FEM Simulation of a MFL System

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Abstract: The paper presents the analysis of a nondestructive testing equipment under investigation. There are two main parts of the research as well as this paper. The first part shows the present state of the developed nondestructive tester based on the Magnetic Flux Leakage (MFL) method, the other part reviews the simulation and the results, which have been made with the principle of the Finite Element method (FEM). The aim now is to define the order of magnitude of the magnetic flux density in the positions of the sensor.

Keywords: Magnetic Flux Leakage method, Vector Finite Element Method, 3D static magnetic field, LabVIEW.

I. INTRODUCTION

The aim of nondestructive testing methods is to obtain some information about the specimen under test without any physical impression of the material. Here, only ferromagnetic materials have been used in the measurements. One of this methods is the so-called Magnetic Flux Leakage method, which detects the gaps, cracks and flaws by the help of the magnetic field supplied by a current source. It is well known that the ferromagnetic materials drive the magnetic flux, but the generated flux emerges the gaps, and this effect can be measured by the appropriate sensor, such as using a little coil or a Hall type sensor.

An equipment has been built up in our laboratory, which can be used as a magnetic flux leakage tester. It can be seen in Fig. 1. The schematic view of the measurement arrangement can be seen in Fig. 2. This measurement system is based on a National Instruments Data Acquisition card (NI-DAQ card connected to the computer through an USB interface) and National Instruments LabVIEW software package [1]. The specimen or the sensor can be positioned in the $x-y$ plane by using LabVIEW commands. There are two kind of cracks, which have been studied, the first one is called ID crack (inner defect), that is the crack and the sensor are in the same side of the specimen, the other is the OD crack (outer defect), i.e. the flaw and the sensor are in the opposite side of the material under test [2]. A U-shaped yoke placed above the specimen is used to generate the magnetic field inside the material under test. The current of coil wounded around the yoke can be a direct current or an alternating current supplied by a computer controlled current amplifier. The function of current can be set by LabVIEW functions. The leakage magnetic flux is measured by a detecting coil or a sensor, and its output is connected to the NI-DAQ card, and the measured signal can be post-processed by LabVIEW procedures or functions. For example, measured signals can be plotted, filtered, saved, or they can be the input of any further mathematical procedures. During the measurement process, the positioning, the signal generation and measurement of signal of the sensors have to be realized simultaneously.

In this case only surface cracks have been analyzed. This research work has two parts. The first one is to build up a Magnetic Flux Leakage based nondestructive testing system in our laboratory, the second is to measure manufactured flaws in a ferromagnetic material, and to simulate this system by FEM to find out the order of magnitude of the measured magnetic flux density.

Here the simulation technique is presented.
II. SIMULATION BY FEM

A. Governing equations

The CAD model of the problem can be seen in Fig. 3. First of all, the measurement system has been modeled as a static magnetic field problem, where the following Maxwell’s equations can be used [3], [4], [5], [6]:

\[ \nabla \times \vec{H} = \vec{J}_0, \text{ in } \Omega_0 \cup \Omega_y \cup \Omega_s, \] 
\[ \nabla \cdot \vec{B} = 0, \text{ in } \Omega_0 \cup \Omega_y \cup \Omega_s, \] 
\[ \vec{H} = \begin{cases} \nu \vec{B}, & \text{in } \Omega_0 \cup \Omega_y, \\ \nu_{fp} \vec{B} + \vec{I}, & \text{in } \Omega_s. \end{cases} \] (3)

Here \( \vec{H} \) is the magnetic field intensity, \( \vec{B} \) is the magnetic flux density, \( \vec{J}_0 \) is the source current density, and \( \nu \) is the reluctivity. The domain of interest has been split into three regions, the air region is denoted by \( \Omega_0 \), where \( \nu = \nu_0 \) (\( \nu_0 \) is the reluctivity of vacuum), the yoke is supposed to be linear, \( \nu = \nu_0 \nu_r \) (\( \nu_r = 1/4000 \)), and it is denoted by \( \Omega_y \), finally \( \Omega_s \) is filled with ferromagnetic material. The second relation of (??) represents the nonlinearity by polarization formulation [7], [8], [9]. Here \( \nu_{fp} \) is the optimal value of reluctivity,

\[ \nu_{fp} = \frac{\nu_{max} + \nu_{min}}{2}, \] (4)

where \( \nu_{max} \) and \( \nu_{min} \) are the maximum and the minimum slope of the nonlinear characteristics, moreover \( \vec{I} \) is a nonlinear residual term determined iteratively by the fixed point technique. The nonlinear characteristics of the specimen can be seen in Fig. 4, which has been approximated by an inverse tangent function.

