frac{1}{r}\, dr = \frac{\mu_0 I l}{2\pi} \left[\ln r \right]_{a_{11}}^{a_{12}}$$

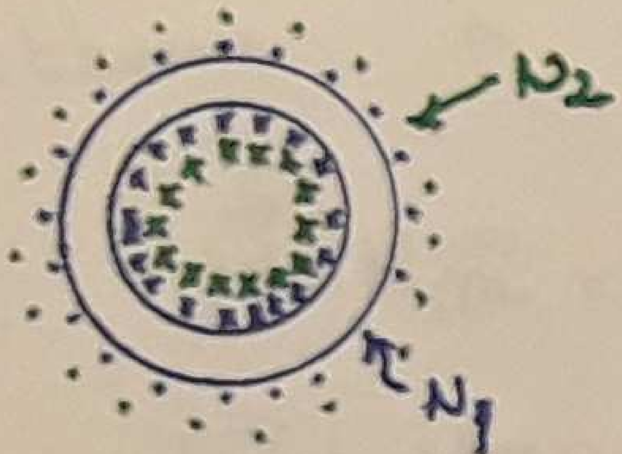
$$= \frac{\mu_0 I l}{2\pi} \ln \frac{a_{12}}{a_{11}}$$

$$\Phi_2 = \frac{\mu_0 I l}{2\pi} \ln \frac{a_{21}}{a_{22}}$$

$$\Phi = \Phi_1 + \Phi_2 = \frac{\mu_0 I l}{2\pi} \ln \frac{a_{12}\, a_{21}}{a_{11}\, a_{22}}$$

$$M = \frac{\Phi}{I} = \frac{\mu_0 l}{2\pi} \ln \frac{a_{12}\, a_{21}}{a_{11}\, a_{22}}$$

Határozzuk meg két, szorosan egymásra csévélt toroid kölcsönös induktivitását, ha a közepes sugár R, a menetszámok N_1 és N_2.



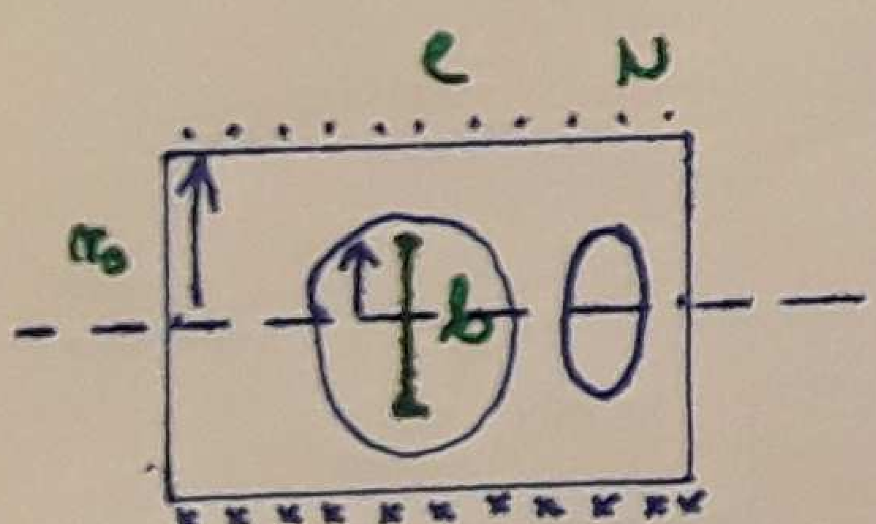
$$H = \frac{N_2 I_2}{2\pi R}$$

$$N_1 \Phi_1 = N_1 \underbrace{\mu_0 \frac{N_2 I_2}{2\pi R}}_{\mu_0 H} A$$

$$M = \frac{N_1 \Phi_1}{I_2} = \mu_0 \frac{N_1 N_2 A}{2\pi R \; l}$$

(szolenoidra hasonló)

Egy N menetszámú, l hosszúságú, r_0 sugarú szolenoidban koncentrikusan egy b sugarú, kör alakú vezetőt helyezünk el. A kör síkja merőleges a szolenoid tengelyére. Számoljuk ki a kölcsönös induktivitást!

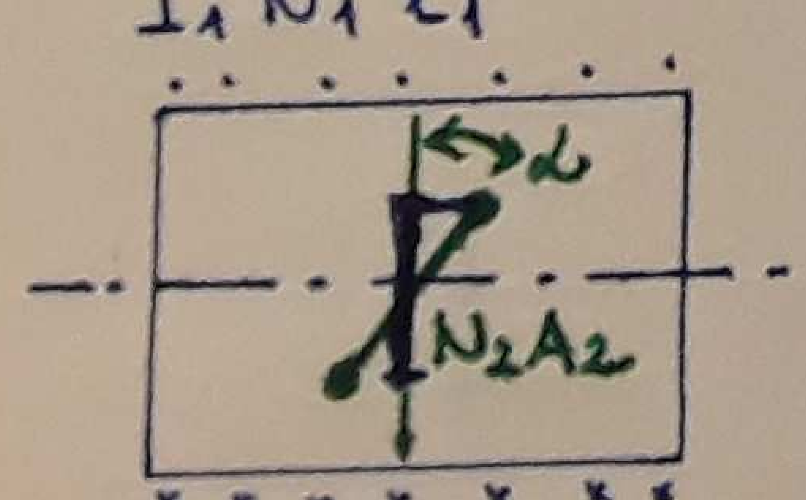


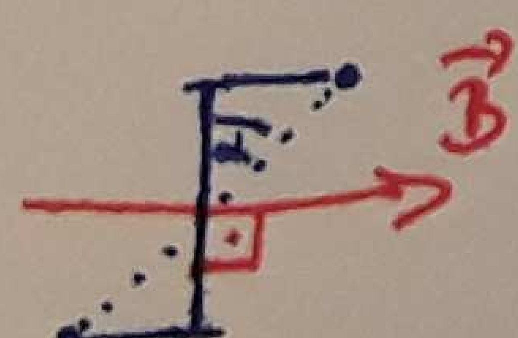
$$H = \frac{NI}{l} \qquad B = \mu_0 \frac{NI}{l} \qquad \Phi = \underbrace{\mu_0 \frac{NI}{l}}_{B} \underbrace{b^2\pi}_{A}$$

$$M = \frac{\Phi}{I} = \mu_0 \frac{N b^2 \pi}{l}$$

Egyenes tekercs belsejében forgatható lehet egy kisebb tekercset. Milyen határok között lehet változtatni a kölcsönös induktivitást?

$$H = \frac{N_1 I_1}{l_1} \qquad B = \mu_0 \frac{N_1 I_1}{l_1}$$





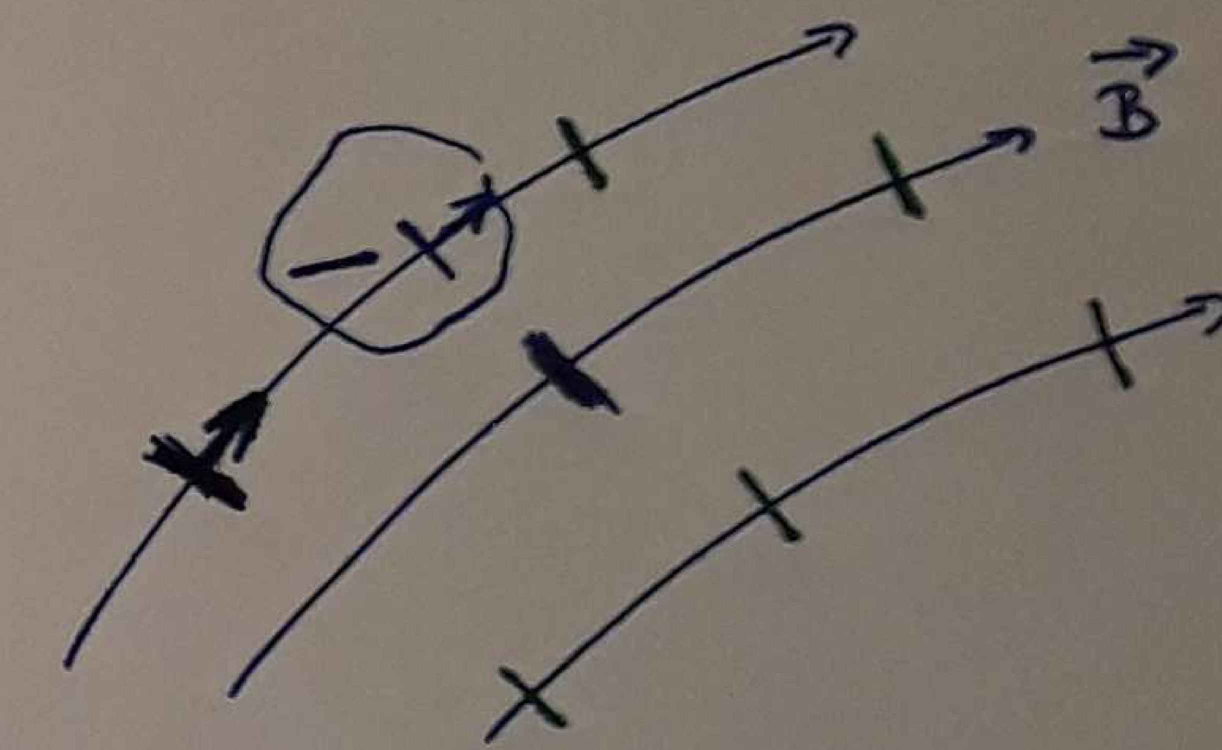
$$\Phi_2 = B \boxed{\cos\alpha \, A_2} = \mu_0 \frac{N_1 I_1}{l_1} \cos\alpha \, A_2$$

$$M = \frac{N_2 \Phi_2}{I_1} = \mu_0 \frac{N_1 N_2 A_2}{l_1} \cos\alpha$$

min. $\alpha = 90°$

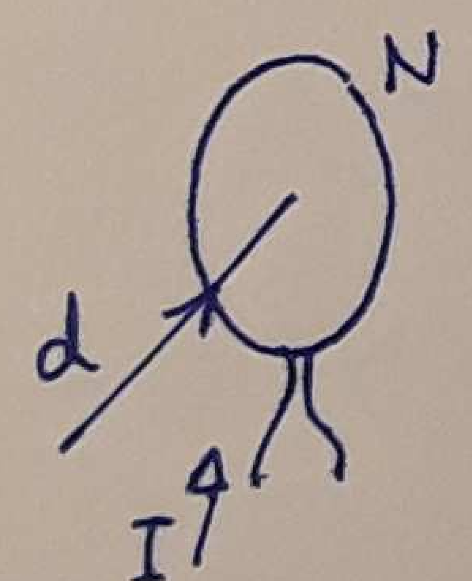
max. $\alpha = 0°$

Kör alakú d átmérőjű, N menetből álló kerettel mágneses tent mérünk, miközben a kereten I áram folyik át. A fellépő nyomaték maximális értéke $\hat{M}$. Mekkora a mágneses indukció értéke?



$$\vec{M} = N\vec{m} \times \vec{B} = NIA\vec{n} \times \vec{B}$$

$$|\vec{M}| = NIAB \sin\alpha \longrightarrow \hat{M} = NIAB \longrightarrow B = \frac{\hat{M}}{IAN}$$



Töltött részecske egyszerre ható elektromos és mágneses térben

- $\vec{E}$ és $\vec{B}$ függetlenül hatnak

$$\vec{F} = q\vec{E} + q\,\vec{v} \times \vec{B}$$

$$= q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Lorentz-erő

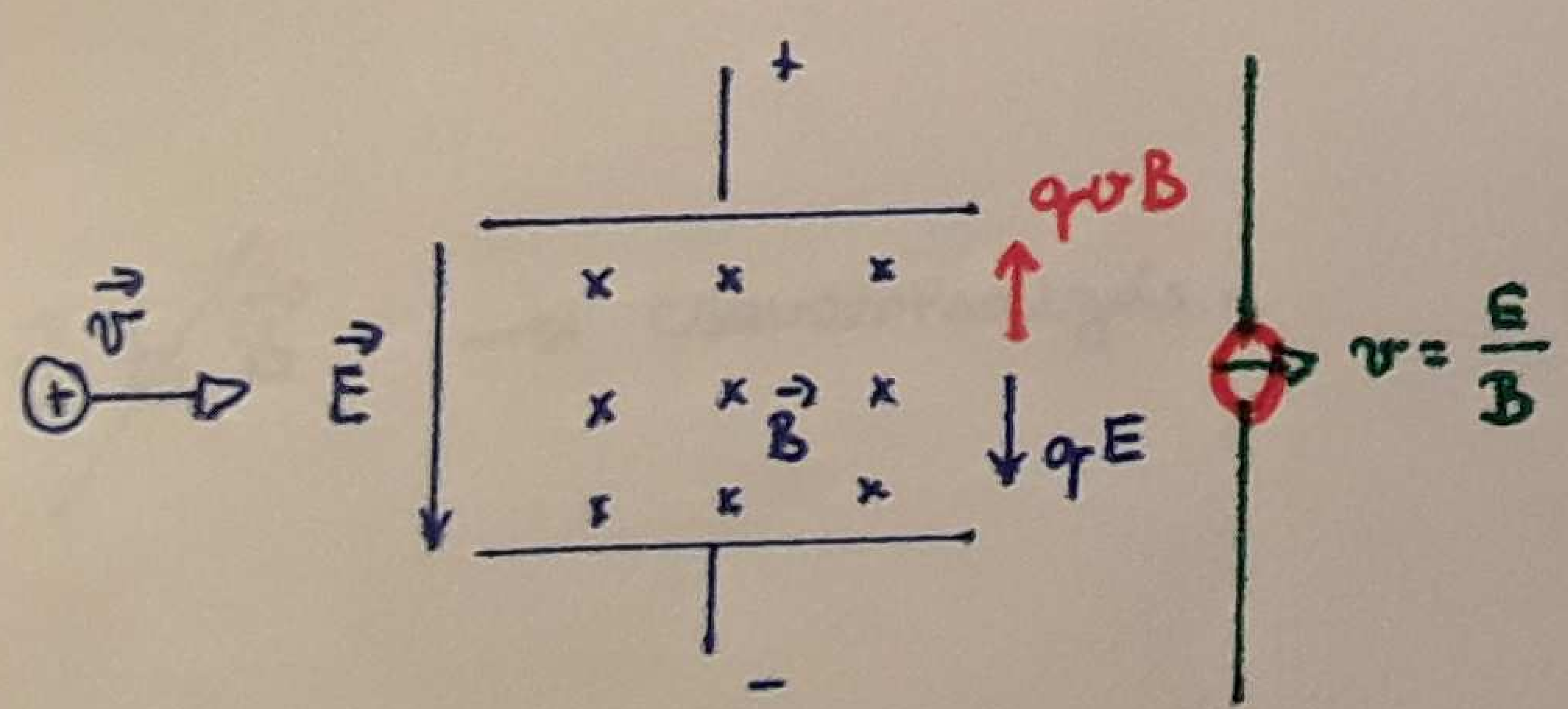
Sebesség-homogenizáló:

$$E = vB$$

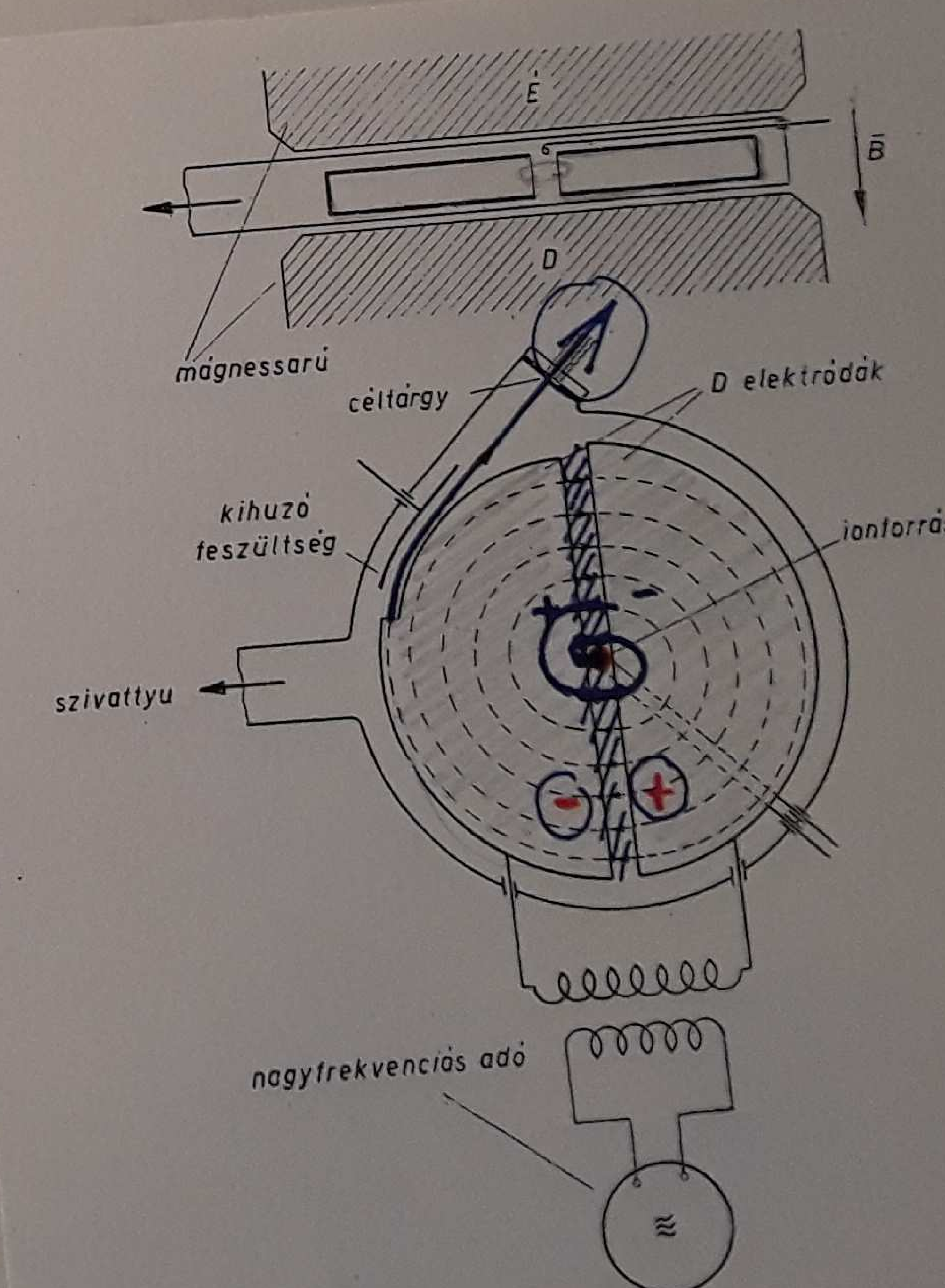
$$\boxed{v = \frac{E}{B}}$$

$$qvB = qE$$

$$vB = E$$



$$E = \frac{U}{d}$$

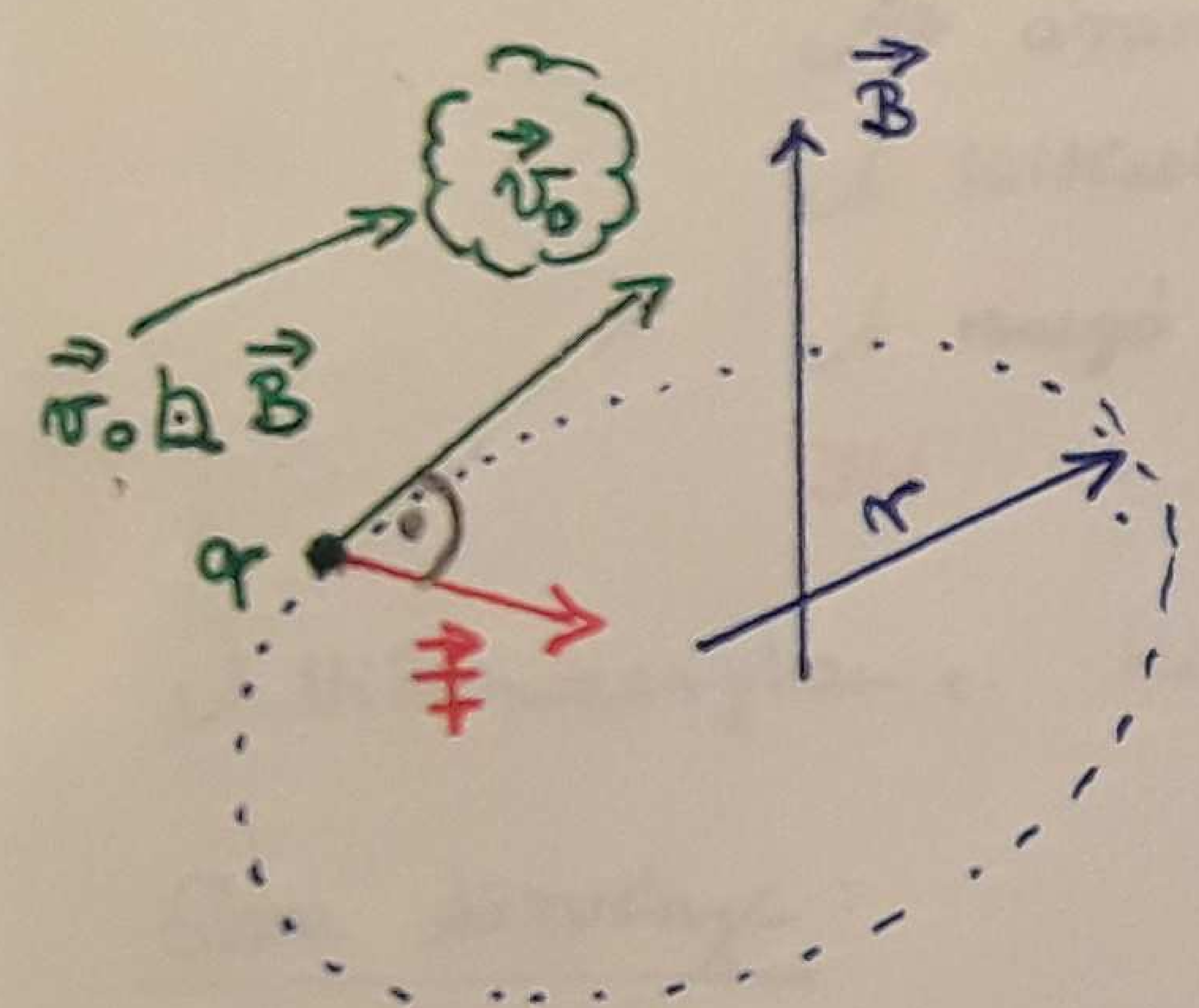


30. ábra. A ciklotron vázlatos rajza

Töltött részecske mozgása homogén mágneses térben

$m\frac{d\vec{v}}{dt} = q\,\vec{v}\times\vec{B} = \vec{F}$ irányváltozik csak, mert $\vec{F} \perp \vec{v}$

KÖRMOZGÁS!



$\frac{mv^2}{r} = qvB \longrightarrow \boxed{r = \frac{mv}{qB}}$

$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\underbrace{\frac{mv}{qB}} = 2\pi\frac{m}{q}\frac{1}{B}$

$\vec{v}_0 \not\perp \vec{B} \rightarrow$ csavarmozgás.

AZ ELEKTROMOS ÉS A MÁGNESES TÉR KAPCSOLATA I.

- ÁRAMLÁSI TÉR
- INDUKCIÓ

ÁRAMLÁSI TÉR

D.C.!!! (STACIONÁRIUS ÁRAMLÁS)

Az áram töltéssel rendelkező részecskék egyirányú mozgása, áramlása!

A töltéseket mozgatni kell - elektromos tér.

A mozgó töltés (áram) mágneses teret kelt maga körül.

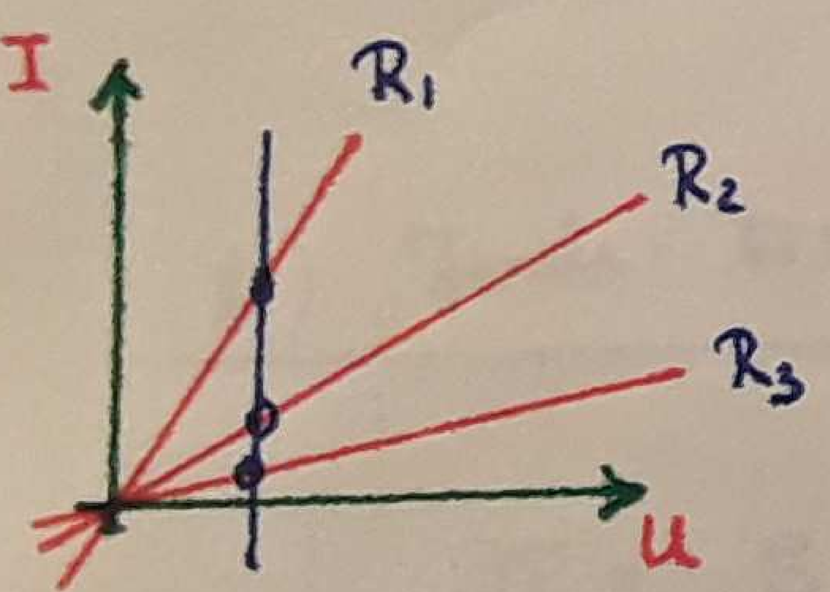
Itt az áramlással foglalkozunk.

A Villamosságtan c. tárgyban tanultakra építünk...

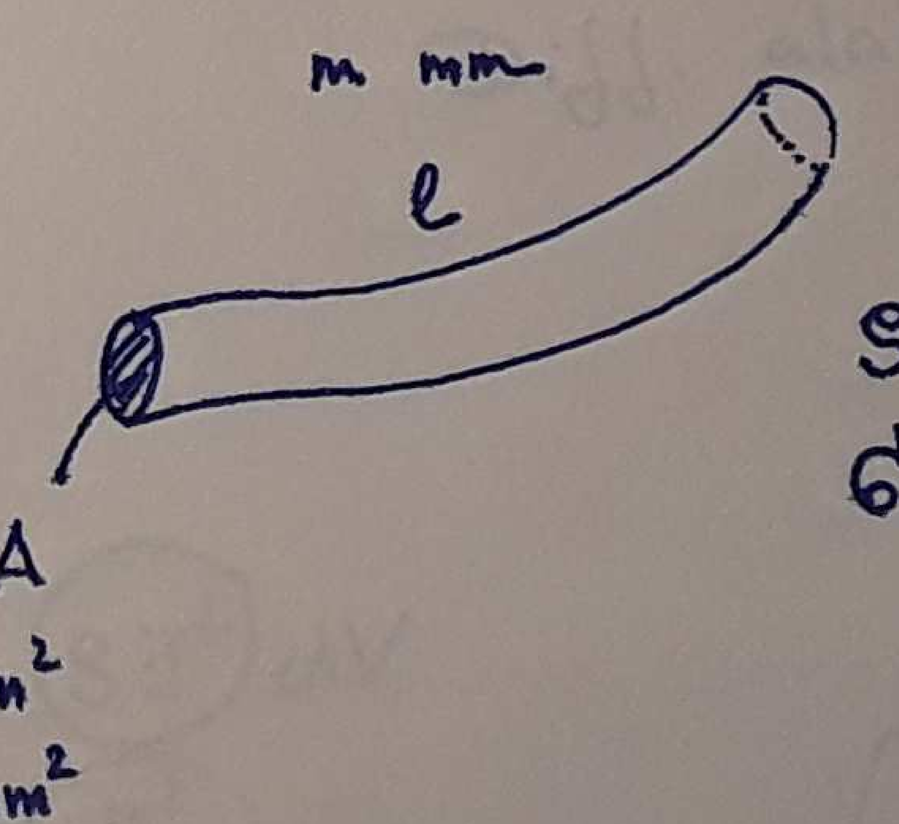
Ohm törvénye:

$$I = \frac{1}{R} U = G U$$

Ω : $\frac{V}{A}$ S : $\frac{A}{V}$



$R_1 < R_2 < R_3$



Méréseknél:

$$R = \rho \frac{l}{A} = \frac{l}{\sigma A}$$

$$1\,\Omega m = \frac{\Omega mm^2}{m} 10^{-6}$$

S/m

Joule-törvény:

$$W = QU \Rightarrow W = ItU = \underbrace{UI}_{P} t$$

$Q = It$

$$P = UI$$

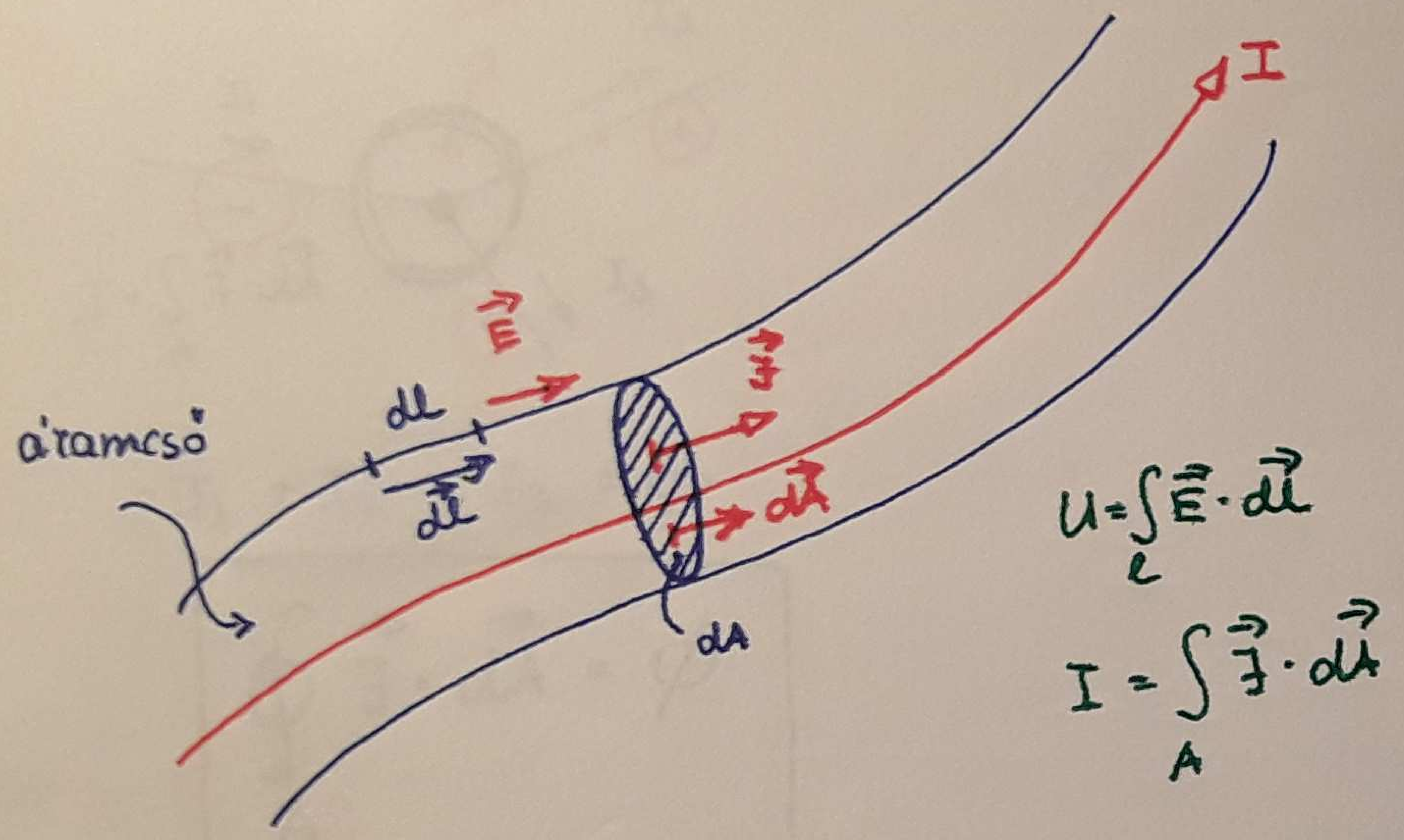
$$P = RI^2 \qquad P = \frac{U^2}{R}$$

pl. réz: $\rho = 0{,}0176 \frac{\Omega mm^2}{m}$

$\sigma = 57 \cdot 10^6$ S/m

Jó vezetők (fémek) $\sigma \sim 10^7$ S/m; Rossz vezetők (föld) $\sigma \sim 10^{-2}$ S/m; Szigetelők $\sigma \sim 10^{-10}$ S/m ... 10^{-17} S/m

Vektoriálisan:



$$U = \int_l \vec{E} \cdot \vec{dl}$$

$$I = \int_A \vec{J} \cdot \vec{dA}$$

a.) Ohm-törvény alakja $(U = RI)$

$U = RI$ infinitezimális

$dU = dR\, dI$

$$E \cancel{dl} = \rho \frac{\cancel{dl}}{\cancel{dA}} J \cancel{dA}$$

$$\boxed{\vec{E} = \rho \vec{J}} \qquad \boxed{\vec{J} = \sigma \vec{E}}$$

Diff. alak.

$$\vec{J} = \sigma(\vec{E} + \vec{E}_b)$$

b.) Joule-törvény alakja $(P = I^2 R)$

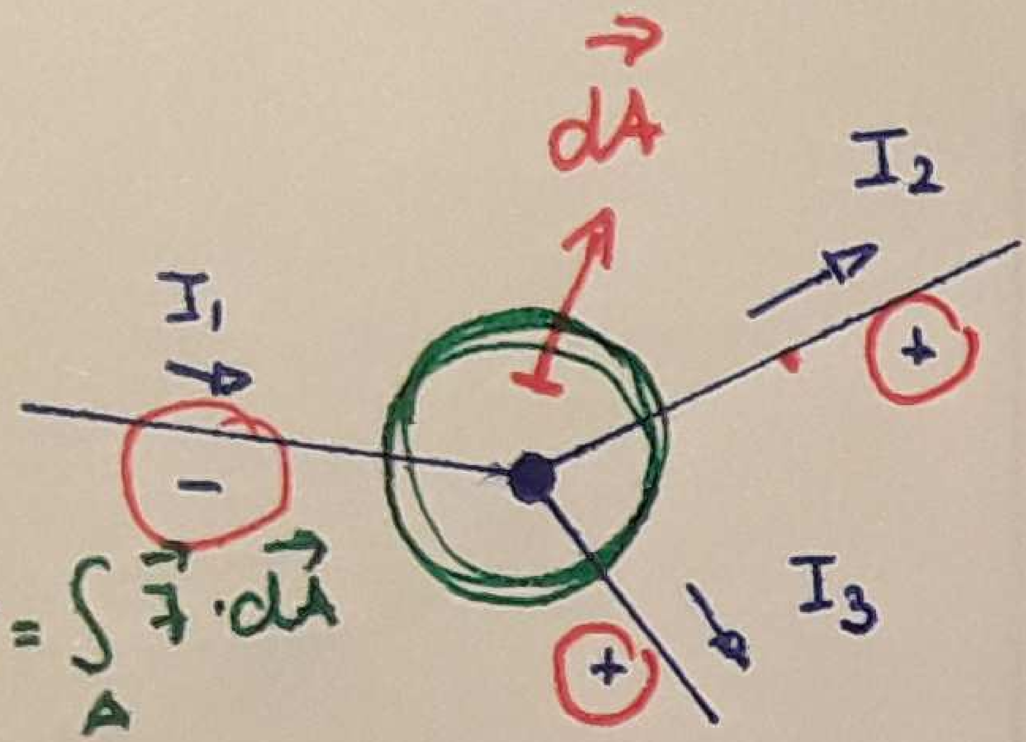
$$dP = \rho \frac{dl}{dA} (J\, dA)^2 = \rho \underbrace{dl\, J^2 dA}_{dV} = \rho J^2 dV$$

$$\frac{J^2}{\sigma} dV$$

$$p = \rho J^2 = \frac{J^2}{\sigma} \quad \left(\frac{W}{m^3}\right)$$

$$P = \int_V p\, dV$$

Kirchhoff csomóponti törvénye



$$I = \int_A \vec{J} \cdot d\vec{A}$$

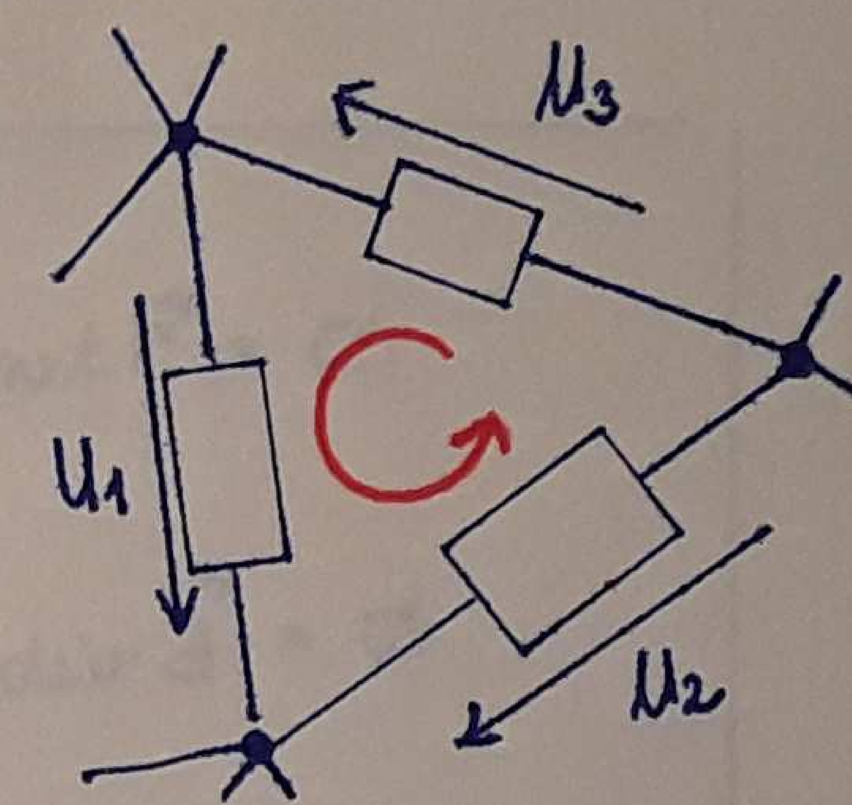
$$-I_1 + I_2 + I_3 = \emptyset$$

$$\boxed{\oint_A \vec{J} \cdot d\vec{A} = \emptyset}$$

$$\sum_k I_k = \emptyset$$

$$\boxed{\operatorname{div} \vec{J} = \emptyset}$$

Kirchhoff huroktörvénye



$$U = \int_l \vec{E} \cdot d\vec{l}$$

$$U_1 - U_2 + U_3 = \emptyset$$

$$\boxed{\oint_l \vec{E} \cdot d\vec{l} = \emptyset}$$

$$\sum_k U_k = \emptyset$$

$$\boxed{\operatorname{rot} \vec{E} = \emptyset}$$

ÖSSZEFOGLALÁS

$$\oint_l \vec{E}\cdot d\vec{l} = 0 \qquad \text{rot}\,\vec{E} = 0$$

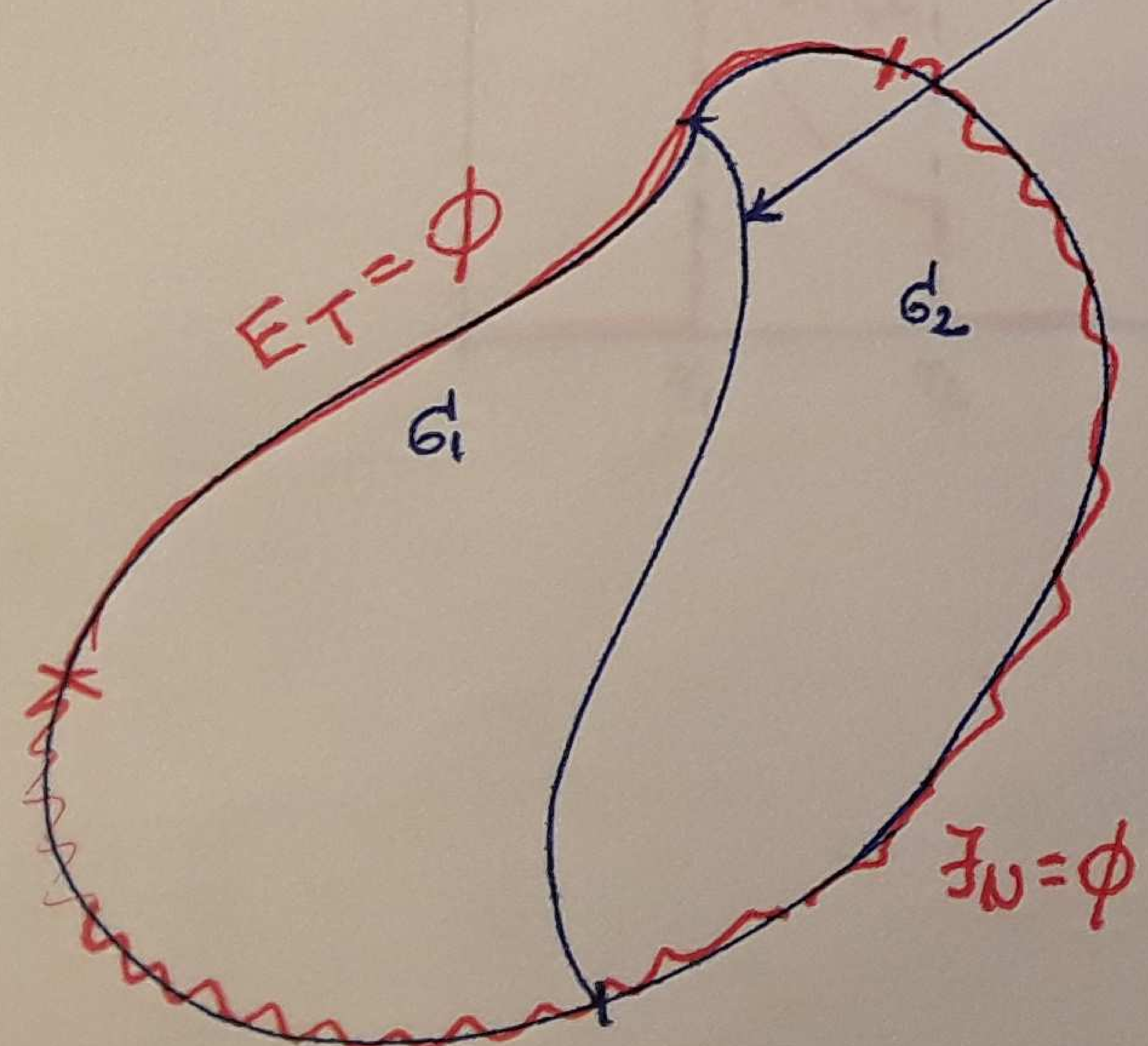
$$\oint_A \vec{J}\cdot d\vec{A} = 0 \qquad \text{div}\,\vec{J} = 0$$

$$\vec{J} = \sigma\vec{E}\,; \quad \vec{J} = \sigma(\vec{E} + \vec{E}_b)$$

Határfeltételek.

E_T folytonos

J_N folytonos



peremfeltétel.

Analógia

$\text{rot}\,\vec{E} = 0$

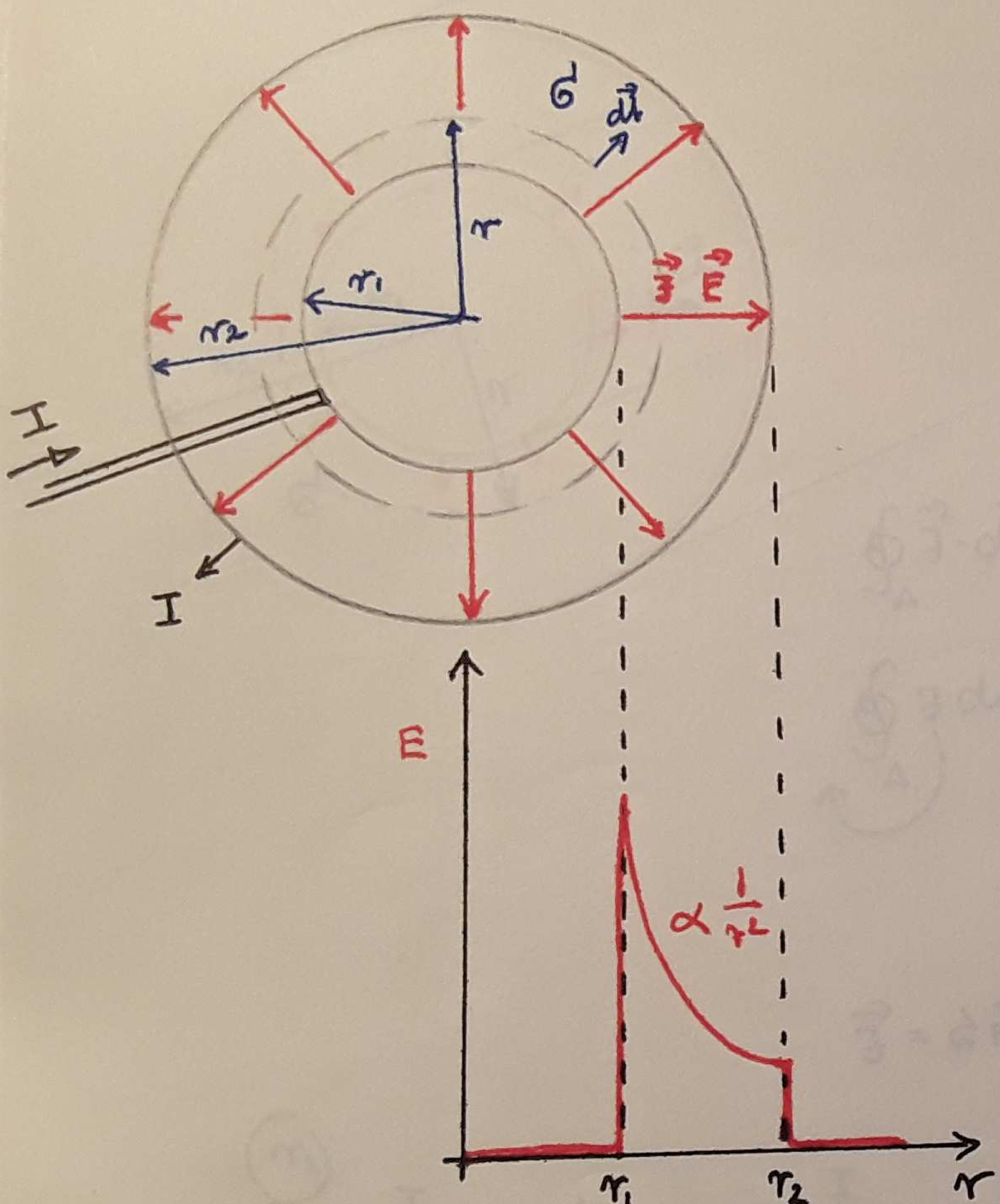
$\text{div}\,\vec{D} = \rho$

$\vec{D} = \varepsilon\vec{E}$

$E_T \quad D_N$

Korábban levezettük!

Vizsgáljuk meg a gömbkondenzátor átvezetését!



$$I \longrightarrow \vec{J} \longrightarrow \vec{E} \longrightarrow U \longrightarrow G$$

$$\vec{J} = \sigma \vec{E}$$

$$\oint_A \vec{J} \cdot d\vec{A} = I$$

$$\oint_A J\, dA = I$$

$$J \oint_A dA = I \longrightarrow J\, 4\pi r^2 = I \longrightarrow \boxed{J = \frac{I}{4\pi r^2}}\ !$$

$$\boxed{E = \frac{I}{4\pi\sigma r^2}}$$

$$U = \int_{r_1\,(r)}^{r_2\,(r_0)} \frac{I}{4\pi\sigma r^2}\, dr = \frac{I}{4\pi\sigma}\left[-\frac{1}{r}\right]_{r_1}^{r_2} = \frac{I}{4\pi\sigma}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

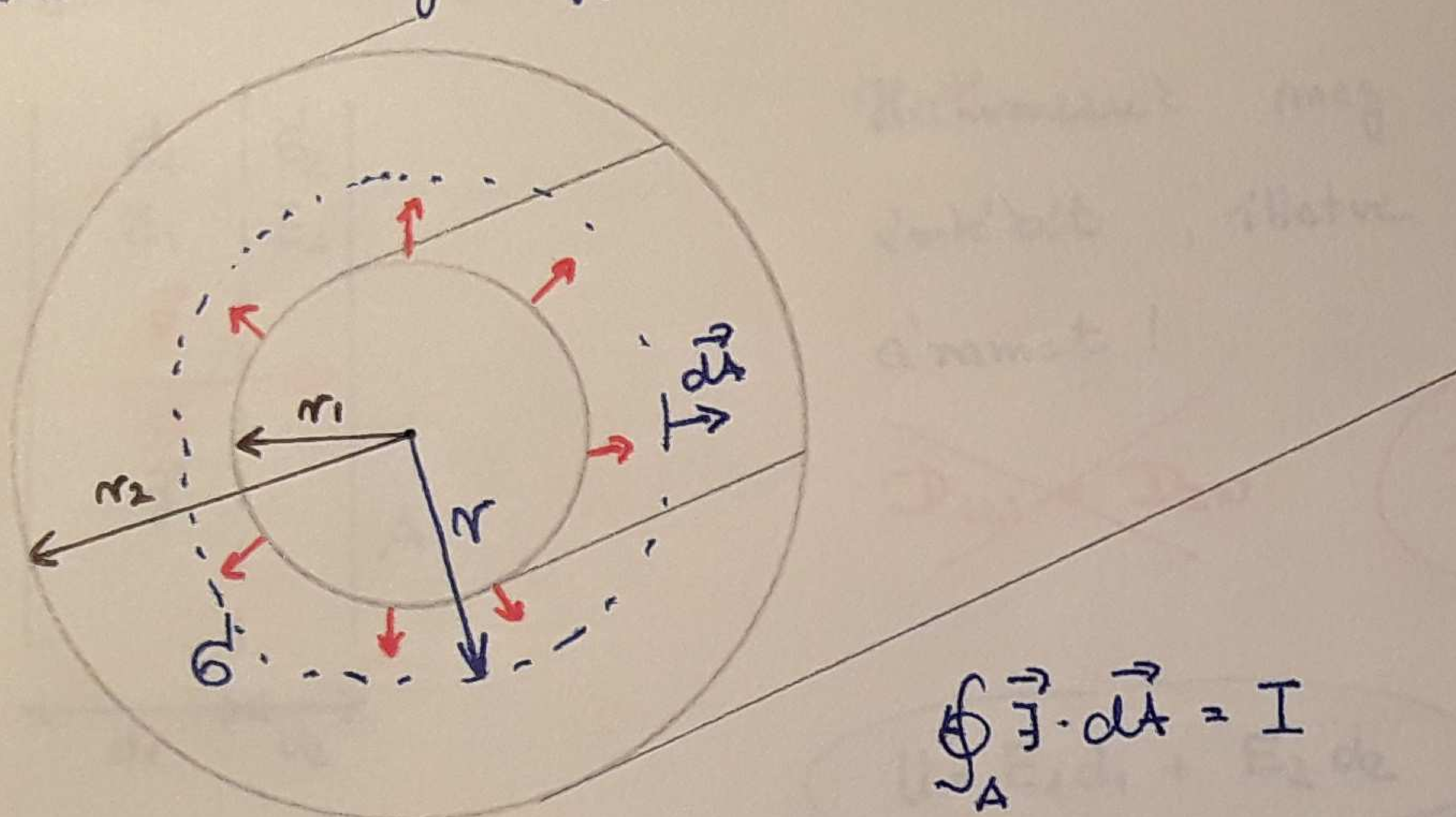
$$\boxed{\varphi(r) = \frac{I}{4\pi\sigma r}}\ !$$

$$G = \frac{I}{U} = \frac{4\pi\sigma}{\frac{1}{r_1} - \frac{1}{r_2}}$$

$$R = \frac{U}{I} = \frac{1}{4\pi\sigma}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Határozzuk meg egy l hosszúságú koaxiális kábel szivárgási ellenállását!

$$I \longrightarrow \vec{J} \longrightarrow \vec{E} \longrightarrow U \longrightarrow R$$



$$\oint_A \vec{J} \cdot \vec{dA} = I$$

$$\oint_A J \, dA = I \qquad J \oint_A dA = I$$

$$J \underbrace{2r\pi l} = I \longrightarrow \boxed{J = \frac{I}{2r\pi l}}$$

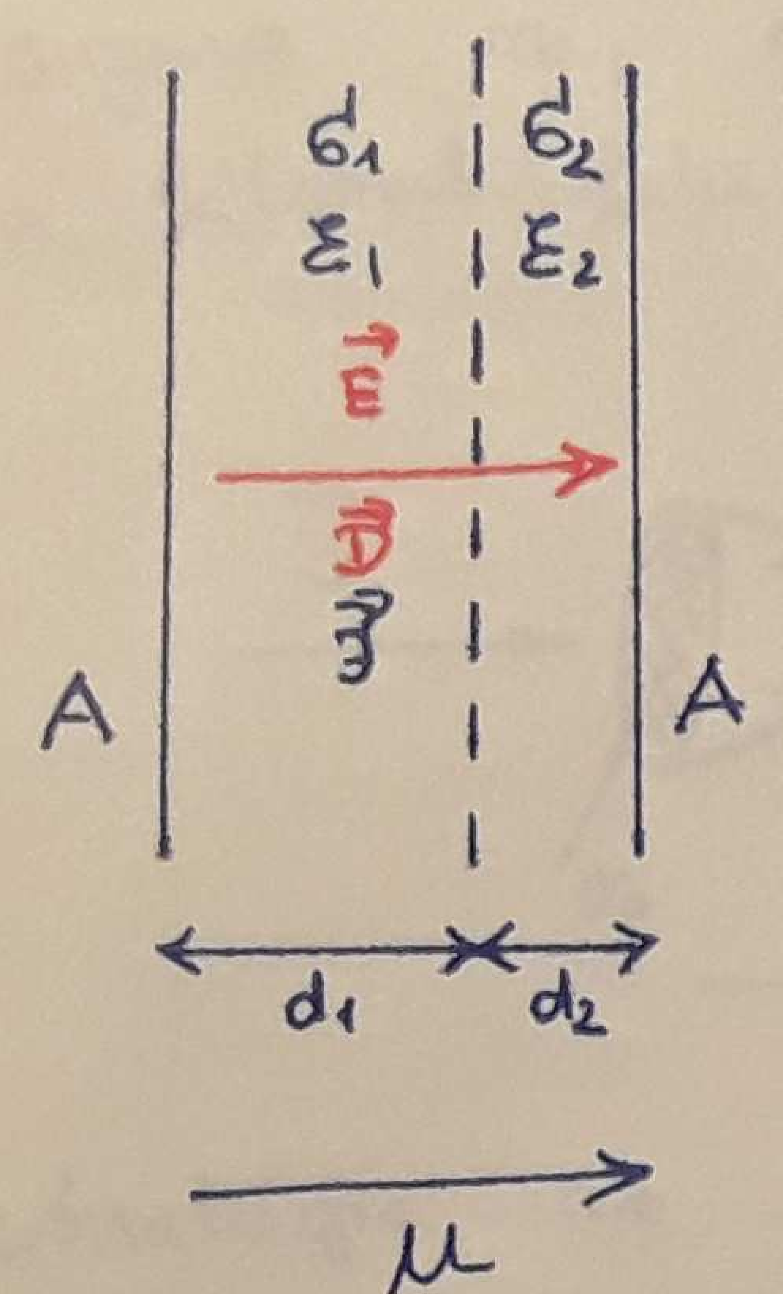
$$\vec{J} = \sigma \vec{E} \longrightarrow \boxed{E = \frac{I}{2r\pi\sigma l}}$$

$$U = \int_{r_1}^{r_2} \frac{I}{2r\pi\sigma l} \, dr = \frac{I}{2\pi\sigma l} \left[\ln r\right]_{r_1}^{r_2} = \frac{I}{2\pi\sigma l} \ln \frac{r_2}{r_1}$$

$$\boxed{\varphi = \frac{I}{2\pi\sigma l} \ln \frac{1}{r}}$$

$$\boxed{R = \frac{U}{I} = \frac{1}{2\pi\sigma l} \ln \frac{r_2}{r_1}}$$

Vizsgáljuk meg az alábbi - nem ideális szigetelővel ellátott - kondenzátort!



Határozzuk meg az egyes rétegekben az elektromos térerősség értékét, illetve a kondenzátoron átfolyó ún. szivárgási áramot!

$\cancel{D_{1N} = D_{2N}}$

$$D_{2N} = D_{1N} + \sigma$$

$$\boxed{J_{1N} = J_{2N}}$$

$$\sigma_1 E_1 = \sigma_2 E_2$$

$$U = E_1 d_1 + E_2 d_2$$

$$E_1 = \frac{\sigma_2}{\sigma_1} E_2$$

$$U = \frac{\sigma_2}{\sigma_1} E_2 d_1 + E_2 d_2 \longrightarrow E_2 \left(\frac{\sigma_2}{\sigma_1} d_1 + d_2 \right) = U$$

$$E_1 = \frac{\sigma_2}{\sigma_1} E_2 = \boxed{\frac{\sigma_2 U}{\sigma_2 d_1 + \sigma_1 d_2}}$$

$$E_2 = \frac{U}{\frac{\sigma_2}{\sigma_1} d_1 + d_2} = \boxed{\frac{\sigma_1 U}{\sigma_2 d_1 + \sigma_1 d_2}}$$

$$J = \sigma_1 E_1 = \frac{\sigma_1 \sigma_2 U}{\sigma_2 d_1 + \sigma_1 d_2}$$

$$J = \sigma_2 E_2 = \frac{\sigma_1 \sigma_2 U}{\sigma_2 d_1 + \sigma_1 d_2}$$

$$I = \int_A \vec{J} \cdot d\vec{A} = \frac{\sigma_1 \sigma_2 U}{\sigma_2 d_1 + \sigma_1 d_2} A$$

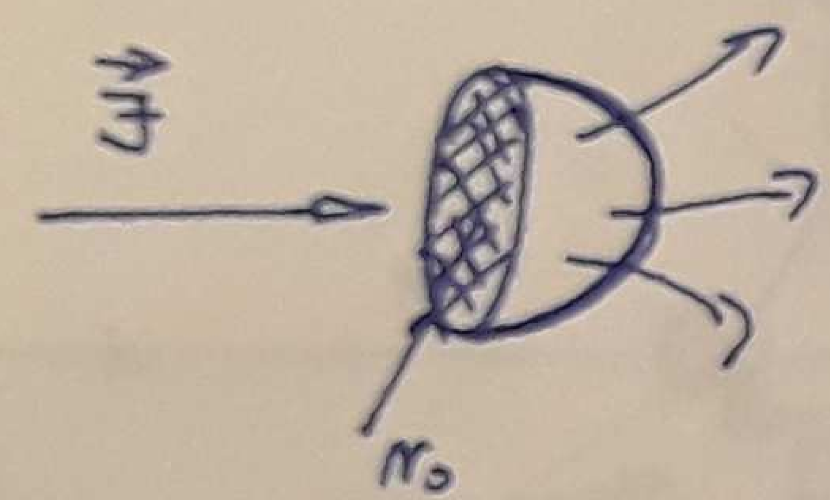
$$\sigma_2 \varepsilon_1 = \sigma_1 \varepsilon_2$$

$$D_{1N} = D_{2N}$$

$$D_1 = \varepsilon_1 E_1 = \frac{\boxed{\sigma_2 \varepsilon_1} U}{\sigma_2 d_1 + \sigma_1 d_2}$$

$$D_2 = \varepsilon_2 E_2 = \frac{\boxed{\varepsilon_2 \sigma_1} U}{\sigma_2 d_1 + \sigma_1 d_2}$$

Homogén áramlási térben az áramsűrűség $\vec{J}$. Tekintsünk benne egy félgömböt, melynek sugara r_0, és a félgömb alapsíkja merőleges az áramlásra. Határozzuk meg a félgömb felületén átáramló áram értékét!



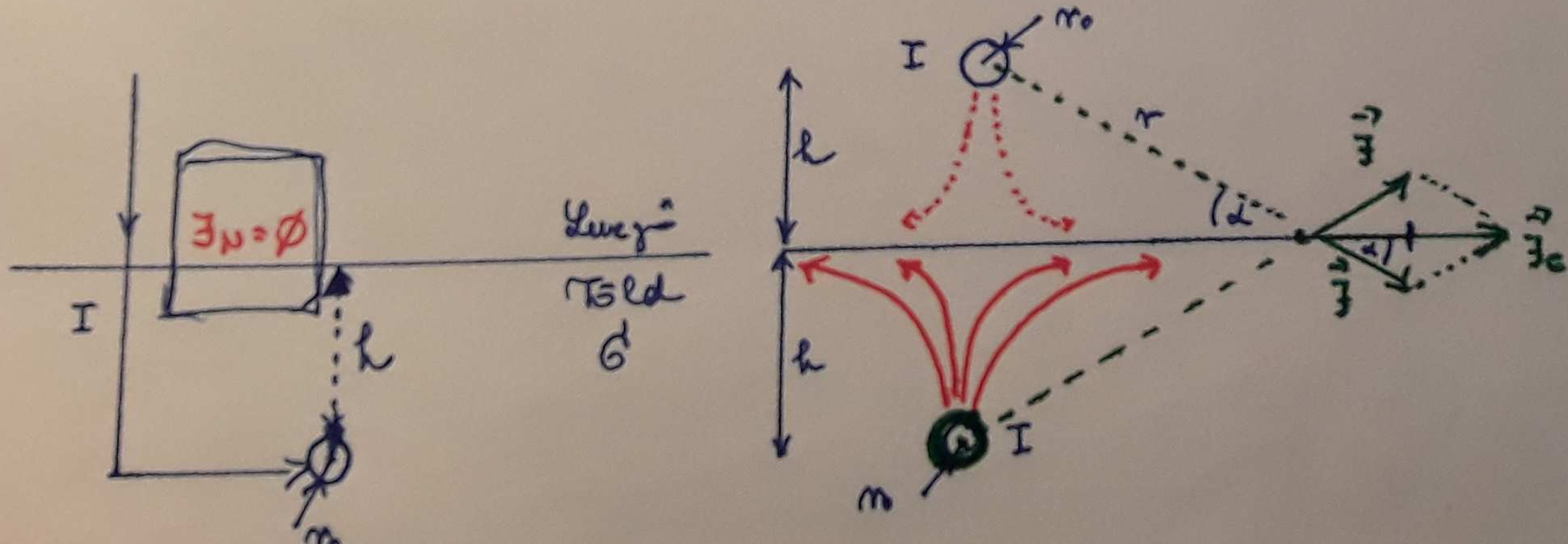
$$I = \int_A \vec{J}\cdot d\vec{A} = J r_0^2 \pi$$

$$\oint_A \vec{J}\cdot d\vec{A} = 0$$

Analógia az elektrosztatikus térrel:

E.:	Q	U	ε	$\vec{D}$	$\vec{E}$	C
St.Á.:	I	U	σ	$\vec{J}$	$\vec{E}$	G

Tükrözés: földelő elektróda tere



$$J = \frac{I}{4\pi r^2}$$

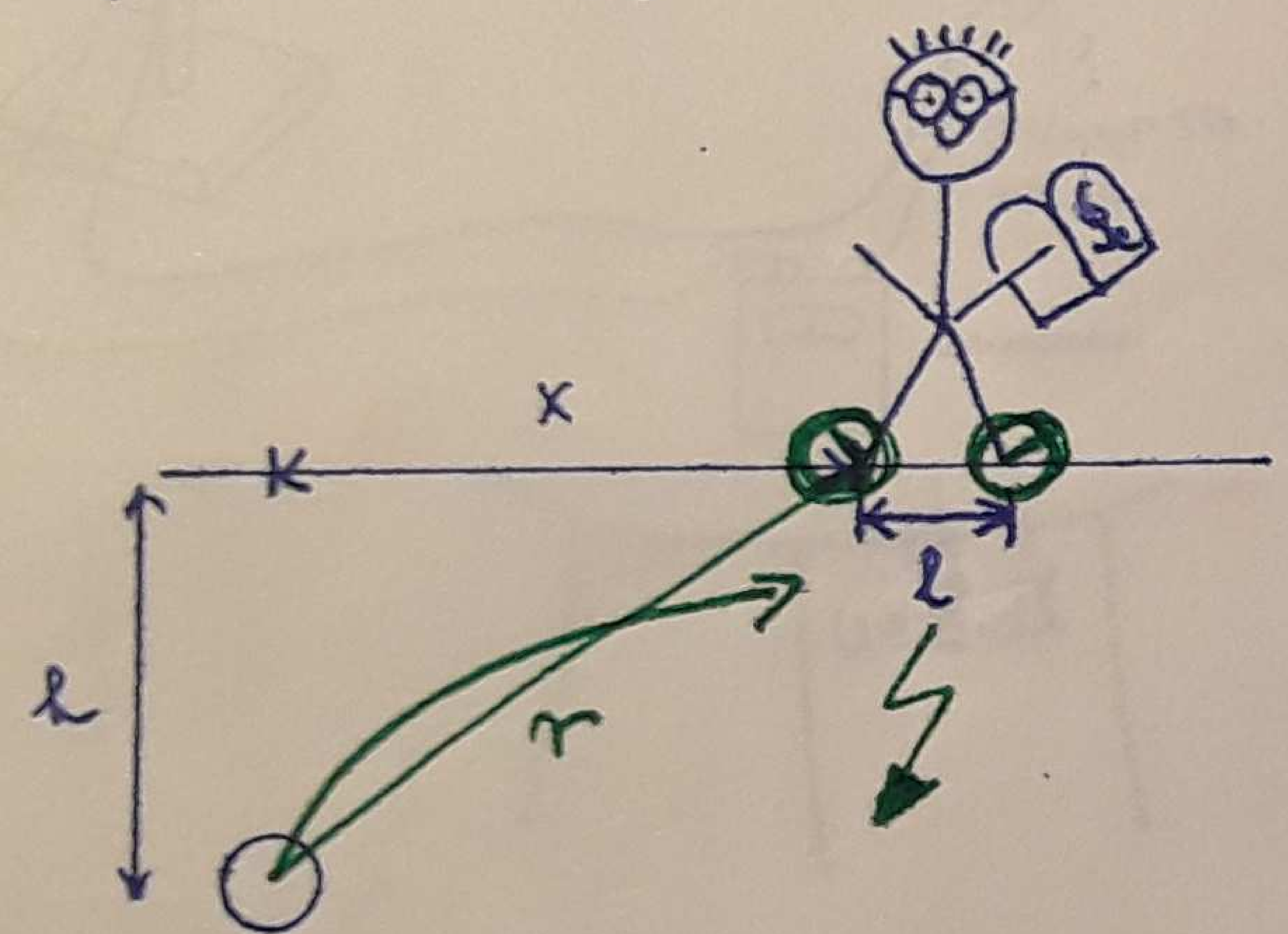
$$J_e = 2J\cos\alpha = \frac{2I}{4\pi r^2}\frac{h}{r}$$

$$\Phi = \frac{I}{4\pi\sigma r_0} + \frac{I}{4\pi\sigma 2h}$$

$$R = \frac{U}{I} = \frac{1}{4\pi\sigma}\left(\frac{1}{r_0} + \frac{1}{2h}\right)$$ – földelési ellenállás

ha $h \to \infty$: $R = \frac{1}{4\pi\sigma r_0}$ szétterjedési ellenállás.

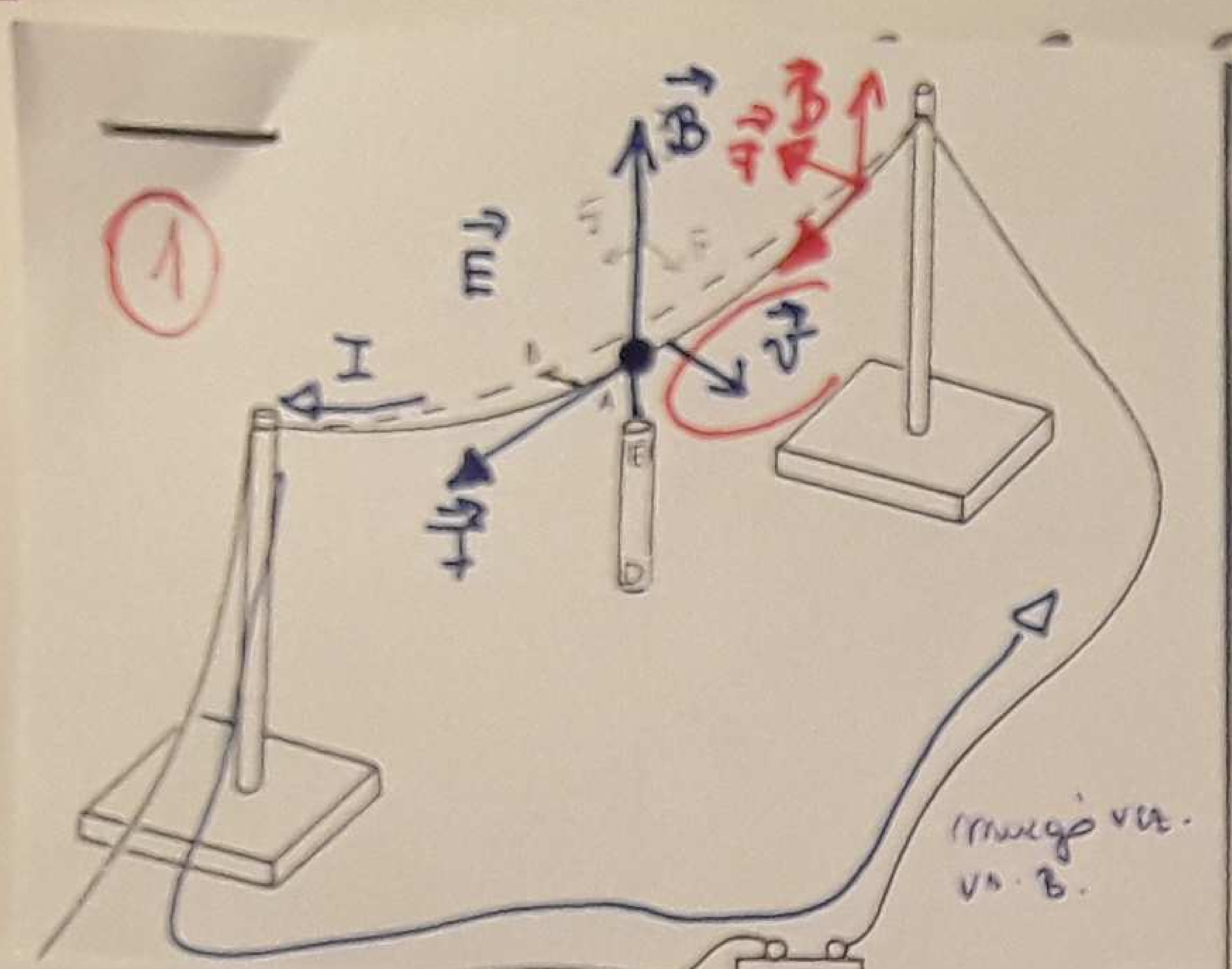
Lépésfeszültség:

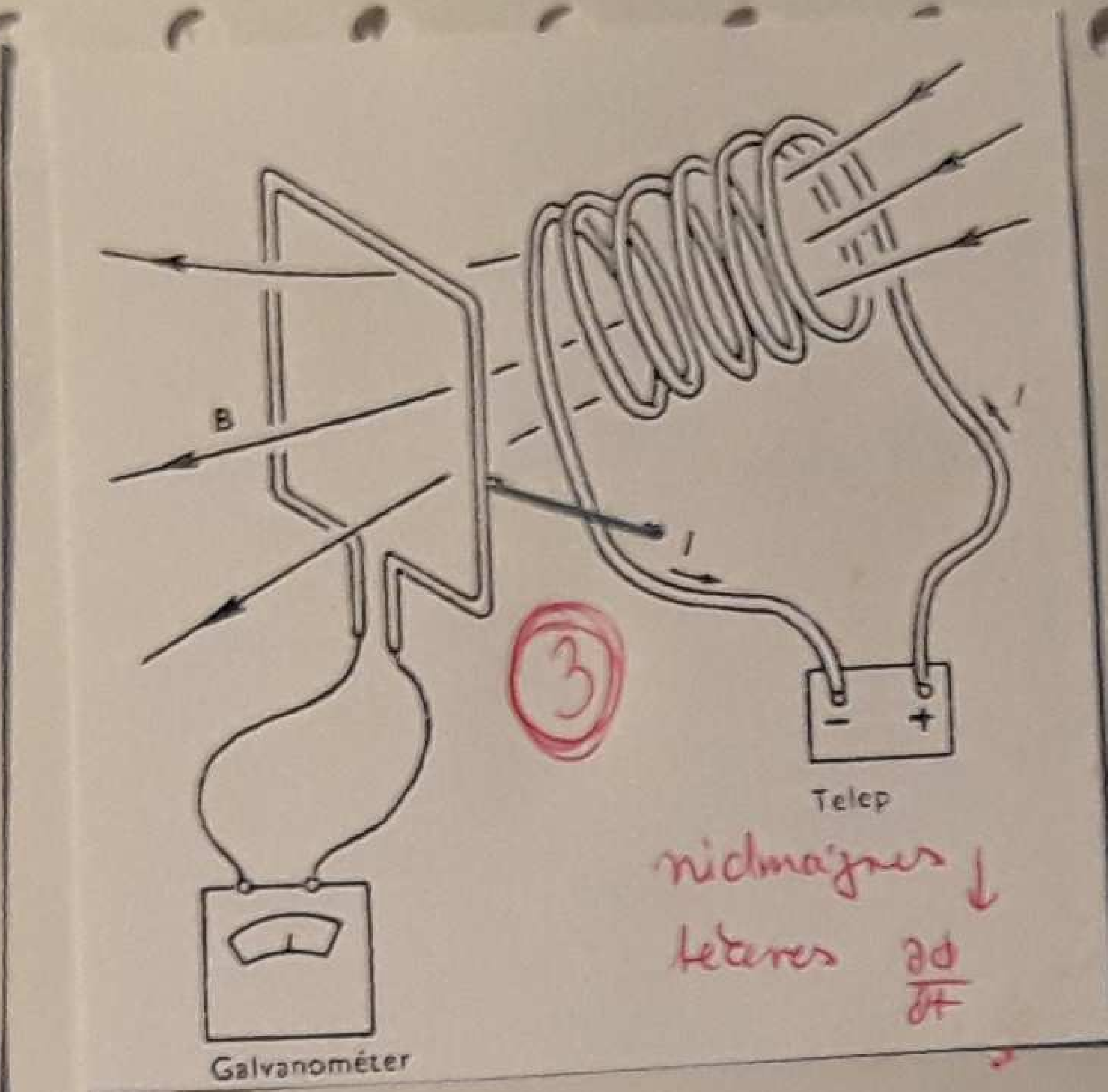


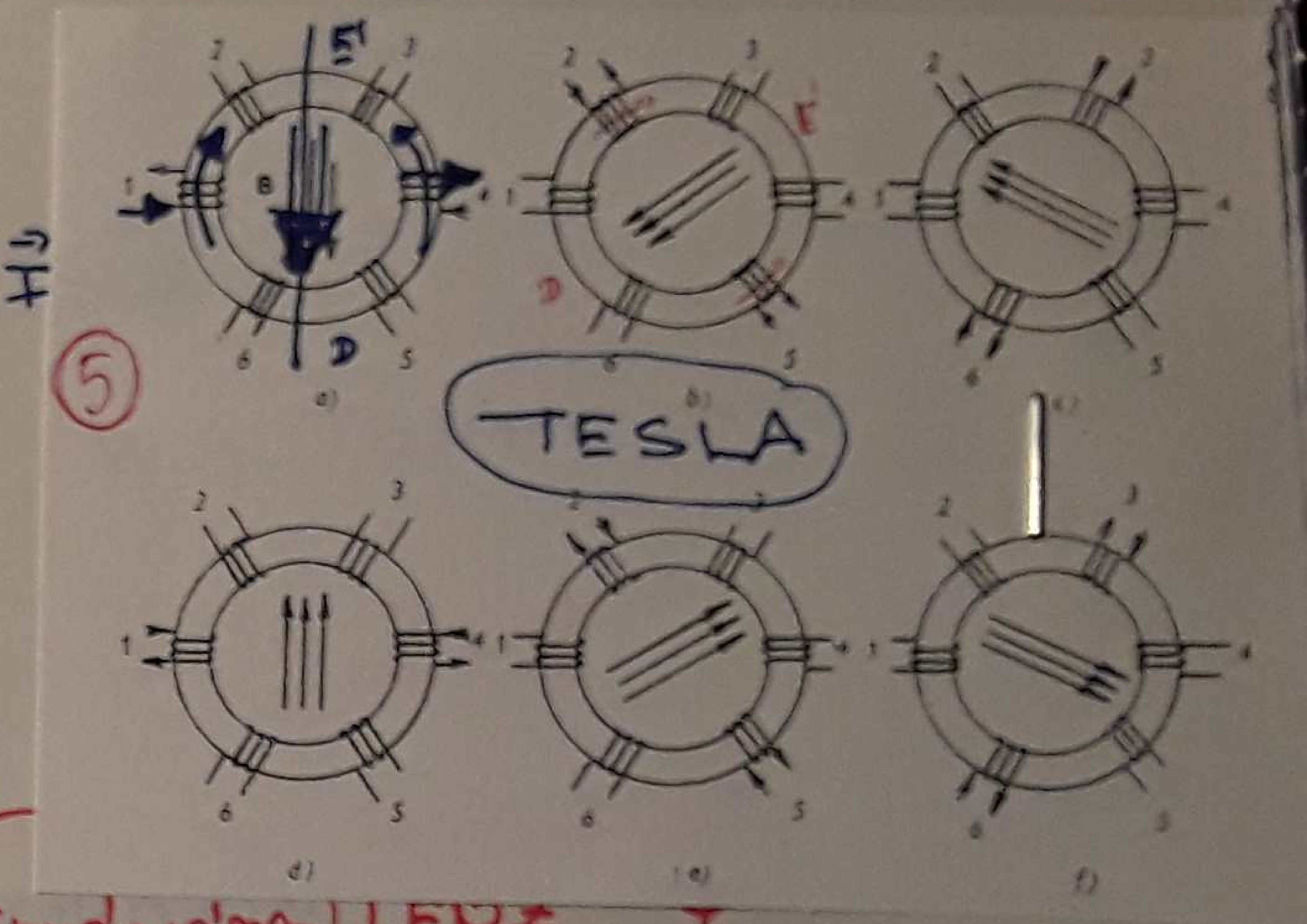
$$\Phi(x) = \boxed{2} \frac{I}{4\pi\sigma} \frac{1}{r} = 2 \frac{I}{4\pi\sigma} \frac{1}{\sqrt{h^2+x^2}}$$

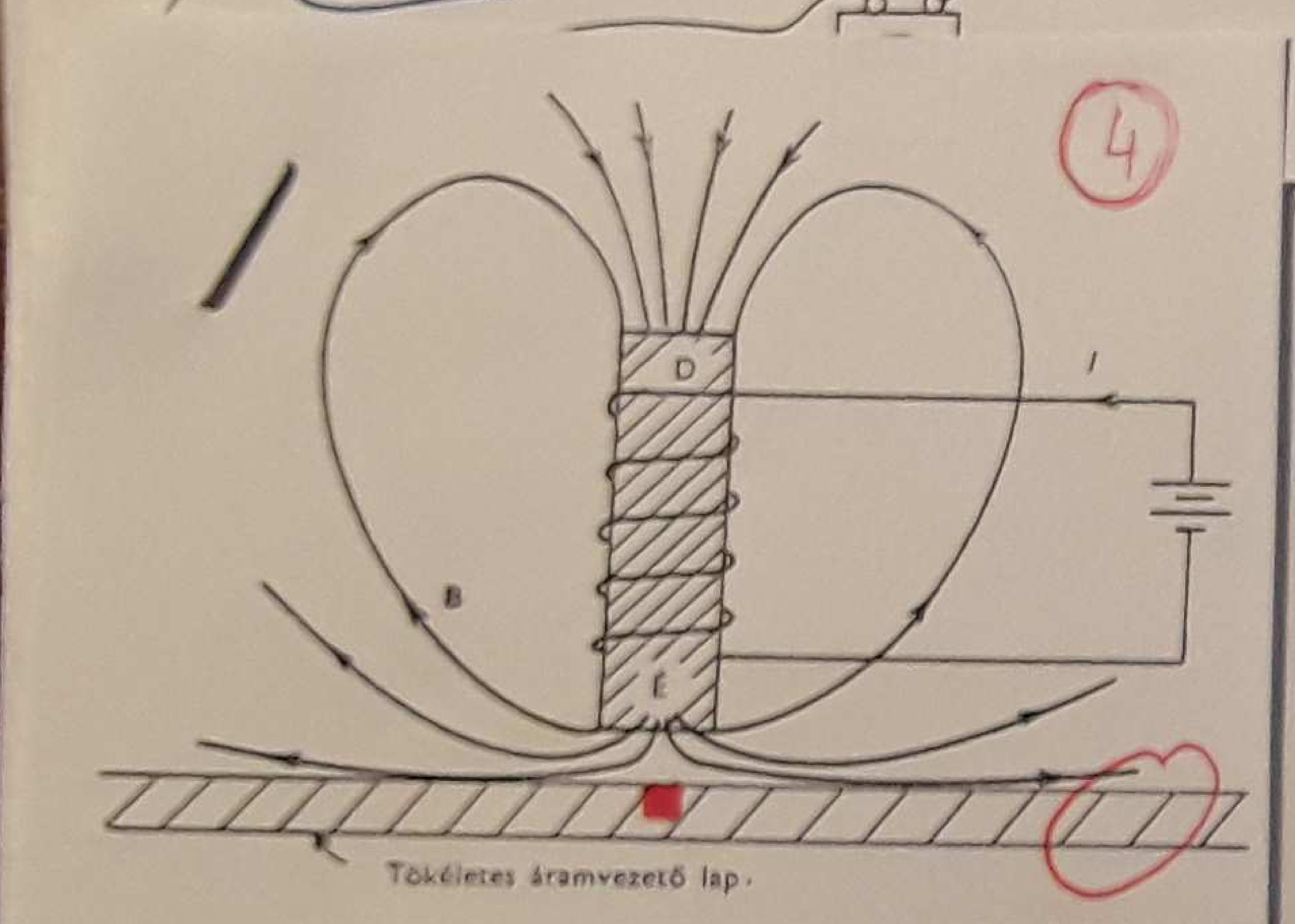
$$U_l = \frac{2I}{4\pi\sigma} \left(\frac{1}{\sqrt{h^2+x^2}} - \frac{1}{\sqrt{h^2+(x+l)^2}} \right)$$

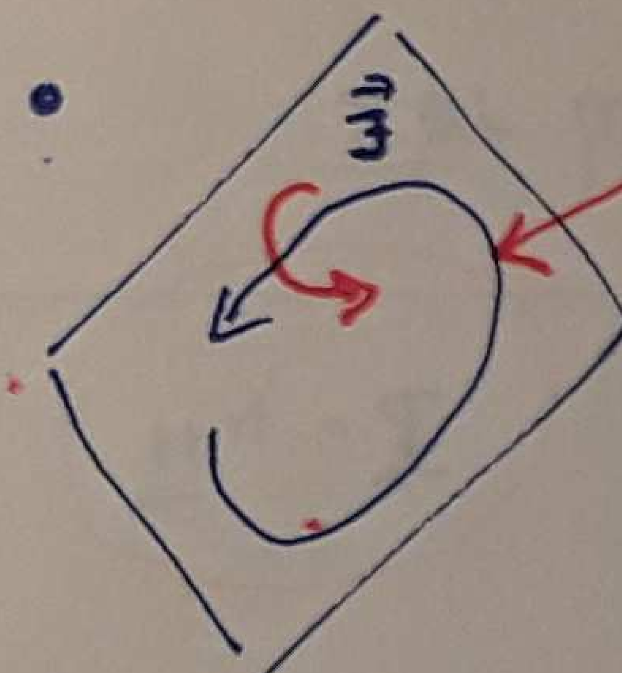
$$\boxed{l \approx 80\,\text{cm}}$$









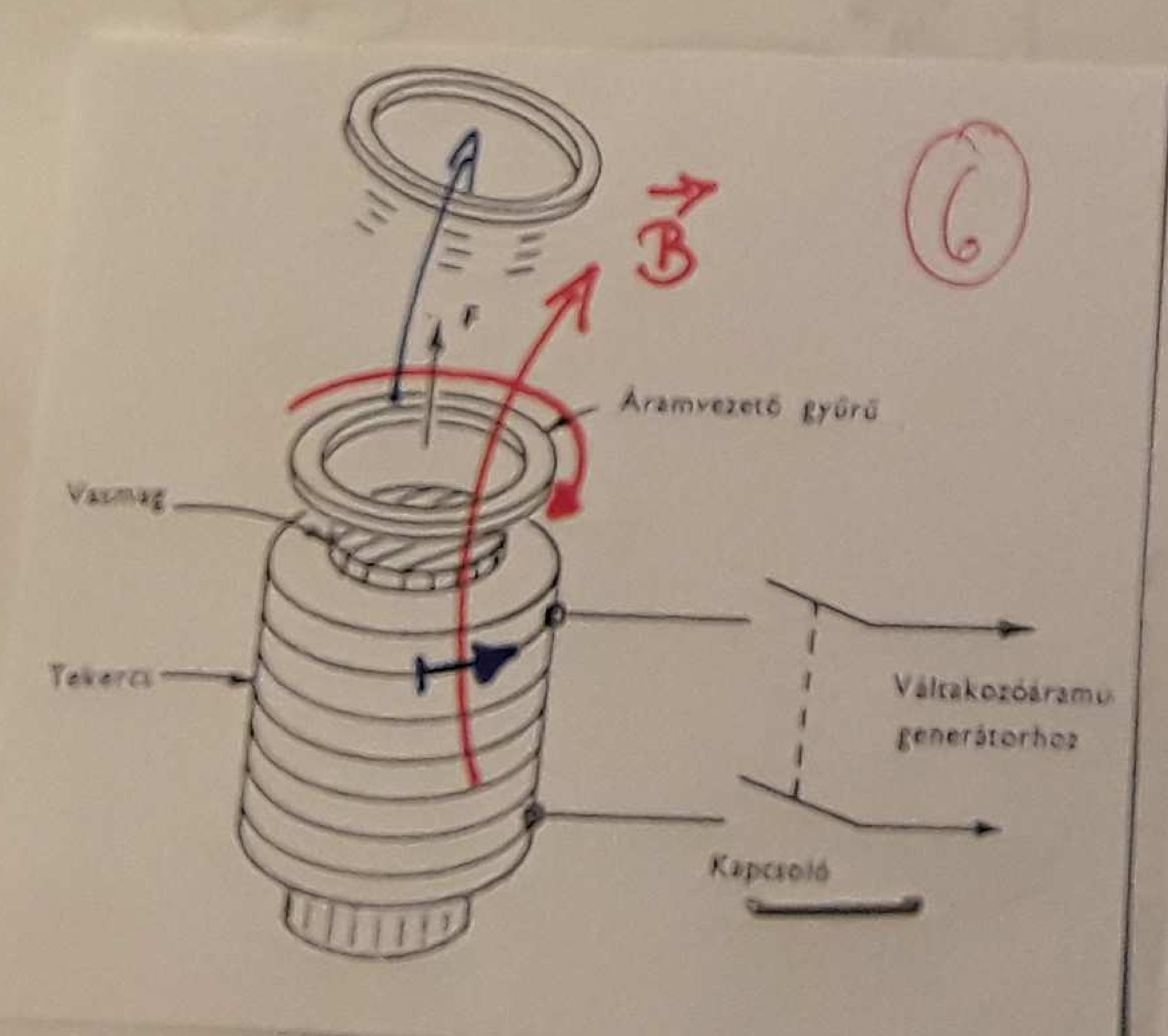


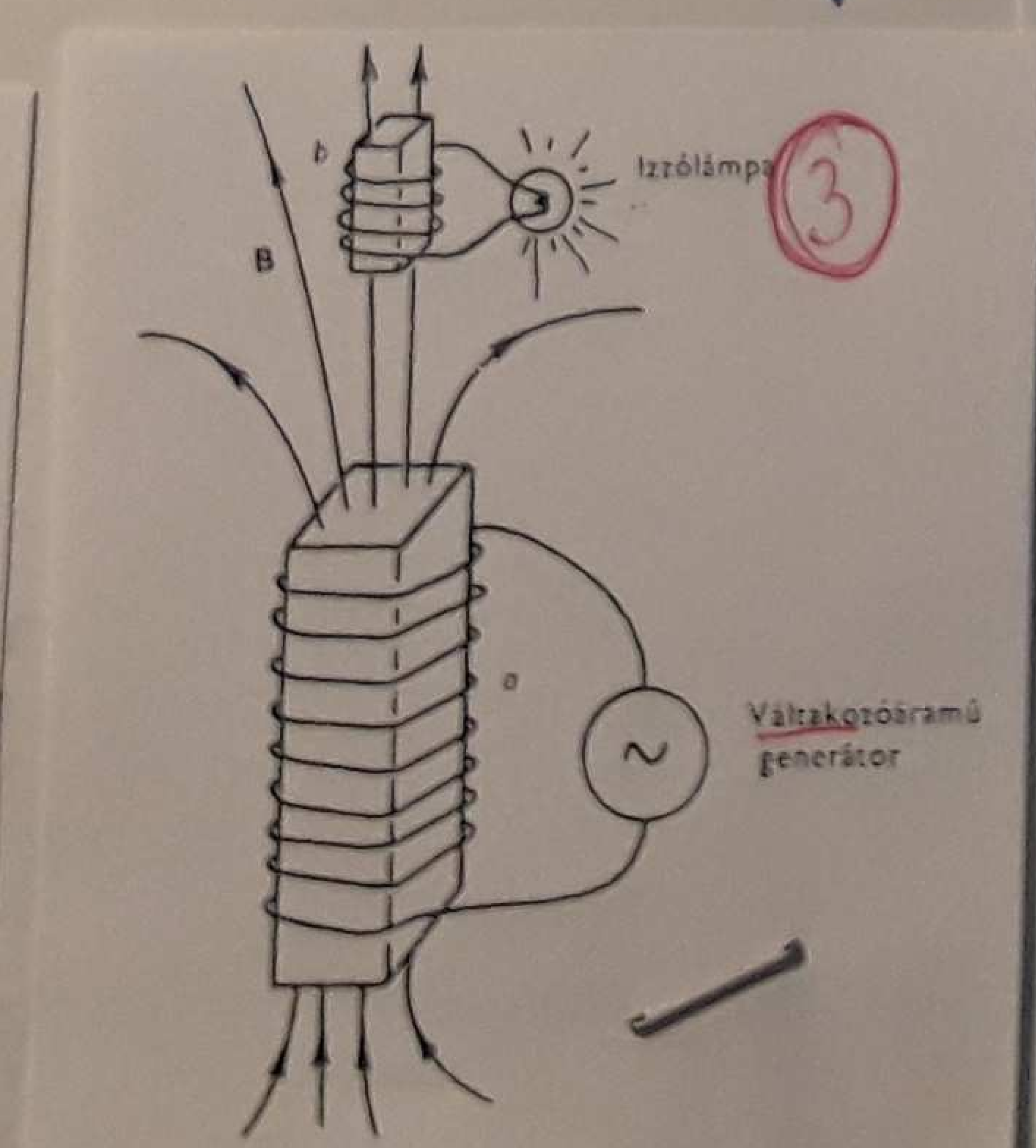
örvényáram LENZ. Lenz

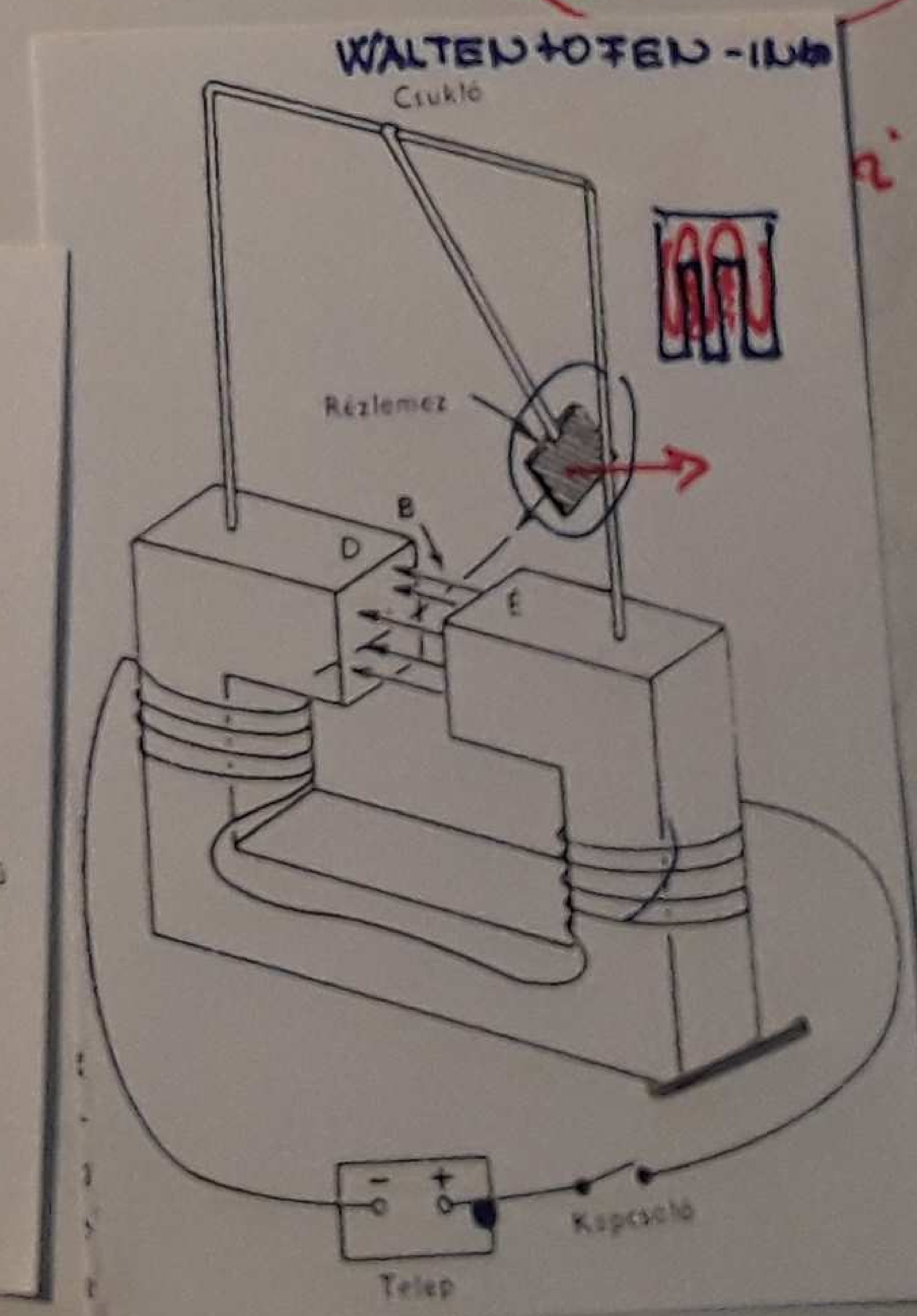
JARADAT

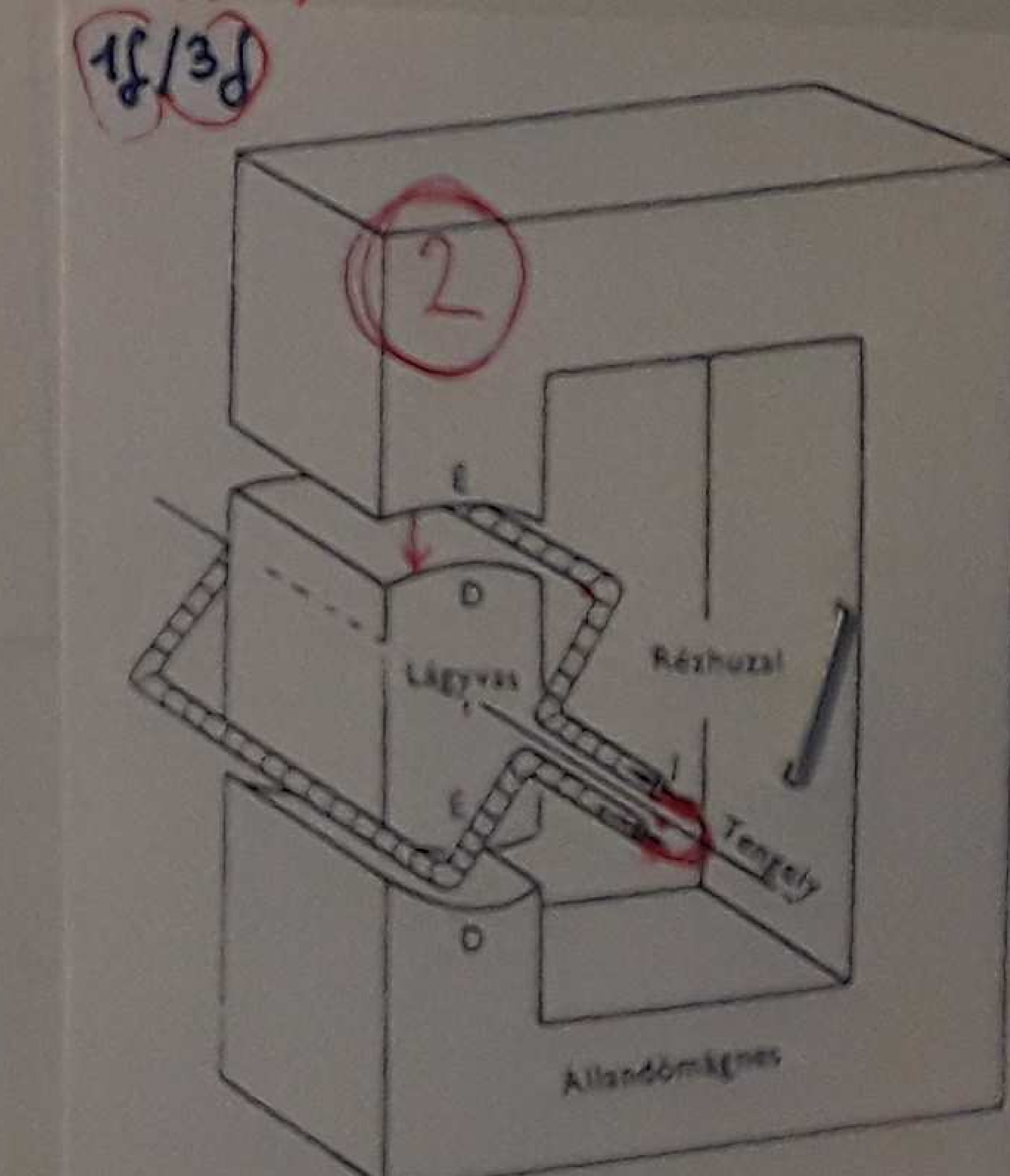
mozgás → $i(t)$

$i \propto \frac{d\Phi}{\dots}$



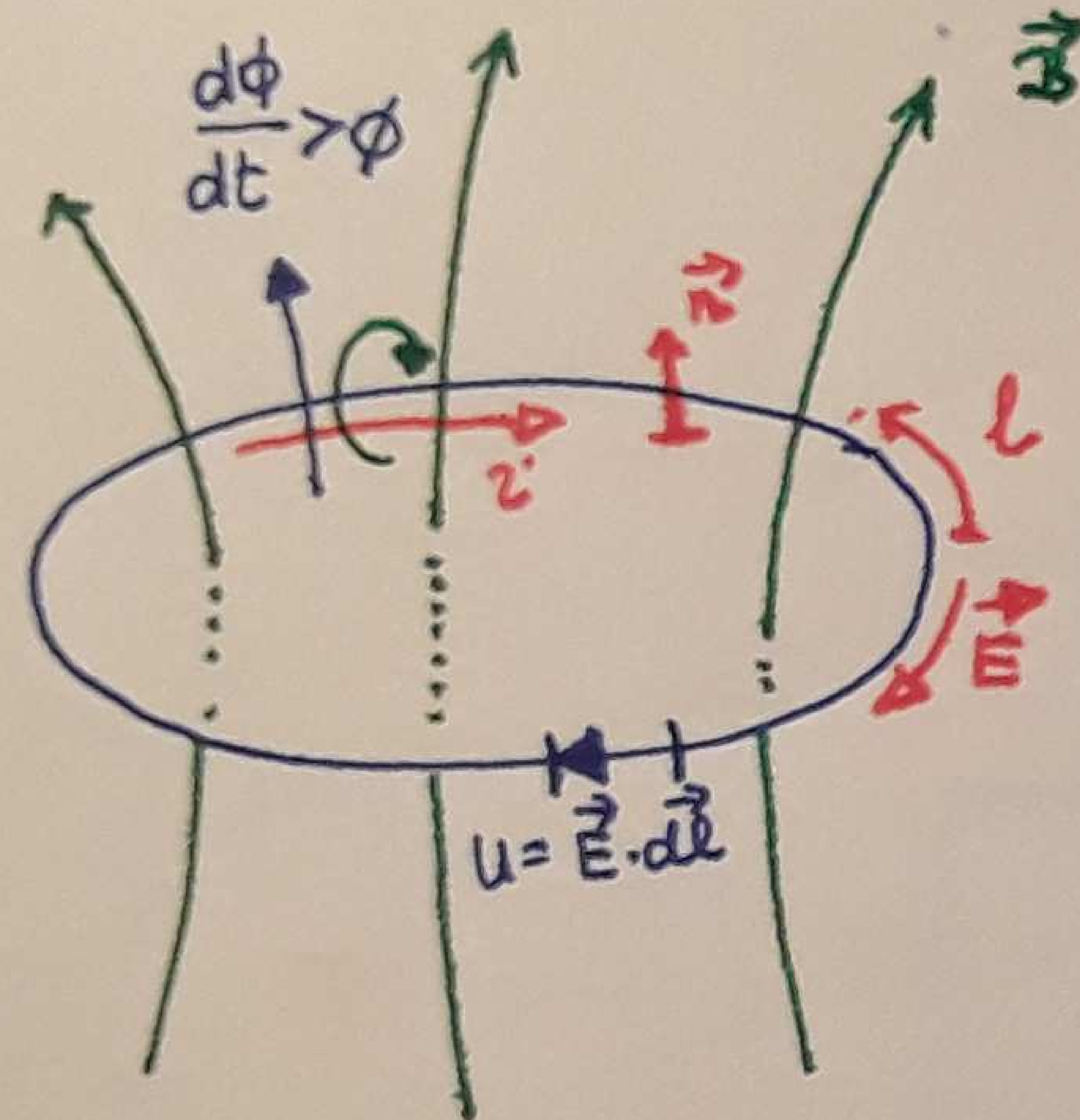






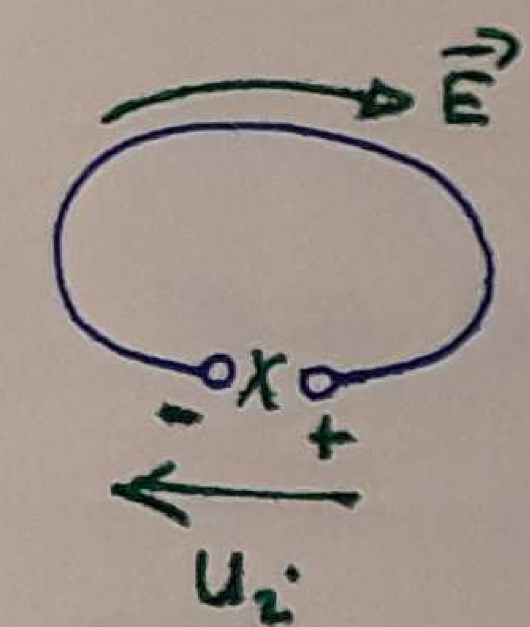
AZ ELEKTROMOS ÉS A MÁGNESES TÉR KAPCSOLATA II.

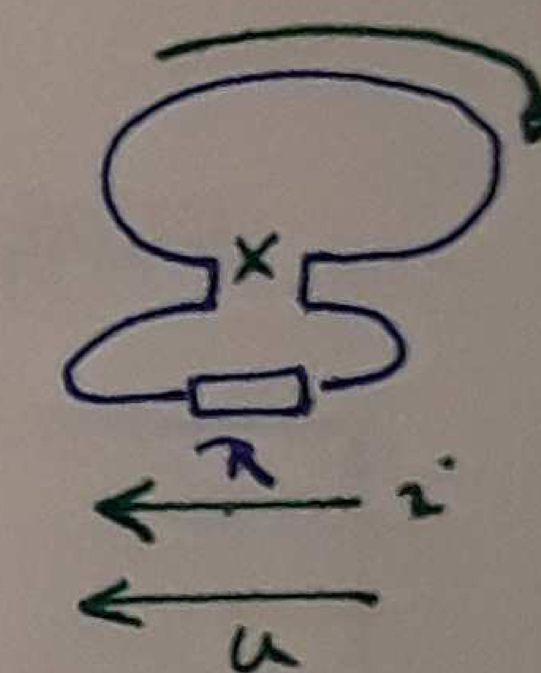
Faraday-féle indukciós törvény

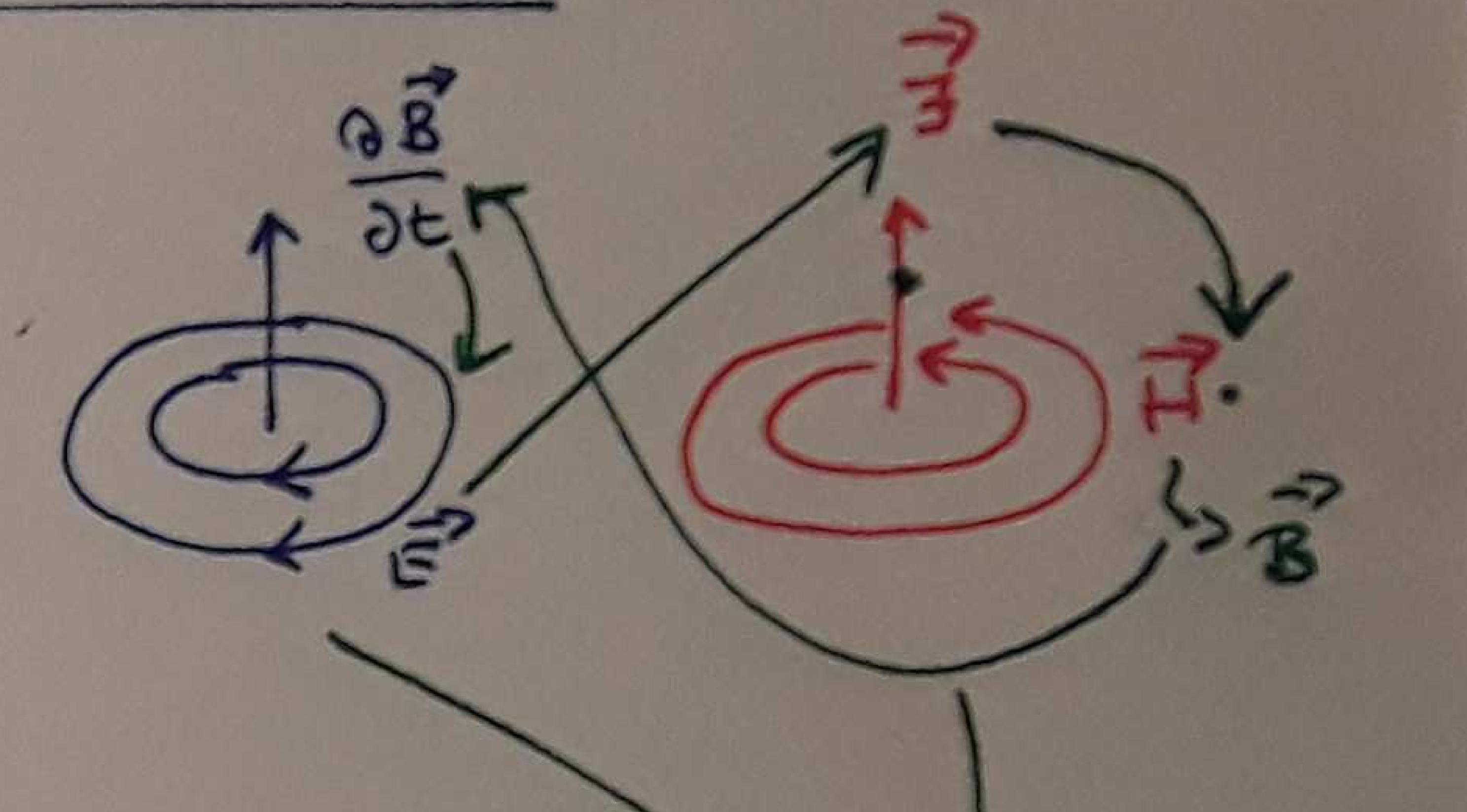


$$\Phi = \int_A \vec{B} \cdot \vec{dA}$$

$$U_i = \oint_l \vec{E} \cdot \vec{dl}$$







$$u_i = -\frac{d\Phi}{dt}$$

$\frac{d\Phi}{dt}$ NYUGALMI

A MOZGÁSI

$$\oint_l \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int_A \vec{B} \cdot \vec{dA}$$

$$\boxed{\oint_l \vec{E} \cdot \vec{dl} = -\int_A \frac{\partial \vec{B}}{\partial t} \cdot \vec{dA}}$$

∂

$$\underbrace{\lim_{\Delta A \to \emptyset} \frac{1}{\Delta A} \oint_l \vec{E} \cdot \vec{dl}}_{\mathrm{rot}\,\vec{E}} = -\frac{\partial \vec{B}}{\partial t}$$

$$\boxed{\mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

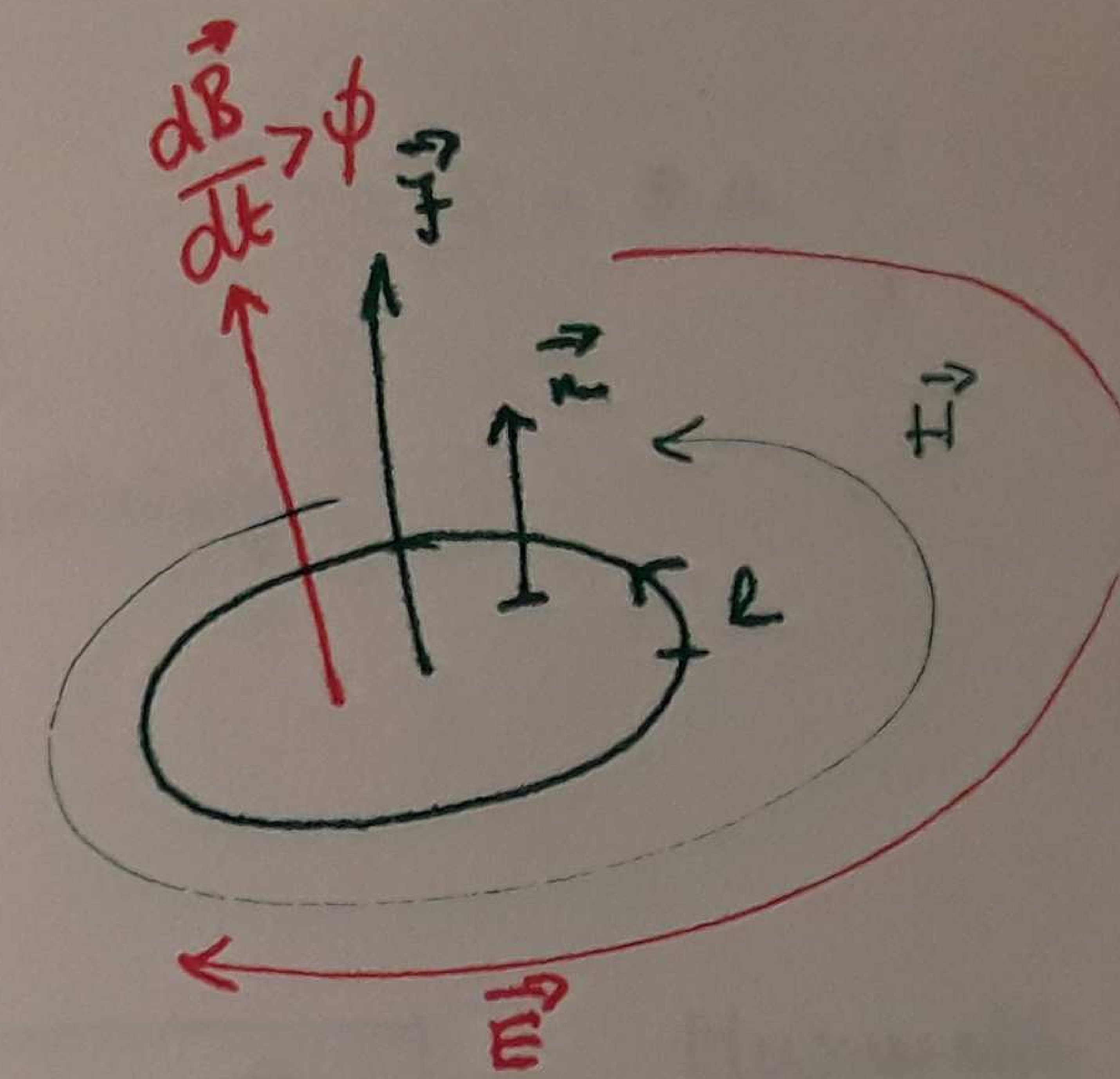
ÖSSZEFOGLALÁS :

$$\oint_l \vec{H} \cdot d\vec{l} = \int_A \vec{J} \cdot d\vec{A} \qquad \operatorname{rot} \vec{H} = \vec{J}$$

$$\oint_l \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_A \vec{B} \cdot d\vec{A} \qquad \operatorname{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_A \vec{B} \cdot d\vec{A} = 0 \qquad \operatorname{div} \vec{B} = 0$$

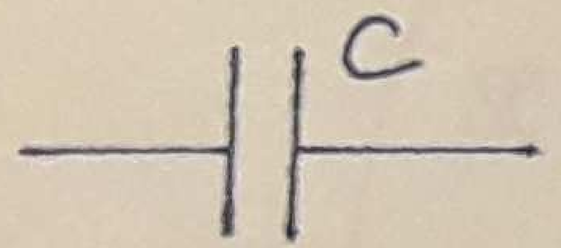
$$\vec{B} = \mu \vec{H} , \qquad \vec{J} = \sigma \vec{E}$$



ÖRVÉNYÁRAMÚ TÉR.

Az eltolási áramsűrűség

A.C. $\qquad Q = CU \qquad C = \varepsilon \frac{A}{d} \qquad \vec{D} = \varepsilon \vec{E}$



$$i = \frac{\Delta Q}{\Delta t} = \frac{C \Delta U}{\Delta t} = \varepsilon \frac{A}{d} \frac{\Delta U}{\Delta t} = \varepsilon \frac{A \Delta E}{\Delta t} = \underbrace{\frac{\Delta D}{\Delta t}}_{\text{áramsűrűség}} A \qquad i = J A$$

ΔE

$$i = \frac{\partial \vec{D}}{\partial t} A \longrightarrow \boxed{i = \int_A \frac{\partial \vec{D}}{\partial t} \cdot d\vec{A}}$$

$$\boxed{\oint_l \vec{H} \cdot d\vec{l} = \int_A \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{A}} \qquad \boxed{\mathrm{rot}\, \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

vez.

Maxwell

Hertz

$$\vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$j\omega \vec{D}$

$2\pi f$

$$\underbrace{\mathrm{div\,rot}\, \vec{H}}_{\equiv 0} = \mathrm{div}\, \vec{J} + \frac{\partial}{\partial t} \mathrm{div}\, \vec{D}$$

$$\boxed{\mathrm{div}\, \vec{J} + \frac{\partial \rho}{\partial t} = 0}$$ folytonossági egyenlet

I. $\oint_l \vec{H} \cdot d\vec{l} = \int_A \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{A}$ $\qquad \mathrm{rot}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

II. $\oint_l \vec{E} \cdot d\vec{l} = -\int_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$ $\qquad \mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

III. $\oint_A \vec{B} \cdot d\vec{A} = \emptyset$ $\qquad \mathrm{div}\,\vec{B} = \emptyset$

IV. $\oint_A \vec{D} \cdot d\vec{A} = \varrho$ $\qquad \mathrm{div}\,\vec{D} = \varrho$

V. $\vec{B} = \mu \vec{H} \qquad \vec{D} = \varepsilon \vec{E} \qquad \vec{J} = \sigma \vec{E}$

Maxwell-egyenletek

Bakelit	$\varepsilon_r \approxeq 5$
Gumi	2 - 3
Papír	3
Olajos papír	4
PVC	3,2
Porcelán	6
Transzformátor olaj	2

Kritikus térerősség

Fajlagos ellenállás

$\mu_r \to$ Diamágnesek $\mu_r < 1$

$\mu_r \to$ Paramágnesek $\mu_r > 1$

$\left.\begin{array}{l} \text{Diamágnesek} \\ \text{Paramágnesek} \end{array}\right\} \mu_r \approx 1$

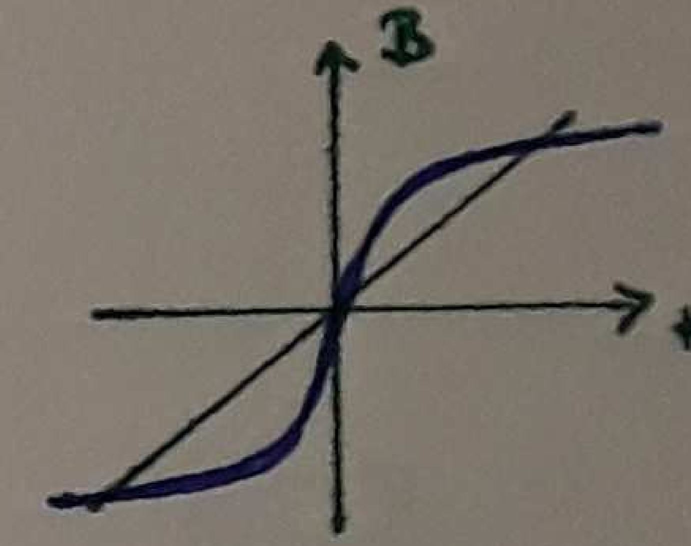
$\mu_r \to$ Ferromágnes $\mu_r \gg 1$

Domének, Barkhausen-jelenség

KÖZEGEK HATÁSA

atomok
molekulák
...
} ⟶ közeg.

KONSTITÚCIÓS RELÁCIÓK : $\vec{D} = \mathcal{D}\{\vec{E}\}$ $\quad \vec{B} = \mathcal{B}\{\vec{H}\}$ $\quad \vec{J} = \mathcal{J}\{\vec{E}\}$



Lin. izotróp : $\vec{D} = \varepsilon \vec{E}$ $\quad D_x = \varepsilon E_x \quad D_y = \varepsilon E_y \quad D_z = \varepsilon E_z$ $\quad \vec{B} = \mu \vec{H}$ $\quad \vec{J} = \sigma \vec{E}$

$\varepsilon_0 \varepsilon_r$ $\quad \mu_0 \mu_r$

Nemlin. izotróp : $\vec{D} = \varepsilon(E)\vec{E}$ $\quad \vec{B} = \mu(H)\vec{H}$ $\quad \vec{J} = \sigma(E)\vec{E}$

Lin. anizotróp

Nemlin. anizotróp

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ $\quad \vec{B} = \mu_0(\vec{H} + \vec{M})$ $\quad \vec{J} = \sigma(\vec{E} + \vec{E}_b)$

khi

pl. $\vec{D} = \varepsilon_0 \underbrace{(1+\chi)}_{\varepsilon_r}\vec{E}$ $\quad \vec{B} = \mu_0 \underbrace{(1+\kappa)}_{\mu_r}\vec{H}$ $\quad$ szuszceptibilitás

$\vec{P} = \varepsilon_0 \chi \vec{E}$ $\quad \vec{M} = \mu_0 \kappa \vec{H}$

permittivitás

dielektromos állandó

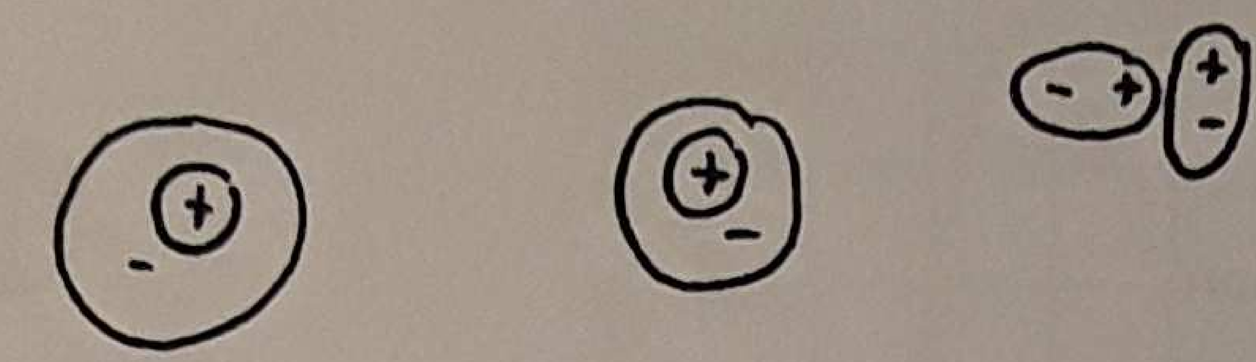
$\varepsilon_0 = 8{,}854 \cdot 10^{-12} \frac{F}{m}$

$\varepsilon_r = 1 \ldots 10$

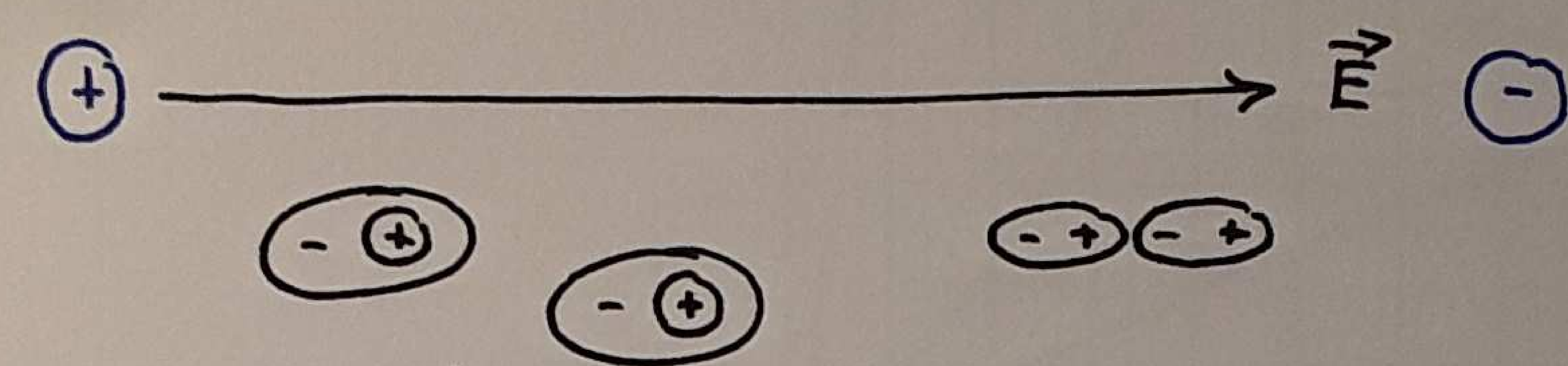
$\mu_0 = 4\pi \cdot 10^{-7} \frac{H}{m}$ $\quad$ permeabilitás

μ_r

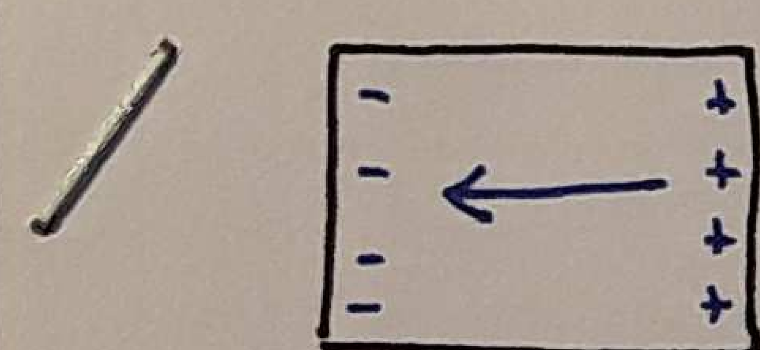
$E = \emptyset$

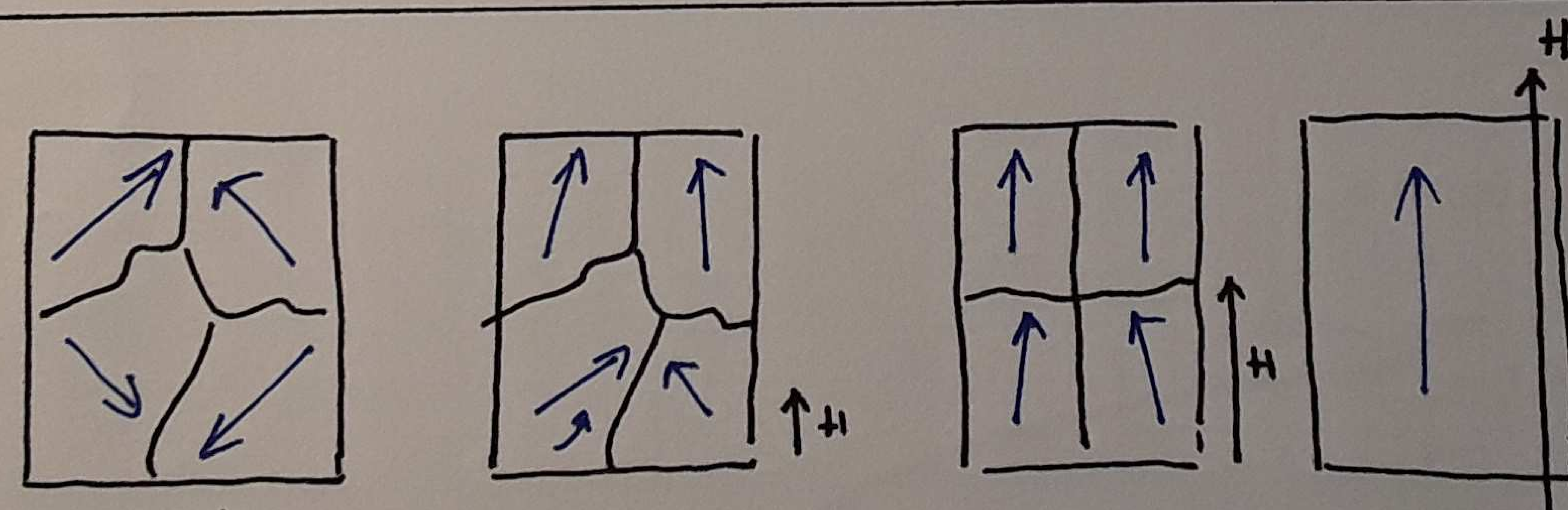


$E > \emptyset$

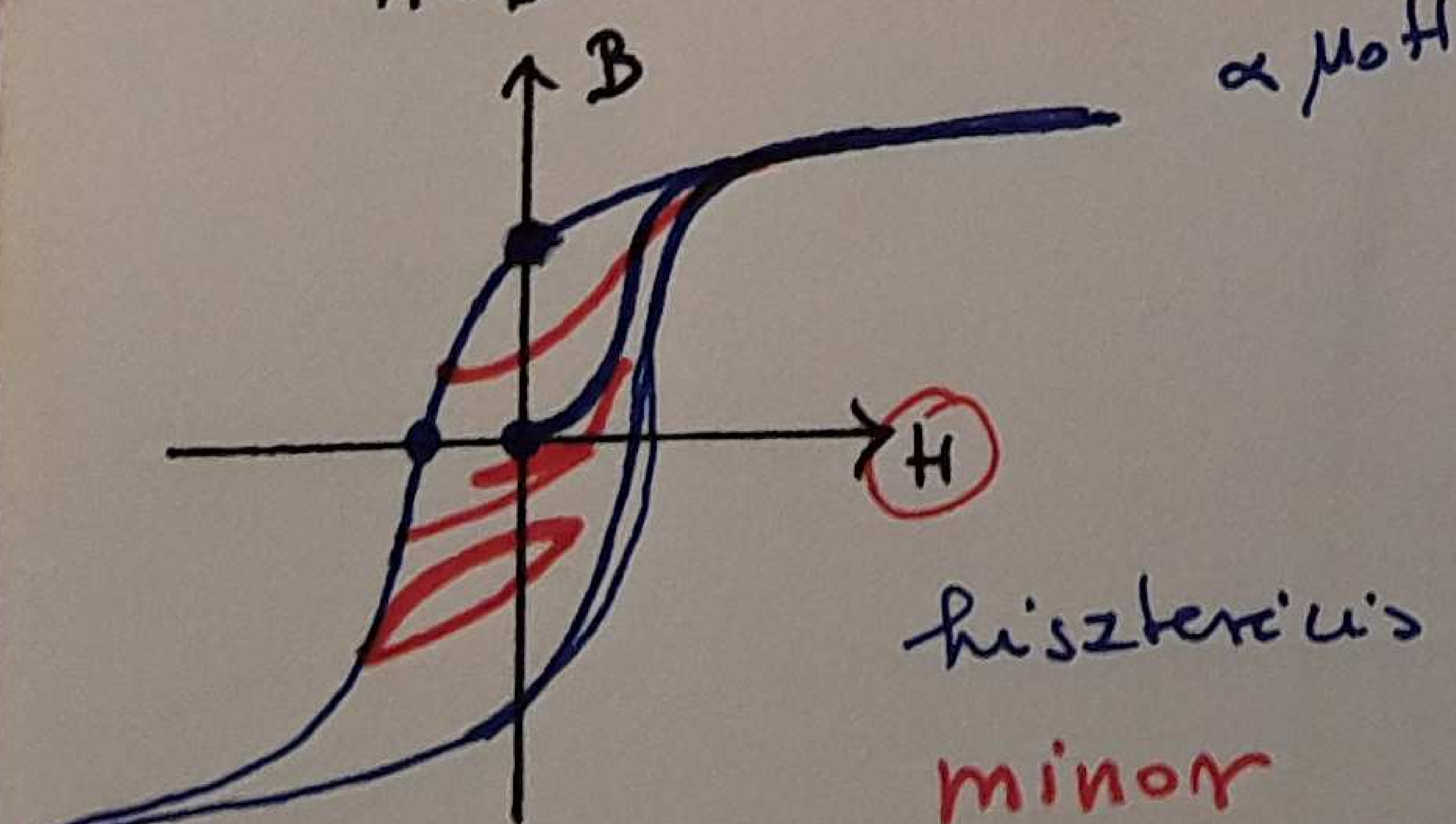


Dipólussá lesz





$H = \emptyset$



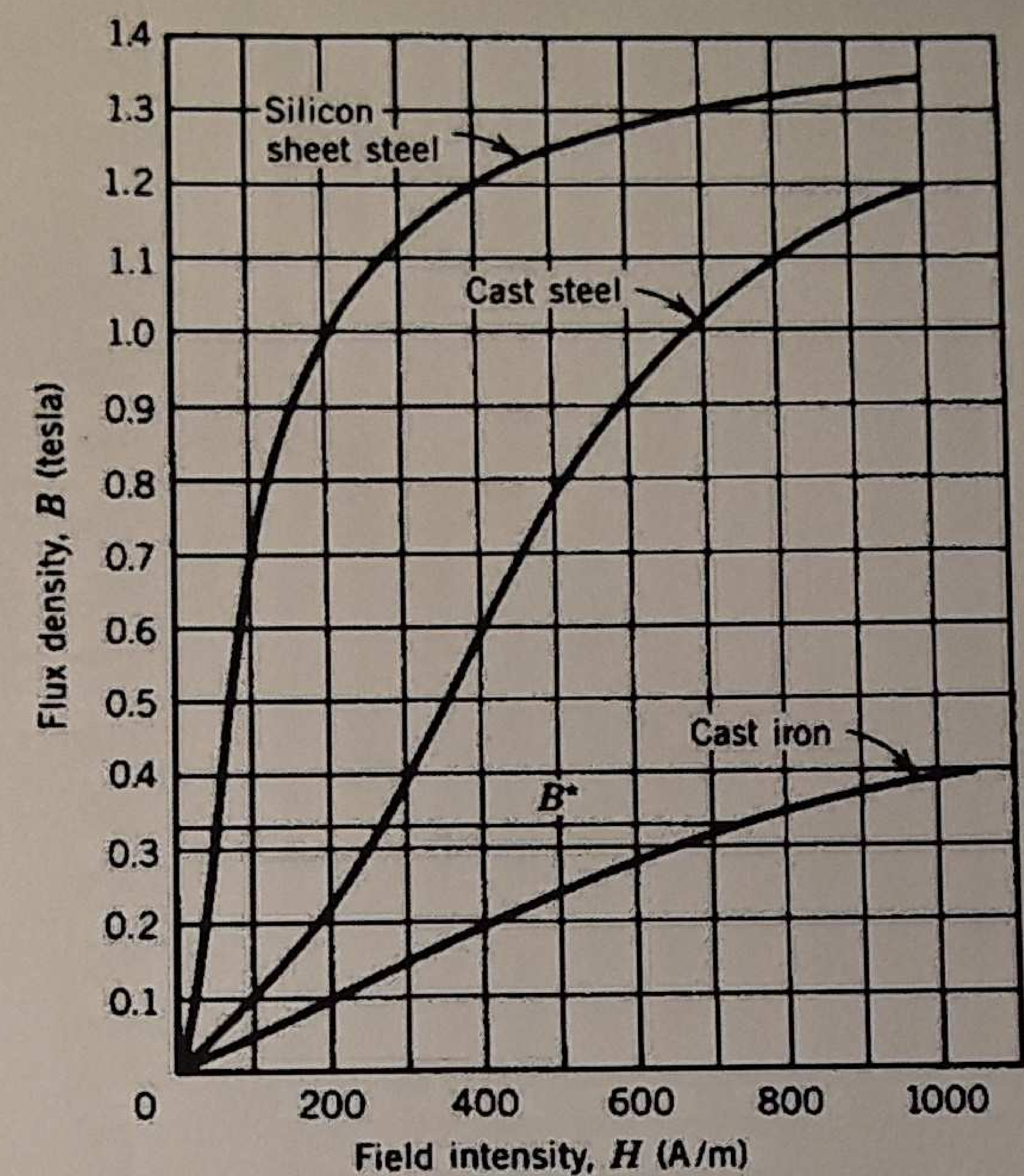
PREISACH

B_r JILES-ATHERTON

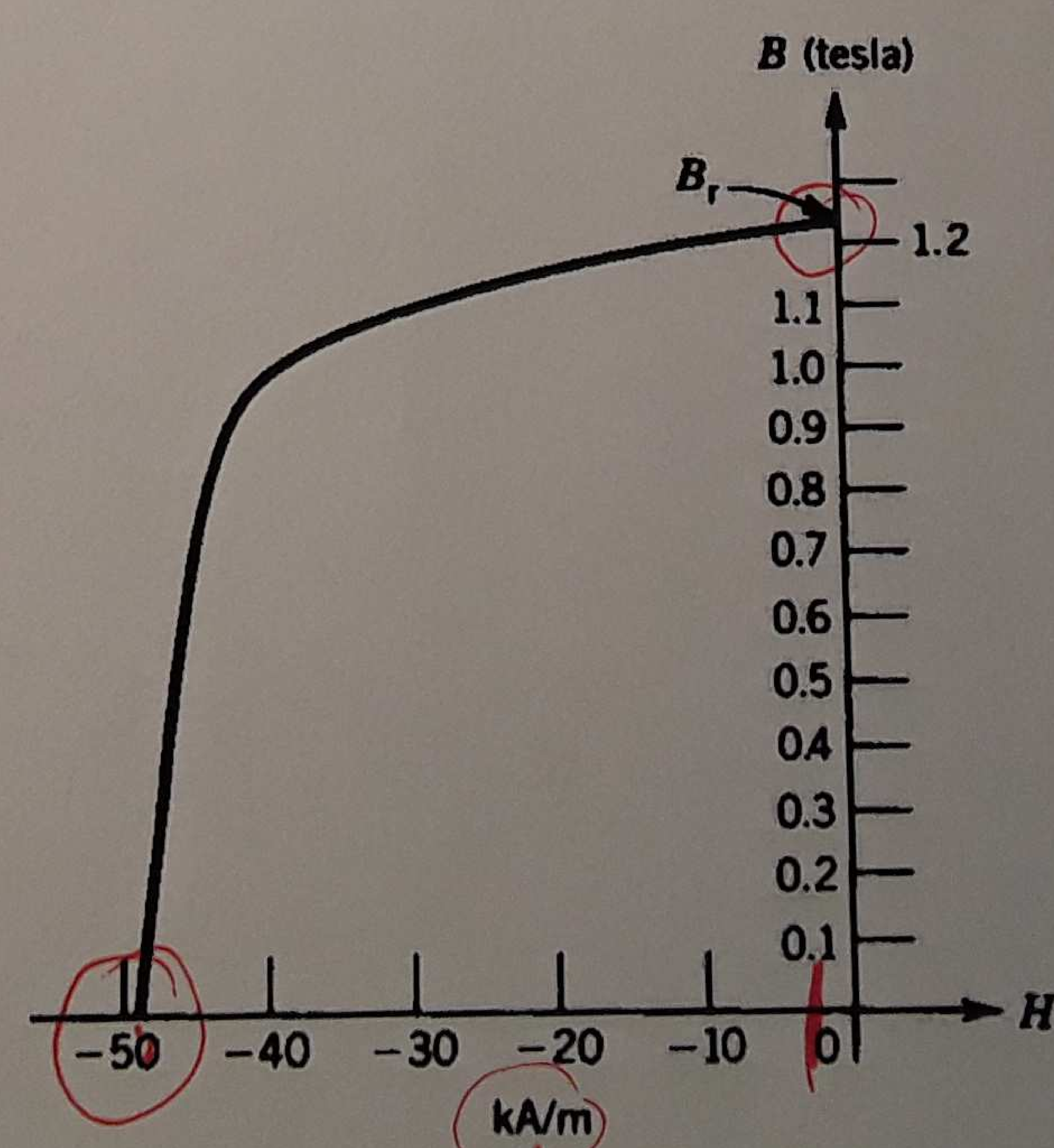
H_c

hiszterézis karakterisztika

minor



Silicon sheet steel
Cast steel
Cast iron
B*
Flux density, B (tesla)
Field intensity, H (A/m)
0
200
400
600
800
1000
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4



B (tesla)
B_r
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
H
-50
-40
-30
-20
-10
0
kA/m

AZ ENERGIA MÉRLEG

$$\mathrm{rot}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad / \cdot \vec{E}$$

$$\mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad / (-\vec{H})$$

$$\underbrace{\vec{E}\cdot \mathrm{rot}\,\vec{H} - \vec{H}\cdot \mathrm{rot}\,\vec{E}}_{-\mathrm{div}(\vec{E}\times\vec{H})} = \vec{E}\cdot\vec{J} + \underbrace{\vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial \vec{B}}{\partial t}}$$

$$\underbrace{\left(\vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial \vec{B}}{\partial t}\right)}_{\frac{\partial w}{\partial t}\ \ \frac{J}{m^3}} + \underbrace{\vec{E}\cdot\vec{J}}_{p\ \ \frac{W}{m^3}} + \mathrm{div}\underbrace{(\vec{E}\times\vec{H})}_{\vec{S}} = 0$$

$\vec{S}$ Poynting-vektor W/m^2

$$\boxed{\frac{\partial w}{\partial t} + p + \mathrm{div}\,\vec{S} = 0}$$

$$\int_V \mathrm{div}\,\vec{v}\,dV = \oint_A \vec{v}\cdot d\vec{A}$$

$$\int_V \left(\vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial \vec{B}}{\partial t}\right) dV + \underbrace{\int_V \vec{E}\cdot\vec{J}\,dV} + \oint_A (\vec{E}\times\vec{H})\cdot d\vec{A} = 0$$

$J = |\vec{J}|$

$$\vec{J} = \sigma(\vec{E} + \vec{E}_b) \rightarrow \vec{E} = \frac{\vec{J}}{\sigma} - \vec{E}_b$$

$$\vec{E}\cdot\vec{J} = \underbrace{\frac{J^2}{\sigma}}_{>0} - \underbrace{\vec{J}\cdot\vec{E}_b}$$

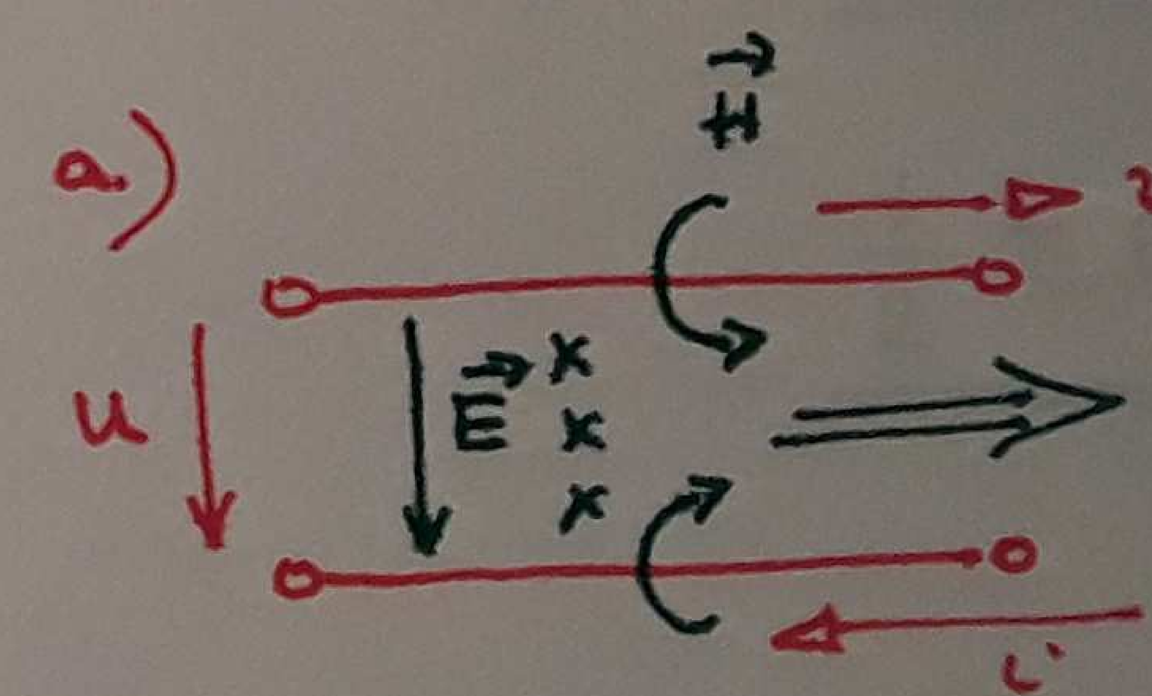
>0 fogy.

<0 gen.

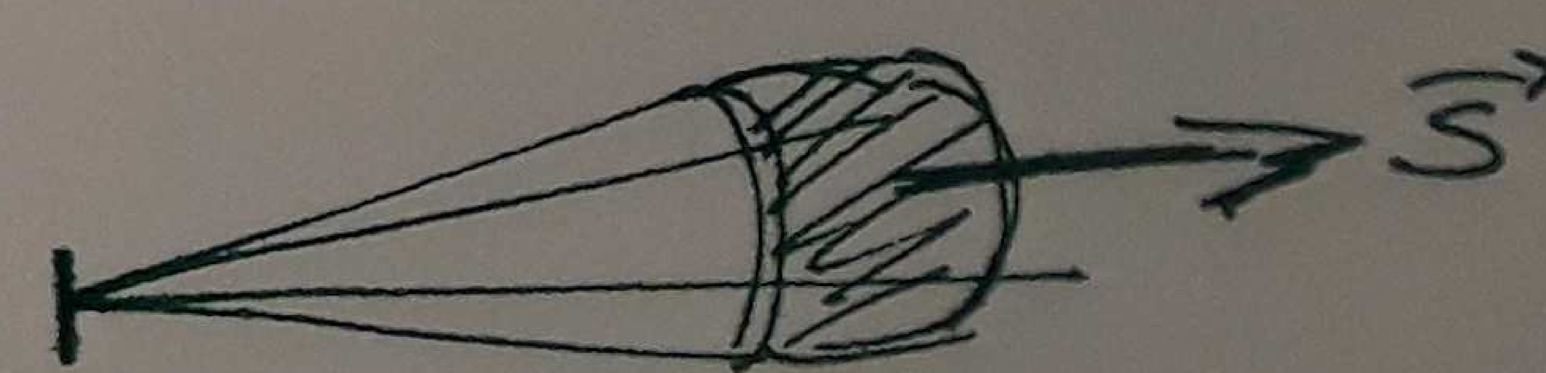
$$I^2R = \frac{I^2}{G}$$

Joule-hő

a)

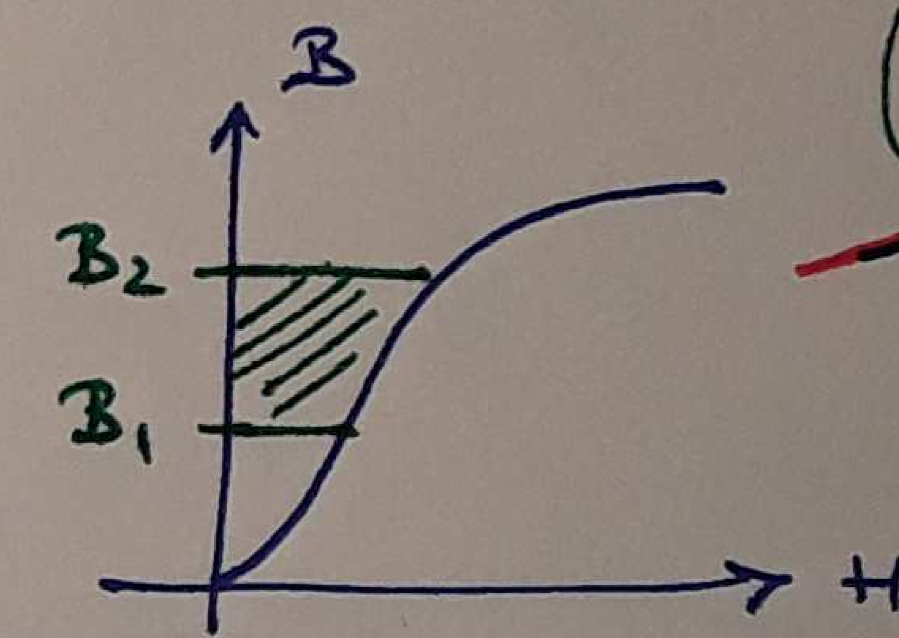


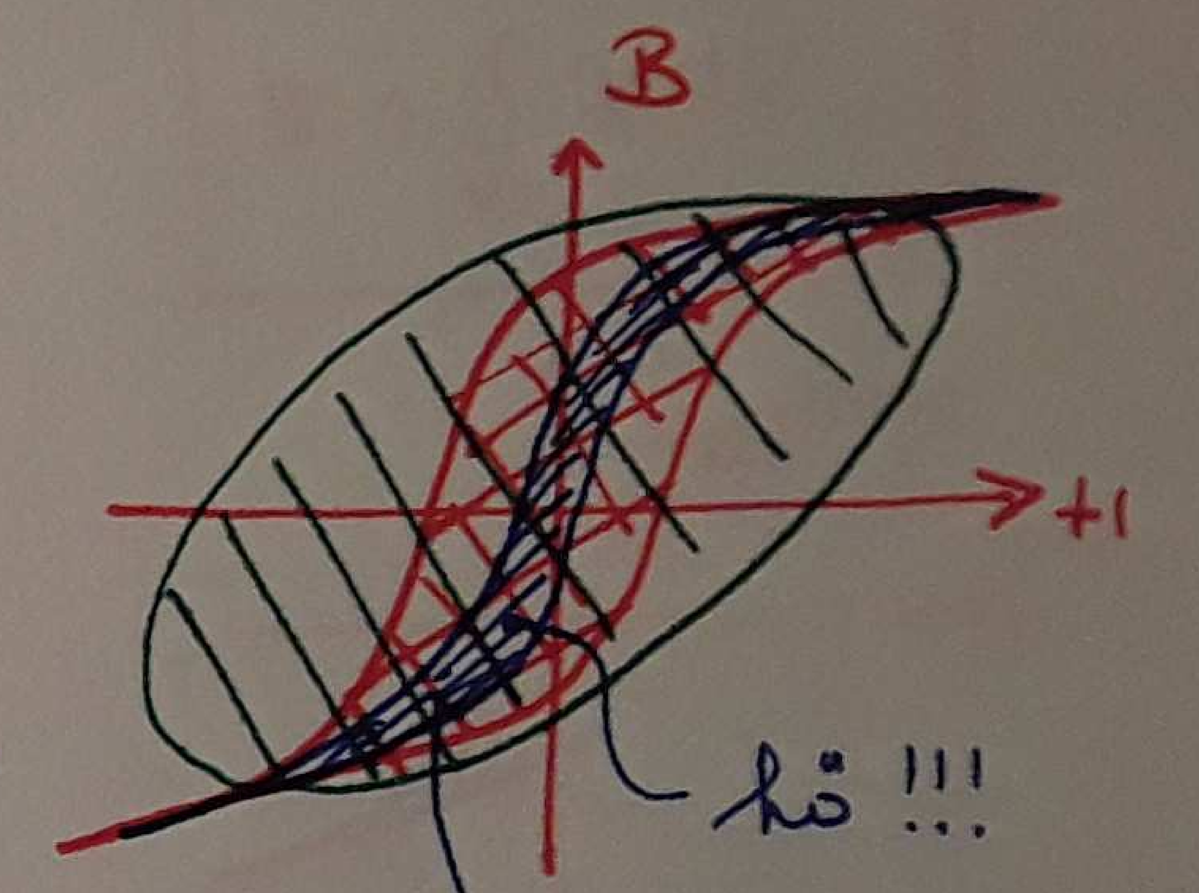
b)



$$\int_V \underbrace{\boxed{\left(\vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial \vec{B}}{\partial t}\right)}}_{\partial w/\partial t} dV = \int_V \frac{\partial w}{\partial t}\, dV$$

$$\Delta w = \int_{t_1}^{t_2} \frac{\partial w}{\partial t}\, dt = \int_{t_1}^{t_2} \left(\vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial \vec{B}}{\partial t}\right) dt = \int_{D_1}^{D_2} \vec{E}\cdot d\vec{D} + \underbrace{\int_{B_1}^{B_2} \vec{H}\cdot d\vec{B}}$$





hő !!!