B. Boundary conditions

It can be seen in Fig. 3 that the flaws are far away from each other, that is they have no got any effect on each other. That is why only one gap can be simulated, and some symmetry planes can be found.

One of them is called \( \Gamma_H \) boundary, the other is the \( \Gamma_B \) boundary, where the following boundary conditions must by satisfied:

\[ \vec{H} \times \vec{n} = \vec{0}, \text{ on } \Gamma_H, \] 
\[ \vec{B} \cdot \vec{n} = 0, \text{ on } \Gamma_B. \] (6)

On the artificial far boundary, which closes the problem region (6) must be prescribed.

III. THE TWO DIMENSIONAL MODEL

The problem has been solved by the help of FEM. First of all, a simplified two dimensional model of an infinite slot has been simulated by using the magnetic vector potential, which is defined by

\[ \vec{B} = \nabla \times \vec{A}, \] (7)

which satisfies (2) exactly. Substituting the definition (7) into the first Maxwell’s equation and using the linearized constitutive relation form lead to the partial differential equations [3], [4]:

\[ \nabla \times (\nu \nabla \times \vec{A}) = \vec{J}_0, \text{ in } \Omega_0 \cup \Omega_y, \] 
\[ \nabla \times (\nu_{fp} \nabla \times \vec{A}) = \vec{J}_0 - \nabla \times \vec{I}, \text{ in } \Omega_s. \] (9)

The boundary conditions belonging to the partial differential equation of a two dimensional static magnetic field problem can be formulated as

\[ (\nu_{fp} \nabla \times \vec{A} + \vec{I}) \times \vec{n} = 0, \text{ on } \Gamma_H, \] 
\[ \vec{n} \times \vec{A} = 0, \text{ on } \Gamma_B. \] (11)

Fig. 5 and Fig. 6 show the \( y \) and the \( z \) components of the magnetic flux density 1mm above the specimen (the peak value of source current is 1A). The width of flaw is 1mm. The location of the gap can be seen easily. This can be compared with the results of the 3D computations shown in Fig. 23, Fig. 24. It is concluded that the magnetic flux of the 2D computations results in a bit larger value than
the components of the 3D simulations. Fig. 7 and Fig. 8 show the same components when the peak value of source current is 2A, and these can be compared with Fig. 23 and Fig. 24.

**IV. THE THREE DIMENSIONAL MODEL**

**A. The vector potential by edge FEM**

In 3D case, the magnetic vector potential is approximated by vector finite elements, and the edge element based FEM has been used [11]. In this case the unknowns are associated to the edges of the FEM mesh. The problem is supposed to be a static magnetic field one again, but the nonlinearity of the specimen has been taken into account by using a the same analytical model as it was presented in section II. The partial differential equation, which solution is not sensitive to Coulomb gauge can be written as [3], [4], [5]:

\[
\nabla \times (\nu \nabla \times \vec{A}) = \nabla \times \vec{T}_0, \quad \text{in } \Omega_0 \cup \Omega_y,
\]

and

\[
\nabla \times (\nu_p \nabla \times \vec{A}) = \nabla \times \vec{T}'_0 - \nabla \times \vec{I}, \quad \text{in } \Omega_s.
\]

Here \(\vec{T}_0\) is the so-called impressed current vector potential, which represents the effect of the source current density through the following equation:

\[
\nabla \cdot \vec{J}_0 = 0,
\]

that is

\[
\nabla \times \vec{T}'_0 = \vec{J}_0.
\]

The impressed current vector potential has to be represented by edge FEM approximation (see next section).

The boundary conditions of the static magnetic field problem are the same as the boundary conditions of the static magnetic field problem of the two dimensional problem (10), (11).

The partial differential equation (13) and the Neumann type boundary condition (10) can be summarized in the following weighted residual formulation [3], [4]:

\[
\int_{\Omega} \vec{W} \cdot (\nabla \times (\nu_p \nabla \times \vec{A})) d\Omega \\
+ \int_{\Omega_H} \vec{W} \cdot ((\nu_p \nabla \times \vec{A} + \vec{I}) \times \vec{n}) d\Gamma \\
= \int_{\Omega} \vec{W} \cdot (\nabla \times \vec{T}_0) d\Omega \\
- \int_{\Omega} \vec{W} \cdot (\nabla \times \vec{I}) d\Omega,
\]

where

\[
\vec{n} \times \vec{W} = \vec{0}, \quad \text{on } \Gamma_B,
\]

and \(\vec{W}\) is a weighting function and the approximation function of the unknown vector potential, as well.