LÁGYMÁGNESEK.

LEMEZELÉS

LIN. $\vec{B} = \mu \vec{H}$

$H = \frac{1}{\mu} B \rightarrow \cancel{\int_{B_1}^{B_2}}$ $\int_0^B \frac{1}{\mu} B\, dB = \frac{1}{\mu}\left[\frac{B^2}{2}\right]_0^B = \frac{B^2}{2\mu}$ $\quad B = \mu H \Rightarrow \frac{\mu^2 H^2}{2\mu} = \frac{1}{2}\mu H^2$

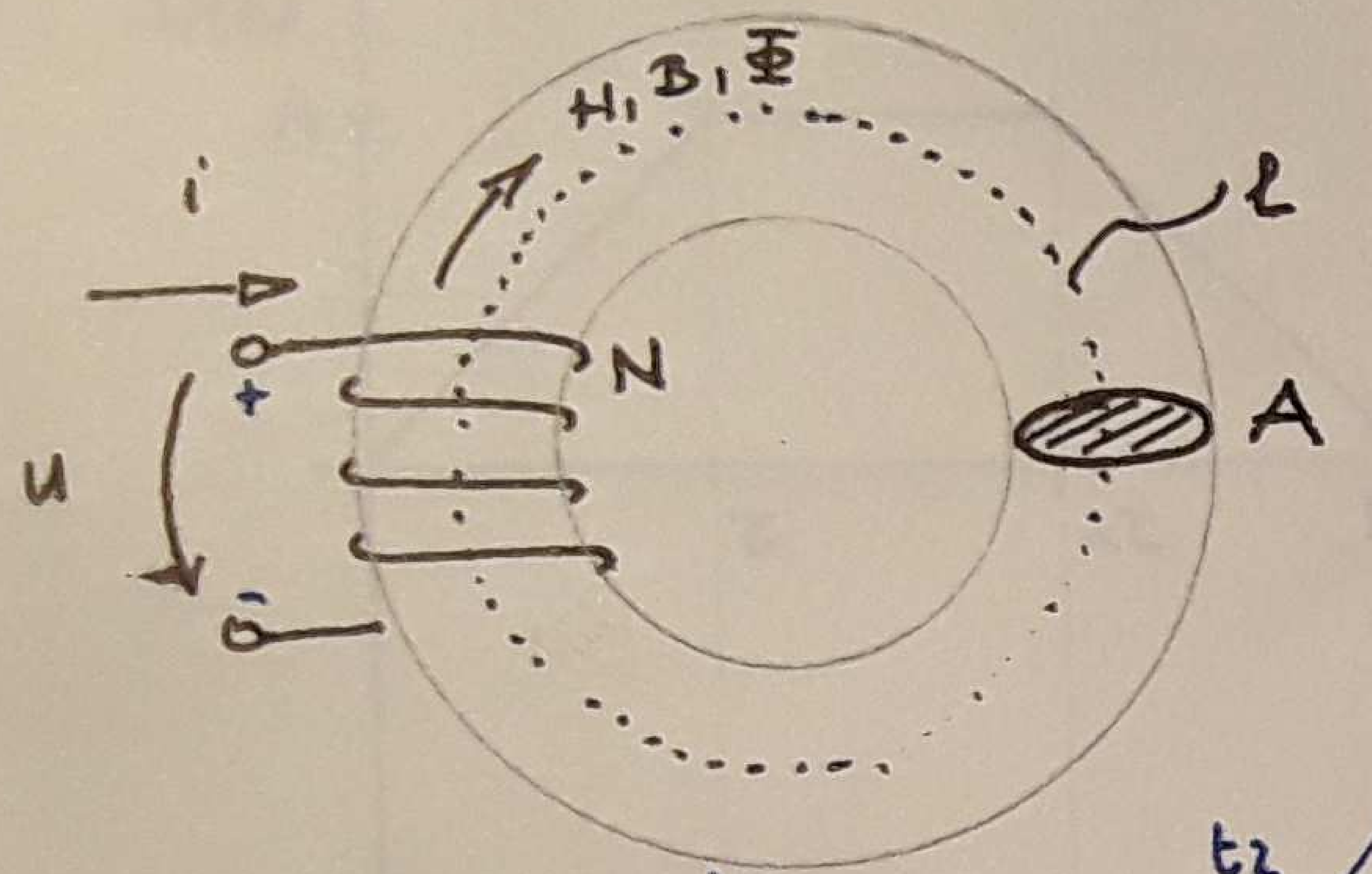
$\vec{D} = \varepsilon \vec{E}$ $\quad \frac{D^2}{2\varepsilon}$ $\quad \frac{1}{2}\varepsilon E^2$

$E = \frac{1}{\varepsilon} D$

$$\boxed{w = \frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2}$$

$$W = \int_V w\, dV$$

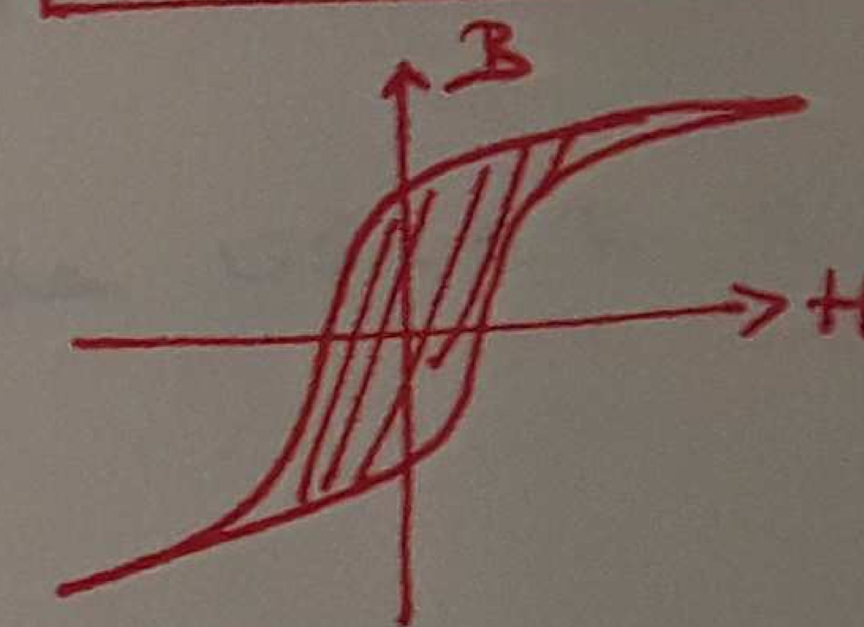
Vizsgáljuk meg a toroid mérőtekercsben kialakuló veszteséget!



$H = \frac{Ni}{l}$ $\quad \Phi = BA$ $\quad \vec{H} \parallel \vec{B}$

$$\boxed{\begin{aligned} w &= \oint H \cdot dB \\ W &= \int_V w\, dV \end{aligned}}$$

$p = p(t) \quad u = u(t) \quad i = i(t)$



$N \frac{d\Phi}{dt}$

$$W = \int_{t_1}^{t_2} p\, dt = \int_{t_1}^{t_2} u i\, dt = \int_{t_1}^{t_2} N \frac{d\Phi}{dt} i\, dt = \int_{\Phi_1}^{\Phi_2} N i\, d\Phi$$

$i = \frac{Hl}{N} \qquad d\Phi = A\,dB$

$$= \int_{B_1}^{B_2} \cancel{N} \frac{Hl}{\cancel{N}} A\, dB = \int_{B_1}^{B_2} H \underbrace{lA}_{V} dB = V \int_{B_1}^{B_2} H dB$$

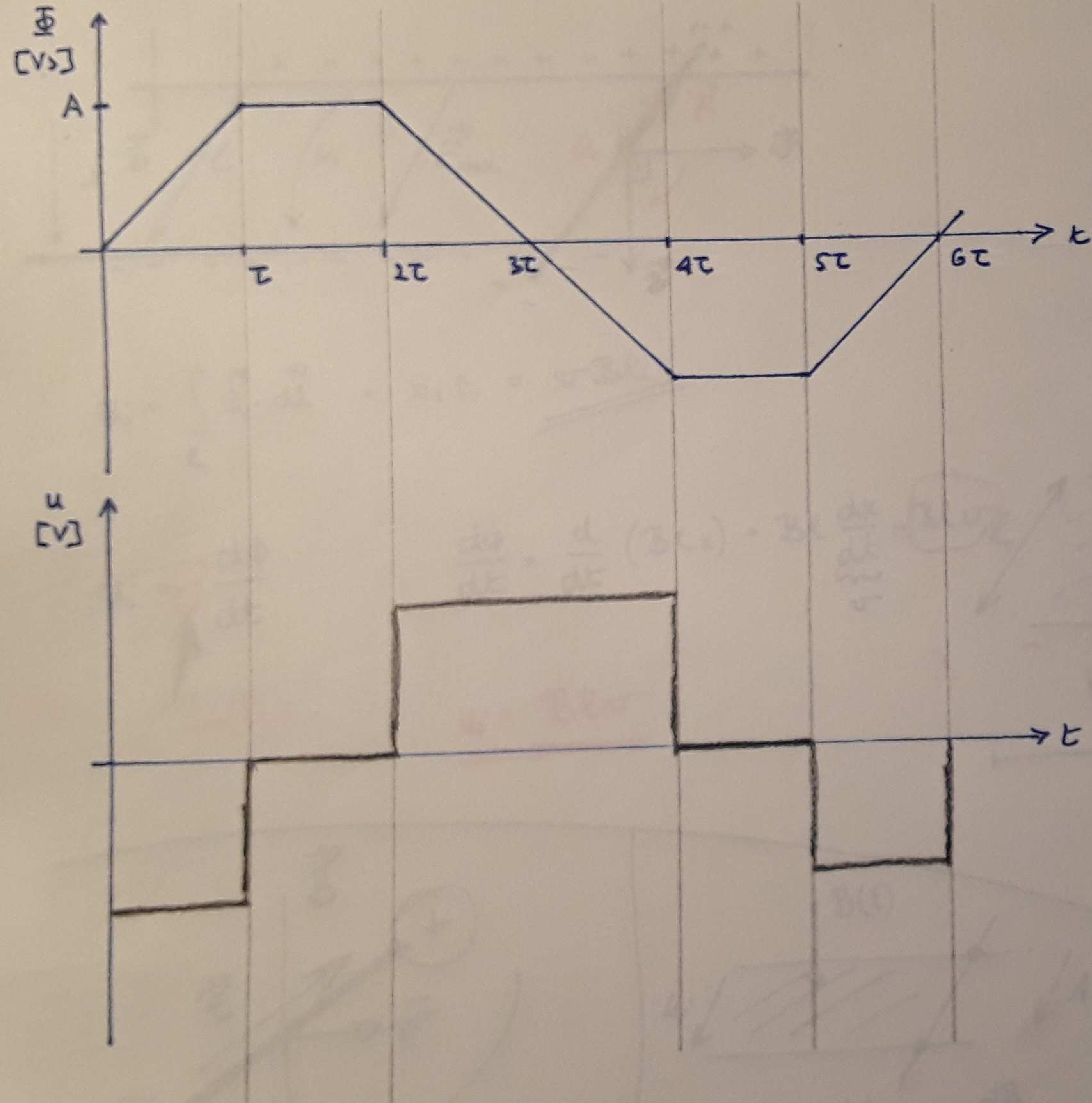
$$\boxed{W = V \underbrace{\oint H dB}_{w}}$$

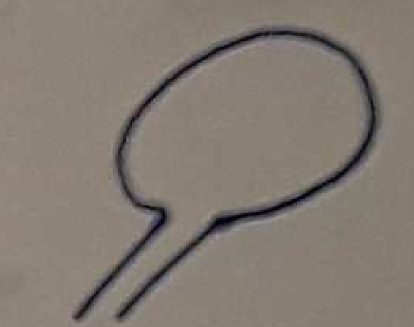
$H = \frac{Ni}{l}$ ✓

$B = \frac{\Phi}{A} = \frac{1}{NA} \int_0^t u(t')\, dt'$ ✓

$u = \frac{d\Phi}{dt} \qquad \Phi = \int_0^t u(t')\, dt'$

Vázoljuk fel a kialakuló indukált feszültséget!





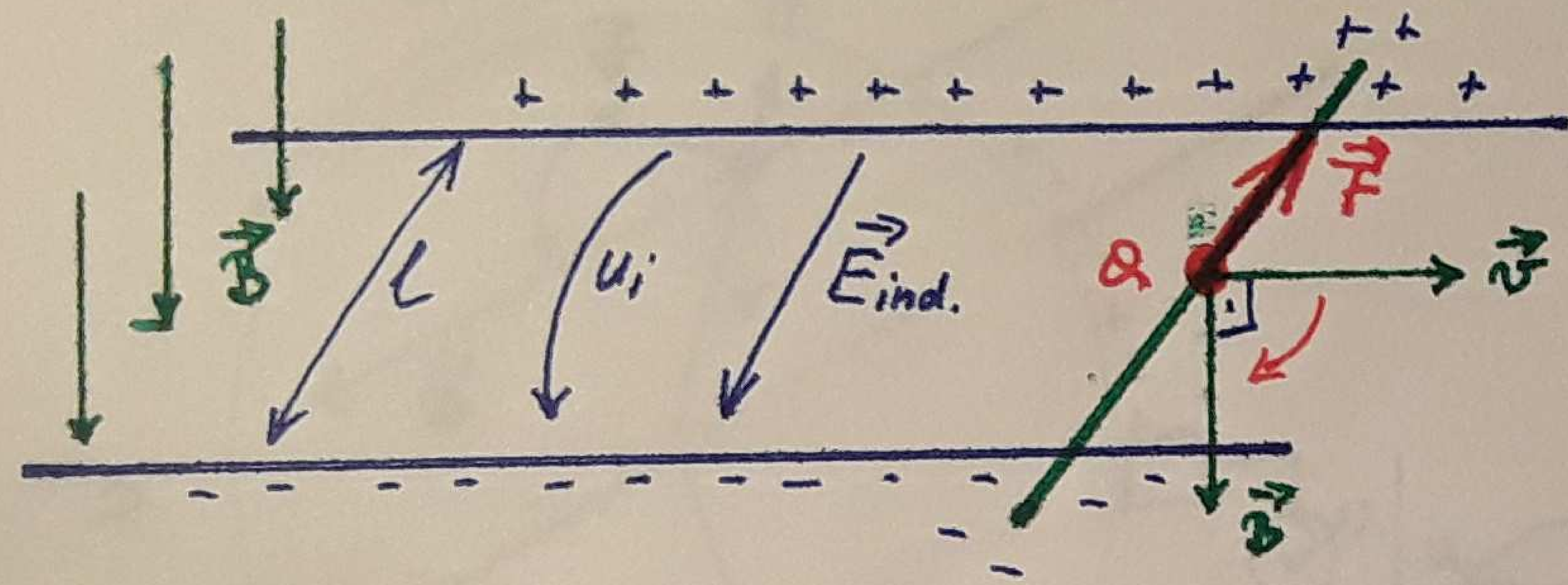
$$u_i(t) = -\frac{d\Phi}{dt}$$

$$\Phi(t) = A\frac{t}{\tau} \quad \text{ha } 0 \le t \le \tau$$

$$\frac{A}{\tau}$$

$$u_i(t) = -N\frac{d\Phi}{dt}$$

Vizsgáljuk meg, milyen irányú és nagyságú indukált feszültség jön létre az alábbi elrendezés esetén!



$$\vec{F} = Q \underbrace{\vec{v} \times \vec{B}}$$

$$\vec{F} = Q\vec{E} \qquad vB \sin 90°$$

$$u_i = \int_l \vec{E} \cdot \vec{dl} = E_i l = vBl$$

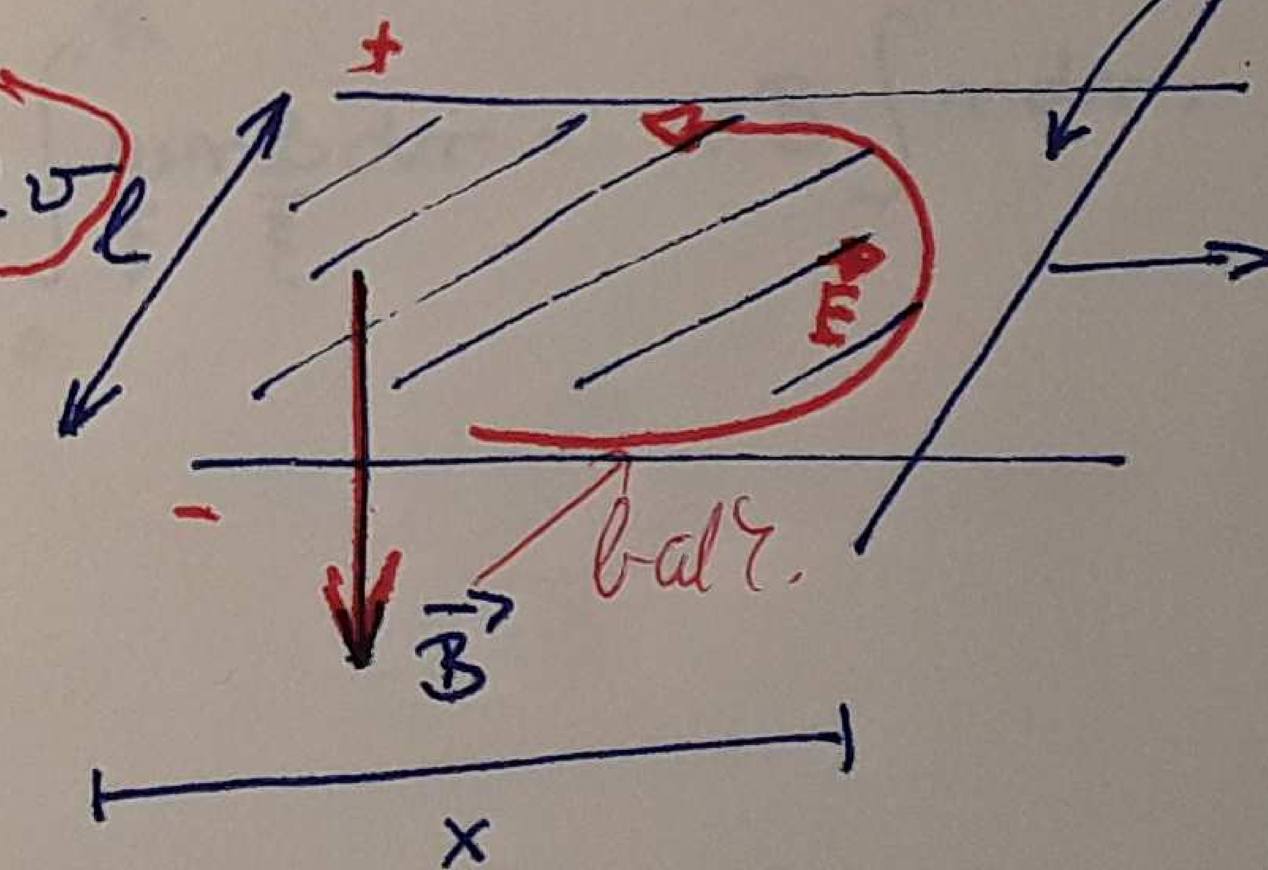
$$\Phi = BA = Blx$$

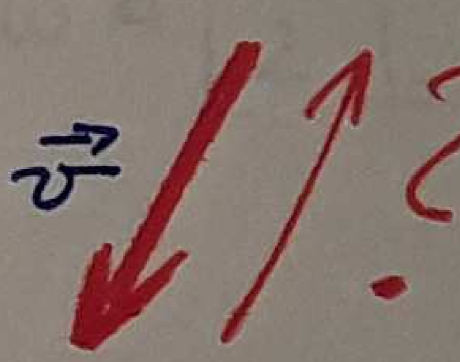
$$u_i = -\frac{d\Phi}{dt}$$

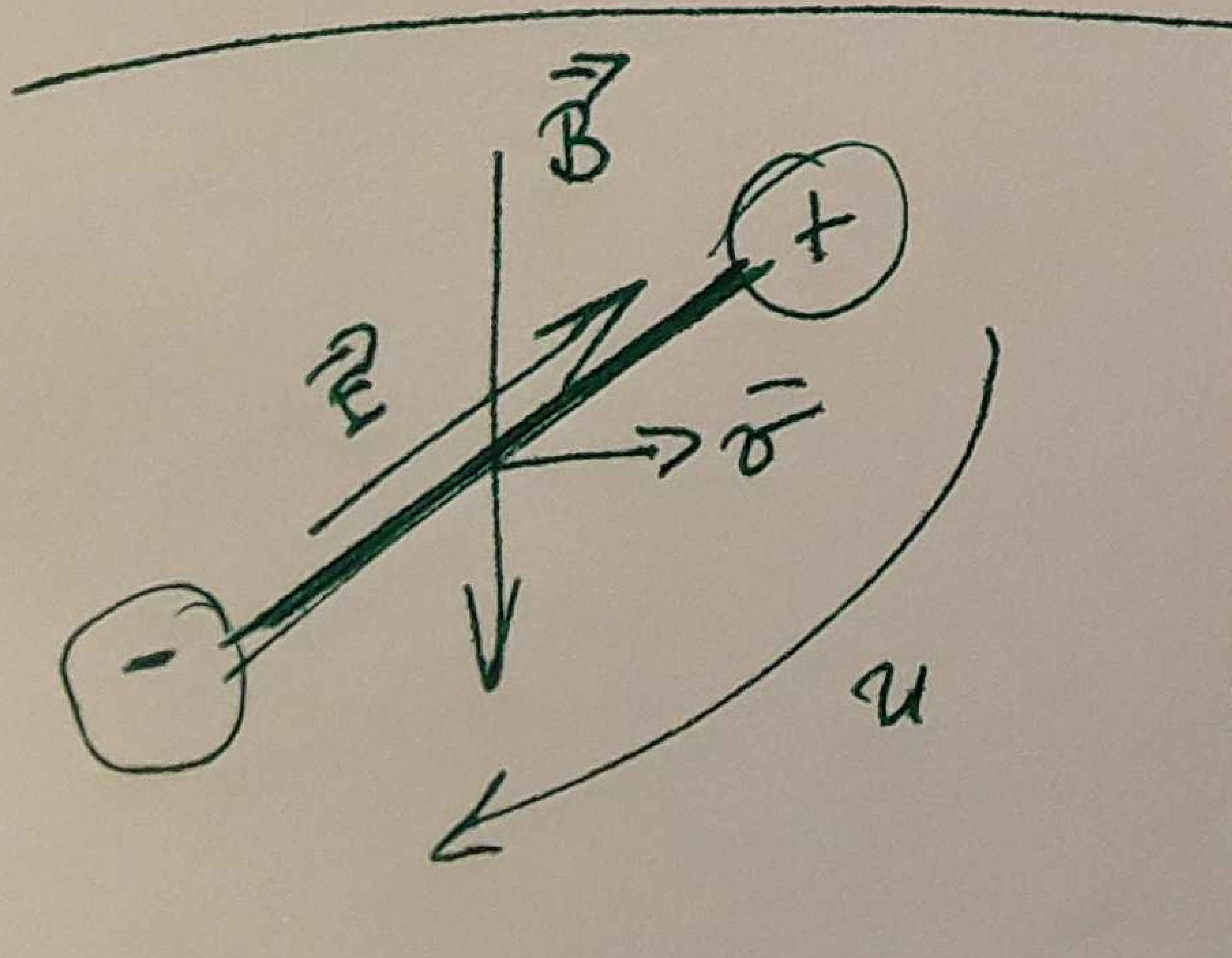
balkéz!

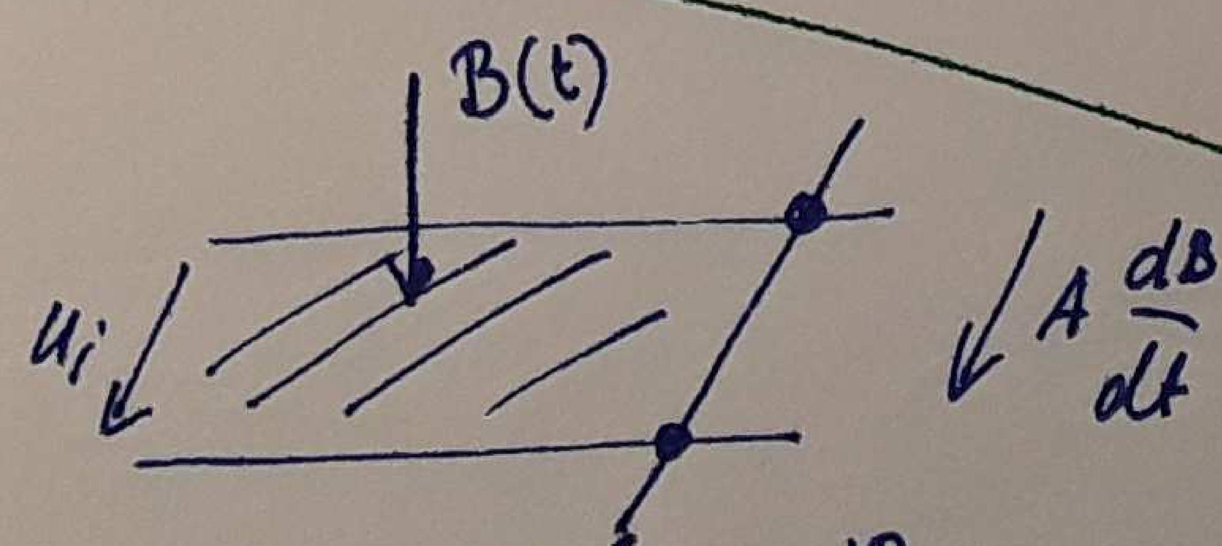
$$\frac{d\Phi}{dt} = \frac{d}{dt}(Blx) = Bl \underbrace{\frac{dx}{dt}}_{v} = Blv$$

$$u_i = Blv$$

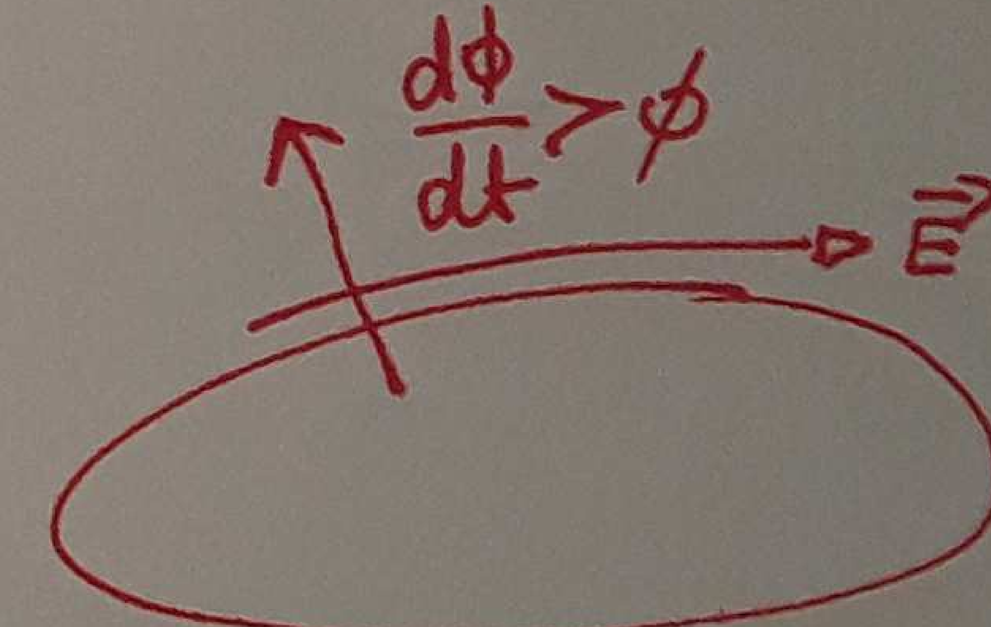




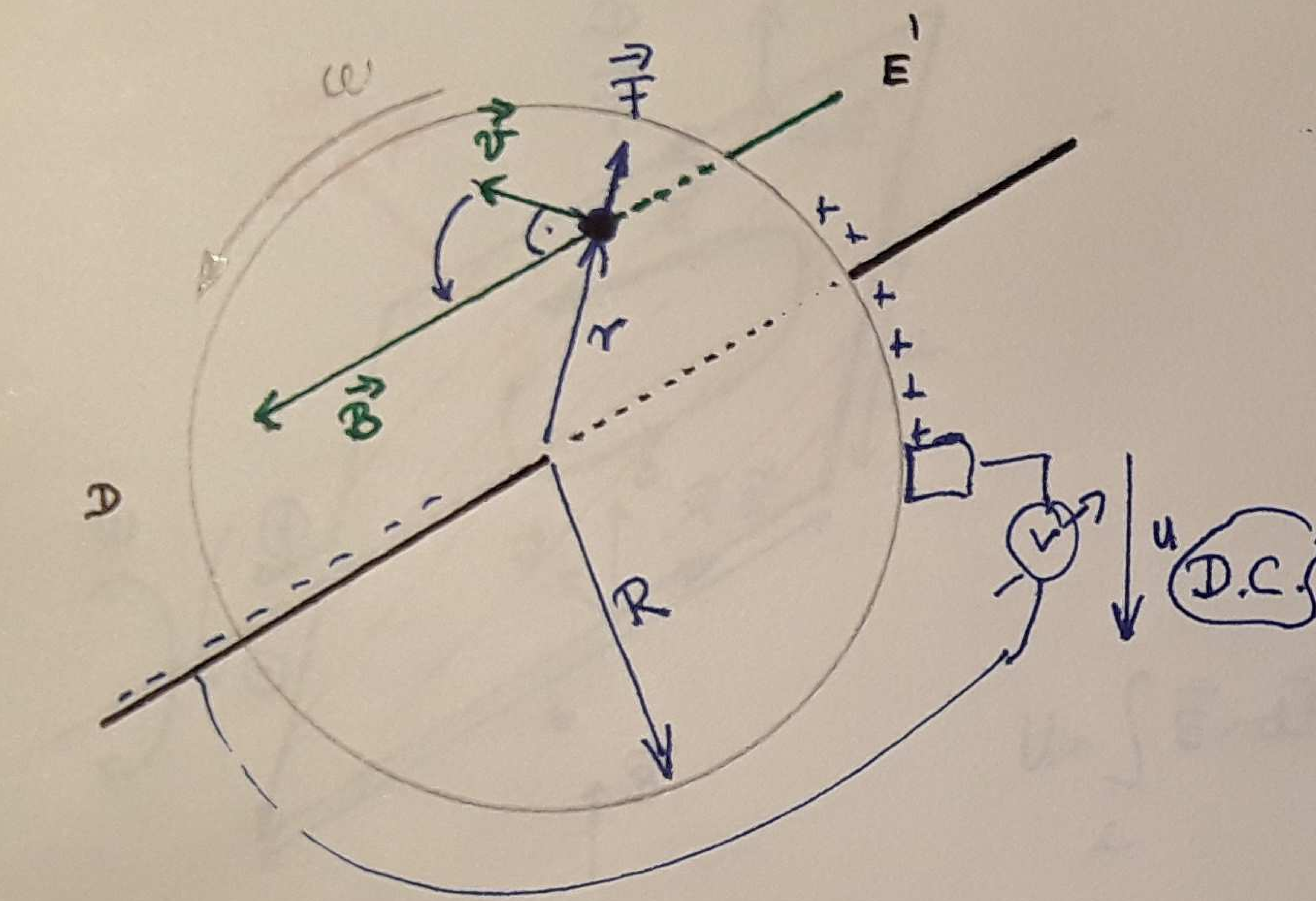


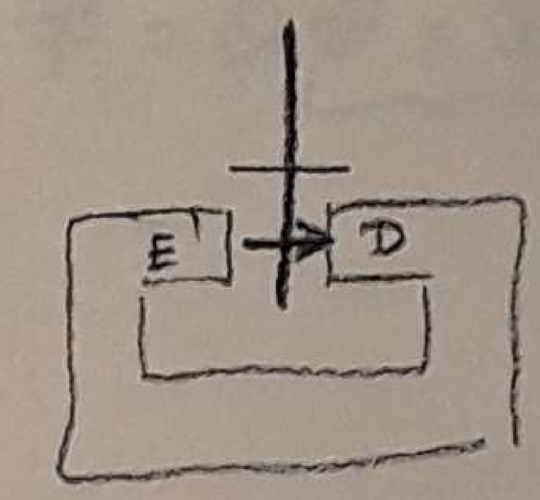


$$u_i = -\frac{d\phi}{dt} = -A\frac{dB}{dt}$$



Vizsgáljuk meg az alábbi, ún. unipoláris generátor működését!



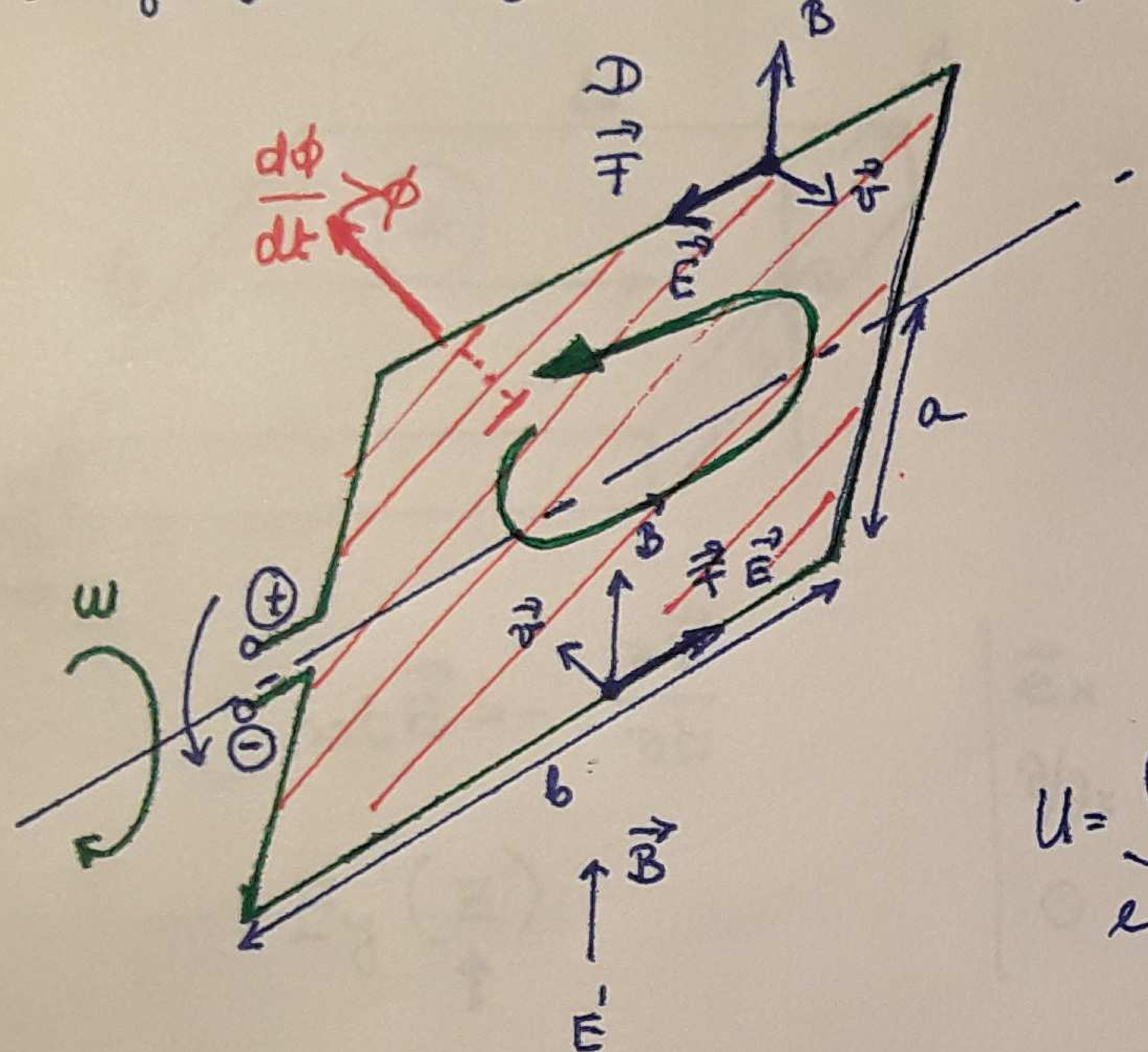


$$\vec{F} = Q\underbrace{\vec{v}\times\vec{B}}_{\cancel{vB\sin 90^\circ}}$$

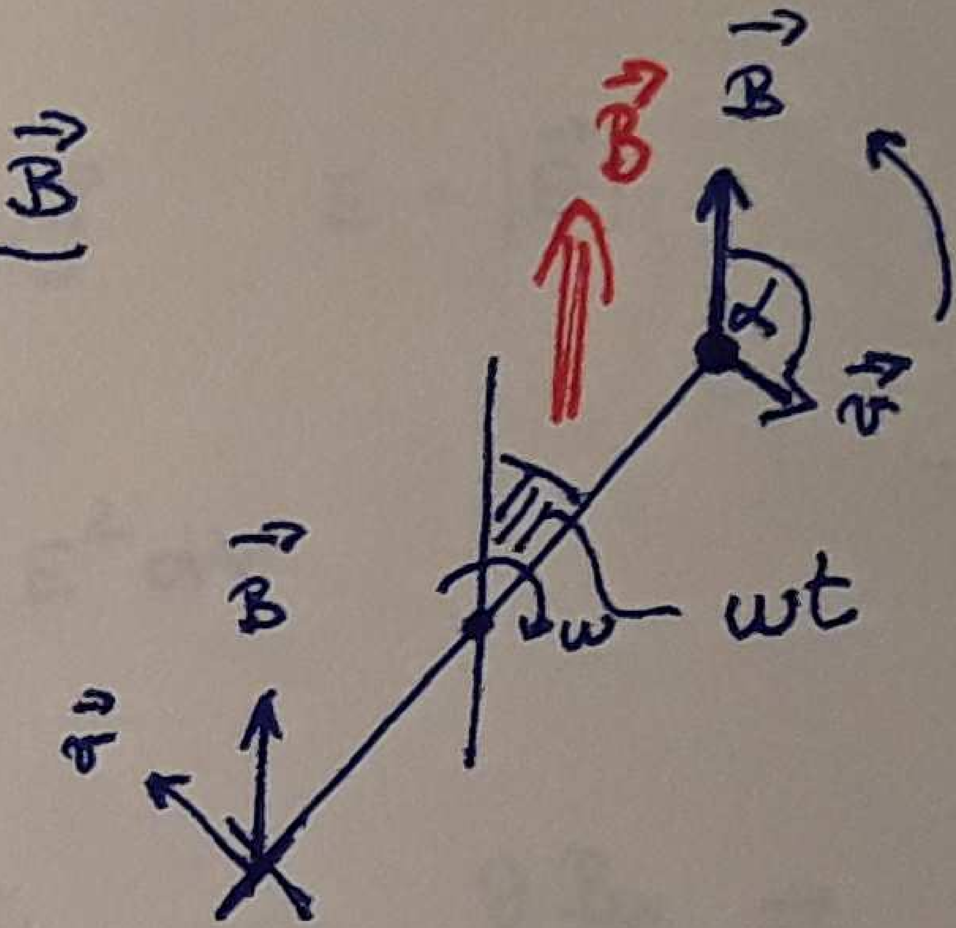
$$E = vB = \underline{\omega r B}$$

$$U = \int_0^R \omega r B\,dr = \omega B\int_0^R r\,dr = \omega B\left[\frac{r^2}{2}\right]_0^R = \frac{\omega B R^2}{2}$$

Vizsgáljuk meg az alábbi generátor működési elvét!



$$\vec F = Q\underbrace{\vec v \times \vec B}$$



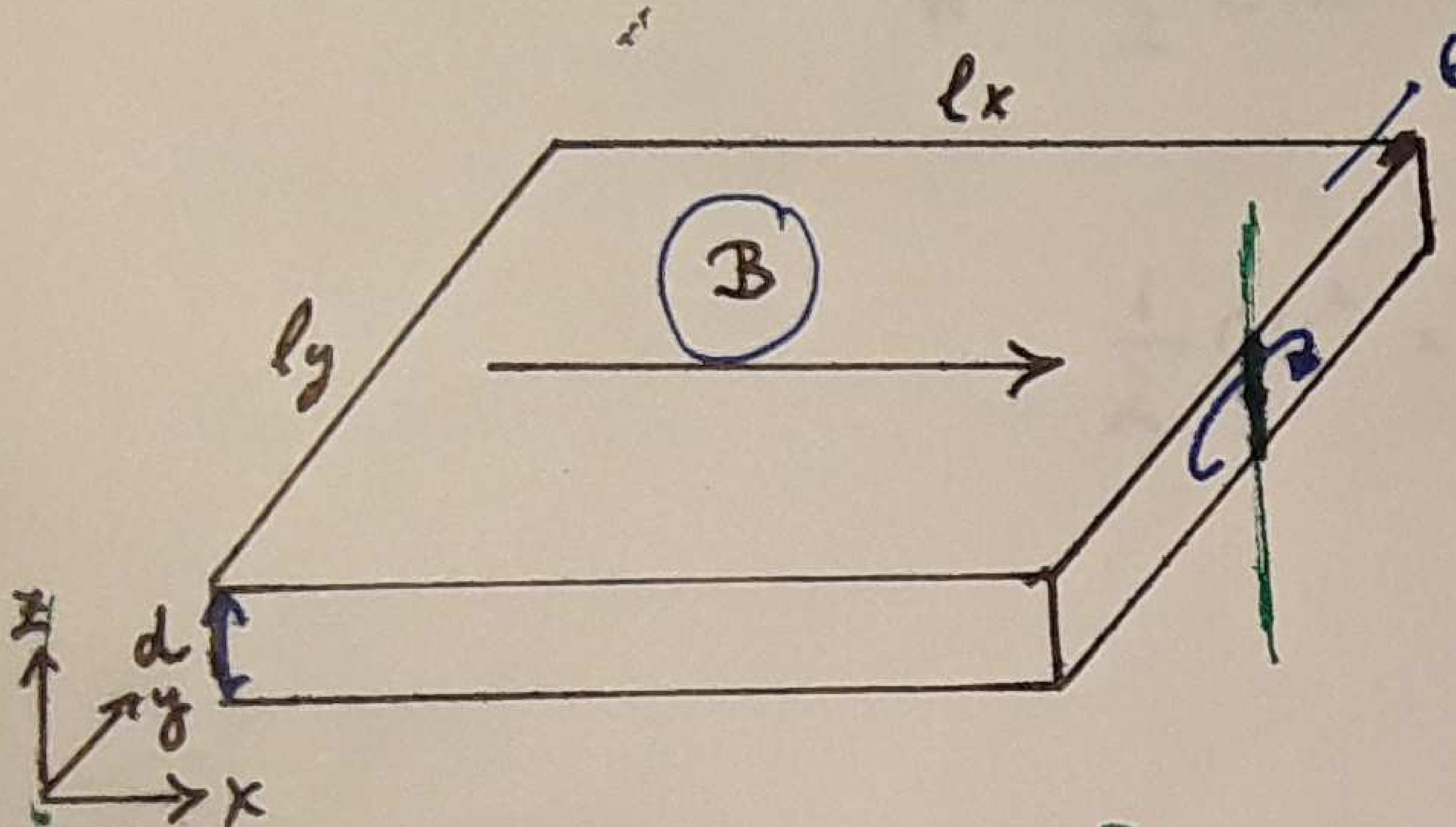
$$U = \int_\ell \vec E \cdot d\vec l = 2\underset{\substack{\uparrow\\ \omega a}}{v} B \underbrace{\sin\alpha}_{\cos\omega t}\, b = \underbrace{2ab}_{A}\,\omega B\cos\omega t = \underline{\underline{AB\omega\cos\omega t}}$$

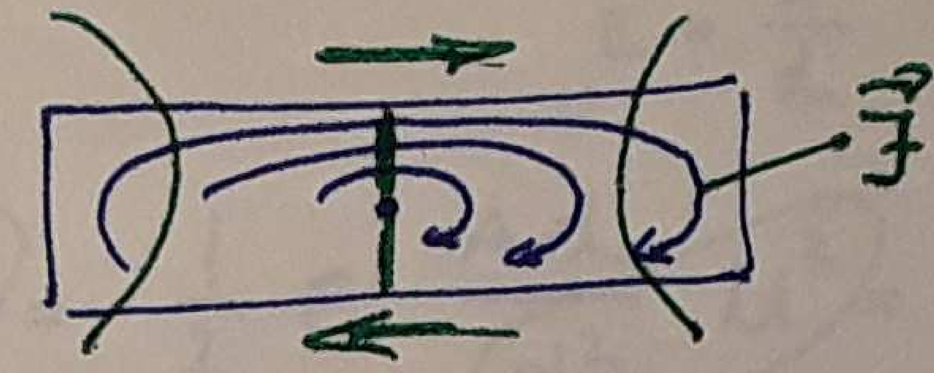
$$u_i = \cancel{-}\frac{d\Phi}{dt}$$

$$\Phi = BA = B\underbrace{2ab}_{A}\sin\omega t$$

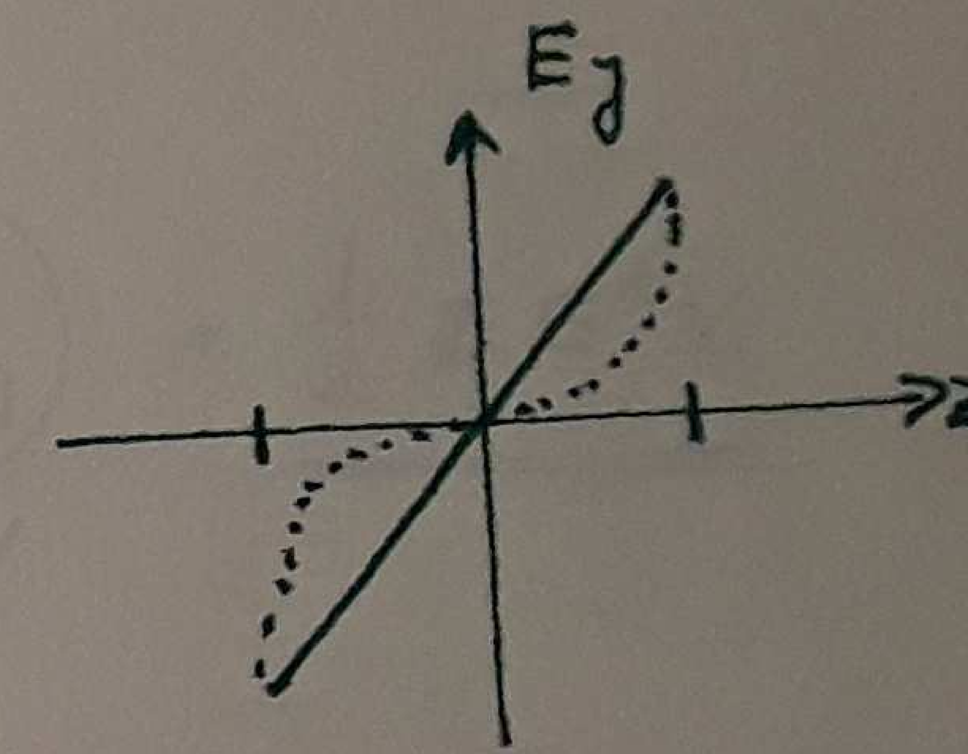
$$\frac{d\Phi}{dt} = +AB\omega\cos\omega t$$

Határozzuk meg az örvényáram-veszteséget a lemezben!





$E = |\vec{E}|$



$$\int_V \underset{\uparrow \atop \sigma\vec{E}}{\vec{J}} \cdot \vec{E}\, dV = \int_V \sigma E^2 dV$$

$$\text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$E_y(\underset{\uparrow}{z})$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix} = -\vec{e}_x \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}\vec{e}_x$$

$$\frac{\partial E_y}{\partial z} = \frac{\partial B_x}{\partial t} \longrightarrow \boxed{E_y = \frac{\partial B_x}{\partial t} z}$$

$$\int_V \sigma E^2 dV = \int_V \sigma \left(\frac{\partial B_x}{\partial t} z\right)^2 \underset{\uparrow \atop dx\,dy\,dz}{dV} = \int_{-d/2}^{d/2} \sigma \left(\frac{\partial B_x}{\partial t}\right)^2 l_x l_y z^2 \cancel{dV}\, dz =$$

$$= \sigma l_x l_y \left(\frac{\partial B_x}{\partial t}\right)^2 \int_{-d/2}^{d/2} z^2 dz = \sigma l_x l_y \left(\frac{\partial B_x}{\partial t}\right)^2 \left[\frac{z^3}{3}\right]_{-d/2}^{d/2} = \sigma l_x l_y \left(\frac{\partial B_x}{\partial t}\right)^2 \underbrace{\frac{d^3/8 + d^3/8}{3}}$$

$$= \sigma \underbrace{l_x l_y d}_{V} \left(\frac{\partial B_x}{\partial t}\right)^2 \cdot \frac{d^2}{12} = V \boxed{\frac{\sigma d^2}{12}\left(\frac{dB_x}{dt}\right)^2} \longrightarrow w$$

$d = 0{,}35\,\text{mm}$

$$\frac{2\frac{d^3}{8}}{3} = \frac{d^3}{12}$$

Síkkondenzátort feltételezve, igazoljuk, hogy

$$W = \frac{1}{2}CU^2 = \underbrace{\frac{1}{2}\varepsilon E^2}_{w} V.$$

$$E = \frac{U}{d}$$

$$\frac{1}{2}CU^2 = \frac{1}{2}\varepsilon \frac{A}{d} U^2 = \frac{1}{2}\varepsilon \frac{Ad}{d^2} U^2 = \frac{1}{2}\varepsilon E^2 \underbrace{Ad}_{V} = \frac{1}{2}\varepsilon E^2 V$$

Szolenoid tekercs esetén igazoljuk, hogy $W = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 H^2 V.$

$$H = \frac{NI}{l}$$

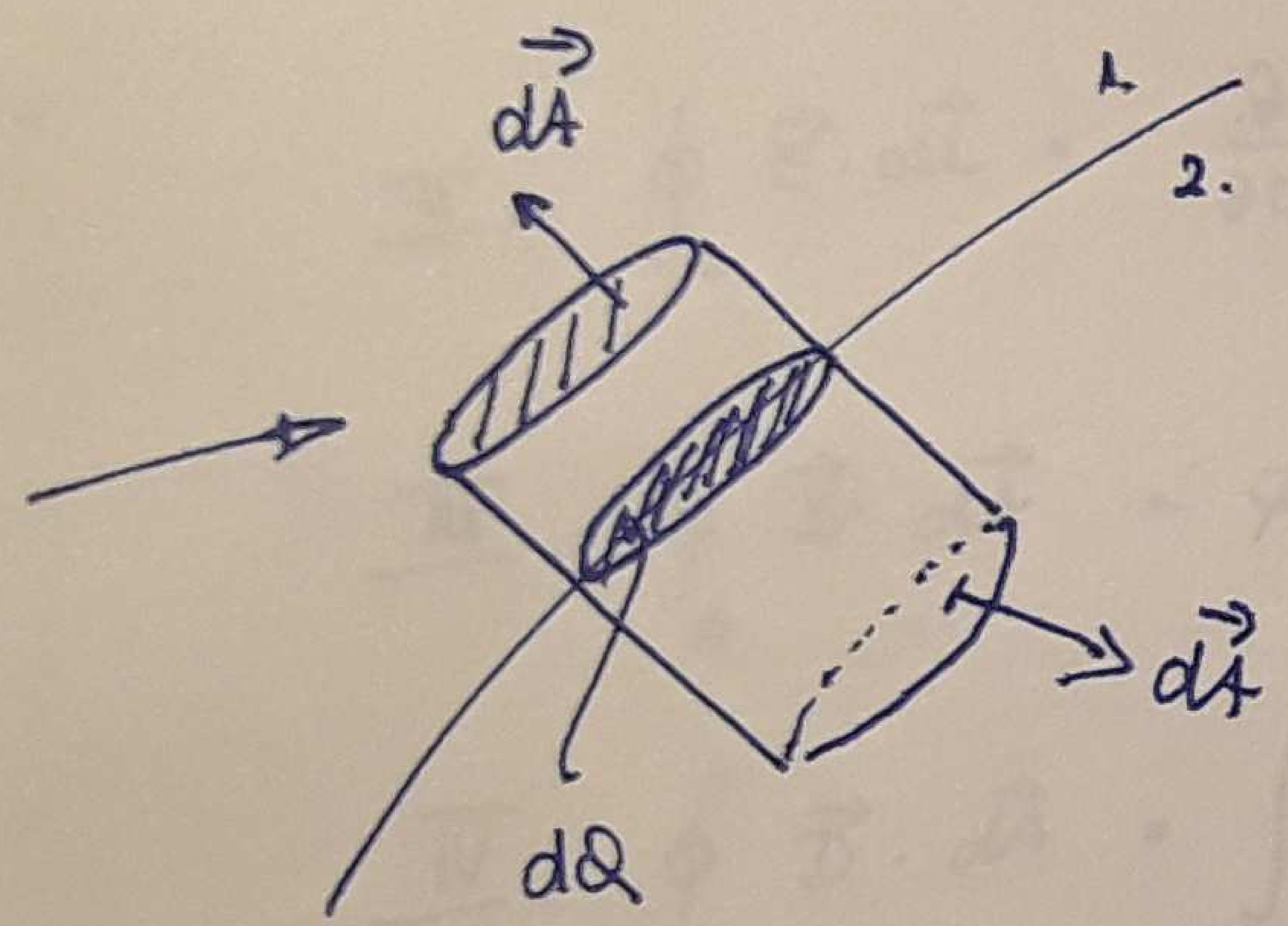
$$\frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l} \cdot I^2 = \frac{1}{2}\mu_0 \underbrace{\frac{N^2 I^2}{l^2}}_{H} Al$$

$$= \frac{1}{2}\mu_0 H^2 \underbrace{Al}_{V} = \frac{1}{2}\mu_0 H^2 V$$

AZ ÁRAMSŰRŰSÉGRE VONATKOZÓ HATÁRFELTÉTEL

Áramlási tér (DC) $\Rightarrow$ J_n folytonos

Váltakozó mágneses és elektromos tér (AC):



$$\operatorname{rot}\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\emptyset = \operatorname{div}\vec{J} + \frac{\partial}{\partial t}\underbrace{\operatorname{div}\vec{D}}_{\rho}$$

$$\int_V \operatorname{div}\vec{v}\,dV = \oint_A \vec{v}\cdot d\vec{A}$$

$$\operatorname{div}\vec{J} + \frac{\partial \rho}{\partial t} = \emptyset$$

$$\oint_A \vec{J}\cdot d\vec{A} + \frac{\partial}{\partial t}\underbrace{\int_V \rho\, dV}_{Q} = \emptyset$$

$$\oint_A \vec{J}\cdot d\vec{A} + \frac{\partial Q}{\partial t} = \emptyset$$

$$dQ = \sigma\, dA$$

$$-J_{1n}\,\cancel{dA} + J_{2n}\,\cancel{dA} + \frac{\partial \sigma}{\partial t}\,\cancel{dA} = \emptyset$$

$$\boxed{J_{2n} = J_{1n} - \frac{\partial \sigma}{\partial t}}$$

A MAXWELL-EGYENLETEK ÖSSZEFOGLALÁSA

1864

I. $\oint_{l} \vec{H}\cdot d\vec{l} = \int_{A} \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{A}$ $\qquad \mathrm{rot}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ $\qquad$ Ampère-féle gerjesztési törvény (ált.)

II. $\oint_{l} \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_{A} \vec{B}\cdot d\vec{A}$ $\qquad \mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\qquad$ Faraday-féle indukciós-törvény

III. $\oint_{A} \vec{B}\cdot d\vec{A} = 0$ $\qquad \mathrm{div}\,\vec{B} = 0$ $\qquad$ A fluxusmegmaradás t.

IV. $\oint_{A} \vec{D}\cdot d\vec{A} = \int_{V} \rho\, dV$ $\qquad \mathrm{div}\,\vec{D} = \rho$ $\qquad$ Gauss-törvény.

V. $\vec{D} = \mathcal{D}\{\vec{E}\}$ $(\vec{D} = \varepsilon\vec{E})$ $\qquad \vec{B} = \mathcal{B}\{\vec{H}\}$ $(\vec{B} = \mu\vec{H})$ $\qquad \vec{J} = \mathcal{J}\{\vec{E}\}$ $(\vec{J} = \sigma\vec{E})$ $\qquad$ Konstitúciós relációk

VI. $\frac{\partial w}{\partial t} = \vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial \vec{B}}{\partial t}$ $\qquad$ energiamérleg

$$w = \frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2$$

1. / 2.

$E_{2t} = E_{1t}$ $\qquad D_{2n} = D_{1n} + \sigma$

$H_{2t} = H_{1t} - K$ $\qquad B_{2n} = B_{1n}$

$J_{2n} = J_{1n} - \frac{\partial \sigma}{\partial t}$

AZ ELEKTRODINAMIKA FELOSZTÁSA

I. $\partial/\partial t = \emptyset \qquad \vec{J} = \emptyset$

ELEKTROSZTATIKA

$\mathrm{rot}\,\vec{E} = \emptyset \qquad \mathrm{div}\,\vec{D} = \varrho \qquad \vec{D} = \mathcal{D}\{\vec{E}\}$

MAGNETOSZTATIKA

$\mathrm{rot}\,\vec{H} = \emptyset \qquad \mathrm{div}\,\vec{B} = \emptyset \qquad \vec{B} = \mathcal{B}\{\vec{H}\}$

II. $\partial/\partial t = \emptyset \qquad \vec{J} \neq \emptyset$

STACIONÁRIUS ÁRAMLÁS

$\mathrm{rot}\,\vec{E} = \emptyset \qquad \mathrm{div}\,\vec{J} = \emptyset \qquad \vec{J} = g\{\vec{E}\}$

STACIONÁRIUS MÁGNESES TÉR

$\mathrm{rot}\,\vec{H} = \vec{J} \qquad \mathrm{div}\,\vec{B} = \emptyset \qquad \vec{B} = \mathcal{B}\{\vec{H}\}$

III. $\partial/\partial t \neq \emptyset \qquad \vec{J} \neq \emptyset$ de $|\partial\vec{D}/\partial t| \approx \emptyset$

ÖRVÉNYÁRAMÚ TÉR

$\mathrm{rot}\,\vec{H} = \vec{J} \qquad \mathrm{rot}\,\vec{E} = -\dot{\vec{B}} \qquad \mathrm{div}\,\vec{B} = \emptyset \qquad \vec{B} = \mathcal{B}\{\vec{H}\} \qquad \vec{J} = \mathcal{J}\{\vec{E}\}$

IV. ÁLT. ESET **ELEKTROMÁGNESES HULLÁMOK**

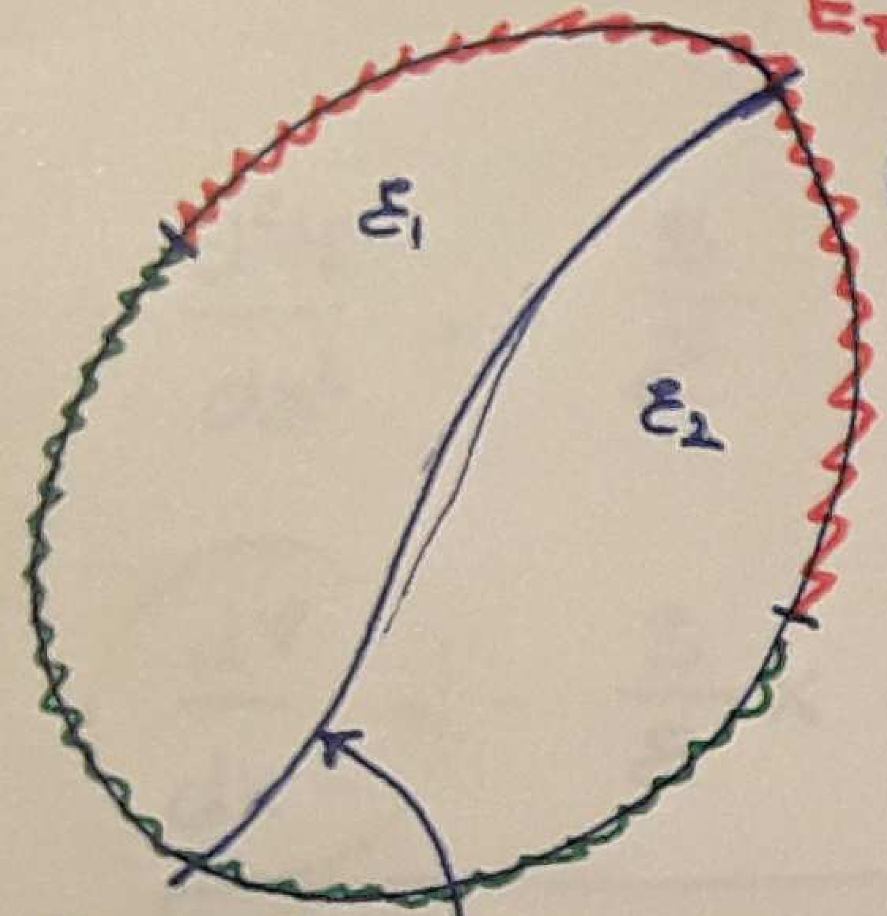
$\mathrm{rot}\,\vec{H} = \vec{J} + \partial\vec{D}/\partial t \qquad \mathrm{rot}\,\vec{E} = -\partial\vec{B}/\partial t \qquad \mathrm{div}\,\vec{B} = \emptyset \qquad \mathrm{div}\,\vec{D} = \varrho$

$\vec{B} = \mathcal{B}\{\vec{H}\} \qquad \vec{D} = \mathcal{D}\{\vec{E}\} \qquad \vec{J} = \mathcal{J}\{\vec{E}\}$

AZ ELEKTROSZTATIKA PARCIÁLIS DIFFERENCIÁLEGYENLETE

$E_T = \emptyset$

Φ



DIRICHLET

$D_N = \emptyset$

$\varepsilon \frac{\partial \varphi}{\partial n} = \emptyset$

E_T és D_N folytonos

NEUMANN

$\mathrm{rot}\,\vec{E} = \emptyset \longrightarrow \vec{E} = -\mathrm{grad}\,\varphi$

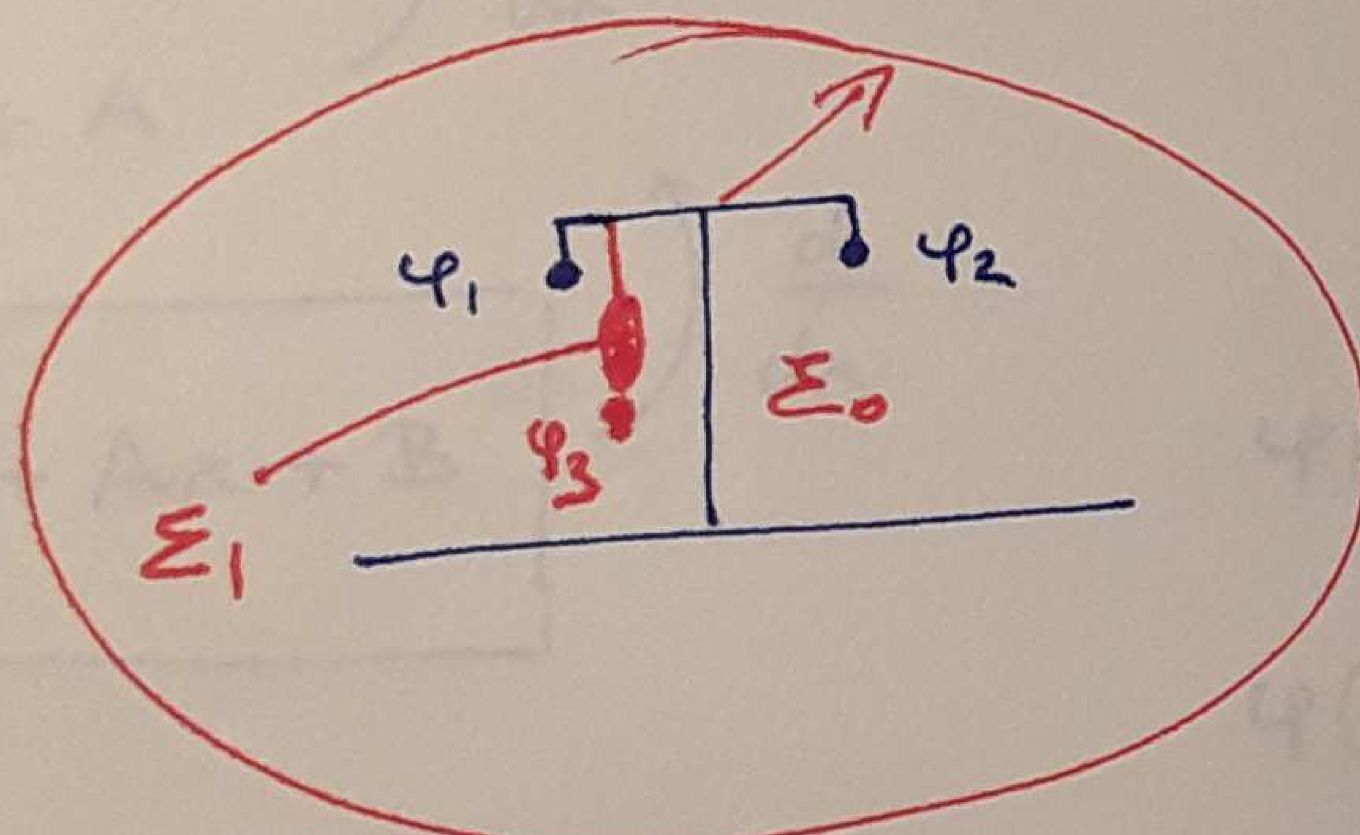
$\mathrm{div}\,\vec{D} = \rho$

$\vec{D} = \varepsilon \vec{E}$

$\vec{D} = -\varepsilon\,\mathrm{grad}\,\varphi$

$$\boxed{-\mathrm{div}(\varepsilon\,\mathrm{grad}\,\varphi) = \rho}$$

Laplace – Poisson



E_T :) φ folytonos

D_N :) $\varepsilon \frac{\partial \varphi}{\partial n}$ folytonos

Végeselem - módszer

HA ε konst. $\quad -\mathrm{div}\,\mathrm{grad}\,\varphi = \frac{\rho}{\varepsilon}$

$$\boxed{\Delta\varphi = -\frac{\rho}{\varepsilon}} \qquad \boxed{\Delta\varphi = \emptyset}$$

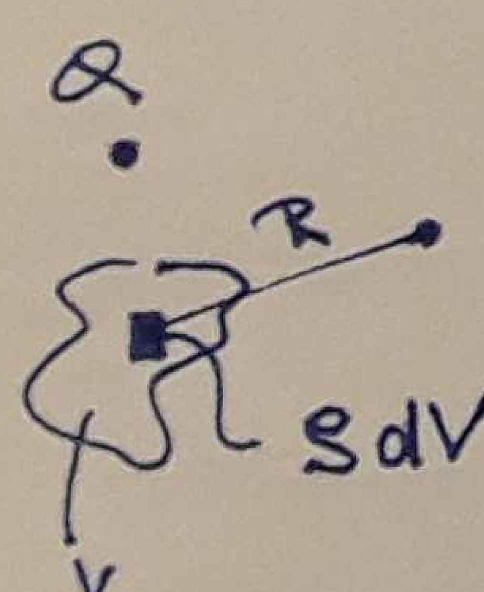
$$\Delta\varphi = -\frac{\rho}{\varepsilon}$$

$\to \varphi(x, y, z)$

$\vec{E} = -\mathrm{grad}\,\varphi$

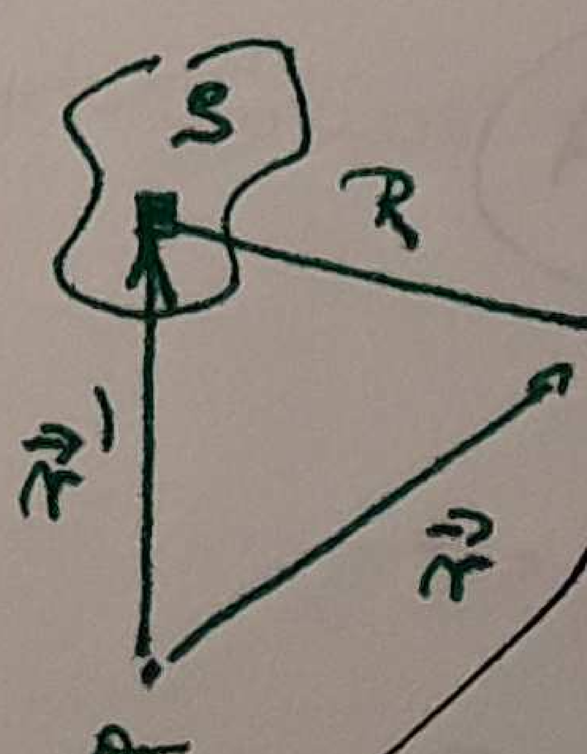
$\vec{D} = \varepsilon\vec{E}$

$w;\ C; \dots$



$$\varphi = \frac{Q}{4\pi\varepsilon R} \to \frac{\rho\,dV'}{4\pi\varepsilon R}$$

$$\boxed{\varphi = \frac{1}{4\pi\varepsilon} \int_V \frac{\rho(\vec{r}')\,dV'}{R}}$$



$\varphi(\vec{r}) = ?$

$\mathrm{rot}\,\vec{E} = \emptyset \qquad \vec{E} = -\mathrm{grad}\,\varphi$

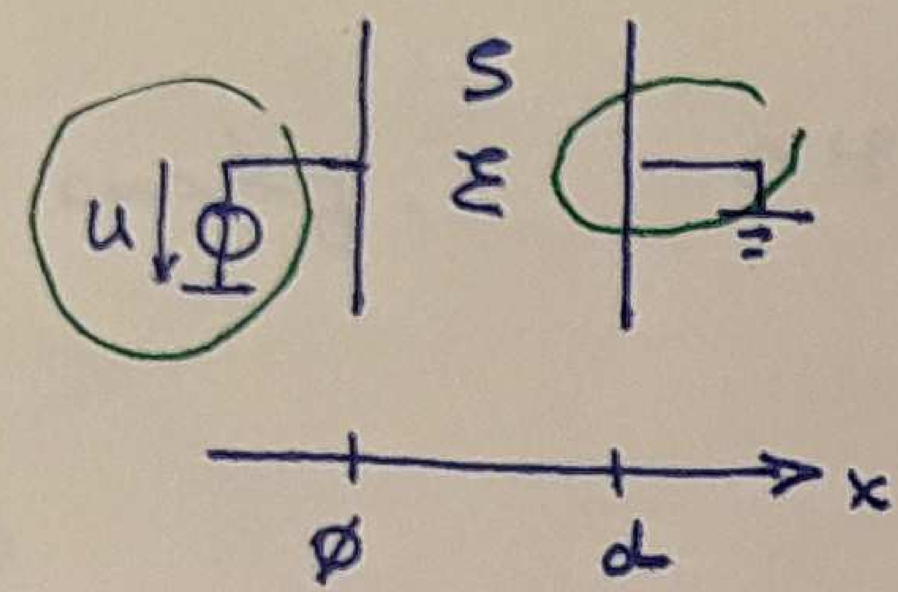
$\mathrm{div}\,\vec{J} = \emptyset \qquad \vec{J} = -\sigma\,\mathrm{grad}\,\varphi$

$\vec{J} = \sigma\vec{E}$

$$\boxed{-\mathrm{div}\,\sigma\,\mathrm{grad}\,\varphi = \emptyset}$$

$\Delta\varphi = \emptyset$

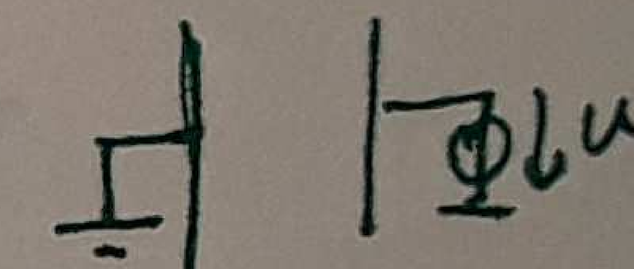
$$\Delta\varphi = -\frac{\varrho}{\varepsilon}$$



$$\varphi = \varphi(x)$$

$$\Delta\varphi = \frac{\partial^2\varphi}{\partial x^2} + \boxed{\cancel{\frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z}}}$$

$$\varphi = \varphi(x, y, z)$$



$$\frac{d^2\varphi}{dx^2} = -\frac{\varrho}{\varepsilon}$$

$$\frac{d}{dx}$$

$$\frac{d\varphi}{dx} = -\frac{\varrho}{\varepsilon}x + A$$

$$\frac{d}{dx}$$

$$\boxed{\varphi = -\frac{\varrho}{2\varepsilon}x^2 + Ax + B}$$

$A = ?$ $B = ?$

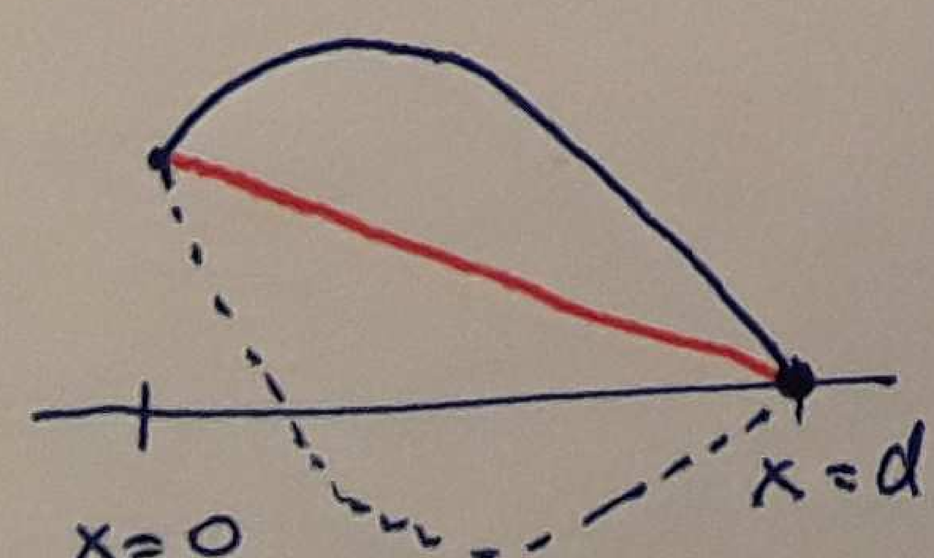
$$\varphi(0) = U = B$$

$$\varphi(d) = \underbrace{\emptyset = -\frac{\varrho}{2\varepsilon}d^2 + Ad + U}$$

$$\frac{\varrho}{2\varepsilon}d^2 - U = Ad$$

$$A = \frac{\varrho}{2\varepsilon}d - \frac{U}{d}$$

$$\varphi = -\frac{\varrho}{2\varepsilon}x^2 + \left[\frac{\varrho}{2\varepsilon}d - \frac{U}{d}\right]x + U$$



$$\varrho = \emptyset \qquad \varphi = -\frac{U}{d}x + U$$

$$\vec{E} = -\operatorname{grad}\varphi = -\frac{d\varphi}{dx}\vec{e_x} = \left(\frac{\varrho}{\varepsilon}x - \left[\frac{\varrho}{2\varepsilon}d - \frac{U}{d}\right]\right)\vec{e_x}$$

$$\varphi(x,y) = U e^{-kx} \cos ky . \qquad \Delta \varphi = \varnothing \ ?$$

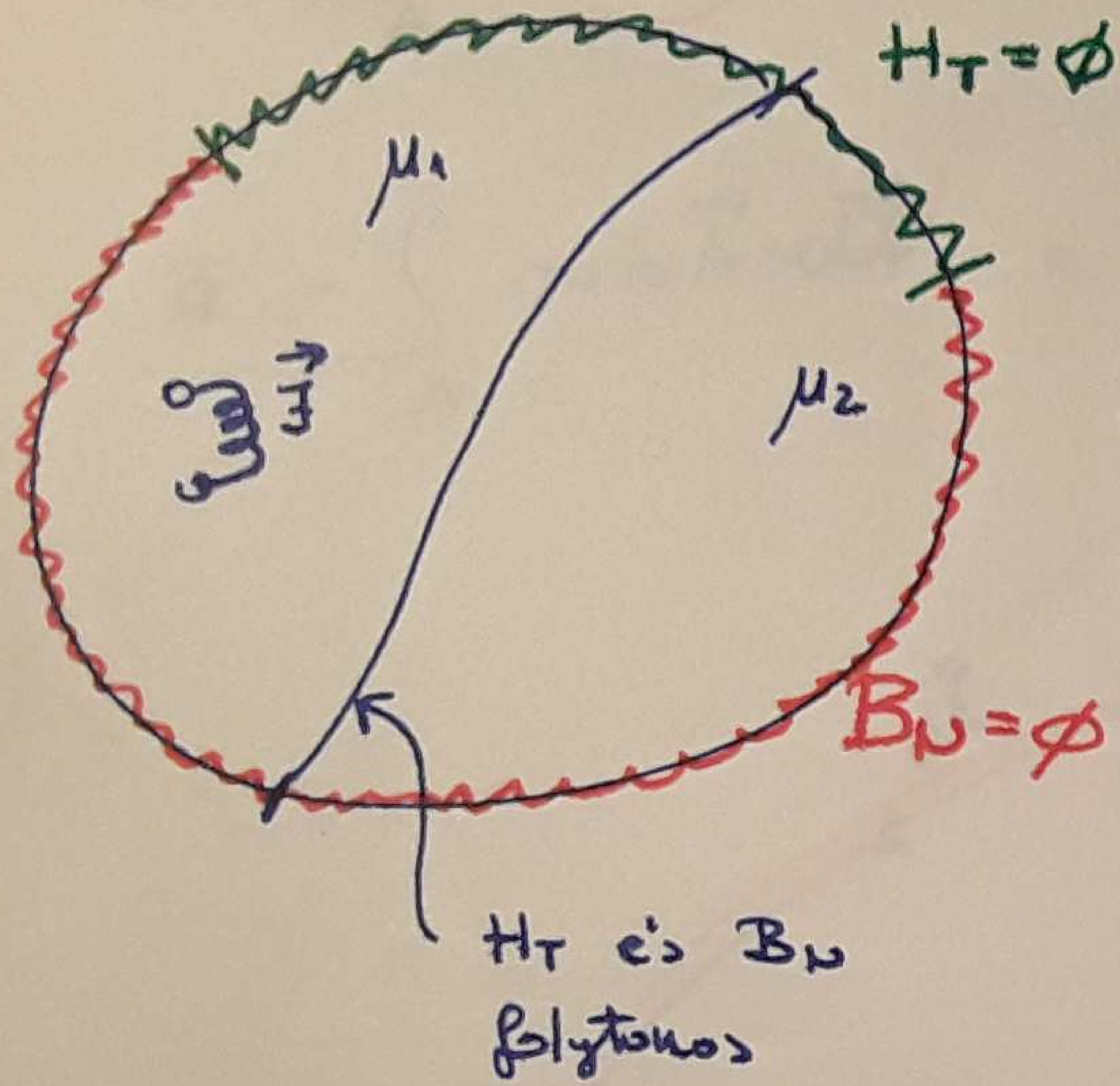
$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \overset{?}{=} \varnothing$$

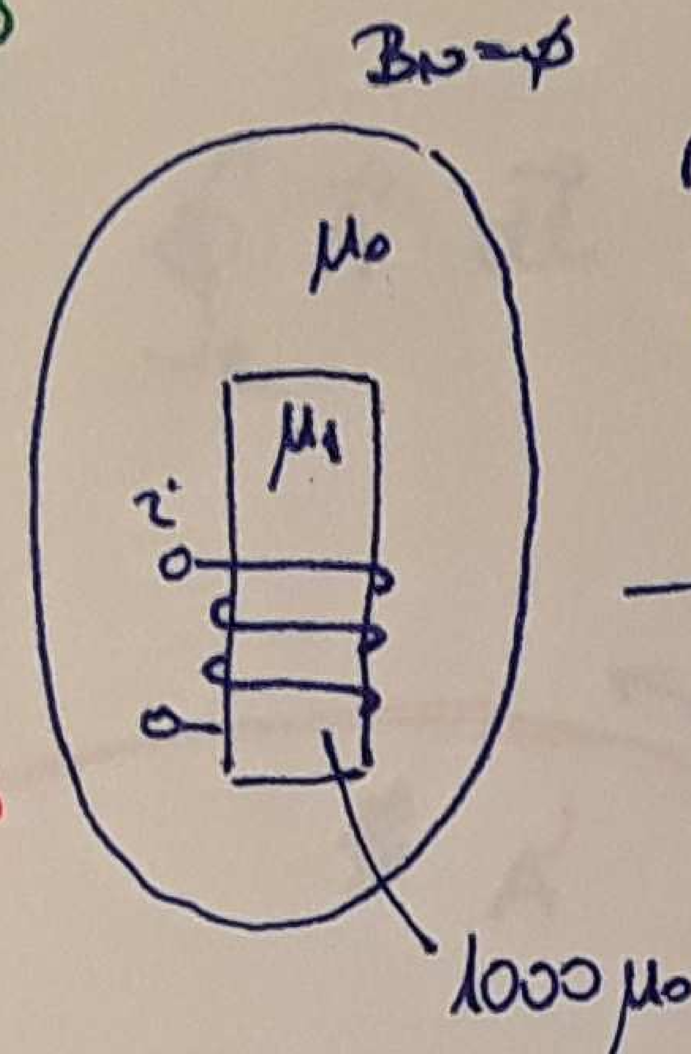
$$\frac{\partial \varphi}{\partial x} = U\left[-k e^{-kx}\right] \cos ky \qquad \frac{\partial^2 \varphi}{\partial x^2} = \underline{U\left[k^2 e^{-kx}\right]} \cos ky$$

$$\frac{\partial \varphi}{\partial y} = U e^{-kx}\left[-k \sin ky\right] \qquad \frac{\partial^2 \varphi}{\partial x^2} = U e^{-kx} \underline{\left[-k^2 \cos ky\right]}$$

$$\varnothing$$

STACIONÁRIUS MÁGNESES TÉR PARCIÁLIS DIFFERENCIÁLEGYENLETE





$\mathrm{rot}\,\vec{H} = \vec{J}$

$\mathrm{div}\,\vec{B} = \emptyset \longrightarrow \boxed{\vec{B} = \mathrm{rot}\,\vec{A}}$

$\vec{B} = \mu\vec{H}$

$\vec{H} = \frac{1}{\mu}\,\mathrm{rot}\,\vec{A}$

$$\boxed{\mathrm{rot}\left[\frac{1}{\mu}\,\mathrm{rot}\,\vec{A}\right] = \vec{J}}$$

B_N:) A_T ← DIRICHLET

H_T:) $\frac{1}{\mu}\,\mathrm{rot}\,A$ ← NEUMANN

$\mathrm{div}\,\mathrm{rot}\,\vec{v} = \emptyset$

$\mathrm{rot}\,\mathrm{rot}\,\vec{A} = \mu\vec{J}$

$\mathrm{grad}\,\underbrace{\mathrm{div}\,\vec{A}}_{\emptyset} - \Delta\vec{A} = \mu\vec{J} \longrightarrow \boxed{\Delta\vec{A} = -\mu\vec{J}}$

Coulomb-mérték.

$\Delta A_x = -\mu J_x$

$\Delta A_y = -\mu J_y$

$\Delta A_z = -\mu J_z$

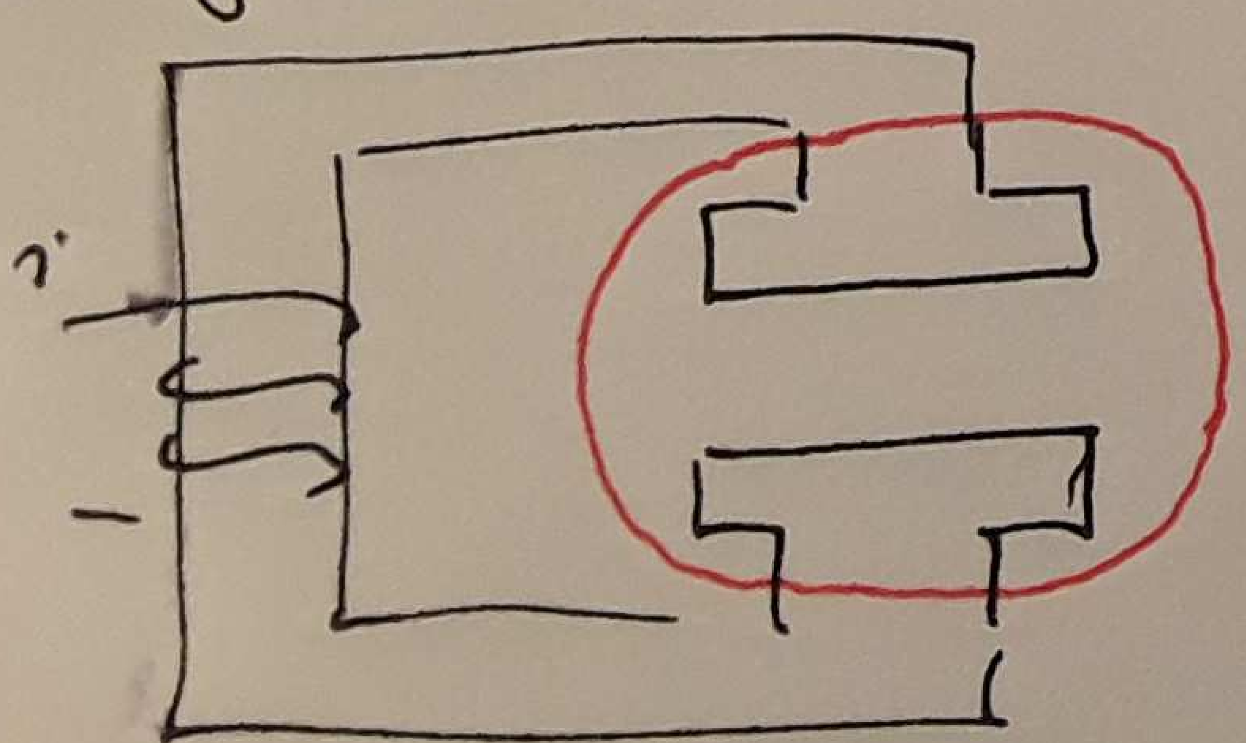
div grad

$\mathrm{rot}\,\vec{H} = \emptyset$

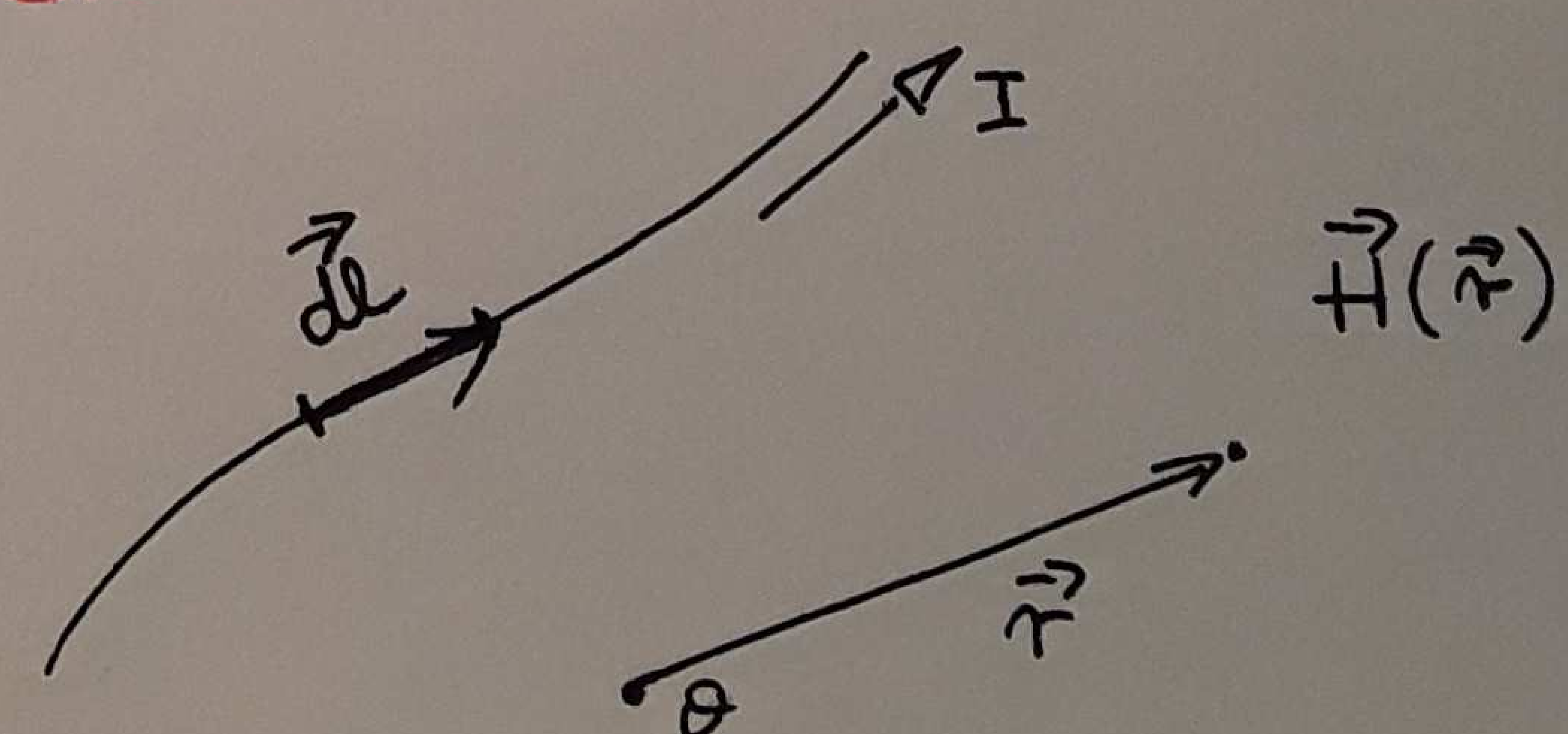
$\mathrm{div}\,\vec{B} = \emptyset$

$\cancel{\vec{B} = \mu\vec{H}}$ + $\vec{B} = \mathcal{B}\{\vec{H}\}$

$\vec{H} = -\mathrm{grad}\,\varphi$.



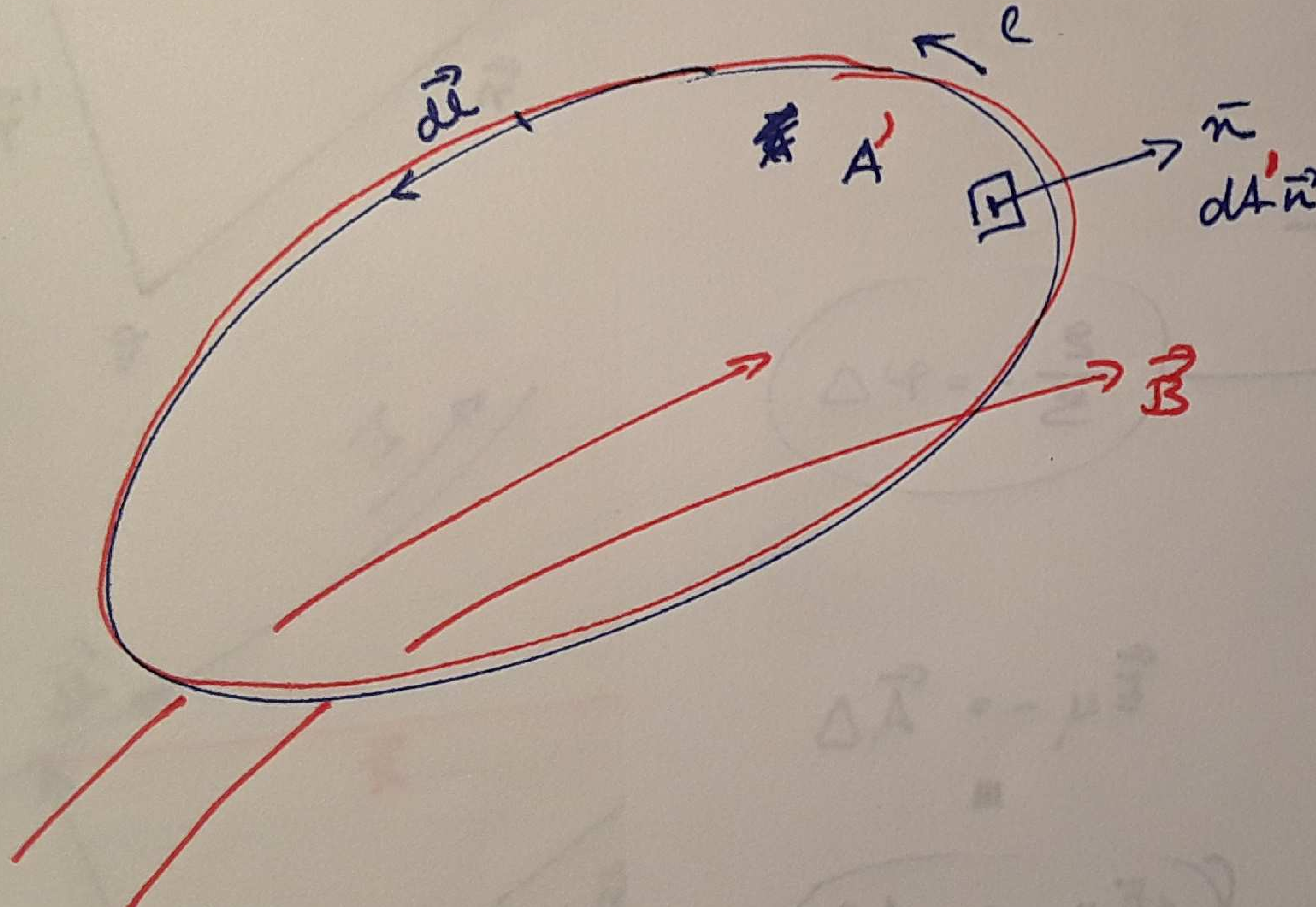
BIOT-SAVART-TÖRVÉNY



$$\Phi = \int_A \vec{B} \cdot d\vec{A}'$$

$$\vec{B} = \text{rot}\,\vec{A}$$

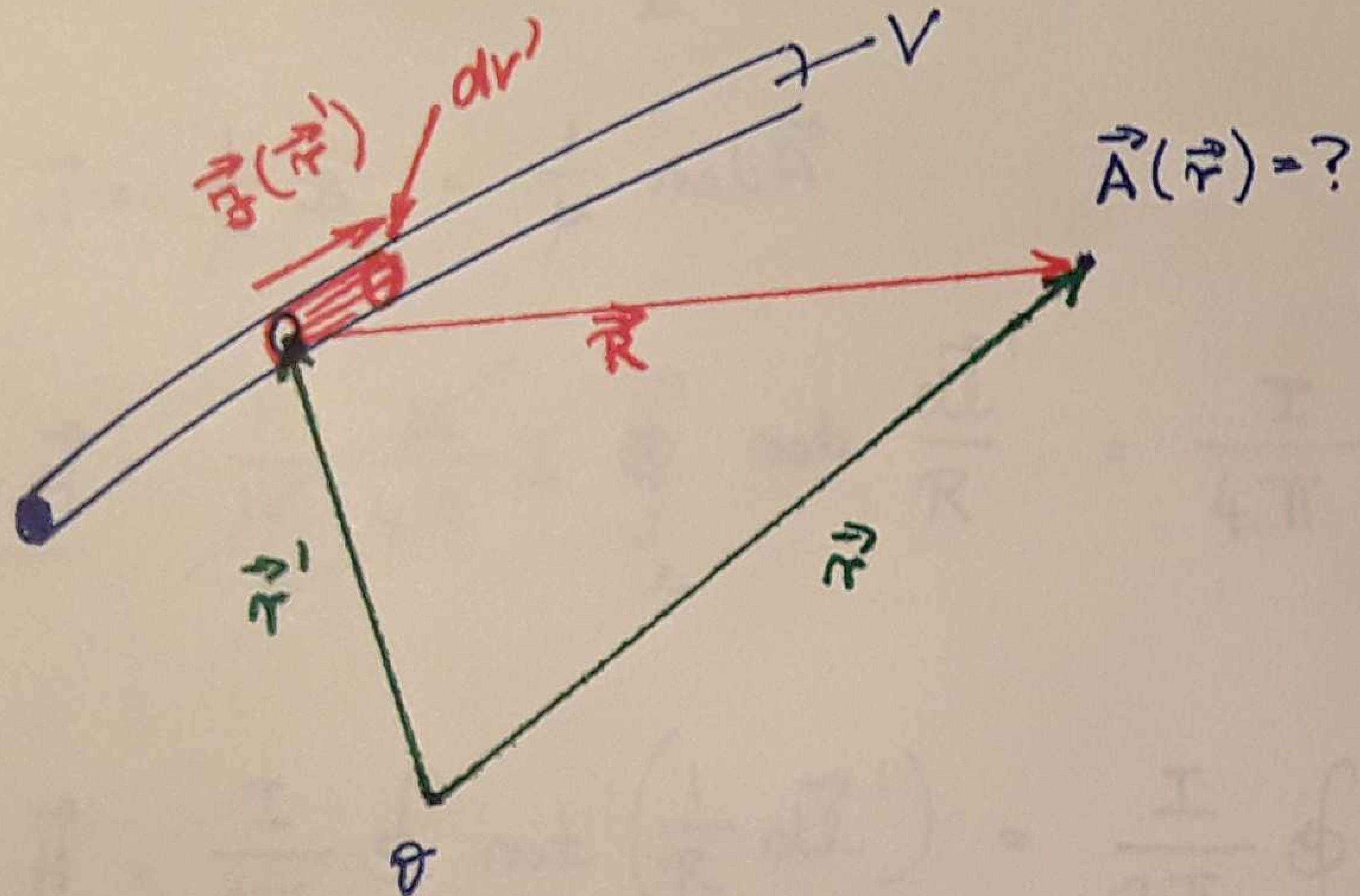
$$\Phi = \int_A \text{rot}\,\vec{A} \cdot d\vec{A}' = \oint_l \vec{A} \cdot d\vec{l}$$



$$\Phi = \int_A \vec{B} \cdot d\vec{A}'$$

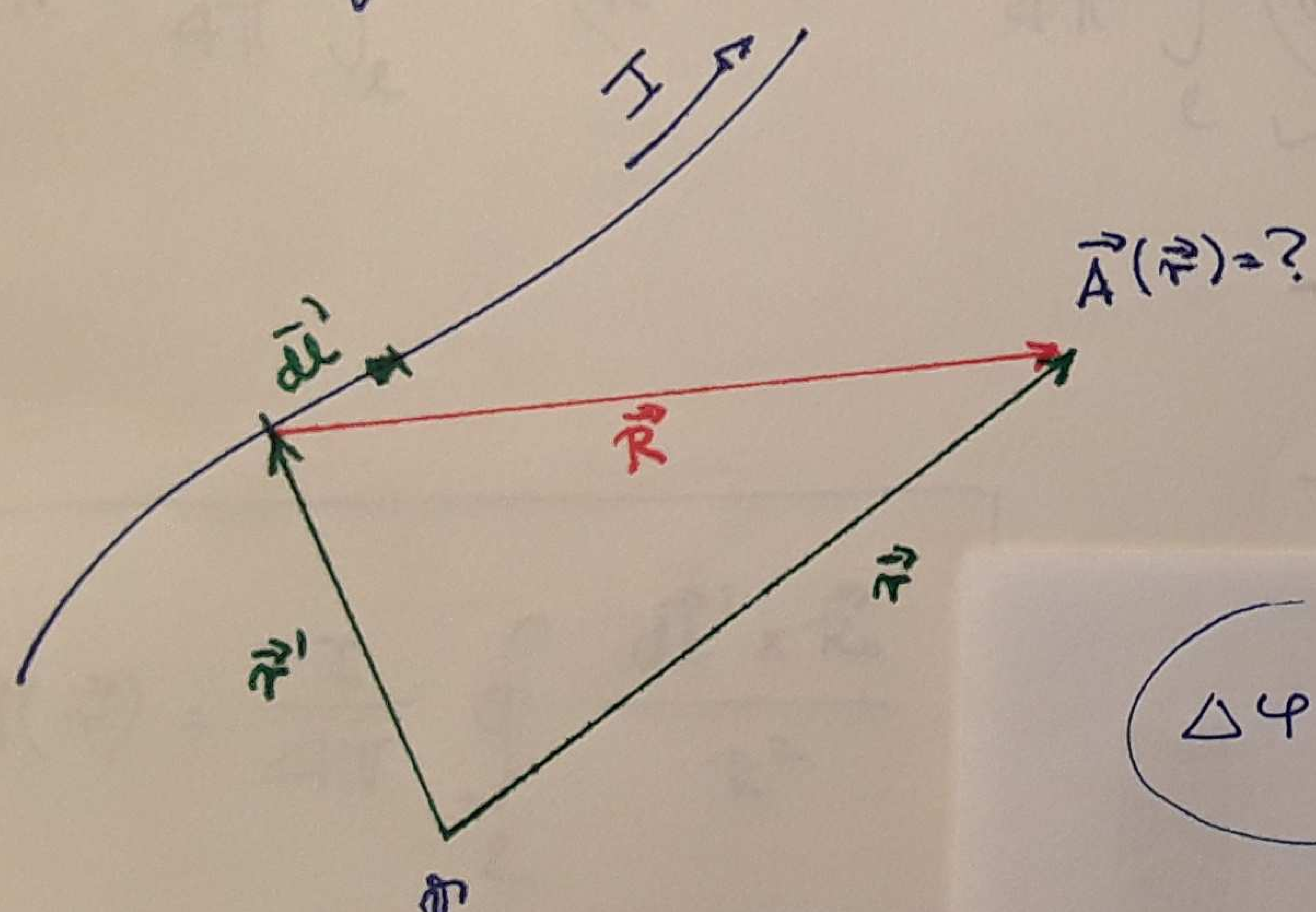
$$\Phi = \oint_l \vec{A} \cdot d\vec{l}$$

A BIOT - SAVART - TÖRVÉNY



$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int_{V} \frac{\vec{J}(\vec{r}')}{R}\, dV' \qquad R = |\vec{r} - \vec{r}'|$$

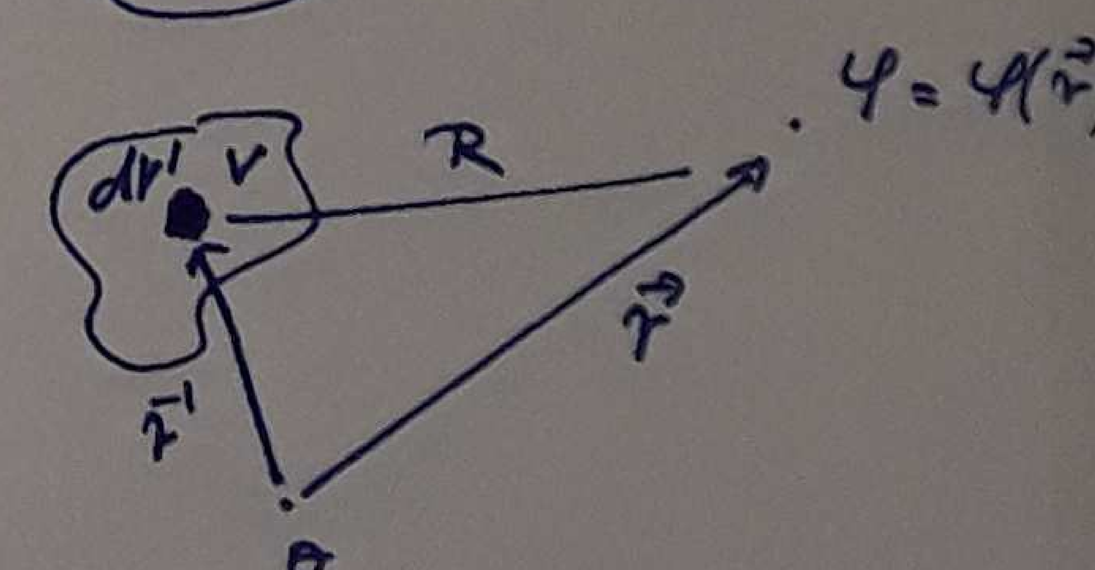
$$\vec{J}\, dV' = \underbrace{J A}_{I}\, d\vec{l}' = I\, d\vec{l}' \qquad \vec{J} \parallel d\vec{l}'$$



$$\boxed{\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\oint_{l} \frac{I\, d\vec{l}'}{R}}$$

$\longrightarrow$ $\vec{H}$

$$\Delta \varphi = -\frac{\rho}{\varepsilon} \longrightarrow \varphi = \frac{1}{4\pi\varepsilon}\int_{V} \frac{\rho(\vec{r}')}{R}\, dV'$$



$$\Delta \vec{A} = -\mu \vec{J}$$

$$\equiv$$

$$\left.\begin{aligned} \Delta A_x &= -\mu J_x \\ \Delta A_y &= -\mu J_y \\ \Delta A_z &= -\mu J_z \end{aligned}\right\} \longrightarrow \vec{A} = \frac{\mu}{4\pi}\int_{V} \frac{\vec{J}(\vec{r}')}{R}\, dV'$$

$$\vec A(\vec r) = \frac{\mu}{4\pi} I \oint_l \frac{d\vec l'}{R}$$

$$\vec H = \frac{1}{\mu}\vec B = \frac{1}{\mu}\,\text{rot}\,\vec A$$

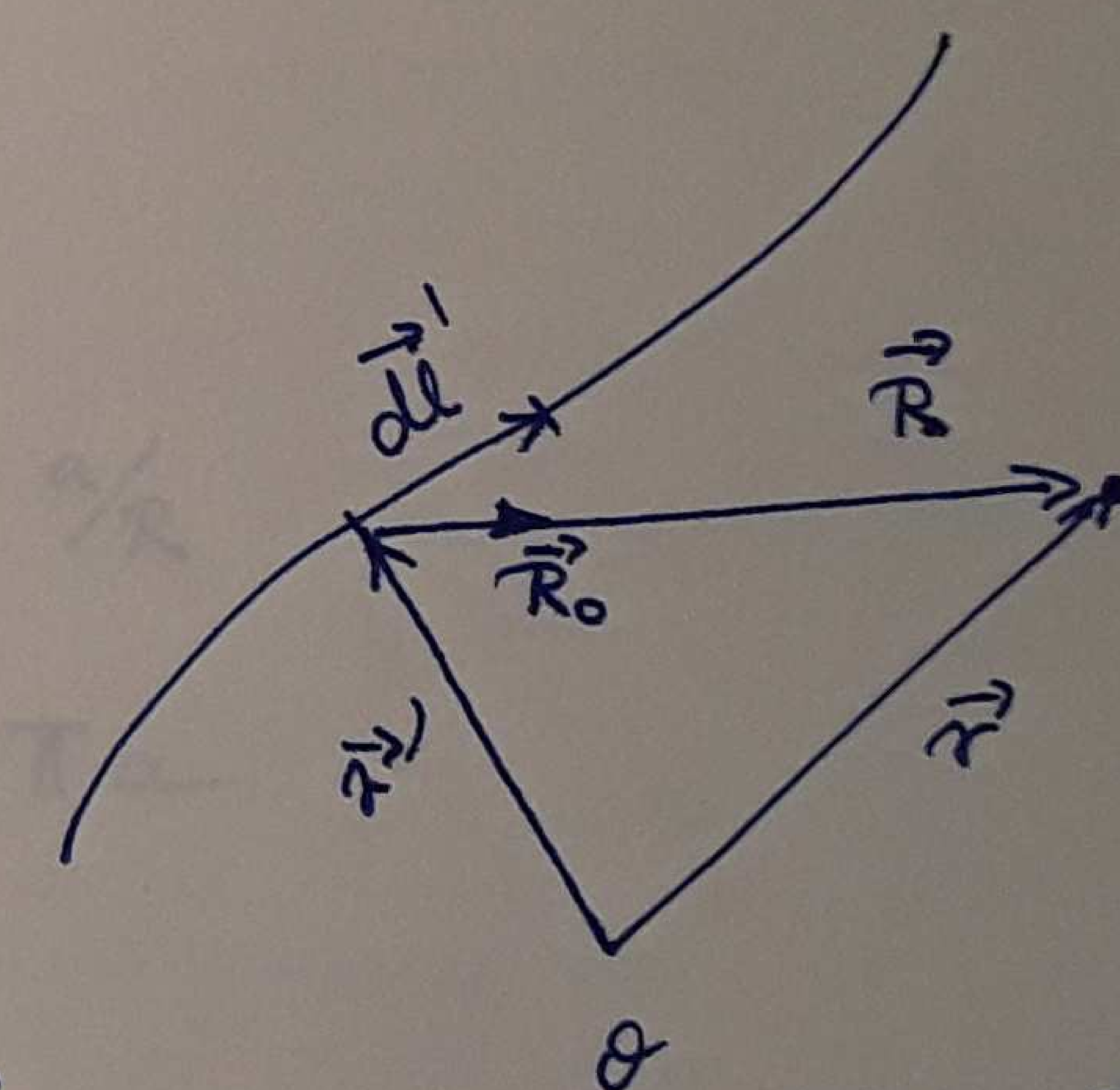
$$\text{rot}(\varphi\vec v) = \text{grad}\,\varphi \times \vec v + \varphi\,\text{rot}\,\vec v$$

$$\text{grad}\frac{1}{R} \times d\vec l' + \cancel{\underbrace{\frac{1}{R}\,\text{rot}\,d\vec l'}_{\emptyset}}$$

$$\vec H = \cancel{\frac{1}{\mu}}\,\frac{\cancel{\mu}}{4\pi} I \oint_l \text{rot}\,\frac{d\vec l'}{R} = \frac{I}{4\pi}\oint_l \text{rot}\Big(\underset{\varphi}{\underset{\uparrow}{\frac{1}{R}}}\,\underset{\vec v}{\underset{\uparrow}{d\vec l'}}\Big)$$

$$\vec H = \frac{I}{4\pi}\oint_l \text{rot}\left(\frac{1}{R}\,d\vec l'\right) = \frac{I}{4\pi}\oint_l \Big(\underbrace{\text{grad}\,\frac{1}{R}}_{-\frac{\vec R_0}{R^2}} \times d\vec l'\Big)$$

$$-\frac{\vec R_0}{R^2}\times d\vec l' = \frac{d\vec l' \times \vec R_0}{R^2}$$



$\vec H(\vec r) = ?$

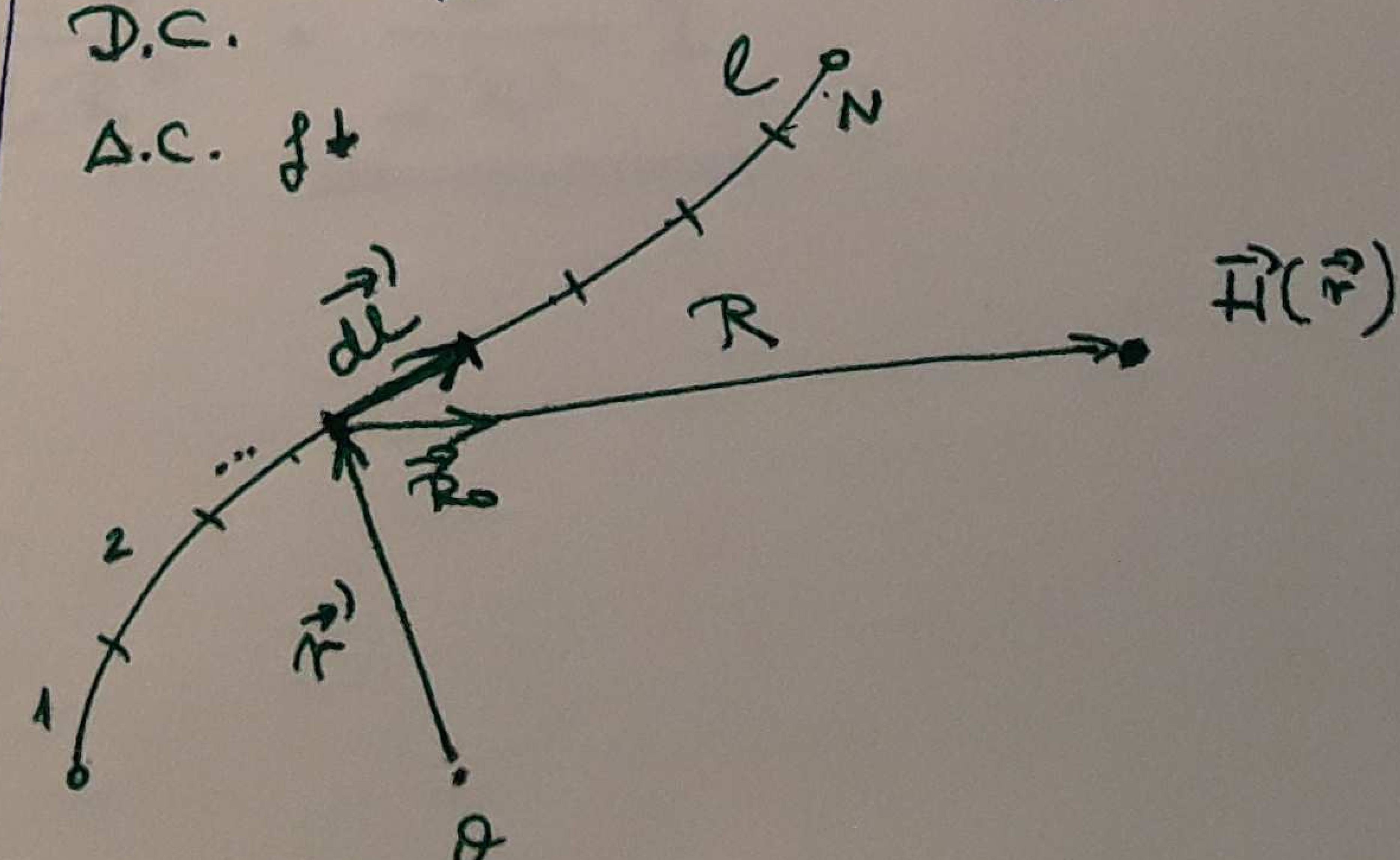
$|\vec R_0| = 1$

$\vec a \times \vec b = -\vec b \times \vec a$

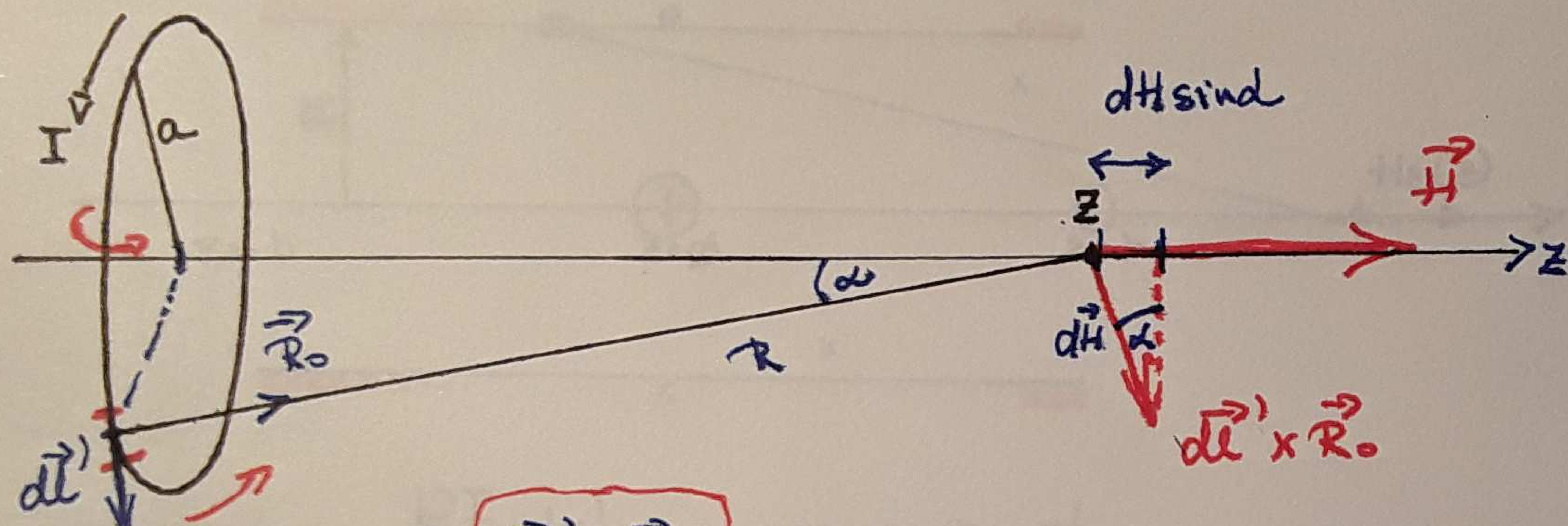
$$\boxed{\vec H(\vec r) = \frac{I}{4\pi}\oint_l \frac{d\vec l' \times \vec R_0}{R^2}}$$

D.C.

A.C. $f\downarrow$



Határozzuk meg a köráram mágneses terét a köráram síkjára merőlegesen, a tengely mentén!



$$d\vec{H} = \frac{I}{4\pi} \frac{d\vec{l} \times \vec{R}_0}{R^2}$$

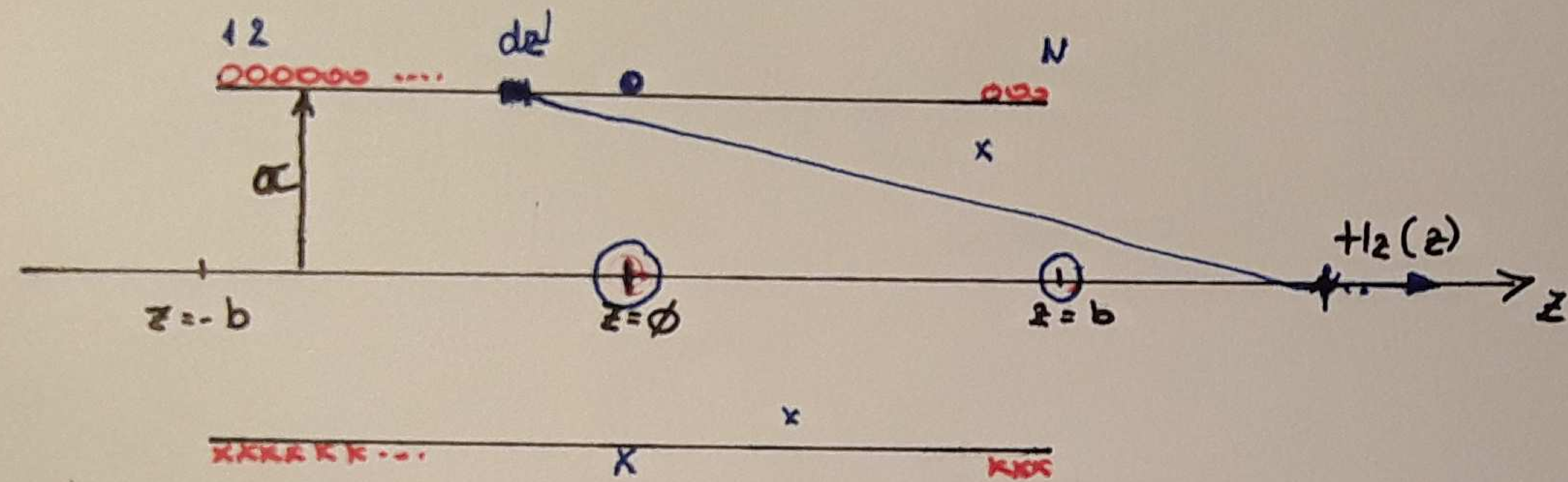
$$H_z(z) = \frac{I}{4\pi} \oint_l \boxed{\frac{d\vec{l}' \times \vec{R}_0}{R^2}}$$

$$= \frac{I}{4\pi} \oint_l \frac{dl' \sin\alpha}{R^2} = \frac{I \sin\alpha}{4\pi R^2} \underbrace{\oint_l dl'}_{} = \frac{I \overbrace{\sin\alpha}^{a/R}}{4\pi R^2} 2\pi a =$$

$$= \frac{I a}{4\cancel{\pi} R^3} 2\cancel{\pi} a = \frac{I a^2}{2R^3} = \underline{\underline{\frac{a^2}{2R^3} I}}$$

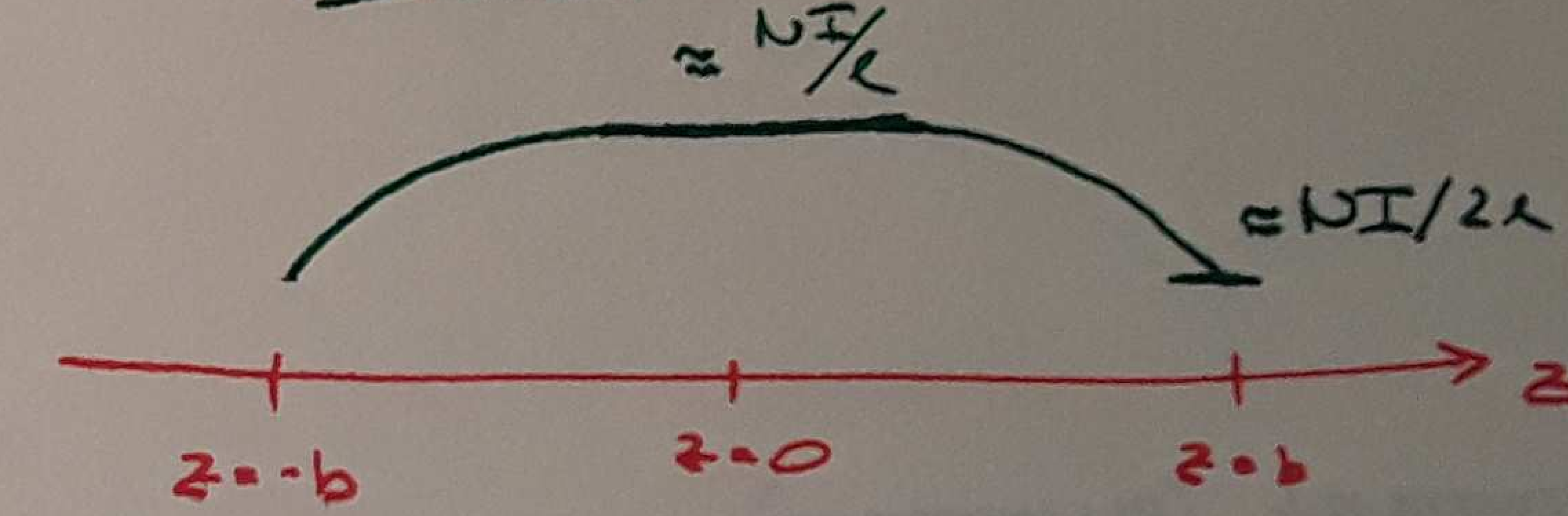
$$\underline{\underline{H_z(z) = \frac{a^2}{2(a^2+z^2)^{3/2}} I}}$$

Határozzuk meg az N menetű, sűrűn tekercselt szolenoid mágneses terét a tengely mentén!



$$H_z(z) = \frac{a^2}{2(a^2+z^2)^{3/2}} I$$

$$\boxed{H \approx \frac{NI}{\ell}} \qquad (\ell = 2b)$$



$$I \Rightarrow \frac{NI}{2b} dz' \qquad z \to z - z'$$

$$dH_z(z) = \frac{a^2}{2\left[a^2+(z-z')^2\right]^{3/2}} \frac{NI}{2b} dz'$$

$$H_z(z) = \int_{-b}^{b} \frac{a^2}{2\left[a^2+(z-z')^2\right]^{3/2}} \frac{NI}{2b} dz' = \frac{a^2 NI}{4b}\left[\frac{z'-z}{a^2\sqrt{(z'-z)^2+a^2}}\right]_{-b}^{+b} =$$

$$= \frac{NI}{4b}\left[\frac{b-z}{\sqrt{(b-z)^2+a^2}} + \frac{b+z}{\sqrt{(b+z)^2+a^2}}\right]$$

$z = \varnothing$

$$H_z(0) = \frac{NI}{4b} \frac{2b}{\sqrt{b^2+\cancel{a^2}}} \approx \frac{NI}{2b} = \frac{NI}{\ell}$$

$\approx 2b/b$

$a \to \varnothing$

$z = b$

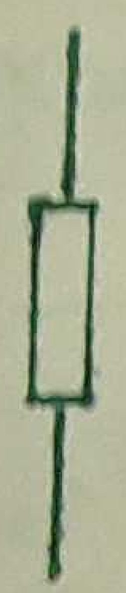
$$H_z(b) = \frac{NI}{4b} \frac{2b}{\sqrt{4b^2+\cancel{a^2}}} \approx \frac{NI}{4b} = \frac{NI}{2\ell}$$

$\frac{2b}{2b}$

$a \to \varnothing \quad (b \gg a)$

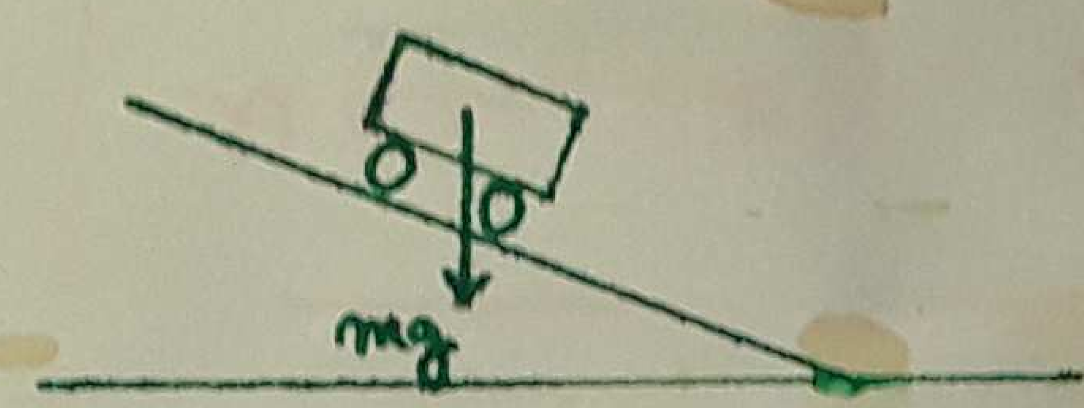
TÁPVONAL-ELMÉLET, TÁVVEZETÉKEK

Koncentrált paraméterű modellek

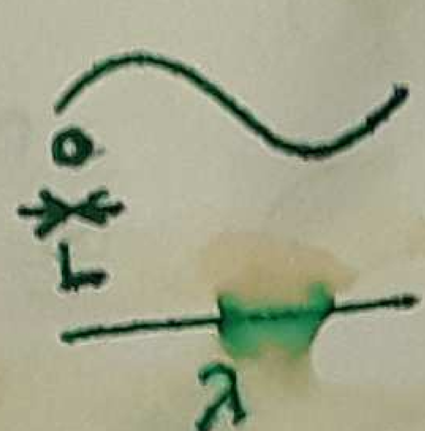


$u = R\,i$

$\uparrow$



$\lambda = \frac{3\cdot 10^8}{50} = 6000\,km$



$\boxed{L \ll \lambda}$

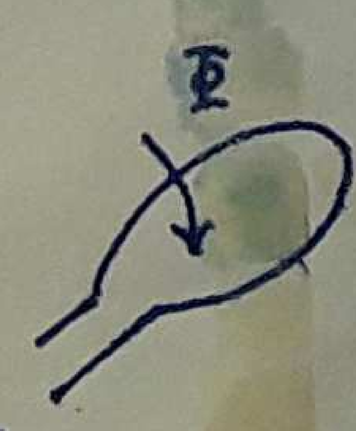
R Ω/m (R')

L H/m (L')

G S/m (G')

C F/m (C')

$S\,\frac{l}{A}$



Elosztott paraméterű modellek

$\left.\begin{matrix} f \\ c \end{matrix}\right\}$ $c = f\lambda$

$\boxed{\lambda = \frac{c}{f}}$

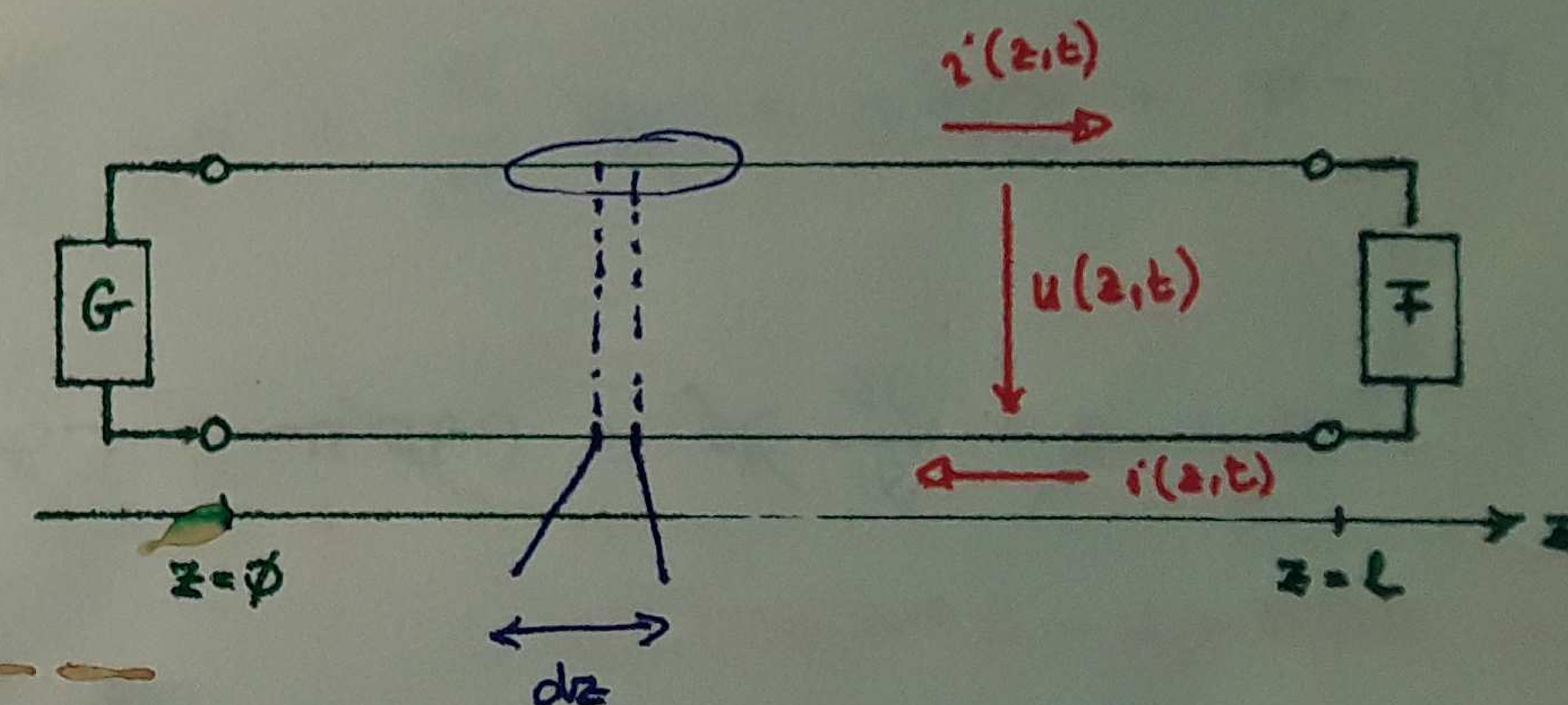
$f\uparrow$ $1\,GHz \rightarrow$

$\lambda = \frac{3\cdot 10^8}{10^9} = 0{,}3\,m$

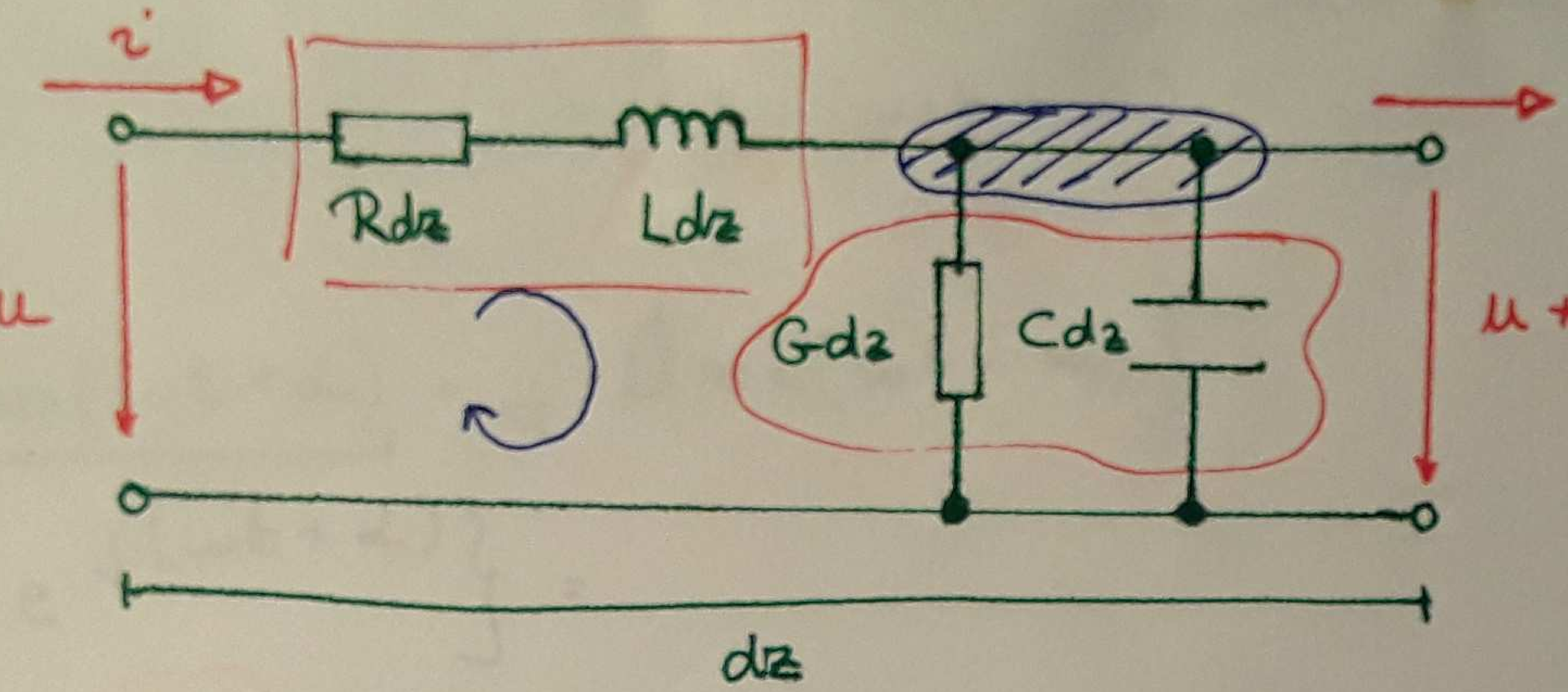
$30\,cm$

$L \overset{!}{\approx} \lambda$

$\vec{H}(\vec{r}, t)$



INFINITEZIMÁLIS



$$i + \Delta i = i + \underbrace{\frac{\partial i}{\partial z} dz}$$

$$u + \Delta u = u + \underbrace{\frac{\partial u}{\partial z} dz}$$

$u = u(z,t) \quad i = i(z,t)$

$i_C = C\frac{du_C}{dt} \quad u_L = L\frac{di_L}{dt}$

TELEGRÁF - EGYENLET

1) $\cancel{-i} + \cancel{i} + \frac{\partial i}{\partial z}\cancel{dz} + G\cancel{dz}\,u + C\cancel{dz}\frac{\partial u}{\partial t} = \phi$

2) $\cancel{-u} + R\cancel{dz}\,i + L\cancel{dz}\frac{\partial i}{\partial t} + \cancel{u} + \frac{\partial u}{\partial z}\cancel{dz} = \phi$

$$\boxed{\frac{\partial i}{\partial z} = -Gu - C\frac{\partial u}{\partial t}}$$

$$\boxed{\frac{\partial u}{\partial z} = -Ri - L\frac{\partial i}{\partial t}}$$

Szinuszos

Komplex

$i \rightarrow I(z)$, $\hat{\underline{I}}$ $\quad u \rightarrow U(z)$, $\hat{U}$

$\frac{\partial i}{\partial t} \rightarrow j\omega I \quad \frac{\partial u}{\partial t} \rightarrow j\omega U$

$u(z,t) \Rightarrow$

$u(t) = \hat{U}\cos(\omega t + \alpha)$

$$\boxed{\hat{\underline{U}} = \hat{U}e^{j\alpha}}$$

$\hat{\underline{U}}(z) = \hat{U}(z)e^{j\alpha(z)}$

$\cancel{\frac{\partial I}{\partial z}}$

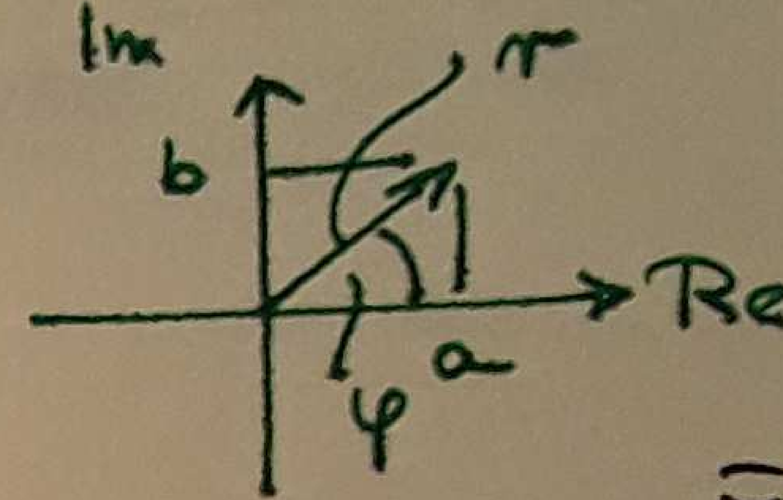
$\frac{dI}{dz} = -GU - j\omega C U$

$\frac{dU}{dz} = -RI - j\omega L I$

$$\frac{dI}{dz} = -\underbrace{(G + j\omega C)}_{\text{ADM.}}U$$

$$\frac{dU}{dz} = -\underbrace{(R + j\omega L)}_{\text{IMP.}}I$$

$u(t) = \hat{U}\cos(\omega t + \alpha)$



$j = \sqrt{-1}$

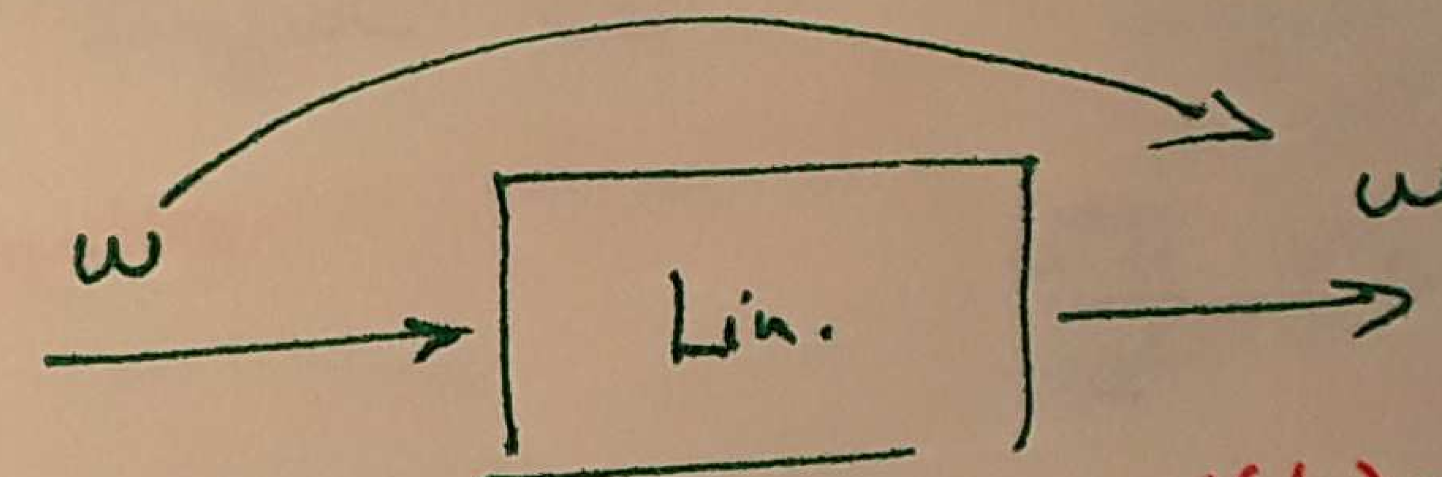
$$\bar{Z} = a + jb = r\cos\varphi + j\,r\sin\varphi = r e^{j\varphi}$$

$$u(t) = \mathrm{Re}\left\{\underbrace{\hat{U}\cos(\omega t + \alpha)} + j\,\hat{U}\sin(\omega t + \alpha)\right\} =$$

$$= \mathrm{Re}\left\{\hat{U} e^{j(\omega t + \alpha)}\right\} =$$

$$= \mathrm{Re}\left\{\hat{U} e^{j\omega t} e^{j\alpha}\right\}$$

$$\boxed{\hat{\bar{U}} = \hat{U} e^{j\alpha}}$$



$u(t) = \hat{U}\cos(\omega t + \alpha)$

$\hat{\bar{U}} = \hat{U} e^{j\alpha}$

$i(t) = \hat{I}\cos(\omega t + \beta)$

A telegráf-egyenletek megoldása speciálisan szinuszos gerjesztés mellett.

$\hat{U}$ U

$$1.)\ \frac{dI}{dz} = -\underline{(G+j\omega C)}\,U \qquad I(z) \longrightarrow i(z,t)$$

$$2.)\ \frac{dU}{dz} = -(R+\cancel{j\omega L})\,I \qquad U(z) \longrightarrow u(z,t)$$

$$1.)\quad U = -\frac{1}{G+j\omega C}\frac{dI}{dz} \rightarrow \frac{dU}{dz} = -\frac{1}{G+j\omega C}\frac{d^2I}{dz^2} \qquad +\frac{1}{G+j\omega C}\frac{d^2I}{dz^2} = +(R+j\omega L)\,I$$

$$\boxed{\frac{d^2I}{dz^2} - \gamma^2 I = \emptyset}$$

Helmholtz

$$\frac{d^2I}{dz^2} - \underbrace{(G+j\omega C)(R+j\omega L)}_{\gamma^2} I = \emptyset$$

$$I(z) \qquad U = -\frac{1}{G+j\omega C}\frac{dI}{dz}$$

$$\gamma = \sqrt{(G+j\omega C)(R+j\omega L)}$$

Terjedési együttható

$$2.)\quad I = -\frac{1}{R+j\omega L}\frac{dU}{dz} \rightarrow \frac{dI}{dz} = -\frac{1}{R+j\omega L}\frac{d^2U}{dz^2} \qquad +\frac{1}{R+j\omega L}\frac{d^2U}{dz^2} = +(G+j\omega C)\,U$$

$$\boxed{\frac{d^2U}{dz^2} - \gamma^2 U = \emptyset}$$

$$\frac{d^2U}{dz^2} - \underbrace{(R+j\omega L)(G+j\omega C)}_{\gamma^2} U = \emptyset$$

$$U(z) \qquad I = -\frac{1}{R+j\omega L}\frac{dU}{dz}$$

$$\gamma = \alpha + j\beta$$

α - csillapítási t.