The second order edge finite elements have been used in this simulation [10], [11], [12].

For short, the value of \(\nu_p\) is equal to \(\nu_0\), or \(\nu_0\nu_r\) in the air region, and in the yoke, respectively, moreover \(\vec{I}\) is zero there.
After using some mathematical identity and using some formulations, the following weak equation can be obtained:

\[
\int_{\Omega} (\nabla \times \vec{W}) \cdot (\nu_{fp} \nabla \times \vec{A}) \, d\Omega = \int_{\Omega} (\nabla \times \vec{W}) \cdot \vec{T}_0 \, d\Omega - \int_{\Omega} (\nabla \times \vec{W}) \cdot \vec{I} \, d\Omega,
\]  

(18)

which solution results in the approximation of the magnetic vector potential, from which the magnetic flux density can be obtained. The magnetic field intensity can be calculated from the magnetic flux density by using the nonlinear model.

B. The calculation of the impressed current vector potential

The following functional can be built up to find out the source term \( \vec{T}_0 \) [3]:

\[
\mathcal{F}\{\vec{T}_0\} = \int_{\Omega} |\nabla \times \vec{T}_0 - \vec{J}_0|^2 \, d\Omega,
\]  

(19)

which has to be minimized. This is equivalent to the following partial differential equation defined in free space:

\[
\nabla \times \nabla \times \vec{J}_0 = \nabla \times \vec{J}_0, \quad \text{in} \ \Omega,
\]  

(20)

and the boundary conditions are

\[
\vec{T}_0 \times \vec{n} = 0, \quad \text{on} \ \Gamma_H,
\]

(21)

\[
\vec{T}_0 \cdot \vec{n} = 0, \quad \text{on} \ \Gamma_B.
\]

(22)

This kind of formulation can be worked out by the help of Comsol Multiphysics easily, and satisfies (15) exactly.

V. THE 3D SIMULATION RESULTS

The FEM simulations have been performed by the Comsol Multiphysics software [6]. The Comsol Multiphysics combined with the functions of Matlab can be used efficiently in the research work, since the geometry, the physics, the finite element mesh (see Fig. 9) and the post-processing can be realized by the functions of Comsol, and the fixed point technique can be developed in the frame of Matlab.

Figure 9: The 3D mesh of the quarter of the problem

Figure 10: The contour plot of \( y \) component

Five different flaws manufactured artificially have been studied. Two of them are rectangular shaped, two of them have circular shape, and one of them is infinite crossing the specimen. The diameter of the circular flaws are 2mm and 3 mm. The longitudinal flaw is 1mm wide and 5mm long, and the width of the cross flaw is 5mm and it is 1mm long.

Figure 11: The \( y \) component without any flaw

Figure 12: The \( z \) component without any flaw

Fig. 10 shows the contour plot of the \( y \) component of the simulated magnetic flux density according to the longitudinal gap. In Fig. 11 and Fig. 12 the simulated results without any flaw can be seen. It can be seen that some variations of the components of the magnetic flux is resulted according to the legs of the yoke. Fig. 13 and Fig. 14 present the \( y \) and the \( z \) components of the simulated magnetic flux density according to the longitudinal gap. Fig. 15 and Fig. 16 present the \( y \) and the \( z \) components of the simulated magnetic flux density according to the cross gap. Fig. 17 and
VI. CONCLUSION

The first stage of the simulations has been finished. The results show unambiguously that the sensor produces the best performance, which can be evaluated above the specimen. With the knowledge of the magnetic flux density close to the surface of the specimen, the type of the sensor can be selected and the amplifier belonging to this can be designed. The next step of this research work is to prepare the measurements, then comparing the results with the simulations.

VII. ACKNOWLEDGEMENT

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REFERENCES

Figure 17. The $y$ component of the gap with diameter of 2mm circle

Figure 18. The $z$ component of the gap with diameter of 2mm circle

Figure 19. The $y$ component of the gap with diameter of 3mm circle

Figure 20. The $z$ component of the gap with diameter of 3mm circle

Figure 21. The $y$ component of the infinite gap with 1A current

Figure 22. The $z$ component of the infinite gap with 1A current

Figure 23. The $y$ component of the infinite gap with 2A current

Figure 24. The $z$ component of the infinite gap with 2A current