β - fázist.

$$\frac{d^2u}{dz^2} - \gamma^2 u = \emptyset \qquad u_h = Me^{\lambda z} \quad \text{sajátérték} \qquad (e^{\lambda z})' = \lambda e^{\lambda z}$$

?

$$u_h' = M\lambda e^{\lambda z}$$
$$u_h'' = M\lambda^2 e^{\lambda z}$$

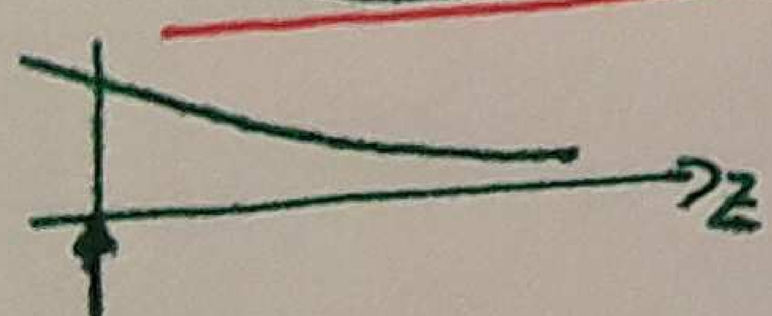
$$\lambda^2 Me^{\lambda z} - \gamma^2 Me^{\lambda z} = \emptyset$$

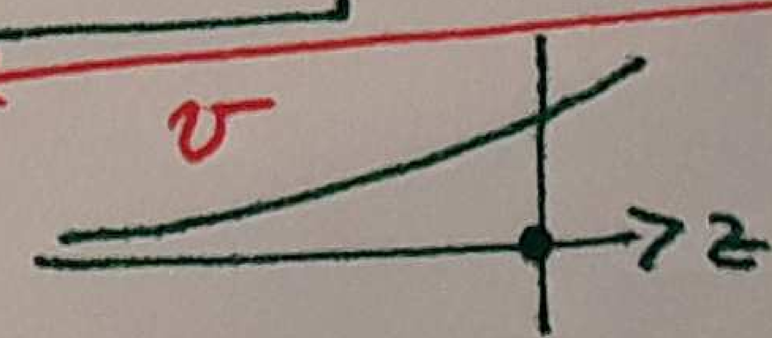
$$\lambda^2 - \gamma^2 = \emptyset \qquad \lambda^2 = \gamma^2 \qquad \lambda_{1,2} = \pm\gamma \quad \begin{cases} M_1 e^{\gamma z} \\ M_2 e^{-\gamma z} \end{cases} \quad \underbrace{M_1 e^{\gamma z}}_{U^-} + \underbrace{M_2 e^{-\gamma z}}_{U^+} \quad !$$

$$\boxed{u(z) = U^+ e^{-\gamma z} + U^- e^{\gamma z}} \qquad \gamma = \alpha + j\beta$$

$$u(z,t) = \mathrm{Re}\left\{ U^+ e^{j\omega t} e^{-(\alpha+j\beta)z} + U^- e^{j\omega t} e^{(\alpha+j\beta)z} \right\} =$$
$$= \mathrm{Re}\left\{ U^+ e^{j\omega t} e^{-\alpha z} e^{-j\beta z} + U^- e^{j\omega t} e^{\alpha z} e^{j\beta z} \right\}$$
$$= \mathrm{Re}\left\{ \underbrace{U^+ e^{-\alpha z}}\, e^{j(\omega t - \beta z)} + \underbrace{U^- e^{\alpha z}}\, e^{j(\omega t + \beta z)} \right\}$$

$v \rightarrow$ $\qquad \leftarrow v$





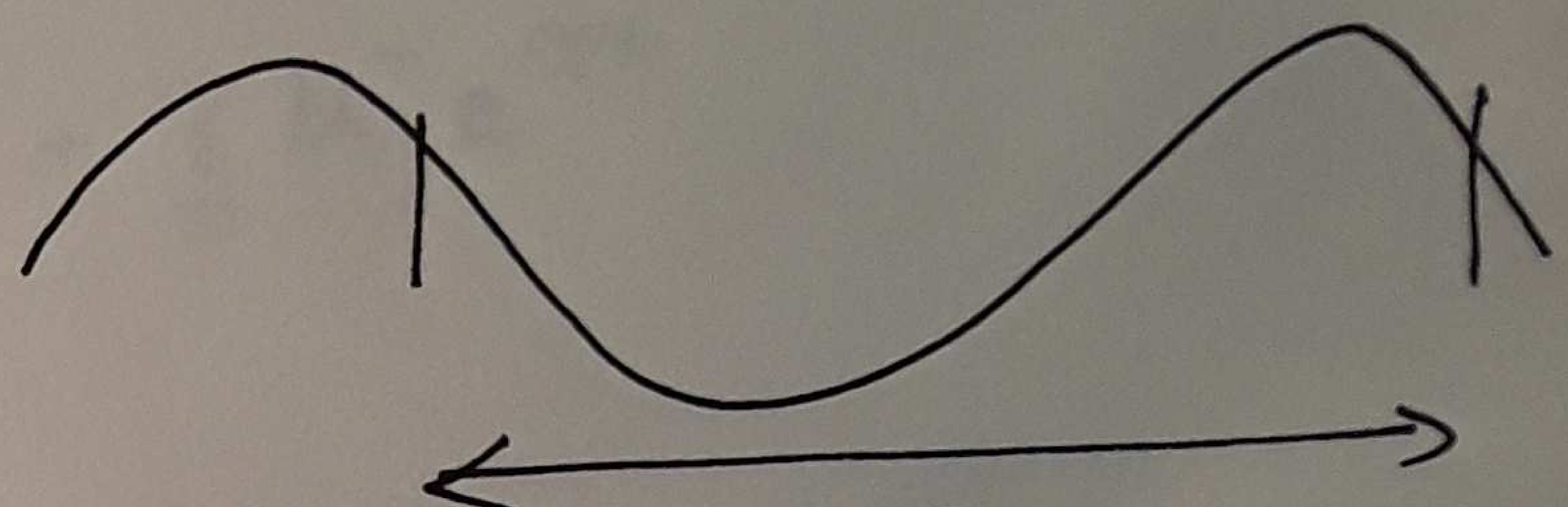
fázissebesség

$$e^{j(\omega t - \beta z)} = e^{j\omega\left(t - \frac{z}{v}\right)}$$

$$\boxed{v = \frac{\omega}{\beta}} \qquad \boxed{\beta = \frac{\omega}{v}}$$

$$\omega t - \beta z + 2\pi = \omega t - \beta(z - \lambda)$$

$$\cancel{\omega t} - \cancel{\beta z} + 2\pi = \cancel{\omega t} - \cancel{\beta z} + \beta\lambda$$



$$2\pi = \beta\lambda$$

$$\boxed{\lambda = \frac{2\pi}{\beta}} \qquad \boxed{v = \frac{\omega}{\beta}}$$

$$I(z) = -\frac{1}{R+j\omega L}\frac{dU}{dz}$$

$$U = \underset{\longrightarrow}{U^+ e^{-\gamma z}} + \underset{\longleftarrow}{U^- e^{\gamma z}}$$

$$\frac{dU}{dz} = -\underline{\gamma} U^+ e^{-\gamma z} + \underline{\gamma} U^- e^{\gamma z}$$

$$I(z) = \frac{\gamma}{R+j\omega L}\left(U^+ e^{-\gamma z} - U^- e^{\gamma z}\right)$$

$$\to \sqrt{\frac{(R+j\omega L)(G+j\omega C)}{(R+j\omega L)^2}} = \sqrt{\frac{G+j\omega C}{R+j\omega L}}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

hullámimpedancia.

$$\gamma \quad Z_0$$

$$\boxed{I(z) = \frac{U^+}{Z_0} e^{-\gamma z} - \frac{U^-}{Z_0} e^{\gamma z}}$$

$$\underbrace{I^+}_{I^+(z)} + \underbrace{I^-}_{I^-(z)}$$

$$\frac{U^+(z)}{I^+(z)} = Z_0$$

$$\frac{U^-(z)}{I^-(z)} = -Z_0$$

$\cancel{U}$

$$\frac{d^2U}{dz^2} - \gamma^2 U = \emptyset$$

$$\boxed{\begin{aligned} U(z) &= \underline{U^+ e^{-\gamma z}} + \underline{U^- e^{\gamma z}} \\ I(z) &= \underline{\frac{U^+}{Z_0} e^{-\gamma z}} - \underline{\frac{U^-}{Z_0} e^{\gamma z}} \end{aligned}}$$

U^+

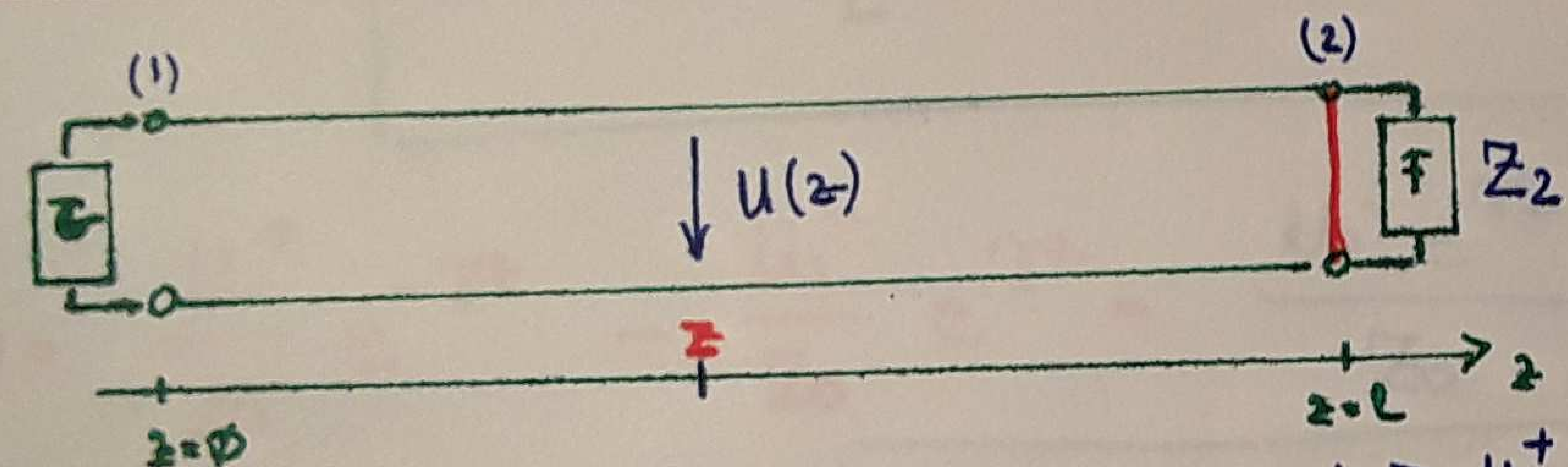
U^-

A feszültség komplex amplitúdójának és az áram komplex amplitúdójának megadása:

$$U(z) = U_1^+ e^{-\gamma z} + U_1^- e^{\gamma z}$$

$$I(z) = \frac{U_1^+}{Z_0} e^{-\gamma z} - \frac{U_1^-}{Z_0} e^{\gamma z}$$

$$\left.\begin{aligned} \gamma &= \sqrt{(R+j\omega L)(G+j\omega C)} \\ Z_0 &= \sqrt{\frac{R+j\omega L}{G+j\omega C}} \end{aligned}\right\} \quad \begin{aligned} U^+ &= ? \\ U^- &= ? \end{aligned}$$



$$U(z) = \underbrace{U_1^+ e^{-\gamma z}}_{U_1^+(z)} + \underbrace{U_1^- e^{\gamma z}}_{U_1^-(z)}$$

$\rightarrow U_1^+$ $\quad \rightarrow U_1^+ e^{-\gamma z}$

$\leftarrow U_1^-$ $\quad \leftarrow U_1^- e^{\gamma z}$

$\rightarrow U_2^+ = U_1^+ e^{-\gamma l} \longrightarrow U_1^+ = U_2^+ e^{\gamma l}$

$\leftarrow U_2^- = U_1^- e^{\gamma l} \longrightarrow U_1^- = U_2^- e^{-\gamma l}$

$$\boxed{\Gamma = \frac{U_2^-}{U_2^+}}$$

Reflexiós t.

$$\left.\begin{aligned} U(l) &= U_2^+ + U_2^- \\ I(l) &= \frac{U_2^+}{Z_0} - \frac{U_2^-}{Z_0} \end{aligned}\right\}$$

U_2, I_2

$$\boxed{Z_2 = \frac{U_2}{I_2}} = \frac{U_2^+ + U_2^-}{\frac{U_2^+}{Z_0} - \frac{U_2^-}{Z_0}} = Z_0 \cdot \frac{1 + \frac{U_2^-}{U_2^+}}{1 - \frac{U_2^-}{U_2^+}}$$

$$U_2^- = \Gamma U_2^+$$

$$\boxed{\Gamma = \frac{U_2^-}{U_2^+} = \frac{Z_2 - Z_0}{Z_2 + Z_0}}$$

$$Z_2 = Z_0 \cdot \frac{1+\Gamma}{1-\Gamma} \longrightarrow$$

$$Z_2(1-\Gamma) = Z_0(1+\Gamma)$$

$$Z_2 - \Gamma Z_2 = Z_0 + \Gamma Z_0$$

$$Z_2 - Z_0 = \Gamma Z_2 + \Gamma Z_0$$

$$Z_2 - Z_0 = \Gamma(Z_2 + Z_0)$$

$$\boxed{\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0}}$$

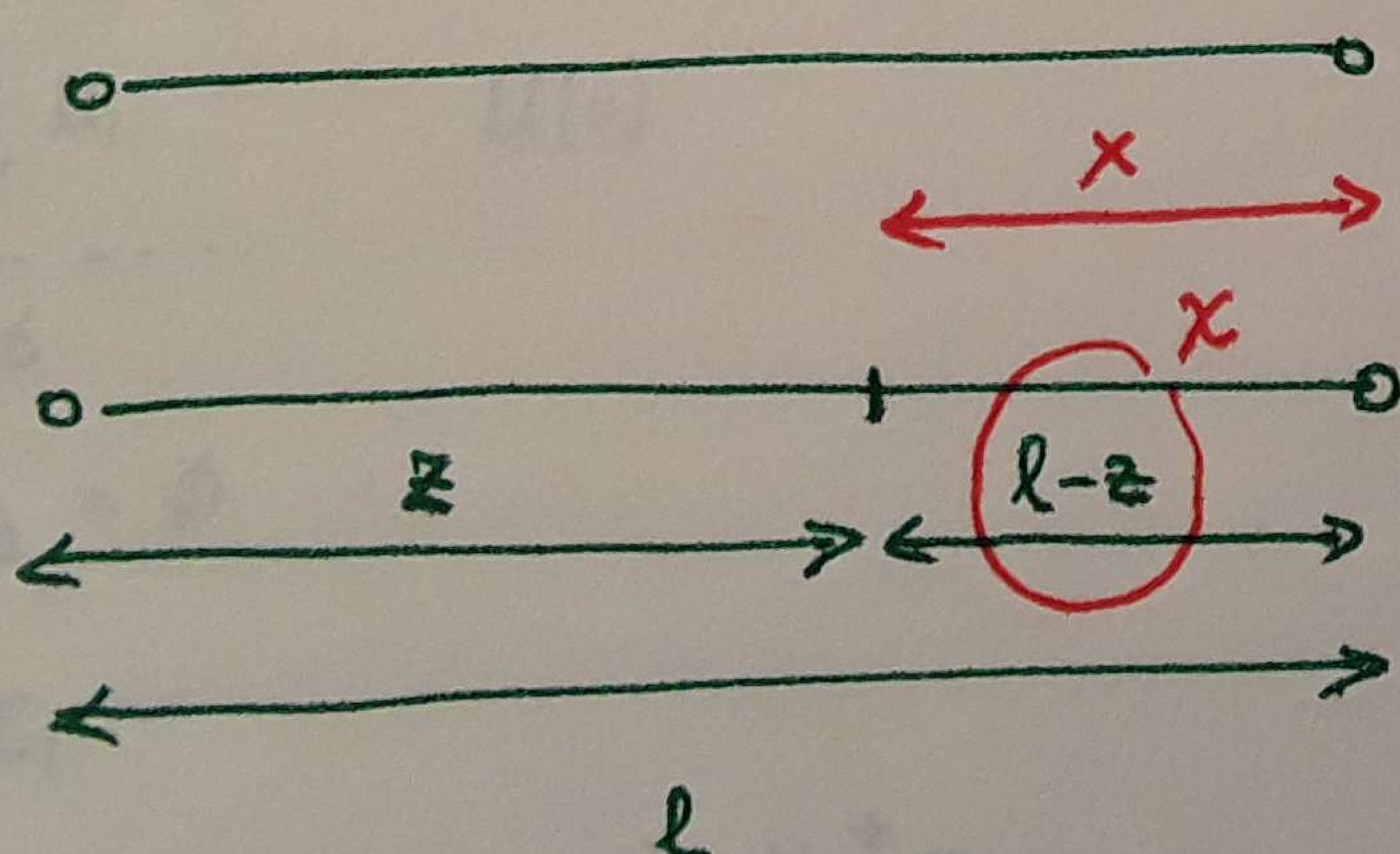
$$U(z) = U_1^+ e^{-\gamma z} + U_1^- e^{\gamma z} = U_2^+ e^{\gamma l} e^{-\gamma z} + U_2^- e^{-\gamma l} e^{\gamma z}$$

$$= \underline{U_2^+} e^{\gamma l} e^{-\gamma z} + \Gamma \underline{U_2^+} e^{-\gamma l} e^{\gamma z}$$

$$\boxed{U(z) = U_2^+ \left[e^{\gamma(l-z)} + \Gamma e^{-\gamma(l-z)} \right]} \qquad U_2^+ = ?$$

$$I(z) = \frac{U_1^+}{Z_0} e^{-\gamma z} - \frac{U_1^-}{Z_0} e^{\gamma z} = \frac{U_2^+ e^{\gamma l}}{Z_0} e^{-\gamma z} - \frac{U_2^- e^{-\gamma l}}{Z_0} e^{\gamma z}$$

$$\boxed{I(z) = \frac{U_2^+}{Z_0} \left[e^{\gamma(l-z)} - \Gamma e^{-\gamma(l-z)} \right]}$$



$$U(z) = U_2^+ \left[e^{\gamma x} + \Gamma e^{-\gamma x} \right]$$

$$I(z) = \frac{U_2^+}{Z_0} \left[e^{\gamma x} - \Gamma e^{-\gamma x} \right]$$

$$U(z) = U_2^+\left[e^{\gamma(l-z)} + \Gamma e^{-\gamma(l-z)}\right]$$

U_2^+ MEGHATÁROZÁSA

$$I(z) = \frac{U_2^+}{Z_0}\left[e^{\gamma(l-z)} - \Gamma e^{-\gamma(l-z)}\right]$$

U_2 ismert

$z = l$

$$U_2 = U_2^+\left[1+\Gamma\right]$$

$$U_2^+ = \frac{U_2}{1+\Gamma} \rightarrow U(z) = \frac{U_2}{1+\Gamma}\left[e^{\gamma(l-z)} + \Gamma e^{-\gamma(l-z)}\right]$$

I_2 ismert

$z = l$

$$Z_2 = \frac{U_2}{I_2}$$

$$I_2 = \frac{U_2^+}{Z_0}\left[1-\Gamma\right]$$

$$U_2^+ = \frac{Z_0 I_2}{1-\Gamma}$$

$U_1 = U_g$ ismert

$z = \emptyset$

$$U_1 = U_2^+\left[e^{\gamma l} + \Gamma e^{-\gamma l}\right]$$

$$U_2^+ = \frac{U_1}{e^{\gamma l} + \Gamma e^{-\gamma l}}$$

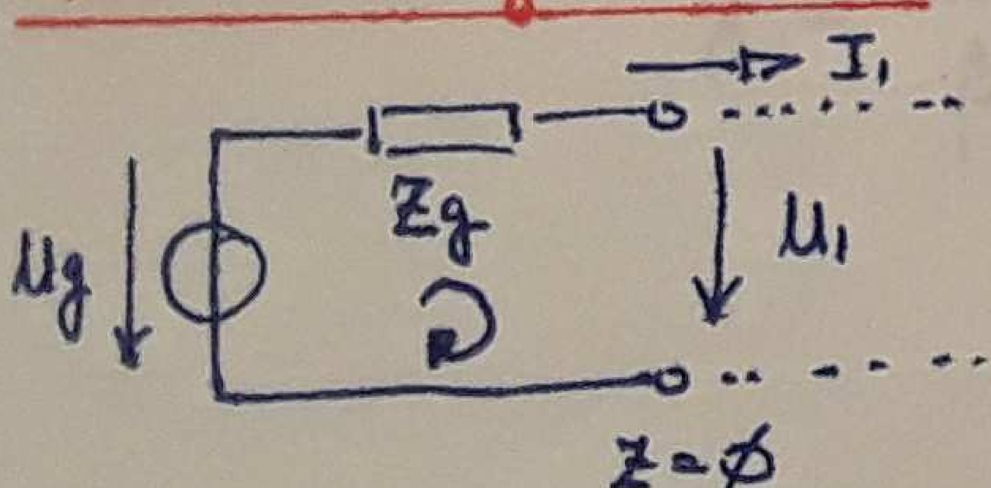
$I_1 = I_g$ ismert

$z = \emptyset$

$$I_1 = \frac{U_2^+}{Z_0}\left[e^{\gamma l} - \Gamma e^{-\gamma l}\right]$$

$$U_2^+ = \frac{Z_0 I_1}{e^{\gamma l} - \Gamma e^{-\gamma l}}$$

Thévenin - generátor



$I(0)$

$U(0)$

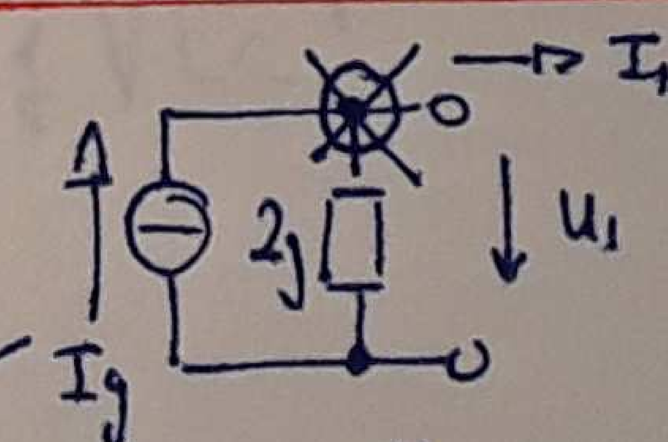
$$Z_g I_1 + U_1 - U_g = \emptyset$$

$$U_g = U_1 + Z_g I_1$$

$$U_g = U_2^+\left[e^{\gamma l} + \Gamma e^{-\gamma l}\right] + \frac{Z_g U_2^+}{Z_0}\left[e^{\gamma l} - \Gamma e^{-\gamma l}\right]$$

$\hookrightarrow U_2^+ = \dots$

Norton - generátor



$$-I_g + \frac{U_1}{Z_g} + I_1 = \emptyset$$

$U_1 = U(0)$

$$I_g = \frac{U_1}{Z_g} + I_1$$

$I_1 = I(0)$

$$I_g = \frac{U_2^+}{Z_g}\left[e^{\gamma l} + \Gamma e^{-\gamma l}\right] + \frac{U_2^+}{Z_0}\left[e^{\gamma l} - \Gamma e^{-\gamma l}\right]$$

$\hookrightarrow U_2^+ \dots$

AZ IDEÁLIS TÁVVEZETÉK

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\boxed{R = \emptyset} \qquad \boxed{G = \emptyset}$$

$$\gamma = \sqrt{j\omega L \cdot j\omega C} = j\omega\sqrt{LC} \quad \begin{cases} \alpha = \emptyset \\ \beta = \omega\sqrt{LC} \end{cases}$$

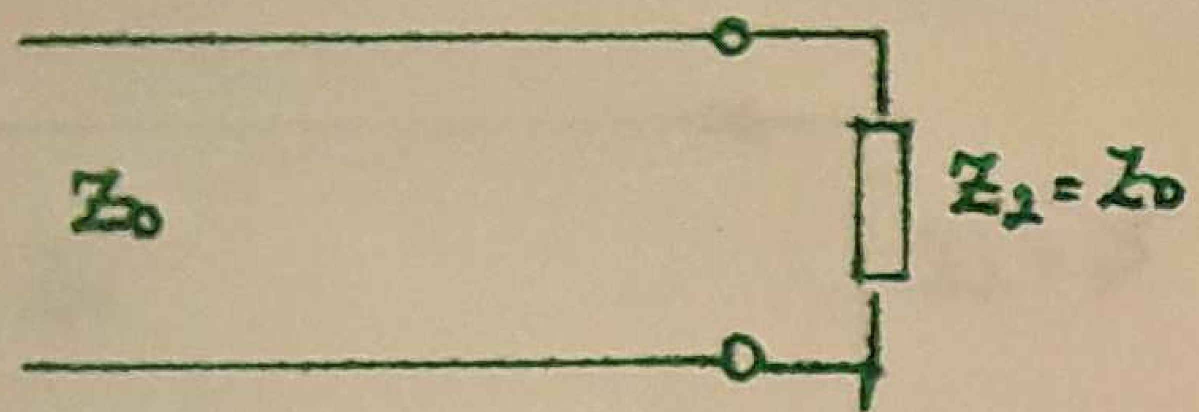
$$Z_0 = \sqrt{\frac{\cancel{j\omega} L}{\cancel{j\omega} C}} = \sqrt{\frac{L}{C}}$$

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} \leq c \quad \left(3 \cdot 10^8 \frac{m}{s}\right)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{2\pi}{2\pi f\sqrt{LC}} = \frac{1}{f\sqrt{LC}} = \frac{v}{f} \qquad \lambda = \frac{v}{f}$$

LEZÁRÁS HULLÁMIMPEDANCIÁVAL

SMITH



$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \emptyset$$

$$U(z) = U_2^+ \left[e^{\gamma(l-z)} + \cancel{\Gamma e^{-\gamma(l-z)}} \right]$$

$$I(z) = \frac{U_2^+}{Z_0} \left[e^{\gamma(l-z)} - \cancel{\Gamma e^{-\gamma(l-z)}} \right]$$

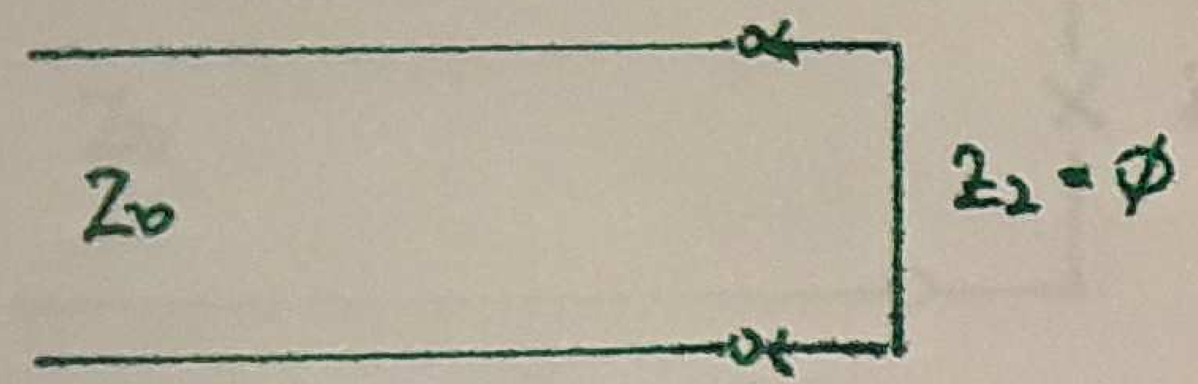
$$U(z) = U_2^+ e^{\gamma(l-z)}$$

$$I(z) = \frac{U_2^+}{Z_0} e^{\gamma(l-z)}$$

Illesztett

IDEÁLIS TÁVVEZETÉK LEZÁRÁSA RÖVIDZÁRRAL.

$\gamma = j\beta$



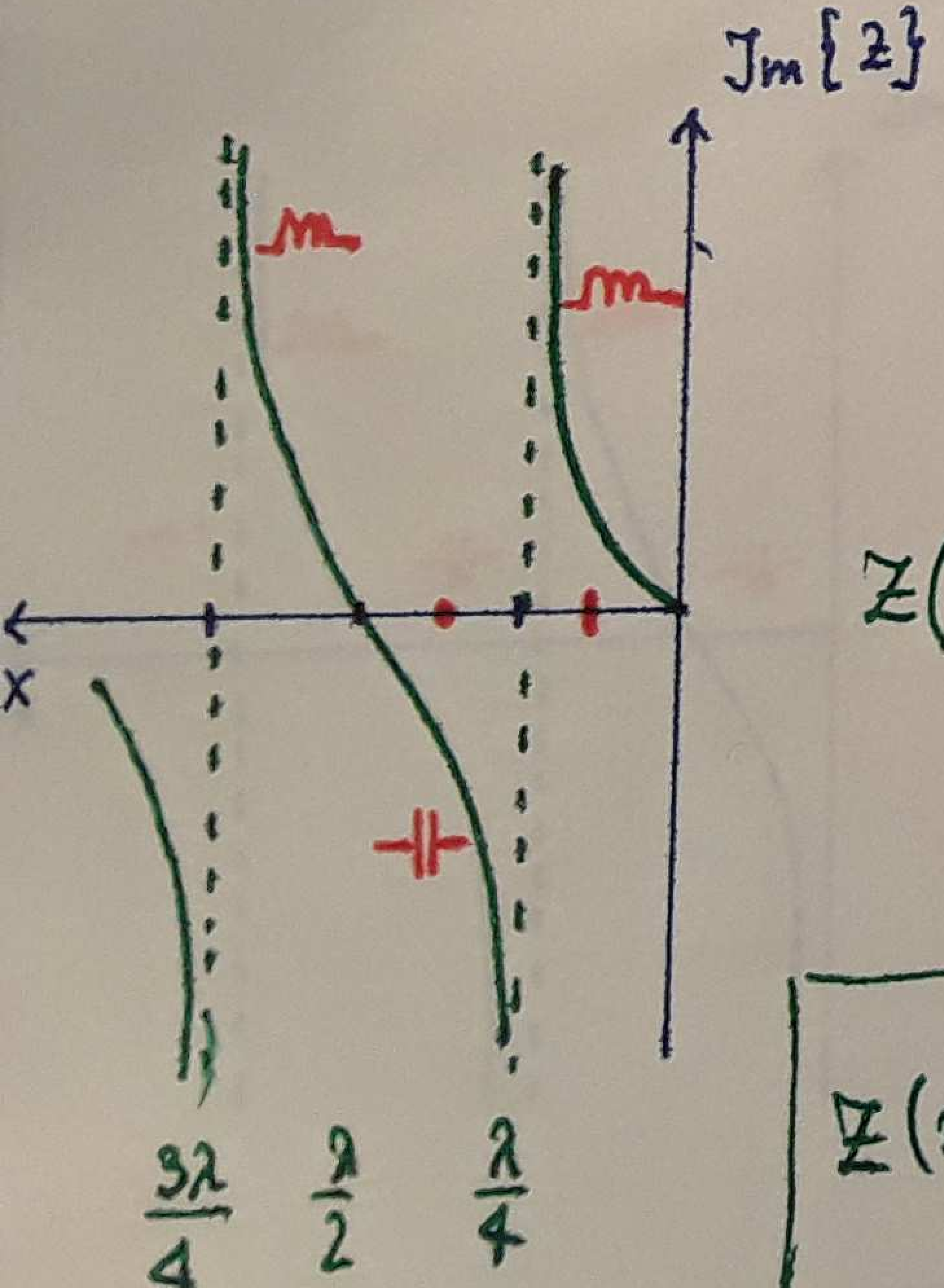
$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{0 - Z_0}{0 + Z_0} = \underline{\underline{-1}}$$

$$\sin\varphi = \frac{e^{j\varphi} - e^{-j\varphi}}{2j}$$

$$\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2}$$

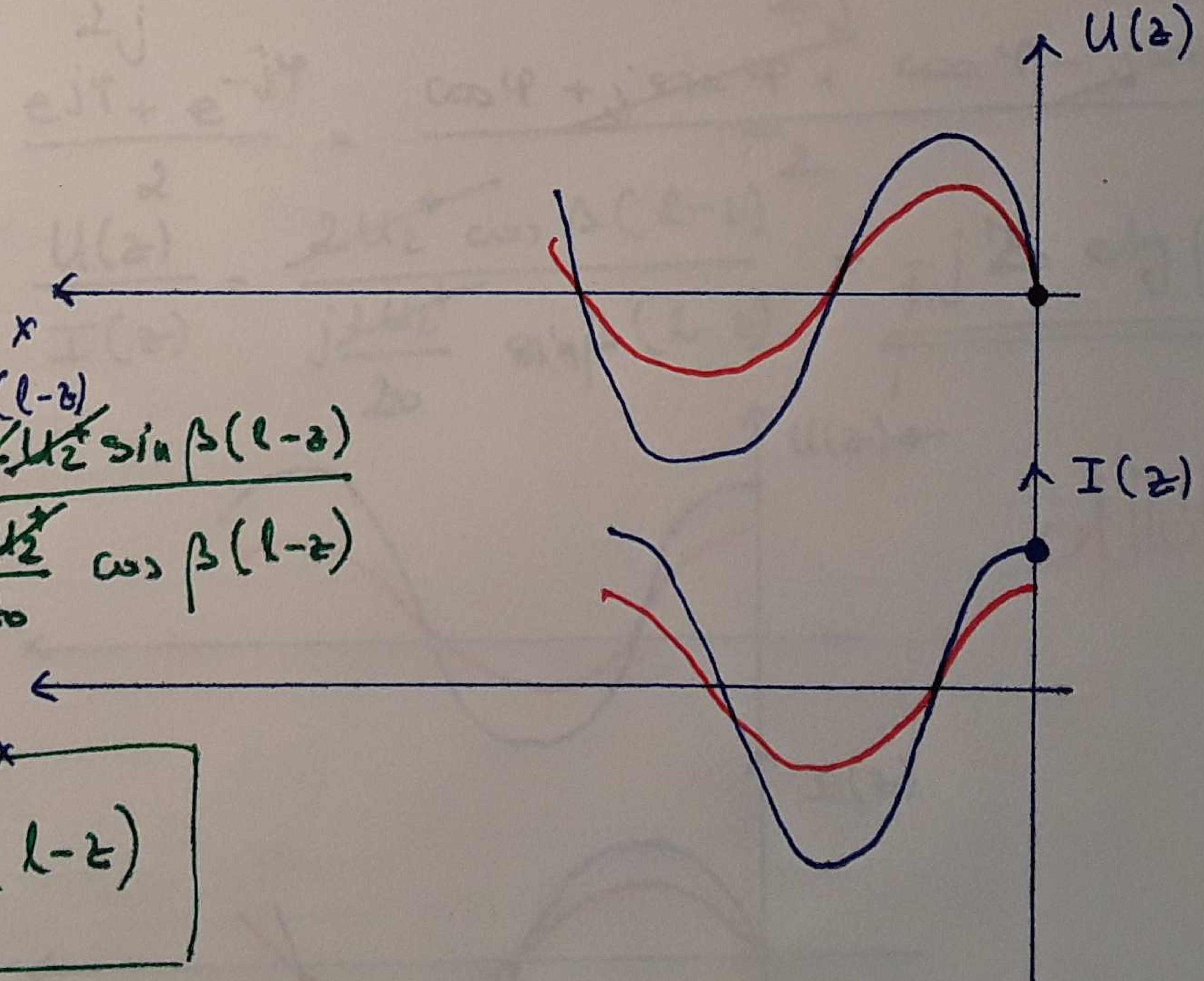
$$U(z) = U_2^+\left[e^{\gamma(l-z)} + \Gamma e^{-\gamma(l-z)}\right] = U_2^+\left[e^{j\beta(l-z)} - 1e^{-j\beta(l-z)}\right]\frac{2j}{2j} = \underline{\underline{j2U_2^+ \sin\beta(l-z)}}$$

$$I(z) = \frac{U_2^+}{Z_0}\left[e^{\gamma(l-z)} - \Gamma e^{-\gamma(l-z)}\right] = \frac{U_2^+}{Z_0}\left[e^{j\beta(l-z)} + e^{-j\beta(l-z)}\right]\frac{2}{2} = \underline{\underline{\frac{2U_2^+}{Z_0}\cos\beta(l-z)}}$$



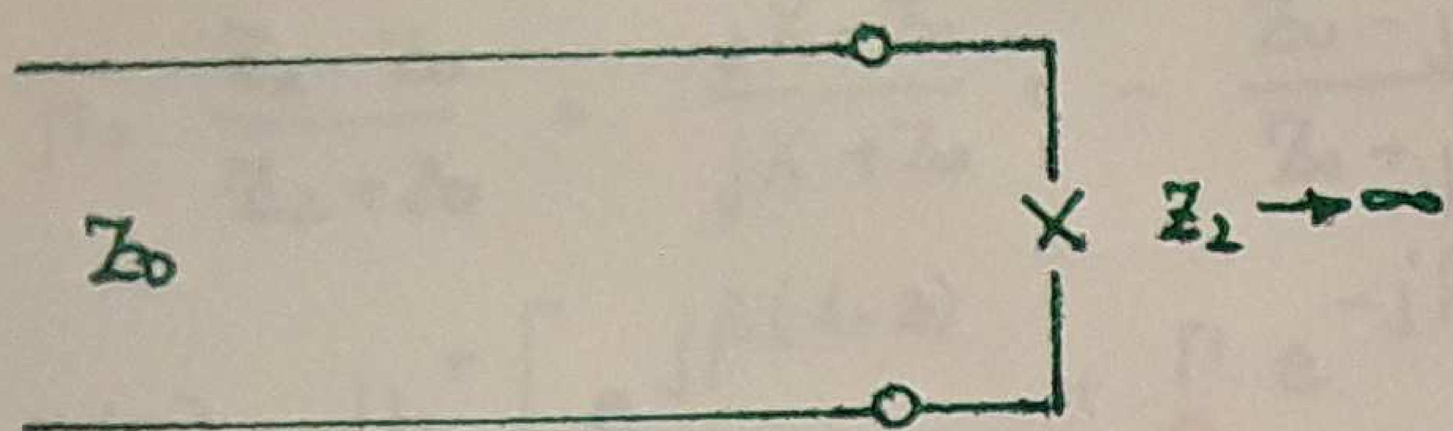
$$Z(z) = \frac{U(z)}{I(z)} = \frac{j\cancel{2}\cancel{U_2^+}\sin\beta(l-z)}{\frac{\cancel{2U_2^+}}{Z_0}\cos\beta(l-z)}$$

$$\boxed{Z(z) = jZ_0 \,\mathrm{tg}\,\beta(l-z)}$$



Állóhullám

IDEÁLIS TÁVVEZETÉK LEZÁRÁSA SZAKADÁSSAL



$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{1 - \cancel{Z_0/Z_2}}{1 + \cancel{Z_0/Z_2}} = 1 \qquad \gamma = j\beta$$

$$U(z) = U_2^+\left[e^{\gamma(l-z)} + \Gamma e^{-\gamma(l-z)}\right] = U_2^+\left[e^{j\beta(l-z)} + e^{-j\beta(l-z)}\right] = \underline{\underline{2U_2^+ \cos\beta(l-z)}}$$

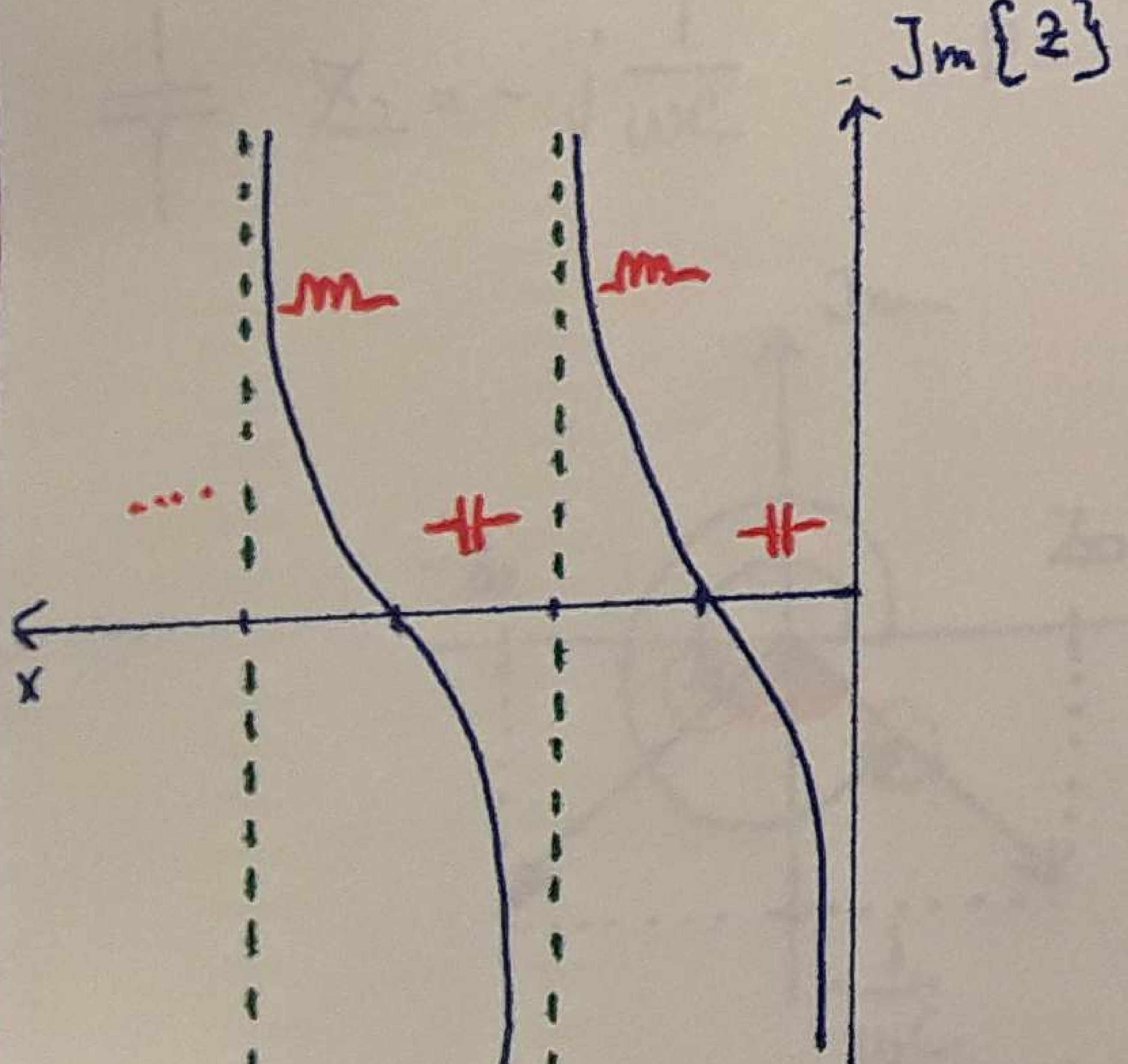
$$I(z) = \frac{U_2^+}{Z_0}\left[e^{\gamma(l-z)} - \Gamma e^{-\gamma(l-z)}\right] = \frac{U_2^+}{Z_0}\left[e^{j\beta(l-z)} - e^{-j\beta(l-z)}\right] = \underline{\underline{\frac{j2U_2^+}{Z_0} \sin\beta(l-z)}}$$

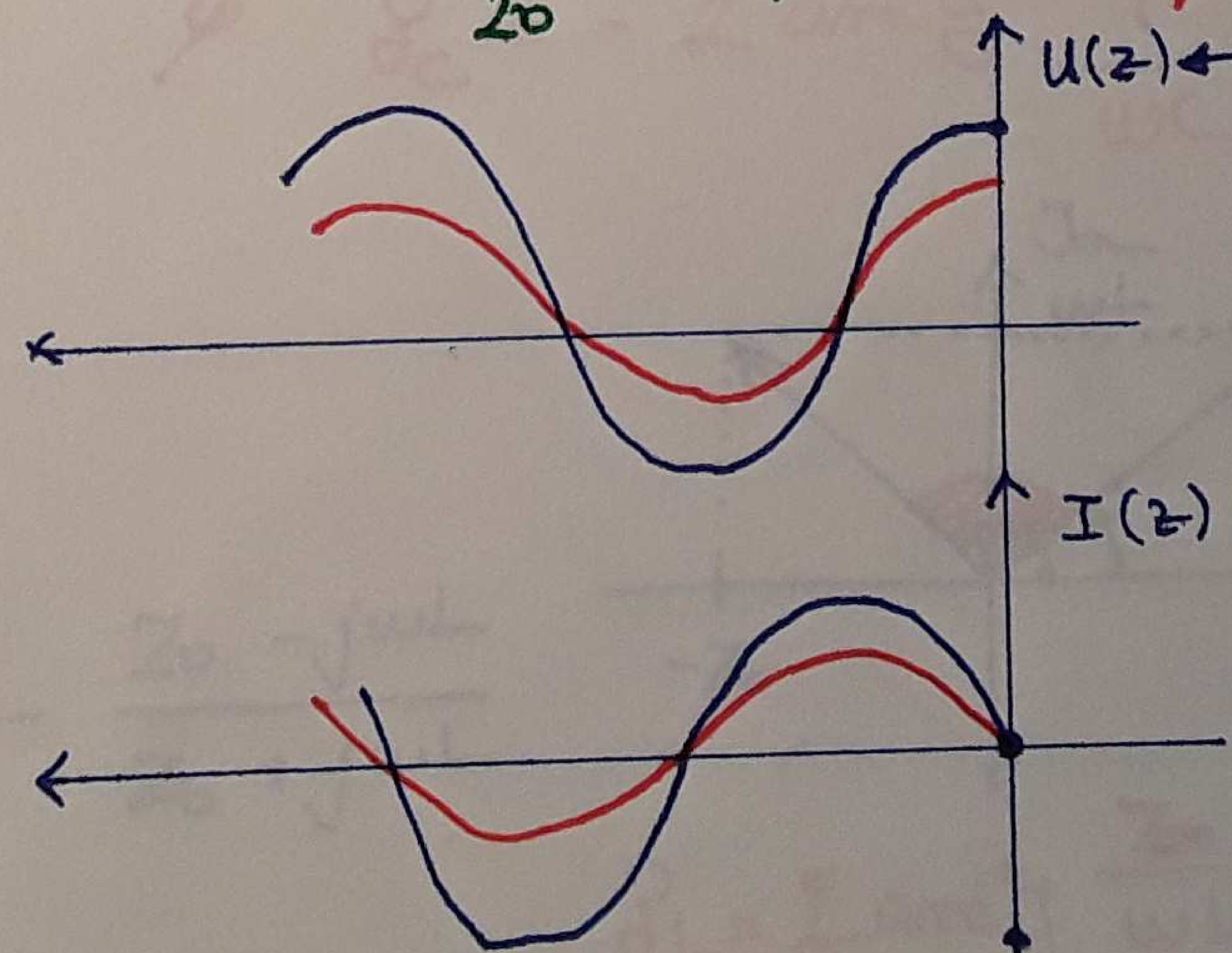
$$\sin\varphi = \frac{e^{j\varphi} - e^{-j\varphi}}{2j} = \frac{\cancel{\cos\varphi} + j\sin\varphi - (\cancel{\cos\varphi} - j\sin\varphi)}{2j} = \frac{\cancel{2j}\sin\varphi}{\cancel{2j}}$$

$$\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2} = \frac{\cos\varphi + \cancel{j\sin\varphi} + \cos\varphi - \cancel{j\sin\varphi}}{2} = \frac{\cancel{2}\cos\varphi}{\cancel{2}}$$

$$Z(z) = \frac{U(z)}{I(z)} = \frac{\cancel{2U_2^+}\cos\beta(l-z)}{j\frac{\cancel{2U_2^+}}{Z_0}\sin\beta(l-z)} = \underline{\underline{-jZ_0 \operatorname{ctg}\beta(l-z)}}$$

$$\frac{1}{j} = -j$$





Állóhullám!

IDEÁLIS TÁVVEZETÉK LEZÁRÁSA REAKTANCIÁVAL

$Z_2 = jX$

$\alpha = \emptyset$

$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{jX - Z_0}{jX + Z_0} = -\frac{Z_0 - jX}{Z_0 + jX} = \boxed{1\, e^{j\delta}}$$

$$U(z) = U_2^+ \left[e^{j\beta(l-z)} + \underset{e^{j\delta}}{\Gamma}\, e^{-j\beta(l-z)} \right] = U_2^+ e^{j\frac{\delta}{2}} \left[e^{-j\frac{\delta}{2}} e^{j\beta(l-z)} + e^{j\frac{\delta}{2}} e^{-j\beta(l-z)} \right] =$$

$$= U_2^+ e^{j\frac{\delta}{2}} \left[e^{j\left[\beta(l-z) - \frac{\delta}{2}\right]} + e^{-j\left[\beta(l-z) - \frac{\delta}{2}\right]} \right] \frac{2}{2}$$

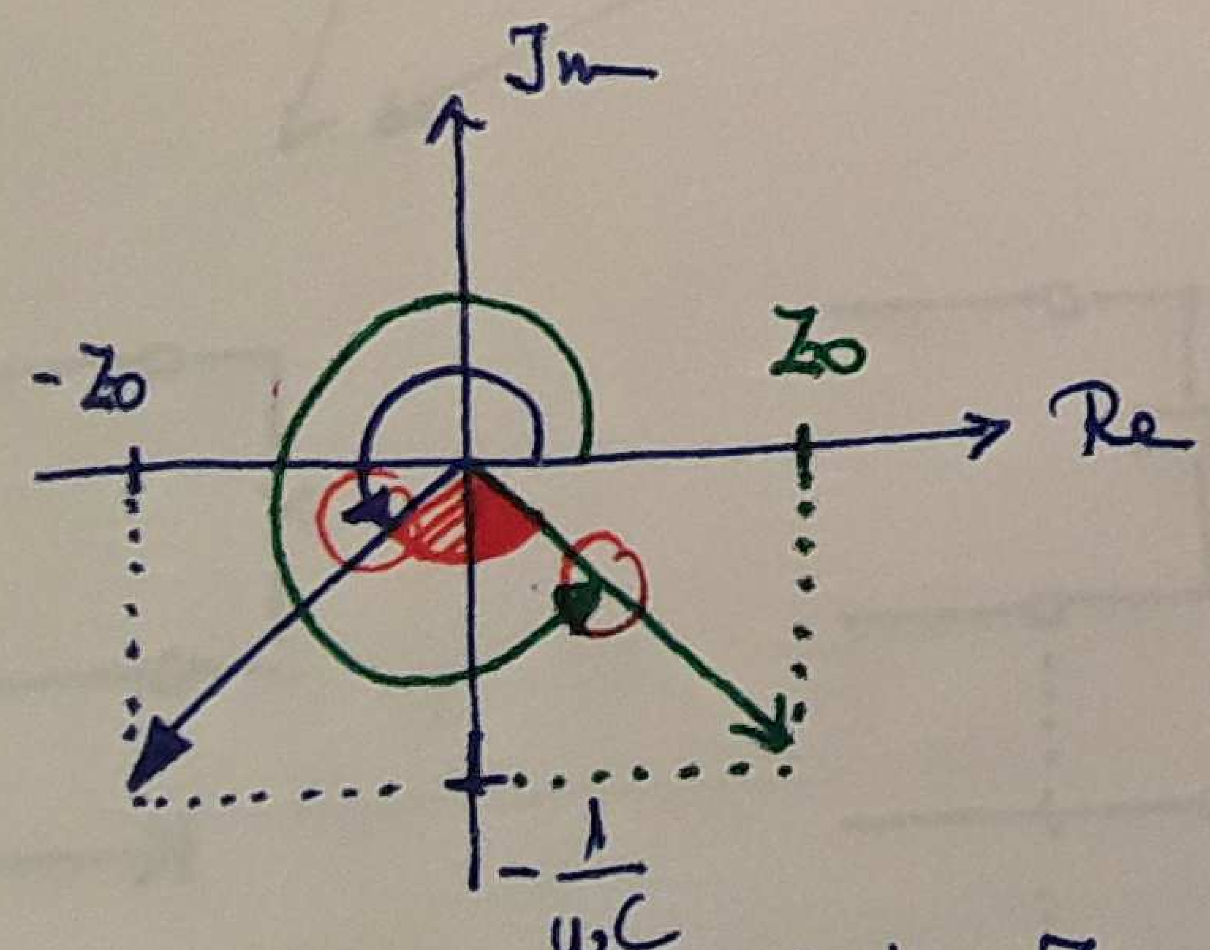
$$= 2 U_2^+ e^{j\frac{\delta}{2}} \cos\left[\beta(l-z) - \frac{\delta}{2}\right]$$

$$I(z) = \frac{j 2 U_2^+}{Z_0} e^{j\delta/2} \sin\left(\beta(l-z) - \frac{\delta}{2}\right)$$

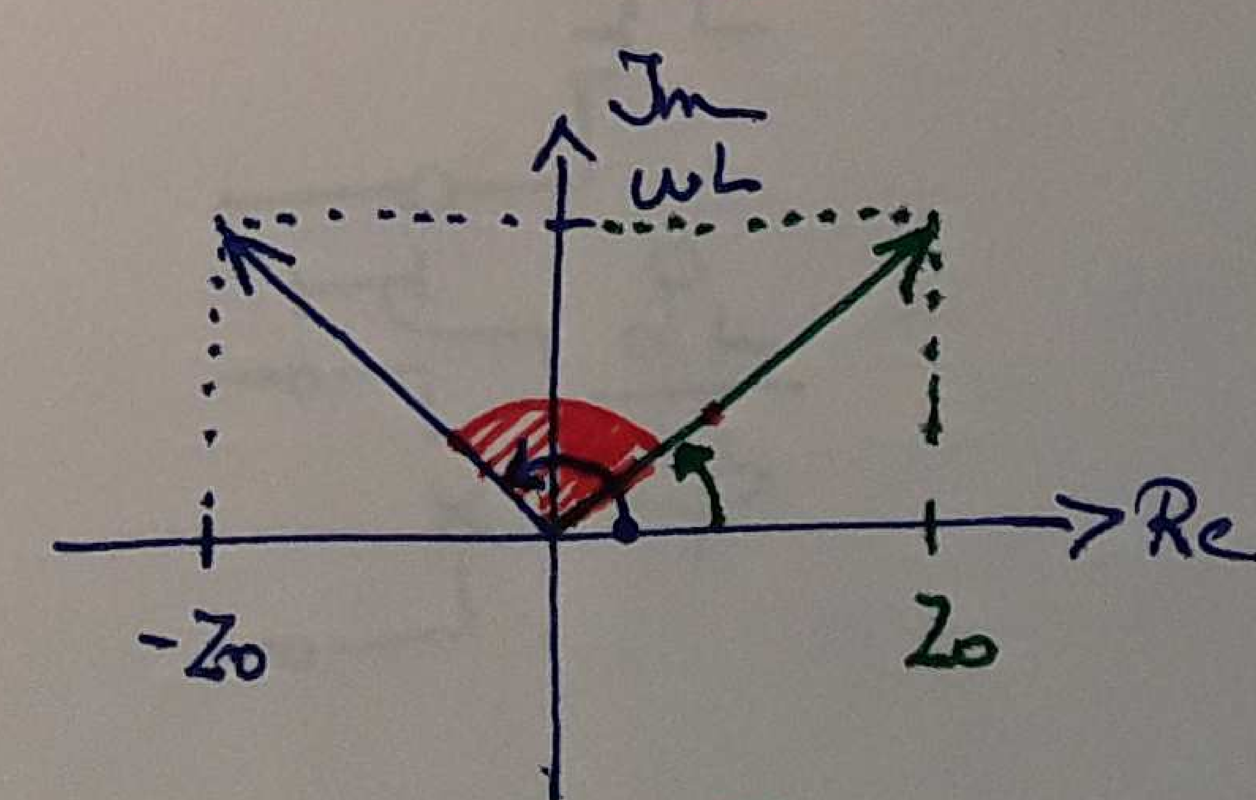
$|X|$

$$Z_2 = -j\frac{1}{\omega C} \qquad \Gamma = \frac{-j\frac{1}{\omega C} - Z_0}{-j\frac{1}{\omega C} + Z_0} = -\frac{Z_0 + j\frac{1}{\omega C}}{Z_0 - j\frac{1}{\omega C}}$$

$$\delta_C = -2\,\mathrm{arctg}\frac{Z_0}{\frac{1}{\omega C}} = -2\,\mathrm{arctg}\,\omega C Z_0$$



$$Z_2 = j\omega L \qquad \Gamma = \frac{j\omega L - Z_0}{j\omega L + Z_0} = -\frac{Z_0 - j\omega L}{Z_0 + j\omega L}$$



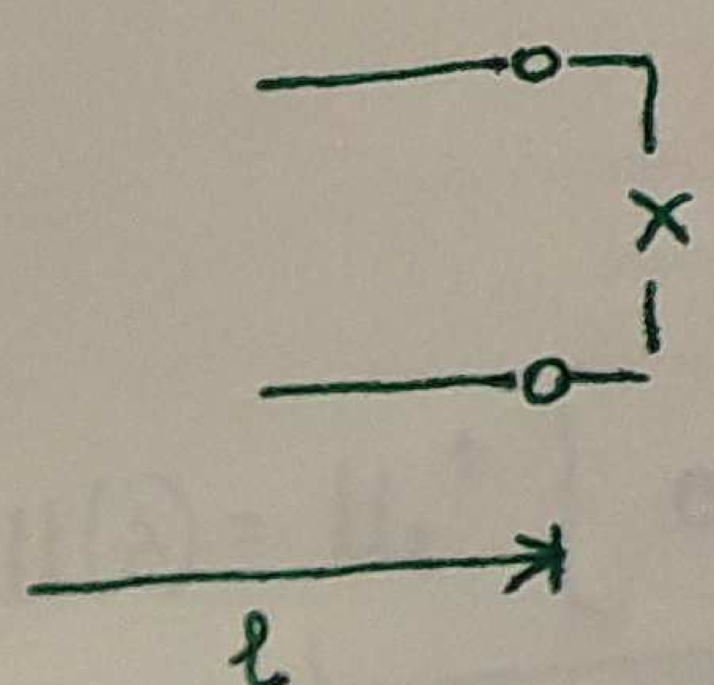
$$\delta_L = 2\,\mathrm{arctg}\frac{Z_0}{\omega L}$$

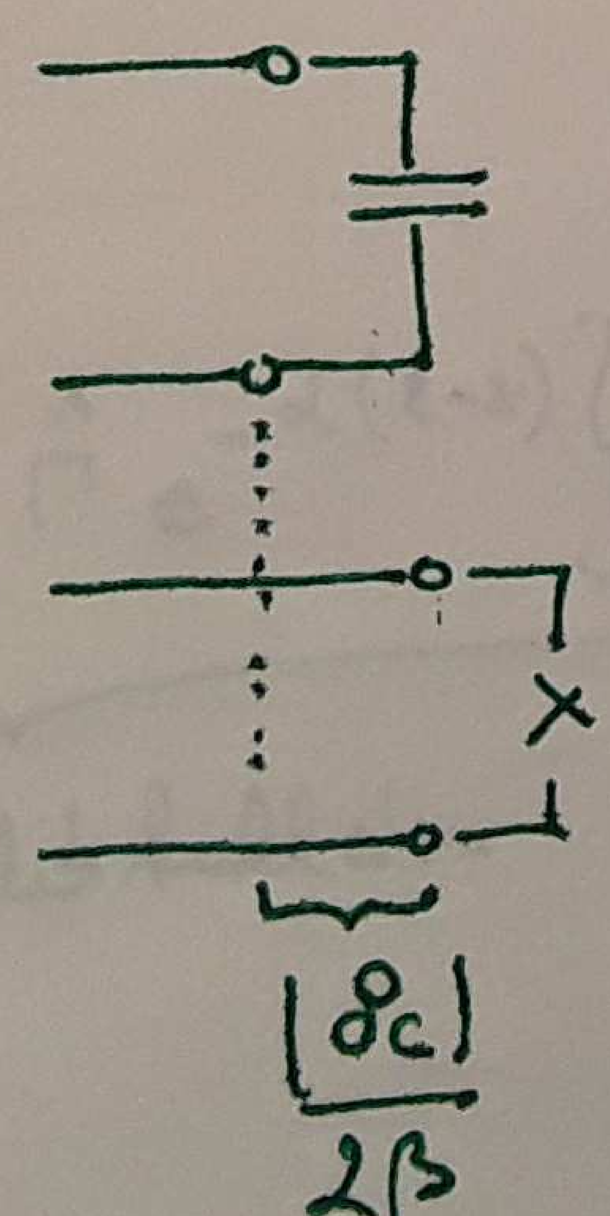
$$u(z) = 2U_2^+ e^{j\frac{\delta}{2}} \cos\underbrace{\left[\beta(l-z) - \frac{\delta}{2}\right]}$$

$$\delta_C = -2\,\mathrm{arctg}\,\omega C Z_0$$

$$\beta(l-z) + \frac{|\delta_C|}{2}$$

$$\beta\left[l - z + \frac{|\delta_C|}{2\beta}\right]$$

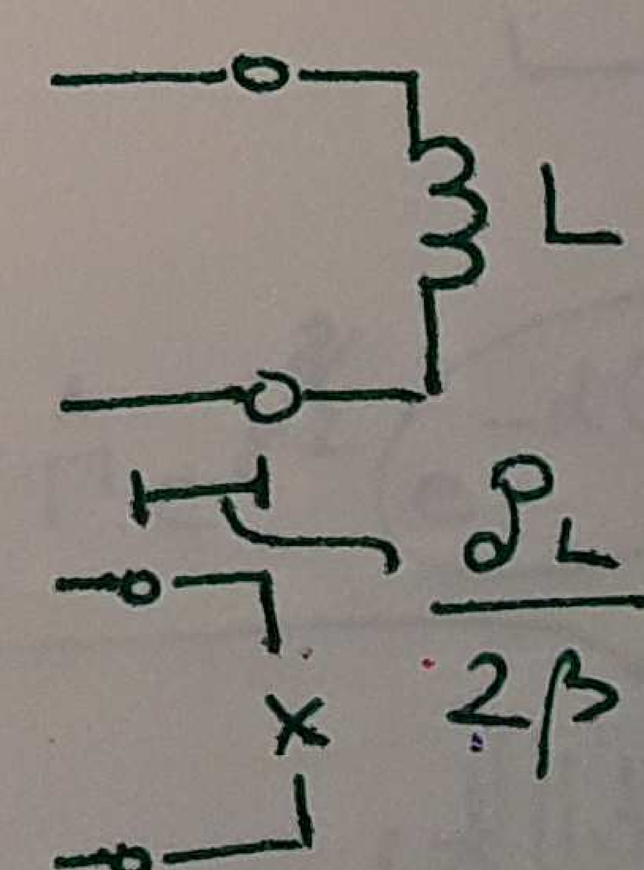




$$\delta_L = 2\,\mathrm{arctg}\,\frac{Z_0}{\omega L}$$

$$\beta(l-z) - \frac{\delta_L}{2}$$

$$\beta\left[l - z - \frac{\delta_L}{2\beta}\right]$$



LEZÁRÁS ÁLTALÁNOS Z_2 IMPEDANCIÁVAL

$$\gamma = \alpha + j\beta$$

$$\Gamma = \hat{\Gamma} e^{j\vartheta}$$

$$U(z) = U_2^+ \left[e^{\gamma(l-z)} + \Gamma e^{-\gamma(l-z)} \right] =$$

$$= U_2^+ \left[e^{(\alpha+j\beta)(l-z)} + \hat{\Gamma} e^{j\vartheta} e^{-(\alpha+j\beta)(l-z)} \right] =$$

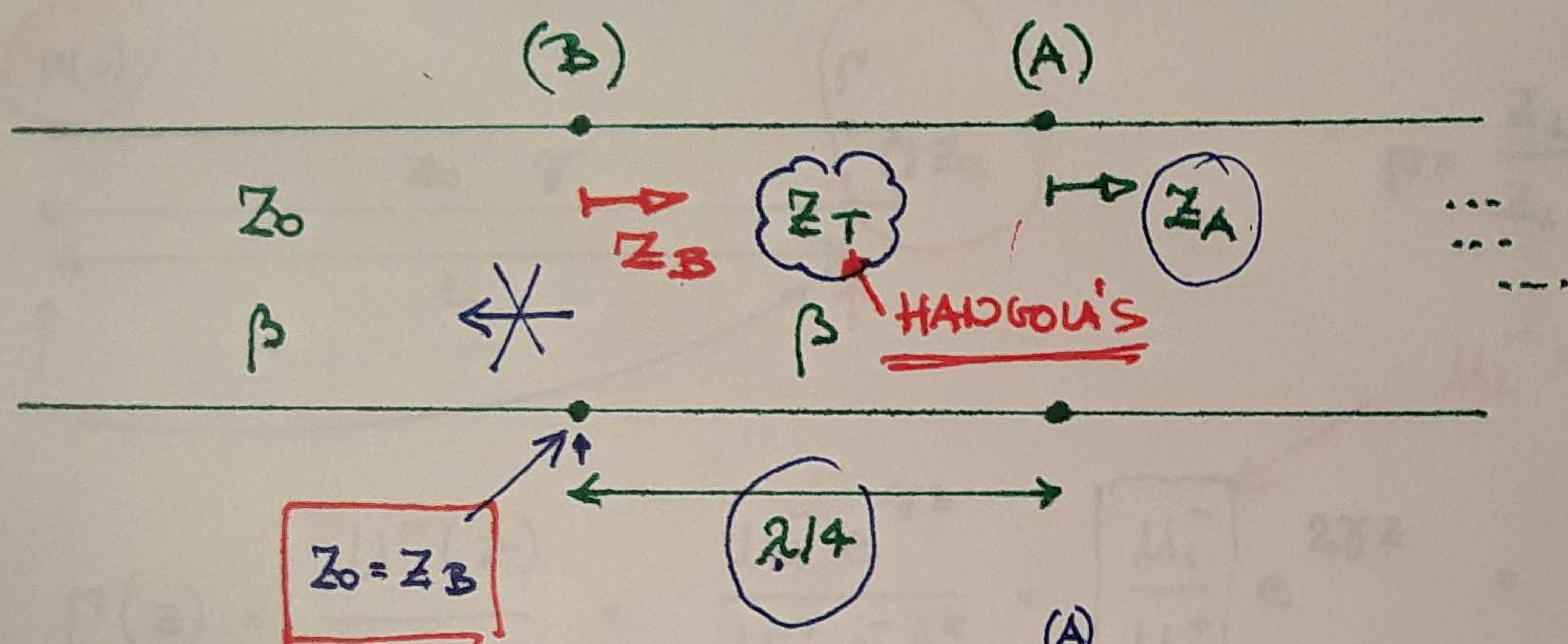
$$= U_2^+ \left[\underline{e^{\alpha(l-z)} e^{j\beta(l-z)}} + \hat{\Gamma} e^{j\vartheta} e^{-\alpha(l-z)} e^{-j\beta(l-z)} \underbrace{- \hat{\Gamma} e^{-\alpha(l-z)} e^{j\beta(l-z)} + \hat{\Gamma} e^{-\alpha(l-z)} e^{j\beta(l-z)}}_{=\emptyset} \right] =$$

$$= U_2^+ \left[\left\{ e^{\alpha(l-z)} - \hat{\Gamma} e^{-\alpha(l-z)} \right\} e^{j\beta(l-z)} + \hat{\Gamma} e^{-\alpha(l-z)} \left\{ \underbrace{e^{j\vartheta}} e^{-j\beta(l-z)} + e^{j\beta(l-z)} \right\} \right] =$$

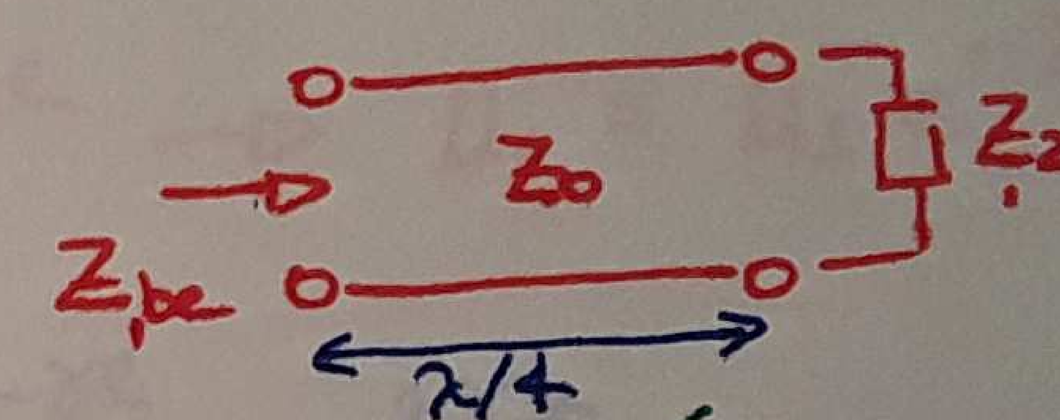
$$= U_2^+ \left[\left\{ e^{\alpha(l-z)} - \hat{\Gamma} e^{-\alpha(l-z)} \right\} e^{j\beta(l-z)} + \hat{\Gamma} e^{j\frac{\vartheta}{2}} e^{-\alpha(l-z)} \underbrace{\left\{ e^{-j\beta(l-z)} e^{+j\frac{\vartheta}{2}} + e^{j\beta(l-z)} e^{-j\frac{\vartheta}{2}} \right\}}_{2 \underbrace{\frac{e^{j\left[\beta(l-z) - \frac{\vartheta}{2}\right]} + e^{-j\left[\beta(l-z) - \frac{\vartheta}{2}\right]}}{2}}_{\cos\left[\beta(l-z) - \vartheta/2\right]}} \right]$$

$$U(z) = \underbrace{U_2^+ \left\{ e^{\alpha(l-z)} - \hat{\Gamma} e^{-\alpha(l-z)} \right\} e^{j\beta(l-z)}}_{\text{Haladó hullám}} + \underbrace{2 U_2^+ \hat{\Gamma} e^{j\frac{\vartheta}{2}} e^{-\alpha(l-z)} \cos\left[\beta(l-z) - \frac{\vartheta}{2}\right]}_{\text{Állóhullám}}$$

A λ/4 HOSSZÚSÁGÚ IMPEDANCIATRANSZFORMÁTOR

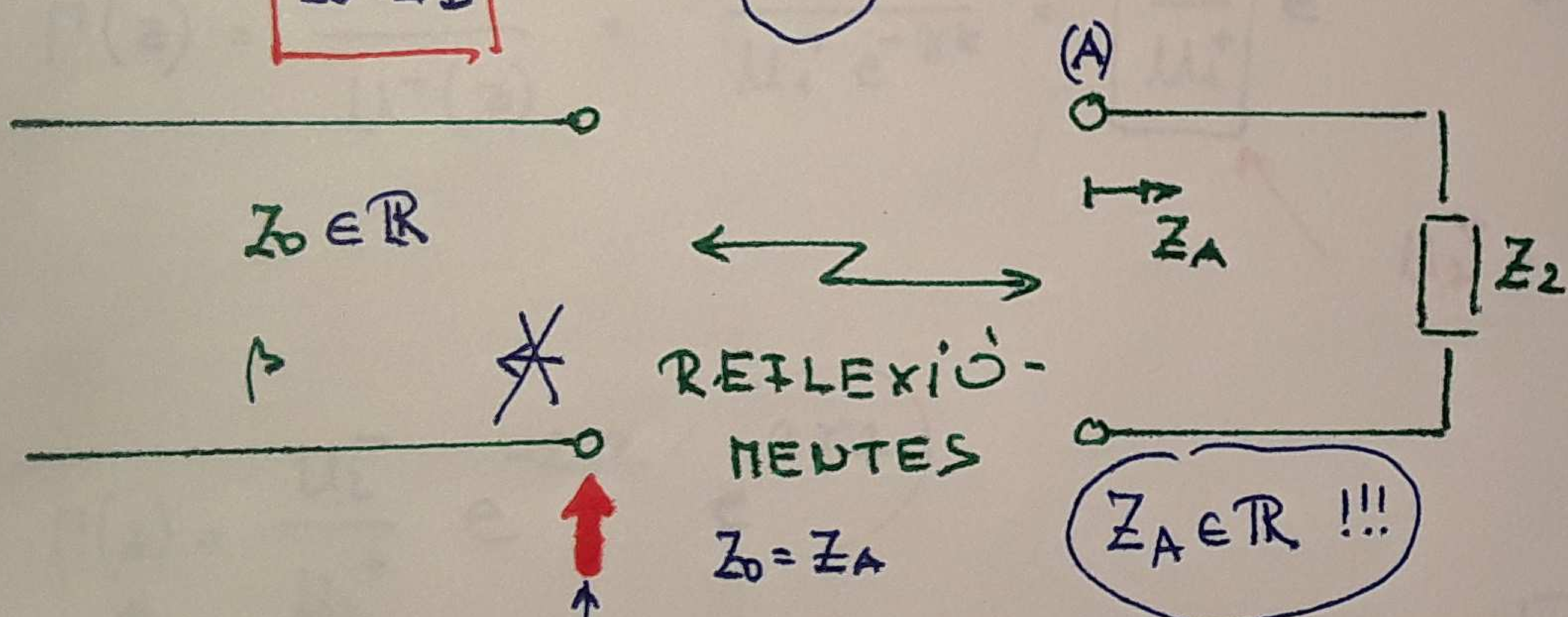


$$Z_{be} = Z_0 \frac{Z_2 + jZ_0 \boxed{\mathrm{tg}\,\beta l}}{Z_0 + jZ_2 \boxed{\mathrm{tg}\,\beta l}}$$



$$\beta l = \frac{2\pi}{\cancel{\lambda}} \frac{\cancel{\lambda}}{4} = \frac{\pi}{2} \qquad \mathrm{tg}\,\frac{\pi}{2} \Rightarrow \infty$$

$$Z_{be} = Z_0 \frac{\cancel{Z_2/\mathrm{tg}\,\beta l} + \cancel{j} Z_0}{\cancel{Z_0/\mathrm{tg}\,\beta l} + \cancel{j} Z_2} = \frac{Z_0^2}{Z_2}$$



$$\boxed{Z_{be} = \frac{Z_0^2}{Z_2}}$$

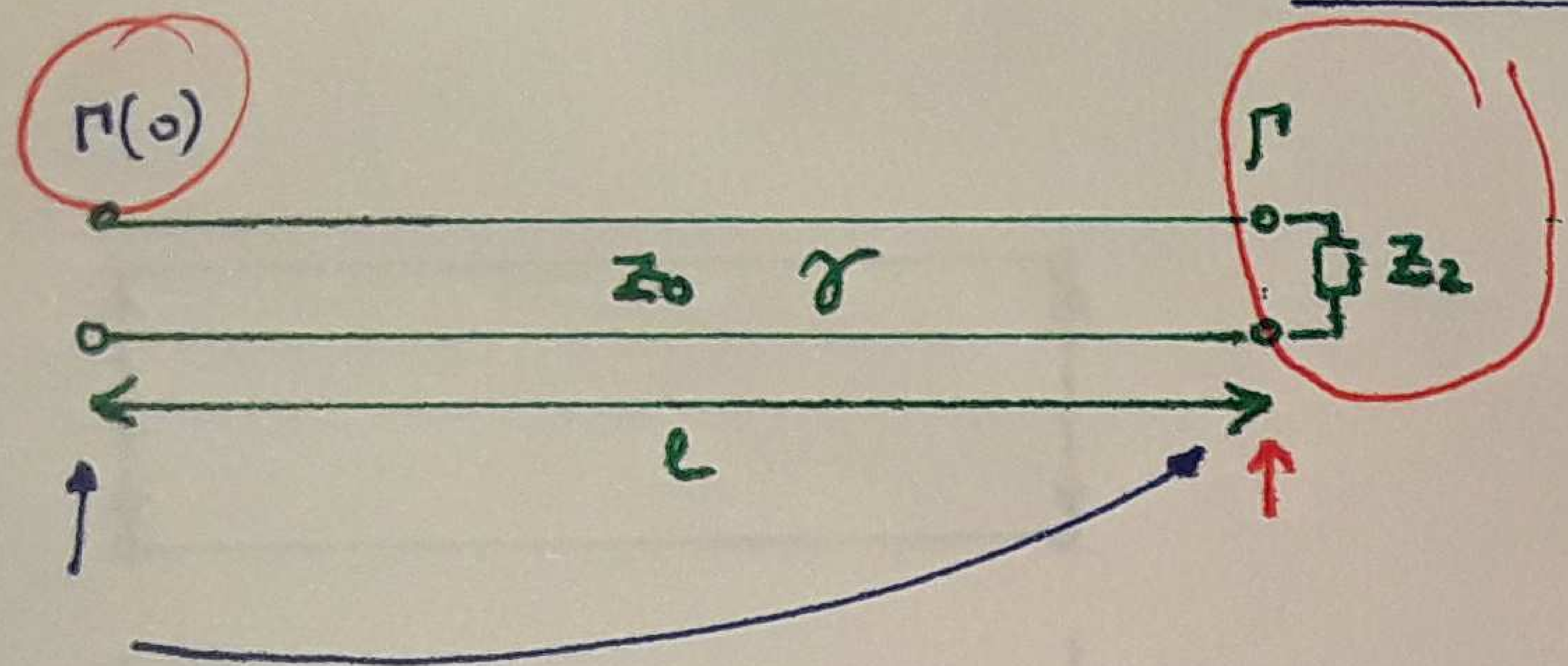
$$\boxed{Z_B = \frac{Z_T^2}{Z_A}}$$

$$Z_T = \sqrt{Z_A \cdot Z_B}$$

$$\boxed{Z_T = \sqrt{Z_A Z_0}}$$

- $Z_A \in \mathbb{R}$
- $Z_0 \in \mathbb{R}$
- ADOTT FR.

REFLEXIÓS TÉNYEZŐ MÉRÉSE



$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0}$$

$$U_2^- = U_1^- e^{\gamma l} \rightarrow U_1^- = U_2^- e^{-\gamma l}$$

$$\Gamma(z) = \frac{U^-(z)}{U^+(z)} = \frac{U_1^- e^{\gamma z}}{U_1^+ e^{-\gamma z}} = \boxed{\frac{U_1^-}{U_1^+}} e^{2\gamma z} = \frac{U_2^- e^{-\gamma l}}{U_2^+ e^{\gamma l}} e^{2\gamma z} \Rightarrow$$

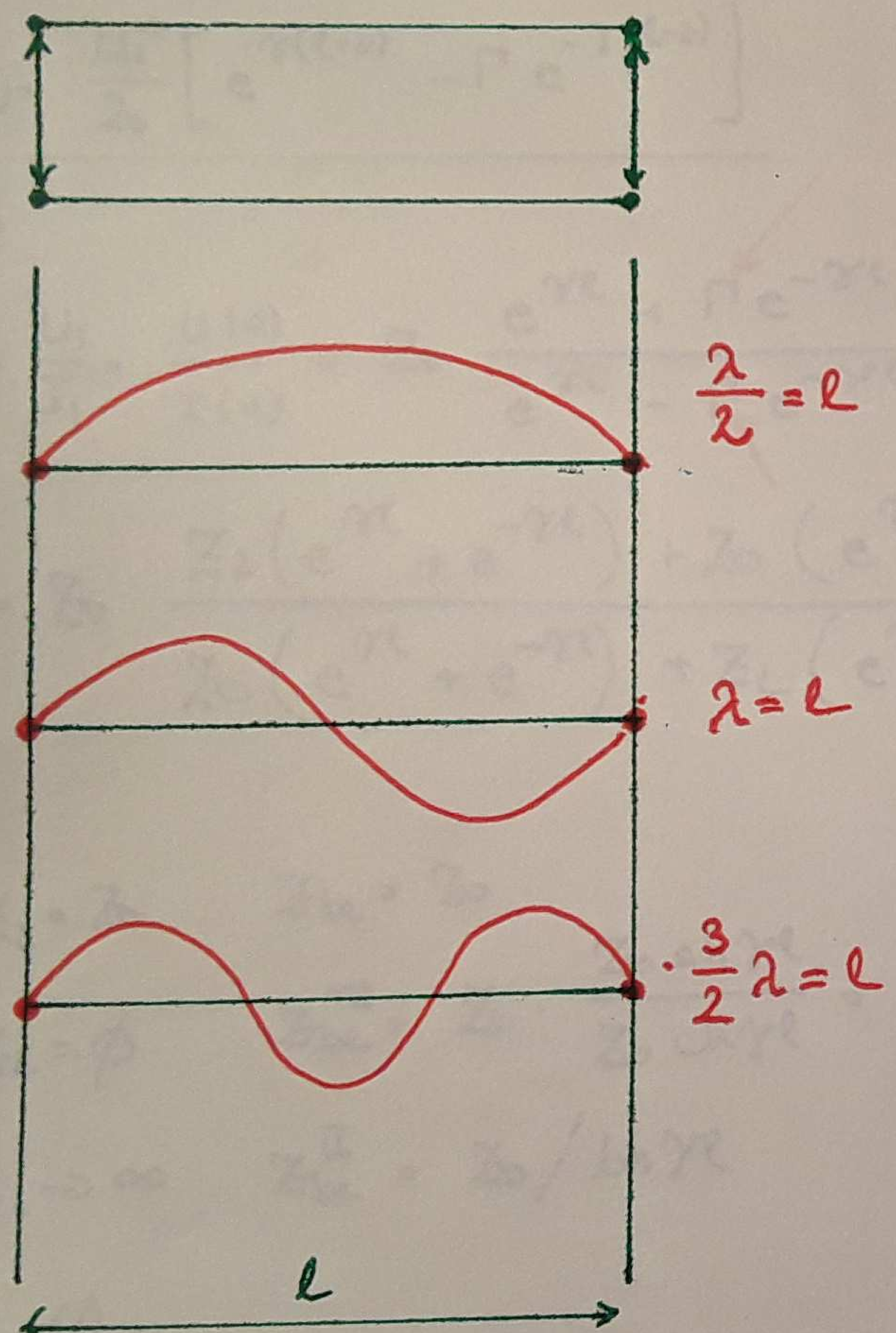
$$U_2^+ = U_1^+ e^{-\gamma l} \rightarrow U_1^+ = U_2^+ e^{\gamma l}$$

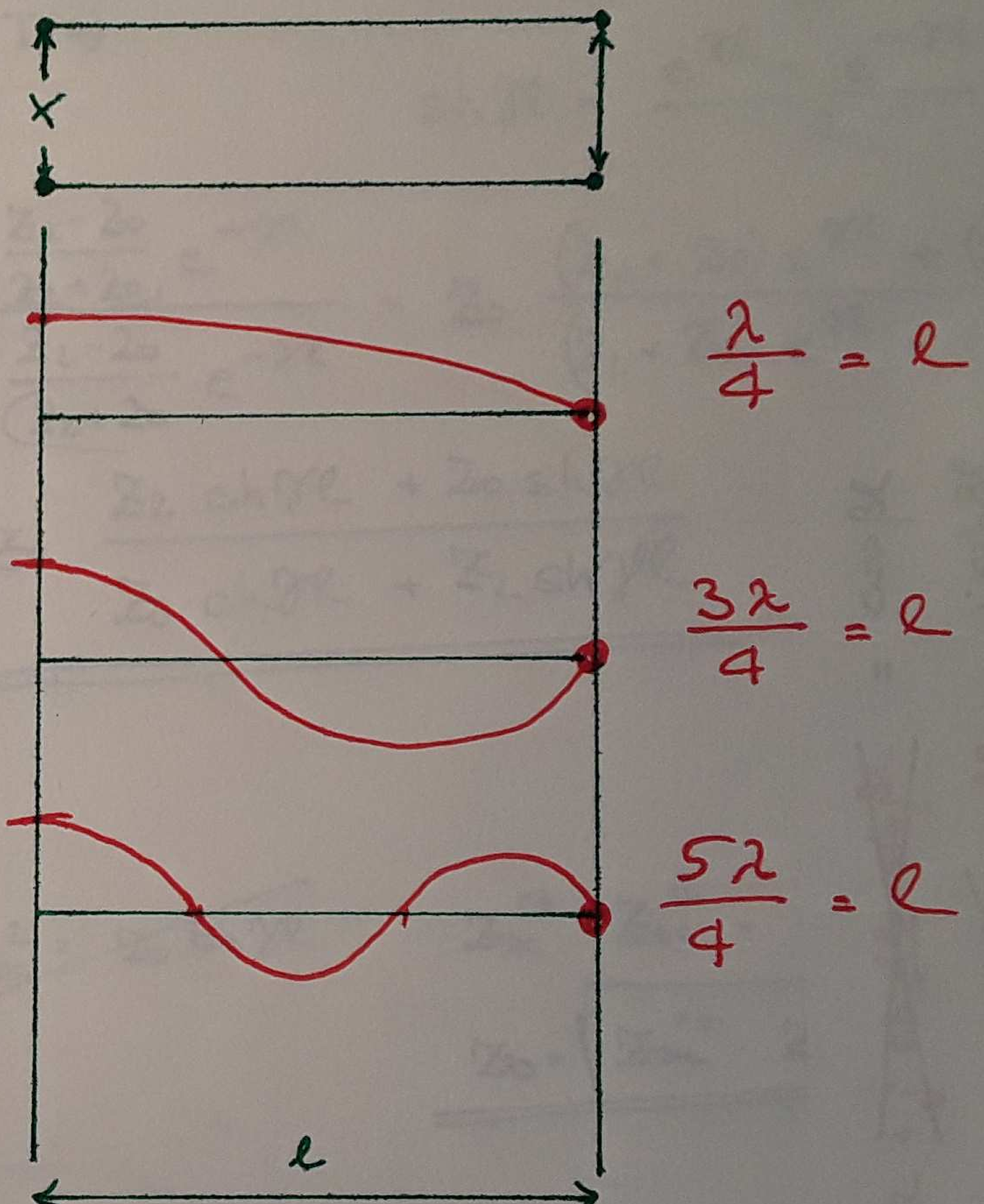
$$\Gamma(z) = \frac{U_2^-}{U_2^+} e^{-2\gamma l} e^{2\gamma z}$$

$$z = 0) \quad \Gamma(0) = \underbrace{\frac{U_2^-}{U_2^+}}_{\Gamma} e^{-2\gamma l} \Rightarrow \boxed{\Gamma(0) = \Gamma e^{-2\gamma l}}$$

$$\Gamma = \Gamma(0) e^{2\gamma l}$$

ÁLLÓHULLÁM A TÁVVEZETÉKEN





$l = \lambda/2 \cdot k \qquad k = 1, 2, 3 \ldots$

$$\boxed{\lambda = \frac{2l}{k}}$$

$v = f\lambda$

$$f = \frac{v}{\lambda} = \frac{kv}{2l}$$

$$l = \frac{2k+1}{4}\lambda \qquad k = 0, 1, 2, \ldots$$

$$\boxed{\lambda = \frac{4l}{2k+1}}$$

$$f = \frac{v}{\lambda} = \frac{v(2k+1)}{4l}$$

A TÁPVONAL BEMENETI IMPEDANCIÁJA

Smith

$$U(z) = U_2^+ \left[e^{\gamma(l-z)} + \Gamma e^{-\gamma(l-z)} \right]$$

$$I(z) = \frac{U_2^+}{Z_0} \left[e^{\gamma(l-z)} - \Gamma e^{-\gamma(l-z)} \right]$$

$$Z(z) = \frac{U(z)}{I(z)}$$

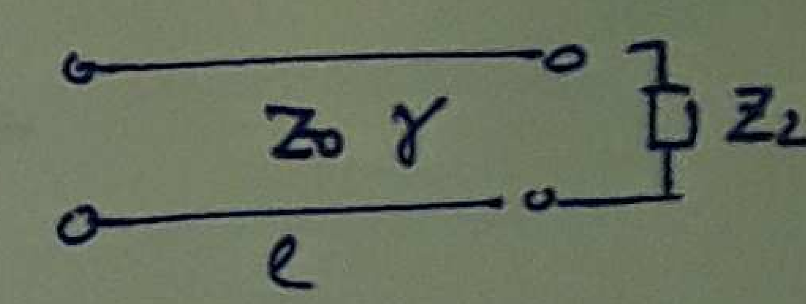
$$\mathrm{ch}\,\gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2}$$

$$\mathrm{sh}\,\gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2}$$

$z = 0$

$$Z_{be} = \frac{U_1}{I_1} = \frac{U(0)}{I(0)} = Z_0 \frac{e^{\gamma l} + \Gamma e^{-\gamma l}}{e^{\gamma l} - \Gamma e^{-\gamma l}} = Z_0 \frac{e^{\gamma l} + \frac{Z_2 - Z_0}{Z_2 + Z_0} e^{-\gamma l}}{e^{\gamma l} - \frac{Z_2 - Z_0}{Z_2 + Z_0} e^{-\gamma l}} = Z_0 \frac{(Z_2 + Z_0) e^{\gamma l} + (Z_2 - Z_0) e^{-\gamma l}}{(Z_2 + Z_0) e^{\gamma l} - (Z_2 - Z_0) e^{-\gamma l}} =$$

$$= Z_0 \frac{Z_2 (e^{\gamma l} + e^{-\gamma l}) + Z_0 (e^{\gamma l} - e^{-\gamma l})}{Z_0 (e^{\gamma l} + e^{-\gamma l}) + Z_2 (e^{\gamma l} - e^{-\gamma l})} = Z_0 \frac{Z_2\, \mathrm{ch}\,\gamma l + Z_0\, \mathrm{sh}\,\gamma l}{Z_0\, \mathrm{ch}\,\gamma l + Z_2\, \mathrm{sh}\,\gamma l}$$



1) $Z_2 = Z_0$ $\quad Z_{be} = Z_0$

2) $Z_2 = 0$ $\quad Z_{be}^{rz} = Z_0 \cdot \frac{Z_0\, \mathrm{sh}\,\gamma l}{Z_0\, \mathrm{ch}\,\gamma l} = Z_0\, \mathrm{th}\,\gamma l$

3) $Z_2 \to \infty$ $\quad Z_{be}^{ü} = Z_0 / \mathrm{th}\,\gamma l$

$\cancel{Z_{be}^2 = Z_0\, \mathrm{th}\,\gamma l}$

$$Z_{be}^{rz} \cdot Z_{be}^{ü} = Z_0^2$$

$$Z_0 = \sqrt{Z_{be}^{rz} \cdot Z_{be}^{ü}}$$

4) $\alpha = 0$

$$Z_{be} = Z_0 \cdot \frac{Z_2 \cos\beta l + Z_0 j \sin\beta l}{Z_0 \cos\beta l + Z_2 j \sin\beta l} = Z_0 \frac{Z_2 + j Z_0\, \mathrm{tg}\,\beta l}{Z_0 + j Z_2\, \mathrm{tg}\,\beta l}$$

$$\mathrm{ch}\,\gamma l = \cancel{\mathrm{ch}\,\alpha l} \cos\beta l \cancel{+ j\, \mathrm{sh}\,\alpha l \sin\beta l} = \cos\beta l$$

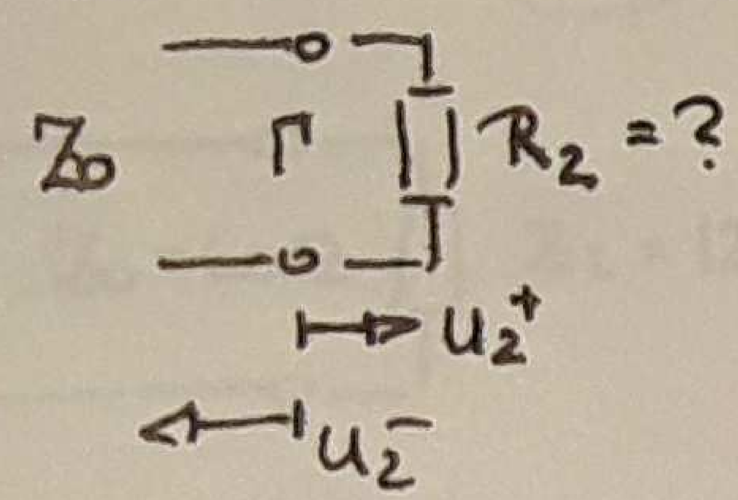
$$\mathrm{sh}\,\gamma l = \cancel{\mathrm{sh}\,\alpha l \cos\beta l} + j\, \cancel{\mathrm{ch}\,\alpha l} \sin\beta l = j \sin\beta l$$

$\mathrm{sh}\,0 = 0$

$\mathrm{ch}\,0 = \cancel{0}\ 1$

$$\gamma = \alpha + j\beta$$

Egy $Z_0 = 600\,\Omega$ hullámellenállású távvezeték R_2 lezáró ellenállásán $U_2^+ = 180\,V$ és $U_2^- = 60\,V$. Mekkora R_2?



$$\boxed{\Gamma = \frac{U_2^-}{U_2^+}} = \frac{60}{180} = \frac{1}{3}$$

$$\Gamma = \frac{R_2 - Z_0}{R_2 + Z_0}$$

$$\Gamma R_2 + \Gamma Z_0 = R_2 - Z_0$$

$$(\Gamma + 1) Z_0 = (1 - \Gamma) R_2$$

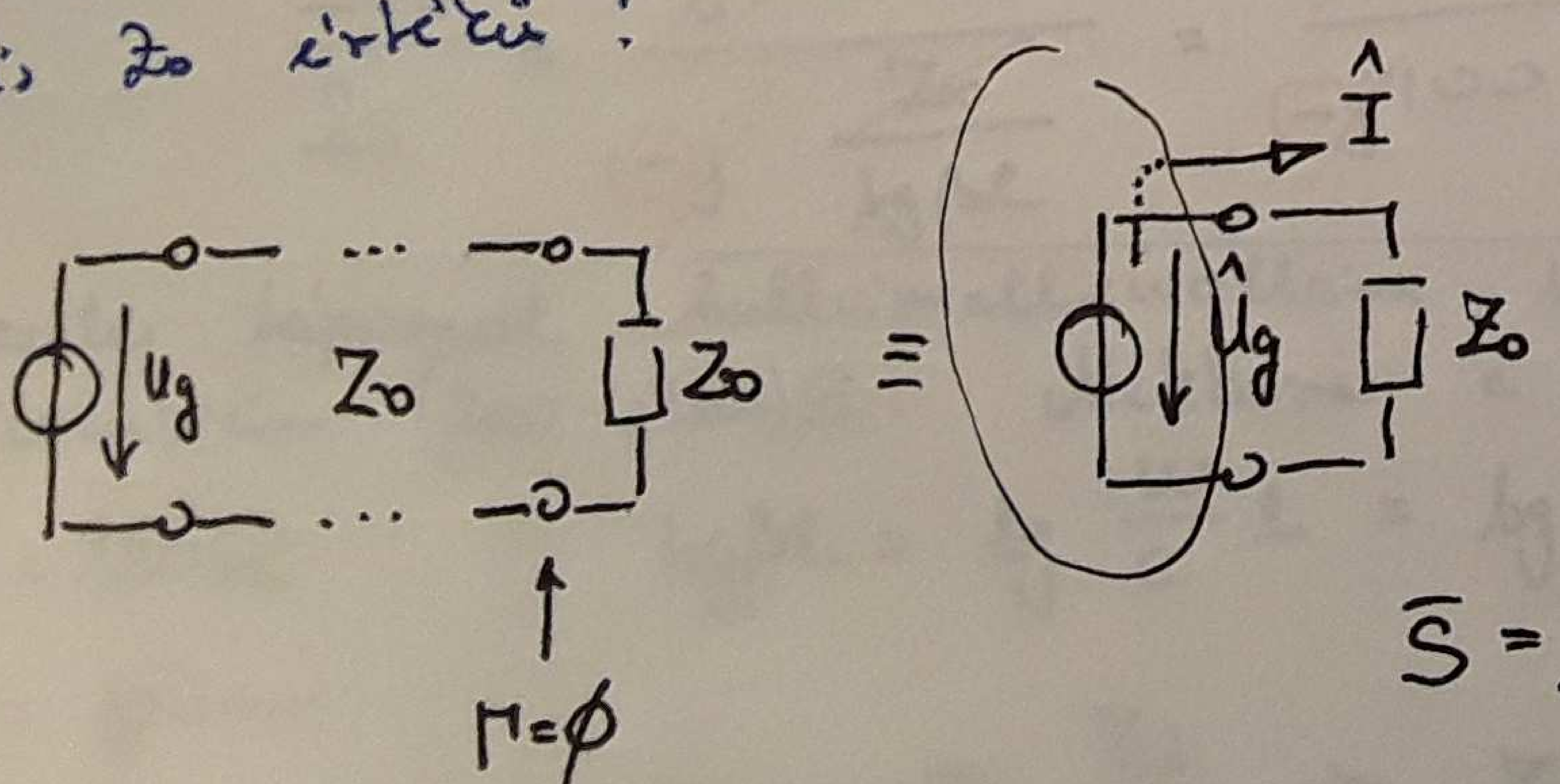
$$\boxed{R_2 = \frac{1+\Gamma}{1-\Gamma} Z_0} = \frac{1 + 1/3}{1 - 1/3}\, 600 = 1200\,\Omega$$

Egy légszigetelésű, ideális távvezeték hosszegységre eső induktivitása mérések alapján $L = \frac{5}{3}$ mH/km. Határozzuk meg a távvezeték hullámellenállását!

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{L}{\frac{1}{v^2 L}}} = \sqrt{v^2 L^2} = vL = 3 \cdot 10^8 \cdot \frac{5}{3}\, 10^{-3}\, \frac{1}{1000} = 500\,\Omega$$

$$v = \frac{1}{\sqrt{LC}} \rightarrow v^2 = \frac{1}{LC} \rightarrow \boxed{C = \frac{1}{v^2 L}}$$

Egy $Z_0 = \boxed{240\,\Omega}$ hullámellenállású ideális távvezetéket $u_g(t) = [120 \cos \omega t]\,V$ feszültséggel tápláljuk. Határozzuk meg a forrás hatásos és meddő teljesítményét, ha a lezárás is Z_0 értékű!



$$\boxed{\bar{S} = -\frac{1}{2} \hat{U}_g \hat{I}^*} = -\frac{1}{2}\, 120^2 \cdot \frac{1}{240} = -30\,VA$$

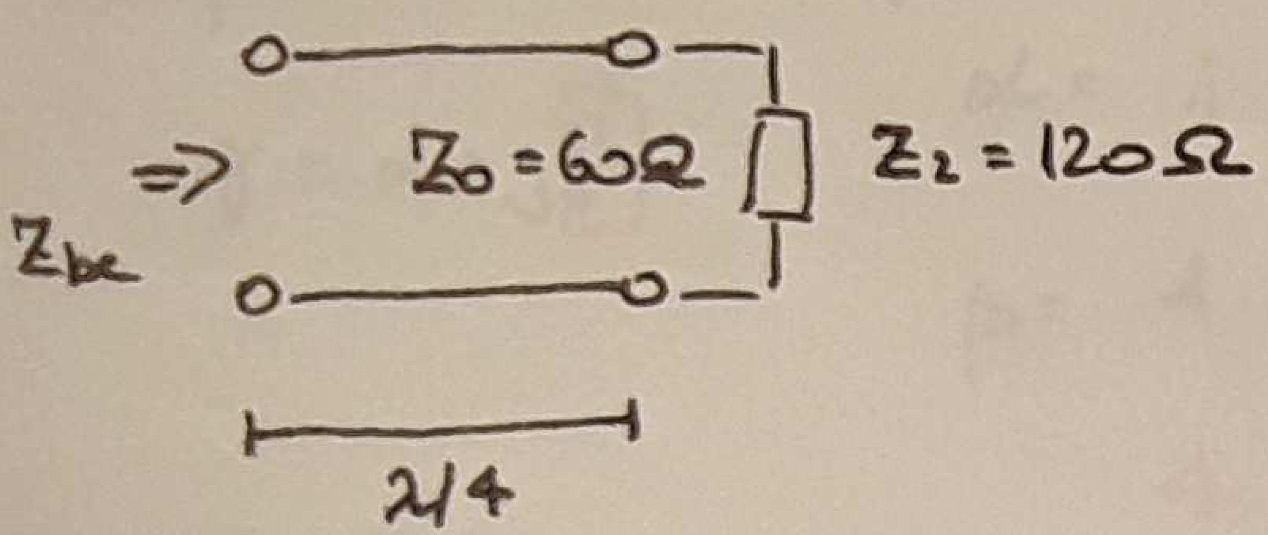
$$\hat{I}^* = \frac{\hat{U}_g^*}{Z_0^*} \Rightarrow -\frac{1}{2} \hat{U}_g \frac{\hat{U}_g^*}{Z_0^*} = -\frac{1}{2} \frac{|\hat{U}_g|^2}{Z_0^*}$$

$$\bar{S} = \cancel{P}$$

$$\overset{W \quad var}{\boxed{\bar{S} = P + jQ}} \rightarrow P = -30\,W$$

$$Q = \emptyset\,var$$

Egy ideális távvezeték hullámellenállása $\underline{60\Omega}$, lezárásának rezisztenciája 120Ω, hossza pedig $\lambda/4$. Határozzuk meg a bemeneti impedanciát!



$$Z_{be} = Z_0 \frac{Z_2 + j\,\boxed{tg\beta l}\cdot Z_0}{Z_0 + j\,\boxed{tg\beta l}\cdot Z_2} = Z_0 \frac{\cancel{Z_2/tg\beta l} + \cancel{j}Z_0}{\cancel{Z_0/tg\beta l} + \cancel{j}Z_2} = \frac{Z_0^2}{Z_2} = \frac{60^2}{120} = \underline{30\Omega}$$

$$tg\beta l = tg\frac{2\pi}{\lambda}\frac{\lambda}{4} = tg\frac{\pi}{2} \to \infty$$

Egy 1km hosszú ideális távvezeték bemenetén mért üresjárási impedancia $\underline{Z_ü = -j100\Omega}$, a rövidzárási bemeneti impedancia pedig $Z_r = j100\Omega$. Számítsuk ki a terjedési együtthatót! $\gamma = j\beta$

$$Z_ü \Rightarrow Z_2 \to \infty \qquad Z_{be} = Z_ü = Z_0 \frac{1 + \cancel{Z_0 j\, tg\beta l / Z_2}}{\cancel{\frac{Z_0}{Z_2}} + j\, tg\beta l} = \frac{Z_0}{j\, tg\beta l} = \boxed{-j\frac{Z_0}{tg\beta l} = -j100}$$

$Z_0 = ?$ $\quad \frac{45°}{180°}\pi$

$\beta = ?$

$Z_0 = 100\Omega$

$$Z_{rz} \Rightarrow Z_2 \to \emptyset \qquad Z_{be} = Z_{rz} = Z_0 \frac{jZ_0\, tg\beta l}{Z_0} = \boxed{jZ_0\, tg\beta l = j100}$$

$$\frac{Z_{rz}}{Z_ü} = \frac{\cancel{j}Z_0\, tg\beta l}{-\cancel{j}\frac{\cancel{Z_0}}{tg\beta l}} = \frac{\cancel{j}100}{-\cancel{j}100} \Rightarrow (tg\beta l)^2 = 1$$

$tg\beta l = 1$ $\quad \cancel{45°} \Rightarrow \underline{\underline{0{,}785}} = \beta l$ (RAD)

$\beta = 0{,}785\ 1/km \qquad \gamma = j\,0{,}785\ 1/km$

Ideális tápvonal hullámellenállása 120Ω, hossza 150m. A bemeneti impedancia 20MHz frekvencián $(200-j120)\Omega$. Mekkora a lezáró impedancia?

$Z_0 = 120\Omega$

$l = 150m$

$f = 20MHz$

$$tg\beta l = tg\frac{\omega}{c}l = tg\left(\frac{2\pi\cdot 20\cdot 10^6}{3\cdot 10^8}150\right) = \emptyset$$

$$Z_{be} = Z_0\frac{Z_2}{Z_0} \Rightarrow Z_{be} = Z_2 \qquad Z_2 = (200 - j120)\Omega$$

14
15
21

Egy távvezeték terjedési együtthatója 100Hz frekvencián $\gamma = (1+j4)\,10^{-3}\ 1/km$. Határozzuk meg a távvezeték csillapítási együtthatóit, fázisegyütthatóját, és a hullám terjedési sebességét!

α $\quad\beta\quad v$

$$\gamma = \alpha + j\boxed{\beta}$$

$$\alpha = 1\cdot 10^{-3}\ \frac{1}{km}$$

$$\beta = 4\cdot 10^{-3}\ \frac{1}{km}$$

$$4\cdot 10^{-6}\ \frac{1}{m}$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} \Rightarrow \cancel{2\pi\cdot 1}$$

$$v = f\cdot\lambda \rightarrow \lambda = \frac{v}{f}$$

$$v = \frac{2\pi f}{\beta} = \frac{2\pi\cdot 100}{4\cdot 10^{-3}\cdot 10^{-3}} = 157\cdot 10^{6}\ \frac{m}{s}$$

$$1{,}57\cdot 10^{8}\ \frac{m}{s}$$

Egy ideális távvonal paraméterei 50Hz-en a következők: $L = 4mH/km$, $C = 9nF/km$. Számítsuk ki a vezeték hullámparamétereit! Z_0 γ

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{4\cdot 10^{-3}\cdot 10^{-3}}{9\cdot 10^{-9}\cdot 10^{-3}}} = 666{,}7\,\Omega$$

$$\gamma = \alpha + j\beta = j\frac{\omega}{c} = j\frac{2\pi f}{c} = j\frac{2\pi\cdot 50}{3\cdot 10^{8}} = j\,1{,}047\cdot 10^{-6}\ \frac{1}{m}$$

Egy $Z_0 = 200\,e^{-j10°}\,\Omega$ hullámimpedanciájú távvonal feszültsége $u(z,t) = 400\cos(\omega t - \beta z)$ V. Írjuk fel az áram egyenletét!

$$Z_0 = \frac{U(z)}{I(z)} \longrightarrow I(z) = \frac{U(z)}{Z_0} = \frac{400}{200\,e^{-j10°}} = 2e^{j10°}A$$

$$i(z,t) = 2\cos(\omega t - \beta z + \underbrace{10°}_{0{,}174})\ A.$$

Egy távvezeték üresjárási impedanciája $Z_ü = 100(1+j)\,\Omega$, rövidzárási impedanciája $Z_{rz} = 100(1-j\sqrt{3})\,\Omega$. Határozzuk meg a hullámimpedanciát!

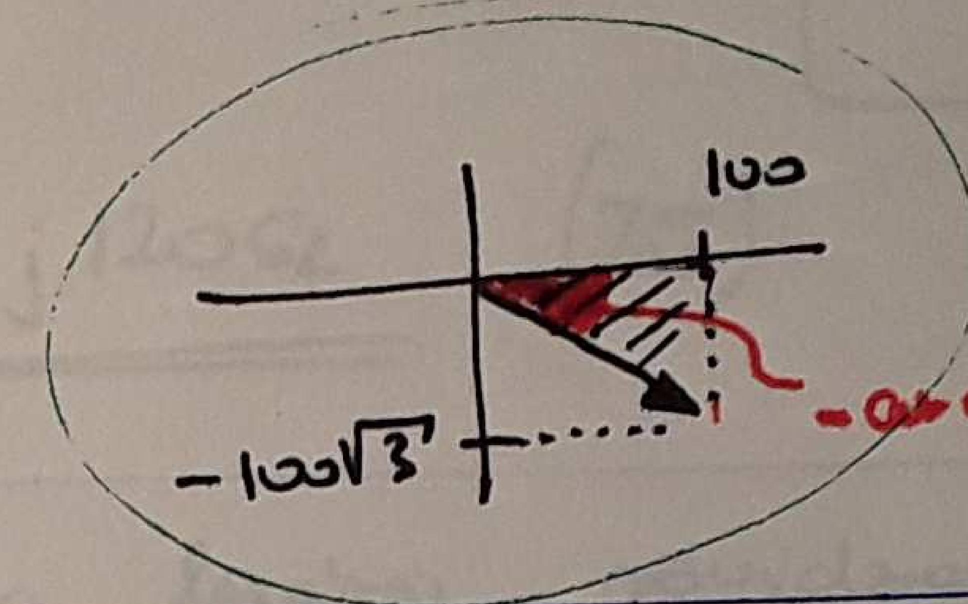
$$Z_ü = 100 + j100 = 100\sqrt{2}\, e^{j45°}\,\Omega$$

$$Z_{rz} = 100 - j\sqrt{3}\,100 = 200\, e^{-j60°}\,\Omega$$

$$Z_0 = \sqrt{Z_ü \cdot Z_{rz}} = \sqrt{100\sqrt{2}\, e^{j45°} \cdot 200\, e^{-j60°}} =$$

$$= 168{,}18\, e^{-j7{,}5°}\,\Omega$$

$$(166{,}74 - j21{,}95)\,\Omega$$



$$-\mathrm{arctg}\,\cancel{\frac{100\sqrt{3}}{100}} = -60°$$

Egy távvezeték adatai 50 Hz-en az alábbiak. Határozzuk meg a hullámparamétereket!

Z_0

γ

$R = 0{,}2155\ \Omega/\text{km}$ $\quad L = 0{,}398\ \text{mH}/\text{km}$ $\quad G = 0{,}28\ \mu\text{S}/\text{km}$ $\quad C = 111{,}8\ \text{nF}/\text{km}$.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{0{,}2155 + j\,2\pi 50 \cdot 0{,}398 \cdot 10^{-3}}{0{,}28 \cdot 10^{-6} + j\,2\pi 50\ 111{,}8 \cdot 10^{-9}}} = \ldots = (73{,}19 - j\,41{,}82)\,\Omega$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \ldots = (1{,}48 + j\,2{,}56) \cdot 10^{-3}\ \frac{1}{\text{km}}$$

Egy $l=\lambda/8$ hosszúságú ideális, 120Ω hullámellenállású tápvonal végén szakadás van.
Határozzuk meg a bemeneti impedanciát!

$$\boxed{Z_{be} = Z_0 \frac{Z_2 + j\cancel{Z_0\, tg\beta l}}{Z_0 + j\cancel{Z_2\, tg\beta l}}}$$

$$tg\beta l = tg \frac{2\pi}{\cancel{\lambda}}\frac{\cancel{\lambda}}{8} = tg\frac{\pi}{4} = 1$$

$$Z_2 \to \infty \qquad Z_{be} = Z_0 \frac{1 + j\cancel{Z_0/Z_2}}{\cancel{Z_0/Z_2} + j} = Z_0/j = \underline{\underline{-j\,120\Omega}} \qquad (Z_ü)$$

Határozzuk meg a bemeneti impedanciát, ha a lezárás rövidzár!

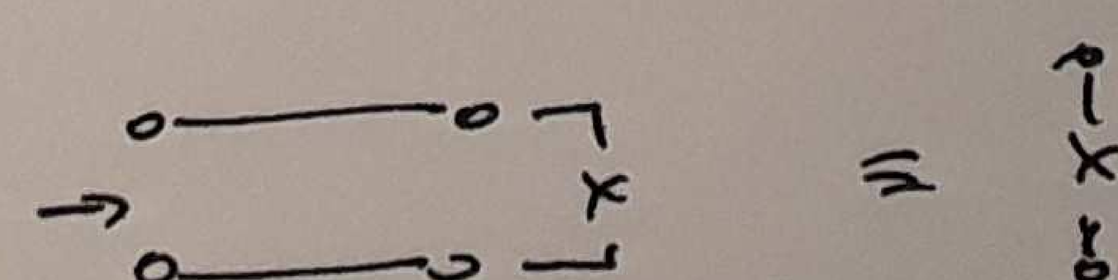
$$Z_2 \to \emptyset \qquad Z_{be} = \cancel{Z_0}\frac{0 + jZ_0}{\cancel{Z_0} + j\emptyset} = \underline{\underline{j\,120\Omega}} \qquad (Z_{rz})$$

$$Z_0 = \sqrt{j120\cdot(-j)120} = \sqrt{120^2} = 120\Omega$$

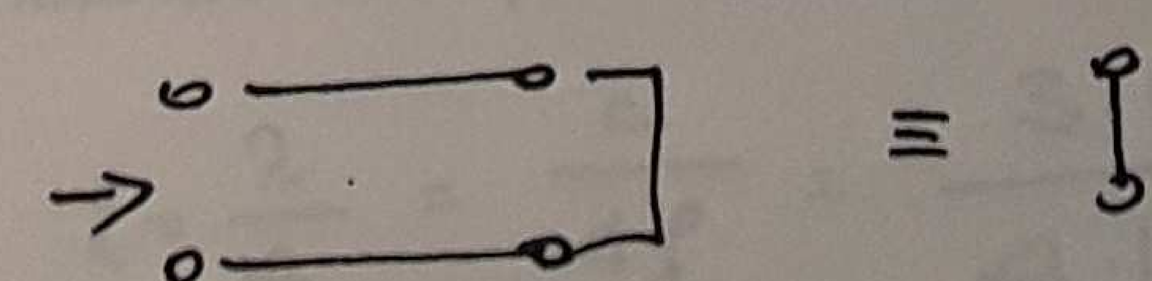
Oldjuk meg a fenti feladatokat, ha $l=\lambda/2$!

$$tg \frac{\cancel{2\pi}}{\cancel{\lambda}}\cdot\frac{\cancel{\lambda}}{\cancel{2}} = tg\,\pi = \emptyset$$

$$Z_ü = Z_0 \frac{1}{\frac{Z_0}{Z_2}} \Rightarrow \infty$$

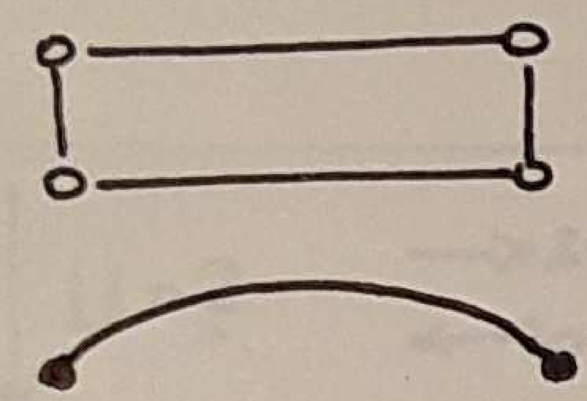


$$Z_{rz} = Z_0 \cdot \frac{\emptyset}{Z_0} = \emptyset$$



Mekkora a mindkét végén rövidrezárt ideális tápvezeték hossza, ha a legkisebb rezonancia frekvencia 300 MHz?

$f\downarrow \quad \lambda\uparrow$



$$l = \frac{\lambda}{2} = \frac{c}{2f} = \frac{3\cdot 10^8}{2\cdot 300\cdot 10^6} = 0{,}5\,m$$

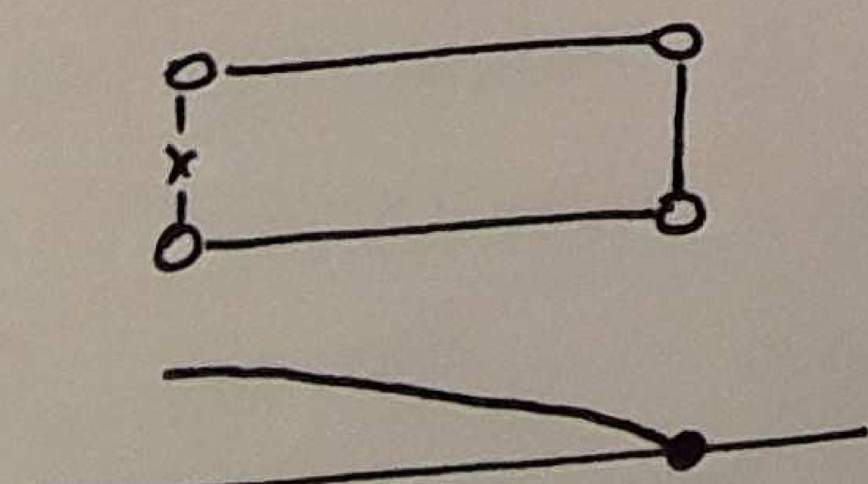
$$c = f\lambda$$

Egy ideális tápvezeték hossza $l = \lambda/4$, lezáró ellenállásának rezisztenciája $210\,\Omega$. Határozzuk meg a hullámellenállást, ha a bemeneti impedancia $60\,\Omega$.

$$Z_{be} = Z_0 \frac{Z_2 + jZ_0 \operatorname{tg}\beta l}{Z_0 + jZ_2 \operatorname{tg}\beta l} \overset{sp.}{=} Z_0 \frac{jZ_0}{jZ_2} = \frac{Z_0^2}{Z_2}$$

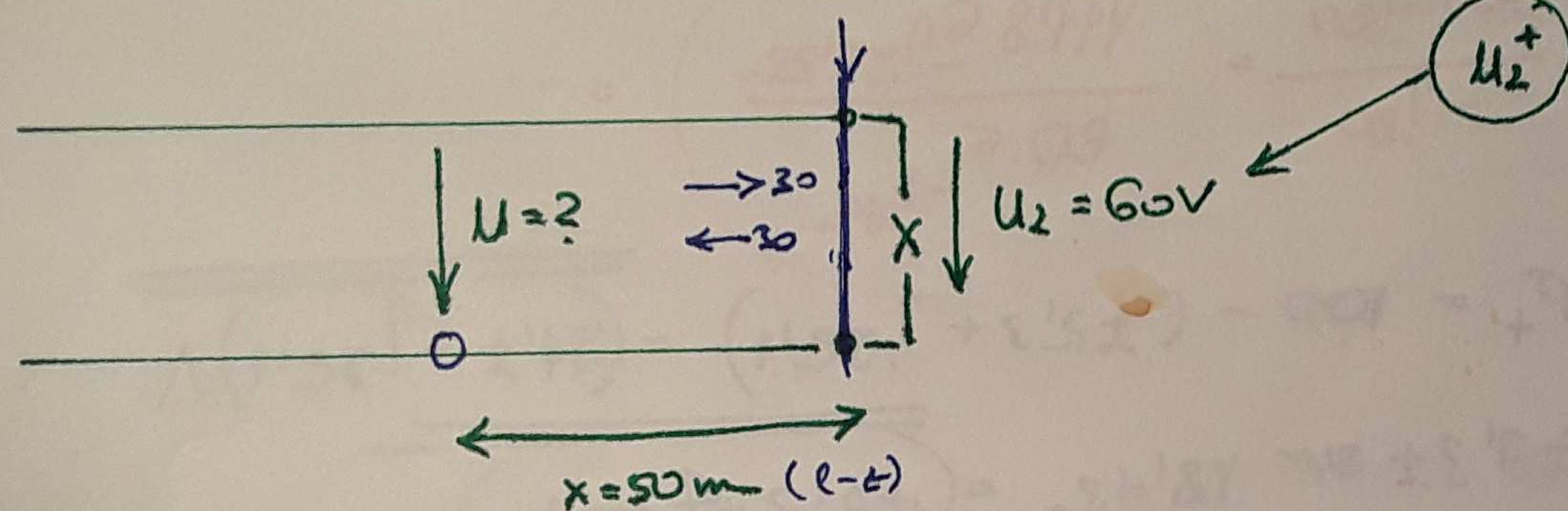
$$Z_{be} = \frac{Z_0^2}{Z_2} \longrightarrow Z_0 = \sqrt{Z_{be}\cdot Z_2} = \sqrt{60\cdot 210} = 112{,}25\,\Omega$$

Mekkora a hossza az egyik végén rövidzárral, a másik végén szakadással lezárt ideális tápvezetéknek, ha a legkisebb rezonancia frekvencia 150 MHz?



$$l = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\cdot 10^8}{4\cdot 150\cdot 10^6} = 0{,}5\,m$$

Egy ideális távvezeték nyitott végén a feszültség csúcsértéke 60V. Számítsuk ki a vezeték végétől $x = 50m$ távolságban a feszültség csúcsértékét, ha a frekvencia 10MHz.



$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0}\Bigg|_{Z_2 \to \infty} = \frac{1 - Z_0/Z_2}{1 + Z_0/Z_2} = \underline{1}$$

$$\frac{U_2^-}{U_2^+}$$

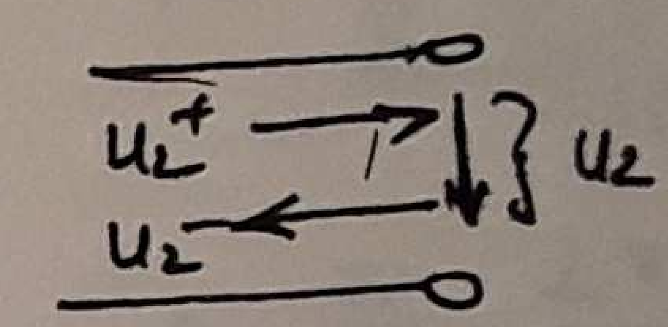
$$\gamma = j\beta = j\frac{2\pi f}{c} = j\frac{2\pi \cdot 10 \cdot 10^6}{3 \cdot 10^8} = \underline{\underline{j\frac{\pi}{15}}}$$

$$U(z) = U_2^+\left[e^{\gamma(l-z)} + \Gamma e^{-\gamma(l-z)}\right]$$

$$z = l)\quad U(l) = \underset{(60)}{U_2} = U_2^+\left[1+1\right] \longrightarrow U_2^+ = \frac{U_2}{2} = \underline{\underline{30V}}$$

$$U(z) = 30\left(\underbrace{\frac{e^{j\beta\cdot 50} + e^{-j\beta\cdot 50}}{2}}_{\cos\beta 50}\right)2 = 60\cos\left(\frac{\pi}{15}50\right) = \underline{\underline{-30V}}$$

Egy $Z_0 = 50\,\Omega$ hullámellenállású távvezetéket $Z_2 = (50 - j50)\,\Omega$ impedanciájú fogyasztó zár le, amelynek feszültsége $u_2(t) = [100 \cos \omega t]$ V. Határozzuk meg a pozitív irányban haladó feszültséghullám komplex csúcsértékét a lezárás helyén! Határozzuk meg a negatív irányban haladó feszültséghullám komplex csúcsértékét is! Adjuk meg a lezárás áramának komplex csúcsértékét!



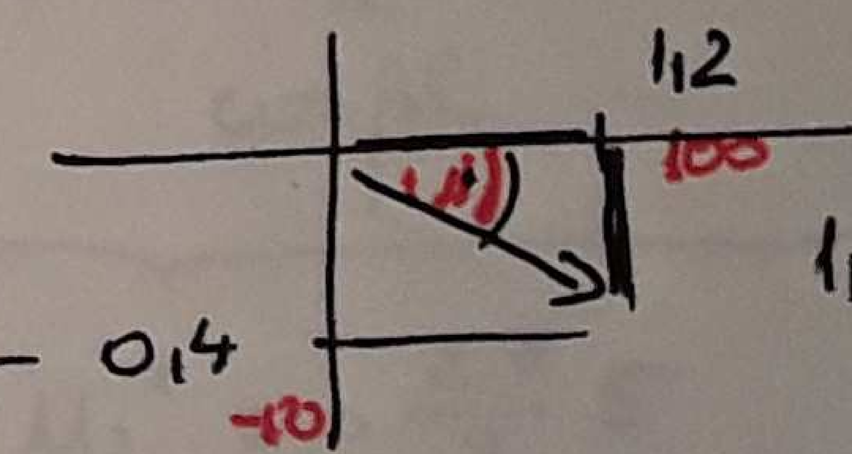
a/ $U(z) = U_2^+ \left[e^{j\beta(l-z)} + \Gamma e^{-j\beta(l-z)} \right]$

$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{\cancel{50} - j50 - \cancel{50}}{50 - j50 + 50} = \frac{-j50}{100 - j50} \cdot \boxed{\frac{100 + j50}{100 + j50}} = \frac{-j5000 + 2500}{100^2 + 50^2} = \underline{0{,}2 - j0{,}4}$$

(*)

$(a + jb)(a - jb) = a^2 - jab + jab + b^2 = a^2 + b^2 = |r|^2$

$z = l)$ $\quad U_2 = \underbrace{U(l)}_{100} = U_2^+ \left[1 + 0{,}2 - j0{,}4 \right]$



$1{,}26\, e^{-j18{,}43^\circ}$

$$U_2^+ = \frac{100}{1{,}2 - j0{,}4} = \frac{100\, e^{j0^\circ}}{1{,}26\, e^{-j18{,}43^\circ}} = \underline{\underline{79{,}37\, e^{+j18{,}43^\circ}\ \text{V}}}$$

$$U_2^+ = 79{,}37 \cos 18{,}43^\circ + j\, 79{,}37 \sin 18{,}43^\circ = \underline{\underline{(75{,}3 + j\,25{,}1^\circ)\ \text{V}}}$$

b/ $U_2 = U_2^+ + U_2^- \longrightarrow U_2^- = U_2 - U_2^+ = 100 - (75{,}3 + j25{,}1) = \underline{\underline{(24{,}7 - j25{,}1)\ \text{V}}}$

c/ $I_2 = \dfrac{U_2}{Z_2} = \dfrac{100}{50 - j50} = \underline{\underline{(1 + j)\ \text{A}}}$.

(*) $\dfrac{-j50}{100 - j50} = \dfrac{50\, e^{-j90^\circ}}{111{,}8\, e^{-j26{,}56^\circ}} = \ldots$

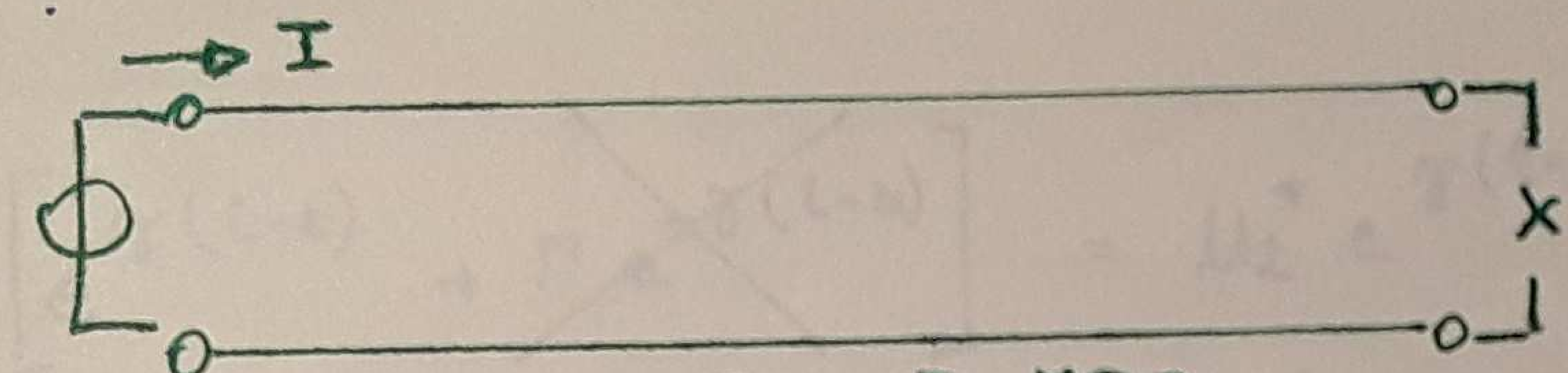
$$I(z) = \frac{U_2^+}{Z_0} \left[e^{j\beta(l-z)} - \Gamma \cdot e^{-j\beta(l-z)} \right] = \frac{U_2^+}{Z_0} \left[1 - \Gamma \right] = \ldots = \underline{\underline{(1 + j)\ \text{A}}}$$

Egy $l=5m$ hosszúságú ideális távvezeték hullámellenállása 75Ω. A vezetéket $100\,MHz$ frekvenciájú szinuszos feszültség forrás táplálja, a csúcsértéke $250V$. A távvezeték szekunder oldala nyitott. Határozzuk meg a forrás áramának csúcsértékét!

$$\Gamma = \left.\frac{Z_2 - Z_0}{Z_2 + Z_0}\right|_{Z_2 \to \infty} = 1$$

$f = 100\,MHz$

$U = 250\,V$



$l = 5m \quad Z_0 = 75\Omega$

$$\beta = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi \cdot 100 \cdot 10^6}{3 \cdot 10^8} = \frac{2}{3}\pi$$

$$U(z) = U_2^+ \left[e^{j\beta(l-z)} + \Gamma e^{-j\beta(l-z)} \right] \rightarrow U(0) = 250 = U_2^+ \cdot 2 \underbrace{\underbrace{\frac{\left[e^{j\beta l} + e^{-j\beta l} \right]}{2}}_{\cos \beta l}}$$

$$I(z) = \frac{U_2^+}{Z_0} \left[e^{j\beta(l-z)} - \Gamma e^{-j\beta(l-z)} \right]$$

$$250 = 2 U_2^+ \cos \frac{2\pi}{3} 5$$

$$\hookrightarrow U_2^+ = \frac{250}{2 \cos\left(\frac{2\pi}{3} 5\right)} = -250V$$

$$I(0) = \frac{-250}{75} \cdot 2j \underbrace{\frac{\left[e^{j\beta l} - e^{-j\beta l} \right]}{2j}}_{\sin \beta l}$$

$$I(0) = -\frac{250}{75} 2j \sin\left(\frac{2\pi}{3} 5\right) = \overbrace{j\,5{,}77A}^{\hat{I}} \qquad 5{,}77 e^{j90°} A$$

Egy hullámimpedanciájával lezárt távvezeték bemenetén a feszültség $[\underline{100\cos\omega t}]$ V, $\omega = 2\pi$ Mrad/s. Határozzuk meg a $z = 300$ m helyen a feszültség időfüggvényét, ha az alábbi adatok ismertek: $Z_0 = 100\, e^{-j30°}\ \Omega$, $\alpha = 10^{-2}\ \frac{1}{m}$, $v = 2 \cdot 10^{8}$ m/s.

$\Gamma = \emptyset.$

$$\beta = \frac{\omega}{v} = \frac{\cancel{2}\pi \cdot 10^{6}}{\cancel{2} \cdot 10^{8}} = \pi \cdot 10^{-2}\ \frac{1}{m}$$

$$U(z) = U_2^+ \left[e^{\gamma(l-z)} + \cancel{\Gamma e^{\gamma(l-z)}} \right] = U_2^+ e^{\gamma(l-z)}$$

$$U(0) = 100 = U_2^+ e^{\gamma l} \longrightarrow U_2^+ = 100\, e^{-\gamma l}$$

$$U(z) = U_2^+ e^{\gamma(l-z)} = 100\, \underbrace{e^{-\gamma l}\ e^{\gamma l}}\ e^{-\gamma z} = 100\, e^{-\gamma z} = 100\, e^{-(\alpha + j\beta) z} =$$

$$= \underbrace{100\, e^{-\alpha z}}\ \underbrace{e^{-j\beta z}} = \underbrace{100\, e^{-0{,}01 \cdot 300}}_{4{,}98} \cdot \underbrace{e^{-j\pi \cdot 10^{-2} \cdot 300}}_{e^{-j3\pi}} = 4{,}98\, e^{-j3\pi}\ \text{V}$$

$$u(z,t) = 4{,}98 \cos(\omega t - 3\pi)\ \text{V}$$

Fejezzük ki az R, L, G, C paraméterekkel azt a feltételt, amely mellett egy veszteséges tápvonal hullámimpedanciája tisztán valós értékű!

$$(a+jb)(a-jb) = a^2 + b^2$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{R+j\omega L}{G+j\omega C} \cdot \frac{G-j\omega C}{G-j\omega C}} =$$

$$= \sqrt{\frac{RG \cancel{-j\omega RC + j\omega LG} + \omega^2 LC}{G^2 + (\omega C)^2}}$$

$$-j\omega RC + j\omega LG = \emptyset$$

$$\cancel{j\omega} RC = \cancel{j\omega} LG$$

$$\boxed{RC = LG} \longrightarrow Z_0 \in \mathbb{R}.$$

Egy ideális távvezeték hullámellenállása 75Ω, hossza 5m. A vezetéket feszültség forrás táplálja : $\hat{U} = 250V$, $f = 100 MHz$. Határozzuk meg a forrás áramát, ha a lezárás rövidzár!

$$\beta = \frac{\omega}{c} = \frac{2\pi \cdot 100 \cdot 10^6}{3 \cdot 10^8} = \frac{2}{3}\pi$$

$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0} = -1$$

$$U(z) = U_2^+ \left[e^{j\beta(l-z)} - e^{-j\beta(l-z)} \right]$$

$$U(0) = 250 = U_2^+ \underbrace{\frac{\left(e^{j\beta l} - e^{-j\beta l}\right)}{2j}}_{\sin\beta l} 2j$$

$$\Rightarrow 250 = U_2^+ \, 2j \sin\beta l \qquad \frac{1}{j} = -j$$

$$U_2^+ = \frac{250}{2j \sin\left(\frac{2\pi}{3} 5\right)} = +j\,144{,}34$$

$$I(z) = \frac{U_2^+}{Z_0} \left[e^{j\beta(l-z)} + e^{-j\beta(l-z)} \right]$$

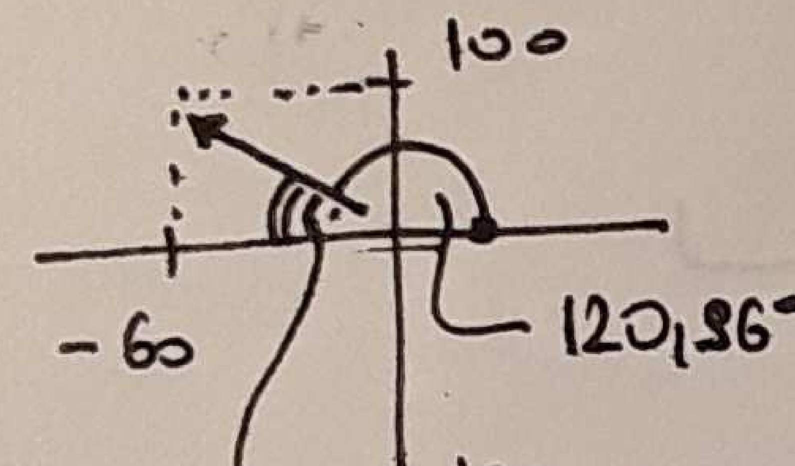
$$I(0) = \frac{j\,144{,}34}{75} \underbrace{\frac{\left[e^{j\beta l} + e^{-j\beta l} \right]}{2}}_{\cos\beta l} 2$$

$$I(0) = 2 \cdot \frac{j\,144{,}34}{75} \cos\frac{2\pi}{3} \cdot 5 = -j\,1{,}92\,A$$

Egy 160Ω hullámellenállású ideális távvezeték bemenetére $u_1(t) = [100\cos\omega t]$ V feszültségű forrás csatlakozik, kimenetére pedig egy $(100+j100)\,\Omega$ impedanciájú fogyasztó. A frekvencia 1MHz, $l = 5$mm. Határozzuk meg a lezárás feszültségének és áramának időfüggvényét!

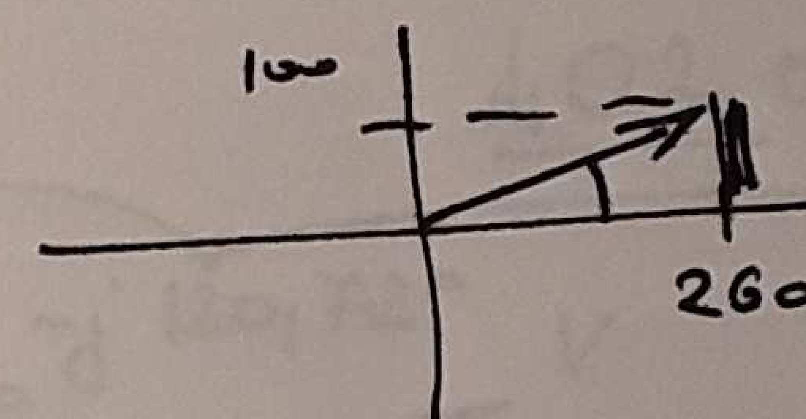
$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{100 + j100 - 160}{100 + j100 + 160} = \cancel{\frac{260 + j100}{}} \quad \frac{-60 + j100}{260 + j100} = \frac{116{,}62\, e^{j120{,}96^\circ}}{278{,}6\, e^{j21{,}04^\circ}} = \underline{0{,}42\, e^{j99{,}92^\circ}} =$$

$$= \underline{-0{,}07 + j0{,}41}$$



$\operatorname{arctg}\frac{100}{60} = 59{,}04^\circ$

$116{,}62\, e^{j120{,}96^\circ}$



$\cancel{2}$

$$\beta = \frac{2\pi f}{c} = \frac{2\pi \cdot 10^6}{3 \cdot 10^8} = \underline{\frac{2\pi}{3} \cdot 10^{-2}}$$

$$\frac{99{,}92^\circ \cdot \pi}{180^\circ} = \underline{\underline{0{,}56\pi}} \qquad \beta l = \frac{2\pi}{3} 5$$

$$u(z) = U_2^+ \left[e^{j\beta(l-z)} + \Gamma e^{-j\beta(l-z)} \right]$$

$$u(0) = 100 = U_2^+ \left[e^{j\beta l} + \Gamma e^{-j\beta l} \right] = U_2^+ \left[e^{j\frac{2\pi}{3}5} + \underbrace{0{,}42\, e^{j99{,}92^\circ} \cdot e^{-j\frac{2\pi}{3}5}}_{} \right] =$$

$$= U_2^+ \left[e^{j\frac{2\pi}{3}5} + 0{,}42 \cdot e^{-j2{,}77\pi} \right] =$$

$$= U_2^+ \left[-0{,}5 - j0{,}866 - 0{,}32 - j0{,}28 \right]$$

$$100 = U_2^+ \left[-0{,}82 - j1{,}146 \right] \longrightarrow U_2^+ = \frac{-100}{0{,}82 + j1{,}15} = \frac{100\, e^{-j90^\circ}}{1{,}41\, e^{j54{,}57^\circ}} = \underline{70{,}92\, e^{-j144{,}57^\circ}}\ \text{V}$$

$$U(z) = U_2^+ \left[e^{j\beta(l-z)} + \Gamma e^{-j\beta(l-z)} \right]$$

$$z=l) \quad U(l) = U_2^+ \left[1 + \Gamma \right] = 70{,}92\, e^{-j\,144{,}51^\circ} \cdot \underbrace{\underbrace{\left(1 + (-0{,}07 + j\,0{,}41)\right)}_{\underbrace{0{,}93 + j\,0{,}41}_{1{,}02\, e^{j\,23{,}79^\circ}}}}$$

$$= 72{,}34\, e^{-j\,120{,}72^\circ} \text{ V}$$

$$u_2(t) = 72{,}34 \cos(\omega t - 120{,}72^\circ) \text{ V}$$

$$I(z) = \frac{U_2^+}{Z_0}\left[1 - \Gamma\right] = \frac{70{,}92\, e^{-j\,144{,}51^\circ}}{160} \left[1 - (-0{,}07 + j\,0{,}41)\right]$$

$z=l$

$$\underbrace{0{,}44\, e^{-j\,144{,}51^\circ} \underbrace{(1{,}07 - j\,0{,}41)}_{1{,}15\, e^{-j\,20{,}96^\circ}}}_{0{,}51 \cdot e^{-j\,165{,}47^\circ} \text{ A}}$$

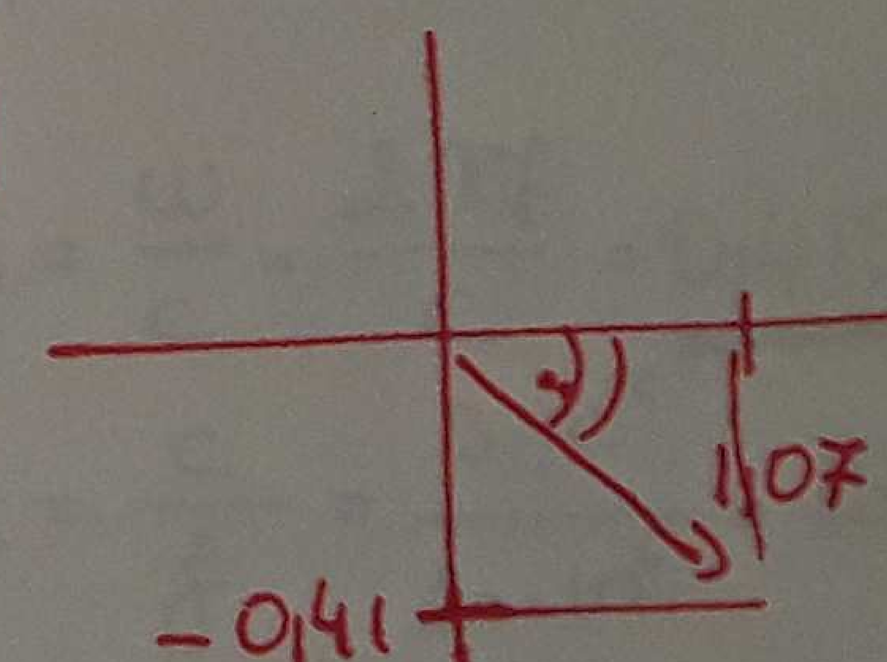
$$i_2(t) = 0{,}51 \cos(\omega t - 165{,}47^\circ) \text{ A}$$

$$\frac{72{,}34\, e^{-j\,120{,}72^\circ}}{0{,}51\, e^{-j\,165{,}47^\circ}}$$

$$141{,}84\, e^{j\,44{,}75^\circ}\ \Omega$$

$$100{,}73 + j\,99{,}86\ \Omega$$

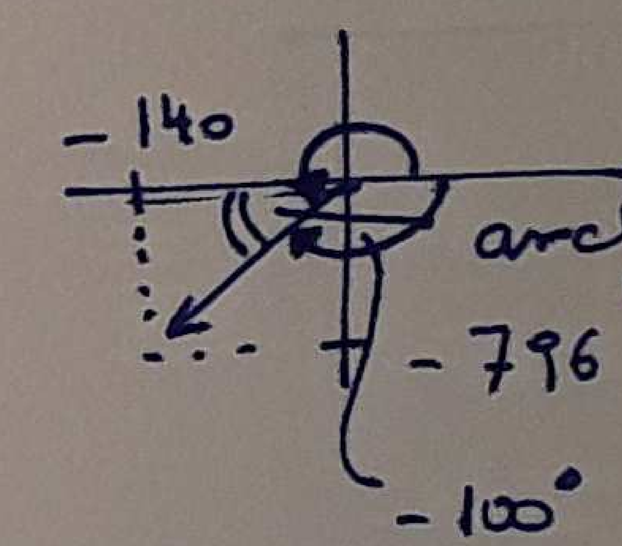
$$100 + j\,100$$



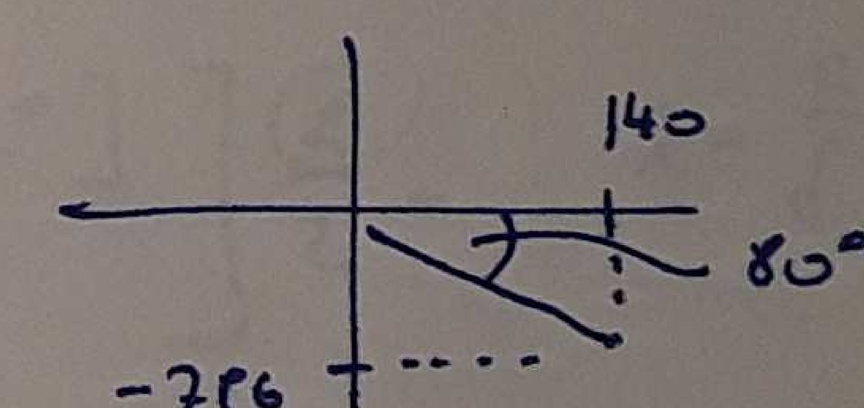
Egy 140Ω hullámellenállású ideális tápvonalat $C_0 = 10\,pF$ kapacitású kondenzátorral zárunk le. A rezonanciafrekvencia 20 MHz. Határozzuk meg a tápvonal hosszát!

$$\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0} = \frac{\frac{1}{j\omega C_0} - Z_0}{\frac{1}{j\omega C_0} + Z_0} = \frac{-j796 - 140}{-j796 + 140} = \frac{\cancel{1}\, e^{-j100°}}{\cancel{1}\, e^{-j80°}} = 1 e^{-j20°}$$

$$\frac{1}{j\omega C_0} = \frac{1}{j2\pi \cdot 20 \cdot 10^6 \cdot 10 \cdot 10^{-12}} = -j\,796\,\Omega$$



$$\mathrm{arctg}\,\frac{796}{140} = 80°$$



$$U(z) = U_2^+\left[e^{j\beta(l-z)} + \Gamma e^{-j\beta(l-z)}\right] =$$

$$= U_2^+\left[e^{j\beta(l-z)} + e^{-j20°} e^{-j\beta(l-z)}\right] =$$

$$= U_2^+ e^{-j10°}\left[e^{j\beta(l-z)} e^{j10°} + e^{-j\beta(l-z)} e^{-j10°}\right] =$$

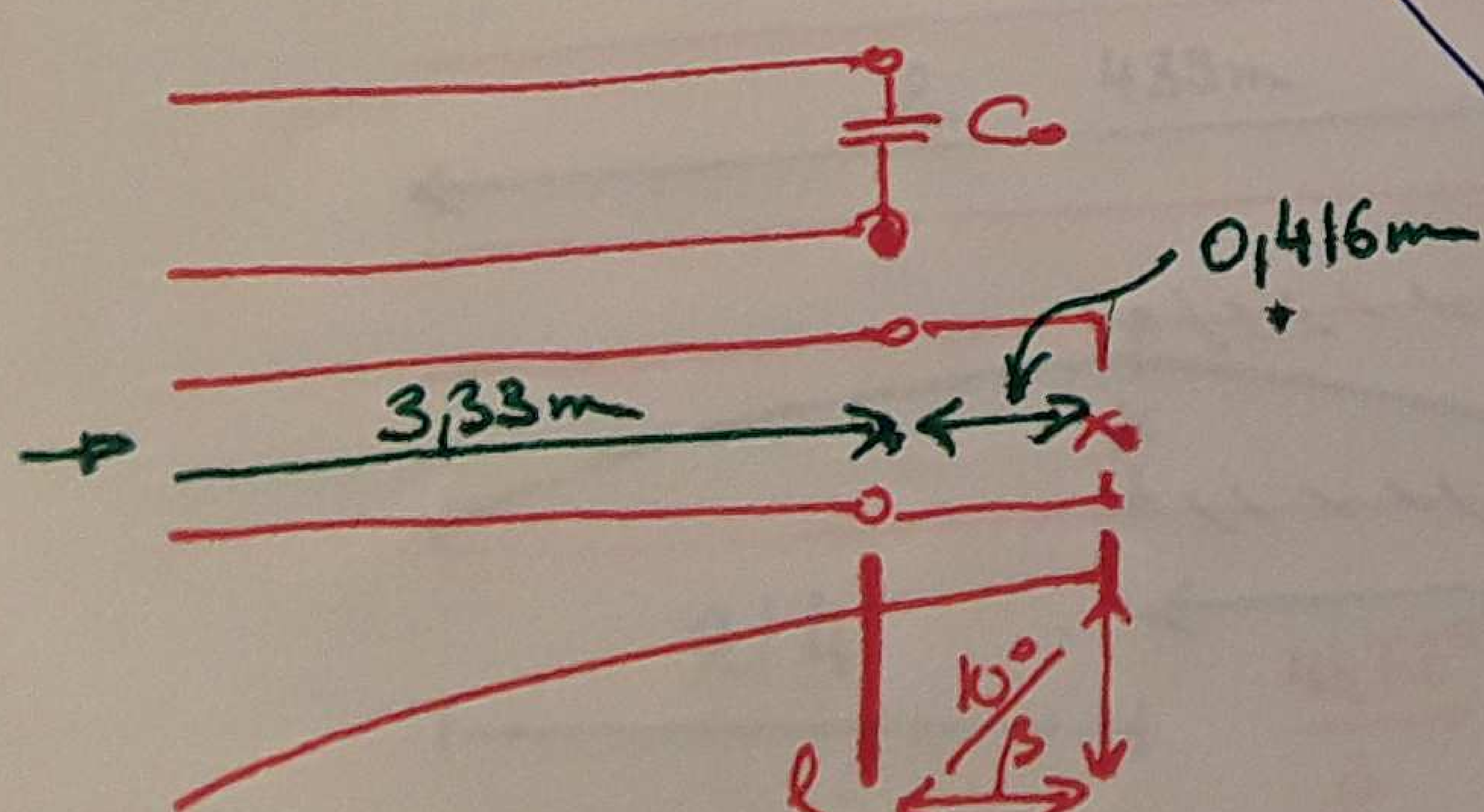
$$= U_2^+ e^{-j10°}\left[e^{j[\beta(l-z)+10°]} + e^{-j[\beta(l-z)+10°]}\right]\frac{2}{2} =$$

$$= 2U_2^+ e^{-j10°} \boxed{\cos}\left[\beta(l-z) + 10°\right]$$

$$\beta = \frac{\omega}{c} = \frac{2\pi f}{c} = 0{,}419$$

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{20 \cdot 10^6} = 15$$

$$\beta(l-z) + 10° = \beta\left[l - z + \frac{10°}{\beta}\right]$$



$$l + \frac{10°}{\beta} = \frac{\lambda}{4}$$

0,416 m

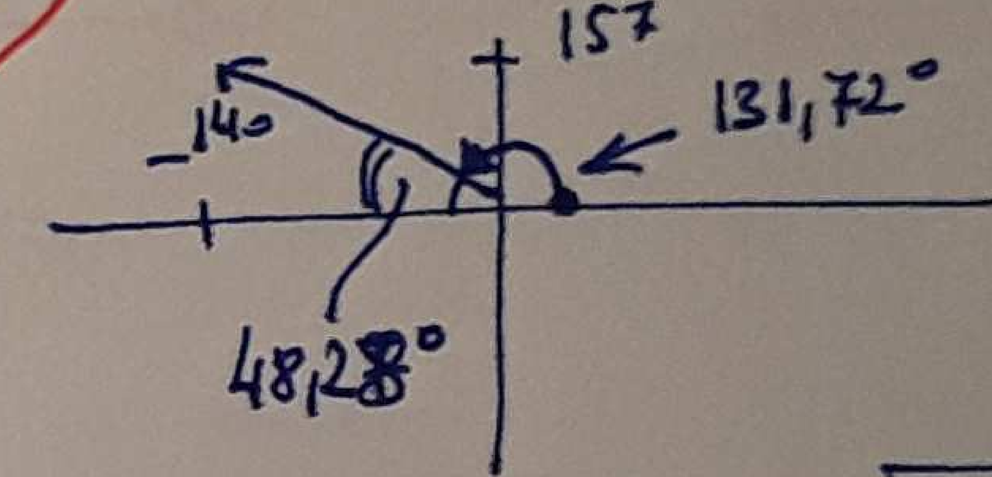
$$l = \frac{\lambda}{4} - \frac{10°}{\beta} = \frac{15}{4} - \frac{10°}{0{,}419} = 3{,}33\,m$$

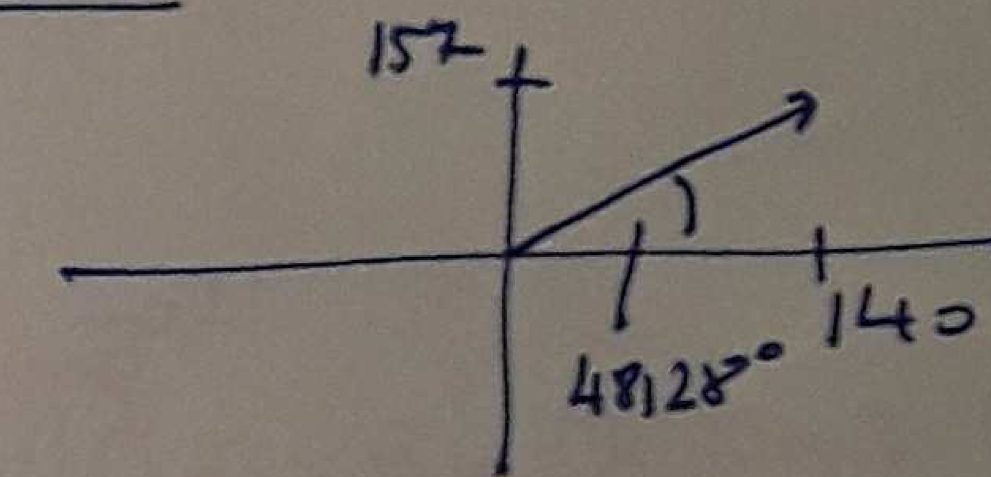
RAD!

Egy 140Ω hullámellenállású ideális tápvonalat $L_0 = 0{,}1\,mH$ induktivitású tekercs zárja. A rezonanciafrekvencia 250 kHz. Határozzuk meg a tápvonal hosszát!

$$j\omega L_0 = j\,2\pi\cdot 250\cdot\cancel{10^3}\cdot 0{,}1\cdot\cancel{10^{-3}} = j\,157\,\Omega$$

$$\Gamma = \frac{j\omega L_0 - Z_0}{j\omega L_0 + Z_0} = \frac{j157-140}{j157+140} = \frac{r\,e^{j131{,}72°}}{r\,e^{j48{,}28°}} = \cancel{e^{j88°}}\ \ 1\,e^{j83{,}44°}$$





$$U(z) = U_2^+\left[e^{j\beta(l-z)} + \Gamma\,e^{-j\beta(l-z)}\right] =$$

$$= U_2^+\left[e^{j\beta(l-z)} + e^{j83{,}44°}\,e^{-j\beta(l-z)}\right] =$$

$$= U_2^+ e^{j41{,}72°}\left[e^{j\beta(l-z)}\,e^{-j41{,}72°} + e^{j41{,}72°}\,e^{-j\beta(l-z)}\right] =$$

$$= U_2^+ e^{j41{,}72°}\underbrace{\left[e^{j[\beta(l-z)-41{,}72°]} + e^{-j[\beta(l-z)-41{,}72°]}\right]\frac{2}{2}}$$

$$= 2U_2^+ e^{j41{,}72°}\,\boxed{\cos}\left(\beta(l-z) - 41{,}72°\right)$$

RAD

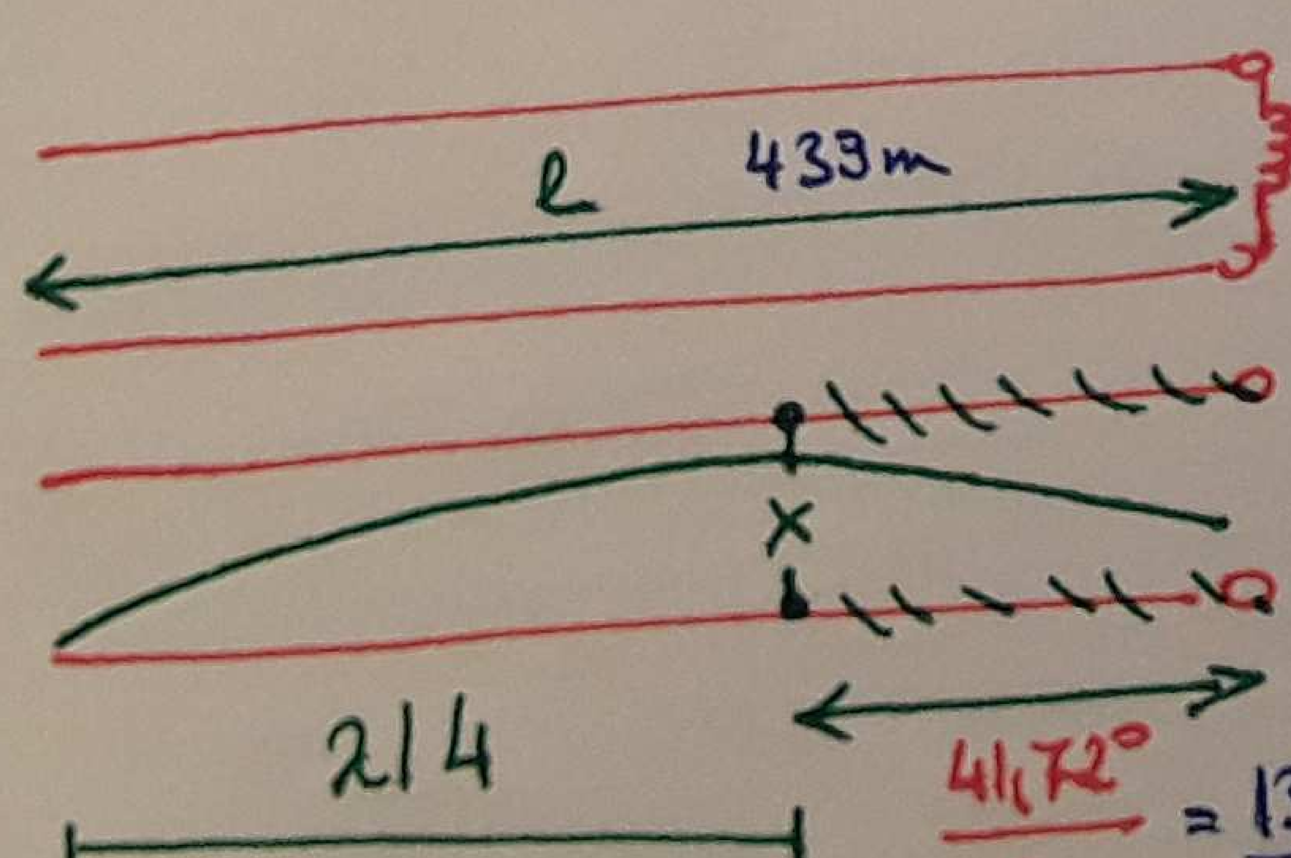
$$\beta(l-z) - 41{,}72°$$

$$\beta\left[l - z - \frac{41{,}72°}{\beta}\right]$$

$$l \pm \frac{\pi/2}{\beta} = \frac{\lambda}{4}$$

$$\lambda = \frac{c}{f} = \frac{3\cdot10^8}{250\cdot10^3} = 1200\,m$$

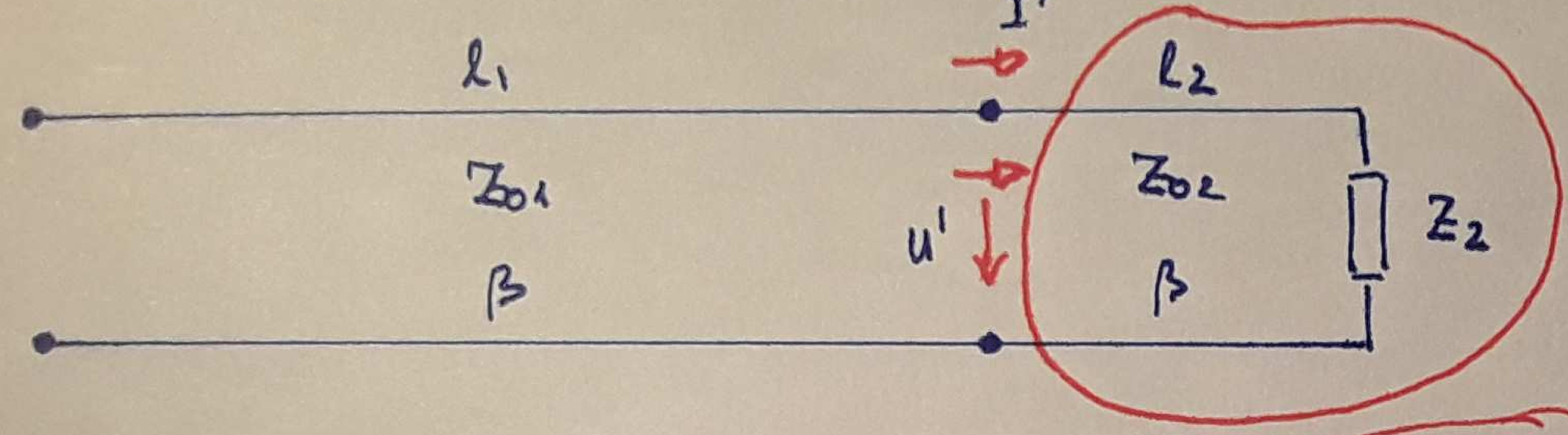
$$\beta = \frac{\omega}{c} = \frac{2\pi\cdot250\cdot10^3}{3\cdot10^8} = 5{,}235\cdot10^{-3}\,\frac{1}{m}$$

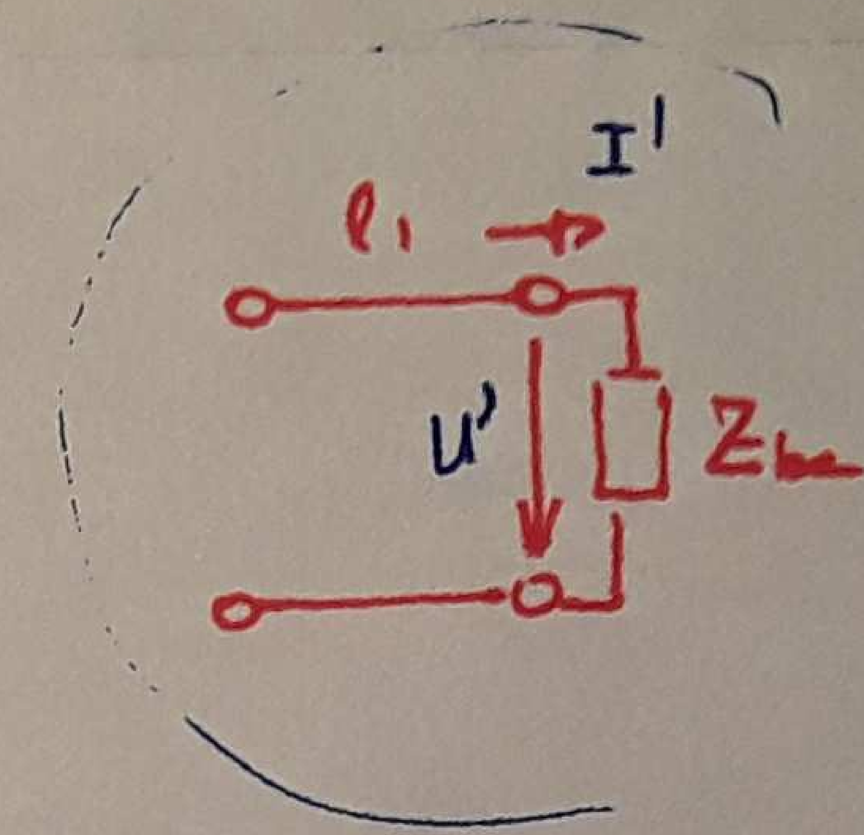


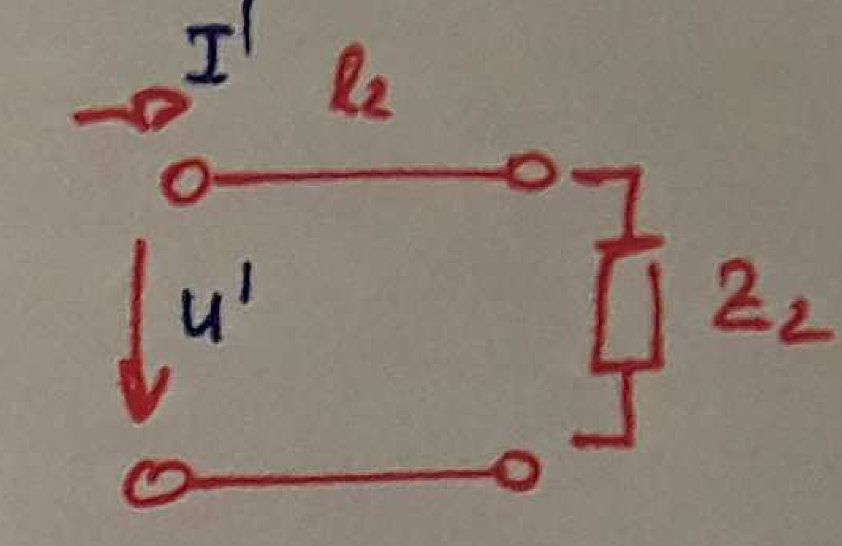
$$\frac{41{,}72°}{\beta} = 139{,}1\,m$$

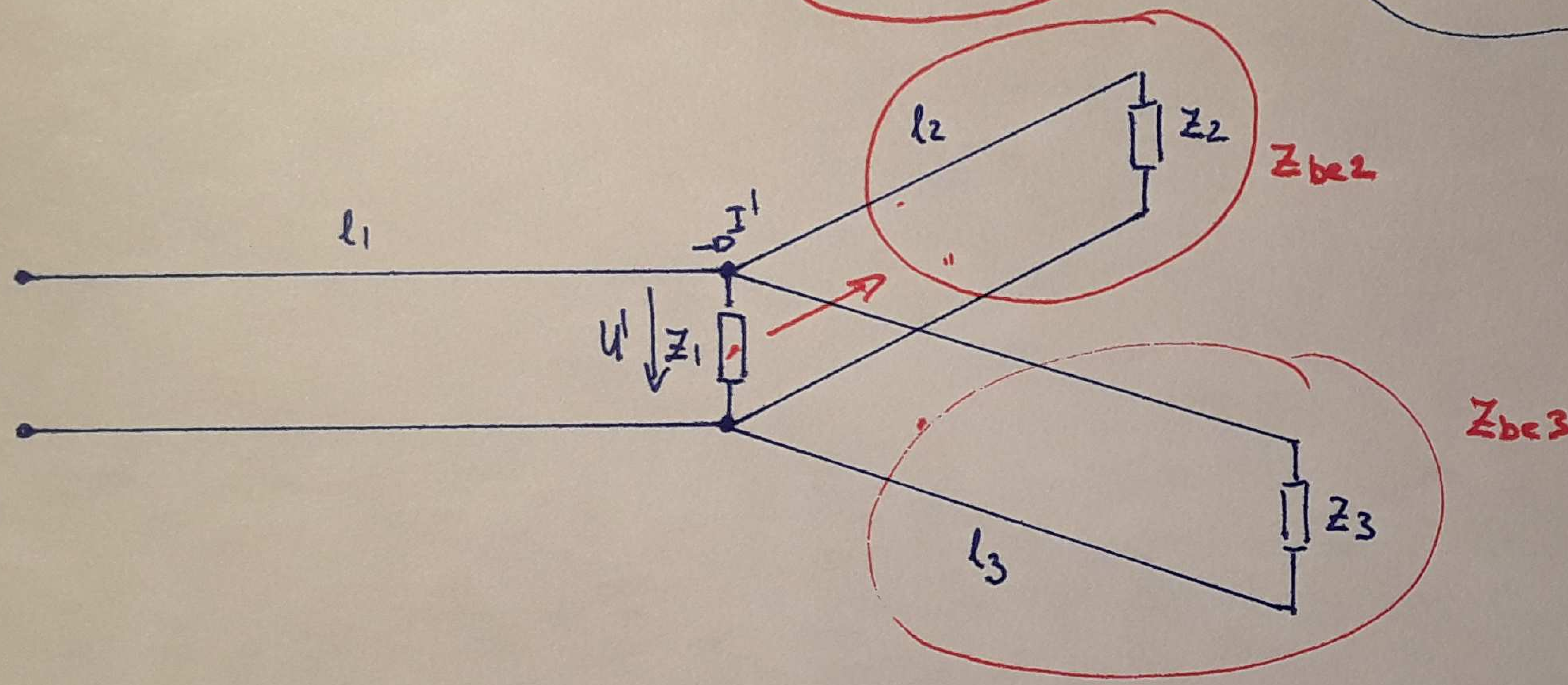
$$\frac{\lambda}{4} = l - \frac{41{,}72°}{\beta}$$

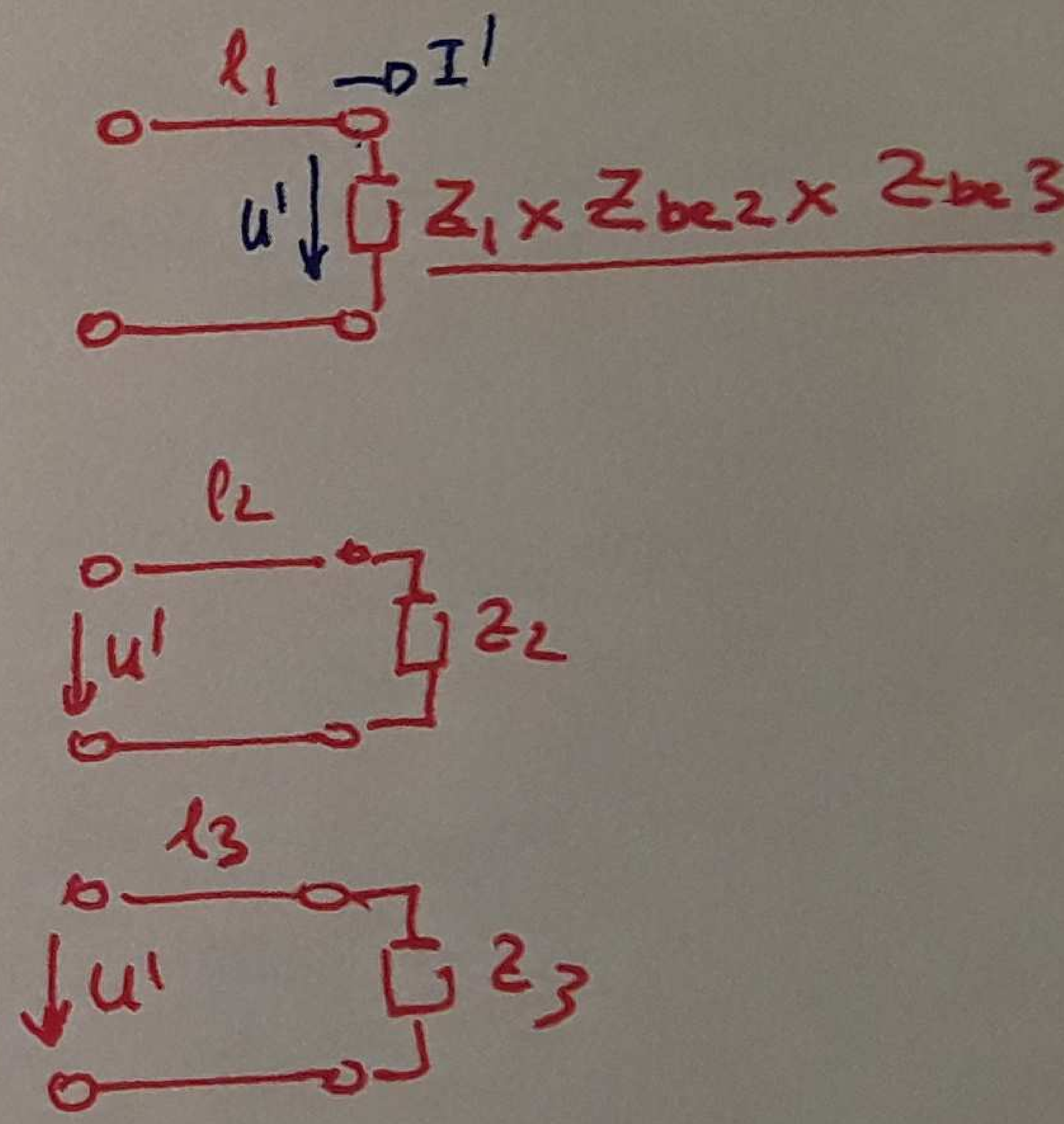
$$l = \frac{\lambda}{4} + \frac{41{,}72°}{\beta} = \frac{1200}{4} + \frac{41{,}72/180\cdot\pi}{5{,}235\cdot10^{-3}} = 439\,m$$


l_1
Z_{01}
β
I'
u'
l_2
Z_{02}
β
Z_2

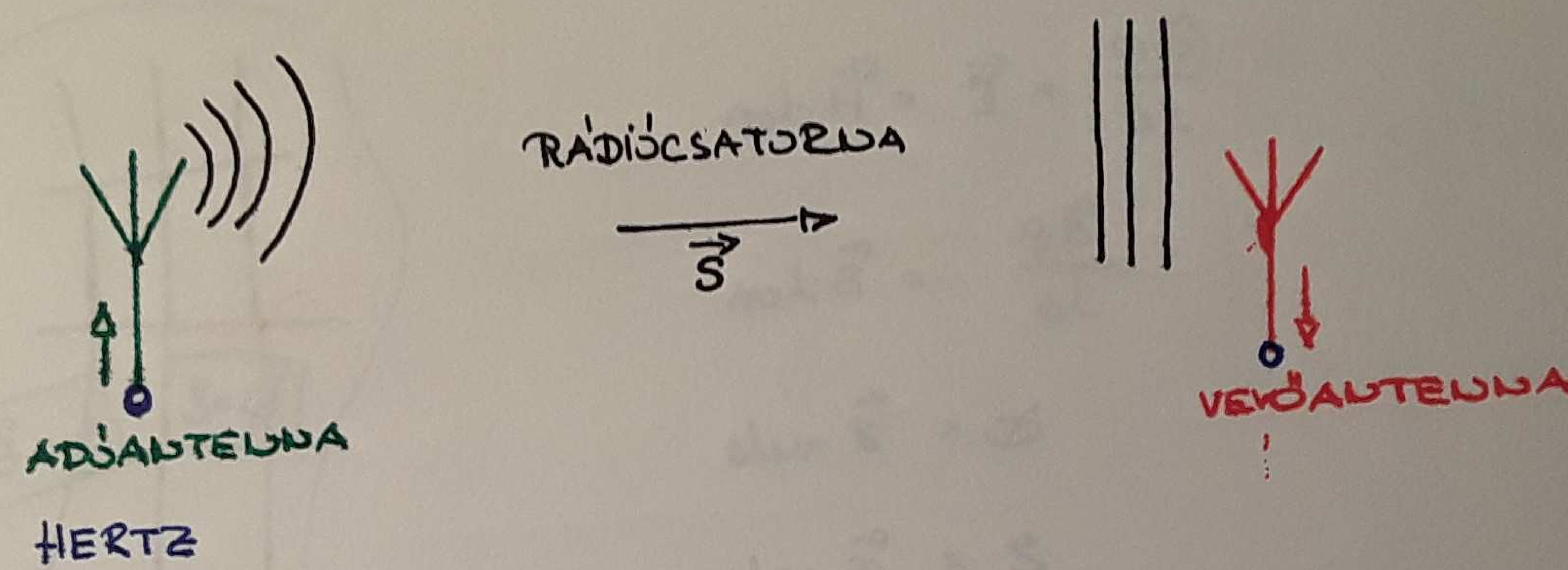

I'
l_1
u'
Z_{be}


I'
l_2
u'
Z_2


l_1
I'
u'
Z_1
l_2
Z_2
Z_{be2}
l_3
Z_3
Z_{be3}

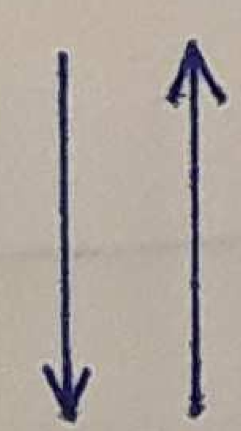

l_1
I'
u'
$Z_1 \times Z_{be2} \times Z_{be3}$
l_2
u'
Z_2
l_3
u'
Z_3

ELEKTROMÁGNESES HULLÁMOK



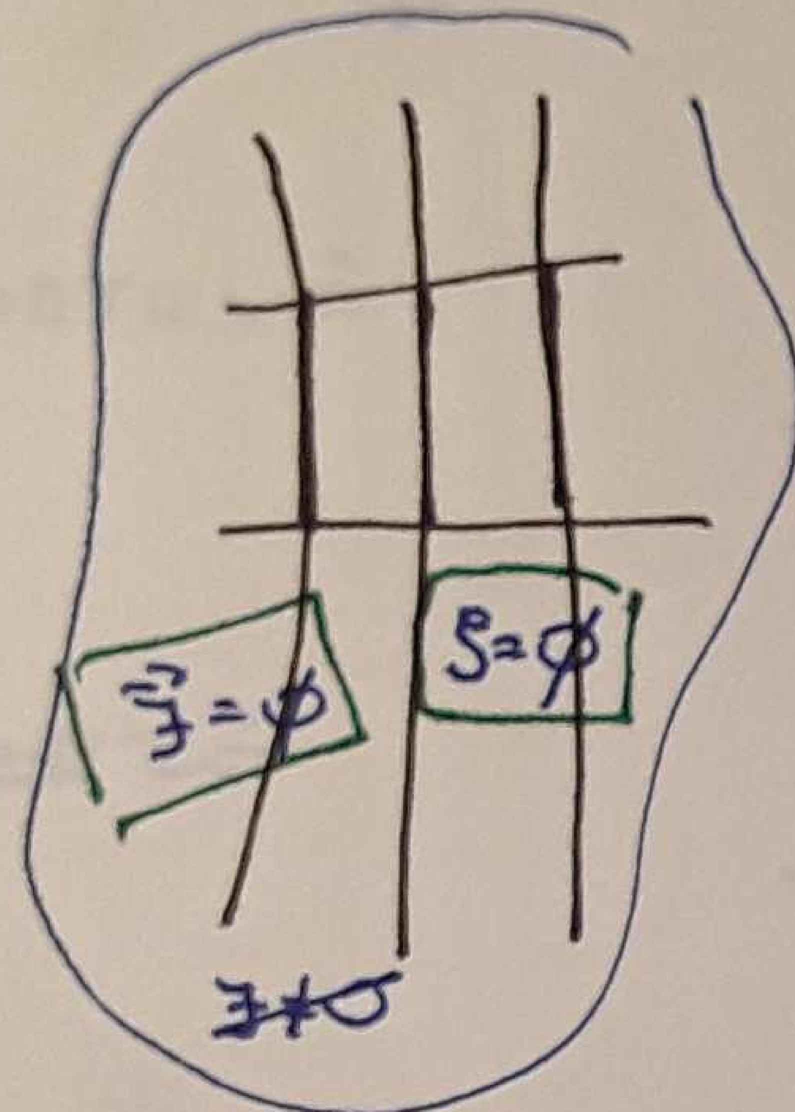
ANTENNA: E.M.-hullám (tér) kisugárzására és vételére szolgál.

- Tápvonal
- Antenna
- Szabad tér (síkhullám)



1)

SÍKHULLÁMOK



$$\mathrm{rot}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\mathrm{div}\,\vec{B} = 0$$

$$\mathrm{div}\,\vec{D} = \rho$$

$$\vec{D} = \varepsilon \vec{E} \qquad \vec{B} = \mu \vec{H} \qquad \boxed{\vec{J} = \sigma \vec{E}}$$

$$\mathrm{rot}\,\vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\mathrm{rot}\,\vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\mathrm{div}\,\vec{H} = 0$$

$$\mathrm{div}\,\vec{E} = 0$$

$$\mathrm{rot}\,\mathrm{rot}\,\vec{E} = -\mu \frac{\partial}{\partial t} \mathrm{rot}\,\vec{H}$$

$$\underbrace{\mathrm{grad}\,\mathrm{div}\,\vec{E}}_{0} - \Delta \vec{E} = -\mu \frac{\partial}{\partial t}\left(\sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right)$$

$$-\Delta \vec{E} = -\mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\cancel{\Delta \vec{E} =}$$

$$\boxed{\Delta \vec{E} - \mu\sigma \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0}$$

$$\mathrm{rot}\,\mathrm{rot}\,\vec{H} = \sigma\, \underline{\mathrm{rot}\,\vec{E}} + \varepsilon \frac{\partial}{\partial t} \underline{\mathrm{rot}\,\vec{E}}$$

$$\mathrm{grad}\,\underline{\mathrm{div}\,\vec{H}} - \Delta \vec{H} = -\sigma\mu \frac{\partial \vec{H}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\boxed{\Delta \vec{H} - \mu\sigma \frac{\partial \vec{H}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0}$$

$$\Delta\vec{E} - \mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = \emptyset$$

$\underline{\sigma = \emptyset}$ SZIGETELŐ

$$\underline{\Delta\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = \emptyset}$$

HULLÁMEGYENLET

$\underline{\varepsilon = \emptyset}$ VEZETŐ

$$\underline{\Delta\vec{E} - \mu\sigma\frac{\partial\vec{E}}{\partial t} = \emptyset}$$

DIFFÚZIÓS EGYENLET

$$\vec{E} = \frac{1}{\sigma + j\omega\varepsilon}\,\mathrm{rot}\,\vec{H}$$

$\hat{=}$ $\vec{H}$

Szinuszos gerjesztés: $\frac{\partial}{\partial t} \to j\omega$

$$\mathrm{rot}\,\vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial\vec{E}}{\partial t}$$

$$\mathrm{rot}\,\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$$

$$\mathrm{div}\,\vec{H} = \emptyset$$

$$\mathrm{div}\,\vec{E} = \emptyset$$

$$\boxed{\begin{aligned} \mathrm{rot}\,\vec{H} &= \sigma\vec{E} + j\omega\varepsilon\vec{E} = (\sigma + j\omega\varepsilon)\vec{E} \\ \mathrm{rot}\,\vec{E} &= -j\omega\mu\vec{H} \\ \mathrm{div}\,\vec{H} &= \emptyset \\ \mathrm{div}\,\vec{E} &= \emptyset \end{aligned}}$$

$$\vec{H} = -\frac{1}{j\omega\mu}\,\mathrm{rot}\,\vec{E}$$

$$-\mathrm{rot}\,\frac{1}{j\omega\mu}\,\mathrm{rot}\,\vec{E} = (\sigma + j\omega\varepsilon)\vec{E}$$

$$-\mathrm{rot}\,\mathrm{rot}\,\vec{E} = j\omega\mu(\sigma + j\omega\varepsilon)\vec{E}$$

$$-\cancel{\mathrm{grad}\,\mathrm{div}\,\vec{E}} + \Delta\vec{E} = j\omega\mu(\sigma + j\omega\varepsilon)\vec{E}$$

$$\boxed{\Delta\vec{E} - j\omega\mu(\sigma + j\omega\varepsilon)\vec{E} = \emptyset}$$

$$\frac{1}{\sigma + j\omega\varepsilon}\,\mathrm{rot}\,\mathrm{rot}\,\vec{H} = -j\omega\mu\vec{H}$$

$$\cancel{\mathrm{grad}\,\mathrm{div}\,\vec{H}} - \Delta\vec{H} = -j\omega\mu(\sigma + j\omega\varepsilon)\vec{H}$$

$$\boxed{\Delta\vec{H} - j\omega\mu(\sigma + j\omega\varepsilon)\vec{H} = \emptyset}$$

LINEÁRISAN POLARIZÁLT SÍKHULLÁM

$$\Delta \vec{E} - j\omega\mu(\sigma + j\omega\varepsilon)\vec{E} = \emptyset$$

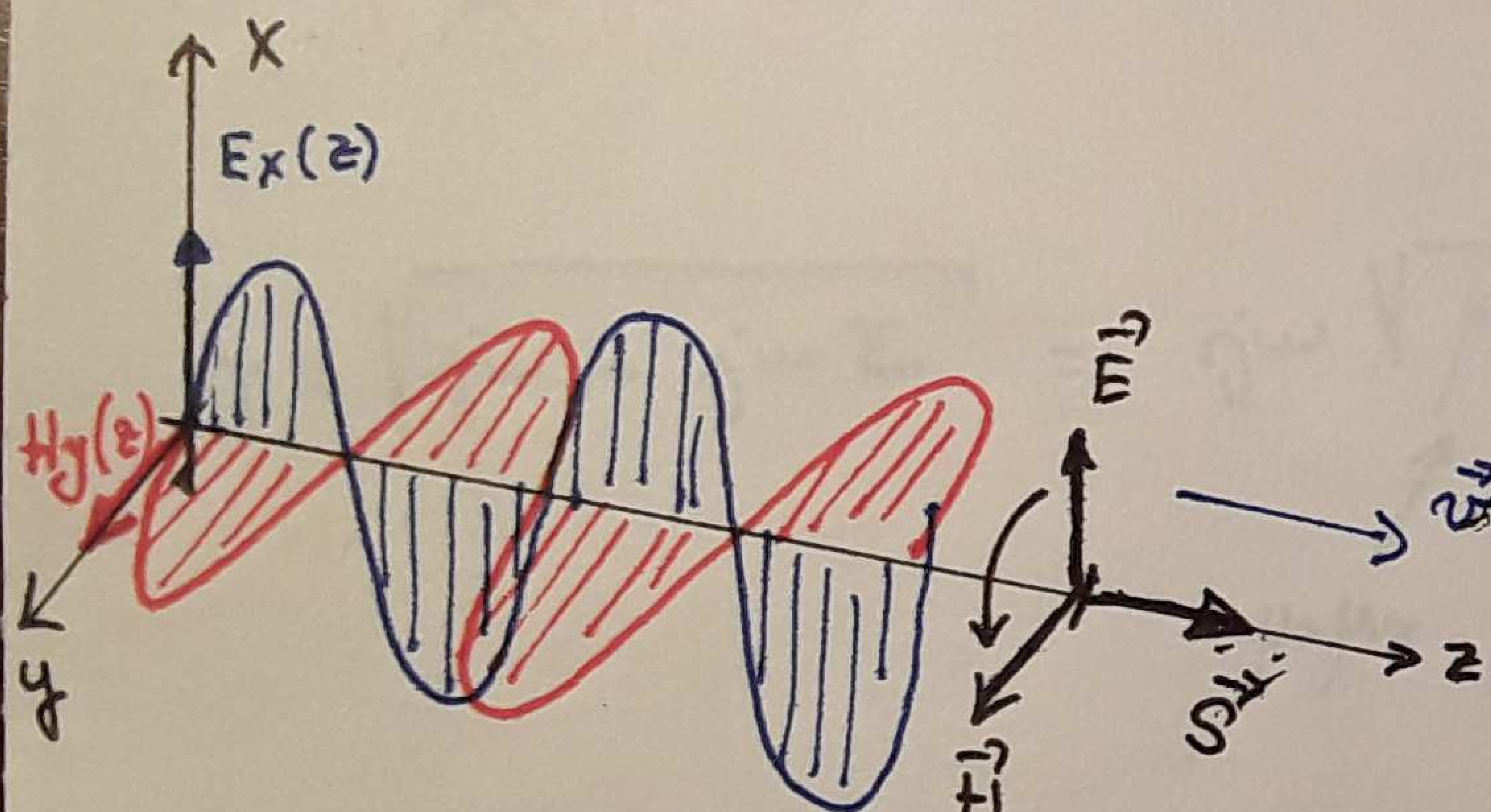
$$\vec{H} = -\frac{1}{j\omega\mu}\,\mathrm{rot}\,\vec{E}$$

$$\Delta \vec{H} - j\omega\mu(\sigma + j\omega\varepsilon)\vec{H} = \emptyset$$

$$\vec{E} = \frac{1}{\sigma + j\omega\varepsilon}\,\mathrm{rot}\,\vec{H}$$

$$\frac{d^2U}{dz^2} - \gamma^2 U = \emptyset$$

$$I = -\frac{1}{R + j\omega L}\frac{dU}{dz}$$



$$\frac{d^2E_x}{dz^2} - \overbrace{j\omega\mu(\sigma + j\omega\varepsilon)}^{\gamma^2}E_x = \emptyset$$

$$H_y = -\frac{1}{j\omega\mu}\frac{dE_x}{dz}$$

R	L	G	C
$\emptyset$	μ	σ	ε

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \qquad Z_0 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$E_y(z) = E_1^+ e^{-\gamma z} + E_1^- e^{\gamma z}$$

$$H_y(z) = \frac{E_1^+}{Z_0} e^{-\gamma z} - \frac{E_1^-}{Z_0} e^{\gamma z}$$

$$U(z) = U_1^+ e^{-\gamma z} + U_1^- e^{\gamma z}$$

$$I(z) = \frac{U_1^+}{Z_0} e^{-\gamma z} - \frac{U_1^-}{Z_0} e^{\gamma z}$$

$$\boxed{\vec{S} = \frac{1}{2}\vec{E} \times \vec{H}^*}$$

IDEÁLIS SZIGETELŐ

$$\gamma = \sqrt{j\omega\mu\,(\cancel{\sigma} + j\omega\varepsilon)}$$

$$\sigma = \emptyset$$

$$Z_0 = \sqrt{\frac{j\omega\mu}{\cancel{\sigma} + j\omega\varepsilon}}$$

$$\gamma = \sqrt{j\omega\mu \; j\omega\varepsilon} = j\omega\sqrt{\mu\varepsilon} = j\omega\underbrace{\sqrt{\mu_0\varepsilon_0}}_{\frac{1}{c}}\sqrt{\mu_r\varepsilon_r} = \frac{j\omega}{c}\sqrt{\mu_r\varepsilon_r}$$

$\mu_0\mu_r$ $\quad$ $\varepsilon_0\varepsilon_r$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad v = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

$$Z_0 = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \underbrace{\sqrt{\frac{\mu_0}{\varepsilon_0}}}_{120\pi\,\Omega \approx 377\,\Omega}\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$\gamma = j\beta = j\,\frac{\omega}{v}$$

IDEÁLIS VEZETŐ

$\varepsilon = \emptyset$ $\qquad \underbrace{|\sigma \vec{E}| \gg |\partial \vec{D}/\partial t|}_{\approx \emptyset}$

$$\gamma = \sqrt{j\omega\mu(\sigma + \cancel{j\omega\varepsilon})}$$

$$Z_0 = \sqrt{\frac{j\omega\mu}{\sigma + \cancel{j\omega\varepsilon}}}$$

$$j = \frac{(1+j)^2}{2} = \frac{1-1+2j}{2} = j$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\gamma = \sqrt{j\omega\mu\sigma} = \sqrt{\frac{(1+j)^2}{2}\omega\mu\sigma} = (1+j)\boxed{\sqrt{\frac{\omega\mu\sigma}{2}}} = \underbrace{\frac{1}{\delta}}_{\alpha} + j\underbrace{\frac{1}{\delta}}_{\beta}$$

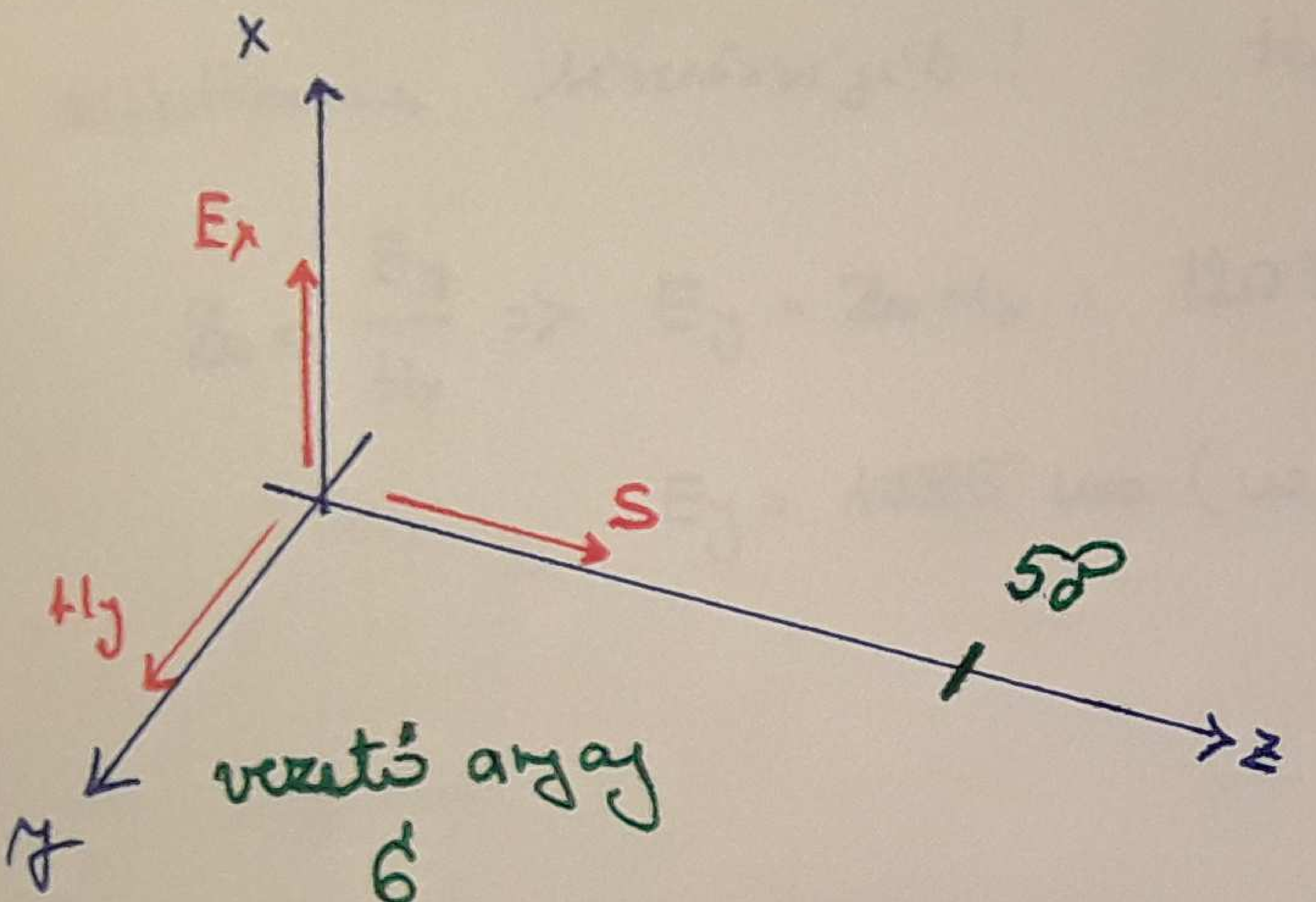
$$\boxed{\delta = \sqrt{\frac{2}{\omega\mu\sigma}}}$$

Behatolási mélység

Szkin mélység

$$Z_0 = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{(1+j)^2}{2}\frac{\omega\mu}{\sigma}} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2\sigma^2}} = \frac{1+j}{\sigma}\sqrt{\frac{\omega\mu\sigma}{2}} = \frac{1+j}{\sigma\delta}$$

VÉGTELEN FÉLTÉR



$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} \qquad \gamma = \underbrace{\frac{1}{\delta}}_{\alpha} + j \underbrace{\frac{1}{\delta}}_{\beta}$$

$$Z_0 = \frac{1+j}{\sigma \delta}$$

$$e^{-t/\tau}\Big|_{\tau} = e^{-1}$$

$$5\tau$$

$$\cancel{E_x(z) = E_1^+}$$

$$E_x(z) = E_1^+ e^{-\gamma z} + \cancel{E_1^- e^{\gamma z}}$$

$$E_x(z) = E_1^+ e^{-\gamma z} = \underbrace{E_1^+ e^{-z/\delta}} e^{-j z/\delta}$$

$$E_x(z,t) = E_1^+ e^{-z/\delta} \cos(\omega t - z/\delta)$$

$$H_y(z) = \frac{E_1^+}{Z_0} e^{-\gamma z}$$

Szabad térben terjedő síkhullám mágneses térerőssége ismert. Adjuk meg az elektromos térerősségét! $H_x(z,t) = 5\cos(\omega t - \beta z)$ A/m.

$$Z_0 = \frac{E_y}{H_x} \Rightarrow E_y = Z_0 H_x = 120\pi \cdot 5 = 1885 \text{ V/m}$$

$$E_y = 1885 \cos(\omega t - \beta z) \text{ V/m}$$

Ismert egy síkhullám két ortogonális összetevője: $E_x = 35 e^{j60^\circ}$ V/m, $H_y = j\,4 \cdot 10^{-3}$ A/m. Határozzuk meg a Poynting-vektor időfüggvényét! Határozzuk meg a közeg hullámimpedanciáját is!

$$S = \frac{1}{2} E_x H_y^* = \frac{1}{2} \cdot 35 e^{j60^\circ} \cdot 4 \cdot 10^{-3} e^{-j90^\circ}$$

$$H_y = j\,4 \cdot 10^{-3} = 4 \cdot 10^{-3} e^{j90^\circ}$$

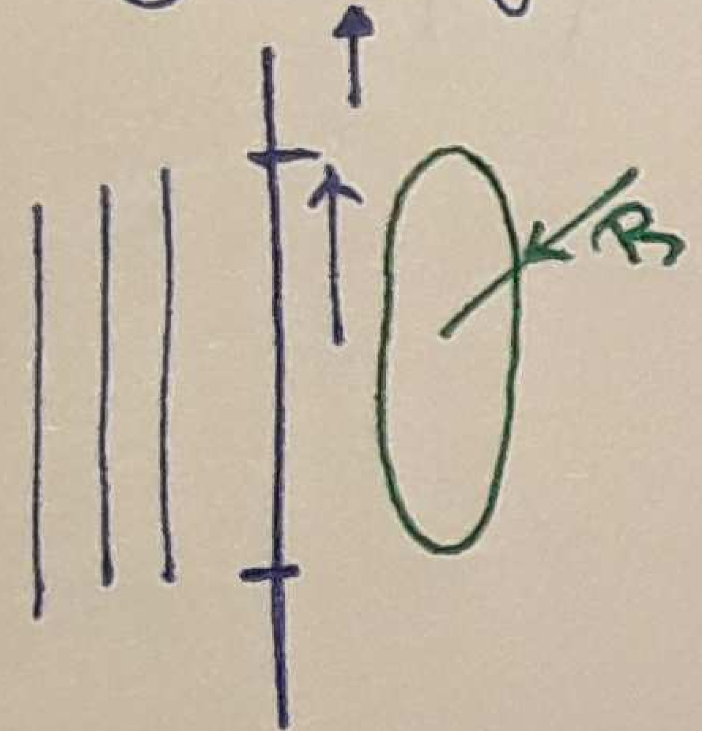
EFF! $= 0{,}07 e^{-j30^\circ}$ W/m²

$$S(t) = 0{,}07 \cos(\omega t - \beta z - 30^\circ) \text{ W/m}^2$$

$$Z_0 = \frac{E_x}{H_y} = \frac{35 e^{j60^\circ}}{4 \cdot 10^{-3} e^{j90^\circ}} = \underline{8750\, e^{-j30^\circ}\ \Omega}$$

Szabadtéri síkhullám elektromos térerőssége $E_y(z,t) = 50\cos(\omega t - \beta z)$ V/m. Határozzuk meg a $z = \text{const.}$ síkban fekvő $R = 2{,}5$ m sugarú kör felületén áthaladó hatásos teljesítményt!

$$\bar{S} = P + jQ$$



$$\bar{S} = \frac{1}{2} E_y H_x^* = \frac{1}{2} E_y \frac{E_y^*}{Z_0^*} = \boxed{\frac{1}{2}\frac{E_y^2}{Z_0}} = \frac{1}{2}\frac{50^2}{120\pi} = \underline{\underline{3{,}32\,\frac{W}{m^2}}}$$

$$Z_0 = \frac{E_y}{H_x} \rightarrow H_x = \frac{E_y}{Z_0}$$

$$PA = PR^2\pi = 3{,}32 \cdot 2{,}5^2 \cdot \pi = \underline{\underline{65{,}2\,W}}$$

$$\bar{S} = \frac{1}{2} E_y H_x^* = \frac{1}{2} Z_0 H_x H_x^* = \boxed{\frac{1}{2} Z_0 H_x^2}$$

$$E_y = Z_0 H_x$$

Veszteséges síkkondenzátorra $u(t) = \hat{U}\cos\omega t$ feszültséget kapcsolunk. Határozzuk meg a lemezek között az áramsűrűség időfüggvényét, ha a lemezek távolsága d, a dielektrikum vezetőképessége σ, relatív permeabilitása ε_r.

$$J = \sigma E + \varepsilon\frac{\partial E}{\partial t} = \sigma\frac{u}{d} + \varepsilon\frac{1}{d}\frac{du}{dt} = \underbrace{\frac{\sigma\hat{U}}{d}\cos\omega t} - \underbrace{\frac{\varepsilon\hat{U}\omega}{d}\sin\omega t}$$

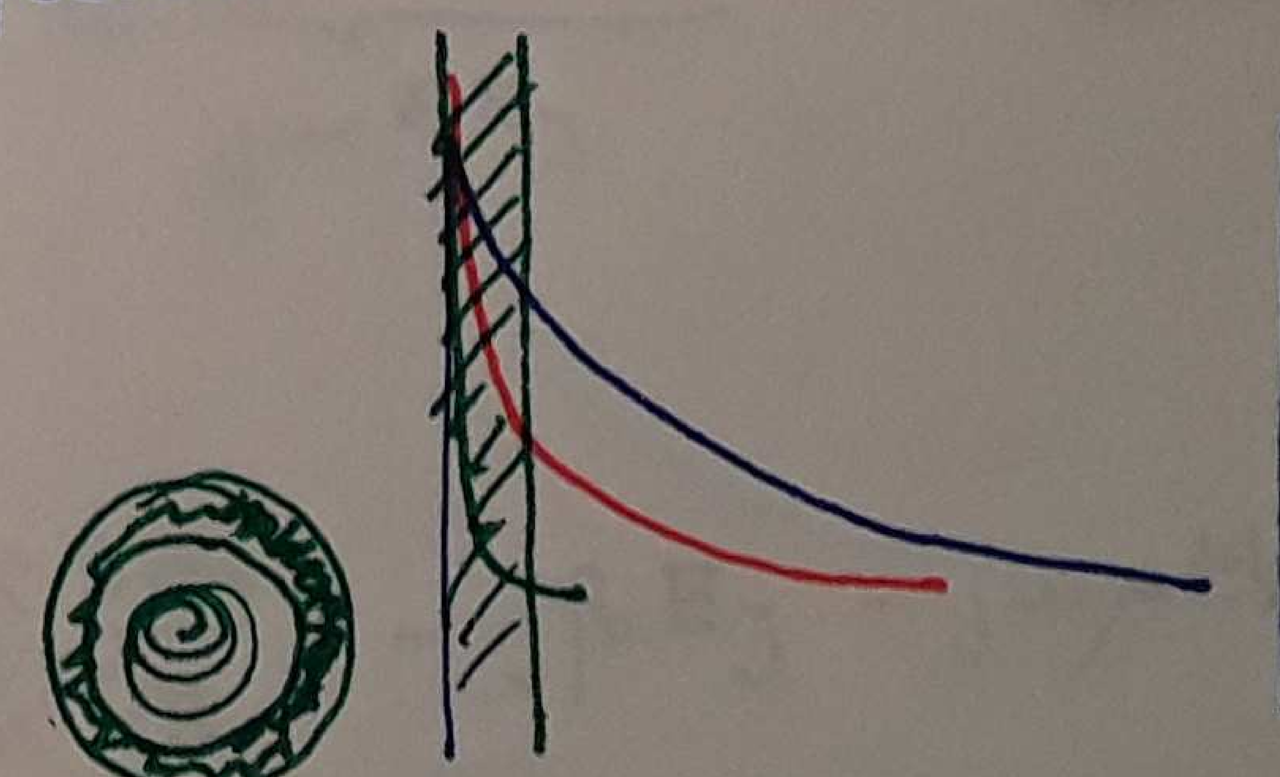
Egy kábel vezetőképessége $\sigma = 12 \cdot 10^{6}\ \mathrm{S/m}$, relatív permeabilitása $\mu_r = 200$. Határozzuk meg a behatolási mélységet $f = 50\,\mathrm{Hz}$ frekvencián!

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{2}{2\pi f \mu_0 \mu_r \sigma}} = \sqrt{\frac{2}{2\pi \, 50 \; 4\pi \cdot 10^{-7} \; 200 \; 12 \cdot 10^{6}}} = 1{,}45\ \mathrm{mm}$$

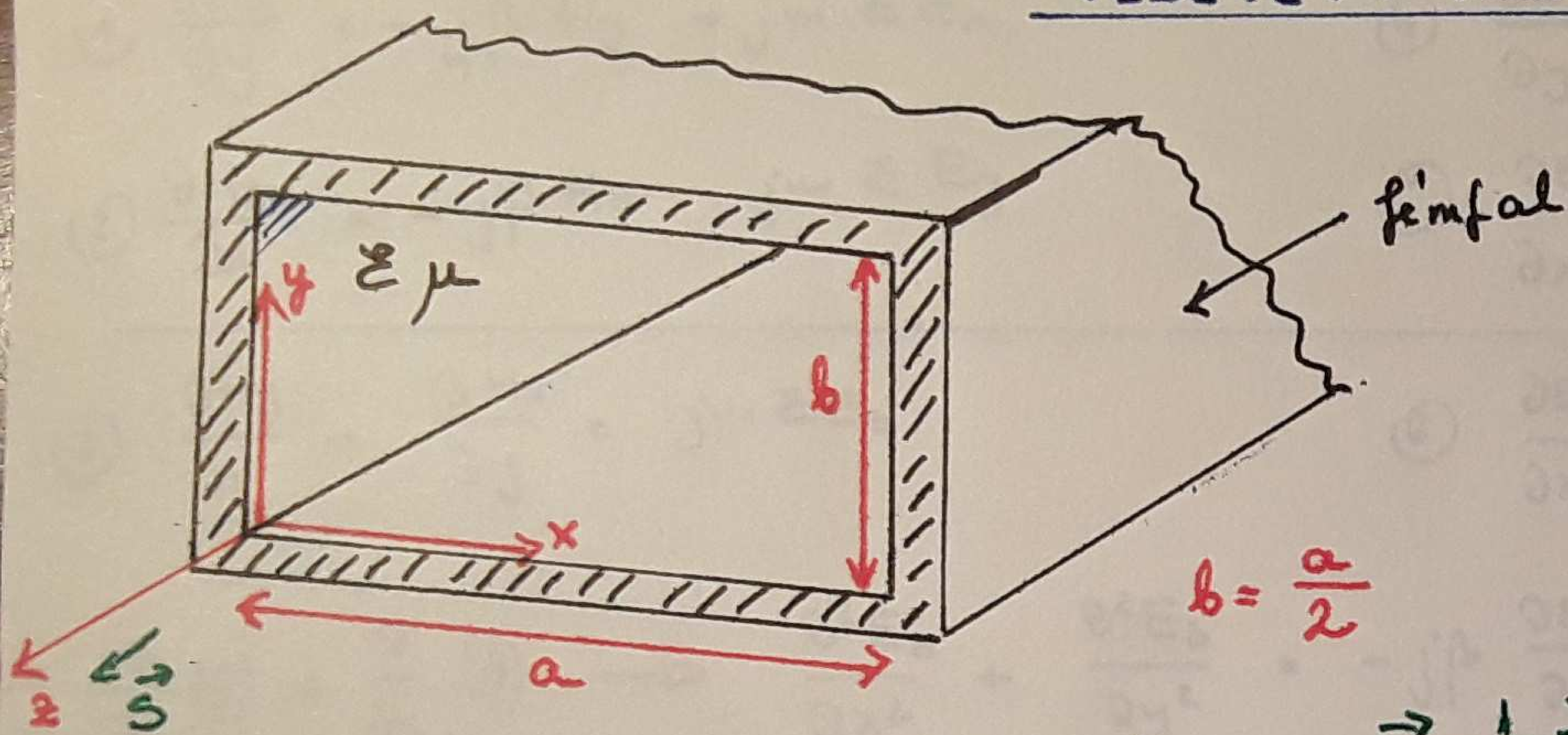
~7mm

Egy végtelen kiterjedésű vezetőben szinuszos áram folyik, a felületen az áramsűrűség $J_0 = 2\ \mathrm{A/mm^2}$. A behatolási mélység 0,8 mm. Határozzuk meg az áramsűrűséget a felülettől 1,2 mm távolságban.

$$J_0 e^{-z/\delta} = 2 e^{-1{,}2/0{,}8} = 0{,}45\ \mathrm{A/mm^2}$$



VEZETETT HULLÁMOK – CSŐTÁPVONALAK



$b = \frac{a}{2}$

$$\begin{cases} \text{rot}\,\vec{H} = \frac{\partial \vec{D}}{\partial t} \quad j\omega \\ \text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \text{div}\,\vec{B} = \emptyset \\ \text{div}\,\vec{D} = \emptyset \\ \vec{B} = \mu \vec{H} \quad \vec{D} = \varepsilon \vec{E} \end{cases}$$

$\partial/\partial t \to j\omega$

$$\boxed{\begin{aligned} \text{rot}\,\vec{H} &= j\omega\varepsilon\vec{E} \\ \text{rot}\,\vec{E} &= -j\omega\mu\vec{H} \\ \text{div}\,\vec{H} &= \emptyset \\ \text{div}\,\vec{E} &= \emptyset \end{aligned}}$$

$\vec{H} \quad \vec{E}$

$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^*$

TRANSZVERZÁLIS

$$\boxed{\begin{aligned} \vec{E}(x,y,z) &= \vec{E}(x,y)\,e^{-j\beta z} \\ \vec{H}(x,y,z) &= \vec{H}(x,y)\,e^{-j\beta z} \end{aligned}}$$

$\gamma = \alpha + j\beta \quad \alpha = \emptyset$

$$\text{rot}\,\vec{H} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = \vec{e}_x \underbrace{\left(\frac{\partial H_z}{\partial y} - \overbrace{\frac{\partial H_y}{\partial z}}^{-j\beta H_y}\right)}_{j\omega\varepsilon E_x} + \vec{e}_y \underbrace{\left(\overbrace{\frac{\partial H_x}{\partial z}}^{-j\beta H_x} - \frac{\partial H_z}{\partial x}\right)}_{j\omega\varepsilon E_y} + \vec{e}_z \underbrace{\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)}_{j\omega\varepsilon E_z}$$

$\boxed{E_z \quad H_z}$

$\boxed{1}\ \boxed{5}\ E_x\ H_y$

② ④ $E_y\ H_x$

$\boxed{1}\ \frac{\partial H_z}{\partial y} = -j\beta H_y + j\omega\varepsilon E_x$

② $\frac{\partial H_z}{\partial x} = -j\beta H_x - j\omega\varepsilon E_y$

③ $\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$

④ $\frac{\partial E_z}{\partial y} = -j\beta E_y - j\omega\mu H_x$

$\boxed{5}\ \frac{\partial E_z}{\partial x} = -j\beta E_x + j\omega\mu H_y$

⑥ $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = j\omega\mu H_z$

$\boxed{E_z \quad H_z} \leftarrow$ P.

$E_x \quad H_x$

$E_y \quad H_y$

(1) $\dfrac{\partial H_z}{\partial y} = -j\beta H_y + j\omega\varepsilon E_x$

(2) $\dfrac{\partial H_z}{\partial x} = -j\beta H_x - j\omega\varepsilon E_y$

(4) $\dfrac{\partial E_z}{\partial y} = -j\beta E_y - j\omega\mu H_x$

(5) $\dfrac{\partial E_z}{\partial x} = -j\beta E_x + j\omega\mu H_y$

(3) $\dfrac{\partial H_y}{\partial x} - \dfrac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$

(6) $\dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = j\omega\mu H_z$

$$\frac{\partial}{\partial x}(5) + \frac{\partial}{\partial y}(4) \longrightarrow \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = -j\beta\frac{\partial E_x}{\partial x} + j\omega\mu\frac{\partial H_y}{\partial x} - j\beta\frac{\partial E_y}{\partial y} - j\omega\mu\frac{\partial H_x}{\partial y}$$

$$-j\beta\underbrace{\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}\right)} + j\omega\mu\underbrace{\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)}_{j\omega\varepsilon E_z}$$

$$\operatorname{div}\vec{E} = \underbrace{\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}} + \frac{\partial E_z}{\partial z} = \emptyset$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = j\beta\underbrace{\frac{\partial E_z}{\partial z}}_{-j\beta E_z} - \omega^2\mu\varepsilon E_z$$

$$\boxed{\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = \underbrace{(\beta^2 - \omega^2\mu\varepsilon)}_{-k^2 \;\longleftarrow\; !} E_z}$$

$$\boxed{\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + k^2 E_z = \emptyset}$$

$E_z = \emptyset$ Dirichlet

$E_z \equiv \emptyset$ Triviális

Nemtriviális megoldás

SAJÁTÉRTÉK

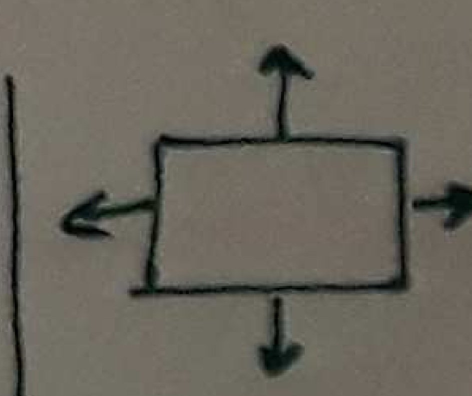
$\underline{\underline{A}}\,\underline{x} = \lambda\underline{x}$

$\dfrac{\partial}{\partial y}(1) + \dfrac{\partial}{\partial x}(2)$

$\vdots$

$$\boxed{k^2 = -(\beta^2 - \omega^2\mu\varepsilon)}$$

$$\boxed{\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k^2 H_z = \emptyset}$$



$\partial H_z / \partial n = \emptyset$ Neumann

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + k^2 E_z = \emptyset \qquad E_z = \emptyset \text{ SIN}$$

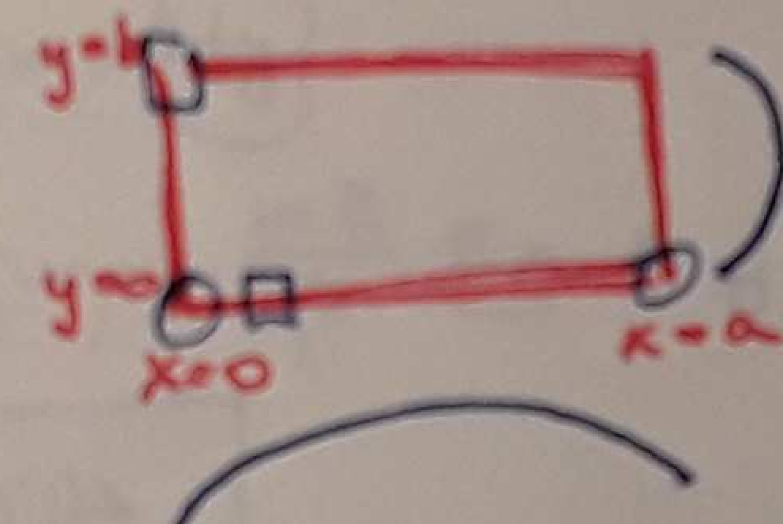
$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k^2 H_z = \emptyset \qquad \frac{\partial H_z}{\partial n} = \emptyset \text{ COS}$$

Szorzatszeparációs alak: $\boxed{E_z(x,y) = X(x)\cdot Y(y)}$

$$\frac{\partial^2}{\partial x^2} XY + \frac{\partial^2}{\partial y^2} XY + k^2 XY = \emptyset$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + k^2 XY = \emptyset \quad / : XY$$

$$\underbrace{\frac{1}{X}\frac{\partial^2 X}{\partial x^2}}_{-k_x^2} + \underbrace{\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}}_{-k_y^2} + k^2 = \emptyset \qquad \boxed{k^2 = k_x^2 + k_y^2}$$



$$\boxed{\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k_x^2} \quad \text{És} \quad \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k_y^2$$

$$\frac{\partial^2 X}{\partial x^2} + k_x^2 X = \emptyset \qquad \frac{\partial^2 Y}{\partial y^2} + k_y^2 Y = \emptyset$$

$$X = Me^{\lambda x} \quad X'' = \lambda^2 M e^{\lambda x} \qquad Y = Ne^{\lambda y}$$

$$\vdots$$

$$\lambda^2 \cancel{Me^{\lambda x}} + k_x^2 \cancel{Me^{\lambda x}} = \emptyset$$

$$\lambda^2 = -k_x^2 \longrightarrow \boxed{\lambda_{12} = \pm j k_x} \qquad \boxed{\lambda_{12} = \pm j k_y}$$

$$X = M_1 e^{jk_x x} + M_2 e^{-jk_x x} \qquad Y = N_1 e^{jk_y y} + N_2 e^{-jk_y y}$$

$$M_2 = M_1^* \qquad N_2 = N_1^*$$

$$X = jMe^{jk_x x} - jMe^{-jk_x x} = -\frac{e^{jk_x x} - e^{-jk_x x}}{j2} M2 = \underline{\underline{-2M \sin k_x x}}$$

$$Y = \underline{\underline{-2N \sin k_y y}}$$

$$E_z(x,y) = X(x)\cdot Y(y) = \boxed{\hat{E}_z \sin k_x x \cdot \sin k_y y} \quad e^{-j\beta z}$$

2M2N

$$M_1 = M_2 \qquad N_1 = N_2$$

$$H_z(x,y) = \hat{H}_z \cos k_x x \cos k_y y \quad e^{-j\beta z}$$

$E_z(x,y,z) = \hat{E}_z \sin k_x x \sin k_y y\, e^{-j\beta z}$

$H_z = \emptyset$

TM H_x H_y

(1) $\frac{\partial H_z}{\partial y} = -j\beta H_y + j\omega\varepsilon E_x$

(5) $\frac{\partial E_z}{\partial x} = -j\beta E_x + j\omega\mu H_y$

(1) $0 = -\cancel{j}\beta H_y + \cancel{j}\omega\varepsilon E_x \longrightarrow \beta H_y = \omega\varepsilon E_x \rightarrow \boxed{H_y = \frac{\omega\varepsilon}{\beta} E_x}$

(5) $k_x \hat{E}_z \cos k_x x \sin k_y y\, e^{-j\beta z} = -j\beta E_x + j\omega\mu H_y$

$k_x \hat{E}_z \cos k_x x \sin k_y y\, e^{-j\beta z} = -j\beta E_x + j\omega\mu \frac{\omega\varepsilon}{\beta} E_x = j\left(\frac{\omega^2\mu\varepsilon}{\beta} - \beta\right)E_x = j\,\frac{\omega^2\mu\varepsilon - \beta^2}{\beta} E_x$

$$\boxed{E_x = -j\,\frac{\beta k_x}{\underbrace{\omega^2\mu\varepsilon - \beta^2}_{k_x^2 + k_y^2}}\,\hat{E}_z \cos k_x x \sin k_y y\, e^{-j\beta z}}$$

$$H_y = -j\,\frac{\omega\varepsilon}{\cancel{\beta}}\,\frac{\cancel{\beta}k_x}{\omega^2\mu\varepsilon - \beta^2}\,\hat{E}_z \cos k_x x \sin k_y y\, e^{-j\beta z}$$

$$\boxed{H_y = -j\,\frac{\omega\varepsilon k_x}{\underbrace{\omega^2\mu\varepsilon - \beta^2}_{k_x^2 + k_y^2}}\,\hat{E}_z \cos k_x x \sin k_y y\, e^{-j\beta z}}$$

(2) $\frac{\partial H_z}{\partial x} = -j\beta H_x - j\omega \varepsilon E_y$

(4) $\frac{\partial E_z}{\partial y} = -j\beta E_y - j\omega\mu H_x$

(2) $0 = -\cancel{j}\beta H_x - \cancel{j}\omega\varepsilon E_y \longrightarrow \beta H_x = -\omega\varepsilon E_y \longrightarrow \boxed{H_x = -\frac{\omega\varepsilon}{\beta} E_y}$

(4) $k_y \hat{E}_z \sin k_x x \cos k_y y \, e^{-j\beta z} = -j\beta E_y - j\omega\mu H_x$

$$\underline{k_y \hat{E}_z \sin k_x x \cos k_y y \, e^{-j\beta z}} = -j\beta E_y + j\omega\mu \frac{\omega\varepsilon}{\beta} E_y = j\left(\frac{\omega^2\mu\varepsilon}{\beta} - \beta\right) E_y = \underline{j \frac{\omega^2\mu\varepsilon - \beta^2}{\beta} E_y}$$

$$\boxed{E_y = -j \frac{\beta k_y}{\omega^2\mu\varepsilon - \beta^2} \hat{E}_z \sin k_x x \cos k_y y \, e^{-j\beta z}}$$

$$H_x = +\frac{\omega\varepsilon}{\cancel{\beta}} j \frac{\cancel{\beta} k_y}{\omega^2\mu\varepsilon - \beta^2} \hat{E}_z \sin k_x x \cos k_y y \, e^{-j\beta z}$$

$$\boxed{H_x = j \frac{\omega\varepsilon k_y}{\omega^2\mu\varepsilon - \beta^2} \hat{E}_z \sin k_x x \cos k_y y \, e^{-j\beta z}}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + k^2 E_z = \emptyset \qquad E_z = \emptyset$$

$\boxed{TM}$

$$E_x = -j \frac{\beta k_x}{k_x^2 + k_y^2} \hat{E}_z \cos \underline{k_x} x \sin \underline{k_y} y \, e^{-j\beta z}$$

$$E_y = -j \frac{\beta k_y}{k_x^2 + k_y^2} \hat{E}_z \sin \underline{k_x} x \cos \underline{k_y} y \, e^{-j\beta z}$$

$$\rightarrow E_z = \hat{E}_z \sin \underline{k_x} x \sin \underline{k_y} y \, e^{-j\beta z}$$

$k_x \neq \emptyset$ És $k_y \neq \emptyset$

$$k_x = \frac{m\pi}{a}$$

$$k_y = \frac{n\pi}{b}$$

$$H_x = j \frac{\omega \varepsilon k_y}{k_x^2 + k_y^2} \hat{E}_z \sin \underline{k_x} x \cos \underline{k_y} y \, e^{-j\beta z}$$

TM_{11} TM_{12} M_{21} ..

m n

MÓDUS.

$$H_y = -j \frac{\omega \varepsilon k_x}{k_x^2 + k_y^2} \hat{E}_z \cos \underline{k_x} x \sin \underline{k_y} y \, e^{-j\beta z}$$

$$\rightarrow H_z = \emptyset$$

$$Z_0 = \frac{E_x}{H_y} = \frac{\beta}{\omega \varepsilon}$$

$$Z_0 = -\frac{E_y}{H_x} = \frac{\beta}{\omega \varepsilon}$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + k^2 H_z = \emptyset \qquad \frac{\partial H_z}{\partial n} = \emptyset$$

$$H_z(x,y,z) = \hat{H}_z \cos \underbrace{k_x x}_{\frac{\pi}{a}x}\ \cancel{\cos k_y y}\ e^{-j\beta z}$$

TE

$E_z = \emptyset$

TE$_{10}$ TE$_{01}$ TE$_{11}$ ...

m n

$$E_x = j\frac{k_y \omega\mu}{k_x^2+k_y^2}\hat{H}_z \cos k_x x \sin k_y y\ e^{-j\beta z} = \emptyset$$

$$E_y = -j\frac{k_x \omega\mu}{k_x^2+k_y^2}\hat{H}_z \sin \underbrace{k_x x}_{\frac{\pi}{a}x}\ \cancel{\cos k_y y}\ e^{-j\beta z}$$

$$H_x = j\frac{k_x \beta}{k_x^2+k_y^2}\hat{H}_z \sin \underbrace{k_x x}_{\frac{\pi}{a}x}\ \cancel{\cos k_y y}\ e^{-j\beta z}$$

$$H_y = j\frac{k_y \beta}{k_x^2+k_y^2}\hat{H}_z \cos k_x x \sin k_y y\ e^{-j\beta z}$$

$$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b}$$

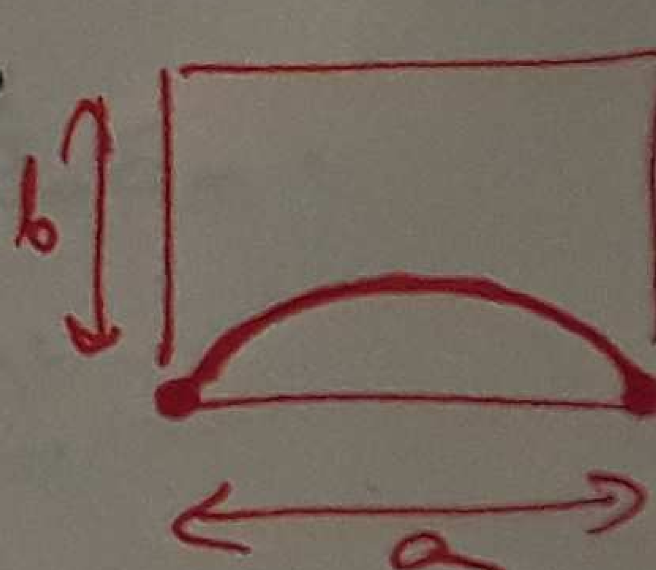
$$\lambda \le \frac{2\cancel{\sqrt{\mu\varepsilon}}}{\sqrt{\left(\frac{m}{a}\right)^2 + \cancel{\left(\frac{n}{b}\right)^2}}} = \frac{2}{\left.\frac{m}{a}\right|_{m=1}} = 2a$$

DOMINÁNS MÓDUS $a > b$

$\mu_r = 1$ $\varepsilon_r = 1$

$$\boxed{\lambda_0 \le 2a}$$

TE10



$$Z_0 = \frac{E_x}{H_y} = \frac{\omega\mu}{\beta}$$

$$Z_0 = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta}$$

$E_x = \emptyset$

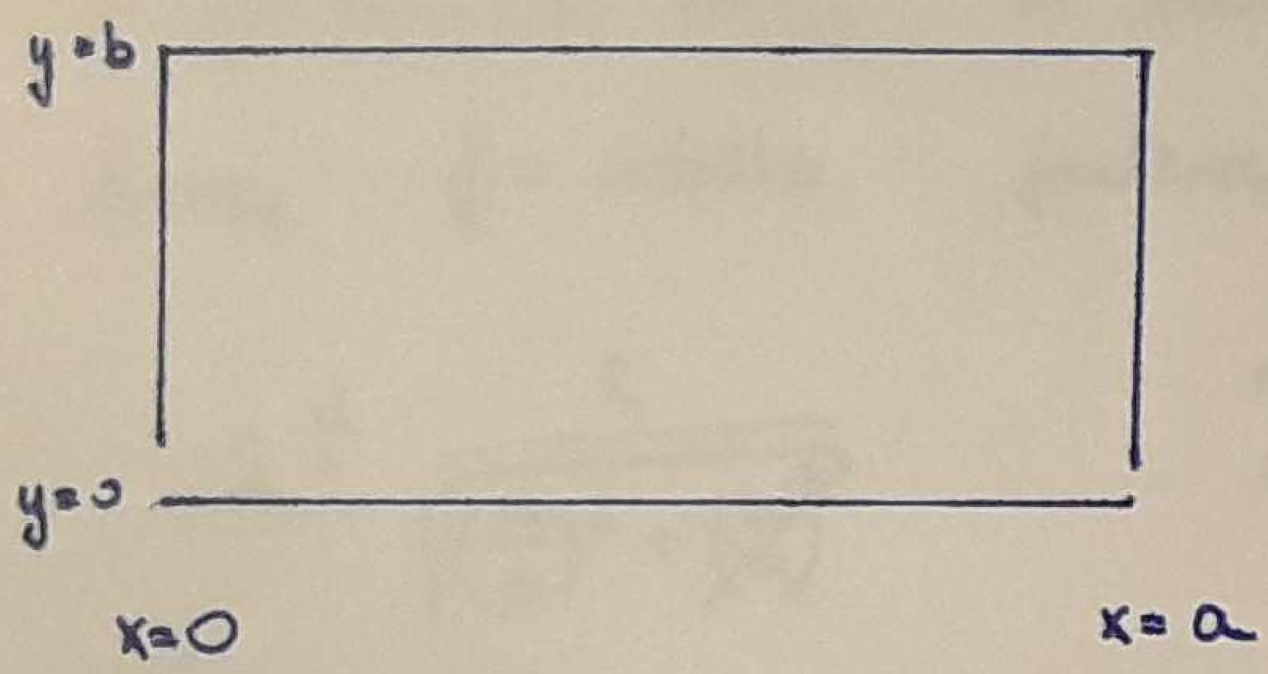
$E_y = \hat{E}_y \sin \frac{\pi}{a}x\ e^{-j\beta z}$

$E_z = \emptyset$

$H_x = \hat{H}_x \sin \frac{\pi}{a}x\ e^{-j\beta z}$

$H_y = \emptyset$

$H_z = \hat{H}_z \cos \frac{\pi}{a}x\ e^{-j\beta z}$



$$E_z(x,y,z) = \hat{E}_z \sin \underbrace{k_x}_{?} x \, \sin \underbrace{k_y}_{?} y \; e^{-j\beta z}$$

TM

$$\sin k_y 0 = 0 \qquad \sin k_y b = 0 \rightarrow \boxed{k_y = \frac{n\pi}{b}} \quad n = 1,2,3,\dots$$

$$\sin \underbrace{k_x 0} = 0 \qquad \sin k_x a = 0 \rightarrow \boxed{k_x = \frac{m\pi}{a}} \quad m = 1,2,3,\dots$$

$m,n \in \mathbb{Z}^+$

$$k^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \underbrace{-\beta^2 + \omega^2 \mu \varepsilon}_{k^2}$$

$$\beta = \sqrt{\underbrace{\omega^2 \mu \varepsilon}_{>0} - \underbrace{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}_{>0}}$$

$\nearrow \beta \in \mathbb{R}$

$\searrow \cancel{\beta \in \mathbb{C}}$ $\quad j\beta \quad e^{-\alpha z}$

$$! \quad \omega^2 \mu \varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\mu_r \varepsilon_r \left(\frac{2\cancel{\pi}}{\lambda}\right)^2 > \left(\frac{m\cancel{\pi}}{a}\right)^2 + \left(\frac{n\cancel{\pi}}{b}\right)^2$$

$$\mu_r \varepsilon_r \frac{4}{\lambda^2} > \left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2$$

$$\frac{\mu_r \varepsilon_r 4}{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} > \lambda^2$$

határhullámhossz

$$\boxed{\lambda \leq \frac{2\sqrt{\mu_r \varepsilon_r}}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}} \qquad f = \frac{c}{\lambda}$$

λ?

$1/c^2 \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$$\omega^2 \mu \varepsilon = (2\pi f)^2 \mu_0 \mu_r \varepsilon_0 \varepsilon_r = \mu_r \varepsilon_r \frac{(2\pi f)^2}{c^2} = \mu_r \varepsilon_r \left(\frac{2\pi f}{c}\right)^2 = \mu_r \varepsilon_r \left(\frac{2\pi}{\lambda}\right)^2$$

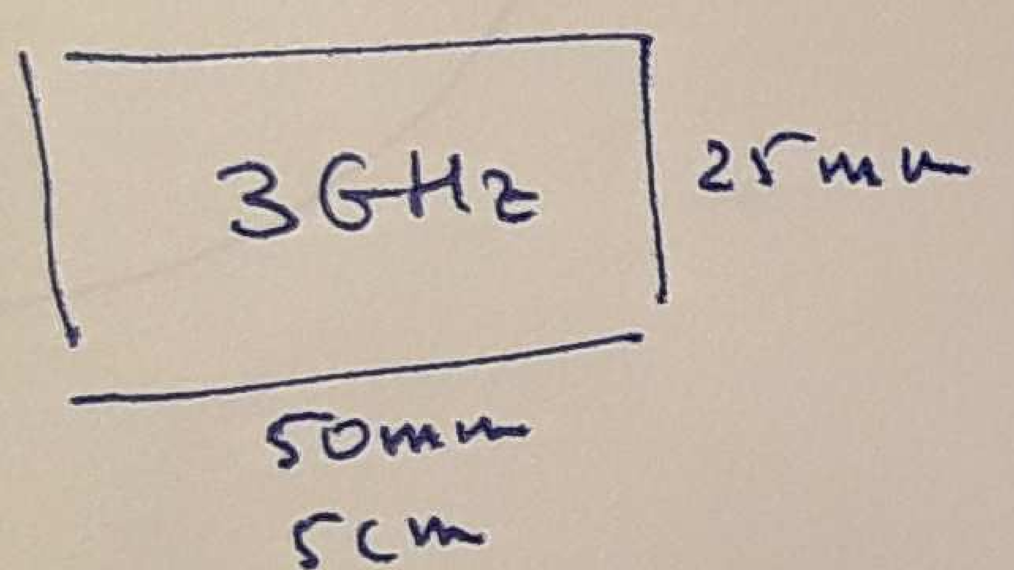
$$\Lambda = \frac{2\pi}{\beta} \qquad \Lambda > \lambda$$

Határozzuk meg a téglalap keresztmetszetű, légtöltésű csőtápvonal „a" méretét úgy, hogy $f = 3\,GHz$ frekvencián a csőben a $\boxed{TE_{10}}$ módus terjedhessen! $(b = 0{,}5a)$

$$\lambda \leq \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \cancel{\left(\frac{n}{b}\right)^2}}} \qquad m = 1,\quad n = 0$$

$$\lambda = \frac{2}{\frac{m}{a}} = \frac{2a}{m} = 2a$$

$$\lambda = \frac{c}{f} \Rightarrow f = \frac{c}{\lambda} = \frac{c}{2a} \longrightarrow a = \frac{c}{2f} = \frac{3 \cdot 10^8}{2 \cdot 3 \cdot 10^9} = 0{,}05\,m$$



$$\boxed{f = \frac{c}{2a}}$$

$$b = 0{,}025\,m$$

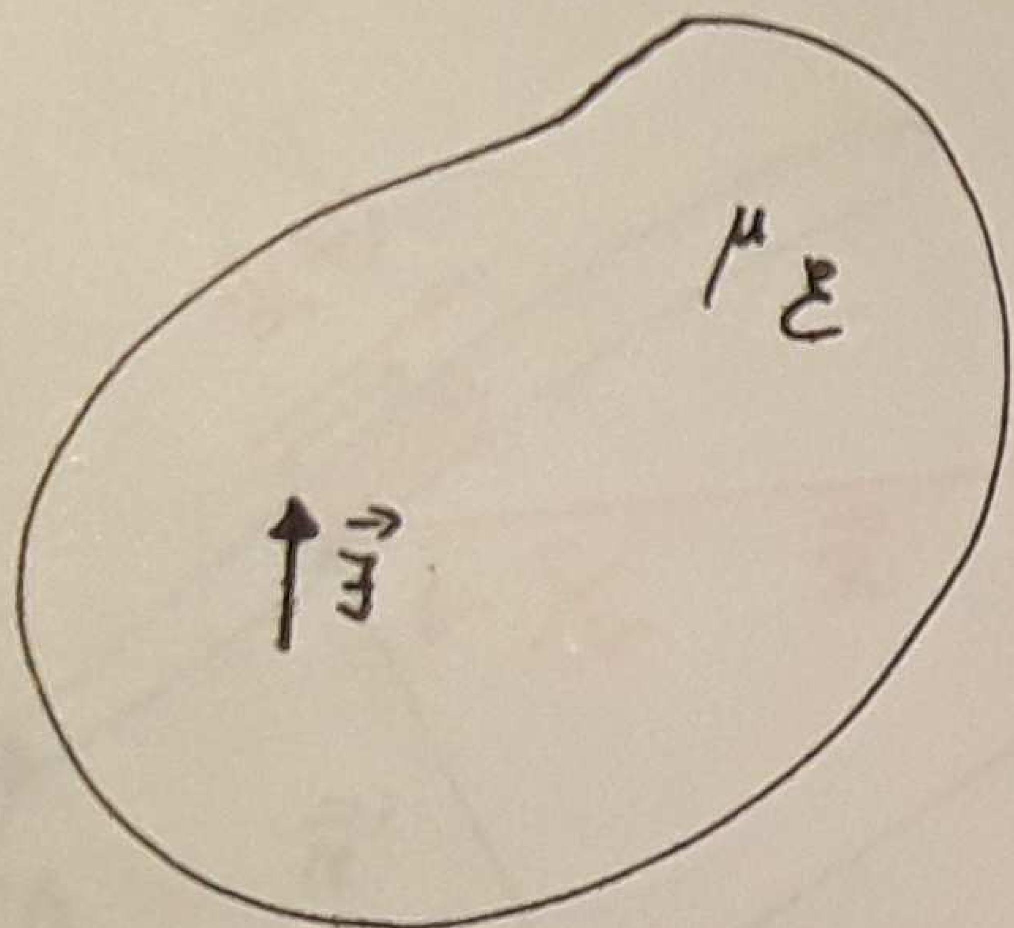
Egy igen hosszú, $a \times b$ méretű, légtöltésű csőtápvonalban TE_{10} módus terjed a pozitív z-tengely irányába. A $z = 0$ helyen: $E = 10 \sin(15{,}71x) \cos 25{,}13t$, $[x] = m$, $[t] = ns$. Írjuk fel az elektromos térerősség kifejezését a $z > 0$ -ra!

$$E = 10 \sin 15{,}71x \cos(25{,}13t - \beta z)\ V/m$$

$$\beta = \sqrt{\omega^2 \mu \varepsilon - k_x^2} = \sqrt{(25{,}13 \cdot 10^9)^2 \cdot 4\pi 10^{-7} \cdot 8{,}85 \cdot 10^{-12} - 15{,}71^2} = 82{,}34\ 1/m$$

$$E = 10 \sin 15{,}71x \cos(25{,}13t - 82{,}34z)\ V/m$$

AZ ANTENNASZÁMÍTÁS ALAPJAI



$$\begin{cases} \mathrm{rot}\,\vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t} \\ \mathrm{rot}\,\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \mathrm{div}\,\vec{B} = 0 \\ \mathrm{div}\,\vec{D} = \rho \\ \vec{B} = \mu \vec{H} \qquad \vec{D} = \varepsilon \vec{E} \end{cases}$$

$\mathrm{div}\,\vec{B} = 0 \longrightarrow \boxed{\vec{B} = \mathrm{rot}\,\vec{A}}$

$\vec{A},\ \varphi$

$$\mathrm{rot}\,\vec{E} = -\frac{\partial}{\partial t}\,\mathrm{rot}\,\vec{A}$$

$$\mathrm{rot}\,\underbrace{\left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right)}_{-\mathrm{grad}\,\varphi} = 0$$

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\mathrm{grad}\,\varphi$$

$$\boxed{\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \mathrm{grad}\,\varphi}$$

$\cancel{\mathrm{div}\,\vec{A} = 0}$

Lorentz-mérték: $\boxed{\mathrm{div}\,\vec{A} = -\mu\varepsilon \dfrac{\partial \varphi}{\partial t}}$

$$\mathrm{rot}\,\frac{1}{\mu}\vec{B} = \vec{J} + \frac{\partial}{\partial t}\varepsilon\vec{E}$$

$$\mathrm{rot}\,\frac{1}{\mu}\,\mathrm{rot}\,\vec{A} = \vec{J} + \varepsilon\frac{\partial}{\partial t}\left(-\frac{\partial \vec{A}}{\partial t} - \mathrm{grad}\,\varphi\right)$$

$$\underbrace{\mathrm{rot}\,\mathrm{rot}\,\vec{A}} = \mu\vec{J} - \mu\varepsilon\frac{\partial^2 \vec{A}}{\partial t^2} - \mu\varepsilon\,\mathrm{grad}\,\frac{\partial \varphi}{\partial t}$$

$$\cancel{\mathrm{grad}\,\mathrm{div}\,\vec{A}} - \Delta\vec{A} = \mu\vec{J} - \mu\varepsilon\frac{\partial^2 \vec{A}}{\partial t^2} \cancel{- \mu\varepsilon\,\mathrm{grad}\,\frac{\partial \varphi}{\partial t}}$$

$$\boxed{\Delta\vec{A} - \mu\varepsilon\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J}}$$

HULLÁMEGYENLET INH.

$\rightarrow \vec{A}$

$$\boxed{\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J}}$$

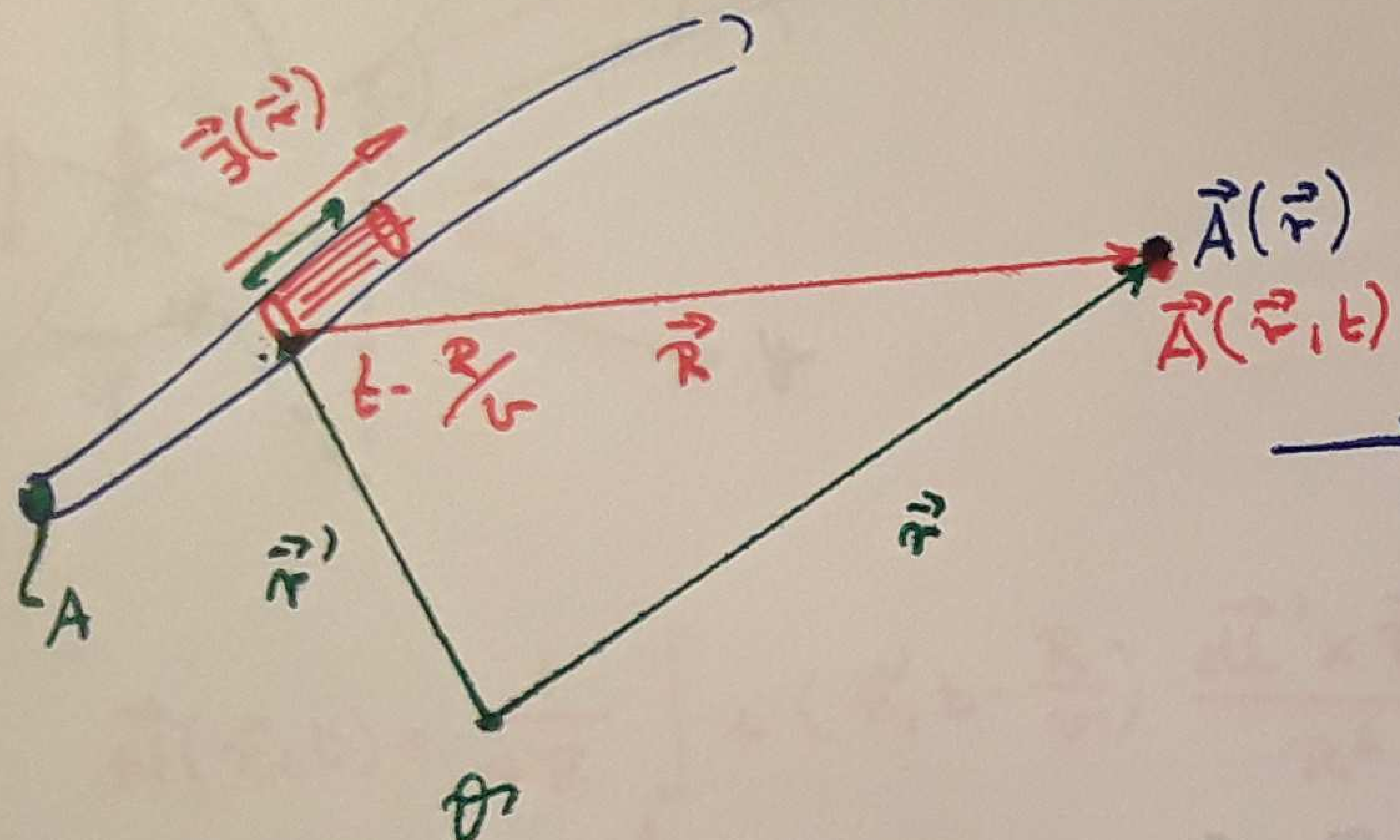
INH. HELMHOLTZ-EGY.

$$\frac{\partial}{\partial t} \rightarrow j\omega$$

$$\frac{\partial^2}{\partial t^2} \rightarrow (j\omega)^2 = -\omega^2$$

RETARDÁLT POTENCIÁL

$$\Delta \vec{A} = -\mu \vec{J}$$



DC.) $\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int_{V'} \frac{\vec{J}(\vec{r}\,')}{R}\, dV'$

$\hookrightarrow \vec{H}$

$$\Delta \vec{A} + \omega^2 \mu\varepsilon \vec{A} = -\mu \vec{J}$$

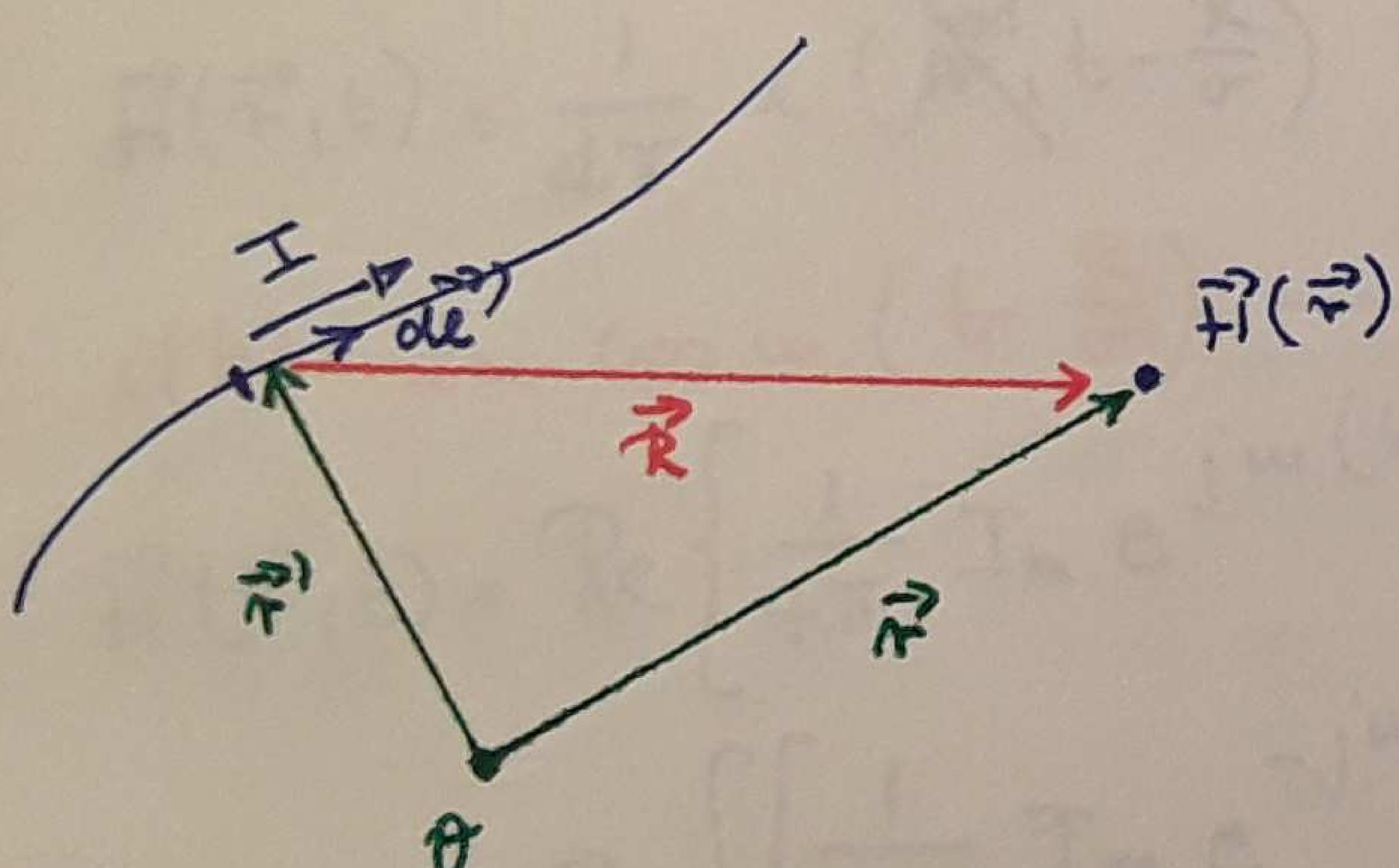
$f\uparrow \Rightarrow$ AC.) $\vec{A}(\vec{r},t) = \frac{\mu}{4\pi}\int_{V'} \frac{1}{R}\, \vec{J}\left(\vec{r}\,', \boxed{t-\frac{R}{v}}\right) dV'$ $\qquad v = \frac{1}{\sqrt{\mu\varepsilon}}$

KESLELTETETT

Biot-Savart-törvény.

DC:) $\vec{H}(\vec{r}) = \frac{I}{4\pi}\oint_l \frac{d\vec{l}\,'\times \vec{R}_0}{R^2}$

$$(uv)' = u'v + uv'$$



$$\vec{H}(\vec{r},t) = \frac{1}{\mu}\operatorname{rot}\vec{A}(\vec{r},t)$$

$$\vec{H}(\vec{r},t) = \frac{1}{4\pi}\oint_l \operatorname{rot}\underbrace{\frac{1}{R}\, I\left(\vec{r}\,', t-\frac{R}{v}\right)}_{\varphi} \underbrace{d\vec{l}\,'}_{\vec{v}} =$$

$$\operatorname{rot}\varphi\vec{v} = \varphi \operatorname{rot}\vec{v} + \operatorname{grad}\varphi \times \vec{v}$$

$$\operatorname{rot}\left[\frac{1}{R}\, I\left(\vec{r}\,', t-\frac{R}{v}\right) d\vec{l}\,'\right] = \frac{1}{R}\, I\left(\vec{r}\,', t-\frac{R}{v}\right) \cancel{\operatorname{rot} d\vec{l}\,'} + \operatorname{grad}\frac{1}{R}\, I\left(\vec{r}\,', t-\frac{R}{v}\right) \times d\vec{l}\,'$$

$\vec{J}\cdot dV'$

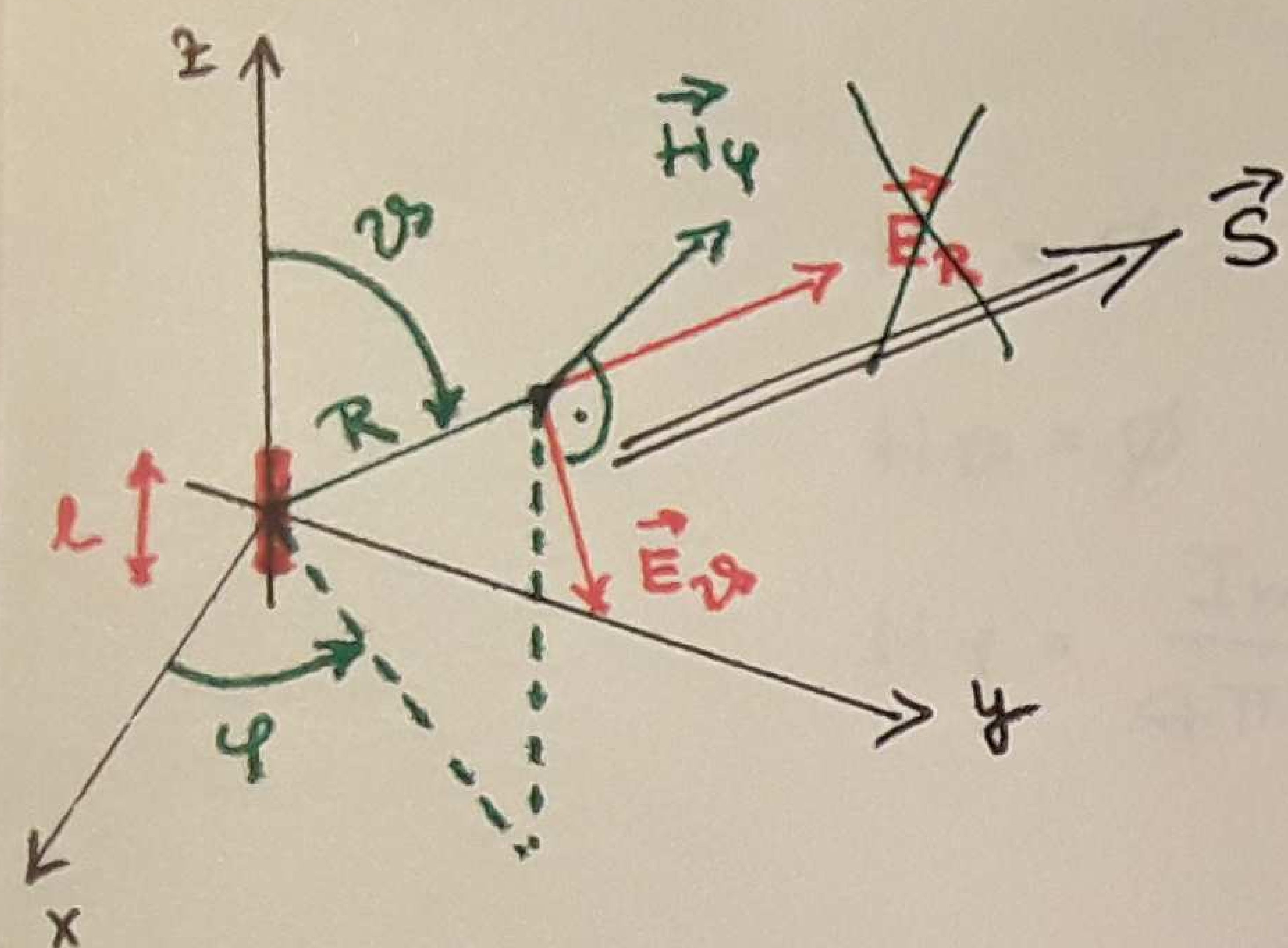
$\cancel{I}$

$\underbrace{\vec{J}\cdot A}_{I}\, d\vec{l} = I\, d\vec{l}\,'$

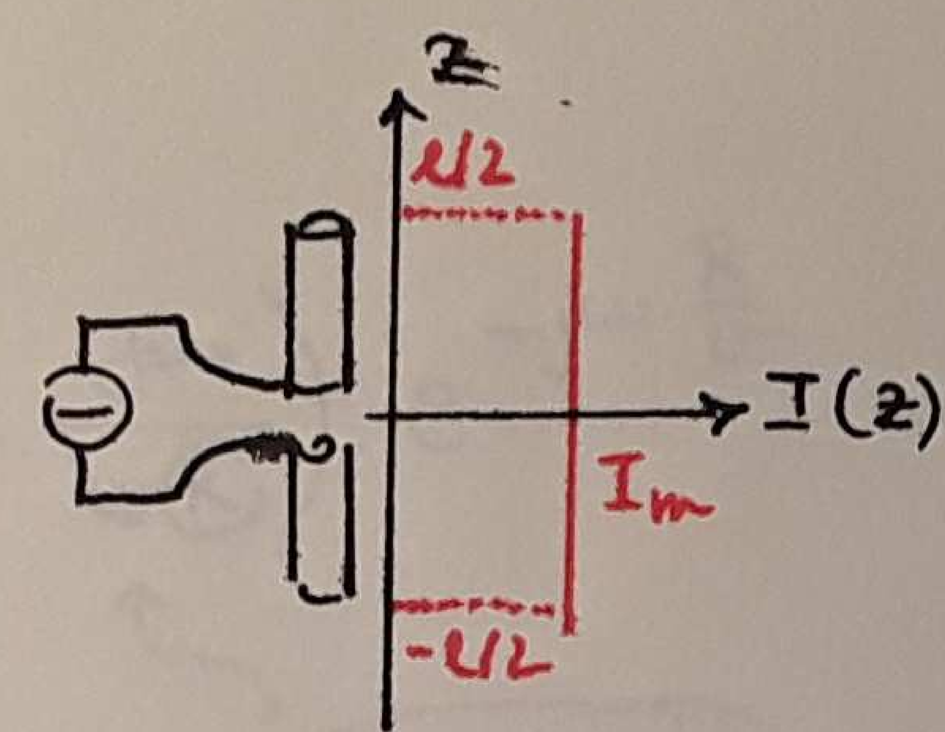
$$\Rightarrow \vec{H}(\vec{r},t) = \frac{1}{4\pi}\oint_l \operatorname{grad}\left[\frac{1}{R}\, I\left(\vec{r}\,', t-\frac{R}{v}\right)\right] \times d\vec{l}\,'$$

$$\frac{\partial}{\partial R}[\ldots] = -\frac{1}{R^2}\, I\left(\vec{r}\,', t-\frac{R}{v}\right) + \frac{1}{R}\, \frac{\partial I\left(\vec{r}\,', t-\frac{R}{v}\right)}{\partial t}\left(-\frac{1}{v}\right)$$

$$\vec{H}(\vec{r},t) = \underbrace{\frac{1}{4\pi}\oint_l \boxed{I}\left(\vec{r}\,', t-\frac{R}{v}\right) \frac{d\vec{l}\,'\times\vec{R}_0}{R^2}}_{\text{KÖZELTÉR}} + \underbrace{\frac{1}{4\pi v}\oint_l \frac{\partial I\left(\vec{r}\,', t-\frac{R}{v}\right)}{\partial t}\, \frac{d\vec{l}\,'\times\vec{R}_0}{R}}_{\text{TÁVOLTÉR}}$$



HERTZ-DIPÓL



$\vec{dl}' \to \vec{l}$

$i(\vec{r}', t) \to i(t)$

$i(t) = I_m \cos\omega t$; $\cancel{\ne}$ $i(t) = I_m \cos\omega\left(t - \frac{R}{v}\right)$

$$\vec{H}(\vec{r}, t) = \frac{1}{4\pi}\int_l i\left(\vec{r}', t - \frac{R}{v}\right)\frac{\vec{dl}' \times \vec{R}_0}{R^2} + \frac{1}{4\pi v}\int_l \frac{\partial i(\vec{r}', t - R/v)}{\partial t}\,\frac{\vec{dl}' \times \vec{R}_0}{R}$$

$$\vec{H}(\vec{r}, t) = \frac{1}{4\pi}\, i\left(\cancel{\vec{r}'}, t - \frac{R}{v}\right)\frac{\vec{l} \times \vec{R}_0}{R^2} + \frac{1}{4\pi v}\,\frac{di(\cancel{\vec{r}'}, t - R/v)}{dt}\,\frac{\vec{l} \times \vec{R}_0}{R}$$

$$i(t) = I_m \cos\omega\left(t - \frac{R}{v}\right) \longrightarrow I_m e^{j\omega(t - R/v)}$$

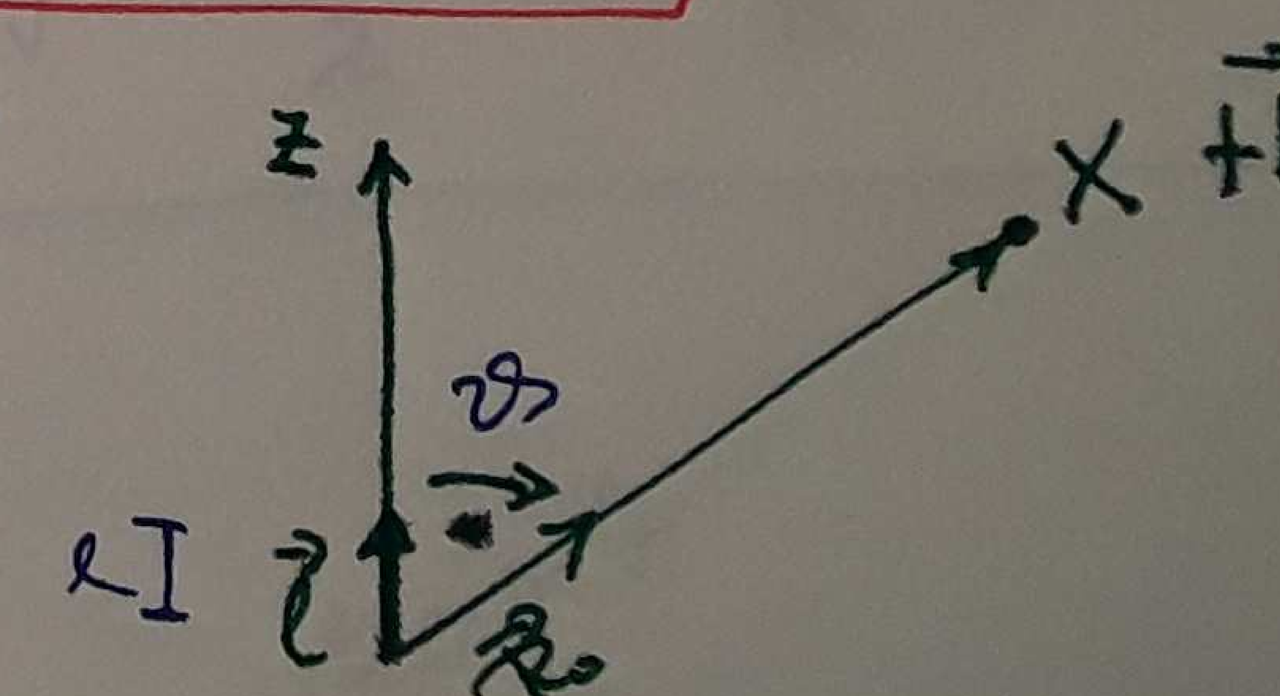
$$\vec{H}(\vec{r}, t) = \mathrm{Re}\left\{\frac{1}{4\pi} I_m e^{j\omega\left(t - \frac{R}{v}\right)}\frac{\vec{l} \times \vec{R}_0}{R^2} + \frac{1}{4\pi v} j\omega I_m e^{j\omega\left(t - \frac{R}{v}\right)}\frac{\vec{l} \times \vec{R}_0}{R}\right\} =$$

$$= \mathrm{Re}\left\{\underbrace{\left[\frac{1}{4\pi} I_m e^{-j\omega\frac{R}{v}}\frac{\vec{l} \times \vec{R}_0}{R^2} + \frac{1}{4\pi v} j\omega I_m e^{-j\omega\frac{R}{v}}\frac{\vec{l} \times \vec{R}_0}{R}\right]}_{\vec{H}(\vec{r})} e^{j\omega t}\right\}$$

$\vec{H} \ne \vec{H}$

$\sin\vartheta \cdot l$

$$\vec{H}(\vec{r}) = \underbrace{\frac{I_m}{4\pi}\left(\frac{1}{R^2} + \frac{j\omega}{vR}\right) e^{-j\omega\frac{R}{v}}}_{\text{SKALÁR}}\underbrace{\left(\vec{l} \times \vec{R}_0\right)}_{\text{VEKTOR}}$$



$$\underline{\text{rot}}\,\vec{H} = j\omega\varepsilon\vec{E}$$

elhagyjuk!

$$H_R = \emptyset$$

$$H_\vartheta = \emptyset$$

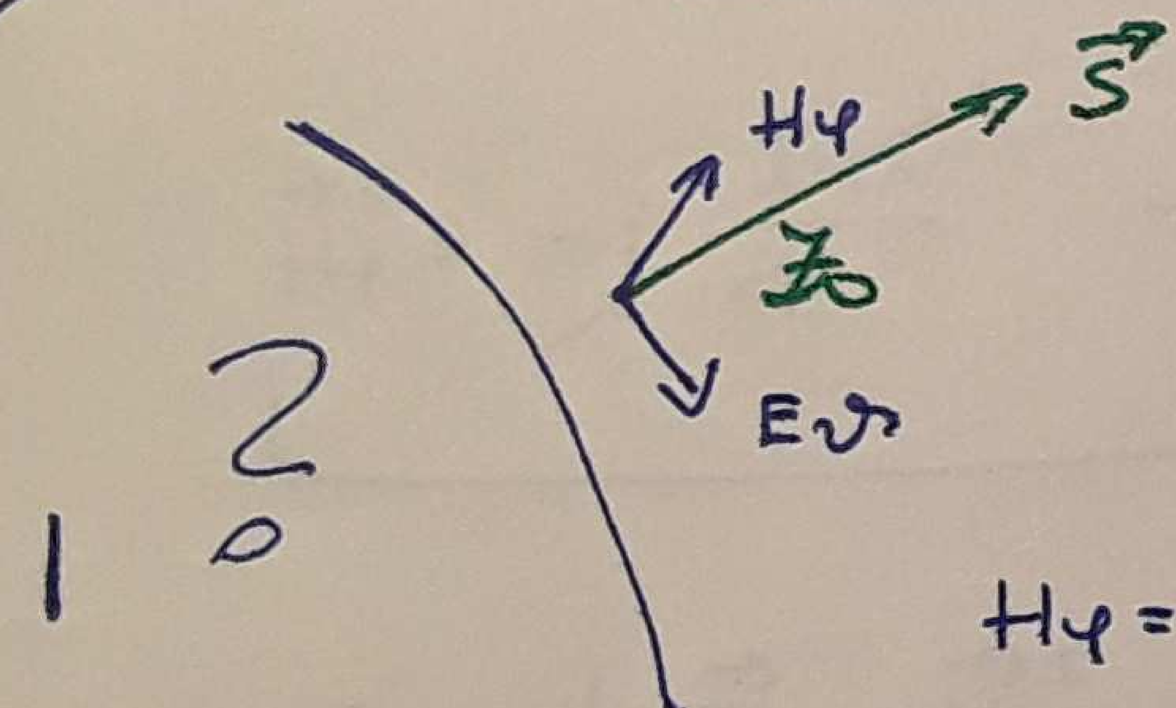
$$H_\varphi = \frac{Iml}{4\pi}\left(\frac{1}{R^2} + \frac{j\omega}{vR}\right) e^{-j\omega\frac{R}{v}} \sin\vartheta$$

közeltér

távoltér

$$\frac{1}{R^2} \qquad \frac{\omega}{vR} = \frac{\beta}{R}$$

~~$\vec{H}(\vec{r}) =$~~



$$E_R = \emptyset$$

$$E_\varphi = \emptyset$$

$$Z_0 = \frac{E_\vartheta}{H_\varphi} = \underline{\underline{120\pi}} \quad \left(\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)$$

$$H_\varphi = \frac{Iml}{4\pi}\frac{j\omega}{vR} e^{-j\omega\frac{R}{v}} \sin\vartheta$$

$$= \frac{Iml}{4\pi}\frac{j\beta}{R} e^{-j\beta R} \sin\vartheta$$

$$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v}$$

$$\boxed{H_\varphi = j\frac{Iml}{2\lambda}\frac{1}{R} e^{-j\beta R} \sin\vartheta}$$

$$\boxed{E_\vartheta = j\frac{60\pi Iml}{\lambda}\frac{1}{R} e^{-j\beta R} \sin\vartheta}$$

$$\vec{S} = \frac{1}{2}\vec{E}_\vartheta \times \vec{H}_\varphi^*$$

$$v = \frac{\omega}{\beta} \Rightarrow \beta = \omega/v$$

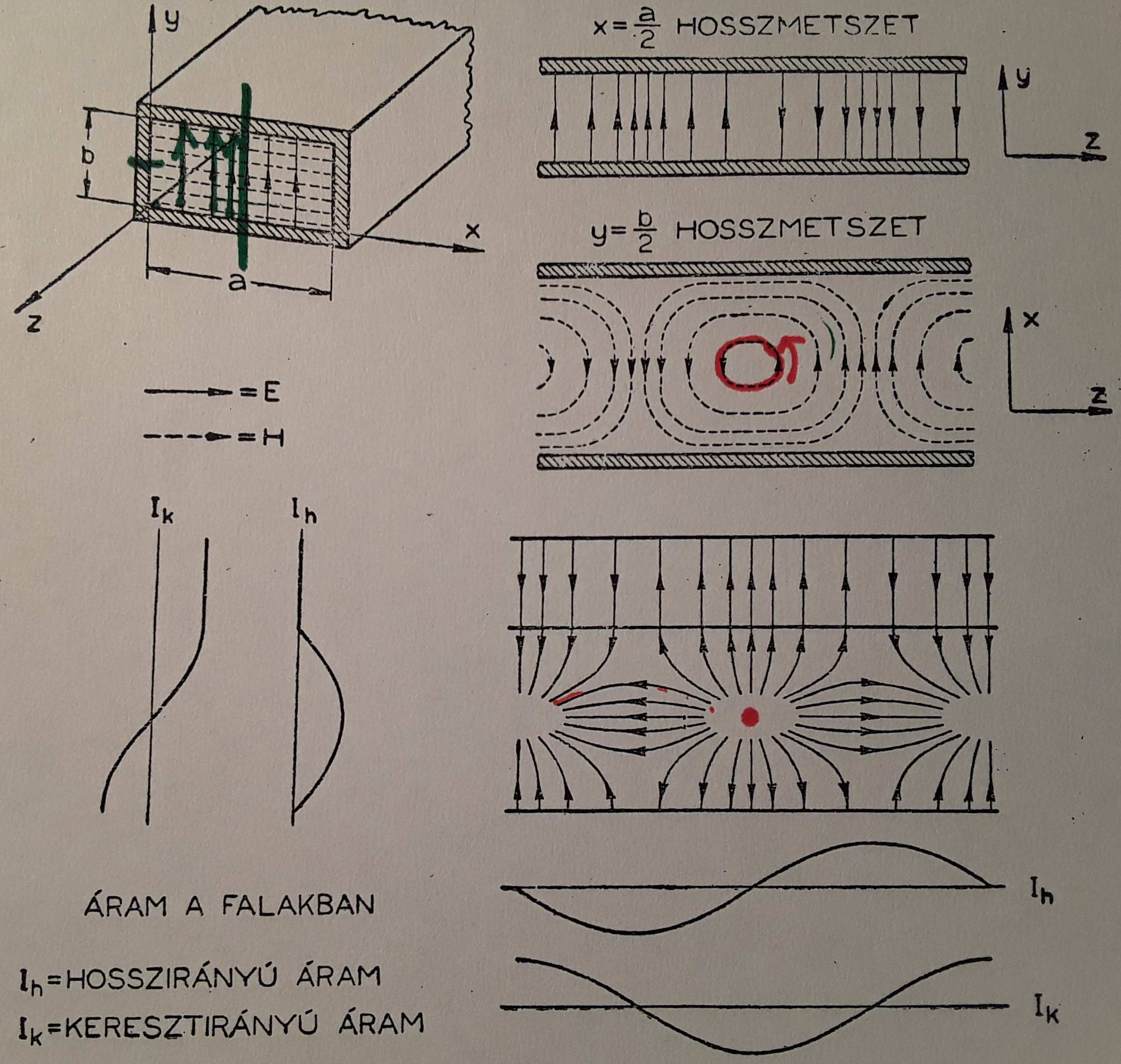
$$\frac{1}{R^2} \qquad \frac{\omega}{v R} = \frac{1}{\beta R} \qquad \cancel{\beta = \frac{\omega}{v}}$$

$$\underbrace{\vec{H}_\varphi^* = \cancel{-j \frac{I_m l}{4}} \; -j \frac{I_m l}{2\lambda} \frac{1}{R} e^{j\beta R} \sin\vartheta \qquad \vec{E}_\vartheta = j \frac{60\pi I_m l}{\lambda} \frac{1}{R} e^{-j\beta R} \sin\vartheta}$$

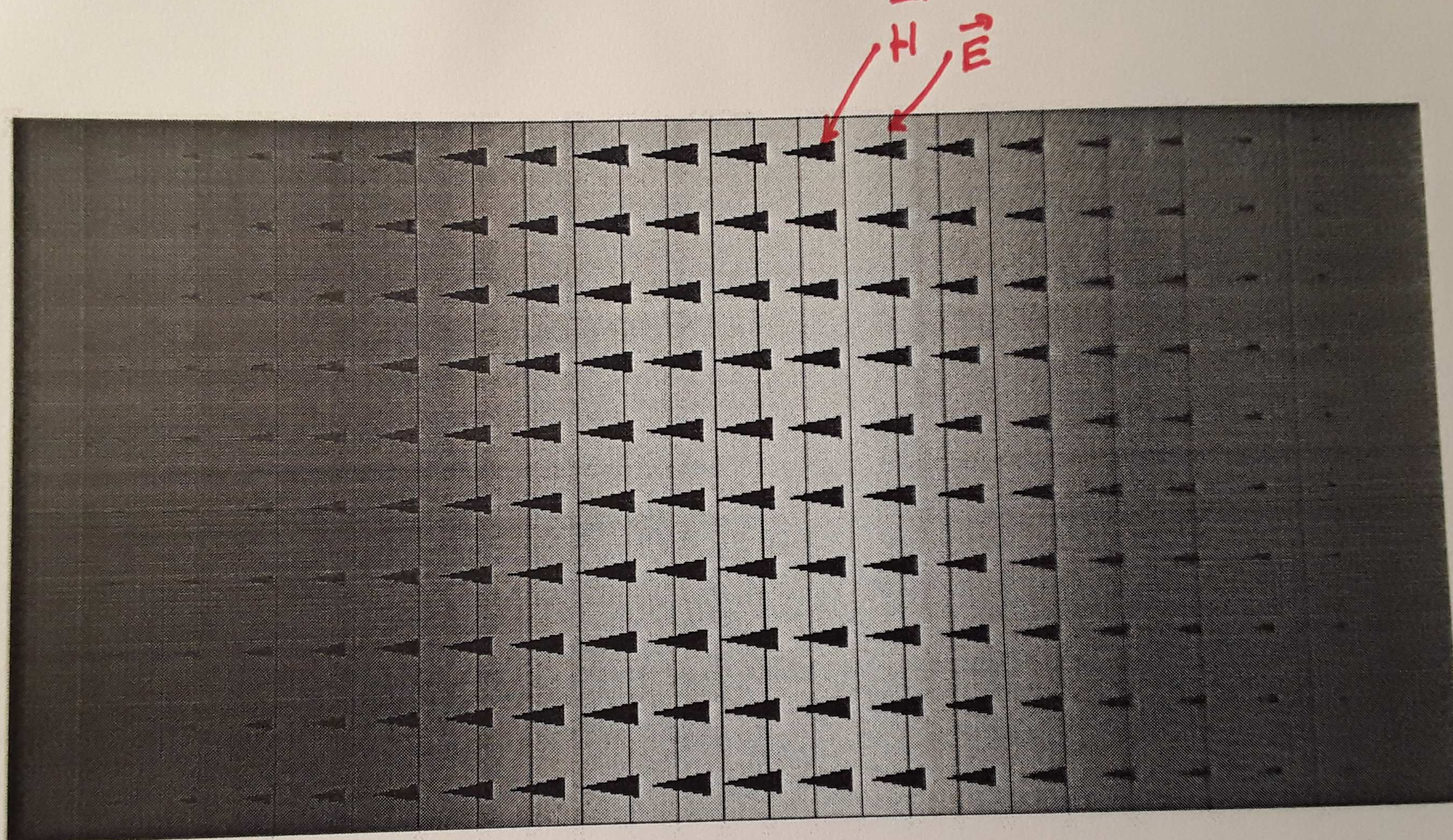
$$\vec{S} = \frac{1}{2} \vec{E}_\vartheta \cdot \vec{H}_\varphi^* = \frac{1}{2} \cancel{j} \frac{60\pi I_m l}{\lambda} \cdot \frac{1}{R} \cancel{e^{-j\beta R}} \sin\vartheta \cdot \left(\cancel{-j} \frac{I_m l}{2\lambda} \frac{1}{R} \cancel{e^{j\beta R}} \sin\vartheta \right)$$

$$= 15\pi I_m^2 l^2 \left(\frac{1}{\lambda}\right)^2 \left(\frac{1}{R}\right)^2 \sin^2\vartheta$$

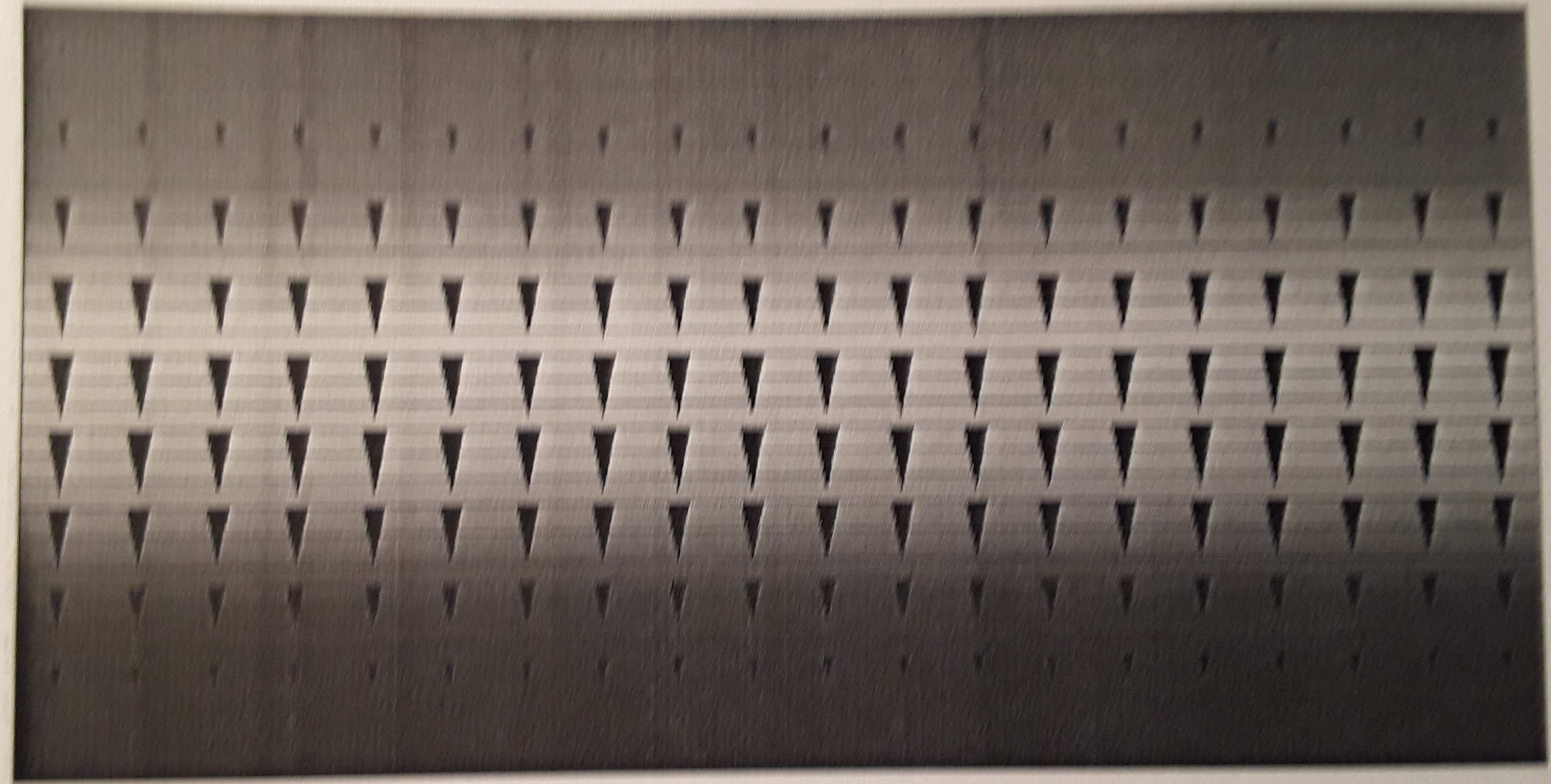
$$\vec{S} = 15\pi \left(\frac{l}{\lambda}\right)^2 I_m^2 \frac{1}{R^2} \sin^2\vartheta$$



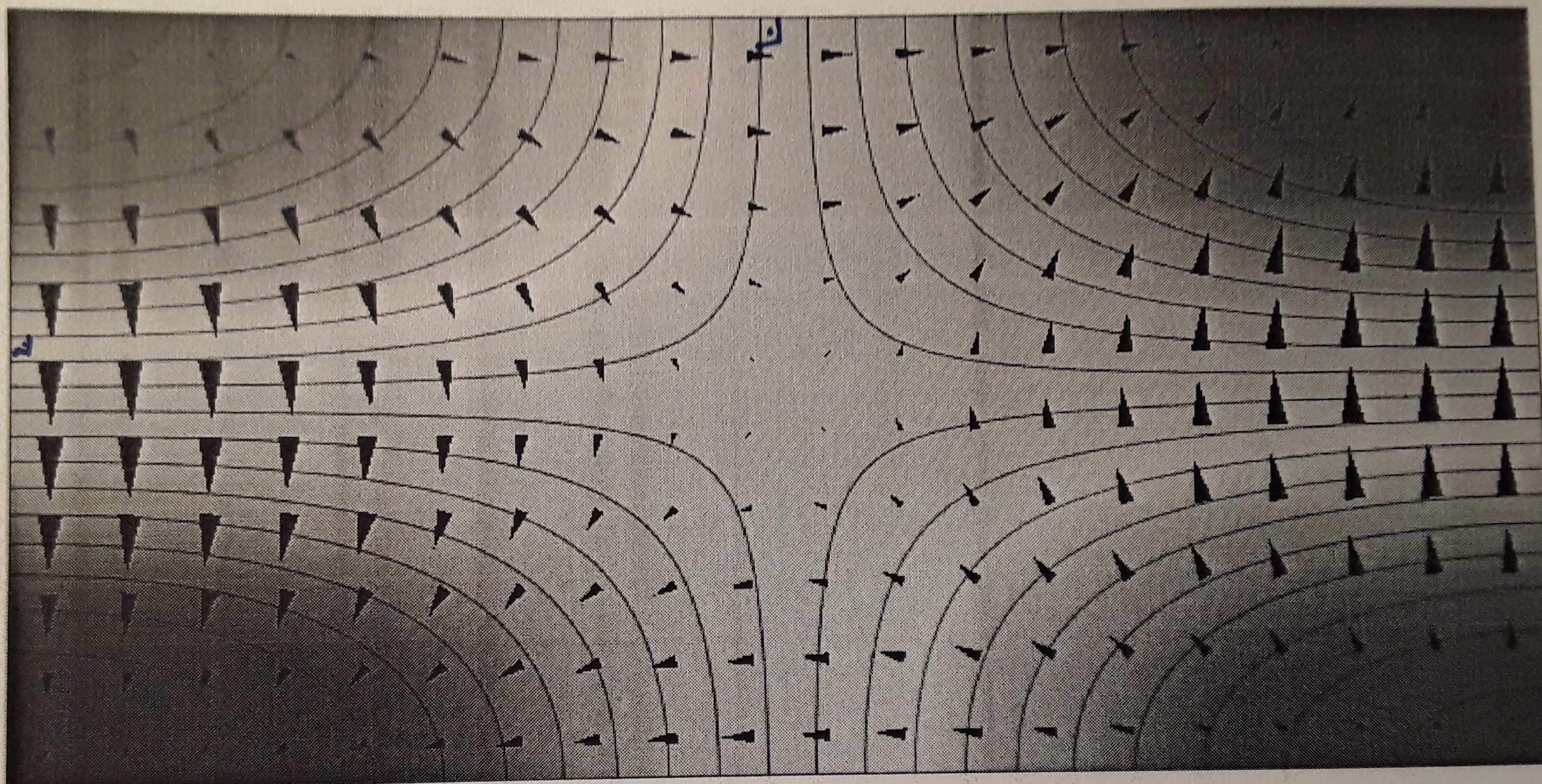
3.32. ábra. Elektromágneses mező és áramok négyszögletes csőtápvonalban TE_{10} módban



TE_{10}

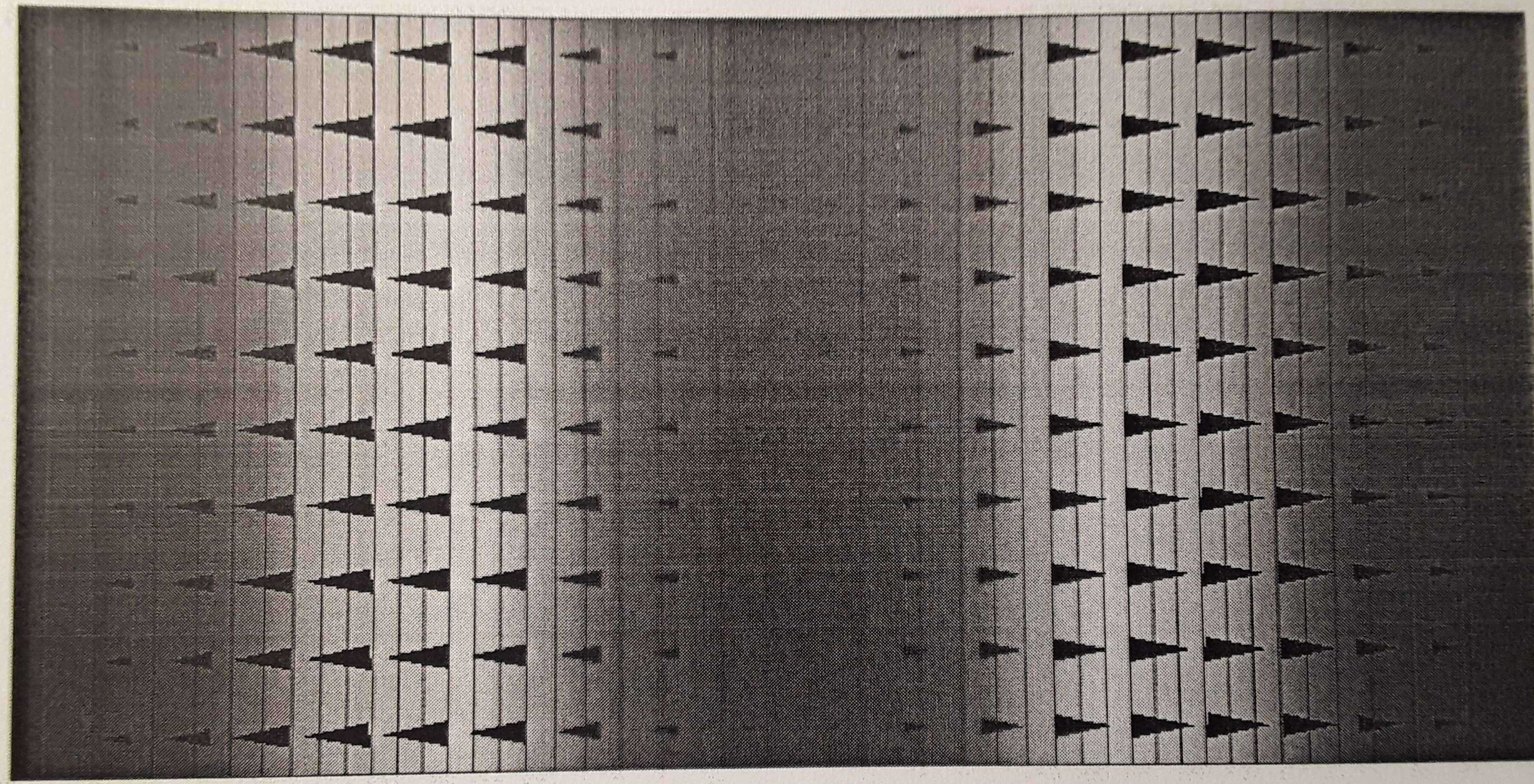


TE01

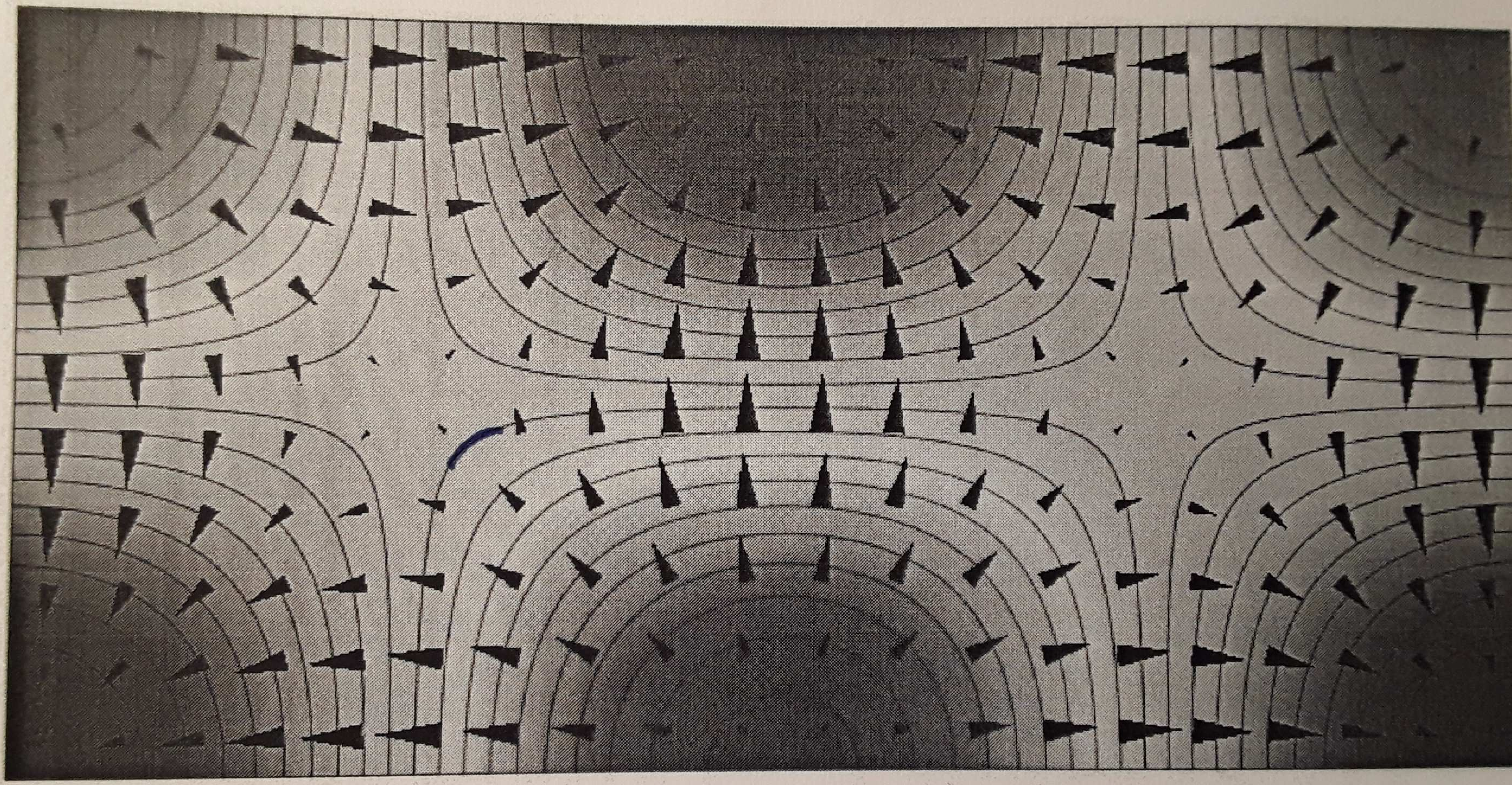


TE11

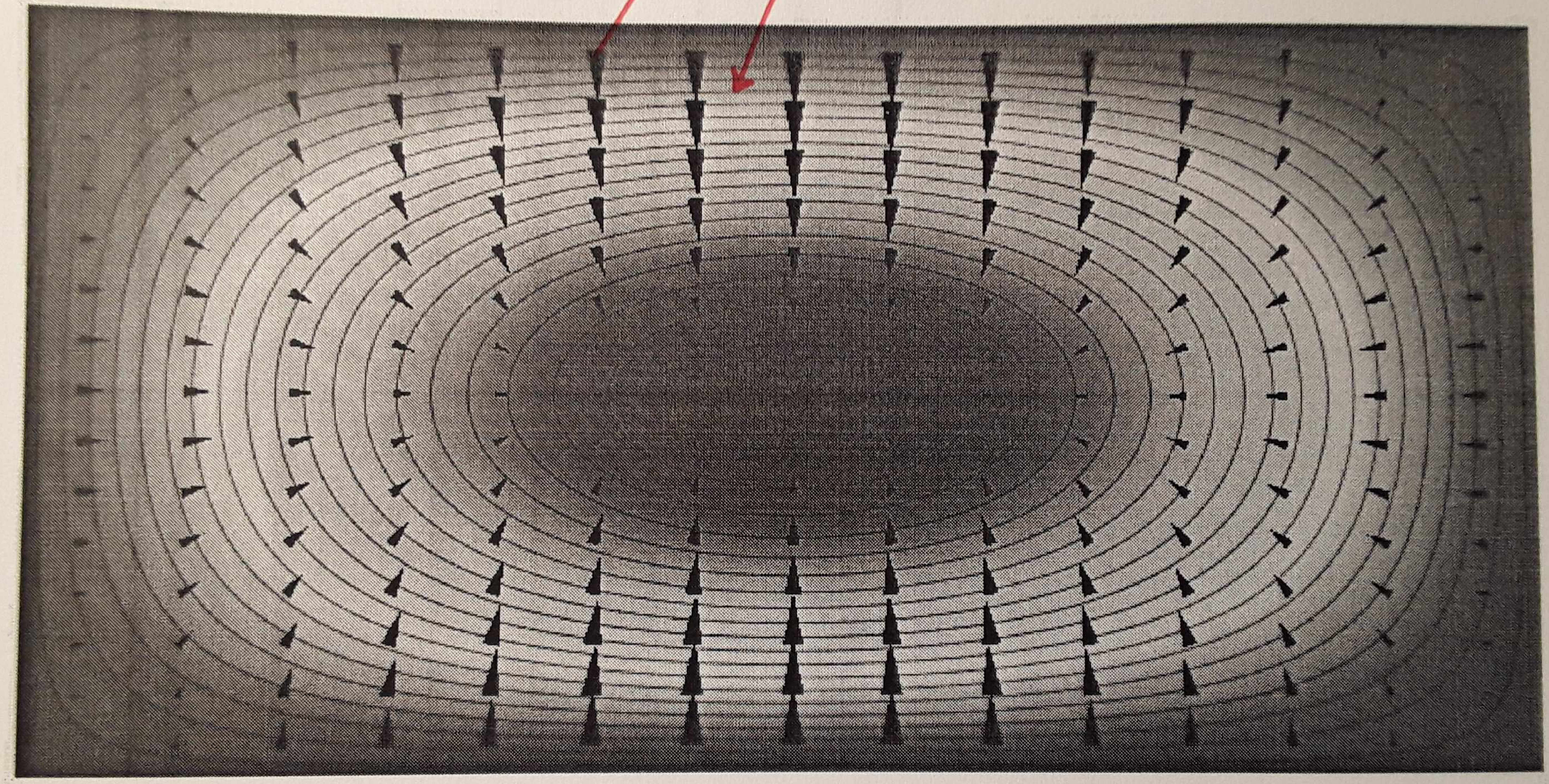
E & H



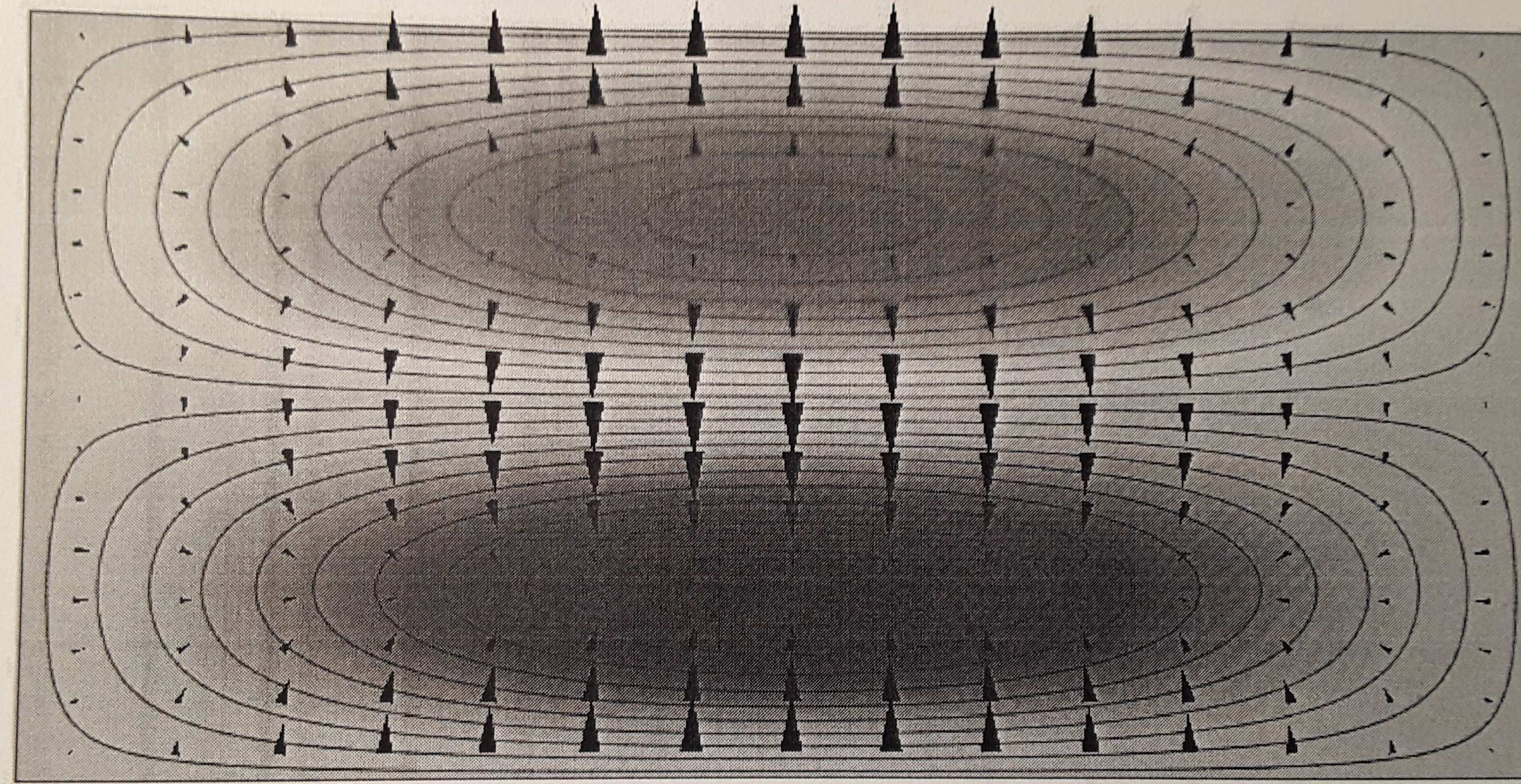
TE20



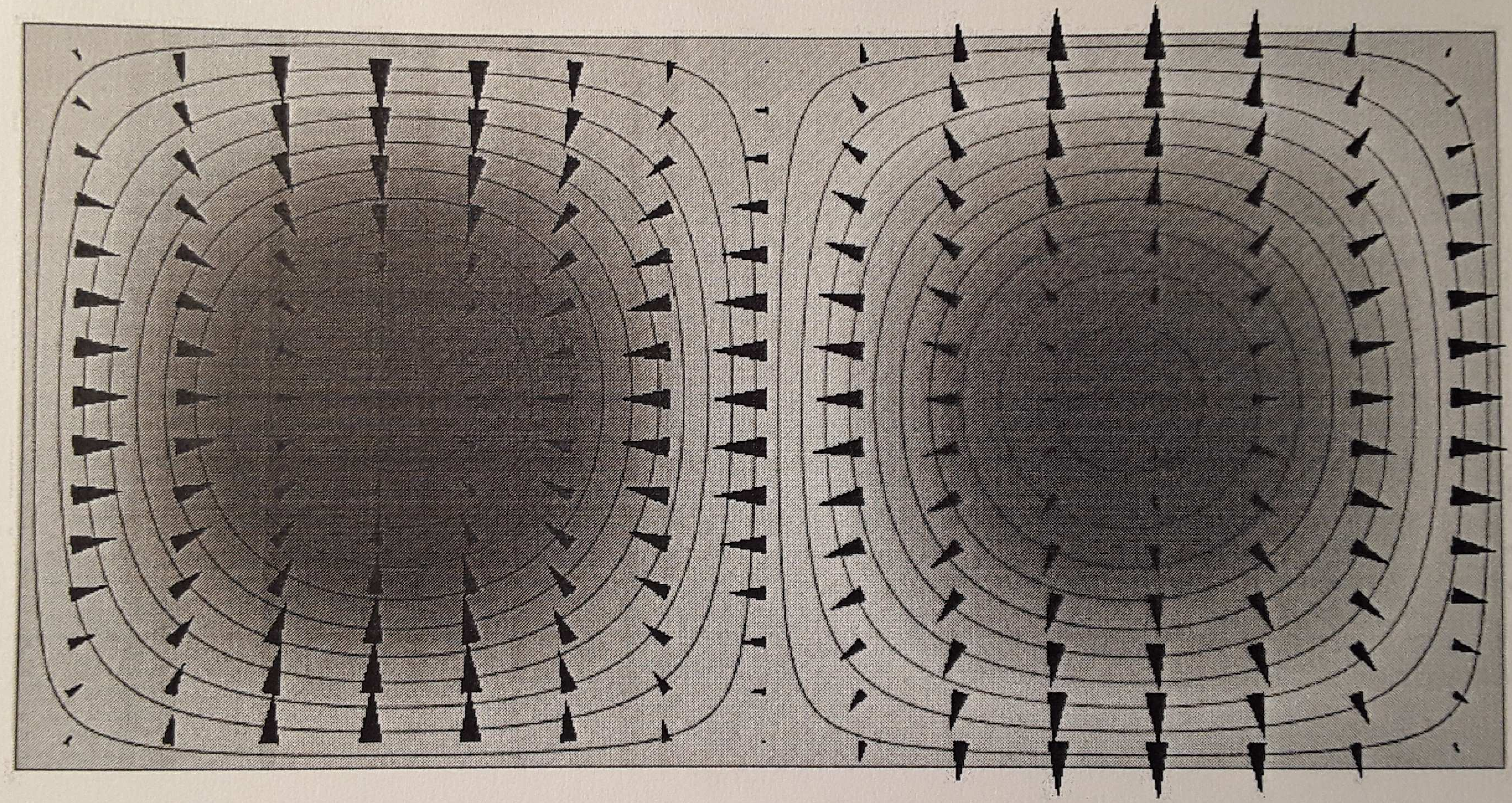
TE21



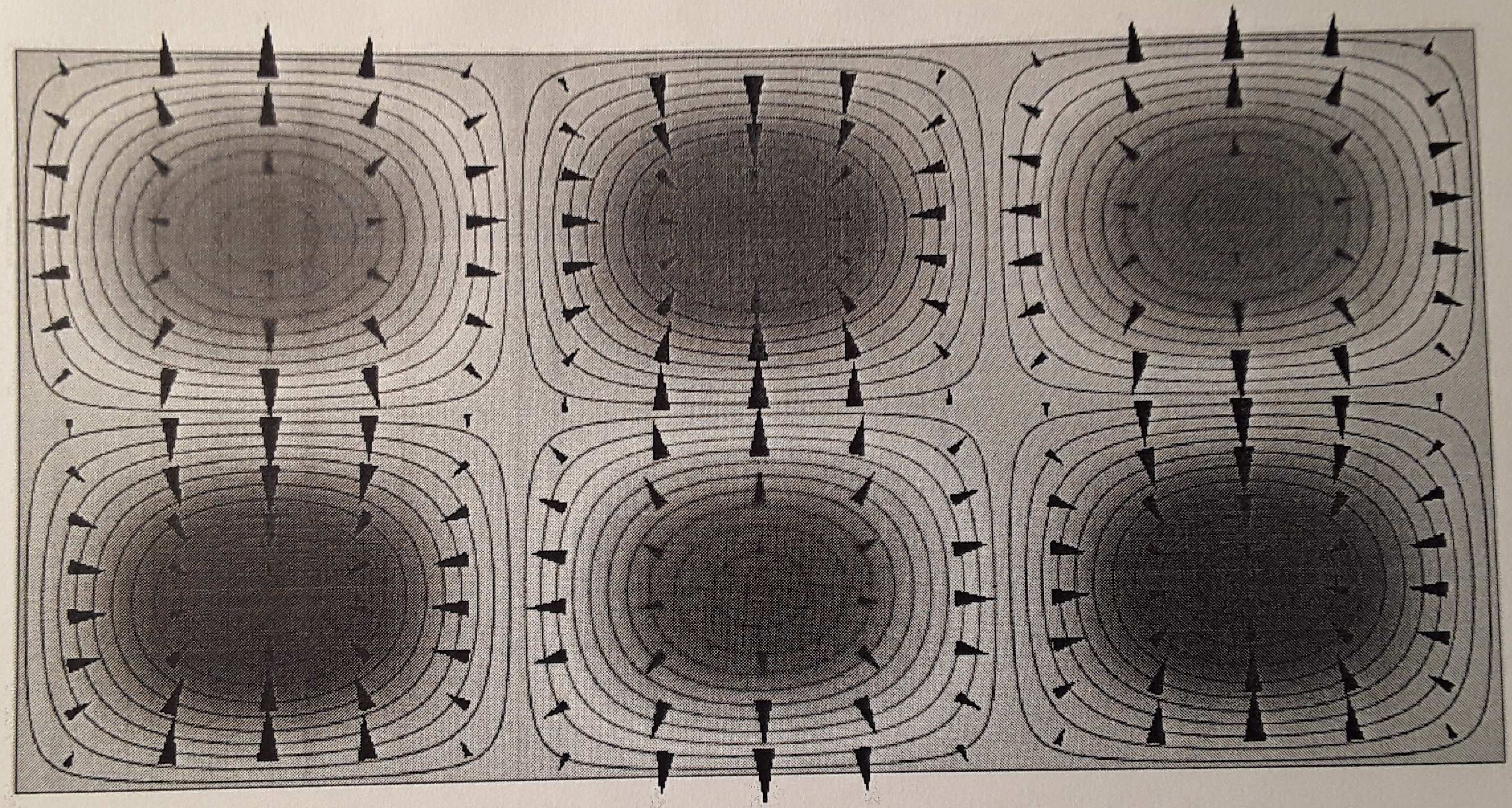
TM11



TM12



TM21



TM32